

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

GRADUATION THESIS

Pareto Front Grid Guided Multi-objective Optimization in Dynamic Pickup and Delivery Problem Considering Two-Sided Fairness

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Abstract

The dynamic delivery problem poses a complex challenge with many practical applications in logistics and transportation. Unlike the static delivery problem, where all order details are known, the dynamic delivery problem deals with continuously evolving information, with only partial data about orders available at any given moment. This thesis presents the Multi-objective dynamic pickup and delivery problem considering two-sided fairness framework, which integrates multiple objectives, including minimizing energy consumption, reducing waiting time, and ensuring fairness for both customers and vehicles. The goal is to minimize overall system costs while balancing the customer experience and the workload of service providers. Previous research has primarily focused on optimizing a single objective or converting other objectives into constraints, which can limit the flexibility and effectiveness of the solutions. Our approach tackles this challenge by introducing PFG-2F, a Pareto front grid-guided multi-objective evolutionary algorithm that incorporates two-sided fairness—ensuring equitable treatment for customers and service providers. The experimental results demonstrate that our method substantially outperforms existing multi-objective and single-objective algorithms specifically designed for the dynamic pickup and delivery problem on Hypervolume and Inverted generational distance metric.

Student

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List of Abbreviations

Abbreviation	Definition
CVRP	Capacitated vehicle routing problem
DPDP	Dynamic pickup and delivery problem
DPDPTW	Dynamic pickup and delivery problem considering time windows
DVRP	Dynamic vehicle routing problem
GA	Genetic algorithm
HV	Hypervolume
IGD	Inverted generational distance
ILP	Integer linear programming
LERK	Leader-based random keys encoding scheme
MODPDPTW	Multi-objective dynamic pickup and delivery problem with time windows considering two-sided fairness
MOEA	Multi-objective evolutionary algorithm
MOEAD	Multi-objective evolutionary algorithm based on decomposition
MOO	Multi-objective optimization
PDP	Pickup and delivery problem
PDPTW	Pickup and delivery problem with time windows
PFG	Pareto front grid
PFG-2F	Pareto front grid guided MOEA considering two-sided fairness
PFG-MOEA	Pareto front grid guided multi-objective evolutionary algorithm
PMX	Partially mapped crossover
SOO	Single-objective optimization
SVRP	Stochastic vehicle routing problem
VRP	Vehicle routing problem
VRPTW	Vehicle routing problem with time windows

INTRODUCTION

The Dynamic pickup and delivery problem (DPDP) represents a significant and intricate extension of the well-known vehicle routing problem, with a primary focus on optimizing the transportation of goods across networked locations. In this context, each vehicle is required to fulfill several transport requests, where each request consists of a pickup point from one customer and a delivery point to another.

In the static variant of DPDP, all transportation requests are available during the planning stage, enabling route optimization prior to vehicle dispatch with the aim of minimizing travel costs and time. By contrast, the dynamic version introduces additional complexity, as order information becomes available incrementally over time. At any given point, only partial knowledge of customer requests is accessible, compelling the system to make rapid decisions based on incomplete data. As a result, routing strategies must be highly adaptive and capable of responding efficiently to continuous environmental changes. DPDP has widespread practical relevance, particularly in modern logistics applications such as express delivery, e-commerce distribution, and supply chain operations. Of particular interest in recent research is the Dynamic pickup and delivery problem considering time windows (DPDPTW), where each transport request must be satisfied within specific time windows. These constraints require that cargo pickups and deliveries occur within predefined intervals, thereby intensifying the complexity of vehicle scheduling and route planning.

Although most existing studies on the DPDP have focused on static scenarios where all requests are known in advance, comparatively little attention has been devoted to the dynamic counterpart. Moreover, solving the DPDP often entails addressing multiple objectives simultaneously, such as minimizing energy consumption and ensuring fairness in resource allocation. Reducing energy usage is critical for enhancing both environmental sustainability and operational cost-efficiency. Concurrently, fairness in service delivery, which ensures that no customer is disproportionately delayed or disadvantaged particularly, is essential for maintaining equity and customer satisfaction. The integration of dynamic demands with multi-objective optimization makes the DPDPTW problem particularly challenging to solve. Algorithms designed for this problem must provide near-instantaneous responses to real-time requests while also maintaining a balanced trade-off among competing objectives.

The main contributions of this thesis are summarized as follows:

- Introduce the Multi-objective dynamic pickup and delivery problem with time windows considering two-sided fairness (MODPDPTW), a multi-objective model for solving the dynamic pickup and delivery problem. This model addresses traditional logistics challenges while integrating considerations related to energy efficiency, customer fairness, and vehicle workload fairness. It aims to deliver equitable service across customers and to distribute operational duties evenly among vehicles, thus promoting both efficiency and sustainability.
- Propose Pareto front grid guided MOEA considering two-sided fairness (PFG-2F), a novel algorithm designed to efficiently navigate the multi-objective solution space. The algorithm combines Pareto front grid (PFG) with random-key-based encoding and fairness-oriented heuristics, allowing for a more effective balance between exploration and exploitation.
- Experiments are conducted across diverse scenarios with varying customer numbers and spatial distributions. Numerical result evaluations demonstrate that PFG-2F outperforms state-of-the-art multi-objective approaches in terms of both Hypervolume (HV) and Inverted generational distance (IGD). Additionally, PFG-2F surpasses several single-objective vehicle routing algorithms that incorporate fairness measures, highlighting the superior performance of the proposed approach.

Thesis outline

The structure is organized as follows:

- **Chapter 1** introduces the foundational concepts, including optimization, multi-objective optimization, and Vehicle routing problem (VRP).
- **Chapter 2** presents DPDP, reviews related works and recent approaches, and introduces the proposed model—Multi-objective dynamic pickup and delivery problem considering two-sided fairness.
- **Chapter 3** proposes a novel method Pareto front grid-guided multi-objective optimization considering two-sided fairness.
- **Chapter 4** presents the experimental results and performance evaluation. The proposed method is compared against state-of-the-art algorithms using metrics such as hypervolume and inverted generational distance.

CHAPTER 1. BACKGROUND

This chapter focuses on explaining the fundamental concepts of optimization, beginning with the definition of optimization and the structure of an optimization problem, including objectives and constraints. Multi-objective optimization is introduced, providing definitions and discussing common approaches used to solve such problems. Finally, this chapter examines VRP, exploring their various variants and highlighting approaches for handling dynamic and real-time scenarios.

1.1 Optimization

Optimization is the process of finding the best solution to a problem from a set of feasible solutions. Formally, an optimization problem can be expressed as finding a decision vector \mathbf{x} that minimizes (or maximizes) an objective function $f(\mathbf{x})$ subject to a set of constraints:

$$\text{minimize } f(\mathbf{x}) \text{ subject to } \mathbf{x} \in X, \quad (1.1)$$

where \mathbf{x} is a vector of decision variables and X is the feasible set defined by the problem's constraints. The function $f(\mathbf{x})$ maps a decision \mathbf{x} to a real value representing the solution's quality. A solution $\mathbf{x}^* \in X$ is optimal if $f(\mathbf{x}^*) \leq f(\mathbf{x})$ for all $\mathbf{x} \in X$ (for a minimization problem). Key concepts include the feasible region (the set X of all \mathbf{x} satisfying constraints), global optimum (an optimal solution with respect to all of X), and local optimum (a solution that is optimal within a neighboring subset of X but not necessarily globally).

In general, optimization problems are types of mathematical problems with goals are to find the best solutions from a solution searching space, these solutions may follow some criteria given by the problem definition. A simple optimization problem will require minimize or maximize a function, represented for cost, profit, time, distance, by constructing solutions subject to certain constraints. The solutions then are used to compute the value of the function. A general structure of an optimization problem includes:

- **Objective function:** a mathematical expression that quantifies the goal of the optimization. It defines the criterion to be minimized or maximized, such as cost, profit, time, or error. The objective function is expressed in terms of the decision variables and serves as the primary metric for evaluating the quality of a solution.

- **Decision variable:** the set of controllable inputs or parameters whose values are to be determined through the optimization process. These variables directly influence the value of the objective function and are subject to the problem's constraints. The solution to the optimization problem is defined by the specific values assigned to these variables.
- **Constraints:** a set of mathematical conditions, usually in form of equalities or inequalities that define the feasible region within which the decision variables must lie. These conditions represent problem's limitations or requirements, as resource capacities, physical laws, or operational restrictions. A feasible solution must satisfy all specified constraints.

1.2 Multi-objective optimization

Many problems involve multiple objectives that must be optimized simultaneously. Multi-objective optimization (MOO), also known as multi-criteria or vector optimization, deals with problems where there are two or more conflicting objectives. Instead of a single objective function, it is replaced by a vector of objective functions. A general multi-objective optimization problem can be formulated as:

$$\begin{aligned} &\text{minimize} \quad \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})) \\ &\text{subject to} \quad \mathbf{x} \in X, \end{aligned} \tag{1.2}$$

where X is the feasible set and $f_i(\mathbf{x})$ represents the i -th objective to be minimized (maximization objectives can be converted to minimization by applying a negative sign). In multi-objective optimization (MOO), the objectives f_1, \dots, f_m are often in conflict, meaning that no single solution is best for all objectives simultaneously. The example of decision space represents X and objective space, for example f_n , f_m is represented in Figure 1.1.

1.2.1 Definitions

Due to trade-offs between conflicting objectives, the goal in multi-objective optimization is typically not to find a single optimal solution, but rather a set of *Pareto optimal* solutions. The concept of Pareto optimality is based on the notion of *dominance* between solutions.

A solution $\mathbf{x}^a \in X$ is said to *dominate* another solution $\mathbf{x}^b \in X$ (denoted $\mathbf{x}^a \prec \mathbf{x}^b$) if the following two conditions are satisfied:

$$f_i(\mathbf{x}^a) \leq f_i(\mathbf{x}^b), \quad \forall i = 1, \dots, m, \quad f_j(\mathbf{x}^a) < f_j(\mathbf{x}^b), \quad \text{for at least one } j \in 1, \dots, m. \tag{1.3}$$

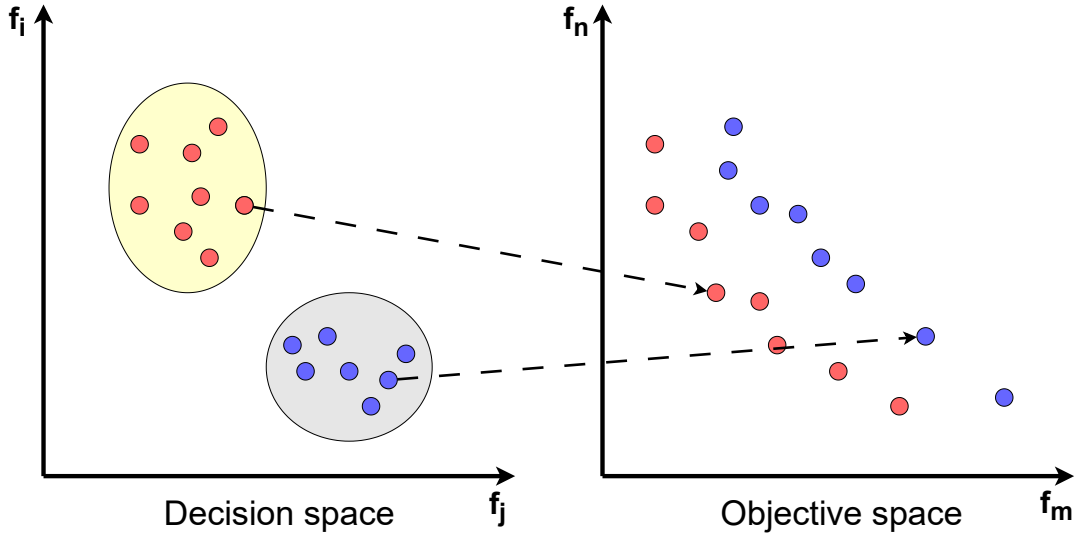


Figure 1.1: Multi-objective searching space.

That is, \mathbf{x}^a is no worse than \mathbf{x}^b in all objectives and strictly better in at least one.

A solution $\mathbf{x}^* \in X$ is said to be *Pareto optimal* if there does not exist any $\mathbf{x} \in X$ such that $\mathbf{x} \prec \mathbf{x}^*$. Formally:

$$\nexists, \mathbf{x} \in X \text{ such that } \mathbf{x} \prec \mathbf{x}^* \quad (1.4)$$

The set of all Pareto optimal solutions in X is called the *Pareto set*, and the corresponding set of objective vectors in the objective space is known as the *Pareto front* [1], as illustrated in Figure 1.2.

By definition, moving from one Pareto optimal solution to another on the Pareto front will improve some objectives but worsen at least one other objective. Decision makers often must choose from the Pareto front according to preference trade-offs.

Reference points in multi-objective optimization are important for measuring the effective of the multi-objective algorithms by evaluating the set of solution produced. Reference points includes:

- **Utopia Point** (also known as the ideal point) is a hypothetical point in objective space defined by $\mathbf{z}^* = (z_1^*, \dots, z_m^*)$, where $z_i^* = \min_{\mathbf{x} \in X} f_i(\mathbf{x})$ is the best achievable value for each objective individually. The utopia point is generally not attainable unless one solution simultaneously optimizes all objectives.
- **Nadir Point** $\mathbf{z}^{nad} = (z_1^{nad}, \dots, z_m^{nad})$, is defined by $z_i^{nad} = \max_{\mathbf{x} \in X_P} f_i(\mathbf{x})$, where X_P is the Pareto set. In other words, the nadir point components are the worst objective values among all Pareto-optimal solutions. The nadir point,

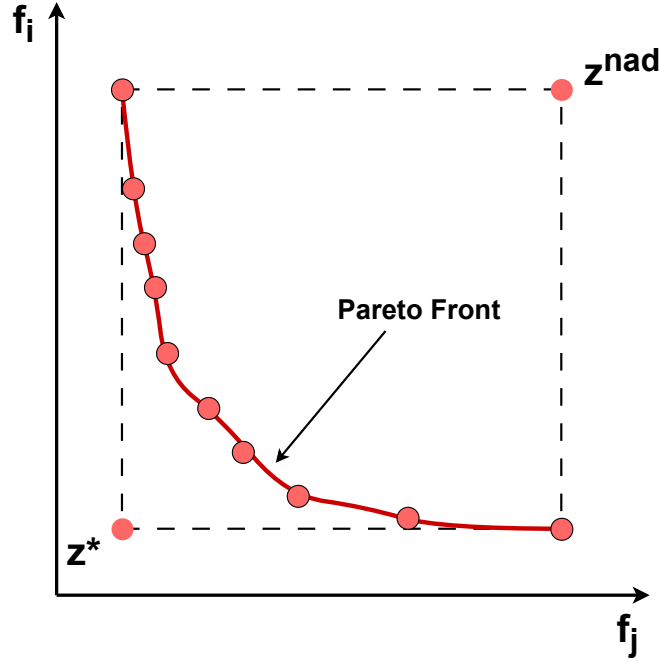


Figure 1.2: Pareto front and reference points.

when all objectives are to be minimized, gives an idea of the extent of the Pareto front.

- **Knee point** is a Pareto-optimal solution where a small improvement in one objective would lead to a large deterioration in at least one other objective. Knee points are often considered good compromise solutions because they exhibit a balanced trade-off among objectives.

