Phan Duc Hung – 20214903 – Group 3

Experimental Report 3
MOMENT OF INERTIA OF THE SYMMETRIC RIGID BODIES

I/Experiment Motivations

- Calculating the moment of the inertia in the symmetric rigid bodies

- Gaining knowledge about the moment of the inertia in the symmetric rigid bodies

II/Experimental Result

1) Measurement of the Rod:

Trial	T (s)
1	2.620
2	2.624
3	2.621
4	2.622
5	2.620
	$\bar{T} = 2.621\,mm$ $\Delta\mathrm{T} = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_i - \bar{T})^2}{5}}}{\sqrt{5}} = 0.0007s$

2) Measurement of the Solid Disk:

Trial	T (s)
1	2.075
2	2.077
3	2.077
4	2.075
5	2.074
	$\bar{T} = 2.076\,mm$ $\Delta\mathrm{T} = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_i - \bar{T})^2}{5}}}{\sqrt{5}} = 0.0005s$

3) Measurement of the Hollow Cylinder:
a) Supported Disk:

Trial	T (s)
1	0.324
2	0.323
3	0.324

4	0.324
5	0.323
	$\bar{T} = 0.324\,mm$ $\Delta T = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_i - \bar{T})^2}{5}}}{\sqrt{5}} = 0.0002s$

b) Supported Disk + Hollow Cylinder:

Trial	T (s)
1	1.128
2	1.126
3	1.127
4	1.127
5	1.126
	$\bar{T} = 1.127\,mm$ $\Delta T = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_i - \bar{T})^2}{5}}}{\sqrt{5}} = 0.0003s$

4) Measurement of the Solid Sphere:

	T (s)
1	2.129
2	2.128
3	2.129
4	2.129
5	2.129
	$\bar{T} = 2.129\,mm$ $\Delta T = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_i - \bar{T})^2}{5}}}{\sqrt{5}} = 0.0002\text{s}$

III/Data Processing

1) The Rod:

a) Moment of inertia obtained by experiment:

$$\overline{I_{cm}} = D_Z\left(\frac{T}{2\pi}\right)^2 = 0.044\left(\frac{2.621}{2\pi}\right)^2 = 7.656 \times 10^{-3}(kg.m^2/s)$$

$$\Rightarrow \Delta I_{cm} = \overline{I_{cm}}\sqrt{\left(2\frac{\Delta T}{T}\right)^2} = (7.656 \times 10^{-3})\sqrt{4\left(\frac{0.0007}{2.621}\right)^2}$$

$$= 0.004 \times 10^{-3}(kg.m^2/s)$$

Hence

$$\boxed{I_{cm} = (7.656 \pm 0.004) \times 10^{-3}(kg.m^2/s)}$$

b) Moment of inertia calculated by the theoretical formula

$$I_{cmTH} = \frac{1}{12}ml^2 = \frac{1}{12} \times 0.240 \times 0.620^2 = 7.688 \times 10^{-3}(kg.m^2/s)$$

The difference between theoretical and experiment result:

$$\%\sigma = \frac{|I_{cmTH} - \overline{I_{cm}}|}{I_{cmTH}} = \frac{|(7.688 - 7.656) \times 10^{-3}|}{7.688 \times 10^{-3}} = 0.416\%$$

2) Solid Disk:

a) Moment of inertia obtained by experiment

$$\overline{I_{cm}} = D_Z\left(\frac{\bar{T}}{2\pi}\right)^2 = 0.044\left(\frac{2.076}{2\pi}\right)^2 = 4.803 \times 10^{-3}(kg.m^2/s)$$

$$\Rightarrow \Delta I_{cm} = \overline{I_{cm}}\sqrt{\left(2\frac{\Delta T}{T}\right)^2} = (4.803 \times 10^{-3})\sqrt{4\left(\frac{0.0005}{2.076}\right)^2}$$

$$= 0.002 \times 10^{-3}(kg.m^2/s)$$

Hence

$$\boxed{I_{cm} = (4.803 \pm 0.002) \times 10^{-3}(kg.m^2/s)}$$

b)

Moment of inertia calculated by the theoretical formula

$$I_{cmTH} = \frac{1}{2}mR^2 = \frac{1}{2} \times 0.795 \times \left(\frac{0.220}{2}\right)^2 = 4.810 \times 10^{-3}(kg.m^2/s)$$

