新 基礎数学　改訂版，大日本図書

5 章 三角関数

3 加法定理とその応用

BASIC

■318

$$\begin{aligned}
\sin 105^\circ &= \sin(60^\circ + 45^\circ) \\
&= \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ \\
&= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\
&= \frac{\sqrt{6}+\sqrt{2}}{4} \\
\cos 105^\circ &= \cos(60^\circ + 45^\circ) \\
&= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\
&= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\
&= \frac{\sqrt{2}-\sqrt{6}}{4} \\
\tan 105^\circ &= \frac{\sin 105^\circ}{\cos 105^\circ} \\
&= \frac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}} \\
&= -\frac{1}{4}(2+4\sqrt{3}+6) \\
&= -2-\sqrt{3}
\end{aligned}$$

■319 (1)

$$\begin{aligned}
\sin\left(\theta + \frac{\pi}{3}\right) &= \sin\theta \cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\theta \\
&= \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta
\end{aligned}$$

■319 (2)

$$\begin{aligned}
\cos\left(\theta + \frac{\pi}{4}\right) &= \cos\theta \cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4} \\
&= \frac{\sqrt{2}}{2}\cos\theta - \frac{\sqrt{2}}{2}\sin\theta
\end{aligned}$$

■320 (1)

$90^\circ < \alpha < 180^\circ$ より

$$\cos\alpha = -\sqrt{1-\left(\frac{3}{4}\right)^2} = -\frac{\sqrt{7}}{4}$$

$270^\circ < \beta < 360^\circ$ より

$$\sin\beta = -\sqrt{1-\left(\frac{1}{3}\right)^2} = -\frac{2\sqrt{2}}{3}$$

■320 (1)

$90^\circ < \alpha < 180^\circ$ より

$$\begin{aligned}
\cos\alpha &= -\sqrt{1-\left(\frac{3}{4}\right)^2} \\
&= -\frac{\sqrt{7}}{4}
\end{aligned}$$

$270^\circ < \beta < 360^\circ$ より

$$\begin{aligned}
\sin\beta &= -\sqrt{1-\left(\frac{1}{3}\right)^2} \\
&= -\frac{2\sqrt{2}}{3}
\end{aligned}$$

加法定理により

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\
&= \frac{3}{4}\cdot\frac{1}{3} + \left(-\frac{\sqrt{7}}{4}\right)\cdot\left(-\frac{2\sqrt{2}}{3}\right) \\
&= \frac{1}{4} + \frac{2\sqrt{14}}{12} \\
&= \frac{1}{4} + \frac{\sqrt{14}}{6}
\end{aligned}$$

■320 (2)

$$\begin{aligned}
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
&= \left(-\frac{\sqrt{7}}{4}\right)\cdot\frac{1}{3} - \frac{3}{4}\cdot\left(-\frac{2\sqrt{2}}{3}\right) \\
&= -\frac{\sqrt{7}}{12} + \frac{6\sqrt{2}}{12} \\
&= \frac{6\sqrt{2}-\sqrt{7}}{12}
\end{aligned}$$

■321 (1)

$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \\ &= \frac{-2+\frac{1}{5}}{1-(-2)\cdot\frac{1}{5}} \\ &= \frac{-2+\frac{1}{5}}{1+\frac{2}{5}} \\ &= \frac{-10+1}{5+2} \\ &= -\frac{9}{7}\end{aligned}$$

■321 (2)

$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \\ &= \frac{\frac{3}{2}+5}{1-\frac{3}{2}\cdot 5} \\ &= \frac{3+10}{2-15} \\ &= -1\end{aligned}$$

$0<\alpha<\frac{\pi}{2}$, $0<\beta<\frac{\pi}{2}$ より $0<\alpha+\beta<\pi$ なので

$$\alpha+\beta = \frac{3}{4}\pi$$

■322

$90^\circ<\alpha<180^\circ$, $\sin\alpha=\frac{2}{3}$ より

$$\begin{aligned}\cos\alpha &= -\sqrt{1-\left(\frac{2}{3}\right)^2} = -\frac{\sqrt{5}}{3} \\ \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ &= 2\cdot\frac{2}{3}\cdot\left(-\frac{\sqrt{5}}{3}\right) = -\frac{4\sqrt{5}}{9} \\ \cos 2\alpha &= 1-2\sin^2\alpha \\ &= 1-2\cdot\frac{4}{9} = \frac{1}{9} \\ \tan 2\alpha &= \frac{\sin 2\alpha}{\cos 2\alpha} \\ &= -4\sqrt{5}\end{aligned}$$

■323

$$\begin{aligned}\cos\frac{\pi}{12} &= \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) \\ &= \cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4} \\ &= \frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}+\sqrt{6}}{4}\end{aligned}$$