1.2.2 Approaches

Solving a multi-objective problem means finding or approximating the Pareto front. If the problem is small and tractable (for example, a multi-objective linear or integer linear program), Pareto optimal solutions can be computed exactly by extending single-objective optimization techniques. For instance, a multi-objective Integer linear programming (ILP) (integer linear program with multiple objective functions) can be tackled by algorithms that systematically explore trade-offs (e.g., using parametric linear programming or variants of branch-and-bound that handle multiple objectives) [2]. However, a common approach is to employ scalarization, converting the multi-objective problem into a series of single-objective problems whose solutions are Pareto optimal for the original problem [1], [2]. This thesis presents an overview of classical scalarization techniques and their corresponding strategies as follows:

- **Weighted Sum:** A weighted sum approach assigns a weight $w_i \geq 0$ to each objective and optimizes a linear combination of the objectives:

$$\min_{\mathbf{x} \in X} \sum_{i=1}^m w_i f_i(\mathbf{x}) \quad (1.5)$$

where $\sum w_i = 1$, $w_i \geq 0$. Weights reflect the relative importance of objectives [1]. In practice, solving the weighted sum formulation for different weight sets (e.g., using an ILP solver when f_i and X are linear) yields points along the Pareto front. This method is easy to implement but has limitations: if the true Pareto front is non-convex in objective space, the weighted sum method cannot obtain those Pareto-optimal solutions that lie on the non-convex parts of the front [2]. Additionally, choosing appropriate weights can be difficult without prior knowledge of the trade-offs.

- **Weighted Tchebycheff:** The weighted Tchebycheff approach is a more general scalarization that can obtain any Pareto optimal solution (given certain conditions) and is especially useful for finding extreme points on a non-convex Pareto front [1]. It uses a reference point, often the utopia point z^* , and minimizes the weighted maximum deviation from this reference point. A common formulation is:

$$\min_{\mathbf{x} \in X} \max_{1 \leq i \leq m} \{w_i |f_i(\mathbf{x}) - z_i^*|\} \quad (1.6)$$

where $w_i > 0$ are scaling weights. In words, this tries to minimize the worst-case (relative) objective value among all objectives. By adjusting the weights w_i , different Pareto solutions can be emphasized. The weighted Tchebycheff method can capture Pareto-optimal solutions that the simple weighted sum method might miss, and it provides a uniform spread of solutions under certain conditions. An augmented version of this method adds a small term $\epsilon \sum_i w_i f_i(\mathbf{x})$ to ensure the obtained solution is truly Pareto optimal (to avoid solutions that only minimize the maximum deviation but are dominated) [1].

- **Decomposition-based Methods:** These strategies involve decomposing the multi-objective problem into a set of simpler subproblems, each focusing on a particular region or trade-off of the Pareto front. For example, one can generate a set of N evenly distributed weight vectors w^1, \dots, w^N in the weight space and solve N single-objective problems (such as weighted sum or Tchebycheff scalarizations for each w^j). This produces a set of N Pareto-optimal solutions approximating the front. Decomposition strategies are at the core of certain evolutionary algorithms (discussed next) where multiple scalarized subprob-

lems are tackled in parallel. The advantage of decomposition is that it can provide a well-spread approximation of the Pareto front by covering different preference regions or weight combinations. Benson's method [2] are examples of mathematical programming approaches that systematically generate Pareto fronts by solving a sequence of subproblems. In summary, scalarization and decomposition techniques convert the multi-objective task into a series of single-objective optimizations, enabling the use of classical solvers or guided search to find Pareto-optimal solutions.

While classical scalarization methods can find Pareto-optimal solutions, they often require solving many subproblems and may struggle with complex, non-linear, or combinatorial objectives. In practice, particularly for difficult problems like vehicle routing, evolutionary algorithms and other metaheuristics are popular for approximating Pareto fronts in a single run [3]. These algorithms evolve a population of candidate solutions and are naturally suited to exploring multiple trade-offs simultaneously. This thesis highlight some common algorithms and frameworks for multi-objective optimization:

NSGA Srinivas and Deb [4] introduced Non-dominated Sorting Genetic Algorithm (NSGA), one of the first genetic algorithms designed for multi-objective optimization. NSGA uses the concept of *non-dominated sorting*: the population of candidate solutions is ranked into tiers ("Pareto fronts") based on dominance. All nondominated individuals are assigned to the first front (Pareto rank 1), then those dominated only by solutions in the first front form the second front (rank 2), and so forth. The GA then applies selection, crossover, and mutation biased toward lower-rank (better) solutions, along with a niche-sharing technique to maintain diversity along the Pareto front [4]. NSGA demonstrated the effectiveness of evolutionary approaches to obtain a spread of Pareto-optimal solutions in one run. However, the original NSGA had some drawbacks: it did not explicitly preserve elite solutions (the best found so far) and it required a sharing parameter to maintain diversity, which can be hard to set.

NSGA-II Deb et al. [5] proposed NSGA-II, which has become one of the most widely used multi-objective evolutionary algorithms. NSGA-II introduced several improvements over NSGA:

- **Elitism:** NSGA-II ensures that Pareto-optimal solutions found are carried over to the next generation. In each iteration, parent and offspring populations are

merged and the best N individuals (for population size N) are selected for the next generation. This retains any superior solutions discovered.

- **Fast Non-dominated Sorting:** NSGA-II uses an efficient sorting algorithm to rank solutions into Pareto fronts with complexity $O(N \log N)$ per generation, improving on the $O(N^2)$ approach of NSGA.
- **Diversity Preservation:** Instead of the niche sharing parameter, NSGA-II uses the *crowding distance* metric. After ranking by Pareto dominance, solutions in the last front that fits in the next generation are sorted by crowding distance (a measure of how isolated a solution is in objective space). Solutions with larger crowding distance (more isolated, contributing to diversity) are preferred. This helps maintain a well-spread Pareto front.

MOEA/D The Multi-objective evolutionary algorithm based on decomposition (MOEA/D), proposed by Zhang and Li [6], introduces a decomposition-based strategy for solving multi-objective optimization problems (MOPs). Unlike Pareto-dominance-based approaches, MOEA/D decomposes a MOP into a set of scalar optimization subproblems using aggregation functions, such as the weighted sum or the Tchebycheff approach.

Let X denote the decision space, and $\mathbf{f} : X \rightarrow \mathbb{R}^m$ represent the vector of m objective functions. MOEA/D defines N weight vectors $\{\mathbf{w}^1, \mathbf{w}^2, \dots, \mathbf{w}^N\}$, where each $\mathbf{w}^j = (w_1^j, \dots, w_m^j)$ satisfies $w_i^j \geq 0$ and $\sum_{i=1}^m w_i^j = 1$. Each weight vector corresponds to a scalar subproblem:

$$\min_{\mathbf{x} \in X} g^j(\mathbf{x} \mid \mathbf{w}^j, \mathbf{z}^*) = \max_{1 \leq i \leq m} \left\{ w_i^j \cdot |f_i(\mathbf{x}) - z_i^*| \right\}, \quad (1.7)$$

where $\mathbf{z}^* = (z_1^*, \dots, z_m^*)$ is the reference point, typically the ideal point in the objective space.

MOEA/D maintains a population $\mathcal{P} = \{\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^N\}$, where each solution \mathbf{x}^j is associated with the weight vector \mathbf{w}^j . The algorithm proceeds as follows:

- **Neighborhood Definition:** For each subproblem j , define a neighborhood $\mathcal{N}(j)$ consisting of the indices of the T closest weight vectors to \mathbf{w}^j based on Euclidean distance.
- **Variation:** For each subproblem j , select parent solutions from $\mathcal{N}(j)$ and apply genetic operators (e.g., crossover and mutation) to generate an offspring \mathbf{y} .

- **Update:** Evaluate \mathbf{y} and update the solutions associated with the subproblems in $\mathcal{N}(j)$. Specifically, for each $k \in \mathcal{N}(j)$, if

$$g^k(\mathbf{y} \mid \mathbf{w}^k, \mathbf{z}^*) < g^k(\mathbf{x}^k \mid \mathbf{w}^k, \mathbf{z}^*),$$

then set $\mathbf{x}^k = \mathbf{y}$.

- **Reference Point Update:** Update the reference point \mathbf{z}^* if necessary, by setting

$$z_i^* = \min\{z_i^*, f_i(\mathbf{y})\}, \quad \text{for all } i = 1, \dots, m.$$

- **External Population Maintenance:** Maintain an external population (EP) to store all non-dominated solutions found during the search process. When a new solution \mathbf{y} is generated, it is compared against the current EP. If \mathbf{y} is not dominated by any member of the EP, it is added to the EP, and any solutions in the EP dominated by \mathbf{y} are removed.

The external population serves as an archive of elite solutions, providing a diverse approximation of the Pareto front. It is beneficial for preserving diversity and guiding the search process, especially in many-objective optimization scenarios.

MOEA/D's decomposition strategy allows for parallel optimization of subproblems, facilitating scalability to problems with a large number of objectives. Its neighborhood-based update mechanism promotes information sharing among similar subproblems, enhancing convergence and diversity. Since its inception, various extensions and improvements have been proposed, incorporating adaptive weight adjustment, hybridization with metaheuristics, and advanced archiving strategies.

1.2.3 Metrics

To evaluate the effectiveness of Pareto fronts generated by multi-objective optimization algorithms, two widely accepted performance metrics are employed: the HV and the IGD. These metrics serve as quantitative indicators for assessing both the convergence of the obtained solutions toward the true Pareto front and their distribution across the objective space—two key aspects of multi-objective optimization quality.

The HV metric, introduced by Zitzler et al. [7], measures the volume of the objective space that is weakly dominated by the non-dominated solutions in the obtained Pareto front, bounded from above by a reference point. It reflects both convergence (how close the solutions are to the true Pareto front) and diversity (how well they spread across the objective space). A higher HV value indicates a

better-quality solution set, as it suggests that the algorithm has not only reached a region close to the optimal front but also captured a diverse range of trade-offs among the objectives. The formal expression of HV is shown in Equation 1.8, and an illustrative example is provided in Figure 1.3a [8].

$$HV = \int_{f_{\min}}^{f_{z^n}} \prod_{i=1}^m (f_i^{z^n} - f_i) df, \quad (1.8)$$

In contrast, the IGD metric, first appeared in [9], evaluates the average distance from a set of reference Pareto-optimal solutions to the closest solutions in the obtained front. It is used primarily to measure convergence, although it also indirectly captures distribution quality by penalizing regions of the true front that are poorly approximated. A lower IGD value indicates that the obtained solutions are closer to the reference front and thus reflect a higher-quality approximation. The formula for computing IGD is given in Equation 1.9, and an example visualization is presented in Figure 1.3b [8].

$$IGD = \frac{1}{|\mathcal{Q}|} \sum_{q \in \mathcal{Q}} \min_{p \in \mathcal{P}} \|q - p\|_2, \quad (1.9)$$

In this equation, \mathcal{Q} denotes the set of reference Pareto-optimal solutions, \mathcal{P} is the set of solutions obtained by the algorithm, and $\|\cdot\|_2$ represents the Euclidean distance. Together, HV and IGD provide a comprehensive framework for evaluating the quality of multi-objective optimization results, supporting both theoretical comparisons and practical performance analysis.

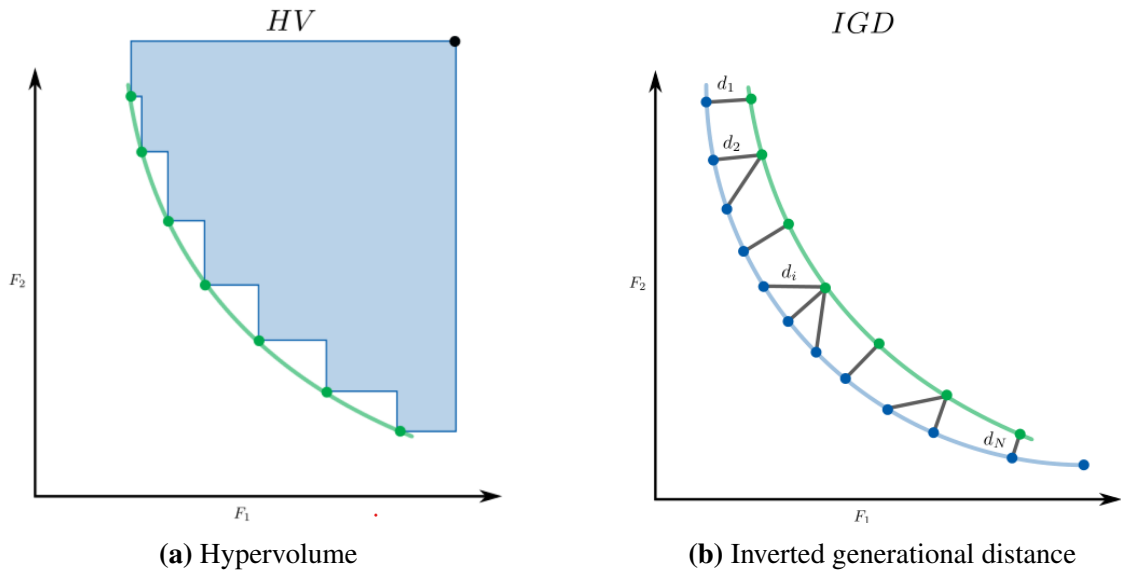


Figure 1.3: Multi-objective optimization metrics [8].

1.3 Vehicle routing problem

VRP, first introduced by Dantzig and Ramser [10], has become a cornerstone of logistics optimization. Over the decades, numerous variants of the VRP have been developed to address the wide variety of real-world constraints and requirements that arise in distribution and transportation [11]. These problem variants extend the classical VRP formulation in order to model practical considerations such as vehicle capacity limits, customer service time windows, uncertainty in demands, and environmental sustainability goals. This thesis highlights several important modern VRP variants below, focusing on their formulation and the real-world motivations behind them. This thesis will separate the VRP based on five types of variants, based on constraint, service type, nature, certainty and objective.

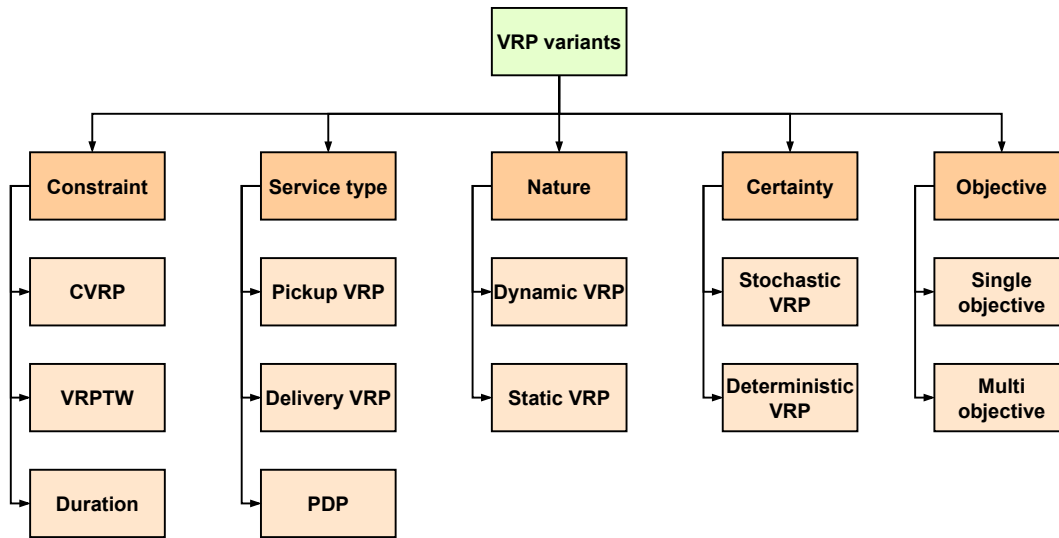


Figure 1.4: Survey on vehicle routing problem.

