GNSS Candidates

1. Explain, briefly, the main error terms contributing to positioning error in GPS satellite navigation. How are these eliminated in differential GPS? What error sources remain?

   The main error terms in positioning:

   1. Satellite related errors
      a. Satellite orbit error
         i. The error is the difference between the position of the satellite in space given by the inaccurate satellite ephemeris and its actual position.
         ii. Post-processing (precise orbit) or differential GPS
      b. Satellite clock error
         i. The satellites' atomic clocks experience noise and clock drift errors.
         ii. Correction model or differential GPS
      c. Relativity effect
         i. In special relativity, the frequency of clocks on the high velocity satellites is slower than clocks on earth.
         ii. In general relativity, the frequency of clocks on the satellites farther away the earth is much faster than clocks on earth.
         iii. Manipulate clock frequency and add adjustment

   2. Signals propagation related errors
      a. Ionospheric delay
         i. The speed and propagation path of signals will be affected when signals travel through ionosphere.
         ii. Correction model (Bent, IRI or Klobuchar) or dual-frequency measurements (iono-free)
      b. Tropospheric delay
         i. The speed and propagation path of signals will be affected when signals travel through troposphere.
         ii. Correction model (Hopfield, Saastamoinen or Black) or differential GPS
      c. Multipath effects
         i. Direct radio signals are interfered by signals reflected from surrounding terrain; buildings, canyon walls, hard ground, etc.
         ii. MEDLL (Multipath Estimation Delay Lock Loop) or filter

   3. Receivers related errors
      a. Receiver antenna phase center error
         i. Correction model
      b. Receiver clock error
         i. Correction model or differential GPS
      c. Receiver internal noise

Differential GPS:
Carrier phase tracking observation equation:
Note that i is the receiver, k is the satellite, L_m is the location, and t is the time.

$$\varphi_{i\ L_m}^{k}(t)\cdot\lambda = \rho_i^k(t) - N_{i\ L_m}^{k}\cdot\lambda - V_{Iono}{}_{i\ L_m}^{k}(t) - V_{Trop}{}_{i\ L_m}^{k}(t) - c\cdot V_{tRi}(t) + c\cdot V_{tS}^{k}(t) + \varepsilon_{i\ L_m}^{j}$$

$\varphi_{i\ L_m}^{k}(t)\cdot\lambda$: the phase of carrier times the wavelength of the carrier
$\rho_i^k(t)$: actual length from the satellite to the receiver
$N_{i\ L_m}^{k}\cdot\lambda$: wavelength dependent adjustment of atmospheric effects (Strongly related to the location)
$V_{Iono}{}_{i\ L_m}^{k}(t)$: wavelength independent adjustment of ionospheric delay (Strongly related to the location)
$V_{Trop}{}_{i\ L_m}^{k}(t)$: wavelength independent adjustment of tropospheric delay (Strongly related to the location)
$c\cdot V_{tRi}(t)$: adjustment of receiver clock error (unrelated to satellite)
$c\cdot V_{tS}^{k}(t)$: adjustment of satellite clock error (unrelated to receiver)
$\varepsilon_{i\ L_m}^{j}$: Gaussian noise

1. Differential GPS between stations (station A and station B)
    a. $\varphi_{A,B\ L_m}^{k}(t)\cdot\lambda = \varphi_{B\ L_m}^{k}(t)\cdot\lambda - \varphi_{A\ L_m}^{k}(t)\cdot\lambda$
    $= \rho_B^k(t) - \rho_A^k(t) + \left(-c\cdot V_{tRB}(t) + \varepsilon_{B\ L_m}^{j} - c\cdot V_{tRA}(t) + \varepsilon_{A\ L_m}^{j}\right)$
    b. Eliminate “Satellite orbit error”, “Satellite clock error”, “Ionospheric delay” and “Tropospheric delay”.
    c. “Receiver clock error” remains.

2. Differential GPS between satellites (satellite I and satellite J)
    a. $\varphi_{A\ L_m}^{I,J}(t)\cdot\lambda = \varphi_{A L_m}^{I}(t)\cdot\lambda - \varphi_{A L_m}^{J}(t)\cdot\lambda$
    $= (\rho_i^k(t) - N_{i\ L_m}^{k}\cdot\lambda - V_{Iono}{}_{i\ L_m}^{k}(t) - V_{Trop}{}_{i\ L_m}^{k}(t) + c\cdot V_{tS}^{k}(t) + \varepsilon_{i\ L_m}^{j}) -$
    $(\rho_i^k(t) - N_{i\ L_m}^{k}\cdot\lambda - V_{Iono}{}_{i\ L_m}^{k}(t) - V_{Trop}{}_{i\ L_m}^{k}(t) + c\cdot V_{tS}^{k}(t) + \varepsilon_{i\ L_m}^{j})$
    b. Eliminate “Receiver clock error”.
    c. “Satellite orbit error”, “Satellite clock error”, “Ionospheric delay” and “Tropospheric delay” remain.

3. Differential GPS between epochs (epoch t_i and epoch t_{i+1})
    a. $\varphi_{A L_m}^{I}(t_i\, , t_{i+1})\cdot\lambda = \varphi_{A L_m}^{I}(t_{i+1})\cdot\lambda - \varphi_{A L_m}^{I}(t_i)\cdot\lambda$
    b. Eliminate “ambiguity of whole cycles”.