Data-Driven Mobility Simulation and Modeling
(CUSP-GX9007)
Solutions
HW #3

Problems – 100 PTS

Problem 1 (25 PTS):
From a photograph one observes that on a level section of highway 10% of the vehicles are trucks, 90% are cars, and that there are 50 vehicles per mile of highway. The trucks travel at 40 mi/hr, and the cars at 50 mi/hr. This highway also has a section with a steep grade on which the speed of the trucks drops to 20 mi/hr, and the speed of the cars to 40 mi/hr. No vehicles enter the observed sections of highways (except at the ends), and the flows are (nearly) stationary.
Determine:
(a) The flow of vehicles on the level section.
(b) The density of vehicles on the grade.
(c) The percent of trucks on the grade as seen on a photograph.
(d) The percent of trucks as seen by a stationary observer on the grade.

a) $q_{level} = \sum k_i v_i = \overset{200}{0.1(50\text{ veh/mi})(40\text{ mi/hr})} + \overset{2250}{0.9(50\text{ veh/mi})(50\text{ mi/hr})} = \underline{\underline{2450\text{ veh/hr}}}$

b) $k_{steep} = k_{trucks} + k_{cars} = \dfrac{q_t}{v_t} + \dfrac{q_c}{v_c} = \dfrac{200\text{ veh/hr}}{20\text{ mi/hr}} + \dfrac{2250\text{ veh/hr}}{40\text{ mi/hr}} = \underline{\underline{66\text{ veh/mile}}}$

c) $\dfrac{k_{truck}}{k_{tot}} = \dfrac{10\text{ veh/mi}}{66\text{ veh/mi}} = 0.15 = \underline{\underline{15\%\text{ ON PHOTO ON GRADE}}}$

d) $\dfrac{q_{truck}}{q_{tot}} = \dfrac{200\text{ vph}}{2450\text{ vph}} = 0.08 = \underline{\underline{8\%\text{ TO OBSERVER ON GRADE}}}$

Problem 2 (15 PTS):
If the speed limit on a highway network is changed so that the average speed drops from v to v' (100km/hr to 80 km/hr, for example), what happens to q and k if people continue to make the same trips every day?

STATE 1 : v, q, k

STATE 2 : v', q', k'

$q = q'$ $\quad$ q IS CONSERVED

$kv = k'v'$

$k' = \dfrac{kv}{v'} = k(>1) \Rightarrow$ K INCREASES

Problem 3 (20 PTS):

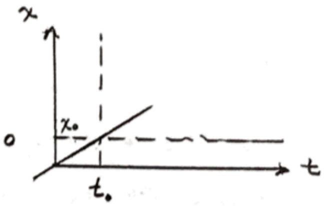
Suppose that the q(t,x) and k(t,x) on a one-directional road are given by the following (in an unspecified system of units):

a) $q(t,x) = q_0 e^{(t/t_0 - x/x_0)}$ and $k(t,x) = k_0 e^{(t/t_0 - x/x_0)}$

b) $q(t,x) = f(x/t)/t$ and $k(t,x) = f(x/t)/x$

c) $\mathrm{q(t,x) = f(x/t)x}$ and $\mathrm{k(t,x) = f(x/t)t}$
$\mathrm{for (t,x) \approx (t_0, x_0) > (0,0)}$

Identify which of these formulas is physically possible, without traffic generation in the vicinity of $(\mathrm{t_0, x_0})$ and which is not (if any) and explain why.



CHECK FOR CONSERVATION: $\frac{\partial q(t,x)}{\partial x} + \frac{\partial k(t,x)}{\partial t} = 0$

a) $\frac{\partial q(t,x)}{\partial x} = q_o e^{t/t_o} e^{-x/x_o}\left(-\frac{1}{x_o}\right)$

$= -\frac{q_o}{x_o} e^{t/t_o - x/x_o}$

$\frac{\partial k(t,x)}{\partial t} = K_o e^{-x/x_o} e^{t/t_o}\left(\frac{1}{t_o}\right) = \frac{k_o}{t_o} e^{t/t_o - x/x_o}$

$\therefore \frac{q_o}{x_o} \cancel{e^{t/t_o - x/x_o}} = \frac{k_o}{t_o} \cancel{e^{t/t_o - x/x_o}} \Rightarrow q_o = k_o \frac{x_o}{t_o} \Rightarrow \frac{q_o}{k_o} = \frac{x_o}{t_o}$