1.3.1 Vehicle routing problems variants

a, Constraint of vehicle routing problem

The classical VRP is defined over a set of customers, each with a specific location and demand, and a fleet of vehicles based at a central depot. The goal is to design a set of routes—one for each vehicle—such that all customers are served exactly once, the total routing cost (e.g., distance or time) is minimized, and operational constraints are respected.

A key variant of this problem is the Capacitated vehicle routing problem (CVRP), where each vehicle has a fixed carrying capacity. This reflects real-world limitations, such as delivery trucks being able to carry only a finite amount of goods. A feasible solution must ensure that the total demand on any route does not exceed the vehicle's capacity, as illustrated in Figure 1.5. CVRP captures the trade-off between route

length and fleet size: while using fewer vehicles can reduce fixed costs, capacity limits prevent overloading, necessitating more or longer routes. This makes CVRP highly applicable to practical distribution tasks as fuel delivery where tanker volume is fixed. It also forms the foundation for numerous advanced VRP variants, which build on the capacity constraint by introducing additional complexities such as time windows, stochastic demands, or environmental objectives.

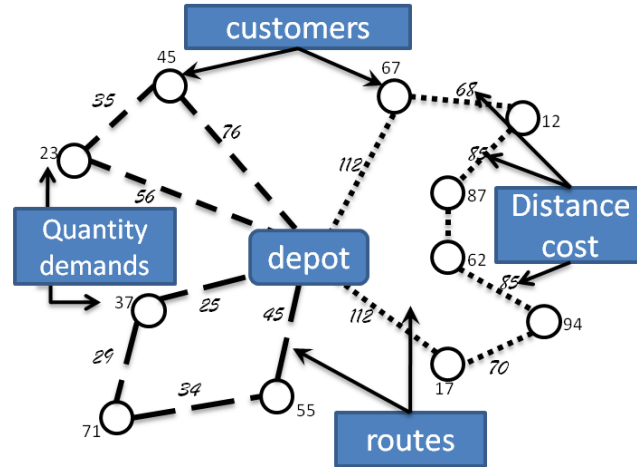


Figure 1.5: Example of capacitated vehicle routing problem.

One of the most common and significant constraints found in extensions of the VRP is the incorporation of service time windows, resulting in a specialized variant known as the Vehicle routing problem with time windows (VRPTW). In this extended model, each customer is assigned a specific time interval, which is defined by an earliest allowable arrival time and a latest permissible arrival time, within which the delivery or service must commence. If a vehicle arrives before the designated time window opens, it is permitted to wait until service can begin; however, arriving after the latest allowed time is strictly prohibited. This constraint increases planning complexity, as it requires careful synchronization of route planning with the availability of each customer. It is also important in real-world operational situations where deliveries must adhere to customer schedules, regulatory restrictions, or predefined contractual service levels, thereby making the problem more representative of actual logistical challenges faced by businesses and transportation systems.

Additionally, the durations of services provided at each customer location must be accounted for, as they significantly impact the feasibility and efficiency of any proposed routing or scheduling solution. These time-related constraints add a layer of complexity to the overall optimization problem by narrowing the solution space and introducing additional dependencies between route segments and time windows.

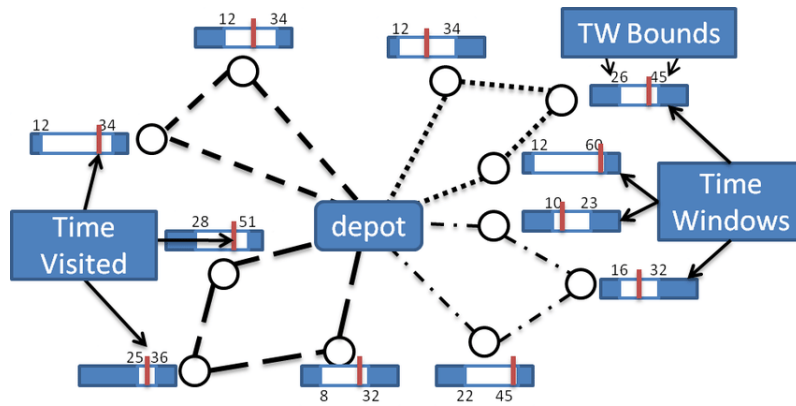


Figure 1.6: Example of vehicle routing problem with time windows [12]

This is especially critical in real-world scenarios involving the transportation of perishable goods, where delays can result in spoilage and financial loss, or in cases where deliveries are restricted to specific time windows due to customer availability or regulatory requirements. As a result, ensuring timely service becomes a key objective, demanding more sophisticated planning and coordination to balance route efficiency with strict temporal constraints.

b, Service types in vehicle routing problem

As VRPs address differing real-world service demands, problems are often categorized based on the type of service operation involved—specifically, whether they involve deliveries, pickups, or both. The four major service types include:

- **Delivery-only VRP:** In the classical case of VRP, vehicles depart from a central depot to deliver goods to customers, visiting each customer exactly once. The objective is typically to minimize the total travel cost while satisfying vehicle capacity and routing constraints.
- **Pickup-only VRP:** Vehicles collect goods (such as waste, returned items, or recyclables) from customers and transport them back to one or more depots or processing centers. The foundational version was introduced by Desaulniers et al. in [13]. This type is commonly encountered in reverse logistics and recycling operations.
- **Combined pickup and delivery VRP:** Each request in this variant specifies both a pickup and a delivery location. The vehicle must visit both locations—pickup first, followed by delivery—as part of the same route. This problem arises in applications such as courier logistics, ride-sharing, and less-than-truckload freight transport. It represents the standard formulation of the pickup and delivery problem and has been extensively studied in the literature [14].

- Simultaneous pickup and delivery VRP: Vehicles serve customers who require both delivery and pickup at the same visit. This problem is well-studied as a specific extension of VRP [15], [16], [17]. This service is provided in a single stop, without returning later. It is commonly used in reusable container logistics, newspaper recycling, or distribution systems with reverse flows.

c, Nature of vehicle routing problem

Many transportation optimization problems, including VRP, have traditionally been studied in a static context where all input data (e.g., customer demands, travel times) are known in advance. However, real-world operations often face uncertainty and change: new customer requests may arrive during the day, traffic conditions can vary, or orders may be cancelled. This gives rise to the Dynamic vehicle routing problem (DVRP) [18], a class of problems where information is revealed or updated in real-time and routes must be adjusted on-the-fly.

In a DVRP, unlike the static VRP, decisions (such as which customer to serve next or which route a vehicle will follow) are made not just once at the start, but continuously as new information arrives over time. Formally, a dynamic routing problem can be seen as a combinatorial optimization problem on a timeline. Initially, a partial set of requests or tasks is known and must be scheduled into vehicle routes. Over time, new requests appear (or other changes occur), requiring the current solution to be updated. The objective is typically to optimize some metric of service (like total travel distance, operational cost, or response time) while feasibly incorporating all requests announced up to that time.

Many dynamic problems also impose real-time constraints; for example, decisions may need to be recomputed within seconds each time an update occurs to be practically useful in dispatching vehicles. One important variant is DPDP [14], which is a dynamic version of the pickup-and-delivery problem. In DPDP, transportation requests consist of a pickup location and a delivery location (e.g., ride-sharing or courier requests) that can arrive dynamically during the routing period. Each new request must be assigned to a vehicle and sequenced into that vehicle's route, respecting any constraints such as capacity or time windows. The dynamic aspect greatly increases the complexity: the algorithm must decide how to insert the pickup and delivery of each new request into an existing set of routes without overly disrupting prior commitments. Dynamic routing problems are inherently challenging because they combine the complexity of NP-hard routing optimization with the uncertainty of online decision-making.

d, Certainty of vehicle routing problem

In the classical deterministic formulation of the VRP, all relevant problem parameters are assumed to be known in advance and remain fixed throughout the planning horizon. This includes customer demands, service times, travel times between locations, and the set of customers to be served. Based on this complete information, a solution is generated that specifies the optimal set of vehicle routes to minimize a given cost function—typically distance, time, or number of vehicles—while satisfying operational constraints such as capacity and service coverage. Deterministic VRP models are computationally tractable and widely studied, serving as the foundation for many algorithmic developments and industrial applications.

However, in real-world logistics and transportation systems, such certainty is rarely attainable. Operational environments are often dynamic and unpredictable: customer demands may fluctuate, traffic conditions may vary unexpectedly, new requests may appear, and some customers might cancel or reschedule. To address these challenges, the Stochastic vehicle routing problem (SVRP) [19], [20] introduces randomness into one or more elements of the problem formulation. In this setting, parameters such as customer demand, travel time, service duration, or customer presence are modeled as random variables, typically governed by known or estimated probability distributions.

The stochastic formulation more accurately captures the nature of real-world routing tasks, where planners must make decisions based on incomplete or probabilistic information. For example, a delivery company may need to dispatch vehicles before knowing the exact quantities that customers will request, or an emergency service provider must plan routes without knowing the exact locations and times of future calls. In these cases, relying solely on a deterministic plan can result in inefficiencies or failures to meet service requirements.

To cope with this uncertainty, SVRP models often incorporate recourse strategies—predefined rules or adaptive mechanisms that allow the solution to adjust in response to actual events during execution. A common example is allowing a vehicle to return to the depot for reloading if encountered demands exceed its remaining capacity. Other strategies include reassigning tasks, rerouting dynamically, or holding buffer capacity in anticipation of stochastic events. These mechanisms aim to enhance the robustness and flexibility of the solution, ensuring acceptable performance across a range of possible realizations of the uncertain parameters.

e, Objective of vehicle routing problem

As concerns about climate change and environmental degradation continue to grow, reducing energy consumption and minimizing carbon emissions have become central objectives in modern logistics and transportation planning. In the context of vehicle routing, this shift has led to the development of environmentally focused problem variants that explicitly aim to lower the ecological impact of transportation operations. One key development is the Green VRP, which incorporates vehicles powered by alternative fuels—such as electric or hybrid vehicles—into the routing model. These energy-constrained vehicles often have limited driving range and must plan visits to refueling or recharging stations [21], [22].

Beyond environmental considerations, sustainability also involves ensuring fairness among the system’s human stakeholders—both service providers and customers. Fairness is increasingly viewed as a vital component for the long-term viability of shared mobility and delivery platforms. Unfair treatment, whether it manifests as an uneven distribution of workload among drivers or excessively long wait times for certain customers, can result in dissatisfaction and attrition [23].

As real-world applications of vehicle routing continue to evolve, the objectives have continued to evolve associated with solving these problems. Traditional VRP have largely focused on single-objective formulations—most commonly minimizing total distance, cost, or fleet size. However, the increasing complexity of modern logistics has necessitated the development of Multi-objective VRP, which aim to optimize two or more conflicting objectives simultaneously. Common objective combinations include minimizing cost and emissions, balancing workload and travel time, or improving service quality while reducing operational risk. Multi-objective optimization methods, such as Pareto-based evolutionary algorithms and goal programming, are widely used to handle these trade-offs [24], [25]. These approaches enable decision-makers to explore a diverse set of efficient solutions and select the most appropriate one based on practical priorities.

1.3.2 Related works

VRP was first introduced by Dantzig and Ramser in 1959 [10], in the context of optimizing gasoline deliveries. Their seminal work laid the foundation for a vast body of research in combinatorial optimization and logistics. Since then, VRP has become a central problem in operations research, inspiring a wide array of variants and solution techniques.

Among the earliest and most significant extensions is the CVRP, which incorporates the realistic constraint that each vehicle can carry only a limited load. This

constraint adds a layer of complexity to the problem, making it more suitable for real-world applications such as retail distribution, waste collection, and freight transportation. The CVRP has been studied extensively, both as a standalone optimization problem and as a baseline for more sophisticated variants.

Research on CVRP has focused on developing exact algorithms (e.g., branch-and-bound, branch-and-cut) and heuristics/metaheuristics such as tabu search, simulated annealing, and genetic algorithms. These approaches attempt to cope with the combinatorial explosion of the solution space as the number of customers and vehicles increases. CVRP has also served as a standard benchmark in many algorithmic studies because of its practical relevance and computational difficulty.

Building upon CVRP, VRPTW introduces temporal constraints that further narrow the solution space. Each customer must be serviced within a specific time interval, and vehicles are not allowed to be late. This model is particularly relevant in industries where service-level agreements or perishability are major concerns, such as courier services, fresh food delivery, and healthcare logistics.

The introduction of benchmark instances for the VRPTW by Solomon in 1987 [26] marked a major milestone in the field. These instances enabled comparative studies across different algorithms and significantly shaped the direction of VRP research in the following decades. The Solomon instances vary in customer distribution and time window tightness, providing a diverse testbed for evaluating both exact and heuristic methods. Recent studies, such as Hieu et al. [27], have also incorporated machine learning techniques to more effectively address both capacity and time window constraints in the VRPTW.

Pickup-only vehicle routing variants have gained attention due to the rise of reverse logistics and sustainability-focused operations. Such models appear frequently in waste collection, recycling, and product return networks [28]. Recent research has explored reverse vehicle routing problems with mixed-location routing optimization, as in the work by Mozhgan et al. [29].

In the combined pickup and delivery vehicle routing problem, the precedence constraint (pickup before delivery for the same request) introduces significant routing complexity. This variant models many practical systems such as taxi-sharing, last-mile logistics, and courier services. Desaulniers et al. [13] provide a comprehensive formulation and solution strategies in their work on the Pickup and delivery problem (PDP).

The simultaneous pickup and delivery variant further complicates route design, as vehicles must handle both flows concurrently at each customer stop. This structure

has found widespread industrial use over the last three decades. Applications include reusable container distribution, newspaper recycling programs, and integrated retail distribution networks. Several studies have extensively reviewed algorithms and modeling techniques for this variant, including those by Polat et al. [30] and Guo and Wang [31].