The difference between theoretical and experiment result:

$$\%\sigma = \frac{|I_{cmTH} - \overline{I_{cm}}|}{I_{cmTH}} = \frac{|(4.810 - 4.803) \times 10^{-3}|}{4.810 \times 10^{-3}} = 1.46\%$$

3) Hollow cylinder:

a) Moment of inertia obtained by experiment

+) Moment of inertia of the supported disk

$$\overline{I_{sp}} = D_Z\left(\frac{T}{2\pi}\right)^2 = 0.044\left(\frac{0.324}{2\pi}\right)^2 = 0.117 \times 10^{-3}(kg.m^2/s)$$

$$\Rightarrow \Delta I_{sp} = \overline{I_{sp}}\sqrt{\left(2\frac{\Delta T}{T}\right)^2} = (0.117 \times 10^{-3})\sqrt{4\left(\frac{0.0002}{0.324}\right)^2}$$

$$= 0.0001 \times 10^{-3}(kg.m^2/s)$$

Then $I_{sp} = (0.117 \pm 0.0001) \times 10^{-3}(kg.m^2/s)$

+) Moment of inertia of the coupled object (supported disk + hollow cylinder)

$$\overline{I_{co}} = D_Z\left(\frac{T}{2\pi}\right)^2 = 0.044\left(\frac{1.127}{2\pi}\right)^2 = 1.416 \times 10^{-3}(kg.m^2/s)$$

$$\Rightarrow \Delta I_{co} = \overline{I_{co}}\sqrt{\left(2\frac{\Delta T}{T}\right)^2} = (1.416 \times 10^{-3})\sqrt{4\left(\frac{0.0003}{1.127}\right)^2}$$

$$= 0.001 \times 10^{-3}(kg.m^2/s)$$

Then $I_{co} = (1.416 \pm 0.001) \times 10^{-3}(kg.m^2/s)$

⇒ Moment of inertia of the hollow cylinder

$$\overline{I_{cm}} = \overline{I_{co}} - \overline{I_{sp}} = (1.416 - 0.117) \times 10^{-3} = 1.299 \times 10^{-3}(kg.m^2/s)$$

$$\Rightarrow \Delta I_{cm} = \sqrt{(\Delta I_{co})^2 + \left(\Delta I_{sp}\right)^2} = \sqrt{(0.0001)^2 + (0.001)^2} = 0.001 \times 10^{-3}(kg.m^2/s)$$

Hence

$$\boxed{I_{cm} = (1.299 \pm 0.001) \times 10^{-3}(kg.m^2/s)}$$

b) Moment of inertia calculated by the theoretical formula

$$I_{cmTH} = mR^2 = 0.780 \times \left(\frac{0.089}{2}\right)^2 = 1.545 \times 10^{-3}(kg.m^2/s)$$

The difference between theoretical and experiment result:

$$\%\sigma = \frac{|I_{cmTH} - \overline{I_{cm}}|}{I_{cmTH}} = \frac{|(1.229 - 1.545) \times 10^{-3}|}{1.545 \times 10^{-3}} = 20.5\%$$

4) Solid sphere:

a) Moment of inertia obtained by experiment

$$\overline{I_{cm}} = D_Z \left(\frac{T}{2\pi}\right)^2 = 0.044 \left(\frac{2.129}{2\pi}\right)^2 = 5.052 \times 10^{-3}(kg.m^2/s)$$

$$\Rightarrow \Delta I_{cm} = \overline{I_{cm}} \sqrt{\left(2\frac{\Delta T}{T}\right)^2} = (5.052 \times 10^{-3}) \sqrt{4\left(\frac{0.0002}{2.129}\right)^2}$$

$$= 0.001 \times 10^{-3}(kg.m^2/s)$$

Then $I_{cm} = (5.052 \pm 0.001) \times 10^{-3}(kg.m^2/s)$

Hence

$$\boxed{I_{cm} = (5.052 \pm 0.001) \times 10^{-3}(kg.m^2/s)}$$

b) Moment of inertia calculated by the theoretical formula

$$I_{cmTH} = \frac{2}{5}mR^2 = \frac{2}{5} \times 2.29 \times \left(\frac{0.146}{2}\right)^2 = 4.881 \times 10^{-3}(kg.m^2/s)$$

