Institute of Scientific Computing
Technical University Braunschweig
Prof. Hermann G. Matthies, Ph.D.

Now we want to consider a series of further examples for incompressible flow. For that purpose we introduce the disretisation for the other three types of boundary conditions, which are introduced on the second assignment. And we want to explore the treatment of obstacles in the internal domain.

1 Further Boundary Conditions and Inner Boundaries

1.1 Free-Slip Conditions

At free-slip conditions the velocity vertical to the wall w_1 should be zero, just as the derivation of the velocity w_2 parallel to the wall in normal direction. In our rectangular domain („staggered grid“), the discrete veclocities vertically to the boundary exactly lie on the boundary, so we can set here (like no-slip conditions):

$$\begin{aligned} u_{0,j} &= 0,\ u_{imax,j} = 0, \quad j = 1, \ldots, jmax, \\ v_{i,0} &= 0,\ v_{i,jmax} = 0, \quad i = 1, \ldots, imax. \end{aligned}$$

The discretisation of the normal derivatives of the horizontal velocities $\frac{\partial v}{\partial n}$ in the boundary point Q (see figure 1) is

$$\frac{\partial v}{\partial n} \approx \frac{v_i - v_a}{\delta x}$$

so that the further boundary conditions

$$u_{i,0} = u_{i,1},\ u_{i,jmax+1} = u_{i,jmax} \quad \text{for } i = 1, \ldots, imax$$

and

$$v_{0,j} = v_{1,j},\ v_{imax+1,j} = v_{imax.j} \quad \text{for } j = 1, \ldots, jmax$$

follows from $\frac{\partial v}{\partial n} = 0$.

For the pressure at the boundary we have

$$p_{0,j} = p_{1,j},\ p_{imax+1,j} = p_{imax,j}$$

for $j = 1, \ldots, jmax$ and

$$p_{i,0} = p_{i,1},\ p_{i,jmax+1} = p_{i,jmax}$$

for $i = 1, \ldots, imax$. F and G must be modified at the boundary:

$$F_{0,j} = u_{0,j},\ F_{imax,j} = u_{imax,j}$$

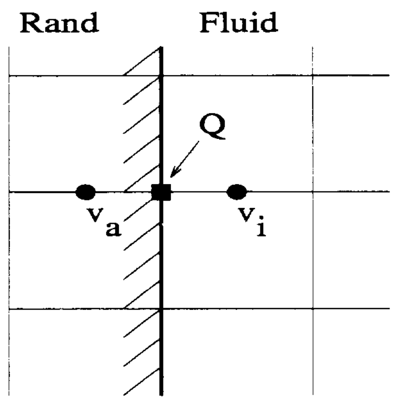


Figure 1: *Boundary consitions*

for $j = 1, \ldots, jmax$ and

$$G_{i,0} = v_{i,0},\ G_{i,jmax} = v_{i,jmax}$$

for $i = 1, \ldots, imax$.

1.2 Outflow Conditions

At outflow conditions the normal derivatives of both velocitiy components are set to zero at the boundary, what means, that the entire velocity should not change in the direction vertically to the boundary. This can be realized in the discrete case, by setting the velocities at the boundaries respectively equal to the adjacent velocities in the domain, i.e.

$$u_{0,j} = u_{1,j},\ u_{imax,j} = u_{imax-1,j},$$

$$v_{0,j} = v_{1,j},\ v_{imax+1,j} = v_{imax,j}$$

for $j = 1, \ldots, jmax$ and

$$u_{i,0} = u_{i,1},\ u_{i,jmax+1} = u_{i,jmax},$$

$$v_{i,0} = v_{i,1},\ v_{i,jmax} = v_{i,jmax-1}$$

for $i = 1, \ldots, imax$. Also the pressure shows no variance in normal-direction:

$$p_{0,j} = p_{1,j},\ p_{imax+1,j} = p_{imax,j}, \quad j = 1, \ldots, jmax,$$

$$p_{i,0} = p_{i,1},\ p_{i,jmax+1} = p_{i,jmax}, \quad i = 1, \ldots, imax.$$

The values of F and G are set to the velocity components respectivly:

$$F_{0,j} = u_{0,j},\ F_{imax,j} = u_{imax,j}, \quad j = 1, \ldots, jmax,$$

$$G_{i,0} = v_{i,0},\ G_{i,jmax} = v_{i,jmax}, \quad i = 1, \ldots, imax.$$

1.3 Inflow Conditions

At inflow conditions the velocity values are fixed given at the boundary. Since these values are different from example to example and at the whole boundary will be also not always constant, we cannot read in these values from the input file. Instead we set these boundary values depending upon the problem in the function `spec_boundary_values`, which is to be called immediately after the function `boundary_values`. To the boundary values of $p\,F$ and G the same formulas are aplied as to outflow condition.

1.4 Solid Inner Walls

To be able to simulate flow around obstacles or to excise subdomains from the rectagular domain, we set the velocities in these subdomains, where no fluid should be, to zero, which corresponds to no-slip conditions. This should be done in the function `spec_boundary_values`. Normally also appropriate conditions at the boundaries of the obstacles for the pressure must be choosen in order to calculate the pressure only in the areas, where also liquid is available. In order to not excessively blow up the program, we want to make no additional modifications with the calculation F of G and in the pressure iteration.

1.5 Tasks

Now the program should be expanded in such a manner, that we can specify exactly one boundary condition at the four walls of our domain. For that purpose we introduce the integer-parameters `wl` for the left, `wr` for the right, `wt` for the upper and `wb` for the lower boundary, which should be set in the function `progpar`. The parameters `wl, wr, wt` and `wb` can be set to the values

- 1 for free-slip conditions,
- 2 for no-slip conditions,
- 3 for inflow or outflow conditions.