Nonetheless, a wide range of research has been conducted to address Dynamic Vehicle Routing Problems (DVRP), which extend the classical VRP by incorporating time-dependent and real-time information. These studies aim to develop methods that can adapt to changes in customer demands, traffic conditions, and other uncertainties during execution. A variety of strategies have been developed in recent researches to handle DVRPs effectively:

- **Rolling horizon / periodic reoptimization:** In this framework, time is divided into planning intervals. At each interval (say every few minutes), the system takes all information available up to that point (including newly arrived requests) and solves a VRP (or Pickup and delivery problem with time windows (PDPTW)) for those requests, treating any future requests as unknown. The solution provides updated routes for the vehicles. When the next interval arrives or a significant event happens, the process is repeated. This approach repeatedly solves static problems as new data comes in, effectively rolling the planning horizon forward [18]. It provides a structured way to balance solution quality and computational effort, as reoptimization can be scheduled at a fixed frequency. However, choosing the right reoptimization frequency is important: too frequent and the system may thrash with little improvement; too infrequent and it may react too slowly to new information.
- **Event-driven reoptimization:** Instead of a fixed schedule, some approaches trigger reoptimization immediately upon certain events, such as the arrival of a new request or a vehicle completing a service [18]. This is a purely reactive strategy: the moment something changes (e.g., a new customer calls in an order), the algorithm updates the vehicle routes to accommodate the change. This can minimize the delay in integrating new requests, potentially improving responsiveness. The challenge is ensuring the computation finishes fast enough to implement decisions in real time, especially if events happen in quick succession.
- **Heuristic insertion strategies:** A widely used class of methods for DVRP and DPDP is to insert new requests directly into existing routes using heuristics, rather than reoptimizing from scratch each time. For example, when a new

pickup-delivery request arrives, one can evaluate the cost of inserting that request at various feasible positions in each vehicle's route and choose the least-cost insertion [14]. This is akin to the insertion heuristics used in static route construction (like the savings algorithm or cheapest insertion), but applied online. In dynamic dial-a-ride problems, insertion heuristics with look-ahead can be used, where the algorithm considers not just the immediate cost but also future implications (sometimes by reserving slack for possible future requests). Insertion methods are typically fast and can handle high volumes of requests, though they may yield myopic decisions compared to a full reoptimization.

- **Metaheuristic adaptation:** Many researchers have adapted metaheuristic algorithms to the dynamic context. For instance, Gendreau et al. [32] implemented a Tabu Search for a real-time VRP: their approach continuously improves the current set of routes and, when a new request arrives, they integrate it into the solution and then continue the Tabu Search from that new state. The search is designed to be fast and anytime, so that it can be interrupted to dispatch vehicles as needed. Similarly, genetic algorithms and ant colony optimization have been modified for dynamic VRPs by running a continuous optimization loop that refines routes and quickly adapts to changes. Montemanni et al. [33], for example, developed an Ant Colony System that repeatedly constructs vehicle routes and can add new requests as pheromone information is updated, making the algorithm “responsive” to real-time events. These metaheuristic approaches often maintain a population or a memory of solutions that they tweak as the problem data evolves, rather than restarting from scratch on update.
- **Assignment and dispatching rules:** In some real-time systems, especially when decisions need to be extremely fast, simpler rule-based strategies are used. For instance, when a new request comes in, assign it to the nearest available vehicle greedily or a vehicle that will become free soonest. While such myopic policies might not yield globally minimal travel cost, they are easy to compute and can be effective when combined with periodic plan improvements. More sophisticated dispatching rules might take into account current route load, detour from current route, or balancing workload among vehicles. These can be seen as a form of rolling decision-making without heavy computation.

Managing dynamic routing in real time often requires careful handling of operational constraints. For example, when reoptimizing, one must consider that vehicles already en route to serve a customer cannot instantly change course to a totally different plan (especially if they have passengers on board in DPDP). Thus, many algorithms enforce route continuity or solution stability constraints, meaning

changes to the plan should be feasible and not overly disruptive. One common practice is to “freeze” a portion of each vehicle’s route that is imminent (e.g., the next immediate service stops) and only allow changes to the part of the route beyond that [18]. This ensures that decisions that are in execution are not reversed by the optimizer, providing consistency in operations.

Over the past decades, a number of academic works have provided frameworks and surveys for dynamic routing problems. Pillac et al. [18] present a comprehensive review of DVRP classifications and solution methods, highlighting the importance of information quality and timing. They categorize approaches by how and when they adapt to new information (periodic vs event-driven, myopic vs anticipatory). Similarly, Berbeglia et al. [14] survey dynamic pickup-and-delivery problems, discussing various models (e.g., allowing advance request announcements versus purely immediate requests) and algorithms applied in the literature. These works suggest that while no one method is universally best, successful dynamic routing systems often share traits: they are fast, flexible, and can handle incremental changes without completely rebuilding the solution from scratch.

As highlighted by Oyola et al. [19], research into the SVRP has expanded significantly, resulting in a diverse array of models that differ based on the source and type of uncertainty, the assumptions about available information, and the complexity of the recourse actions allowed. Their survey classifies SVRP variants into categories such as demand uncertainty, travel time uncertainty, and customer presence uncertainty, and further distinguishes between offline (preplanned) and online (real-time adaptive) solution approaches. This growing body of work reflects the increasing need for vehicle routing systems that are not only cost-efficient but also reliable and adaptable in dynamic, uncertain environments.

As logistics operations become more reliant on real-time data, IoT integration, and responsive service delivery, the role of stochastic models in vehicle routing continues to grow. SVRP thus represents a critical evolution of the classical VRP, bridging the gap between theoretical optimization and operational resilience in uncertain and time-sensitive logistics contexts.

Erdogan and Miller-Hooks [21] formalized the Green vehicle routing problem to account for these constraints, while also considering the strategic placement of alternative energy infrastructure. Further developments such as Electric vehicle routing problem extend this framework by incorporating battery capacity limits, charging durations, and load-dependent energy consumption. The objective functions in these models often go beyond traditional cost or distance minimization

to include metrics such as total energy usage or greenhouse gas emissions. These enhancements reflect a growing recognition that sustainable routing solutions must balance economic efficiency with environmental responsibility, especially as logistics providers adopt low-emission vehicle fleets to comply with urban regulations and corporate sustainability goals.

To address fairness concerns, recent research [34]–[40] has proposed various optimization models that integrate fairness metrics into the route planning process. These approaches aim to avoid scenarios where certain drivers are overburdened while others are underutilized, thereby promoting workforce retention and operational stability. Building on this foundation, Kang [41] introduced a two-sided fairness model for dynamic vehicle routing in ride-sharing systems. This framework simultaneously considers customer fairness—by minimizing average waiting times—and provider fairness—by balancing task assignments among drivers. By embedding fairness directly into the objective function, such models contribute to a more equitable and resilient mobility system, reinforcing the goals of sustainable urban logistics.

In summary, this chapter has laid the groundwork for understanding optimization in the context of complex decision-making problems. It began by defining the general structure of an optimization problem, including its objectives and constraints, and extended this foundation to the domain of multi-objective optimization, where trade-offs between conflicting goals must be carefully managed. The discussion then focused on the VRP, introducing its classical form as well as various real-world extensions, including different service constraints, the problem’s nature (dynamic or static), the level of certainty, and the objective structure—whether single or multi-objective. Special attention was given to dynamic and multi-objective variants, which present additional challenges due to the need for rapid and adaptive solution methods. The concepts introduced in this chapter serve as the theoretical basis for the algorithmic models developed in subsequent chapters.

CHAPTER 2. MULTI-OBJECTIVE DYNAMIC PICKUP AND DELIVERY PROBLEM CONSIDERING TWO-SIDED FAIRNESS

This chapter presents the development of a model called the Multi-objective dynamic pickup and delivery problem with time windows considering two-sided fairness (MODPDPTW), which extends the classical Dynamic pickup and delivery problem (DPDP). Recent approaches to solving the DPDP are reviewed to provide context and highlight current research trends. Building upon these foundations, an ILP formulation for the proposed model is introduced to capture the multi-objective nature and fairness considerations of the problem.

2.1 Dynamic pickup and delivery problem

The DPDP was first introduced by Psaraftis in [42] as a significant extension of the classical PDP and PDPTW. This formulation captures essential features of real-world logistics by incorporating the dynamic, time-dependent nature of request arrivals into the vehicle routing framework. Unlike static models where all customer requests are known in advance, the DPDP allows new pickup and delivery requests to arise during route execution, requiring real-time updates to vehicle schedules. It combines the complexity of managing both pickup and delivery tasks with constraints such as customer time windows and vehicle capacities, making it a more realistic and challenging variant of traditional routing problems. Cai et al. [43] proposed a new, more practical DPDP model in which large volumes of cargo—such as raw materials, semi-finished, and finished products—must be transported among factories during the manufacturing process. Their model introduces additional real-world constraints, including dock availability, time windows, vehicle capacity, and last-in-first-out loading rules.

An illustrative example of the DPDP is shown in Figure 2.1. The left side of the figure represents the static version of the PDP, where all pickup and delivery requests are known in advance and time is not explicitly considered. In contrast, the right side illustrates the dynamic version, where each request is associated with an arrival time and specific time windows for service. This dynamic setting introduces additional complexity, as the routing decisions must be continuously updated in response to newly arriving requests while still satisfying time window and capacity constraints.

This section review recent studies related to the DPDP. These studies are classified into two primary categories: exact methods and approximate methods [44]. Exact algorithms guarantee optimal solutions to the problem; however, their ap-

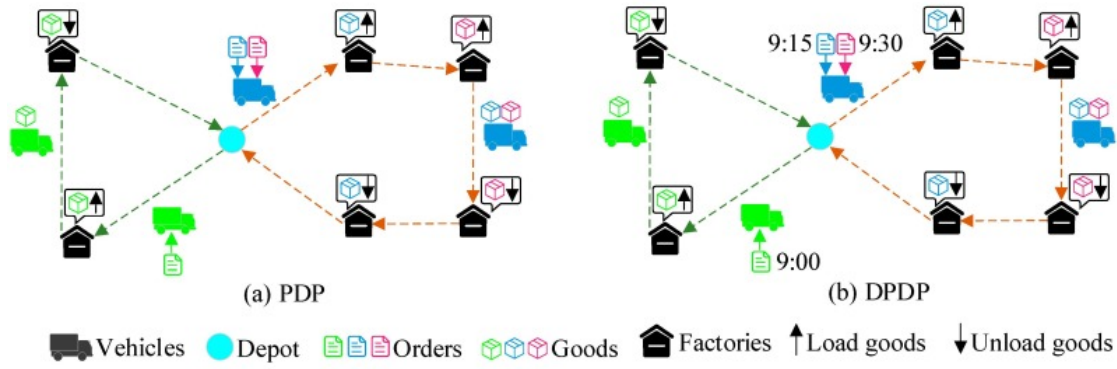


Figure 2.1: Example of dynamic pickup and delivery problem [44].

plicability to dynamic problems is often limited due to their high computational complexity. In contrast, approximate algorithms, while not ensuring optimality, can yield high-quality solutions within a reasonable computational time. Therefore, this thesis further categorize approximate methods into two main approaches: learning-based techniques and metaheuristic algorithms.

Learning-based algorithms for DPDP integrate machine learning techniques with optimization methods to allow systems to learn decision making strategies in dynamic environments, where customer demands and operational conditions evolve over time. In the research [45], the authors introduce the dynamic delivery problem with time constraints, specifically the bi-objective time-dependent dynamic pick up and delivery problem (TDPDPLP). This problem has two objectives: minimizing administrative costs and minimizing total late penalties. The study proposes a comprehensive multi-objective reinforcement learning method to solve the bi-objective TDPDPLP, reducing GPU usage during both training and deployment while producing models that can perform efficiently on various problem instances. In the study [46], the authors focus on minimizing the cost for DPDP. They introduce a hierarchical optimization framework to address large-scale DPDPs more effectively. An upper-level agent is designed to dynamically divide the DPDP into sub-problems of varying sizes, optimizing vehicle routes to achieve globally better solutions. A lower-level agent is also developed to solve each sub-problem efficiently by combining the advantages of classical operational research methods with reinforcement learning-based strategies. Li et al. [47] explore the DPDP with constraints on time, last-in-first-out, and back-to-depot, to minimize transportation costs. They introduce a novel graph-based relational learning method for large-scale industry applications, where vehicle relationships in a dynamic dispatching environment are represented through attention-driven graph convolution. Additionally, they propose a constraint embedding technique to further enhance the efficiency of inference time.

Metaheuristic algorithms have been widely applied to solve various combinatorial optimization problems. The main advantage of these methods is their ability to provide high-quality solutions in an acceptable time. The DPDP has made considerable progress, especially with the incorporation of autonomous delivery robots (ADRs) and electric vehicles to improve logistics efficiency and sustainability. Recent research, such as that by [48], has focused on DPDPs involving ADRs. Their study considers energy consumption and recharging strategies to address the unpredictable nature of order arrivals and peak demand periods. They introduce an e-assignment algorithm that reassigns previously scheduled orders when a new order arrives. In supply chain management, Zhou et. al [49] proposed a memetic algorithm combining genetic algorithms and local search strategies to solve real-world DPDP. Their extensive experiments demonstrated the effectiveness of this approach in minimizing delivery costs and handling periodic order releases. The authors in [50] introduce a decomposition-based multiobjective evolutionary algorithm to address the complexity of DPDPs with constraints like time windows and vehicle capacity. This approach is a hybrid of multiobjective optimization and tabu search to enhance solution diversity and avoid local optima.

The study [51] introduces a variant of the DPDP that incorporates electric vehicles and time windows, addressing issues like task allocation, routing, and queue scheduling at service sites. A mixed integer linear programming (MILP) model is developed to minimize travel distance and queue time. Additionally, the authors propose an adaptive hybrid neighborhood search algorithm to solve large-scale instances. The experimental results demonstrate the algorithm's effectiveness in finding competitive solutions. In [41], the authors focus on improve the fairness between service provider and customer in DVRP. However, they have main limitations as not considering multi-objective optimization to optimize effectively all the objectives, the encoding scheme and crossover, mutation is still simple, not focus on constrained problem.

In vehicle routing systems involving both providers and customers, fairness is a critical factor for ensuring system sustainability. A lack of fairness can lead to the departure of both providers and customers, thereby undermining the longevity of the system. To establish a balanced system that incorporates fairness, recent studies [34]–[40] have developed methods aimed at optimizing utility while enhancing fairness among service providers. However, these studies have typically addressed fairness from a single perspective, focusing exclusively on either providers or customers. Recent research [41] has developed a system that considers two-sided fairness for both customers and providers by employing a Genetic algorithm (GA)

specifically designed to optimize all relevant objectives. Despite this advancement, multi-objective approaches that leverage techniques such as Multi-objective evolutionary algorithm (MOEA) to enhance the search for optimal solutions have not been thoroughly explored. These approaches hold the potential to simultaneously optimize multiple conflicting objectives, thereby providing a more comprehensive solution that balances fairness and utility more effectively.

To address these limitations, the thesis introduce the MODPDPTW, wherein a vehicle system serves customer requests that arrive randomly and unpredictably. This problem encompasses multiple objectives, including minimizing consumption costs, improving service quality, and balancing demand among customers and vehicles, making it a nonconvex and NP-hard challenge. In this context, the proposed method employs a multi-objective framework that integrates fairness considerations for both providers and customers within the vehicle routing system. By utilizing advanced multi-objective optimization techniques, this work enhances the search process for optimal solutions, ensuring a more equitable distribution of resources and services. This approach overcomes the limitations of previous single-objective and two-sided fairness methods, providing a more robust and balanced solution for sustainable vehicle routing systems.

2.2 Formulation of multi-objective dynamic pickup and delivery problem considering two-sided fairness

This section formulates the MODPDPTW problem to schedule a fleet of vehicles to service customer demands in a dynamic environment where demands arrive randomly during the system's operation. The goal is to enhance the provider's revenue while ensuring fairness by balancing the workload among vehicles and waiting times among customers.

