5 章 三角関数
1　三角比とその応用
BASIC

■255 (1)

斜辺 $\sqrt{2^2 + 1^2} = \sqrt{5}$ より

$$\sin \alpha = \frac{1}{\sqrt{5}}, \cos \alpha = \frac{2}{\sqrt{5}}, \tan \alpha = \frac{1}{2}$$

■255 (2)

$\sqrt{7-3} = 2$ より

$$\sin \alpha = \frac{\sqrt{3}}{\sqrt{7}}, \cos \alpha = \frac{2}{\sqrt{7}}, \tan \alpha = \frac{\sqrt{3}}{2}$$

■255 (3)

$\sqrt{13^2 - 12^2} = 5$ より

$$\sin \alpha = \frac{5}{13}, \cos \alpha = \frac{12}{13}, \tan \alpha = \frac{5}{12}$$

■256 (1)

$$\sin 60° \cos 30° - \cos 60° \sin 30°$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2}$$
$$= \frac{3}{4} - \frac{1}{4}$$
$$= \frac{1}{2}$$

■256 (2)

$$\cos 30° \cos 60° + \sin 30° \sin 60°$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{2}$$

■256 (3)

$$\frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$$
$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$$
$$= \frac{1}{2}(\sqrt{3} - 1)^2$$
$$= \frac{1}{2}(4 - 2\sqrt{3})$$
$$= 2 - \sqrt{3}$$

■257 (1)

$$\sin 6° = 0.1045$$

■257 (2)

$$\cos 33° = 0.8387$$

■257 (3)

$$\tan 84° = 9.5144$$

■258

距離を x とする
$$\tan 22° = \frac{634}{x}$$
$$x = \frac{634}{6404}$$
$$= 1569.3 \cdots$$

$1569m$

■259 (1)

$$\sin 81° = \cos(90° - 81°) = \cos 9°$$

■259 (2)

$$\cos 56° = \sin(90° - 56°)$$
$$= \sin 34°$$

■259 (3)

$$\tan 77° = \frac{1}{\tan(90° - 77°)}$$
$$= \frac{1}{\tan 13°}$$

■260 (1)

$$\sin 45° \cos 135° - \cos 45° \sin 135°$$
$$= \frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$$
$$= -1$$

■260 (2)

$$\frac{\tan 45° - \tan 150°}{1 + \tan 45° \tan 150°} = \frac{1 - \left(-\frac{1}{\sqrt{3}}\right)}{1 + 1 \cdot \left(-\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$
$$= \frac{1}{2}(\sqrt{3} + 1)^2$$
$$= 2 + \sqrt{3}$$

■260 (3)

$$\cos 120° \cos 150° + \tan 120° \sin 150°$$
$$+ \sin 120° \tan 135°$$
$$= \left(-\frac{1}{2}\right) \cdot \left(-\frac{\sqrt{3}}{2}\right) + (-\sqrt{3}) \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot (-1)$$
$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$
$$= -\frac{3}{4}\sqrt{3}$$

■261 (1)

$$\sin 100° = \sin (180° - 80°)$$
$$= \sin 80°$$
$$= 0.9848$$

■261 (2)

$$\cos 176° = \cos (180° - 4°)$$
$$= -\cos 4°$$
$$= -0.9976$$

■261 (3)

$$\tan 111° = \tan (180° - 69°)$$
$$= -\tan 69°$$
$$= -2.6051$$

■262 (1)

$0° < \alpha < 90°$ より

$$\cos \alpha = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$$
$$\tan \alpha = \frac{1}{\sqrt{15}}$$

■262 (2)

$90° < \alpha < 180°$ より

$$\cos \alpha = -\sqrt{1 - \left(\frac{1}{4}\right)^2}$$
$$= -\frac{\sqrt{15}}{4}$$
$$\tan \alpha = -\frac{1}{\sqrt{15}}$$

■262 (3)

$90° < \alpha < 180° + 90°$ より

$$\sin \alpha = \sqrt{1 - \left(-\frac{5}{6}\right)^2}$$
$$= \frac{\sqrt{11}}{6}$$
$$\tan \alpha = -\frac{\sqrt{11}}{5}$$

■263 (1)

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$
$$1 + \frac{1}{9} = \frac{1}{\cos^2 \alpha}$$
$$\cos^2 \alpha = \frac{9}{10}$$

$0° < \alpha < 90°$ より

$$\sin \cos \alpha = \frac{3}{\sqrt{10}}$$
$$\sin \alpha = \tan \alpha \cdot \cos \alpha$$
$$= \frac{1}{3} \cdot \frac{3}{\sqrt{10}}$$
$$= \frac{1}{\sqrt{10}}$$

■263 (2)