■324

$$\begin{aligned}\sin^2\frac{\alpha}{2} &= \frac{1-\cos\alpha}{2} \\ &= \frac{1+\frac{4}{5}}{2} = \frac{9}{10}\end{aligned}$$

$\frac{\pi}{2}<\frac{\alpha}{2}<\frac{3}{4}\pi$ より

$$\begin{aligned}\sin\frac{\alpha}{2} &= \frac{3}{\sqrt{10}} \\ \cos\frac{\alpha}{2} &= -\sqrt{1-\left(\frac{3}{\sqrt{10}}\right)^2} = -\frac{1}{\sqrt{10}} \\ \tan\frac{\alpha}{2} &= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = -3\end{aligned}$$

■325 (1)

$$\begin{aligned}\cos 5\theta\sin 2\theta &= \frac{1}{2}\{\sin(5\theta+2\theta)-\sin(5\theta-2\theta)\} \\ &= \frac{1}{2}(\sin 7\theta-\sin 3\theta)\end{aligned}$$

■325 (2)

$$\begin{aligned}\sin 3\theta\sin 2\theta &= -\frac{1}{2}\{\cos(3\theta+2\theta)-\cos(3\theta-2\theta)\} \\ &= -\frac{1}{2}(\cos 5\theta-\cos\theta)\end{aligned}$$

■325 (3)

$$\begin{aligned}\cos 4\theta\cos\theta &= \frac{1}{2}\{\cos(4\theta+\theta)+\cos(4\theta-\theta)\} \\ &= \frac{1}{2}(\cos 5\theta+\cos 3\theta)\end{aligned}$$

■325 (4)

$$\begin{aligned}\sin 3\theta\cos 7\theta &= \frac{1}{2}\{\sin(3\theta+7\theta)+\sin(3\theta-7\theta)\} \\ &= \frac{1}{2}(\sin 10\theta-\sin 4\theta)\end{aligned}$$

■326 (1)

$$\begin{aligned}\sin 5\theta + \sin\theta &= 2\sin\frac{5\theta+\theta}{2}\cos\frac{5\theta-\theta}{2}\\ &= 2\sin 3\theta\cos 2\theta\end{aligned}$$

■326 (2)

$$\begin{aligned}\cos 6\theta + \cos 2\theta &= 2\cos\frac{6\theta+2\theta}{2}\sin\frac{6\theta-2\theta}{2}\\ &= 2\cos 4\theta\sin 2\theta\end{aligned}$$

■326 (3)

$$\begin{aligned}\cos\theta - \cos 5\theta &= -2\sin\frac{\theta+5\theta}{2}\sin\frac{\theta-5\theta}{2}\\ &= 2\sin 3\theta\sin 2\theta\end{aligned}$$

■326 (4)

$$\begin{aligned}\sin 2\theta - \sin 3\theta &= 2\cos\frac{2\theta+3\theta}{2}\sin\frac{2\theta-3\theta}{2}\\ &= -2\cos\frac{5}{2}\theta\sin\frac{\theta}{2}\end{aligned}$$

■327 (1)

$$\begin{aligned}y &= \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\\ &= \sin\left(x+\frac{\pi}{3}\right)\end{aligned}$$

■327 (2)

$$\begin{aligned}y &= 2\sin x - 2\cos x\\ &= 2(\sin x - \cos x)\\ &= 2\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)\end{aligned}$$

■328

$$\begin{aligned}y &= \sqrt{3}\sin x + \cos x\\ &= 2\sin\left(x+\frac{\pi}{6}\right)\end{aligned}$$

$\frac{\pi}{6} \leqq x + \frac{\pi}{6} < \frac{13}{6}\pi$ なので

$$x+\frac{\pi}{6} = \frac{\pi}{2} \text{ すなわち } x = \frac{\pi}{3} \text{ で最大値 } 2$$

$$x+\frac{\pi}{6} = \frac{3}{2}\pi \text{ すなわち } x = \frac{4}{3}\pi \text{ で最小値 } -2$$

CHECK

■329 (1)

$$90^\circ < \alpha < 180^\circ \text{より}$$

$$\cos\alpha = -\sqrt{1-\left(\frac{\sqrt{2}}{3}\right)^2} = -\frac{\sqrt{7}}{3}$$

$$90^\circ < \beta < 180^\circ \text{より}$$

$$\sin\beta = \sqrt{1-\left(-\frac{2}{5}\right)^2} = \frac{\sqrt{21}}{5}\text{なので}$$

