```
f[a_, b_, ε_, x_, y_] = a * x^2 + b * y^2 + ε * x^4 - 1;

ellplus[a_, b_, ε_, x_] = (√(1 - a x^2 - x^4 ε))/(√b);

ellminus[a_, b_, ε_, x_] = -(√(1 - a x^2 - x^4 ε))/(√b);

gradf[a_, b_, ε_, x_, y_] = Grad[f[a, b, ε, x, y], {x, y}];
```

```
nextCoord[a_, b_, startθ_, startα_, ε_, n_] :=
 Module[{start, s, startx, starty, startt, endx,
   endy, angle, list, ss, xx, yy, u, slope, v, ϕ, plot, coord, gr1},
  start = NSolve[{f[a, b, ε, x, y] == 0, x == t * Cos[startθ],
     y == t * Sin[startθ], t ≥ 0}, {x, y, t}, Reals];
  s = {x, y, t} /. start;
  startx = s[[1, 1]];
  starty = s[[1, 2]];
  startt = s[[1, 3]];
  coord = {{startx, starty}};
  angle = startα;
  j = 1;
  list = {}; (* stores position angles and trajectory angles*)
  While[j ≤ n,
   ss = NSolve[{f[a, b, ε, x, y] == 0, x == startx + t * Cos[angle],
      y == starty + t * Sin[angle], t ≥ 0}, {x, y, t}, Reals];
   xx = ss[[All, 1, 2]];
   yy = ss[[All, 2, 2]];
   (*Update for next iteration of while loop*)
   startx =
    If[startx ≤ xx[[1]] + 0.0000001 && startx ≥ xx[[1]] - 0.0000001, xx[[2]], xx[[1]]];
   starty = If[starty ≤ yy[[1]] + 0.0000001 && starty ≥ yy[[1]] - 0.0000001,
     yy[[2]], yy[[1]]];
   endx = If[startx ≤ xx[[1]] + 0.0000001 && startx ≥ xx[[1]] - 0.0000001,
     xx[[2]], xx[[1]]];
   endy = If[starty ≤ yy[[1]] + 0.0000001 && starty ≥ yy[[1]] - 0.0000001,
     yy[[2]], yy[[1]]];
   coord = Append[coord, {startx, starty}];
   u = {endx - startx, endy - starty};
   v = gradf[a, b, ε, startx, starty].{{0, -1}, {1, 0}};
   ϕ = VectorAngle[u, v];
   list = Append[list, {ArcTan[startx, starty], ϕ}];