The real-world map, where a typical MODPDPTW operates, is represented as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Here, \mathcal{V} denotes the set of all nodes (locations), and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of directed edges (roads connecting different locations). A directed edge from position i to j , denoted as $(i, j) \in \mathcal{E}$, exists if there is a path enabling travel from i to j . Then, d_{ij} is the distance of the path that the vehicle travels from i to j . Let V be a set of K homogeneous vehicles, each with capacity Q , departing from and returning to a common depot daily. Denote T as the system's operation time. At any time $t \in T$, each vehicle $v \in V$ is defined by its location l_v^t , and a set of customer demands R^t arises at different locations. Each request $r \in R^t$ is represented by a tuple $\langle l_r^p, l_r^d, q_r, sp_r^b, sp_r^e, sd_r^b, sd_r^e \rangle$, where l_r^p and l_r^d are the pickup and delivery locations, q_r is the load of the cargo, $[sp_r^b, sp_r^e]$ is the soft

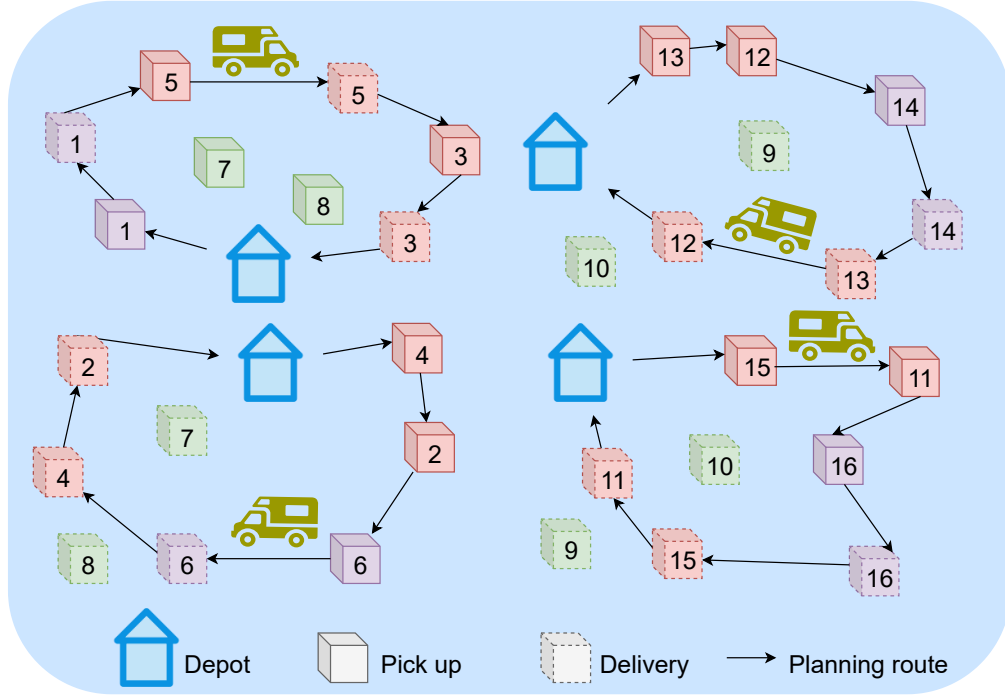


Figure 2.2: Example of solution to MODPDPTW.

pickup time window, and $[sd_r^b, sd_r^e]$ is the soft delivery time window. Regarding the soft time windows, vehicles arriving early wait without penalty, but arriving after the window incurs a delay penalty. Once a vehicle picks up the cargo for request r , it must complete the corresponding delivery. However, vehicles can sequence multiple pickups before subsequent deliveries, provided the total load does not exceed their capacity. This part defines the following variables to monitor the scheduling of vehicles:

- x_r^v is a binary variable that equals 1 if demand r is served by vehicle v .
- $y_{r,ij}^v$ is a binary variable that equals 1 if vehicle v traverses arc (i, j) while serving demand r .
- τ_r^p represents the time when the cargo for demand r is picked up.
- τ_r^d represents the time when the cargo for demand r is fully delivered.

Energy consumption. The MODPDPTW involves a fleet of fuel-powered vehicles that depart from a depot to serve customers. Based on the mechanical power [22], the constant mechanical power P_{ij}^v for vehicle $v \in V$ traveling edge $(i, j) \in \mathcal{E}$ without considering the gradient of the terrain is calculated as follows:

$$P_{ij}^v = \frac{1}{2} \cdot c_d \cdot \rho \cdot S \cdot \gamma^3 + (m_v + \tilde{Q}_{ij}^v) \cdot g \cdot c_r \cdot \gamma, \quad (2.1)$$

where c_d is the aerodynamic drag coefficient, ρ is the air density, S is the frontal area of the vehicle, g is the gravitational constant, c_r is the rolling friction coefficient, m_v is the curb weight of vehicle, and γ is the speed of the vehicle. Additionally, \tilde{Q}_{ij}^v is the load of the vehicle v in edge (i, j) .

The fuel consumption of the vehicle $v \in V$ when traversing arc $(i, j) \in \mathcal{E}$ is denoted by L_{ij}^v and is calculated as follows:

$$L_{ij}^v = \frac{\xi}{\kappa \cdot \psi} \cdot \left(\omega \cdot R + \frac{P_{ij}^v}{\eta} \right) \cdot \frac{d_{ij}}{\gamma}, \quad (2.2)$$

where ξ is the fuel-to-air mass ratio, κ is the heating value of fuel, ψ is the fuel rate conversion factor, and ω is the engine friction factor. Additionally, R and η represent the technical parameters of the engine and the drive train efficiency, respectively.

Therefore, energy consumption costs are associated with fuel usage during pickups and deliveries. Based on the referenced energy consumption models, the energy consumption costs EC for the assigned vehicles are calculated as:

$$f_1 = \sum_{t \in T} \sum_{r \in R^t} \sum_{v \in V} \sum_{(i,j) \in \mathcal{E}} y_{r,ij}^v L_{ij}^v. \quad (2.3)$$

Waiting time. Furthermore, the quality of the service is assessed based on the response time to requests. The total waiting time for each demand is defined as follows:

$$WT_r = \max(\tau_r^p - sp_r^e, 0) + \max(\tau_r^d - sd_r^e, 0), \forall r \in R^t, t \in T, \quad (2.4)$$

where $\max(\tau_r^p - sp_r^e, 0)$ and $\max(\tau_r^d - sd_r^e, 0)$ represent the waiting time of the request before being picked up and delivered, respectively. The system's objective is to minimize the waiting time, ensuring that customers receive prompt attention immediately after making a request. This is particularly significant in contexts where the quality of goods deteriorates over time. Therefore, this research optimizes the maximum waiting time of a request, calculated as follows:

$$f_2 = \max_{r \in R^t, t \in T} WT_r. \quad (2.5)$$

Customer-based Fairness. Fairness is increasingly important in real-world vehicle routing and scheduling. Fairness is addressed from two perspectives: customer-based, focusing on balancing each customer's waiting time, and vehicle-based,

aiming to distribute the workload among vehicles evenly. Each customer demand has two-time windows corresponding to the pickup and delivery moments. When a vehicle arrives at the pickup or delivery location before the start of the time window, it waits until the task can be performed. However, if the vehicle arrives after the end of the time window, the customers must wait, leading to a decrease in service quality. Then, customer-based fairness is quantified by the variability in wait times across all requests, as follows:

$$f_3 = \sqrt{\frac{1}{n} \sum_{t \in T} \sum_{r \in R^t} \left(WT_r - \frac{1}{n} \sum_{t \in T} \sum_{r \in R^t} WT_r \right)^2}, \quad (2.6)$$

where $n = \sum_{t \in T} |R^t|$ is the number of customer demands.

Vehicle-provider Fairness. The workload of each vehicle is calculated based on the total distance traveled by that vehicle. Let DT_v represent the workload of vehicle v , which is defined as follows:

$$DT_v = \sum_{t \in T} \sum_{r \in R^t} \sum_{(i,j) \in \mathcal{E}} y_{r,ij}^v d_{ij}, \forall v \in V. \quad (2.7)$$

Instead of focusing solely on profit or revenue, model aims to prevent any particular vehicle from having an excessive workload, thereby achieving a more balanced allocation of trips. Therefore, vehicle-based fairness is evaluated based on the disparity in the workload of the vehicles, calculated as follows:

$$f_4 = \sqrt{\frac{1}{K} \sum_{v \in V} \left(D_v - \frac{1}{K} \sum_{v \in V} D_v \right)^2}. \quad (2.8)$$

$$\text{minimize } \{f_1, f_2, f_3, f_4\}, \quad (2.9a)$$

$$\text{subject to: } \sum_{v \in V} x_r^v = 1, \forall r \in R^t, t \in T, \quad (2.9b)$$

$$y_{r,ij}^v \leq x_r^v, \forall (i,j) \in \mathcal{E}, r \in R^t, t \in T, v \in V, \quad (2.9c)$$

$$\tilde{q}_{ij}^{v,t} \leq Q, \forall (i,j) \in \mathcal{E}, t \in T, v \in V, \quad (2.9d)$$

$$\tau_r^p \geq sp_r^b, \forall r \in R^t, t \in T, \quad (2.9e)$$

$$\tau_r^d \geq sd_r^b, \forall r \in R^t, t \in T. \quad (2.9f)$$

The aim of (2.9a) is to minimize energy consumption costs and the maximum time waiting among customers while maintaining a balance between customer waiting times and workload distribution across vehicles. Constraint (2.9b) ensures that each customer is served exactly once. Constraint (2.9c) stipulates that vehicle v only fulfills request r if r is assigned to v . Constraint (2.9d) ensures that the load on each vehicle does not exceed its capacity at any point in time. Constraints (2.9e) and (2.9f) guarantee that service times, including pickups and deliveries, adhere to the beginning of each customer's designated time window.

In conclusion, this chapter has introduced the Multi-objective dynamic pickup and delivery problem considering two-sided fairness, an extension of the classical DPDP that incorporates fairness and multiple objectives. A review of recent approaches to solving DPDP was provided to establish the research context and identify prevailing solution strategies. Building on this foundation, an ILP formulation was presented to formally define the proposed model, capturing both the dynamic nature of the problem and the dual fairness objectives. This formulation serves as a critical step toward designing effective algorithms capable of addressing the complexities of real-world, fair, and dynamic pickup and delivery systems.

CHAPTER 3. PARETO FRONT GRID GUIDED MULTI-OBJECTIVE OPTIMIZATION CONSIDERING TWO-SIDED FAIRNESS

This chapter presents an effective method for solving the MODPDPTW, named Pareto front grid-guided multi-objective optimization with two-sided fairness (PFG-2F). The approach integrates Pareto front grid (PFG) generation with fairness heuristics and an ordering rule based on time window urgency. To explore the solution space, it applies combinatorial operators including Partially mapped crossover (PMX) and Swap mutation.

3.1 Overview

The overview methods, including all components are described in Figure 3.1. To handle and optimize all dynamically appearing requests, each request is stored in a request database as it arrives. At every decision timestep, the system processes all currently unserved requests. The algorithm takes as input the pickup and delivery locations of these requests and initializes a population by randomly shuffling the request sequences. Each individual in the population is then evaluated, and non-dominated sorting is applied to extract a Pareto front for that generation.

If the stopping condition has not been met, the algorithm proceeds to generate a Pareto front grid (PFG) using the current ideal point z^* and nadir point z^{nad} . This grid guides the selection process, after which crossover and mutation operators—guided by selected heuristics—are applied to generate offspring. The resulting offspring are then added to the population for the next iteration.

Once the stopping condition is satisfied (i.e., max generation is reached), a representative solution is selected for execution. This solution typically balances multiple objectives—such as minimizing energy cost, reducing customer waiting time, or ensuring fairness among customers and service providers. In most cases, a knee point is selected as the final solution, as described in Subsection 1.2.1. The selected solution is then passed to the system controller, which routes the vehicles accordingly, completing the operations for the current timestep.

3.2 Individual representation

In [41], the Leader-based random keys encoding scheme (LERK) is introduced to represent complete solutions. Specifically, leader keys determine the sequence of service providers, and the subsequent assignment of all requests follows this ordering. This random key encoding strategy has been shown to be effective in generating diverse solutions within the objective space. In order to promote equitable

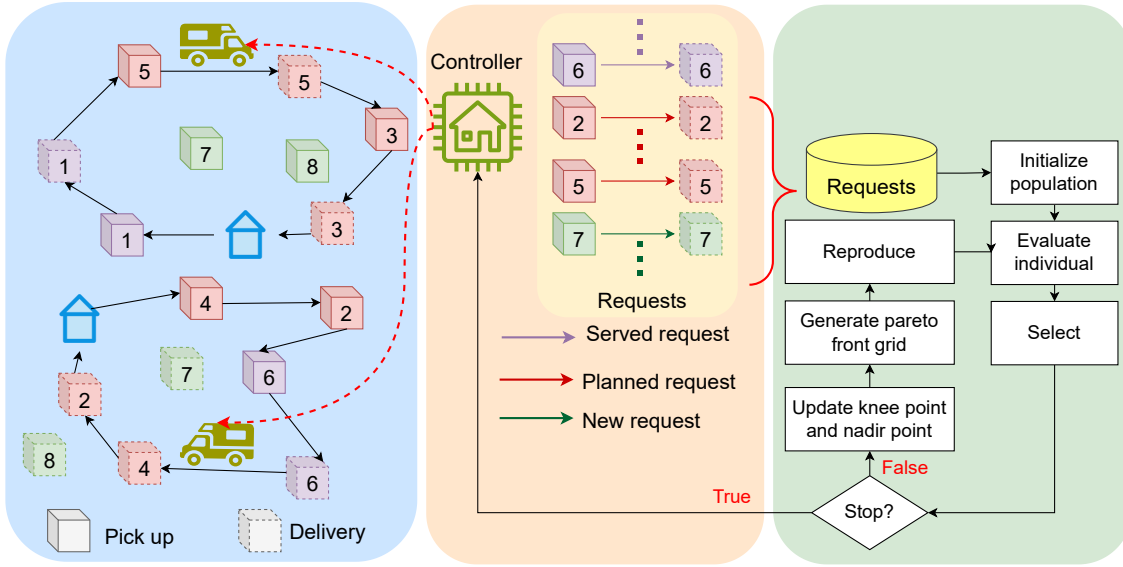


Figure 3.1: The overview of the proposed algorithm.

distribution of workload among vehicle providers, this thesis propose an enhanced version of LERK that incorporates heuristic initialization. Figure 3.2 demonstrates an example of individual representation. Our methodology comprises two main components: (1) *Fairness heuristics initialization* and (2) an *Urgent time ordering rule*. The former ensures a balanced allocation of requests across all vehicles, while the latter reorders requests based on the urgency of their time windows. By initially distributing requests equally among vehicles and then prioritizing more urgent time windows, the approach fosters fairness among providers and helps ensure that tightly constrained time windows are served promptly.

3.2.1 Fairness heuristics initialization

Consider a system with K vehicles, each operated by a distinct service provider and collectively responsible for fulfilling a set of customer requests. In multi-objective vehicle routing problems—especially those involving dynamic and evolving demand—ensuring fairness in workload distribution across vehicles is not only a practical concern but also a key sustainability objective. Unbalanced assignments, where some vehicles are overloaded while others are underutilized, can lead to provider dissatisfaction and reduce long-term system stability.