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\ &= \frac{\sqrt{2}}{3}\cdot\left(-\frac{2}{5}\right) + \frac{\sqrt{21}}{5}\cdot\left(-\frac{\sqrt{7}}{3}\right) \\ &= -\frac{2\sqrt{2}}{15} - \frac{7\sqrt{3}}{15}\end{aligned}$$

■329 (2)

$$\begin{aligned}\cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\ &= -\frac{\sqrt{7}}{3}\cdot\left(-\frac{2}{5}\right) + \frac{\sqrt{2}}{3}\cdot\left(\frac{\sqrt{21}}{5}\right) \\ &= \frac{2\sqrt{7}}{15} + \frac{\sqrt{42}}{15}\end{aligned}$$

■330

$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \\ &= \frac{\frac{1}{4}-3}{1+\frac{1}{4}\cdot 3} \\ &= \frac{1-12}{4+3} \\ &= -\frac{11}{7}\end{aligned}$$

■331 (1)

$$\begin{aligned}\sin 2\alpha &= 2\sin\alpha\cos\alpha \\ &= 2\cdot\frac{4}{5}\cdot\frac{3}{5} \\ &= \frac{24}{25} \\ \cos 2\alpha &= 1-2\sin^2\alpha \\ &= 1-2\cdot\left(\frac{4}{5}\right)^2 \\ &= 1-\frac{32}{25} \\ &= -\frac{7}{25} \\ \tan 2\alpha &= \frac{\sin 2\alpha}{\cos 2\alpha} \\ &= \frac{24}{25}\div\left(-\frac{7}{25}\right) \\ &= -\frac{24}{7}\end{aligned}$$

■332 (1)

$$\begin{aligned}\sin^2\frac{\alpha}{2} &= \frac{1-\cos\alpha}{2} \\ &= \frac{1-\frac{1}{4}}{2} \\ &= \frac{3}{8}\end{aligned}$$

$\frac{3}{4}\pi < \frac{\alpha}{2} < \pi$ より $\sin\frac{\alpha}{2} > 0$ なので

$$\sin\frac{\alpha}{2} = \sqrt{\frac{3}{8}} = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4}$$

■332 (2)

$$\begin{aligned}\cos\frac{\alpha}{2} &= -\sqrt{1-\left(\frac{\sqrt{6}}{4}\right)^2} \\ &= -\frac{\sqrt{10}}{4}\end{aligned}$$

■332 (3)

$$\begin{aligned}\tan\frac{\alpha}{2} &= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ &= -\frac{\sqrt{6}}{\sqrt{10}} = -\frac{\sqrt{15}}{5}\end{aligned}$$

■333

$$
\begin{aligned}
(\text{左辺}) &= \cos^4\theta - \sin^4\theta \\
&= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) \\
&= 1 \cdot \cos 2\theta \\
&= (\text{右辺})
\end{aligned}
$$

■334 (1)

$$
\begin{aligned}
&2\sin(\theta + 120^\circ)\cos(30^\circ - \theta) \\
&= \sin\{(\theta + 120^\circ) + (30^\circ - \theta)\} + \\
&\sin\{(\theta + 120^\circ) - (30^\circ - \theta)\} \\
&= \sin 150^\circ + \sin(2\theta + 90^\circ) \\
&= \frac{1}{2} + \cos 2\theta
\end{aligned}
$$

■334 (2)

$$
\begin{aligned}
&\cos\frac{2\theta + 3\pi}{4}\cos\frac{2\theta - 3\pi}{4} \\
&= \frac{1}{2}\left\{\cos\left(\frac{2\theta + 3\pi}{4} + \frac{2\theta - 3\pi}{4}\right)\right. \\
&\quad \left. + \cos\left(\frac{2\theta + 3\pi}{4} - \frac{2\theta - 3\pi}{4}\right)\right\} \\
&= \frac{1}{2}\left(\cos\frac{4\theta}{4} + \cos\frac{6\pi}{4}\right) \\
&= \frac{1}{2}\left(\cos\theta + \cos\frac{3}{2}\pi\right) \\
&= \frac{1}{2}(\cos\theta + 0) \\
&= \frac{1}{2}\cos\theta
\end{aligned}
$$

■335 (1)

$$
\begin{aligned}
&\sin 100^\circ + \sin 40^\circ \\
&= 2\sin\frac{100^\circ + 40^\circ}{2} \\
&\quad \cdot \cos\frac{100^\circ - 40^\circ}{2} \\
&= 2\sin 70^\circ \cos 30^\circ \\
&= 2\sin 70^\circ \cdot \frac{\sqrt{3}}{2} \\
&= \sqrt{3}\sin 70^\circ
\end{aligned}
$$

