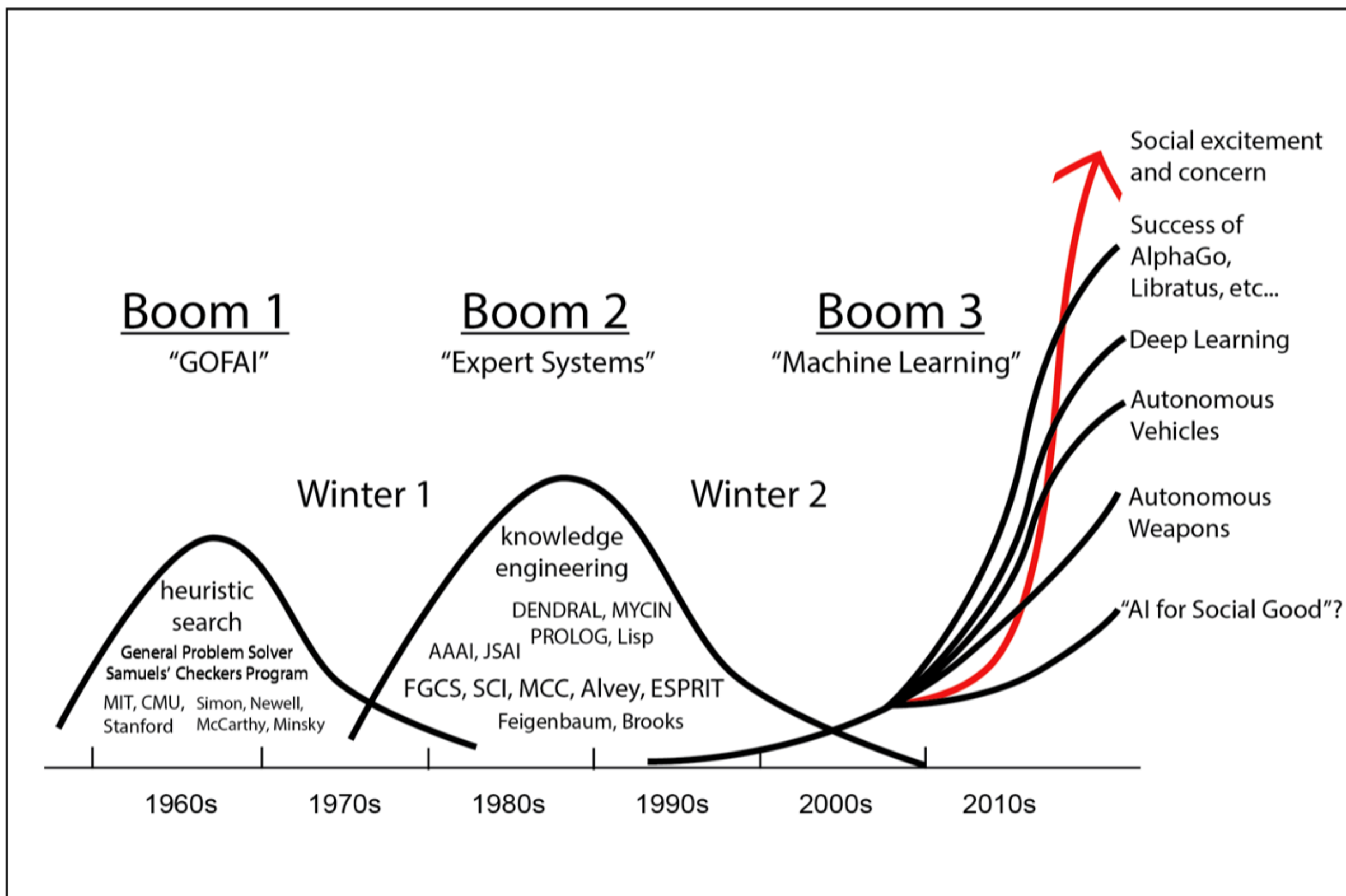
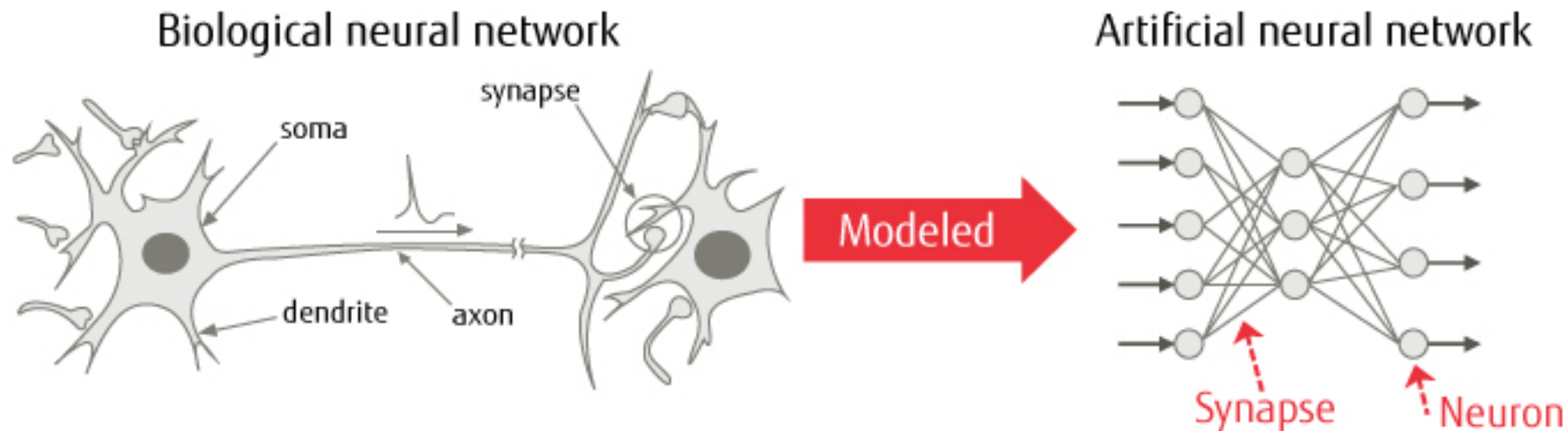