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$
$$1 + 4 = \frac{1}{\cos^2 \alpha}$$
$$\cos^2 \alpha = \frac{1}{5}$$

$90° < \alpha < 180°$ より

$$\cos\alpha = -\frac{1}{\sqrt{5}}$$
$$\sin\alpha = -2 \cdot \left(-\frac{1}{\sqrt{5}}\right)$$
$$= \frac{2}{\sqrt{5}}$$

■264 (1)

正弦定理より $\dfrac{a}{\sin 30°} = \dfrac{4}{\sin 45°}$

$$a = 4 \cdot \sqrt{2} \cdot \frac{1}{2}$$
$$= 2\sqrt{2}$$

■264 (2)

正弦定理より $\dfrac{2}{\sin 45°} = \dfrac{\sqrt{3}}{\sin C}$

$$\sin C = \sqrt{3} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{6}}{4}$$

■264 (3)

正弦定理より $\dfrac{a}{\sin 45°} = \dfrac{5}{\sin 30°}$

$$a = 5 \cdot 2 \cdot \frac{1}{\sqrt{2}}$$
$$= 5\sqrt{2}$$

■265

正弦定理より半径を r とすると $2r = \dfrac{a}{\sin 60°}$

$$r = \frac{1}{2} \cdot a \cdot \frac{2}{\sqrt{3}}$$
$$= \frac{\sqrt{3}}{3}a$$

■266 (1)

余弦定理より $a^2 = 3 + 16 - 2 \cdot 4 \cdot \sqrt{3} \cdot \cos 30°$

$$= 19 - 8\sqrt{3} \cdot \frac{\sqrt{3}}{2}$$
$$= 7$$
$$a = \sqrt{7}$$

■266 (2)

余弦定理より $b^2 = 6 + 3 - 2 \cdot \sqrt{6}\sqrt{3} \cdot \cos 135°$

$$= 9 + 6$$
$$= 15$$
$$b = \sqrt{15}$$

■266 (3)

余弦定理より $c^2 = 4 + 27 - 2 \cdot 2 \cdot 3\sqrt{3} \cdot \cos 150°$

$$= 31 + 18$$
$$= 49$$
$$c = 7$$

■267

$$\cos A = \frac{16 + 25 - 4}{2 \cdot 4 \cdot 5}$$
$$= \frac{37}{40}$$
$$\cos B = \frac{4 + 25 - 16}{2 \cdot 2 \cdot 5}$$
$$= \frac{13}{20}$$
$$\cos C = \frac{4 + 16 - 25}{2 \cdot 2 \cdot 4}$$
$$= -\frac{5}{16}$$

■268 (1)

$$\triangle ABC = \frac{1}{2} \cdot 5 \cdot 7 \cdot \sin 45°$$
$$= \frac{35}{4}\sqrt{2}$$

■268 (2)

$$\triangle ABC = \frac{1}{2} \cdot 2 \cdot 3 \cdot \sin 150°$$
$$= \frac{3}{2}$$

■269

$$\triangle ABC = \frac{1}{2} \cdot 7 \cdot c \cdot \sin 30°$$
$$9 = \frac{7}{2}c \cdot \frac{1}{2}$$
$$c = \frac{36}{7}$$

■270 (1)

$$\cos C = \frac{25 + 36 - 81}{2 \cdot 5 \cdot 6}$$
$$= \frac{-20}{2 \cdot 5 \cdot 6}$$
$$= -\frac{1}{3}$$

■270 (2)

$0° < c < 180°$ より

$$\sin c = \sqrt{1 - \left(-\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$$

■270 (3)

$$S = \frac{1}{2} \cdot 5 \cdot 6 \cdot \sin C$$
$$= 15 \cdot \frac{2\sqrt{2}}{3}$$
$$= 10\sqrt{2}$$

■271 (1)

$$\cos A = \frac{41 + 64 - 25}{2 \cdot 7 \cdot 8} = \frac{11}{14}$$

$0° < A < 180°$ より

$$\sin A = \sqrt{1 - \left(\frac{11}{14}\right)^2} = \frac{\sqrt{75}}{14} = \frac{5\sqrt{3}}{14}$$
$$\triangle ABC = \frac{1}{2} \cdot 7 \cdot 8 \cdot \frac{5\sqrt{3}}{14}$$
$$= 10\sqrt{3}$$

■271 (2)

$$\cos A = \frac{9 + 16 - 4}{2 \cdot 3 \cdot 4} = \frac{7}{8}$$

$0° < A < 180°$ より

$$\sin A = \sqrt{1 - \left(\frac{7}{8}\right)^2} = \frac{\sqrt{15}}{8}$$
$$\triangle ABC = \frac{1}{2} \cdot 3 \cdot 4 \cdot \frac{\sqrt{15}}{8}$$
$$= \frac{3}{4}\sqrt{15}$$