   angle = angle + 2 * ϕ;
   j++;
  ];
  plot = Plot[{ellplus[a, b, ε, x], ellminus[a, b, ε, x]}, {x, -1/Sqrt[a] - .5,
     1/Sqrt[a] + .5}, Axes → False, PlotStyle → Black, AspectRatio → Automatic];
  gr1 = Show[plot,
    Graphics[{Line[coord]}]
    (*,Graphics[{PointSize[Large],Red,Thickness[Large],Arrow[{{ s[[1,1]]+1*Cos[startα],
         s[[1,2]]+1*Cos[startα]},{s[[1,1]],s[[1,2]]}}]}]*)];
  gr2 = Graphics[Map[Point, list], Frame → True, FrameLabel → {θ, α},
    AspectRatio → Automatic];
  GraphicsRow[{gr1, gr2}]
 ]
```

```
Manipulate[
 nextCoord[a, b, startθ, startα, ε, n],
 "starting position",
 {{startθ, .45, ""}, 0, 2 * Pi, .01, Appearance → "Labeled", ImageSize → Tiny},
 "starting angle",
 {{startα, Pi, ""}, 0, 2 * Pi, .01, Appearance → "Labeled", ImageSize → Tiny},
 "a",
 {{a, 1, ""}, .1, 2, .01, Appearance → "Labeled", ImageSize → Tiny},
 "b",
 {{b, 1, ""}, .1, 2, .01, Appearance → "Labeled", ImageSize → Tiny},
 "ε",
 {{ε, 0, ""}, 0, 100, .01, Appearance → "Labeled", ImageSize → Tiny},
 "num reflections",
 {{n, 1, ""}, 1, 200, 1, Appearance → "Labeled", ImageSize → Tiny},
 SaveDefinitions → True
]
```
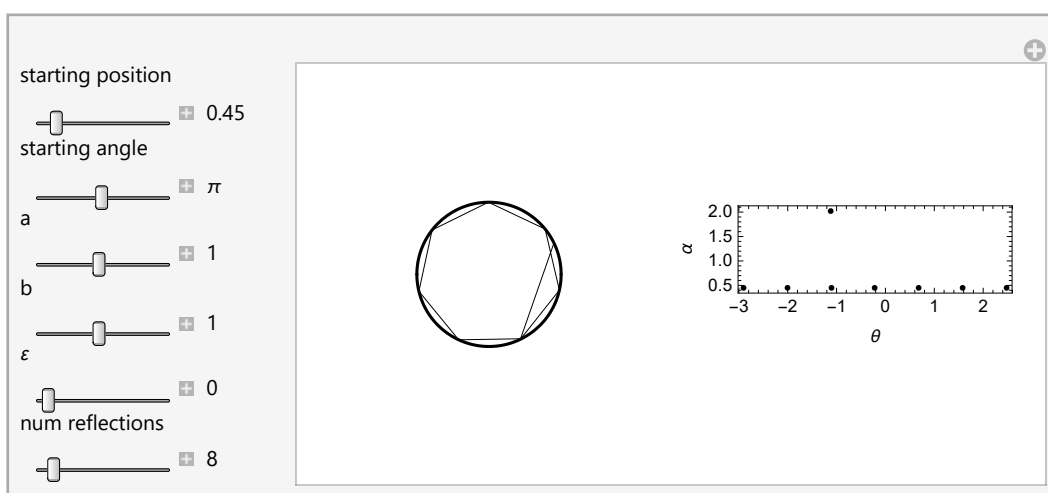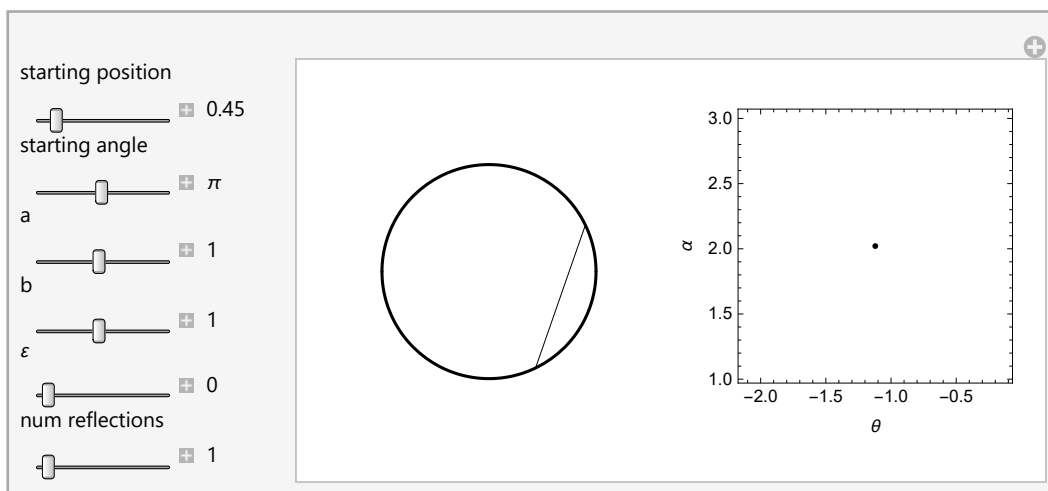


```
gradf[1, 1.5, 0, 1, 0].{{0, -1}, {1, 0}}
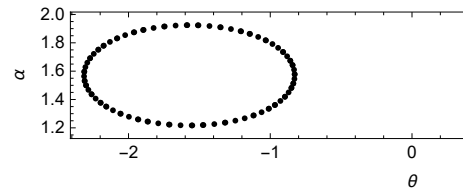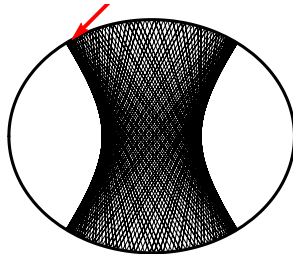```

{0., -2.}

```
nextCoord[1, 1.5, 2.26, 5.28, 0.1 , 20]
```

$Aborted

```
p1 = gr2;
```

```
nextCoord[1, 1.5, 2.26, 5.28 - .2, 0 , 200]
```



```
Pi
```

π

```
p2 = gr2;
```

```
Show[p1, p2, p3, p4, p5, p6, p7, p8, p9, p10, p11, p12, p13, p14, p15]
```