Thoughts on Deep Learning

모두의 연구소
- 2018. 10. 24 -



Gets much deeper



	The second AI boom (1988)	The third AI boom (2015)
Layer	3 layers	7 layers
Neuron	29	1.1 million
Synapse	232	730 million

the neural network used in a mobile robot Fujitsu Laboratories
The object recognition network Fujitsu Laboratories developed in 2015

Representation Power

MLP1 - Universal Approximator:

It can approximate with any desired non-zero amount of error a family of functions that includes all **continuous function** on a closed and bounded subset of \mathbb{R}^n , and **any function** mapping from any **finite dimensional discrete** space to another

Hornik (1989), Cybenko (1989)

Representation Power

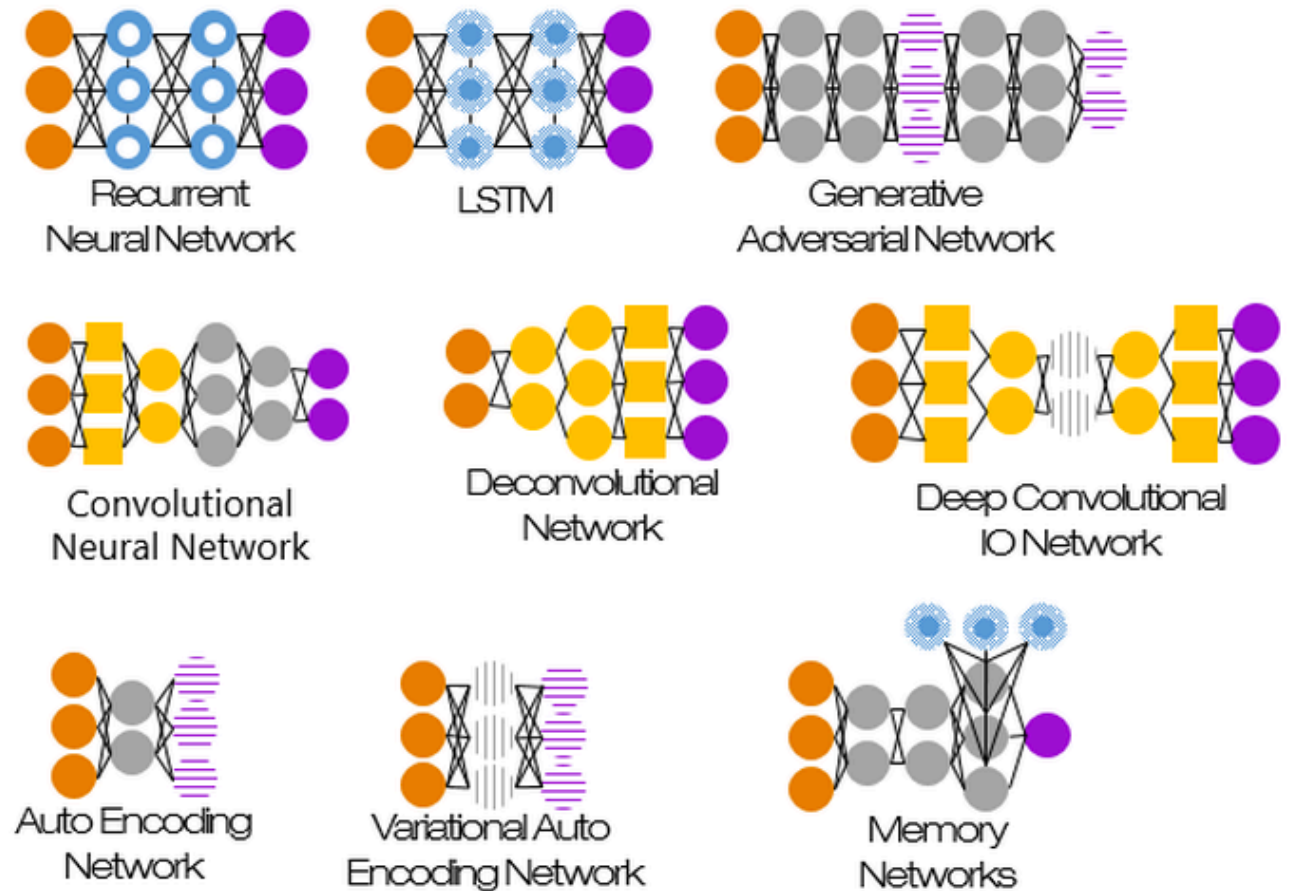
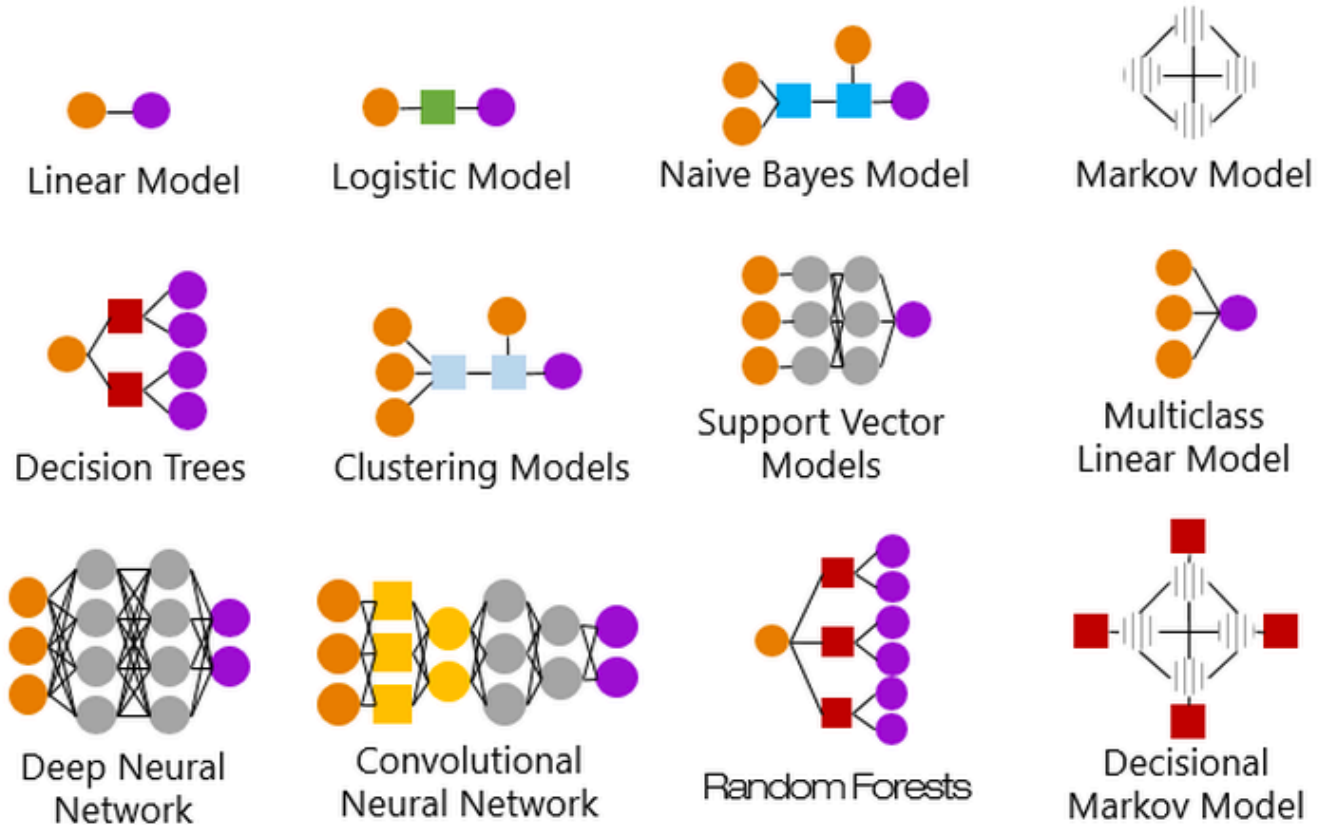
Limit of theorem:

- Does not discuss the learnability of the neural network.
- Does not guarantee that a training algorithm will find the correct function generating training data.
- Does not state how large the hidden layer should be.

Representation Power

Why we go deep?

There exist neural networks with many layers of bounded size cannot be approximated by networks with fewer layers unless these layers are *exponentially* large.



