新 基礎数学　改訂版，大日本図書

5 章 三角関数

1　三角比とその応用

BASIC

■255 (1)

斜辺 $\sqrt{2^2+1^2}=\sqrt{5}$ より

$$\sin\alpha=\frac{1}{\sqrt{5}},\cos\alpha=\frac{2}{\sqrt{5}},\tan\alpha=\frac{1}{2}$$

■255 (2)

$\sqrt{7-3}=2$ より

$$\sin\alpha=\frac{\sqrt{3}}{\sqrt{7}},\cos\alpha=\frac{2}{\sqrt{7}},\tan\alpha=\frac{\sqrt{3}}{2}$$

■255 (3)

$\sqrt{13^2-12^2}=5$ より

$$\sin\alpha=\frac{5}{13},\cos\alpha=\frac{12}{13},\tan\alpha=\frac{5}{12}$$

■256 (1)

$$\begin{aligned}&\sin 60^\circ\cos 30^\circ-\cos 60^\circ\sin 30^\circ\\&=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{1}{2}\cdot\frac{1}{2}\\&=\frac{3}{4}-\frac{1}{4}\\&=\frac{1}{2}\end{aligned}$$

■256 (2)

$$\begin{aligned}&\cos 30^\circ\cos 60^\circ+\sin 30^\circ\sin 60^\circ\\&=\frac{\sqrt{3}}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\\&=\frac{\sqrt{3}}{2}\end{aligned}$$

■256 (3)

$$\begin{aligned}&\frac{\tan 60^\circ-\tan 45^\circ}{1+\tan 60^\circ\tan 45^\circ}\\&=\frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1}\\&=\frac{1}{2}(\sqrt{3}-1)^2\\&=\frac{1}{2}(4-2\sqrt{3})\\&=2-\sqrt{3}\end{aligned}$$

■257 (1)

$$\sin 6^\circ=0.1045$$

■257 (2)

$$\cos 33^\circ=0.8387$$

■257 (3)

$$\tan 84^\circ=9.5144$$

■258

距離を x とする

$$\begin{aligned}\tan 22^\circ&=\frac{634}{x}\\x&=\frac{634}{6404}\\&=1569.3\cdots\end{aligned}$$

$1569m$

■259 (1)

$$\sin 81^\circ=\cos(90^\circ-81^\circ)=\cos 9^\circ$$

■259 (2)

$$\begin{aligned}\cos 56^\circ&=\sin(90^\circ-56^\circ)\\&=\sin 34^\circ\end{aligned}$$

■259 (3)

$$\begin{aligned}\tan 77^\circ&=\frac{1}{\tan(90^\circ-77^\circ)}\\&=\frac{1}{\tan 13^\circ}\end{aligned}$$

■260 (1)

$$\begin{aligned}&\sin 45^\circ\cos 135^\circ-\cos 45^\circ\sin 135^\circ\\&=\frac{1}{\sqrt{2}}\cdot\left(-\frac{1}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\\&=-1\end{aligned}$$

■260 (2)

$$\begin{aligned}\frac{\tan 45^\circ - \tan 150^\circ}{1+\tan 45^\circ \tan 150^\circ} &= \frac{1-\left(-\frac{1}{\sqrt{3}}\right)}{1+1\cdot\left(-\frac{1}{\sqrt{3}}\right)} \\ &= \frac{\sqrt{3}+1}{\sqrt{3}-1} \\ &= \frac{1}{2}(\sqrt{3}+1)^2 \\ &= 2+\sqrt{3}\end{aligned}$$

■260 (3)

$$\begin{aligned}&\cos 120^\circ \cos 150^\circ + \tan 120^\circ \sin 150^\circ \\ &+ \sin 120^\circ \tan 135^\circ \\ =&\left(-\frac{1}{2}\right)\cdot\left(-\frac{\sqrt{3}}{2}\right)+(-\sqrt{3})\cdot\frac{1}{2}+\frac{\sqrt{3}}{2}\cdot(-1) \\ =&\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2} \\ =&-\frac{3}{4}\sqrt{3}\end{aligned}$$