■335 (2)

$$
\begin{aligned}
&\cos 100^\circ - \cos 20^\circ \\
&= -2\sin\frac{100^\circ + 20^\circ}{2} \\
&\quad \cdot \sin\frac{100^\circ - 20^\circ}{2} \\
&= -2\sin 60^\circ \sin 40^\circ \\
&= -2 \cdot \frac{\sqrt{3}}{2}\sin 40^\circ \\
&= -\sqrt{3}\sin 40^\circ
\end{aligned}
$$

■336

$$
\begin{aligned}
y &= 3\sin x - \sqrt{3}\cos x \\
&= 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right) \\
&= 2\sqrt{3}\sin\left(x - \frac{\pi}{6}\right)
\end{aligned}
$$

STEP UP

■337 (1)

$$
\begin{aligned}
(\text{左辺}) &= \frac{1}{\tan\theta} - \frac{1-\tan^2\theta}{2\tan\theta} \\
&= \frac{2-1+\tan^2\theta}{2\tan\theta} \\
&= \frac{1+\tan^2\theta}{2\tan\theta} \\
&= \frac{1}{2\tan\theta\cdot\cos^2\theta} \\
&= \frac{1}{2\sin\theta\cos\theta} \\
&= \frac{1}{\sin 2\theta} = (\text{右辺})
\end{aligned}
$$

■337 (2)

$$
\begin{aligned}
(\text{左辺}) &= \frac{1+\sin 2x-\cos 2x}{1+\sin 2x+\cos 2x} \\
&= \frac{1+2\sin x\cos x-(1-2\sin^2 x)}{1+2\sin x\cos x+(2\cos^2 x-1)} \\
&= \frac{2\sin x\cos x+2\sin^2 x}{2\sin x\cos x+2\cos^2 x} \\
&= \frac{2\sin x(\cos x+\sin x)}{2\cos x(\sin x+\cos x)} \\
&= \tan x = (\text{右辺})
\end{aligned}
$$

■338 (1)

$$
\begin{aligned}
\cos 2x &= \sin x \\
1-2\sin^2 x &= \sin x \\
2\sin^2 x+\sin x-1 &= 0 \\
(2\sin x-1)(\sin x+1) &= 0 \\
\sin x &= \frac{1}{2},\ -1
\end{aligned}
$$

$0 \leqq x < 2\pi$ より

$$x = \frac{\pi}{6},\ \frac{5}{6}\pi,\ \frac{3}{2}\pi$$

■338 (2)

$$
\begin{aligned}
\sin 2x &\geqq \sqrt{3}\cos x \\
2\sin x\cos x &\geqq \sqrt{3}\cos x \\
\cos x\left(\sin x-\frac{\sqrt{3}}{2}\right) &\geqq 0
\end{aligned}
$$

これを満たすのは、

$$\text{(i)}\ \begin{cases}\cos x \geqq 0 \\ \sin x \geqq \frac{\sqrt{3}}{2}\end{cases}$$

または

$$\text{(ii)}\ \begin{cases}\cos x \leqq 0 \\ \sin x \leqq \frac{\sqrt{3}}{2}\end{cases}$$

$0 \leqq x < 2\pi$ において、(i) は

$$\begin{cases}0 \leqq x \leqq \frac{\pi}{2},\ \frac{3}{2}\pi \leqq x < 2\pi \\ \frac{\pi}{3} \leqq x \leqq \frac{2}{3}\pi\end{cases}$$

または、(ii) は

$$\begin{cases}\frac{\pi}{2} \leqq x \leqq \frac{3}{2}\pi \\ 0 \leqq x \leqq \frac{\pi}{3},\ \frac{2}{3}\pi \leqq x < 2\pi\end{cases}$$

したがって、

$$\frac{\pi}{3} \leqq x \leqq \frac{\pi}{2},\quad \frac{2}{3}\pi \leqq x \leqq \frac{3}{2}\pi$$

■338 (3)

$$
\begin{aligned}
\sin 3x &= \sin x \\
\sin(2x+x) &= \sin x \\
\sin 2x\cos x+\sin x\cos 2x &= \sin x \\
2\sin x\cos^2 x+\sin x(2\cos^2 x-1) &= \sin x \\
\sin x(4\cos^2 x-2) &= 0 \\
\sin x = 0\quad \text{または}\quad \cos x &= \pm\frac{1}{\sqrt{2}}
\end{aligned}
$$

$0 \leqq x < 2\pi$ より

$$x = 0,\ \pi,\ \frac{\pi}{4},\ \frac{3}{4}\pi,\ \frac{5}{4}\pi,\ \frac{7}{4}\pi$$