So there are
Various Networks

Why Deep Learning is so special?

Two Important Concepts:

Computation Graph
Auto-grad

Computation Graph Abstraction

Allows us us to *easily* construct *arbitrary* networks, *evaluate* their predictions for given inputs and compute gradients for their parameters with respect to *arbitrary* scalar losses.

Computation Graph Abstraction

a neural network

=

An (arbitrary) DAG

A computation graph is a representation of an arbitrary mathematical computation as a graph.

It is a directed acyclic graph(**DAG**) in which

- **nodes:** mathematical operations or (bound) variables
- **edges:** the flow of intermediary values between nodes

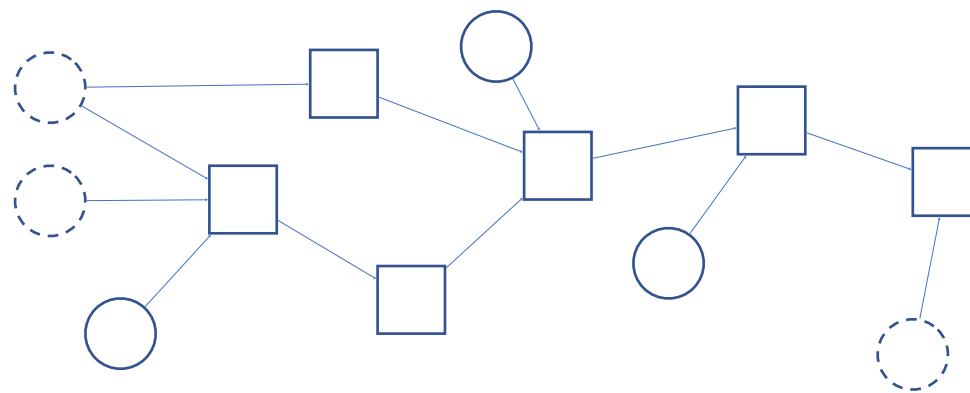
The graph structure defines the **order** of the computation in terms of the dependencies between the different component

Computation Graph Abstraction

a neural network

=

An (arbitrary) DAG

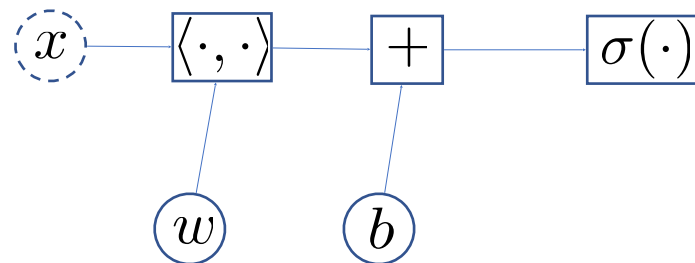


1. **Solid Circles:** parameters (to be estimated or found)
2. **Dashed Circles:** vector inputs/outputs (given as a training example)
3. **Squares:** compute nodes (functions, often continuous/differentiable)

Computation Graph Abstraction

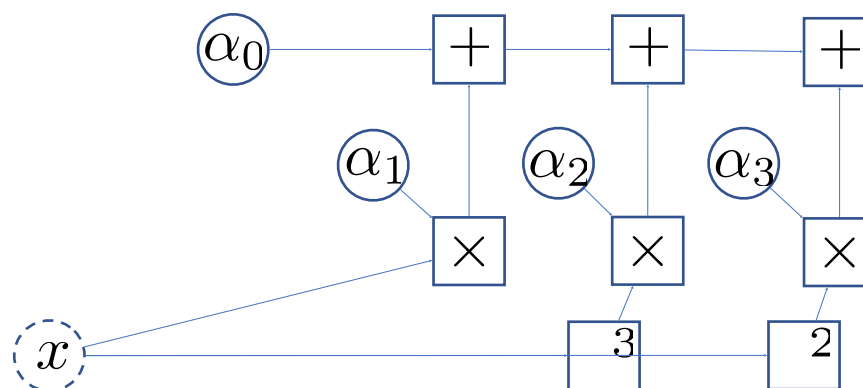
1. Logistic regression

$$p_{\theta}(y = 1|x) = \sigma(w^{\top}x + b) = \frac{1}{1 + \exp(-w^{\top}x - b)}$$