The difference between theoretical and experiment result:

$$\%\sigma = \frac{|I_{cmTH} - \overline{I_{cm}}|}{I_{cmTH}} = \frac{|(4.881 - 5.052) \times 10^{-3}|}{4.881 \times 10^{-3}} = 3.5\%$$

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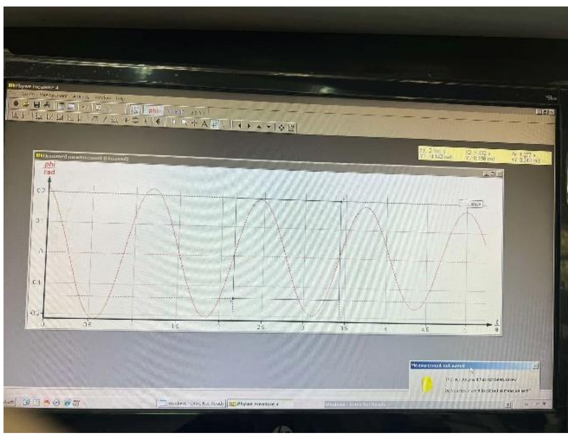
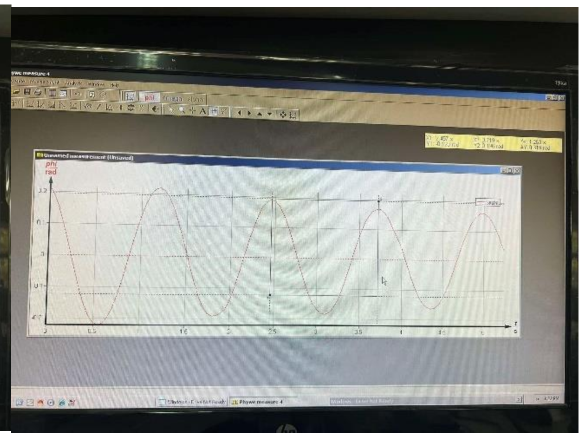
EXPERIMENT REPORT 4:
DETERMINATION OF GRAVITATIONAL ACCELERATRION USING SIMPLE PENDULUM OSCILLATION WITH PC INTERFACE

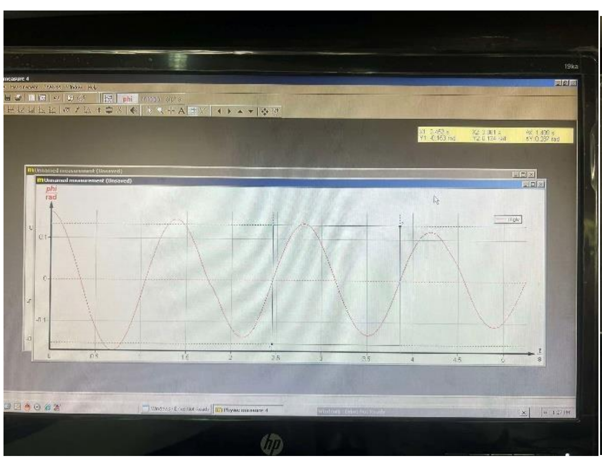
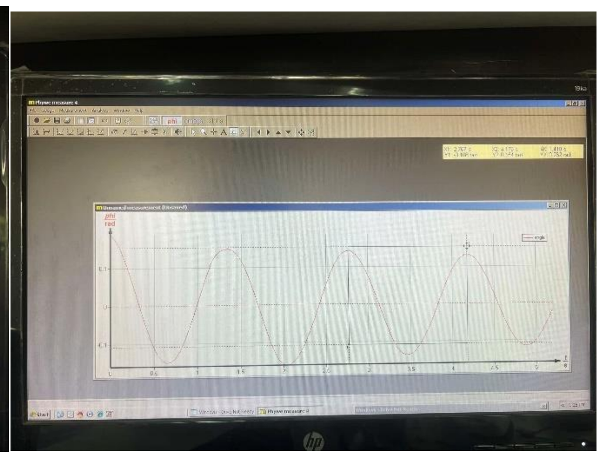
I) Experimental results:

1) Pendulum with vertical oscillation plan

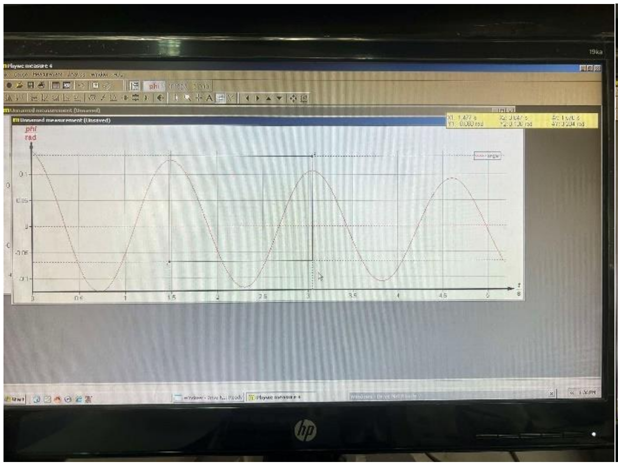
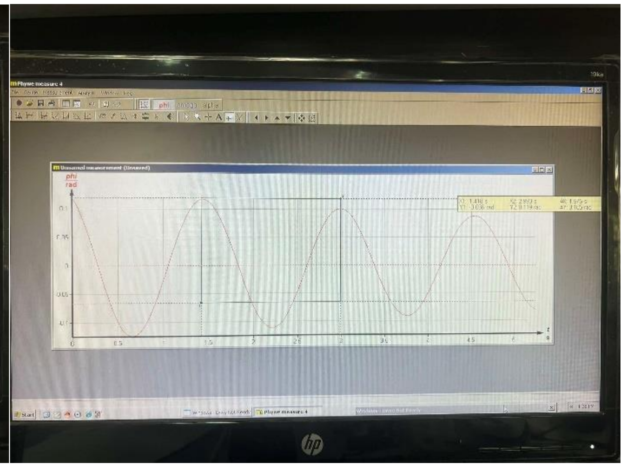
L = 350:

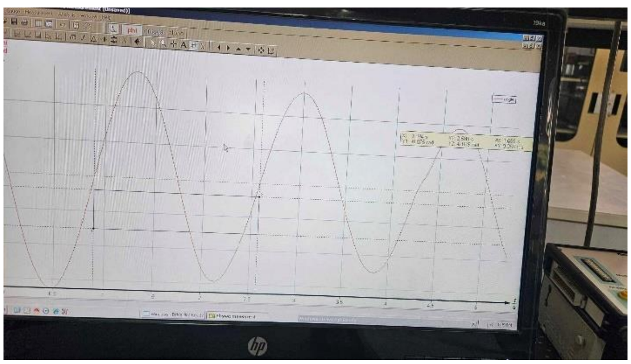
L = 450:

L = 550:

L = 650:

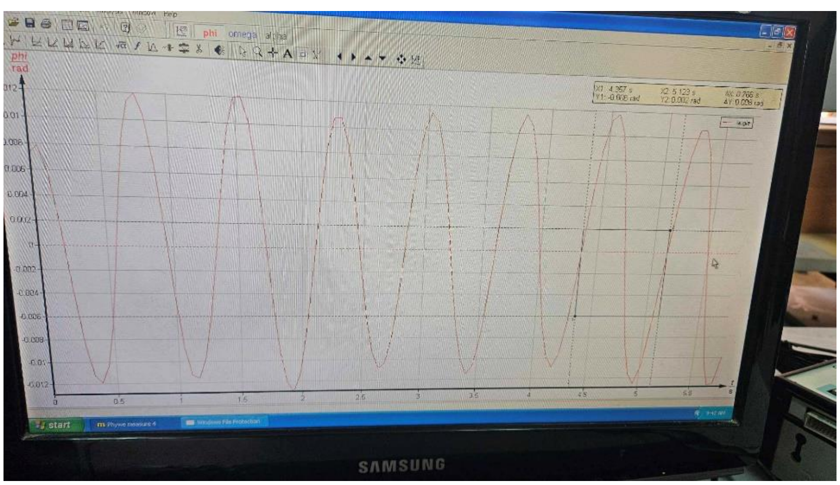


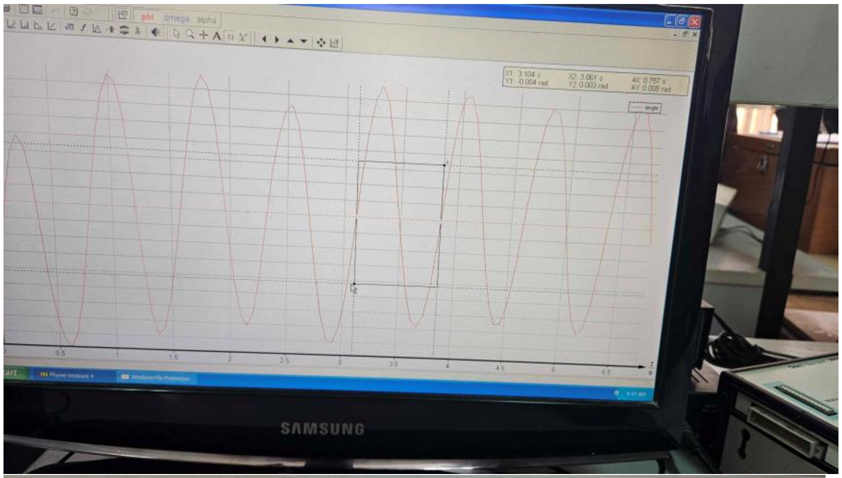
Trial	L = 0.350 (m) T_1 (s)	L = 0.450 (m) T_2 (s)	L = 0.550 (m) T_3 (s)
1	1.287	1.409	1.575
2	1.277	1.410	1.550
3	1.263	1.419	1.570
4	1.267	1.418	1.575
5	1.267	1.408	1.560
	$\overline{T_1} = \frac{\sum_{i=1}^{5} T_{1i}}{5}$ =1.272s	$\overline{T_2} = \frac{\sum_{i=1}^{5} T_{2i}}{5}$ =1.413s	$\overline{T_3} = \frac{\sum_{i=1}^{5} T_{3i}}{5} = 1.566$s
	$\Delta T_1 = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_{1i}-\overline{T_1})^2}{5}}}{\sqrt{5}}$	$\Delta T_2 = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_{2i}-\overline{T_2})^2}{5}}}{\sqrt{5}}$	$\Delta T_3 = \frac{\sqrt{\frac{\sum_{i=1}^{5}(T_{3i}-\overline{T_3})^2}{5}}}{\sqrt{5}}$

	=0.004(s)	=0.002(s)	=0.004(s)

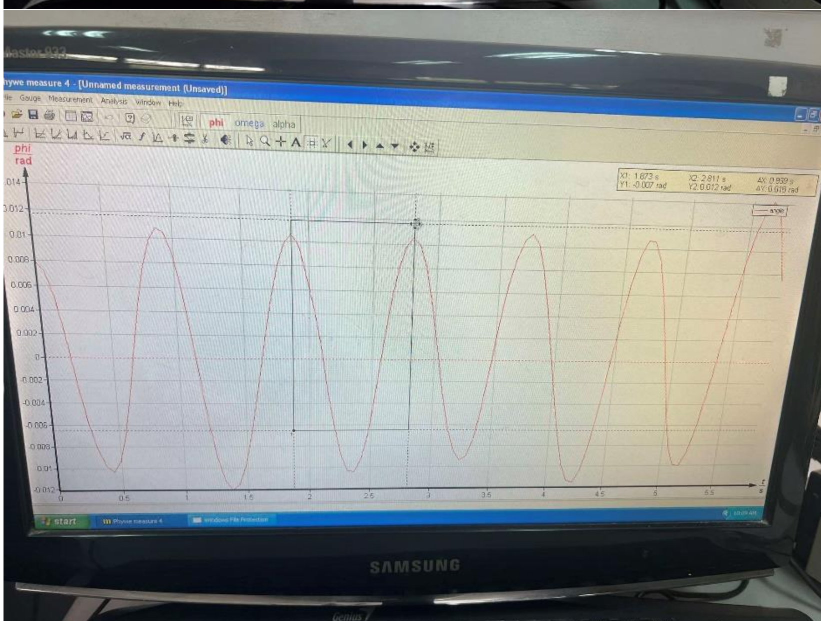
Trial	L = 0.650 (m) T_4 (s)
1	1.677
2	1.689
3	1.689
4	1.673
5	1.685
	$\overline{T_4} = \frac{\sum_{i=1}^{5} T_{4i}}{5}$ =1.683s
	$\Delta T_4 = \frac{\sqrt{\frac{\sum_{i=1}^{5}\left(T_{4i} - \overline{T_4}\right)^2}{5}}}{\sqrt{5}}$ =0.003(s)

