1. a $\sum D^{-l} = D^{-1}+D^{-1}+D^{-2}+D^{-3}+D^{-2}+D^{-3} \le 1$

$D > 2$.

As prefix-free code is a subset of uniquely decodable code, the average length of uniquely decodable code can be ~~longer~~ than prefix-free code.

shorter.

∴ the lower bound will be larger.

b.

	①	②	③	④
$\frac{1}{3}$	0	1	00	10
$\frac{1}{3}$	10	01	01	11
$\frac{1}{4}$	110	000	10	01
$\frac{1}{12}$	111	001	11	00

$I = -\sum p_i \log_2 p_i = 1.86$.

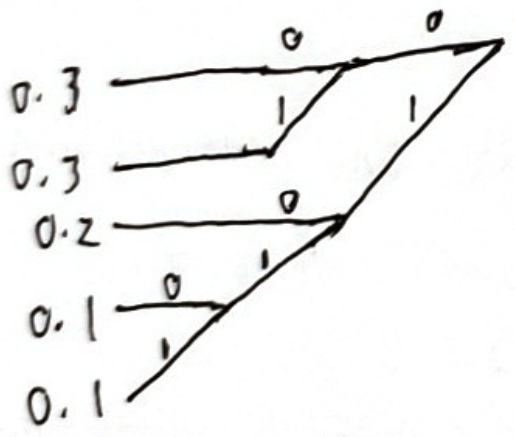
$L_{\frac{1}{3}} = \log_2 3 = 1.58$.

$L_{\frac{1}{4}} = \log_2 4 = 2$ $L_{\frac{1}{12}} = \log_2 12 = 3.58$.

It can be seen that for ①, ④, $\frac{1}{3}$'s code length longer than shannon.

for ①, ②, one $\frac{1}{3}$ and $\frac{1}{4}$ code length longer than shannon.

c.



00

01

10

110

111

$E[L] = 2.2$.

$p' = (\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{8}\ \frac{1}{8})$ $H(p') = 2.25$.

$E(L) = 3\times\frac{1}{4}\times 2 + 2\times\frac{1}{8}\times 3 = 2.25$.

d. I_1: $2^{-1}+2^{-2}+2^{-2} = 1 \le 1$ ∴ can

I_2: $2^{-2}+2^{-2}+2^{-1}+2^{-3} = \frac{7}{4} < 1$ ∴ can

I_3: $3^{-1}+4\times 3^{-2} = \frac{7}{9} < 1$ ∴ can

I_4: $8\times 3^{-2}+3\times 3^{-3} = 1 \le 1$ ∴ can.

2. a. length of sequence is 10000, start with 1.
run length sequence length is 4246.

b. the length of optimum binary stream is 13344

c. ① It can be seen from figure 2-c-compression_ratio that the compression ratio become lower when α increase. That's because when the α is lower, the sequence will keep seable from a long time, which means the $\log_2 L$ will be smaller than the length L. But when alpha becomes higher, the sequence change rapidly. As a result, the compression ratio becomes lower.

② It can be seen from 2-c-pmf that $\alpha = 0.01$ has a wide length distribution, but $\alpha = 0.5$ and $\alpha = 0.95$ only have short length, and the shorter length have the higher possibility.

3. b. ① it can be seen from 3-b that the compression ratio of adaptive golomb encoder is smaller than ideal encoder. Because ideal encoder use the smallest possible value to express each run-length value, but adaptive golomb encoder need to estimator $m = 2^k$. That will lead to diviation from the optimal encoder, due to error and other reason exsiting.

② According to 3.b2, Nmax should set to 2, And A should set to twice the mean of run-length values.

4. b, c: From figure 4-b which is the entropy and figure 4-c which ~~the~~ is then entropy rate, it's easy to see that arithmetic coder has low entropy and entropy rate. Because this coder make full use of the correlation between symbols, which will greatly reduce the entropy, since markov-1 sequence have great realibility between neighbour.

d. It can be seen that arithmetic ~~to~~ encoder has higher compression ratio. As arthmetic enceder can more fully utilize the symbol frequency distribution due to it can adjust the model based on the actual statitical characteristic of data. But run-length enceder / golomb can not do this.
Besides, arthmetic encoder is lossless than run-length encoder. As a result, run-length / golomb need more sequence to keep the integrity of origin data than arthmetic encoder.