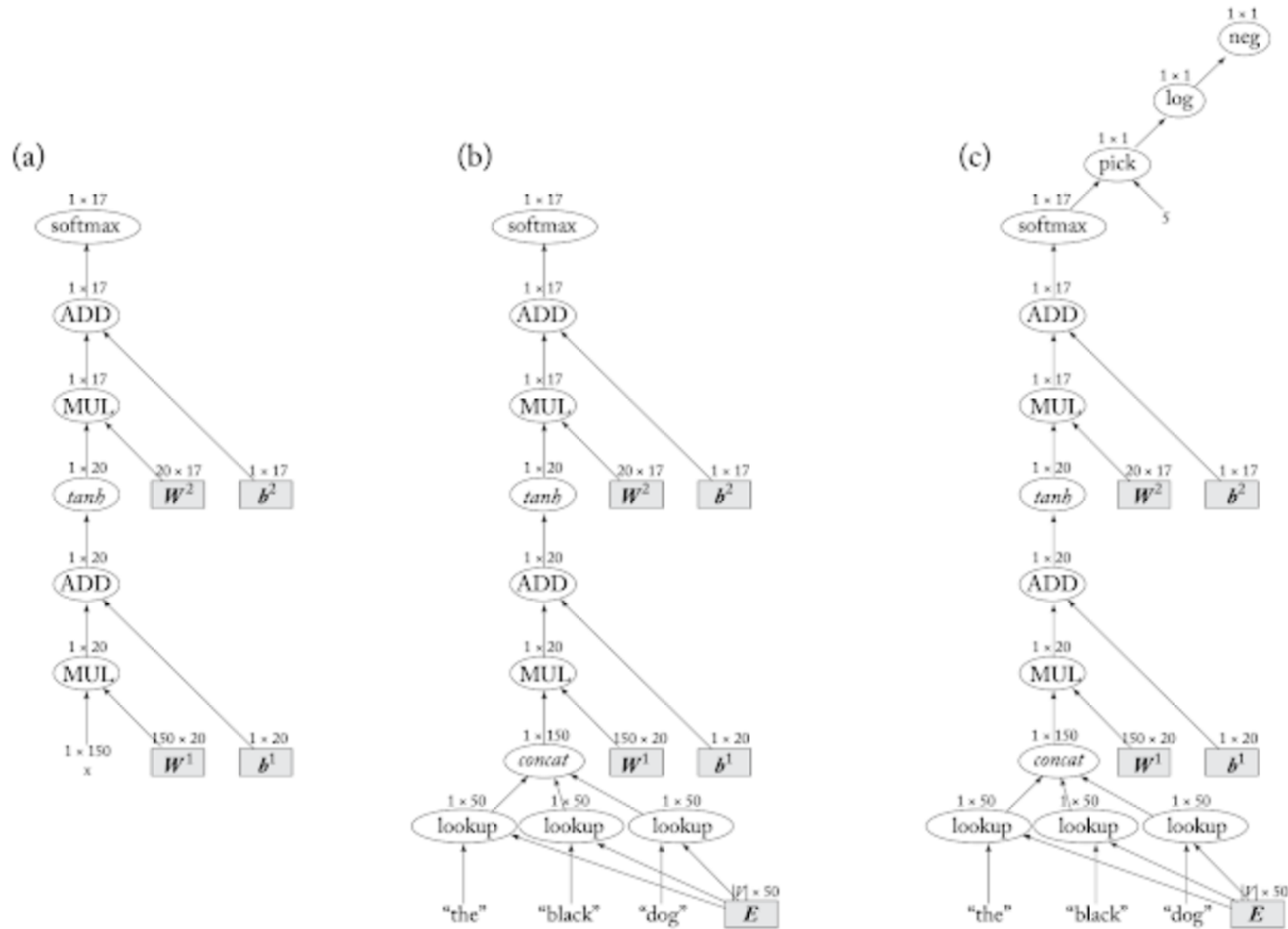
2. 3rd-order polynomial function

$$y = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$$



Computation Graph Abstraction

Examples from Deep Neural Network



Computation Graph

**Forward
Computation**

**Backward
Computation**

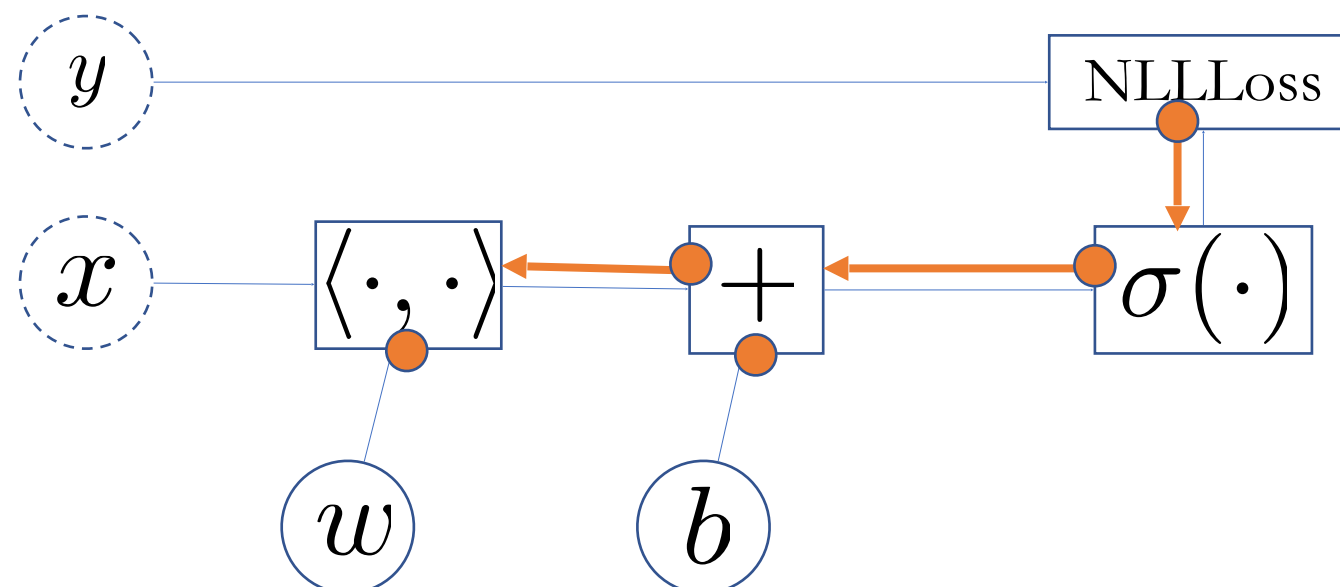
Forward Computation

Algorithm 5.3 Computation graph forward pass.

```
1: for  $i = 1$  to  $N$  do  
2:   Let  $a_1, \dots, a_m = \pi^{-1}(i)$   
3:    $v(i) \leftarrow f_i(v(a_1), \dots, v(a_m))$ 