■261 (1)

$$\begin{aligned}\sin 100^\circ &= \sin(180^\circ - 80^\circ) \\ &= \sin 80^\circ \\ &= 0.9848\end{aligned}$$

■261 (2)

$$\begin{aligned}\cos 176^\circ &= \cos(180^\circ - 4^\circ) \\ &= -\cos 4^\circ \\ &= -0.9976\end{aligned}$$

■261 (3)

$$\begin{aligned}\tan 111^\circ &= \tan(180^\circ - 69^\circ) \\ &= -\tan 69^\circ \\ &= -2.6051\end{aligned}$$

■262 (1)

$0^\circ < \alpha < 90^\circ$ より

$$\begin{aligned}\cos\alpha = \sqrt{1-\left(\frac{1}{4}\right)^2} &= \frac{\sqrt{15}}{4} \\ \tan\alpha &= \frac{1}{\sqrt{15}}\end{aligned}$$

■262 (2)

$90^\circ < \alpha < 180^\circ$ より

$$\begin{aligned}\cos\alpha &= -\sqrt{1-\left(\frac{1}{4}\right)^2} \\ &= -\frac{\sqrt{15}}{4} \\ \tan\alpha &= -\frac{1}{\sqrt{15}}\end{aligned}$$

■262 (3)

$90^\circ < \alpha < 180^\circ + 90^\circ$ より

$$\begin{aligned}\sin\alpha &= \sqrt{1-\left(-\frac{5}{6}\right)^2} \\ &= \frac{\sqrt{11}}{6} \\ \tan\alpha &= -\frac{\sqrt{11}}{5}\end{aligned}$$

■263 (1)

$$\begin{aligned}1+\tan^2\alpha &= \frac{1}{\cos^2\alpha} \\ 1+\frac{1}{9} &= \frac{1}{\cos^2\alpha} \\ \cos^2\alpha &= \frac{9}{10}\end{aligned}$$

$0^\circ < \alpha < 90^\circ$ より

$$\begin{aligned}\sin\cos\alpha &= \frac{3}{\sqrt{10}} \\ \sin\alpha &= \tan\alpha\cdot\cos\alpha \\ &= \frac{1}{3}\cdot\frac{3}{\sqrt{10}} \\ &= \frac{1}{\sqrt{10}}\end{aligned}$$

■263 (2)

$$\begin{aligned}1+\tan^2\alpha &= \frac{1}{\cos^2\alpha} \\ 1+4 &= \frac{1}{\cos^2\alpha} \\ \cos^2\alpha &= \frac{1}{5}\end{aligned}$$

$90° < \alpha < 180°$ より

$$\cos\alpha = -\frac{1}{\sqrt{5}}$$

$$\begin{aligned}\sin\alpha &= -2\cdot\left(-\frac{1}{\sqrt{5}}\right)\\ &= \frac{2}{\sqrt{5}}\end{aligned}$$

■264 (1)

正弦定理より $\dfrac{a}{\sin 30°} = \dfrac{4}{\sin 45°}$

$$\begin{aligned}a &= 4\cdot\sqrt{2}\cdot\frac{1}{2}\\ &= 2\sqrt{2}\end{aligned}$$

■264 (2)

正弦定理より $\dfrac{2}{\sin 45°} = \dfrac{\sqrt{3}}{\sin C}$

$$\begin{aligned}\sin C &= \sqrt{3}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\\ &= \frac{\sqrt{6}}{4}\end{aligned}$$

■264 (3)

正弦定理より $\dfrac{a}{\sin 45°} = \dfrac{5}{\sin 30°}$

$$\begin{aligned}a &= 5\cdot 2\cdot\frac{1}{\sqrt{2}}\\ &= 5\sqrt{2}\end{aligned}$$

■265

正弦定理より半径を r とすると $2r = \dfrac{a}{\sin 60°}$

$$\begin{aligned}r &= \frac{1}{2}\cdot a\cdot\frac{2}{\sqrt{3}}\\ &= \frac{\sqrt{3}}{3}a\end{aligned}$$