■338 (4)

$$
\begin{aligned}
\sin x+\sin 3x &= \sin 2x+\sin 4x \\
2\sin 2x\cos x &= 2\sin 3x\cos x \\
\cos x(\sin 3x-\sin 2x) &= 0 \\
2\cos x\cos\frac{5x}{2}\sin\frac{x}{2} &= 0
\end{aligned}
$$

これを満たすのは

$$\cos x = 0\quad \text{または}$$

$$\cos\frac{5}{2}x = 0\quad \text{または}$$

$$\sin\frac{x}{2} = 0$$

$0 \leqq \frac{5}{2}x < 5\pi,\ 0 \leqq \frac{x}{2} < \pi$ より

$$\begin{aligned} x &= \frac{\pi}{2},\ \frac{3}{2}\pi \\ &\text{または} \\ \frac{5}{2}x &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2} \\ &\text{または} \\ \frac{x}{2} &= 0 \end{aligned}$$

したがって

$$x = 0, \frac{\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{3\pi}{2}, \frac{9\pi}{5}$$

■338 (5)

$$\begin{aligned} \cos x + \cos 2x + \cos 3x &= 0 \\ \cos 2x + (\cos x + \cos 3x) &= 0 \\ \cos 2x + 2\cos 2x \cos x &= 0 \\ \cos 2x(1 + 2\cos x) &= 0 \end{aligned}$$

これを満たすのは

$$\begin{aligned} \cos 2x = 0 \quad &\text{または} \\ \cos x = -\frac{1}{2} & \end{aligned}$$

$0 \leqq 2x < 4\pi$ より

$$\begin{aligned} 2x &= \frac{\pi}{2},\ \frac{3}{2}\pi,\ \frac{5}{2}\pi,\ \frac{7}{2}\pi \\ &\text{または} \\ x &= \frac{2}{3}\pi,\ \frac{4}{3}\pi \end{aligned}$$

したがって

$$x = \frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi$$

■339 (1)

$$\begin{aligned} y &= \sin^2 x \\ &= \frac{1}{2}(1 - \cos 2x) \\ &= \frac{1}{2} - \frac{1}{2}\cos 2x \end{aligned}$$

■339 (2)

$$\begin{aligned} y &= \cos x + \cos\left(x - \frac{\pi}{3}\right) \\ &= 2\cos\frac{x + x - \frac{\pi}{3}}{2}\cos\frac{x - \left(x - \frac{\pi}{3}\right)}{2} \\ &= 2\cos\left(x - \frac{\pi}{6}\right) \cdot \cos\frac{\pi}{6} \\ &= \sqrt{3}\cos\left(x - \frac{\pi}{6}\right) \end{aligned}$$

■339 (3)

$$\begin{aligned} y &= 2\sin x \cos\left(x + \frac{\pi}{6}\right) \\ &= \sin\left(2x + \frac{\pi}{6}\right) + \sin\left(-\frac{\pi}{6}\right) \\ &= \sin\left\{2\left(x + \frac{\pi}{12}\right)\right\} - \frac{1}{2} \end{aligned}$$

■340 (1)

$C = \pi - A - B$ なので

$$\begin{aligned} &\sin 2A + \sin 2B \\ &= 2\sin(A + B)\cos(A - B) \\ &= 2\sin(\pi - C)\cos(A - B) \\ &= 2\sin C\cos(A - B) \end{aligned}$$

提示された式を変形すると

$$\begin{aligned} &\sin A\cos A + \sin B\cos B \\ &= \sin A\cos B + \sin B\cos A \\ &\sin A(\cos A - \cos B) \\ &\quad - \sin B(\cos A - \cos B) = 0 \\ &(\cos A - \cos B)(\sin A - \sin B) = 0 \end{aligned}$$

これより

$$\begin{aligned} &\cos A = \cos B \\ &\text{または} \\ &\sin A = \sin B \end{aligned}$$

$0 < A < \pi,\ 0 < B < \pi$ より

$$\begin{aligned} &A = B \\ &\text{または} \\ &A = \pi - B \end{aligned}$$

$A=\pi-B$ のとき $A+B=\pi$ となり $C=0$ となるため不適。

$$\therefore A=B$$

したがって、$CA=CB$ の二等辺三角形である。

■340 (2)

$$\begin{aligned}&\cos 2A+\cos 2B\\&=2\cos(\pi-A-B)\\&\cos^2 A-\sin^2 A+\cos^2 B-\sin^2 B\\&=-2\cos(A+B)\\&=-2(\cos A\cos B-\sin A\sin B)\\&(\cos^2 A+2\cos A\cos B+\cos^2 B)\\&-(\sin^2 A+2\sin A\sin B+\sin^2 B)=0\\&(\cos A+\cos B)^2-(\sin A+\sin B)^2=0\\&(\cos A+\cos B+\sin A+\sin B)\\&\cdot(\cos A+\cos B-\sin A-\sin B)=0\end{aligned}$$