For simplification purpose we use an equal value for both inflow and outflow conditions, because the boundary values for $F\,G$ and p are equal in both cases. At inflow conditions the velocity values at the boundaries are overwritten in the time loop in the function `spec_boundary_values`.

Task 1 1. *Expand the function* `progpar` *by setting the boundary-flags* `wl, wr, wt` *and* `wb`*.*

2. *Extend the function* `boundary_values` *in such a way, that depending upon initialization of the boundary value flags* `wl, wr, wt` *and* `wb` *the correct boundary values for u and v according to the above formulas are set.*

3. *Write the function*

    [U, V] = spec_boundary_values(U, V, imax, jmax, problem),

   where - dependent on the respective problem (characterized by the parameter `problem`*) - the inflow conditions and the conditions for internal boundaries are implemented. With „the Driven Cavity“the special condition $u = 1$ at the top boundary must be taken up here. For further problems a specification follows in the next paragraph. The parameter* `problem` *is to be set in the initialization file* `progpar`.

2 Further Examples

With the modifications of the current program from the first paragraph we can process now a whole series of further examples. For that purpose a new input file must be made available and the problem specific internal boundary values and inflow conditions must be taken up in the function „`spec_boundary_values` “So the following flow can be simulated.

2.1 The Von Karmann Vortex Street

We consider a flow in a canal in x-direction, which meets on a square stumbling-block (see figure 2) with a slanting velocity profile.

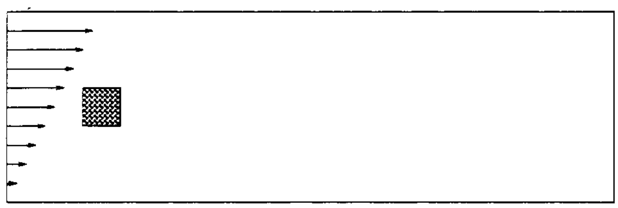


Figure 2: *von Karmann vortex street*

At the lefthand side the fluid streams in and on the right side the fluid streams out again, while at the upper and lower edge no friction should be available (free-slip conditions). In the function „`spec_boundary_values`“ these boundary values are to be implemented. The slanting flow profile is implemented by suitable boundary values u and v:

$$u_{0,j} = j/jmax,\ v_{0,j} = -v_{1,j}, \quad j = 1, \ldots, jmax,$$

and the obstacle, which is to fill out a fifth of the canal width, by the appropriate velocity values for $u_{i,j}$ and $v_{i,j}$ within this domain model. We choose the values for the parameters as follows:

`imax = 64`	`jmax = 16`	`delx = 0.2`	`dely = 0.2`
`delt = 0.02`	`T_end = 0.2`		
`epsi = 0.01`	`omg = 1.7`	`alpha = 0.9`	`itermax = 150`
`GX = 0.0`	`GY = 0.0`	`nu = 1.3414e-5`	
`U_I = 1.0`	`V_I = 0.0`	`P_I = 0.0`	
`wl = 3`	`wr = 3`	`wt = 1`	`wb = 1`

2.2 Flow Above a Stair

In a canal, which spreads to one side, streams a fluid with a constant velocity (see figure 3). On top and below the fluid should stick respectively.

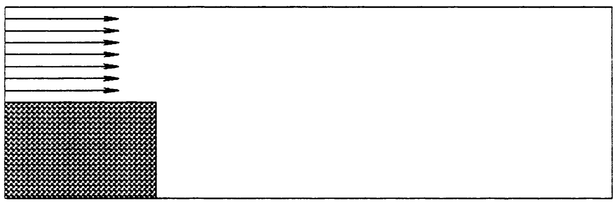


Figure 3: *Flow above a stair*

The following settings are to be taken up in „`spec_boundary_values`“:

$$\begin{array}{lll} u_{0,j} = 1.0, & v_{0,j} = -v_{1,j}, & j = jmax/2+1, \dots, jmax, \\ u_{i,j} = 0.0, & i = 0, \dots, jmax/2, & j = 1, \dots, jmax/2, \\ v_{i,j} = 0.0, & i = 1, \dots, jmax/2, & j = 0, \dots, jmax/2. \end{array}$$

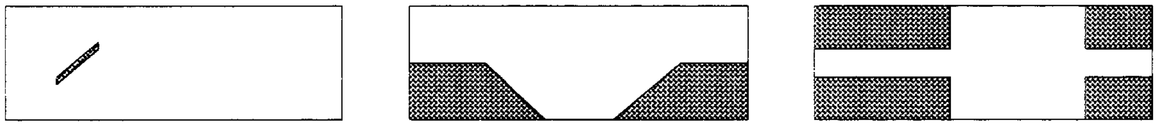
Further the following parameter values are to select:

`imax = 64`	`jmax = 16`	`delx = 0.2`	`dely = 0.2`
`delt = 0.02`	`T_end = 0.2`		
`epsi = 0.01`	`omg = 1.7`	`alpha = 0.12`	`itermax = 150`
`GX = 0.0`	`GY = 0.0`	`nu = 0.1`	
`U_I = 1.0`	`V_I = 0.0`	`P_I = 0.0`	
`wl = 3`	`wr = 3`	`wt = 2`	`wb = 2`

At the initialization of the velocity values in `initgrid` u should be set to 0 in the lower half.

2.3 A Selection of Further Examples

Choose at least one example from the following three, and consider a completely new example, which you want to process.



Wirbelstraße mit schrägem Balken

Strömung über eine tiefergelegte Straße

Kanal mit Erweiterung

Figure 4: *Further examples*

At the examples mentioned above the fluid should flow in respectively to the left with a constant rate of speed in x-direction and flow out to the right again, while on top and below no friction should work and no-slip conditions at the internal boundaries.