■266 (1)

余弦定理より $a^2 = 3 + 16 - 2\cdot 4\cdot\sqrt{3}\cdot\cos 30°$

$$\begin{aligned}&= 19 - 8\sqrt{3}\cdot\frac{\sqrt{3}}{2}\\ &= 7\\ a &= \sqrt{7}\end{aligned}$$

■266 (2)

余弦定理より $b^2 = 6 + 3 - 2\cdot\sqrt{6}\sqrt{3}\cdot\cos 135°$

$$\begin{aligned}&= 9 + 6\\ &= 15\\ b &= \sqrt{15}\end{aligned}$$

■266 (3)

余弦定理より $c^2 = 4 + 27 - 2\cdot 2\cdot 3\sqrt{3}\cdot\cos 150°$

$$\begin{aligned}&= 31 + 18\\ &= 49\\ c &= 7\end{aligned}$$

■267

$$\begin{aligned}\cos A &= \frac{16+25-4}{2\cdot 4\cdot 5}\\ &= \frac{37}{40}\\ \cos B &= \frac{4+25-16}{2\cdot 2\cdot 5}\\ &= \frac{13}{20}\\ \cos C &= \frac{4+16-25}{2\cdot 2\cdot 4}\\ &= -\frac{5}{16}\end{aligned}$$

■268 (1)

$$\begin{aligned}\triangle ABC &= \frac{1}{2}\cdot 5\cdot 7\cdot\sin 45°\\ &= \frac{35}{4}\sqrt{2}\end{aligned}$$

■268 (2)

$$\begin{aligned}\triangle ABC &= \frac{1}{2}\cdot 2\cdot 3\cdot\sin 150°\\ &= \frac{3}{2}\end{aligned}$$

■269

$$\begin{aligned}\triangle ABC &= \frac{1}{2}\cdot 7\cdot c\cdot\sin 30°\\ 9 &= \frac{7}{2}c\cdot\frac{1}{2}\\ c &= \frac{36}{7}\end{aligned}$$

■270 (1)

$$\begin{aligned}\cos C &= \frac{25+36-81}{2\cdot5\cdot6}\\ &= \frac{-20}{2\cdot5\cdot6}\\ &= -\frac{1}{3}\end{aligned}$$

■270 (2)

$0° < c < 180°$ より

$$\sin c = \sqrt{1-\left(-\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$$

■270 (3)

$$\begin{aligned}S &= \frac{1}{2}\cdot5\cdot6\cdot\sin C\\ &= 15\cdot\frac{2\sqrt{2}}{3}\\ &= 10\sqrt{2}\end{aligned}$$

■271 (1)

$$\cos A = \frac{41+64-25}{2\cdot7\cdot8} = \frac{11}{14}$$

$0° < A < 180°$ より

$$\sin A = \sqrt{1-\left(\frac{11}{14}\right)^2} = \frac{\sqrt{75}}{14} = \frac{5\sqrt{3}}{14}$$

$$\begin{aligned}\triangle ABC &= \frac{1}{2}\cdot7\cdot8\cdot\frac{5\sqrt{3}}{14}\\ &= 10\sqrt{3}\end{aligned}$$

■271 (2)

$$\cos A = \frac{9+16-4}{2\cdot3\cdot4} = \frac{7}{8}$$

$0° < A < 180°$ より

$$\sin\ A = \sqrt{1-\left(\frac{7}{8}\right)^2} = \frac{\sqrt{15}}{8}$$

$$\begin{aligned}\triangle ABC &= \frac{1}{2}\cdot3\cdot4\cdot\frac{\sqrt{15}}{8}\\ &= \frac{3}{4}\sqrt{15}\end{aligned}$$

CHECK

■272 (1)

対辺 $\sqrt{3^2-(\sqrt{2})^2} = \sqrt{7}$ より

$$\sin\alpha = \frac{\sqrt{7}}{3},\quad \cos\alpha = \frac{\sqrt{2}}{3},\quad \tan\alpha = \frac{\sqrt{7}}{\sqrt{2}}$$

■272 (2)