合成公式を用いると

$$\begin{aligned}&\left\{\sqrt{2}\sin\left(A+\frac{\pi}{4}\right)+\sqrt{2}\sin\left(B+\frac{\pi}{4}\right)\right\}\\&\cdot\left\{\sqrt{2}\sin\left(A-\frac{\pi}{4}\right)+\sqrt{2}\sin\left(B-\frac{\pi}{4}\right)\right\}=0\end{aligned}$$

これより

$$\sin\left(A+\frac{\pi}{4}\right)=-\sin\left(B+\frac{\pi}{4}\right)$$
または
$$\sin\left(A-\frac{\pi}{4}\right)=-\sin\left(B-\frac{\pi}{4}\right)$$

$0<A<\pi,\ 0<B<\pi$ より

$$A+\frac{\pi}{4}=2\pi-\left(B+\frac{\pi}{4}\right)$$
または
$$A-\frac{\pi}{4}=-\left(B-\frac{\pi}{4}\right)$$

整理すると

$$A+B=\frac{3}{2}\pi$$
または
$$A+B=\frac{\pi}{2}$$

$A+B=\frac{3}{2}\pi$ は不適。

$$A+B=\frac{\pi}{2}$$
$$\therefore C=90^\circ$$

したがって、$C=90^\circ$ の直角三角形である。

■341

$$\begin{aligned}&\sin\alpha+\sin\beta+\sin\gamma\\&=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\end{aligned}$$

$\gamma=\pi-\alpha-\beta$ を代入すると

$$\begin{aligned}(\text{左辺})&=\sin\alpha+\sin\beta\\&\quad+\sin(\pi-\alpha-\beta)\\&=\sin\alpha+\sin\beta+\sin(\alpha+\beta)\end{aligned}$$

一方、

$$\begin{aligned}(\text{右辺})&=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\\&\quad\cdot\cos\left(\frac{\pi}{2}-\frac{\alpha+\beta}{2}\right)\\&=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\alpha+\beta}{2}\\&=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\\&\quad\cdot\left(\sin\frac{\alpha}{2}\cos\frac{\beta}{2}+\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\right)\\&=4\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\cos^2\frac{\beta}{2}\\&\quad+4\cos^2\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\beta}{2}\\&=2\sin\alpha\cdot\frac{1+\cos\beta}{2}\\&\quad+2\sin\beta\cdot\frac{1+\cos\alpha}{2}\\&=\sin\alpha(1+\cos\beta)+\sin\beta(1+\cos\alpha)\\&=\sin\alpha+\sin\alpha\cos\beta\\&\quad+\sin\beta+\sin\beta\cos\alpha\\&=\sin\alpha+\sin\beta+\sin(\alpha+\beta)\end{aligned}$$

よって、(左辺) = (右辺) となり、等式は示された。

■342 (1)

$$
\begin{aligned}
(与式) &= \sin 10^\circ \sin 50^\circ \sin 70^\circ \\
&= \frac{1}{2}(\cos 40^\circ - \cos 60^\circ)\sin 70^\circ \\
&= \frac{1}{2}\left(\cos 40^\circ - \frac{1}{2}\right)\sin 70^\circ \\
&= \frac{1}{2}\sin 70^\circ \cos 40^\circ - \frac{1}{4}\sin 70^\circ \\
&= \frac{1}{2}\cdot\frac{1}{2}(\sin 110^\circ + \sin 30^\circ) - \frac{1}{4}\sin 70^\circ \\
&= \frac{1}{4}\sin(180^\circ - 70^\circ) + \frac{1}{4}\cdot\frac{1}{2} - \frac{1}{4}\sin 70^\circ \\
&= \frac{1}{4}\sin 70^\circ + \frac{1}{8} - \frac{1}{4}\sin 70^\circ \\
&= \frac{1}{8}
\end{aligned}
$$

■342 (2)

$$
\begin{aligned}
&\sin 80^\circ - \sin 20^\circ - \sin 40^\circ \\
&= 2\cos 50^\circ \sin 30^\circ - \sin 40^\circ \\
&= 2\cos 50^\circ \cdot \frac{1}{2} - \sin 40^\circ \\
&= \cos 50^\circ - \sin 40^\circ \\
&= \cos(90^\circ - 40^\circ) - \sin 40^\circ \\
&= \sin 40^\circ - \sin 40^\circ \\
&= 0
\end{aligned}
$$

