

Matrix Analysis Homework 6

Tao

tao@example.com

School of Mathematics

Q1

Proposition 1. $\mathbf{A} \in M(n, \mathbb{C})$ is diagonalizable if and only if all generalized eigenvectors of matrix \mathbf{A} are eigenvectors of matrix \mathbf{A} .

Proof. All generalized eigenvectors of matrix \mathbf{A} is eigenvectors of matrix \mathbf{A} .

\Leftrightarrow For all eigenvalues λ of the matrix \mathbf{A} ,

$$\text{Ker}((\mathbf{A} - \lambda \mathbf{I})^n) = \text{Ker}(\mathbf{A} - \lambda \mathbf{I}).$$

\Leftrightarrow For all eigenvalues λ of the matrix \mathbf{A} , algebraic multiplicity of λ equals to the geometric multiplicity of λ .

$\Leftrightarrow \mathbf{A}$ is diagonalizable.

□