斜辺 $\sqrt{1^2+(\sqrt{5})^2} = \sqrt{6}$ より

$$\sin\alpha = \frac{\sqrt{5}}{\sqrt{6}},\quad \cos\alpha = \frac{1}{\sqrt{6}},\quad \tan\alpha = \sqrt{5}$$

■273 (1)

$\triangle ABD$ において $\angle ABD = 150°$ かつ $AB = DB$

$DB\cos 30° = \sqrt{3}$

$DB = \sqrt{3}\cdot\dfrac{2}{\sqrt{3}} = 2$

$\therefore AB = 2$

■273 (2)

$$\begin{aligned}\tan 15° &= \frac{CD}{AB+BC}\\ &= \frac{1}{2+\sqrt{3}}\\ &= 2-\sqrt{3}\end{aligned}$$

■274

$$\begin{aligned}BH &= 815\sin 34°\\ &= 815\times0.5592\\ &= 455.748 \approx 455.7 \qquad \therefore 456m\end{aligned}$$

■275 (1)

$$\begin{aligned}&\sin60°\cos30° + \cos120°\sin150° + \sin135°\cos180°\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)\cdot\frac{1}{2} + \frac{1}{\sqrt{2}}\cdot(-1)\\ &= \frac{3}{4}-\frac{1}{4}-\frac{\sqrt{2}}{2}\\ &= \frac{1-\sqrt{2}}{2}\end{aligned}$$

■275 (2)

$$\begin{aligned}
&\frac{\tan 30^\circ - \tan 135^\circ - \tan 180^\circ}{1+\tan 120^\circ \cdot \tan 45^\circ}\\
&= \frac{\frac{1}{\sqrt{3}} - (-1) - 0}{1+(-\sqrt{3})\cdot 1}\\
&= \frac{1+\sqrt{3}}{\sqrt{3}(1-\sqrt{3})}\\
&= \frac{(1+\sqrt{3})^2}{\sqrt{3}(1-3)} = \frac{4+2\sqrt{3}}{-2\sqrt{3}}\\
&= -\frac{2+\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}+3}{3}
\end{aligned}$$

■276

$90^\circ < \alpha < 180^\circ$ なので $\sin\alpha > 0$

$$\begin{aligned}
\sin\alpha &= \sqrt{1-\left(-\frac{2}{5}\right)^2}\\
&= \frac{\sqrt{21}}{5}\\
\tan\alpha &= -\frac{\sqrt{21}}{2}
\end{aligned}$$

■277

$$\begin{aligned}
1+\tan^2\alpha &= \frac{1}{\cos^2\alpha}\\
1+9 &= \frac{1}{\cos^2\alpha}\\
\cos^2\alpha &= \frac{1}{10}
\end{aligned}$$

$90^\circ < \alpha < 180^\circ$ なので $\cos\alpha < 0$

$$\begin{aligned}
\cos\alpha &= -\frac{1}{\sqrt{10}}\\
\sin\alpha &= \tan\alpha \cdot \cos\alpha\\
&= (-3)\cdot\left(-\frac{1}{\sqrt{10}}\right)\\
&= \frac{3}{\sqrt{10}}
\end{aligned}$$

■278 (1)

$$A = 180^\circ - (105^\circ + 30^\circ) = 45^\circ$$

正弦定理より

$$\begin{aligned}
&2R = \frac{\sqrt{6}}{\sin 45^\circ}, \quad c = 2R\sin 30^\circ\\
&2R = \sqrt{6}\cdot\sqrt{2} = 2\sqrt{3}\\
&\therefore R = \sqrt{3}\\
&\quad c = 2\sqrt{3}\cdot\frac{1}{2} = \sqrt{3}
\end{aligned}$$

■278 (2)

余弦定理より

$$\begin{aligned}
c^2 &= 3^2+5^2-2\cdot 3\cdot 5\cdot\cos 120^\circ\\
&= 9+25-30\cdot\left(-\frac{1}{2}\right)\\
&= 34+15 = 49\\
\therefore c &= 7 \quad (c>0)
\end{aligned}$$