THIS IS PHYSICALLY POSSIBLE IF $\frac{q_o}{k_o} = \frac{x_o}{t_o}$

$q(t,x) = \frac{\partial N(t,x)}{\partial t} \Rightarrow N(t,x) = t_o q_o e^{t/t_o - x/x_o} + C$

$k(t,x) = \frac{\partial N(t,x)}{\partial x} \Rightarrow N(t,x) = x_o k_o e^{t/t_o - x/x_o} + C$

EQUIVALENT SINCE $t_o q_o = x_o k_o$

REPRESENTS VEH. WITH CONSTANT SPEED.

b) $\frac{\partial q(t,x)}{\partial x} = \frac{1}{t^2} f'\left(\frac{x}{t}\right)$

$\frac{\partial k(t,x)}{\partial t} = -\frac{1}{t^2} f'\left(\frac{x}{t}\right)$

$\Rightarrow \frac{1}{t^2} f'\left(\frac{x}{t}\right) - \frac{1}{t^2} f'\left(\frac{x}{t}\right) = 0$

THEREFORE THIS IS ALSO PHYSICALLY POSSIBLE.

$q(t,x) = \frac{\partial N(t,x)}{\partial t} = \frac{f\left(\frac{x}{t}\right)}{t} \Rightarrow N(t,x) = \int \frac{f\left(\frac{x}{t}\right)}{t} dt$

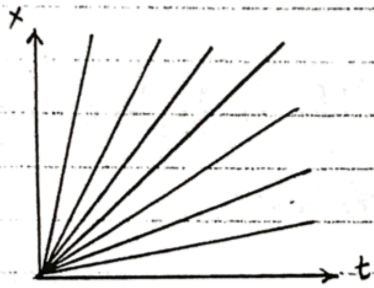
$k(t,x) = \frac{\partial N(t,x)}{\partial x} = \frac{f\left(\frac{x}{t}\right)}{x} \Rightarrow N(t,x) = \int \frac{f\left(\frac{x}{t}\right)}{x} dx$

SO WE KNOW THAT: $\frac{\partial}{\partial t}\left[\ ?\ \right] = \frac{f\left(\frac{x}{t}\right)}{t}$ AND $\frac{\partial}{\partial t}\left[\ ?\ \right] = \frac{f\left(\frac{x}{t}\right)}{x}$

BY SUBSTITUTING $u = \frac{x}{t}$: $N(t,x) = \int f(u)\frac{1}{u}\,du$

$N(t,x) = F(u)$ WHERE $\frac{d}{du}F(u) = \frac{f(u)}{u}\,du$

$N(t,x) = F\left(\frac{x}{t}\right)$.



THUS $N(t,x)$ IS A FUNCTION OF $\frac{x}{t}$ ONLY, AND IS DEPICTED BY A SET OF TRAJECTORIES FORMING A "FAN." THIS MIGHT REPRESENT PLATOON DISPERSION ON A <u>VERY</u> LARGE x AND t SCALE.

C) $\frac{\partial q(t,x)}{\partial x} = f\left(\frac{x}{t}\right) + \frac{x}{t} f'\left(\frac{x}{t}\right)$

$\frac{\partial k(t,x)}{\partial t} = f\left(\frac{x}{t}\right) - \frac{x}{t^3} f'\left(\frac{x}{t}\right)$

CHECK CONSERVATION:

$2f\left(\frac{x}{t}\right) + f'\left(\frac{x}{t}\right)\left(\frac{x}{t} - \frac{x}{t^3}\right) \stackrel{?}{=} 0$ NO.

THEREFORE, <u>NOT</u> PHYSICALLY POSSIBLE, DESPITE UNITS OF q AND k BEING PLAUSIBLE, WITH RATIO OF q/k GIVING UNITS OF x/t.

<u>NOTE</u>: THIS PROBLEM ILLUSTRATES THAT PROPERLY DEFINED q AND k FUNCTIONS MUST PROVIDE CONSERVATION, AND MUST ALLOW ONE TO CONSTRUCT MEANINGFUL VEHICLE TRAJECTORIES AND N-CURVES.

Problem 4 (35 PTS):

A vehicle (A) traveling on a freeway joins a 1/2-mile queue that contains 100 vehicles at a time t = 0 min. Vehicles in this queue pass through the bottleneck at a rate of 50 veh/min. When there is no queue, vehicles travel (in free flow) at a rate of 1 mile/min, provided that the flow satisfies: $q < q_{max} = 100$ veh/min. Do the following:

(a) Determine the delay and the time in queue for our hypothetical vehicle.

