

5 章 三角関数

1 三角比とその応用 BASIC

■255 (1)

斜辺 $\sqrt{2^2 + 1^2} = \sqrt{5}$ より

$$\sin \alpha = \frac{1}{\sqrt{5}}, \cos \alpha = \frac{2}{\sqrt{5}}, \tan \alpha = \frac{1}{2}$$

■255 (2)

$\sqrt{7-3} = 2$ より

$$\sin \alpha = \frac{\sqrt{3}}{\sqrt{7}}, \cos \alpha = \frac{2}{\sqrt{7}}, \tan \alpha = \frac{\sqrt{3}}{2}$$

■255 (3)

$\sqrt{13^2 - 12^2} = 5$ より

$$\sin \alpha = \frac{5}{13}, \cos \alpha = \frac{12}{13}, \tan \alpha = \frac{5}{12}$$

■256 (1)

$$\begin{aligned} & \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{3}{4} - \frac{1}{4} \\ &= \frac{1}{2} \end{aligned}$$

■256 (2)

$$\begin{aligned} & \cos 30^\circ \cos 60^\circ + \sin 30^\circ \sin 60^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

■256 (3)

$$\begin{aligned} & \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ} \\ &= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1} \\ &= \frac{1}{2}(\sqrt{3} - 1)^2 \\ &= \frac{1}{2}(4 - 2\sqrt{3}) \\ &= 2 - \sqrt{3} \end{aligned}$$

■257 (1)

$$\sin 6^\circ = 0.1045$$

■257 (2)

$$\cos 33^\circ = 0.8387$$

■257 (3)

$$\tan 84^\circ = 9.5144$$

■258

距離を x とする

$$\begin{aligned} \tan 22^\circ &= \frac{634}{x} \\ x &= \frac{634}{\tan 22^\circ} \\ &= 1569.3 \dots \end{aligned}$$

$$1569m$$

■259 (1)

$$\sin 81^\circ = \cos(90^\circ - 81^\circ) = \cos 9^\circ$$

■259 (2)

$$\begin{aligned} \cos 56^\circ &= \sin(90^\circ - 56^\circ) \\ &= \sin 34^\circ \end{aligned}$$

■259 (3)

$$\begin{aligned} \tan 77^\circ &= \frac{1}{\tan(90^\circ - 77^\circ)} \\ &= \frac{1}{\tan 13^\circ} \end{aligned}$$

■260 (1)

$$\begin{aligned} & \sin 45^\circ \cos 135^\circ - \cos 45^\circ \sin 135^\circ \\ &= \frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \\ &= -1 \end{aligned}$$

■260 (2)

$$\begin{aligned}\frac{\tan 45^\circ - \tan 150^\circ}{1 + \tan 45^\circ \tan 150^\circ} &= \frac{1 - \left(-\frac{1}{\sqrt{3}}\right)}{1 + 1 \cdot \left(-\frac{1}{\sqrt{3}}\right)} \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\ &= \frac{1}{2}(\sqrt{3} + 1)^2 \\ &= 2 + \sqrt{3}\end{aligned}$$

■260 (3)

$$\begin{aligned}&\cos 120^\circ \cos 150^\circ + \tan 120^\circ \sin 150^\circ \\ &+ \sin 120^\circ \tan 135^\circ \\ &= \left(-\frac{1}{2}\right) \cdot \left(-\frac{\sqrt{3}}{2}\right) + (-\sqrt{3}) \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot (-1) \\ &= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \\ &= -\frac{3}{4}\sqrt{3}\end{aligned}$$

■261 (1)

$$\begin{aligned}\sin 100^\circ &= \sin (180^\circ - 80^\circ) \\ &= \sin 80^\circ \\ &= 0.9848\end{aligned}$$

■261 (2)

$$\begin{aligned}\cos 176^\circ &= \cos (180^\circ - 4^\circ) \\ &= -\cos 4^\circ \\ &= -0.9976\end{aligned}$$

■261 (3)

$$\begin{aligned}\tan 111^\circ &= \tan (180^\circ - 69^\circ) \\ &= -\tan 69^\circ \\ &= -2.6051\end{aligned}$$

■262 (1)

$0^\circ < \alpha < 90^\circ$ より

$$\begin{aligned}\cos \alpha &= \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4} \\ \tan \alpha &= \frac{1}{\sqrt{15}}\end{aligned}$$

■262 (2)

$90^\circ < \alpha < 180^\circ$ より

$$\begin{aligned}\cos \alpha &= -\sqrt{1 - \left(\frac{1}{4}\right)^2} \\ &= -\frac{\sqrt{15}}{4} \\ \tan \alpha &= -\frac{1}{\sqrt{15}}\end{aligned}$$

■262 (3)

$90^\circ < \alpha < 180^\circ + 90^\circ$ より

$$\begin{aligned}\sin \alpha &= \sqrt{1 - \left(-\frac{5}{6}\right)^2} \\ &= \frac{\sqrt{11}}{6} \\ \tan \alpha &= -\frac{\sqrt{11}}{5}\end{aligned}$$

■263 (1)

$$\begin{aligned}1 + \tan^2 \alpha &= \frac{1}{\cos^2 \alpha} \\ 1 + \frac{1}{9} &= \frac{1}{\cos^2 \alpha} \\ \cos^2 \alpha &= \frac{9}{10}\end{aligned}$$

$0^\circ < \alpha < 90^\circ$ より

$$\begin{aligned}\sin \cos \alpha &= \frac{3}{\sqrt{10}} \\ \sin \alpha &= \tan \alpha \cdot \cos \alpha \\ &= \frac{1}{3} \cdot \frac{3}{\sqrt{10}} \\ &= \frac{1}{\sqrt{10}}\end{aligned}$$