面積 S は

$$\begin{aligned}
S &= \frac{1}{2}\cdot 3\cdot 5\cdot\sin 120^\circ\\
&= \frac{15}{2}\cdot\frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{4}
\end{aligned}$$

■278 (3)

余弦定理より

$$\begin{aligned}
\cos B &= \frac{7^2+3^2-8^2}{2\cdot 7\cdot 3}\\
&= \frac{49+9-64}{42} = \frac{-6}{42} = -\frac{1}{7}
\end{aligned}$$

$0^\circ < B < 180^\circ$ より $\sin B > 0$ なので

$$\sin B = \sqrt{1-\left(-\frac{1}{7}\right)^2} = \sqrt{\frac{48}{49}} = \frac{4\sqrt{3}}{7}$$

面積 S は

$$\begin{aligned}
S &= \frac{1}{2}\cdot 7\cdot 3\cdot\frac{4\sqrt{3}}{7}\\
&= 6\sqrt{3}
\end{aligned}$$

STEP UP

■279

$$\begin{cases} a = b\cos C + c\cos B & \dots(1) \\ b = c\cos A + a\cos C & \dots(2) \\ c = a\cos B + b\cos A & \dots(3) \end{cases}$$

(3) $\times c-$ (1) $\times a$ より

$$\begin{aligned} c^2 - a^2 &= (ac\cos B + bc\cos A) \\ &\quad - (ab\cos C + ac\cos B) \\ c^2 - a^2 &= bc\cos A - ab\cos C \quad \dots(4) \end{aligned}$$

(4) $+$ (2) $\times b$ より

$$\begin{aligned} &(c^2 - a^2) + b^2 = \\ &(bc\cos A - ab\cos C) + \\ &(bc\cos A + ab\cos C) \\ b^2 + c^2 - a^2 &= 2bc\cos A \\ \therefore a^2 &= b^2 + c^2 - 2bc\cos A \end{aligned}$$

■280 (1)

$$\begin{aligned} AH &= 10\sqrt{2} \times \frac{1}{2} = 5\sqrt{2} \\ \tan\angle OAH &= \frac{9}{5\sqrt{2}} = \frac{9\sqrt{2}}{10} \\ &\approx 1.2726\cdots \approx 1.27 \\ \therefore \angle OAH &\approx 52^\circ \end{aligned}$$

■280 (2)

$$\begin{aligned} HM &= 5 \\ \tan\angle OMH &= \frac{9}{5} = 1.8 \\ \therefore \angle OMH &\approx 61^\circ \end{aligned}$$

■280 (3)

$$\begin{aligned} OB &= \sqrt{9^2 + (5\sqrt{2})^2} \\ &= \sqrt{81 + 50} = \sqrt{131} \end{aligned}$$

余弦定理より

$$\begin{aligned} \cos\angle BOC &= \frac{OB^2 + OC^2 - BC^2}{2 \cdot OB \cdot OC} \\ &= \frac{131 + 131 - 10^2}{2 \cdot 131} \\ &= \frac{162}{262} \approx 0.6183\dots \\ \therefore \angle BOC &\approx 52^\circ \end{aligned}$$

■281

正弦定理より, $2R = \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\therefore \sin A = \dfrac{a}{2R}$

これより, 面積 S は

$$\begin{aligned} S &= \frac{1}{2}bc\sin A \\ &= \frac{bc}{2} \cdot \frac{a}{2R} \\ &= \frac{abc}{4R} \quad \dots(\text{証明終}) \end{aligned}$$

また, $a = 2R\sin A, b = 2R\sin B, c = 2R\sin C$ を $S = \frac{abc}{4R}$ に代入すると

$$\begin{aligned} S &= \frac{1}{4R} \cdot (2R\sin A) \cdot (2R\sin B) \cdot (2R\sin C) \\ &= 2R^2 \sin A \sin B \sin C \quad \dots(\text{証明終}) \end{aligned}$$

■282 (1)