To address this challenge, the proposed method introduces a fairness-enhancing heuristic at the initialization stage of the algorithm. The core idea is to assign customer requests to vehicles in a balanced way, such that each vehicle is initially responsible for approximately the same number of customers or requests. This ensures a fair starting point before further refinement is performed through the evolutionary process.

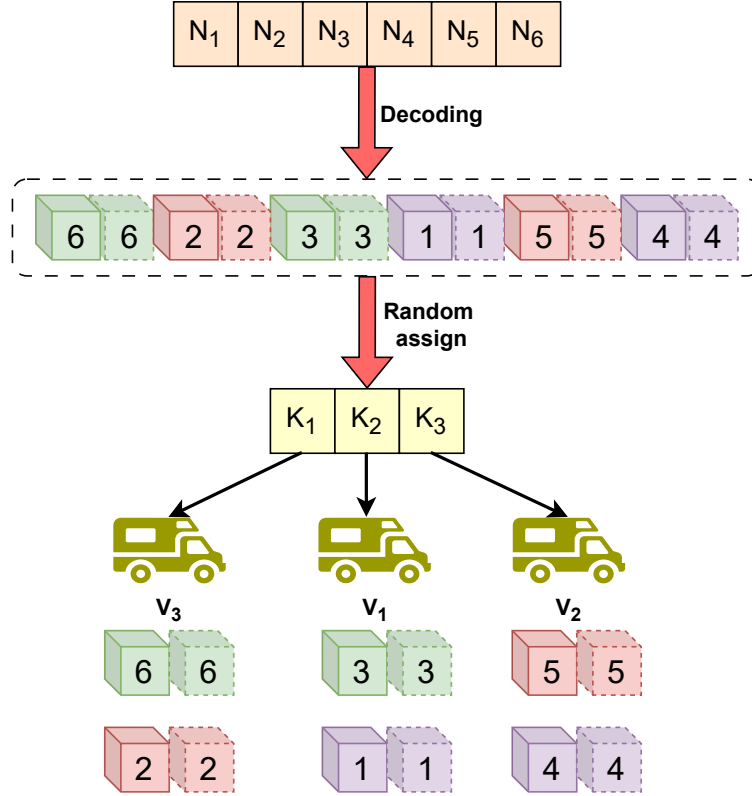


Figure 3.2: Individual representation.

Although this initial assignment aims to be equitable, it is not necessarily optimal in terms of routing efficiency or other objectives (e.g., energy consumption, waiting time). Therefore, the solution is further improved using evolutionary operations such as crossover and mutation, which explore the search space while preserving the fairness structure as much as possible.

The pseudocode in Algorithm 1 outlines the initial fairness-based assignment strategy among all vehicles.

Algorithm 1 Fair request assignment among vehicles

Input: Set of customer requests \mathcal{R} , number of vehicles K

Output: Initial assignment of requests to vehicle routes $\{V_1, V_2, \dots, V_K\}$

- 1: Initialize empty vehicle routes: $\mathcal{V} \leftarrow [V_1, V_2, \dots, V_K]$;
 - 2: Shuffle the request set \mathcal{R} randomly to ensure fairness;
 - 3: Initialize counter $i \leftarrow 0$;
 - 4: **for all** $r \in \mathcal{R}$ **do**
 - 5: $k \leftarrow (i \bmod K) + 1$;
 - 6: Assign request r to vehicle route V_k ;
 - 7: $i \leftarrow i + 1$;
 - 8: **end for**
 - 9: **return** \mathcal{V}
-

This heuristic ensures that each vehicle is initially assigned roughly $\lceil |\mathcal{R}|/K \rceil$ requests. Such an approach improves fairness among providers from the outset, laying a strong foundation for later optimization steps. As the evolutionary process proceeds, these assignments are modified using genetic operators such as crossover and mutation, which enable exploration of diverse routing configurations while maintaining workload balance constraints when necessary.

This fairness-first initialization plays a crucial role in the proposed multi-objective framework by improving both social equity among service providers and the quality of solutions obtained throughout the optimization process.

3.2.2 Urgent time ordering rule

In dynamic vehicle routing problems with time window constraints, each customer request is decomposed into two service nodes: a pickup node and a delivery node. At any decision time t , a vehicle $k \in K$ is assigned a sequence of such service nodes, denoted by $\mathbf{p}_k \subseteq N^t$, where N^t represents the set of all active pickup and delivery nodes at time t .

To improve the feasibility of time-constrained routes, urgent time ordering rule is applied to each vehicle's assigned sequence, illustrated in Figure 3.3. This heuristic prioritizes nodes whose service time windows are close to expiring.

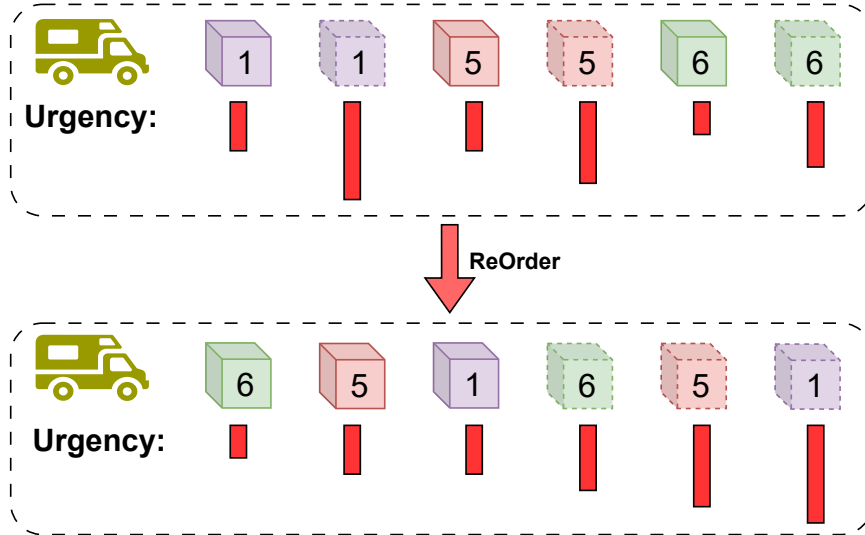


Figure 3.3: Urgent time ordering rule.

For each node $i \in \mathbf{p}_k$, let e_i denote the end of its associated time window. The urgency of node i is defined as the remaining time from the current time t to the end of its time window:

$$\text{urgency}(i) = e_i - t,$$

where smaller urgency values indicate that the node is approaching the end of its allowable service period and should therefore be serviced earlier.

Using this urgency metric, the assigned node sequence \mathbf{p}_k is reordered in ascending order of urgency to form a new prioritized sequence $\tilde{\mathbf{p}}_k$, defined as:

$$\tilde{\mathbf{p}}_k = \text{ReOrder}(\mathbf{p}_k, t), \quad (3.1)$$

where $\text{ReOrder}(\cdot)$ is a sorting function applies urgency criterion to individual nodes.

Algorithm 2 ReOrder nodes based on time window urgency

Input: Assigned node sequence \mathbf{p}_k for vehicle k , current time t

Output: Reordered node sequence $\tilde{\mathbf{p}}_k$

- 1: Initialize empty list $\tilde{\mathbf{p}}_k \leftarrow []$;
- 2: **for** each node $i \in \mathbf{p}_k$ **do**
- 3: Compute urgency score:

$$\text{urgency}(i) \leftarrow e_i - t;$$

- 4: **end for**
 - 5: Sort \mathbf{p}_k in ascending order of $\text{urgency}(i)$;
 - 6: Assign the sorted list to $\tilde{\mathbf{p}}_k$;
 - 7: **return** $\tilde{\mathbf{p}}_k$;
-

3.3 Pareto front grid generation

Pareto front grid guided multi-objective evolutionary algorithm (PFG-MOEA), first proposed in [52], has proven effective for identifying high-quality solutions in multi-objective optimization. The strategy leverages Pareto Front to improve the searching approach to guide the evolutionary process by partitioning the objective space into grids and retaining leading individuals within each grid cell. This method focuses the search on the most promising regions of the objective space, thereby enhancing both the quality of the solutions and the efficiency of the search.

The algorithm employs two well-established reference points in the multi-objective space: the ideal point z^* and the nadir point z^{nad} . Both points are updated at each evolutionary iteration. The ideal point represents the optimal values for each objective, highlighting solutions on the Pareto front that demonstrate significant trade-offs among objectives. In contrast, the nadir point corresponds to the worst objective values within the Pareto optimal set. These reference points are utilized in each generation to construct the PFG, as illustrated in Figure 3.4. The generation of PFG is detailed in Algorithm 3.

Algorithm 3 Generate PFG

Input: population pop , the number of segments GK , the number of objectives m , the idea point $z^* = [z_1^*, \dots, z_m^*]$, the nadir point $z^n = [z_1^{nad}, \dots, z_m^{nad}]$, a small positive value σ

Output: Pareto front grid $\{PFG_j^i\}, \forall j = 1, \dots, GK, i = 1, \dots, m$

```

1: for  $j \leftarrow 1$  to  $m$  do
2:    $d_j \leftarrow (z_j^{nad} - z_j^* + 2\sigma) / GK$ ;
3: end for
4: for all  $x \in \text{pop}$  do
5:   for  $j \leftarrow 1$  to  $m$  do
6:      $\text{Grid}_j(x) \leftarrow \left\lceil \frac{f_j(x) - z_j^* + \sigma}{d_j} \right\rceil$ ;
7:   end for
8: end for
9: for  $i \leftarrow 1$  to  $m$  do
10:  for  $j \leftarrow 1$  to  $GK$  do
11:     $S_i(j) \leftarrow$  individuals depending on the  $j$ -th segment of the  $i$ -th objective;
12:     $g_{\min} \leftarrow \min\{\text{Grid}(S_i(j))\}$ ;
13:    for all  $x \in S_i(j)$  do
14:      if  $\text{Grid}_j(x) = g_{\min}$  then
15:         $PFG_j^i \leftarrow PFG_j^i \cup \{x\}$ ;
16:      end if
17:    end for
18:  end for
19: end for

```

3.4 Reproduction

To reproduce the original solution and seek improved outcomes, this thesis incorporates a reproduction strategy consisting of selection, crossover, and mutation tailored for permutation-based representations. The selection process is based on Pareto front grid generation, as described in Subsection 3.4.1, to balance convergence and diversity in the population. Crossover is performed using PMX, which preserves permutation validity while combining genetic material from two parent solutions, as detailed in Subsection 3.4.2. Mutation is applied through a swap mutation operator, which randomly selects two positions in a permutation and exchanges their values to introduce local variation and maintain diversity, as explained in Subsection 3.4.3.

3.4.1 Selection

PFG is employed to generate grids that partition the objective space, thereby grouping solutions into different regions. Within each grid, selection is conducted via non-dominated sorting, and the ideal point z^* is then computed. Any solutions lying farthest from the hyperplane defined by z^* are removed. Then, two solutions

are randomly chosen from the mating pool P to serve as parents. To foster diversity both within the same PFG and across the entire population, crossover is governed by a probabilistic rate ϵ , as shown in Equation 3.2. With probability ϵ , the parents are drawn from consecutive grids (i.e., $PFG_i^j \cup PFG_{i+1}^j$); otherwise, they are taken from the entire population pop . Therefore, P is define as

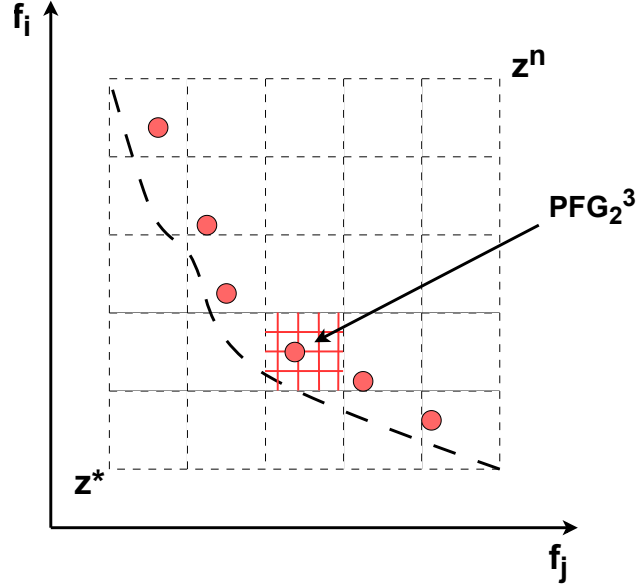


Figure 3.4: Pareto front grid generation.

$$P = \begin{cases} PFG_i^j \cup PFG_{i+1}^j, & \text{if } \text{rand} < \epsilon, \\ \text{pop}, & \text{otherwise.} \end{cases} \quad (3.2)$$

where rand is a random variable following a uniform distribution range $[0, 1]$.

3.4.2 Crossover

This thesis employs the crossover operator PMX, a recombination method specifically designed for genetic algorithms operating on permutation-based representations, as illustrated in Figure 3.5. The PMX operator is particularly well-suited for combinatorial optimization problems in which each solution is naturally expressed as a sequence or permutation, such as travelling salesman problem, vehicle routing, scheduling, or sequencing tasks.

The purpose of PMX is to generate offspring that inherit genetic material from two parent solutions while preserving the permutation structure—i.e., ensuring each gene appears exactly once. This is especially important in routing problems where each pickup–delivery request must be served exactly once.

The PMX procedure consists of the following steps:

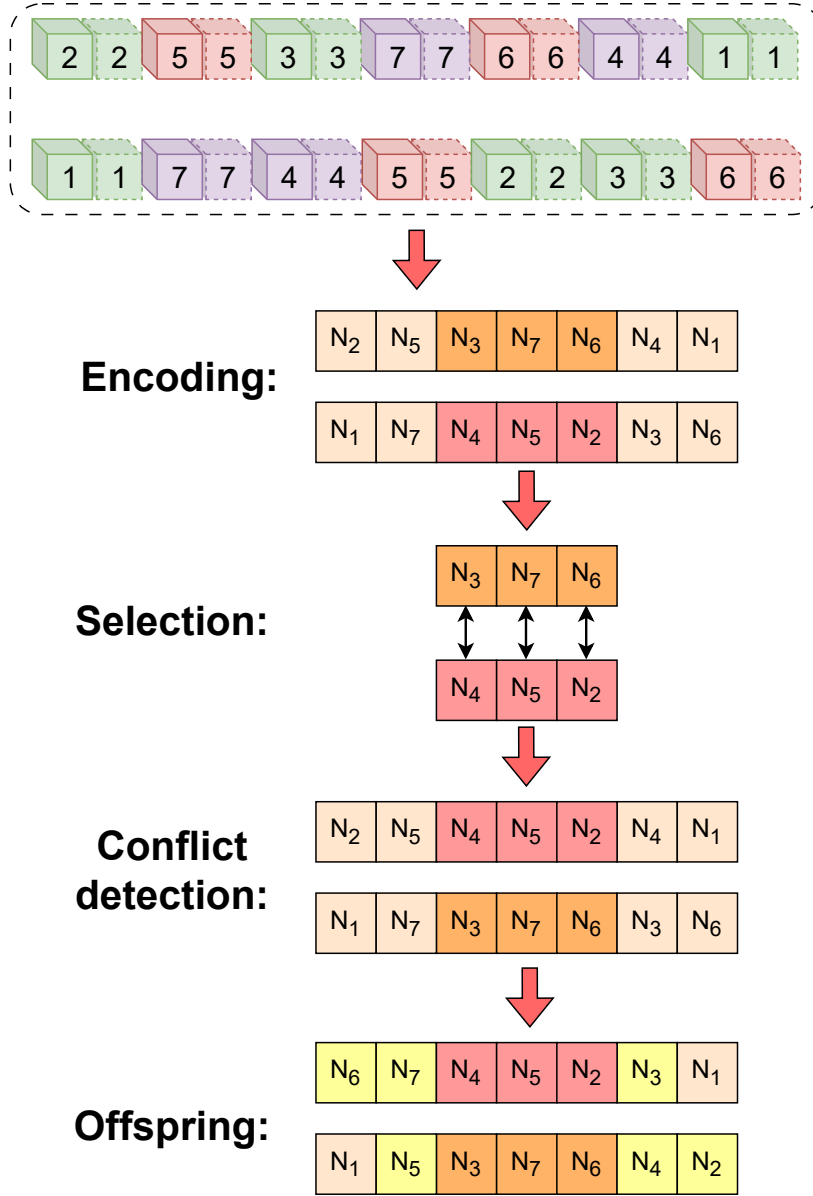


Figure 3.5: An example of partially mapped crossover.