■263 (2)

$$\begin{aligned}1 + \tan^2 \alpha &= \frac{1}{\cos^2 \alpha} \\ 1 + 4 &= \frac{1}{\cos^2 \alpha} \\ \cos^2 \alpha &= \frac{1}{5}\end{aligned}$$

$90^\circ < \alpha < 180^\circ$ より

$$\begin{aligned}\cos \alpha &= -\frac{1}{\sqrt{5}} \\ \sin \alpha &= -2 \cdot \left(-\frac{1}{\sqrt{5}}\right) \\ &= \frac{2}{\sqrt{5}}\end{aligned}$$

■264 (1)

$$\begin{aligned}\text{正弦定理より } \frac{a}{\sin 30^\circ} &= \frac{4}{\sin 45^\circ} \\ a &= 4 \cdot \sqrt{2} \cdot \frac{1}{2} \\ &= 2\sqrt{2}\end{aligned}$$

■264 (2)

$$\begin{aligned}\text{正弦定理より } \frac{2}{\sin 45^\circ} &= \frac{\sqrt{3}}{\sin C} \\ \sin C &= \sqrt{3} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{6}}{4}\end{aligned}$$

■264 (3)

$$\begin{aligned}\text{正弦定理より } \frac{a}{\sin 45^\circ} &= \frac{5}{\sin 30^\circ} \\ a &= 5 \cdot 2 \cdot \frac{1}{\sqrt{2}} \\ &= 5\sqrt{2}\end{aligned}$$

■265

$$\begin{aligned}\text{正弦定理より半径を } r \text{ とすると } 2r &= \frac{a}{\sin 60^\circ} \\ r &= \frac{1}{2} \cdot a \cdot \frac{2}{\sqrt{3}} \\ &= \frac{\sqrt{3}}{3}a\end{aligned}$$

■266 (1)

$$\begin{aligned}\text{余弦定理より } a^2 &= 3 + 16 - 2 \cdot 4 \cdot \sqrt{3} \cdot \cos 30^\circ \\ &= 19 - 8\sqrt{3} \cdot \frac{\sqrt{3}}{2} \\ &= 7 \\ a &= \sqrt{7}\end{aligned}$$

■266 (2)

$$\begin{aligned}\text{余弦定理より } b^2 &= 6 + 3 - 2 \cdot \sqrt{6} \sqrt{3} \cdot \cos 135^\circ \\ &= 9 + 6 \\ &= 15 \\ b &= \sqrt{15}\end{aligned}$$

■266 (3)

$$\begin{aligned}\text{余弦定理より } c^2 &= 4 + 27 - 2 \cdot 2 \cdot 3\sqrt{3} \cdot \cos 150^\circ \\ &= 31 + 18 \\ &= 49 \\ c &= 7\end{aligned}$$

■267

$$\begin{aligned}\cos A &= \frac{16 + 25 - 4}{2 \cdot 4 \cdot 5} \\ &= \frac{37}{40} \\ \cos B &= \frac{4 + 25 - 16}{2 \cdot 2 \cdot 5} \\ &= \frac{13}{20} \\ \cos C &= \frac{4 + 16 - 25}{2 \cdot 2 \cdot 4} \\ &= -\frac{5}{16}\end{aligned}$$

■268 (1)

$$\begin{aligned}\triangle ABC &= \frac{1}{2} \cdot 5 \cdot 7 \cdot \sin 45^\circ \\ &= \frac{35}{4} \sqrt{2}\end{aligned}$$

■268 (2)

$$\begin{aligned}\triangle ABC &= \frac{1}{2} \cdot 2 \cdot 3 \cdot \sin 150^\circ \\ &= \frac{3}{2}\end{aligned}$$

■269

$$\begin{aligned}\triangle ABC &= \frac{1}{2} \cdot 7 \cdot c \cdot \sin 30^\circ \\ 9 &= \frac{7}{2} c \cdot \frac{1}{2} \\ c &= \frac{36}{7}\end{aligned}$$

■270 (1)

$$\begin{aligned}\cos C &= \frac{25 + 36 - 81}{2 \cdot 5 \cdot 6} \\ &= \frac{-20}{2 \cdot 5 \cdot 6} \\ &= -\frac{1}{3}\end{aligned}$$

■270 (2)

$0^\circ < c < 180^\circ$ より

$$\sin c = \sqrt{1 - \left(-\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$$

■270 (3)

$$\begin{aligned}S &= \frac{1}{2} \cdot 5 \cdot 6 \cdot \sin C \\ &= 15 \cdot \frac{2\sqrt{2}}{3} \\ &= 10\sqrt{2}\end{aligned}$$

■271 (1)

$$\cos A = \frac{41 + 64 - 25}{2 \cdot 7 \cdot 8} = \frac{11}{14}$$

$0^\circ < A < 180^\circ$ より

$$\begin{aligned}\sin A &= \sqrt{1 - \left(\frac{11}{14}\right)^2} = \frac{\sqrt{75}}{14} = \frac{5\sqrt{3}}{14} \\ \triangle ABC &= \frac{1}{2} \cdot 7 \cdot 8 \cdot \frac{5\sqrt{3}}{14} \\ &= 10\sqrt{3}\end{aligned}$$

■271 (2)

$$\cos A = \frac{9 + 16 - 4}{2 \cdot 3 \cdot 4} = \frac{7}{8}$$

$0^\circ < A < 180^\circ$ より

$$\begin{aligned}\sin A &= \sqrt{1 - \left(\frac{7}{8}\right)^2} = \frac{\sqrt{15}}{8} \\ \triangle ABC &= \frac{1}{2} \cdot 3 \cdot 4 \cdot \frac{\sqrt{15}}{8} \\ &= \frac{3}{4}\sqrt{15}\end{aligned}$$