2) Pendulum with inclined oscillation plan

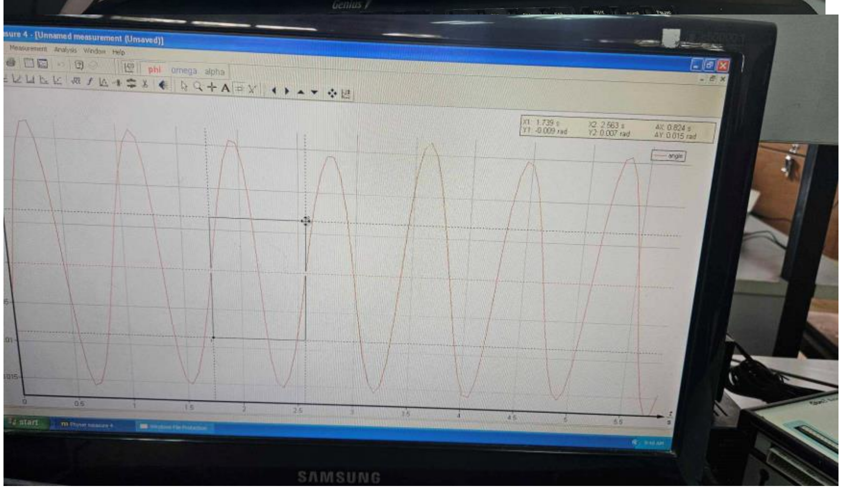




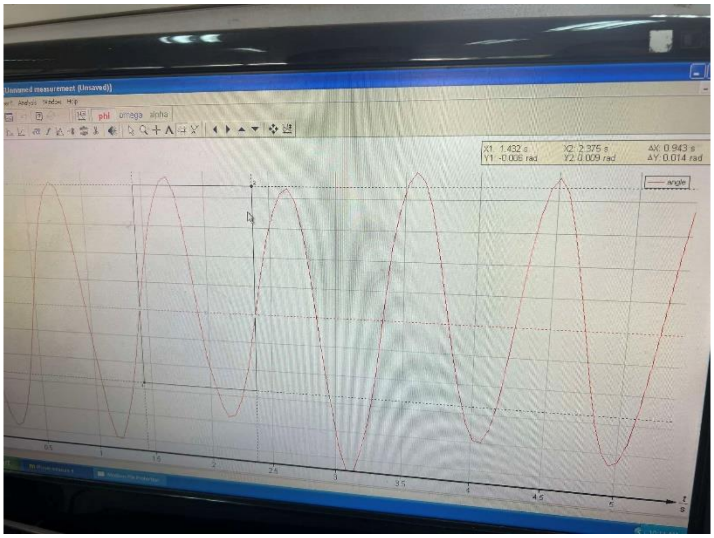
phi omega alpha
X1: 3.104 s
Y1: -0.004 rad
X2: 3.861 s
Y2: 0.003 rad
ΔX: 0.757 s
ΔY: 0.008 rad
angle
SAMSUNG



hywe measure 4 - [Unnamed measurement (Unsaved)]
File Gauge Measurement Analysis Window Help
phi omega alpha
phi
rad
X1: 1.873 s
Y1: -0.007 rad
X2: 2.811 s
Y2: 0.012 rad
ΔX: 0.939 s
ΔY: 0.018 rad
angle
start
SAMSUNG



sure 4 - [Unnamed measurement (Unsaved)]
Measurement Analysis Window Help
phi omega alpha
X1: 1.739 s
Y1: -0.009 rad
X2: 2.563 s
Y2: 0.007 rad
ΔX: 0.824 s
ΔY: 0.015 rad
angle
start
SAMSUNG



Trial	$\theta = 0^0$ T_1 (s)	$\theta = 15^0$ T_2 (s)	$\theta = 35^0$ T_3 (s)
1	0.766	0.773	0.820
2	0.761	0.768	0.830
3	0.766	0.782	0.825
4	0.775	0.778	0.825
5	0.757	0.778	0.825
	$\overline{T_1} = \frac{\sum_{i=1}^{5} T_{1i}}{5}$ =0.765(s)	$\overline{T_2} = \frac{\sum_{i=1}^{5} T_{2i}}{5}$ =0.776	$\overline{T_3} = \frac{\sum_{i=1}^{5} T_{3i}}{5}$ =0.825
	$\Delta T_1 = \frac{\sqrt{\frac{\sum_{i=1}^{5}\left(T_{1i} - \overline{T_1}\right)^2}{5}}}{\sqrt{5}}$ =0.003	$\Delta T_2 = \frac{\sqrt{\frac{\sum_{i=1}^{5}\left(T_{2i} - \overline{T_2}\right)^2}{5}}}{\sqrt{5}}$ =0.002	$\Delta T_3 = \frac{\sqrt{\frac{\sum_{i=1}^{5}\left(T_{3i} - \overline{T_3}\right)^2}{5}}}{\sqrt{5}}$ =0.001