1. **Permutation encoding:** Each chromosome encodes a complete route as a permutation of request indices, where each index corresponds to a pickup–delivery pair. This representation is appropriate for applying PMX.
2. **Crossover point selection:** Two crossover points are selected uniformly at random along the length of the parent chromosomes, defining a contiguous mapping section.
3. **Mapping section inheritance:** The subsequence between the crossover points from the first parent is copied directly into corresponding positions of offspring.

4. **Conflict resolution via mapping:** For each gene in the same section of the second parent that conflicts with those already copied (i.e., duplicates), a mapping is created. This mapping is used to resolve conflicts by identifying substitute genes that maintain permutation validity.
5. **Completion of the offspring:** The remaining positions in the offspring are filled using genes from the second parent. If a gene is not already present in the offspring, it is inserted directly; otherwise, the mapping is applied recursively until a valid gene is found.

The PMX operator thus ensures that the resulting offspring is a valid permutation, preserving both relative order and positional information from the parents. This balance of structure preservation and genetic diversity makes PMX highly effective for evolutionary approaches to permutation-constrained problems.

3.4.3 Mutation

Swap mutation is a simple yet effective genetic operator used to introduce variation in solution populations during the optimization process. In the context of MODPDPTW, each solution is typically represented as a sequence of pickup and delivery nodes that must respect precedence constraints—i.e., each pickup must occur before its corresponding delivery—and vehicle capacity limits.

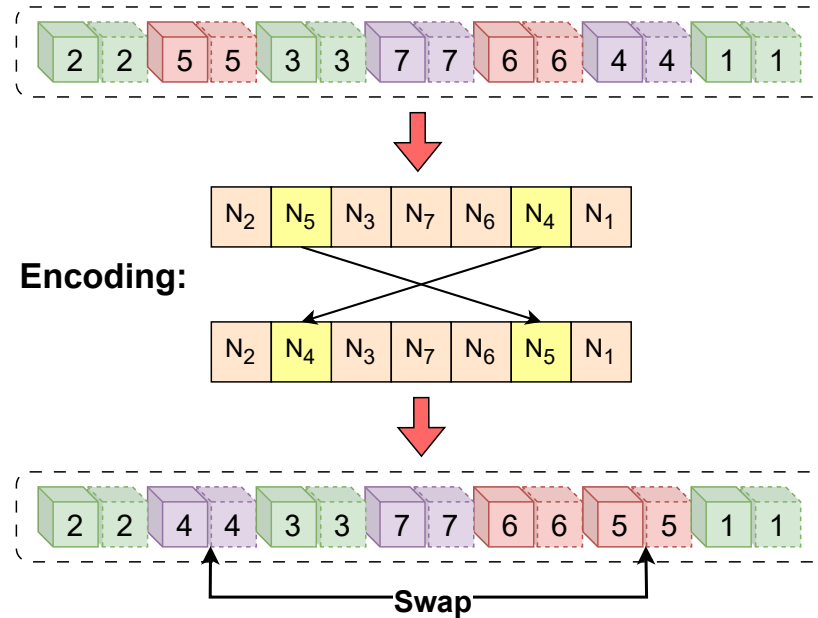


Figure 3.6: An example of swap mutation.

The Swap mutation operator selects two distinct positions in the solution sequence and exchanges their values, as represented in Figure 3.6. However, due to

the precedence requirements in MODPDPTW, a direct swap may result in infeasible solutions. To maintain feasibility, the mutation process is constrained as follows:

- A pickup node can only be swapped with another pickup node, and similarly, a delivery node with another delivery node.
- Swapping a pickup and its corresponding delivery is not allowed unless their relative order is preserved.
- After mutation, the solution is checked for precedence, capacity and time windows feasibility; if violated, the swap is discarded or repaired.

By applying these rules, Swap mutation helps explore the solution space without disrupting the problem's structural constraints. It enhances diversity in the population and aids in escaping local optima during the evolutionary search.

In conclusion, this chapter has presented an effective solution method for MODPDPTW, called PFG-2F. The proposed approach combines PFG generation with fairness-aware heuristics and a time window urgency-based ordering rule to guide the search toward high-quality and balanced solutions. To explore the solution space effectively, the method employs combinatorial genetic operators, specifically PMX for recombination and swap mutation for local diversification. Together, these components form a robust framework for addressing the complex, multi-objective, and fairness-sensitive nature of MODPDPTW.

CHAPTER 4. SIMULATION EXPERIMENTS

In this chapter, experiments are conducted on multiple datasets, including large-scale instances with a high number of requests to simulate real-world scenarios. The proposed algorithm is compared against state-of-the-art MOO algorithms using the HV and IGD metrics. The results demonstrate the effectiveness and real-time adaptability of the proposed method.

4.1 Experimental settings

This section describe the experimental setup used to evaluate the proposed approach. Specifically, this section presents details about the dataset, the runtime environment in which the experiments were conducted, and the baseline methods used for comparison. This provides a clear context for interpreting the results in next section.

4.1.1 Datasets

The dataset is developed based on PDPTW [53], ensuring authenticity and diversity to reflect real-world scenarios. It consists of 180 instances tailored for time slot routing and is categorized into three types: 100 requests, 200 requests, and 400 requests. Each instance is named using the format `[dis][category]_[type]-[index]`, where `dis` indicates the customer distribution type of the instance, including R for random, C for clustered, and RC for combined, `category` represents the difficulty level based on time windows (1 for narrower and 2 for wider and more flexible), and `type` denotes the number of locations in the dataset (for example, a 100 request instance includes 200 locations). The number of vehicle-providers is scaled up to fit for each dynamic circumstances in the dataset. Hence, this work creates 18 different distribution to evaluate the effectiveness of all algorithms. Examples of 100 request instance with clustered, random and combined distributions are illustrated at Figure 4.1.

All parameters used in Section 2.2 are set up to be adapted to real world scenarios, this work uses the configuration to calculate energy consumption as the setup described in [22]. The problem parameters are set as follow: $c_d = 0.7$, $c_r = 0.01$, $\xi = 1$, $\eta = 0.36$, $m_v = 3.2$ tons, $\psi = 737$ liters/gram, $\gamma = 40$ km/h, $\kappa = 44$ kJ/gram, $g = 9.81$ m/s², $\rho = 1.20$ kg/m³, $\omega = 0.2$ kJ/(rev · liter), $S = 3.912$ m², $R = 165$ liters · rev/s. These values are used consistently in all subsequent calculations.

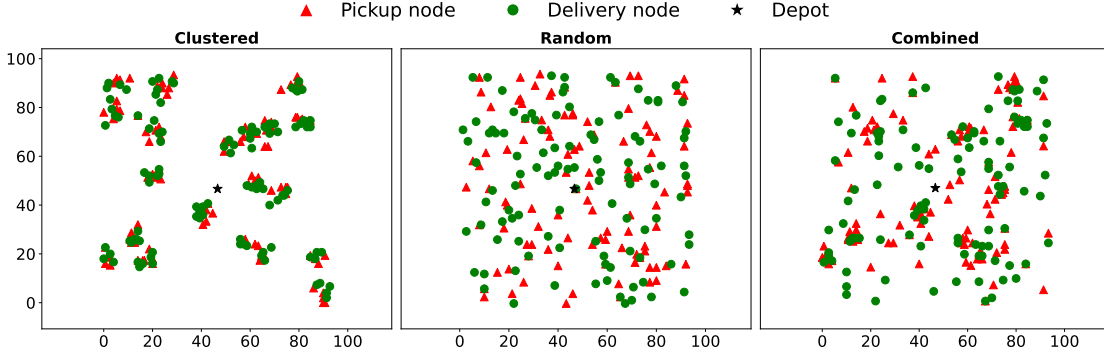


Figure 4.1: Examples of 100 requests instances with different distributions.

4.1.2 Runtime environment

All experiments in this thesis were conducted using Kaggle’s cloud-based computing environment. The notebook environment provided access to a CPU with 4 cores and 16 GB of RAM, which was sufficient for executing the proposed models and evaluating their performance on benchmark datasets. This setup allowed for reproducible and scalable experimentation without the need for dedicated local hardware resources.

4.1.3 Compared algorithms

This thesis evaluates the performance of PFG-2F against the original PFG-MOEA [52] and other state-of-the-art multi-objective baseline algorithms, including NSGA-II [54], MOEA/D [6], and MOEA/D-DE [55]. To assess the effectiveness of the Random-Key encoding scheme, the thesis develops a variant of PFG-2F named PFG-2F-PM, which employs a permutation encoding scheme combined with Partially Mapped Crossover (PMX) [56]. This variant is compared to the proposed algorithm, referred to as PFG-2F-RK, to determine the impact of the encoding strategy on overall performance.

For a fair and consistent evaluation, all compared MOO algorithms are configured with the same initial population, the same number of evaluations per generation, and an equal number of generations. This setup ensures a balanced comparison by eliminating advantages that could arise from randomly favorable initial populations, which may lead to faster convergence or better early results. By standardizing these parameters, the evaluation focuses solely on the effectiveness of each algorithm’s optimization strategy rather than variability in initialization.

To evaluate effectiveness, the HV and IGD metrics (as defined in Subsection 1.2.1) are computed separately for each instance and averaged over 10 independent runs. Specifically, for HV, the reference point is set as the worst objective

values observed across all solutions for each instance. For IGD, the reference Pareto front is approximated by collecting all non-dominated solutions found by all algorithms across all runs.

In addition, this thesis implements the FairGA algorithm proposed by Kang et al. [41], which incorporates two-sided fairness considerations in solving DVRP, to further benchmark the proposed method under fairness-aware settings.

4.2 Experimental results

This section explains how parameters in proposed algorithm are selected based on effectiveness. The comparison with baseline methods is organized into two categories: MOO approaches, evaluated using HV and IGD metrics, and Single-objective optimization (SOO) approaches, assessed by the percentage improvement in each individual objective.

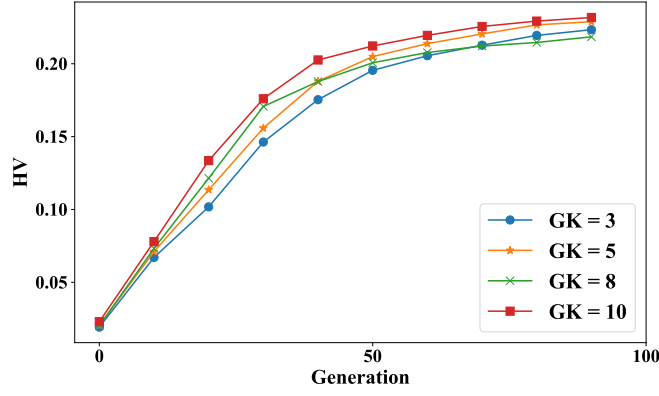
4.2.1 Parameter tuning

To investigate the impact of the parameter GK on the performance of PFG in the objective and solution space decomposition process, this section conduct a fine-tuning experiment. The parameter GK determines the number of grid segments used to partition the objective space, which directly affects how the PFG are constructed. Figure 4.2 presents the convergence trends in terms of hypervolume (HV) for different GK values. From the results, we observe that $GK = 5$ consistently yields the best overall performance across most test instances. This balance stems from the way GK influences the granularity of the decomposition:

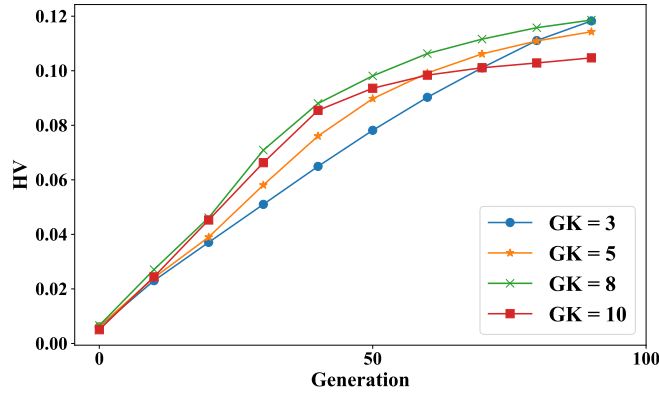
When GK is set to a small value, the number of PFGs is limited, resulting in each grid region covering a broader portion of the objective space. Consequently, more solutions fall into each region, increasing the competition during environmental selection. Only a few solutions with superior convergence are retained, which reduces diversity across the population due to high selection pressure.

Conversely, using a large GK value results in a finer grid, generating a greater number of PFGs. Each region then contains fewer solutions, typically close to one another in objective space. Although this setup improves population diversity by spreading solutions across many small regions, it weakens convergence since many selected solutions may be similar and suboptimal in terms of quality.

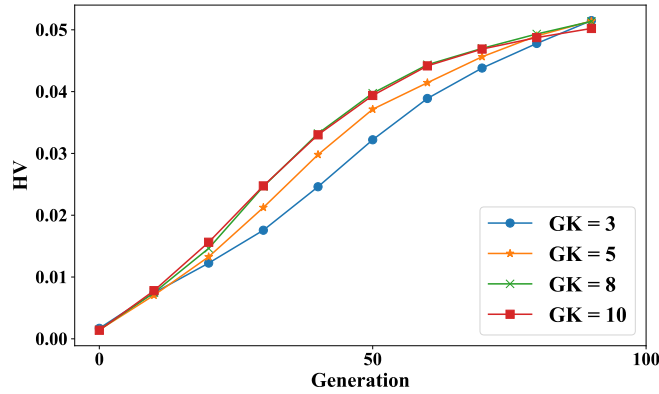
Thus, selecting an appropriate GK is essential for maintaining a good trade-off between convergence and diversity. Our findings suggest that $GK = 5$ strikes this balance effectively, leading to improved overall performance.



(a) GK performance in 100 requests.



(b) GK performance in 200 requests.



(c) GK performance in 400 requests.

Figure 4.2: Parameter GK performance analysis.