```

Backward Computation

- Automatic differentiation (autograd)
 - Reverse-sweep the DAG starting from the loss function node.
 - Iteratively multiplies the Jacobian of each OP node until the leaf nodes of the parameters.
 - As expensive as forward computation with a constant overhead: $O(N)$, where N : # of nodes.



Backward Computation

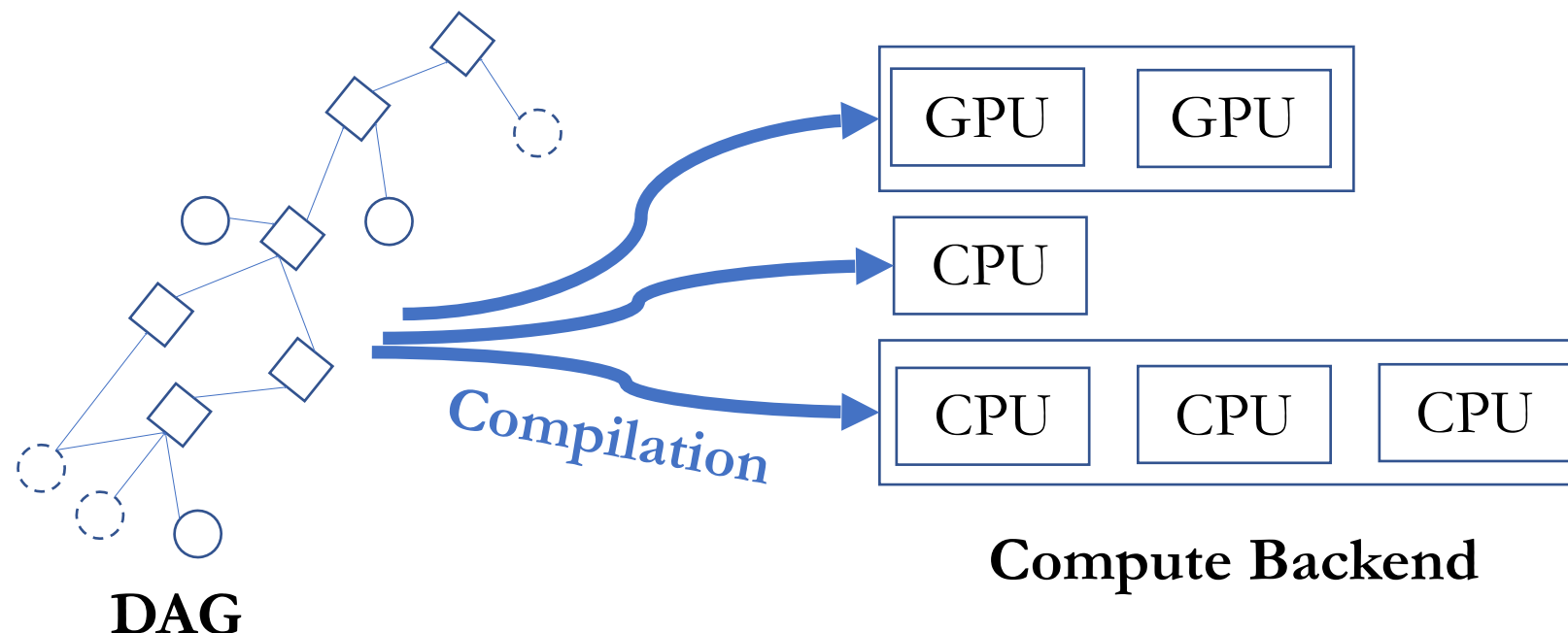
- Automatic differentiation (autograd)
 - Implement the Jacobian-vector product of each OP node:

$$\begin{bmatrix} \frac{\partial L}{\partial x_1} \\ \vdots \\ \frac{\partial L}{\partial x_d} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_{d'}}{\partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_1}{\partial x_d} & \cdots & \frac{\partial F_{d'}}{\partial x_d} \end{bmatrix} \begin{bmatrix} \frac{\partial L}{\partial F_1} \\ \vdots \\ \frac{\partial L}{\partial F_{d'}} \end{bmatrix}$$

- Can be implemented efficiently without explicitly computing the Jacobian.
- The same implementation can be reused every time the OP node is called.

Backward Computation

- Practical Implications – Automatic differentiation (autograd)
 - Unless a complete new OP is introduced, no need to manually derive the gradient
 - Nice de-coupling of specification (front-end) and implementation (back-end)
 1. [Front-end] Design a neural network by creating a DAG.
 2. [Back-end] The DAG is “compiled” into an efficient code for a target compute device.



Reference: cho, 2018

Backward Computation

Algorithm 5.4 Computation graph backward pass (backpropagation).

1: $d(N) \leftarrow 1$ $\triangleright \frac{\partial N}{\partial N} = 1$
2: **for** $i = N-1$ **to** 1 **do**
3: $d(i) \leftarrow \sum_{j \in \pi(i)} d(j) \cdot \frac{\partial f_j}{\partial i}$ $\triangleright \frac{\partial N}{\partial i} = \sum_{j \in \pi(i)} \frac{\partial N}{\partial j} \frac{\partial j}{\partial i}$

Do you know deep learning?

Summary: the equations of backpropagation

$$\delta^L = \nabla_a C \odot \sigma'(z^L) \quad (\text{BP1})$$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l) \quad (\text{BP2})$$

$$\frac{\partial C}{\partial b_j^l} = \delta_j^l \quad (\text{BP3})$$

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l \quad (\text{BP4})$$

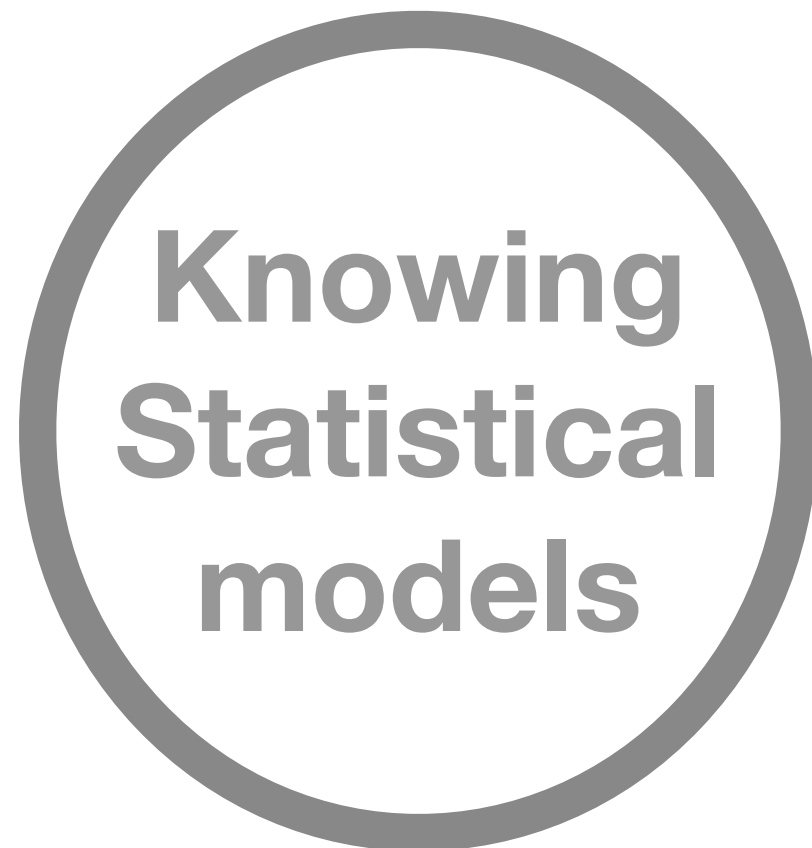
$$\begin{aligned} C &= \frac{1}{2|T|} \sum_{(X,t) \in T} \sum_p (n_p^3 - t_p)^2 \\ \frac{\partial C}{\partial w_{00}^3} &= \frac{1}{|T|} \sum_{(X,t) \in T} \sum_p (n_p^3 - t_p) \times \frac{\partial}{\partial w_{00}^3} (n_p^3) \\ &= \frac{1}{|T|} \sum_{(X,t) \in T} \sum_p (n_p^3 - t_p) \times \frac{\partial}{\partial w_{00}^3} (a(z_p^3)) \\ &= \frac{1}{|T|} \sum_{(X,t) \in T} \sum_p (n_p^3 - t_p) \times (n_p^3(1 - n_p^3)) \times \frac{\partial}{\partial w_{00}^3} (z_p^3) \\ &= \frac{1}{|T|} \sum_{(X,t) \in T} \sum_p (n_p^3 - t_p) \times (n_p^3(1 - n_p^3)) \times \frac{\partial}{\partial w_{00}^3} (w_{0p}^3 n_0^2 + w_{1p}^3 n_1^2 + b_p^3) \end{aligned}$$