■343

$$
\begin{aligned}
y &= 3\sin^2 x + 2\sqrt{3}\sin x\cos x + \cos^2 x \\
&= 1 + 2\sin^2 x + \sqrt{3}\sin 2x \\
&= 1 + (1 - \cos 2x) + \sqrt{3}\sin 2x \\
&= \sqrt{3}\sin 2x - \cos 2x + 2 \\
&= 2\sin\left(2x - \frac{\pi}{6}\right) + 2
\end{aligned}
$$

$0 \leqq x < 2\pi$ より $-\frac{\pi}{6} \leqq 2x - \frac{\pi}{6} < \frac{23}{6}\pi$ であるから

$$2x - \frac{\pi}{6} = \frac{\pi}{2},\ \frac{5}{2}\pi$$

すなわち $x = \dfrac{\pi}{3},\ \dfrac{4}{3}\pi$ で最大値 4 をとる。

$$2x - \frac{\pi}{6} = \frac{3}{2}\pi,\ \frac{7}{2}\pi$$

すなわち $x = \dfrac{5}{6}\pi,\ \dfrac{11}{6}\pi$ で最小値 0 をとる。

■344

$$
\begin{aligned}
f(x) &= a\sin x + b\cos x \\
&= \sqrt{a^2 + b^2}\sin(x + \alpha)
\end{aligned}
$$

$x = \frac{\pi}{3}$ で最大値 2、$x = \frac{4}{3}\pi$ で最小値 -2 をとるので

$$\sqrt{a^2 + b^2} = 2$$

また、α の 1 つは

$$
\begin{cases}
\frac{\pi}{3} + \alpha = \frac{\pi}{2} \\
\frac{4}{3}\pi + \alpha = \frac{3}{2}\pi
\end{cases}
$$

これを解いて

$$\alpha = \frac{\pi}{6}$$

よって

$$
\begin{aligned}
f(x) &= 2\sin\left(x + \frac{\pi}{6}\right) \\
&= 2\sin x\cos\frac{\pi}{6} + 2\cos x\sin\frac{\pi}{6} \\
&= \sqrt{3}\sin x + \cos x \\
\therefore a &= \sqrt{3},\ b = 1
\end{aligned}
$$

■345 (1)

$$
\begin{aligned}
\sin x + \cos x &= \frac{1}{\sqrt{2}} \\
\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\
\sin\left(x + \frac{\pi}{4}\right) &= \frac{1}{2}
\end{aligned}
$$

$0 \leqq x < 2\pi$ より $\frac{\pi}{4} \leqq x + \frac{\pi}{4} < \frac{9}{4}\pi$ なので

$$
\begin{aligned}
x + \frac{\pi}{4} &= \frac{5}{6}\pi,\ \frac{13}{6}\pi \\
\therefore x &= \frac{7}{12}\pi,\ \frac{23}{12}\pi
\end{aligned}
$$

■345 (2)

$$
\begin{aligned}
\sin x - \sqrt{3}\cos x + \sqrt{2} &= 0 \\
2\sin\left(x - \frac{\pi}{3}\right) &= -\sqrt{2} \\
\sin\left(x - \frac{\pi}{3}\right) &= -\frac{1}{\sqrt{2}}
\end{aligned}
$$

$0 \leqq x < 2\pi$ より $-\frac{\pi}{3} \leqq x - \frac{\pi}{3} < \frac{5}{3}\pi$ なので

$$x - \frac{\pi}{3} = -\frac{\pi}{4},\ \frac{5}{4}\pi$$
$$\therefore x = \frac{\pi}{12},\ \frac{19}{12}\pi$$

■346

$$\begin{aligned}
y &= 2\sin x + \cos\left(x + \frac{\pi}{6}\right) \\
&= 2\sin x + \cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6} \\
&= 2\sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x \\
&= \frac{3}{2}\sin x + \frac{\sqrt{3}}{2}\cos x \\
&= \frac{\sqrt{3}}{2}(\sqrt{3}\sin x + \cos x) \\
&= \frac{\sqrt{3}}{2}\cdot 2\sin\left(x + \frac{\pi}{6}\right) \\
&= \sqrt{3}\sin\left(x + \frac{\pi}{6}\right)
\end{aligned}$$

PLUS

■347 (1)

$$\begin{aligned}\sqrt{2}\cos x &= 1\\ \cos x &= \frac{1}{\sqrt{2}}\\ x &= \frac{\pi}{4}+2n\pi,\ \frac{7}{4}\pi+2n\pi \quad (n\text{ は整数})\end{aligned}$$

■347 (2)

$$\begin{aligned}\sqrt{3}\tan x &= 1\\ \tan x &= \frac{1}{\sqrt{3}}\\ x &= \frac{\pi}{6}+n\pi \quad (n\text{ は整数})\end{aligned}$$