4.2.2 Compare with MOO algorithms

Table 4.1 demonstrates the average HV values of the evaluated algorithms across various scenarios. The results indicate that the proposed algorithm consistently outperforms existing approaches in nearly all instances. Among the compared methods, the algorithms incorporating the Pareto front grid and equal workload distribution among vehicles exhibit substantial improvements over the remaining

approaches. Notably, PFG-2F with the Random-key encoding scheme achieves a marginal improvement over the Permutation encoding scheme. This improvement can be attributed to the inherent limitations of genetic operators in the Permutation encoding scheme, which are less effective in the reproduction process compared to the equal workload division strategy. In contrast, the Random-key encoding scheme offers greater flexibility and a more effective exploration of the solution space. Although PFG-MOEA follows an evolutionary trend guided by the Pareto front grid, fairness objectives are not explicitly considered. When the number of requests remains constant, PFG-2F-RK demonstrates the most significant improvement in scenarios where customer locations exhibit a clustered distribution. With customer locations are randomly distributed, the proposed algorithm continues to maintain superior performance relative to other approaches. However, when combining both random and clustered distributions, the fairness mechanism of PFG-2F-RK encounters challenges, leading to a reduction in performance gains. Despite these challenges, the proposed algorithm remains more effective across the majority of instances. Furthermore, as the number of requests increases, its performance remains stable despite capacity and resource constraints. This finding highlights the robustness and adaptability of PFG-2F-RK in handling complex and dynamic problem environments.

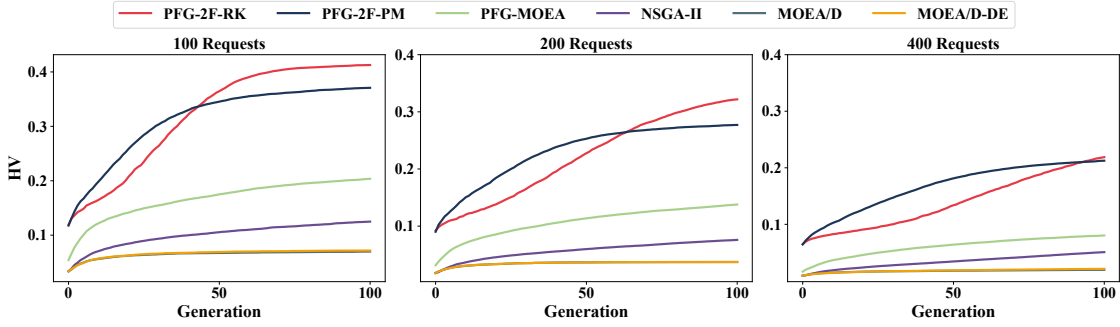
Table 4.1: HV values of algorithms

		NSGA-II	MOEA/D	MOEA/D-DE	PFG-MOEA	PFG-2F-PM	PFG-2F-RK
100	C	0.138 \pm 0.03	0.077 \pm 0.03	0.081 \pm 0.02	0.209 \pm 0.04	0.412 \pm 0.05	0.463 \pm 0.06
	R	0.081 \pm 0.02	0.038 \pm 0.01	0.038 \pm 0.01	0.150 \pm 0.03	0.287 \pm 0.04	0.325 \pm 0.07
	RC	0.043 \pm 0.02	0.018 \pm 0.01	0.020 \pm 0.01	0.086 \pm 0.02	0.212 \pm 0.03	0.223 \pm 0.04
200	C	0.113 \pm 0.04	0.067 \pm 0.03	0.064 \pm 0.03	0.192 \pm 0.05	0.331 \pm 0.05	0.370 \pm 0.06
	R	0.077 \pm 0.02	0.037 \pm 0.01	0.039 \pm 0.01	0.132 \pm 0.03	0.273 \pm 0.03	0.316 \pm 0.02
	RC	0.052 \pm 0.02	0.018 \pm 0.01	0.021 \pm 0.01	0.074 \pm 0.03	0.211 \pm 0.04	0.203 \pm 0.05
400	C	0.124 \pm 0.04	0.066 \pm 0.02	0.071 \pm 0.02	0.209 \pm 0.04	0.369 \pm 0.06	0.406 \pm 0.06
	R	0.070 \pm 0.02	0.036 \pm 0.01	0.036 \pm 0.01	0.131 \pm 0.03	0.271 \pm 0.05	0.324 \pm 0.05
	RC	0.057 \pm 0.02	0.023 \pm 0.01	0.022 \pm 0.01	0.081 \pm 0.03	0.215 \pm 0.02	0.231 \pm 0.04

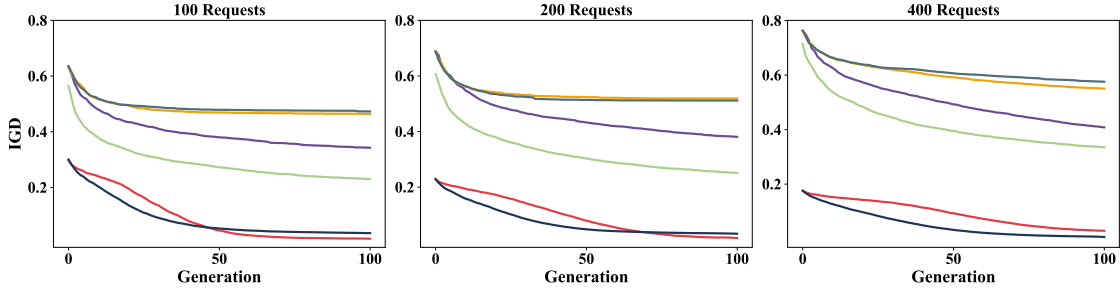
Table 4.2: IGD values of algorithms

		NSGA-II	MOEA/D	MOEA/D-DE	PFG-MOEA	PFG-2F-PM	PFG-2F-RK
100	C	0.353 \pm 0.05	0.481 \pm 0.07	0.468 \pm 0.06	0.258 \pm 0.04	0.039 \pm 0.03	0.020 \pm 0.02
	R	0.369 \pm 0.06	0.517 \pm 0.07	0.523 \pm 0.07	0.241 \pm 0.05	0.033 \pm 0.04	0.017 \pm 0.02
	RC	0.435 \pm 0.09	0.586 \pm 0.12	0.558 \pm 0.10	0.298 \pm 0.06	0.006 \pm 0.01	0.026 \pm 0.01
200	C	0.325 \pm 0.08	0.450 \pm 0.08	0.454 \pm 0.09	0.201 \pm 0.07	0.030 \pm 0.03	0.010 \pm 0.01
	R	0.372 \pm 0.07	0.505 \pm 0.08	0.504 \pm 0.08	0.247 \pm 0.07	0.024 \pm 0.02	0.018 \pm 0.01
	RC	0.401 \pm 0.10	0.590 \pm 0.11	0.541 \pm 0.10	0.370 \pm 0.09	0.004 \pm 0.00	0.035 \pm 0.02
400	C	0.350 \pm 0.06	0.488 \pm 0.07	0.470 \pm 0.07	0.232 \pm 0.05	0.038 \pm 0.04	0.017 \pm 0.02
	R	0.402 \pm 0.06	0.513 \pm 0.08	0.530 \pm 0.09	0.265 \pm 0.06	0.040 \pm 0.03	0.017 \pm 0.02
	RC	0.389 \pm 0.09	0.551 \pm 0.08	0.552 \pm 0.09	0.335 \pm 0.08	0.008 \pm 0.01	0.026 \pm 0.02

Additionally, convergence of the algorithms is visualized in Figure 4.3a. Although PFG-2F-PM demonstrates superior exploration capability during the first 50 generations compared to the proposed algorithm, it quickly becomes trapped in local optima. In contrast, PFG-2F-RK continues to improve, although at a decreasing rate as it approaches convergence. The MOEA/D and MOEA/D-DE algorithms exhibit early convergence, often settling in local optima. Meanwhile, PFG-MOEA and NSGA-II continue to improve beyond the 100th generation; however, their progress remains marginal compared to the proposed algorithm in terms of both convergence speed and solution quality. Similarly, this part presents the average IGD values and convergence trends in Table 4.2 and Figure 4.3b, respectively. The improvement achieved by PFG-2F-RK follows a similar pattern to that observed for HV. However, PFG-2F-PM outperforms the proposed algorithm on datasets where customer distribution combines both random and clustered patterns.



(a) HV convergence trend of PFG-2F compared to other benchmarks.



(b) IGD convergence trend of PFG-2F compared to other benchmarks.

Figure 4.3: HV and IGD convergence across different instances.

Figure 4.4 illustrates the effectiveness of PFG-2F-RK across 18 scenarios. Each scenario combines 100, 200, or 400 requests with one of six distributions (C1, C2, R1, R2, RC1, RC2). As the number of requests increases, performance tends to decline because the algorithm must handle a larger search space with the same number of generations. Among all distributions, C2 yields the most favorable outcomes, while those with narrower time windows exhibit more variability. The worst performance occurs under R1, primarily due to the randomness of the data and tighter time constraints.

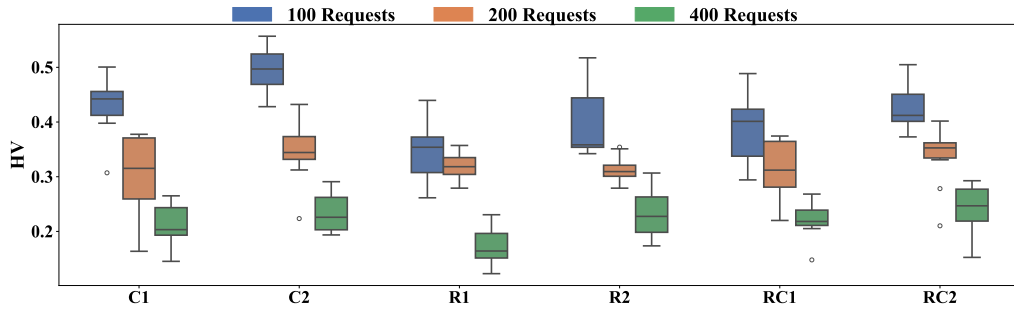


Figure 4.4: Effectiveness of PFG-2F over all instances.

4.2.3 Compare with SOO algorithms

To evaluate the effectiveness of PFG-2F in comparison to single-objective algorithms, this work benchmarks it against FairGA [41]. FairGA is specifically designed to optimize the total distance traveled while incorporating two-sided fairness considerations. It achieves this by integrating two fairness objectives related to reproduction alongside its primary distance optimization.

This thesis assesses improvement using Equation 4.1 across the four objectives: EC, VF, CF, and MWT. Since PFG-2F generates a set of non-dominated solutions due to its multi-objective nature, this work selects a single representative solution for comparison. The PFG-2F algorithm chooses the solution that corresponds to the intersection of the hyperplane created by the obtained Pareto front and the line connecting the ideal and nadir points. This solution is deemed effective for balancing the objective functions. If no such solution is found, the algorithm selects the solution nearest to the ideal point.

The improvement of algorithm A compared to algorithm B for instance i is $\%imp(A, B, i)$, which is calculated as follows:

$$\%imp(A, B, i) = \frac{f(B, i) - f(A, i)}{f(B, i)} \times 100\% \quad (4.1)$$

where $f(\cdot)$ is the considered objective value.

Table 4.3: Improvement of PFG-2F compared to FairGA

Dis	EC imp (%)	VF imp (%)	CF imp (%)	MWT imp (%)
C1	15.42	69.15	39.74	40.11
C2	18.74	58.04	45.75	36.18
R1	16.29	64.83	28.85	34.46
R2	14.56	59.95	58.11	49.19
RC1	17.62	64.90	31.35	33.60
RC2	15.85	64.40	58.87	52.21

Table 4.3 presents the percentage improvements of PFG-2F over FairGA across different instances. The results indicate a significant enhancement in the total distance objective, which is the primary focus of FairGA. Additionally, PFG-2F demonstrates superior performance in the remaining three objectives: customer fairness (CF), vehicle fairness (VF), and time window constraint satisfaction (MWT). These findings highlight the advantage of employing a multi-objective approach, as PFG-2F effectively balances and improves multiple objectives simultaneously, as evidenced by the increased HV and IGD.

In conclusion, the proposed algorithm demonstrates superior performance across multiple datasets, including large-scale instances designed to reflect real-world complexity. When evaluated using HV and IGD metrics, the algorithm consistently outperforms all state-of-the-art MOO methods. Notably, it achieves at least a 15% improvement in objective performance compared to SOO baselines, while also maintaining strong real-time adaptability. These results confirm the robustness and effectiveness of the proposed approach in addressing complex, dynamic optimization problems.

CONCLUSIONS

This thesis introduces the Multi-objective dynamic pickup and delivery problem with time windows, which captures the complexities of real-world logistics systems operating under dynamic and uncertain conditions. Unlike static formulations, this problem requires vehicles to make routing decisions incrementally as new transportation requests arrive over time. To reflect realistic logistics objectives, the proposed framework incorporates multiple goals: minimizing energy consumption to promote sustainability, reducing customer waiting times to enhance service quality, and ensuring fairness in workload distribution across both customers and service providers. These objectives mirror key concerns in on-demand delivery services, ride-sharing, and urban distribution networks, where both efficiency and equity are critical.

To address these challenges, a novel algorithm called PFG-2F is proposed, which combines Pareto front grid generation with fairness-aware heuristics and random-key-based solution encoding. The algorithm is designed to efficiently explore the multi-objective solution space while remaining responsive to dynamic inputs. Experimental evaluations conducted across a variety of benchmark scenarios demonstrate that PFG-2F significantly outperforms existing state-of-the-art multi-objective algorithms, achieving better results in both hypervolume and inverted generational distance. These results highlight the algorithm's ability to generate diverse, high-quality solutions with superior convergence properties. Looking ahead, future research may investigate the integration of machine learning-based heuristics or predictive models to further enhance the adaptability and decision-making capabilities of the system, enabling even more efficient and intelligent vehicle-task allocation in complex, time-sensitive logistics environments.

Publications and related papers

The main research results and methodologies developed throughout this thesis have been peer-reviewed and officially accepted for presentation and publication at:

1. **Phan Hung**, Bui Trong Duc, Nguyen Thi Tam, and Huynh Thi Thanh Binh. "Pareto Front Grid Guided Multiobjective Optimization in Dynamic Pickup and Delivery Problem Considering Two-Sided Fairness". In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO '25)*, 2025. (Full paper presentation, conference **ranked A**).

2. Ha Minh Hieu, **Phan Hung**, Tran Duc Chinh, Van Duc Cuong, Dao Van Tung, and Huynh Thi Thanh Binh. “Alimentation Deep Multiple Optimal Ant Colony Optimization to Solve Vehicle Routing Problem with Time Windows”. In *Proceedings of the Genetic and Evolutionary Computation Conference Companion (GECCO '24 Companion)*, 2024. DOI: 10.1145/3638530.3664050. (Poster presentation, conference **ranked A**).

Parts of this thesis have been presented and awarded:

3. **Second prize** at The 42st Student Research Conference, Hanoi University of Science and Technology, in the field of AI Applications, Blockchain, and Big Data, academic year: 2024-2025.
4. **Third prize** at The 41st Student Research Conference, Hanoi University of Science and Technology, in the field of AI Applications, Blockchain, and Big Data, academic year: 2023-2024.

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