Only when p is 0 will there be a w_{00}^3 term, otherwise the whole thing will be multiplied by 0

$$\begin{aligned} &= \frac{1}{|T|} \sum_{(X,t) \in T} (n_0^3 - t_0) \times (n_0^3(1 - n_0^3)) \times \frac{\partial}{\partial w_{00}^3} (w_{00}^3 n_0^2 + w_{10}^3 n_1^2 + b_0^3) \\ &= \frac{1}{|T|} \sum_{(X,t) \in T} (n_0^3 - t_0) \times (n_0^3(1 - n_0^3)) \times (n_0^2) \\ &= \frac{1}{|T|} \sum_{(X,t) \in T} \frac{\partial C_{Xt}}{\partial n_0^3} \times \frac{\partial n_0^3}{\partial z_0^3} \times n_0^2 \end{aligned}$$

Backprop?

Deep learning Frameworks!!



Algorithm

**Dedicated algorithm
for model**

**Knowing
Statistical
models**

Algorithm

**Dedicated algorithm
for model**

Model Interpretation

**Theories developed
for interpretation**

**Knowing
Statistical
models**

Algorithm

**Dedicated algorithm
for model**

**Fill the lack of
information with
assumption**

Assumption

**Knowing
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models**

Model Interpretation

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Model Interpretation

**Theories developed
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**Existence
Estimability
Optimality**

Theory

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Algorithm

**Dedicated algorithm
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Model Interpretation

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Statistical
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**Fill the lack of
information with
assumption**

**Existence
Estimability
Optimality**

Assumption

Statistical Flaws

Causality

Simpson's Paradox

Theory

Algorithm

Back Propagation

**Knowing
Deep
Learning**

Algorithm

Back Propagation

Model Interpretation

*Blackbox Model
No need to interpret*

Knowing
Deep
Learning

Algorithm

Back Propagation

Model Interpretation

*Blackbox Model
No need to interpret*

**Knowing
Deep
Learning**

No Assumption

Assumption

Algorithm

Back Propagation

Model Interpretation

*Blackbox Model
No need to interpret*

**Knowing
Deep
Learning**

No Assumption

Assumption

No Clear Theory

Theory

Algorithm

Back Propagation

Model Interpretation

*Blackbox Model
No need to interpret*

**Knowing
Deep
Learning**

No Assumption

No Clear Theory

Assumption

Only Prediction

Only Correlation

No bias concept

Theory

**What is *IMPORTANT* is
matrix operation!!**

**Understanding
network**

=

**Know every details
about input & output**

Batch normalization Fully Connected Pooling
Layer normalization Dropout
Spectral normalization Masking Convolution
Skip Connection Padding Dilated convolution
Variational Inference Adversarial Network
Re-parametrization trick Inverse autoregression GRU
Attention mechanism Memory Cell
Self attention
Residual Block

AND
Various Ideas!!

Understanding
network

=

Know every details
about input & output

**In practice, there are techniques
like...**

Practicalities

- Optimization algorithm:
 - Adam(Kingma and Ba, 2014): Effective & Robust to the choice of learning rate
- Initializer: magnitude of random variable matters
 - Xavier Initializer: generally good.

$$W \sim U\left(-\frac{\sqrt{6}}{\sqrt{d_{in} + d_{out}}}, \frac{\sqrt{6}}{\sqrt{d_{in} + d_{out}}}\right)$$

- He: good for deep network

$$W \sim N\left(0, \frac{2}{d_{in}}\right)$$

Practicalities

- Restart & Ensemble: Do if resource allows
- Vanishing gradients:
 - Shallower network
 - Batch normalization
 - Special architecture such as LSTM or GRU
- Exploding gradients:
 - Gradient clipping
- Saturated neurons:
 - Change initializer
 - Change learning rate
 - Scale input values
 - Layer normalization

$$g_h = \frac{\tanh(\mathbf{h})}{\|\tanh(\mathbf{h})\|}$$

Practicalities

- Dead neurons: (especially for ReLU)
 - Reduce learning rate
- Shuffling
 - Crucial and double check if done when training
- Learning rate scheduling
 - Reduce learning rate