Trial	$\theta = 45^0$ T_4 (s)	$\theta = 65^0$ T_5 (s)
1	0.939	1.287
2	0.929	1.296
3	0.943	1.278

4	0.942	1.291
5	0.948	1.273
	$\overline{T_4} = \frac{\sum_{i=1}^{5} T_{4i}}{5}$ =0.940	$\overline{T_5} = \frac{\sum_{i=1}^{5} T_{5i}}{5}$ =1.285
	$\Delta T_4 = \frac{\sqrt{\frac{\sum_{i=1}^{5}\left(T_{4i} - \overline{T_4}\right)^2}{5}}}{\sqrt{5}}$ =0.003	$\Delta T_5 = \frac{\sqrt{\frac{\sum_{i=1}^{5}\left(T_{5i} - \overline{T_5}\right)^2}{5}}}{\sqrt{5}}$ =0.004

II) Data treatment:

1) Determination of the oscillation period of a thread pendulum as a function of the pendulum length:

Pendulum with vertical oscillation plane: $T = 2\pi\sqrt{\frac{l}{g}} \rightarrow g = l\left(\frac{2\pi}{T}\right)^2$

a. L_1 = 0.350m

$$\overline{g_1} = L_1\left(\frac{2\pi}{T_1}\right)^2 = 0.350\left(\frac{2\pi}{1.272}\right)^2 = 8.54\ (m/s^2)$$

$$\Delta g_1 = \overline{g_1}\sqrt{\left(-\frac{2\Delta T_1}{\overline{T_1}}\right)^2 + \left(\frac{\Delta l_1}{\overline{l_1}}\right)^2} = 0.06\ (m/s^2)$$

$$\boxed{g_1 = \overline{g_1} \pm \Delta g_1 = 8.54 \pm 0.06\ (m/s^2)}$$

b. L_2 = 0.450m

$$\overline{g_2} = L_2\left(\frac{2\pi}{T_2}\right)^2 = 0.450\left(\frac{2\pi}{1.413}\right)^2 = 8.90\ (m/s^2)$$

$$\Delta g_2 = \overline{g_2}\sqrt{\left(-\frac{2\Delta T_2}{\overline{T_2}}\right)^2 + \left(\frac{\Delta l_2}{\overline{l_2}}\right)^2} = 0.03\ (m/s^2)$$

Hence,

$$\boxed{g_2 = \overline{g_2} \pm \Delta g_2 = 8.90 \pm 0.03\ (m/s^2)}$$

c. $L_3 = 0.550m$

$$\overline{g_3} = L_3\left(\frac{2\pi}{T_3}\right)^2 = 0.550\left(\frac{2\pi}{1.566}\right)^2 = 8.85\ (m/s^2)$$

$$\Delta g_3 = \overline{g_3}\sqrt{\left(-\frac{2\Delta T_3}{\overline{T_3}}\right)^2 + \left(\frac{\Delta l_3}{\overline{l_3}}\right)^2} = 0.05\ (m/s^2)$$

Hence,

$$\boxed{g_3 = \overline{g_3} \pm \Delta g_3 = 8.85 \pm 0.05\ (m/s^2)}$$

d. $L_4 = 0.650m$

$$\overline{g_4} = L_4\left(\frac{2\pi}{T_4}\right)^2 = 0.650\left(\frac{2\pi}{1.683}\right)^2 = 9.06\ (m/s^2)$$

$$\Delta g_4 = \overline{g_4}\sqrt{\left(-\frac{2\Delta T_4}{\overline{T_4}}\right)^2 + \left(\frac{\Delta l_4}{\overline{l_4}}\right)^2} = 0.03\ (m/s^2)$$