例題の結果（$S = \frac{1}{2}ab\sin C$ 等）を用いて

$$\begin{aligned} S &= \frac{1}{2} \cdot 12 \cdot 14 \cdot \sin 60^\circ \\ &= 6 \cdot 14 \cdot \frac{\sqrt{3}}{2} \\ &= 42\sqrt{3} \end{aligned}$$

■282 (2)

$\triangle ABD$ で余弦定理より

$$\begin{aligned} \cos A &= \frac{4^2 + 9^2 - 7^2}{2 \cdot 4 \cdot 9} \\ &= \frac{16 + 81 - 49}{72} = \frac{48}{72} = \frac{2}{3} \end{aligned}$$

$0^\circ < A < 180^\circ$ より $\sin A > 0$ なので

$$\sin A = \sqrt{1 - \left(\frac{2}{3}\right)^2} = \frac{\sqrt{5}}{3}$$

$\triangle CBD$ で余弦定理より

$$\begin{aligned} \cos C &= \frac{8^2 + 3^2 - 7^2}{2 \cdot 8 \cdot 3} \\ &= \frac{64 + 9 - 49}{48} = \frac{24}{48} = \frac{1}{2} \end{aligned}$$

$0^\circ < C < 180^\circ$ より $\sin C > 0$ なので

$$\sin C = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

四角形 $ABCD$ の面積は

$$\begin{aligned} S &= \triangle ABD + \triangle CBD \\ &= \frac{1}{2} \cdot 4 \cdot 9 \cdot \frac{\sqrt{5}}{3} + \frac{1}{2} \cdot 8 \cdot 3 \cdot \frac{\sqrt{3}}{2} \\ &= 6\sqrt{5} + 6\sqrt{3} \end{aligned}$$

■283

正弦定理より外接円の半径を R とすると

$$\sin B = \frac{b}{2R}, \quad \sin C = \frac{c}{2R}$$

余弦定理より

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

これらを等式の左辺に代入すると

$$\begin{aligned} (\text{左辺}) &= b\,(b - a\cos C) - c\,(c - a\cos B) \\ &= b\left(b - a \cdot \frac{a^2 + b^2 - c^2}{2ab}\right) \\ &\quad - c\left(c - a \cdot \frac{c^2 + a^2 - b^2}{2ca}\right) \\ &= \frac{b}{2R} \cdot \frac{2b^2 - (a^2 + b^2 - c^2)}{2b} \\ &\quad - \frac{c}{2R} \cdot \frac{2c^2 - (c^2 + a^2 - b^2)}{2c} \\ &= \frac{1}{4R}\left\{(b^2 - a^2 + c^2) - (c^2 - a^2 + b^2)\right\} \\ &= 0 \end{aligned}$$

よって等式は成り立つ.

■284 (1)

$$\begin{aligned} \sin A &= 2\cos B \sin C \\ \frac{a}{2R} &= 2 \cdot \frac{c^2 + a^2 - b^2}{2ca} \cdot \frac{c}{2R} \\ a &= \frac{c^2 + a^2 - b^2}{a} \\ a^2 &= c^2 + a^2 - b^2 \\ b^2 &= c^2 \\ \therefore b &= c \quad (b, c > 0) \end{aligned}$$

よって, $AB = AC$ の二等辺三角形

■284 (2)

$$\begin{aligned} \tan A : \tan B &= a : b \\ b\tan A &= a\tan B \\ b \cdot \frac{\sin A}{\cos A} &= a \cdot \frac{\sin B}{\cos B} \\ b\sin A\cos B &= a\sin B\cos A \\ b \cdot \frac{a}{2R} \cdot \frac{c^2 + a^2 - b^2}{2ca} &= a \cdot \frac{b}{2R} \cdot \frac{b^2 + c^2 - a^2}{2bc} \\ \frac{c^2 + a^2 - b^2}{c} &= \frac{b^2 + c^2 - a^2}{c} \\ c^2 + a^2 - b^2 &= b^2 + c^2 - a^2 \\ 2a^2 &= 2b^2 \\ \therefore a &= b \quad (a, b > 0) \end{aligned}$$

よって, $CA = CB$

の二等辺三角形