(b) Determine the (average) density of vehicles in the queue.

(c) Plot a triangular flow-density relation for our freeway that will be consistent with the given data.

(d) If the (free) flow upstream of the bottleneck is 80 veh/min, determine the location of the end of the queue (in miles upstream of the bottleneck) 1 minute after the arrival of vehicle A. Solve this with the help of a picture, drawn to scale, as follows:

(1) Construct the virtual arrival curve, the bottleneck departure curve and the back-of-queue curve, starting with vehicle A.

(2) Identify on the picture the vehicle (B) that joins the queue at t = 1 min.

(3) Determine the distance in queue for such vehicle.

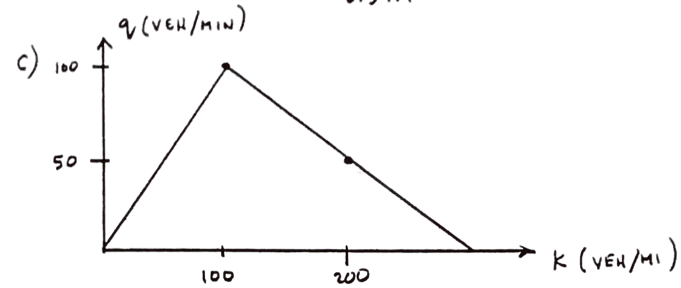
1/2 - MILE QUEUE, CONTAINS 100 VEH AT $t=0$, BN → 50 VEH/MIN
NO QUEUE: 1 MILE/MIN, $q < q_{max} = 100$ VEH/MIN

a) TIME IN QUEUE: $t_q = \frac{100 \text{ VEH}}{50 \text{ VEH/MIN}} = \underline{\underline{2 \text{ MIN}}} = \frac{W}{1 - \frac{V_q}{V_f}}$

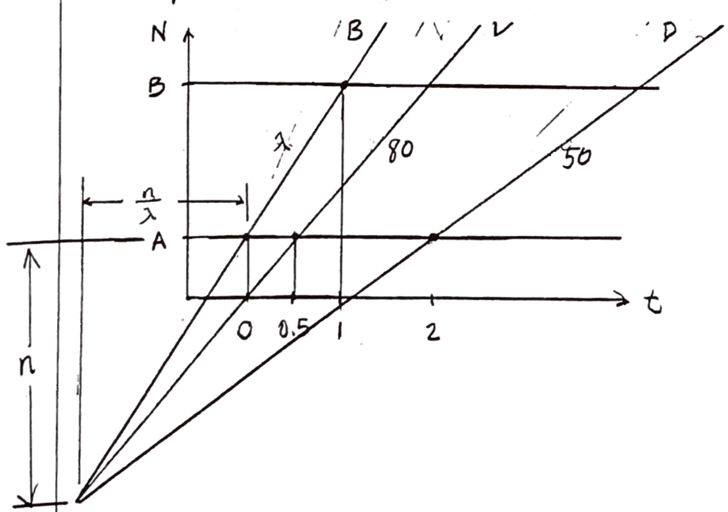
DELAY: $W = t_q\left(1 - \frac{V_q}{V_f}\right) = 2 \text{ MIN}\left(1 - \frac{\left(\frac{0.5 \text{ MI}}{2 \text{ MIN}}\right)}{1 \text{ MI/MIN}}\right) = \underline{\underline{1.5 \text{ MIN}}}$

b) DENSITY IN QUEUE: $\frac{100 \text{ VEH}}{0.5 \text{ MI}} = \underline{\underline{200 \text{ VEH/MI}}}$

c)



d) $q_{UPSTREAM} = 80$ VEH/MIN



$\frac{n}{\lambda} + 2 = \frac{n}{80} + 1.5 = \frac{n}{50}$

$30n = 1.5(80)(50)$

$\underline{n = 200 \text{ VEH}}$

$\frac{200}{\lambda} + 2 = \frac{200}{50}$ $\quad \underline{\lambda = 100 \text{ VEH/MIN}}$

AT $t = 1$: $n = 200 + 100 = 300$ VEH

$t_Q = 300\left(\frac{1}{50} - \frac{1}{100}\right)$

$= 3$ MIN

$l_Q = 3 \text{ MIN}\,(V_q)$

$= 3 \text{ MIN}\left(\frac{0.5 \text{ MI}}{2 \text{ MIN}}\right)$

$= \underline{\underline{0.75 \text{ MILE}}}$