■347 (3)

$$\begin{aligned}2\cos^2 x &= \sin x+1\\ 2(1-\sin^2 x) &= \sin x+1\\ 2\sin^2 x+\sin x-1 &= 0\\ (2\sin x-1)(\sin x+1) &= 0\\ \sin x &= \frac{1}{2},\ -1\end{aligned}$$

n を整数として

$$\begin{aligned}x = \frac{\pi}{6}+2n\pi,\ &\frac{5}{6}\pi+2n\pi,\\ &\frac{3}{2}\pi+2n\pi\end{aligned}$$

■347 (4)

$$\begin{aligned}\tan x &= \sqrt{2}\cos x\\ \frac{\sin x}{\cos x} &= \sqrt{2}\cos x\\ \sin x &= \sqrt{2}\cos^2 x\\ \sin x &= \sqrt{2}(1-\sin^2 x)\\ \sqrt{2}\sin^2 x+\sin x-\sqrt{2} &= 0\\ (\sqrt{2}\sin x-1)(\sin x+\sqrt{2}) &= 0\end{aligned}$$

$\sin x+\sqrt{2}\neq 0$ より

$$\sin x = \frac{1}{\sqrt{2}}$$

n を整数として

$$x = \frac{\pi}{4}+2n\pi,\ \frac{3}{4}\pi+2n\pi$$

■347 (5)

$$\begin{aligned}2\cos\left(x-\frac{\pi}{3}\right) &= -1\\ \cos\left(x-\frac{\pi}{3}\right) &= -\frac{1}{2}\end{aligned}$$

n を整数として

$$\begin{aligned}x-\frac{\pi}{3} = &\frac{2}{3}\pi+2n\pi,\\ &\frac{4}{3}\pi+2n\pi\end{aligned}$$

これより

$$\begin{aligned}x = &\pi+2n\pi,\\ &\frac{5}{3}\pi+2n\pi\end{aligned}$$

■347 (6)

$$\begin{aligned}\sqrt{2}\sin x &\leqq 1\\ \sin x &\leqq \frac{1}{\sqrt{2}}\\ -\frac{5}{4}\pi+2n\pi \leqq x &\leqq \frac{\pi}{4}+2n\pi \quad (n\text{ は整数})\end{aligned}$$

■347 (7)

$$\begin{aligned}\tan x &\geqq 1\\ \frac{\pi}{4}+n\pi \leqq x &< \frac{\pi}{2}+n\pi \quad (n\text{ は整数})\end{aligned}$$

■347 (8)

$$\begin{aligned}\sqrt{3}\sin x-\cos x &> 1\\ 2\sin\left(x-\frac{\pi}{6}\right) &> 1\\ \sin\left(x-\frac{\pi}{6}\right) &> \frac{1}{2}\end{aligned}$$

$\frac{\pi}{6}+2n\pi < x-\frac{\pi}{6} < \frac{5}{6}\pi+2n\pi$ より

$$\frac{\pi}{3}+2n\pi < x < \pi+2n\pi \quad (n\text{ は整数})$$

■348

$$\begin{aligned}&\sin x+\sin 2x+\sin 3x+\sin 4x\\ &= \frac{\sin 2x\sin\frac{5}{2}x}{\sin\frac{x}{2}}\end{aligned}$$

両辺に $\sin\frac{x}{2}$ を掛けると

$$
\begin{aligned}
&(\text{左辺}) = \sin\frac{x}{2}(\sin x + \sin 2x + \sin 3x + \sin 4x) \\
&= \sin\frac{x}{2}\sin x + \sin\frac{x}{2}\sin 2x \\
&\quad + \sin\frac{x}{2}\sin 3x + \sin\frac{x}{2}\sin 4x \\
&= \frac{1}{2}\left(\cos\frac{x}{2} - \cos\frac{3}{2}x\right) \\
&\quad + \frac{1}{2}\left(\cos\frac{3}{2}x - \cos\frac{5}{2}x\right) \\
&\quad + \frac{1}{2}\left(\cos\frac{5}{2}x - \cos\frac{7}{2}x\right) \\
&\quad + \frac{1}{2}\left(\cos\frac{7}{2}x - \cos\frac{9}{2}x\right) \\
&= \frac{1}{2}\left(\cos\frac{x}{2} - \cos\frac{9}{2}x\right) \\
&= \sin\frac{\frac{9}{2}x + \frac{x}{2}}{2}\sin\frac{\frac{9}{2}x - \frac{x}{2}}{2} \\
&= \sin\frac{5}{2}x\sin 2x = (\text{右辺})
\end{aligned}
$$