Hence,

$$\boxed{g_3 = \overline{g_3} \pm \Delta g_3 = 9.06 \pm 0.03\ (m/s^2)}$$

2) **Determination of the gravitational acceleration as a function of the inclination of the pendulum force:**
Length of the pendulum: l = 0.150 (m)
Pendulum with inclined oscillation plane:

$$T = 2\pi\sqrt{\frac{l}{g cos\theta}} \rightarrow g = \frac{l}{cos\theta}\left(\frac{2\pi}{T}\right)^2$$

a. $\theta_1 = 0^0$

$$\overline{g_1} = \frac{l}{\cos\theta_1}\left(\frac{2\pi}{T_1}\right)^2 = \frac{0.150}{\cos\theta_1}\left(\frac{2\pi}{0.765}\right)^2 = 10.12\ (m/s^2)$$

$$\Delta g_1 = \overline{g_1}\sqrt{\left(-\frac{2\Delta T_1}{\overline{T_1}}\right)^2 + \left(\frac{\Delta l_1}{\overline{l_1}}\right)^2 + \left(\frac{\Delta\cos\theta_1}{\cos\theta_1}\right)^2} = 0.10\ (m/s^2)$$

Hence,

$$\boxed{g_1 = \overline{g_1} \pm \Delta g_1 = 10.11 \pm 0.10\ (m/s^2)}$$

b. $\theta_2 = 15^0$

$$\overline{g_2} = \frac{l}{\cos\theta_2}\left(\frac{2\pi}{T_2}\right)^2 = \frac{0.150}{\cos\theta_2}\left(\frac{2\pi}{0.776}\right)^2 = 10.19\ (m/s^2)$$

$$\Delta g_2 = \overline{g_2}\sqrt{\left(-\frac{2\Delta T_2}{\overline{T_2}}\right)^2 + \left(\frac{\Delta l_2}{\overline{l_2}}\right)^2 + \left(\frac{\Delta\cos\theta_2}{\cos\theta_2}\right)^2} = 0.09\ (m/s^2)$$

Hence,

$$\boxed{g_2 = \overline{g_2} \pm \Delta g_2 = 10.19 \pm 0.09\ (m/s^2)}$$

c. $\theta_3 = 35^0$

$$\overline{g_3} = \frac{l}{\cos\theta_3}\left(\frac{2\pi}{T_3}\right)^2 = \frac{0.150}{\cos\theta_3}\left(\frac{2\pi}{0.825}\right)^2 = 10.62\ (m/s^2)$$

$$\Delta g_3 = \overline{g_3}\sqrt{\left(-\frac{2\Delta T_3}{\overline{T_3}}\right)^2 + \left(\frac{\Delta l_3}{\overline{l_3}}\right)^2 + \left(\frac{\Delta\cos\theta_3}{\cos\theta_3}\right)^2} = 0.08\ (m/s^2)$$

Hence,

$$\boxed{g_3 = \overline{g_3} \pm \Delta g_3 = 10.62 \pm 0.08\ (m/s^2)}$$

d. $\theta_4 = 45^0$

$$\overline{g_4} = \frac{l}{\cos\theta_4}\left(\frac{2\pi}{T_4}\right)^2 = \frac{0.150}{\cos\theta_4}\left(\frac{2\pi}{0.940}\right)^2 = 9.47\ (m/s^2)$$

$$\Delta g_4 = \overline{g_4}\sqrt{\left(-\frac{2\Delta T_4}{\overline{T_4}}\right)^2 + \left(\frac{\Delta l_4}{\overline{l_4}}\right)^2 + \left(\frac{\Delta\cos\theta_4}{\cos\theta_4}\right)^2} = 0.09\ (m/s^2)$$

Hence,

$$\boxed{g_4 = \overline{g_4} \pm \Delta g_4 = 9.47 \pm 0.09\ (m/s^2)}$$

e. $\theta_5 = 65^0$

$$\overline{g_5} = \frac{l}{\cos\theta_5}\left(\frac{2\pi}{T_5}\right)^2 = \frac{0.150}{\cos\theta_5}\left(\frac{2\pi}{1.285}\right)^2 = 8.49\ (m/s^2)$$

$$\Delta g_5 = \overline{g_5}\sqrt{\left(-\frac{2\Delta T_5}{\overline{T_5}}\right)^2 + \left(\frac{\Delta l_5}{\overline{l_5}}\right)^2 + \left(\frac{\Delta\cos\theta_5}{\cos\theta_5}\right)^2} = 0.08\ (m/s^2)$$

Hence,

$$\boxed{g_5 = \overline{g_5} \pm \Delta g_5 = 8.49 \pm 0.08\ (m/s^2)}$$