Exponential Decay Model

Let t denote time since symptom onset (in days), and let $c(t)$ be the viral concentration at time t on the natural scale (copies/mL). When the concentration of pathogen shed $c(t)$ is subject to exponential decay, it decreases at a rate a_0 proportional to the current value. Define:

$$c(t) = c_0 \cdot e^{-a_0 t}$$

Where c_0 is the concentration of pathogen shed at symptom onset.
Taking logarithm base 10:

$$\log_{10} c(t) = \frac{\log c_0 - a_0 t}{\log 10} = \frac{c_0}{\log 10} - \frac{a_0 t}{\log 10}$$

Viral Load Observation Model

The observed viral load values (on $\log_{10}$ scale) are modeled as:

$$Y_{\text{viral}}(t) \sim \mathcal{N}\left(\mu_{\text{viral}}(t), \sigma^2_{\text{viral}}\right)$$

with mean:

$$\mu_{\text{viral}}(t) = \frac{c_0}{\log 10} - \frac{a_0 t}{\log 10}$$

Ct Value Observation Model

Assuming a standard curve allows for DNA quantification, we model a linear relationship between Ct values and $\log_{10}$viral concentration.

$$\log_{10} c(t) = \beta_0 + \beta_1 \cdot \text{Ct}(t) \quad \Longrightarrow \quad \text{Ct}(t) = \frac{\log_{10} c(t) - \beta_0}{\beta_1}$$

Therefore, the observed Ct values are modeled as:

$$Y_{\text{Ct}}(t) \sim \mathcal{N}\left(\mu_{\text{Ct}}(t), \sigma^2_{\text{Ct}}\right)$$

with mean:

$$\mu_{\text{Ct}}(t) = \frac{\frac{c_0}{\log 10} - \frac{a_0 t}{\log 10} - \beta_0}{\beta_1}$$

standard deviation:

$$\sigma_{\text{Ct}}(t) = \frac{\sigma_{\text{viral}}}{|\beta_1|}$$

Transformation from Ct to Log Viral Load

Given a Ct value x, we can back-transform to estimated viral load:

$$\log_{10} c(t) = \beta_0 + \beta_1 \cdot x$$