Matrix Analysis Homework 6

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Proposition 1. $\mathbf{A} \in M(n, \mathbb{C})$ is diagonalizable if and only if all generalized eigenvectors of matrix $\mathbf{A}$ is eigenvectors of matrix $\mathbf{A}$.

Proof. All generalized eigenvectors of matrix $\mathbf{A}$ is eigenvectors of matrix $\mathbf{A}$.

$\Leftrightarrow$ For all eigenvalues λ of the matrix $\mathbf{A}$,

$$\mathrm{Ker}\left((\mathbf{A}-\lambda\mathbf{I})^n\right) = \mathrm{Ker}\left(\mathbf{A}-\lambda\mathbf{I}\right).$$

$\Leftrightarrow$ For all eigenvalues λ of the matrix $\mathbf{A}$, algebraic multiplicity of λ equals to the geometric multiplicity of λ.

$\Leftrightarrow$ $\mathbf{A}$ is diagonalizable.

□