

Cache Memories

15-213: Introduction to Computer Systems

11th Lecture, June 9, 2022

Instructor:

Kyle Liang

Today

- Cache memory organization and operation
- Performance impact of caches
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

Activity Time

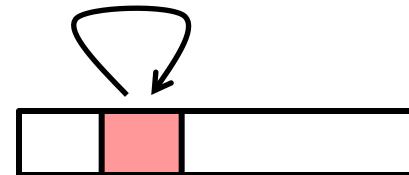
■ Do Activity 1-2

Recall: Locality

- **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently

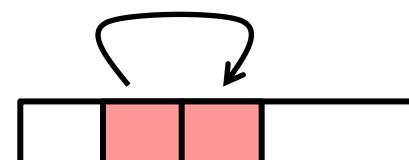
- **Temporal locality:**

- Recently referenced items are likely to be referenced again in the near future



- **Spatial locality:**

- Items with nearby addresses tend to be referenced close together in time



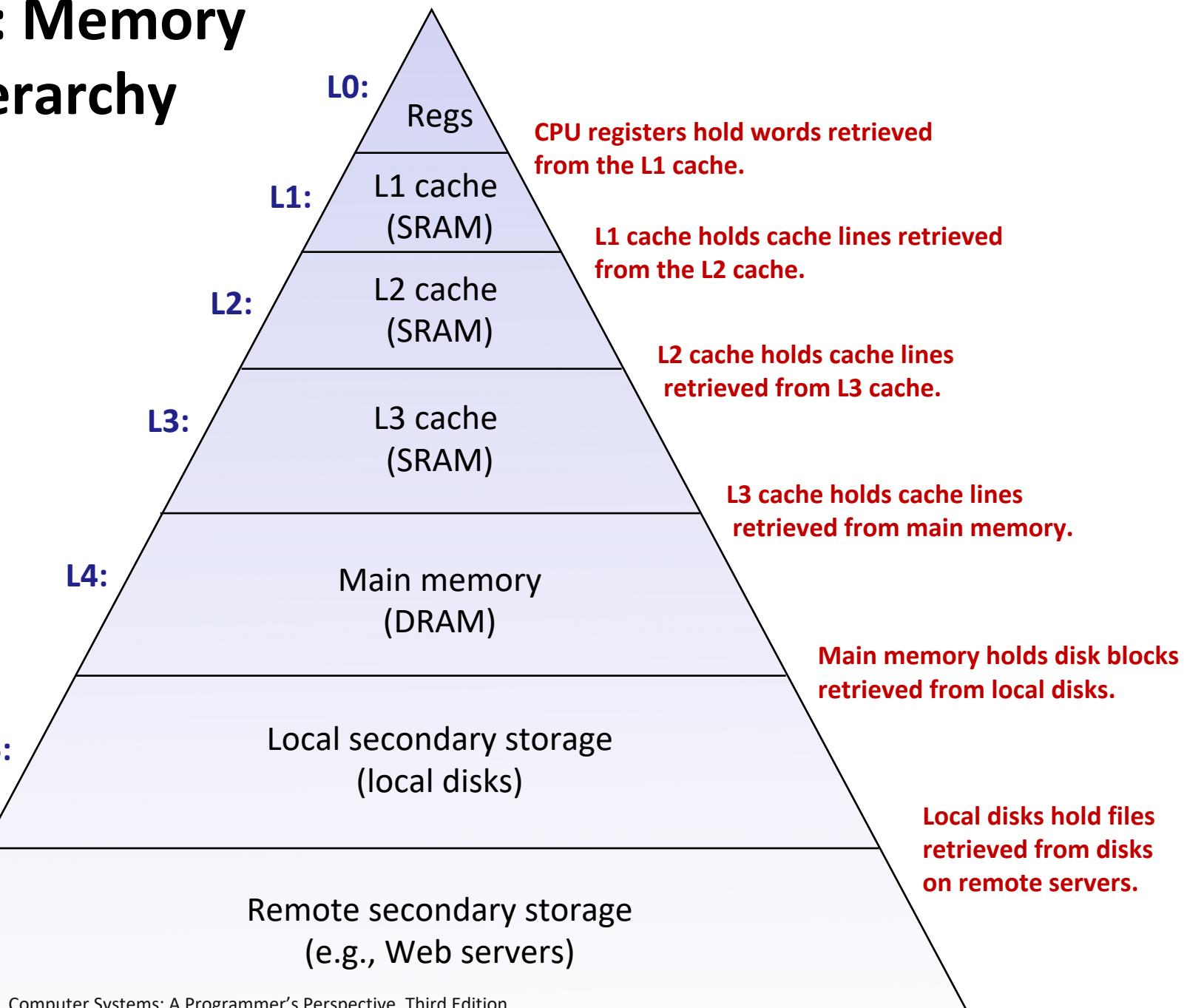
Recall: Memory Hierarchy

Smaller,
faster,
and
costlier
(per byte)
storage
devices

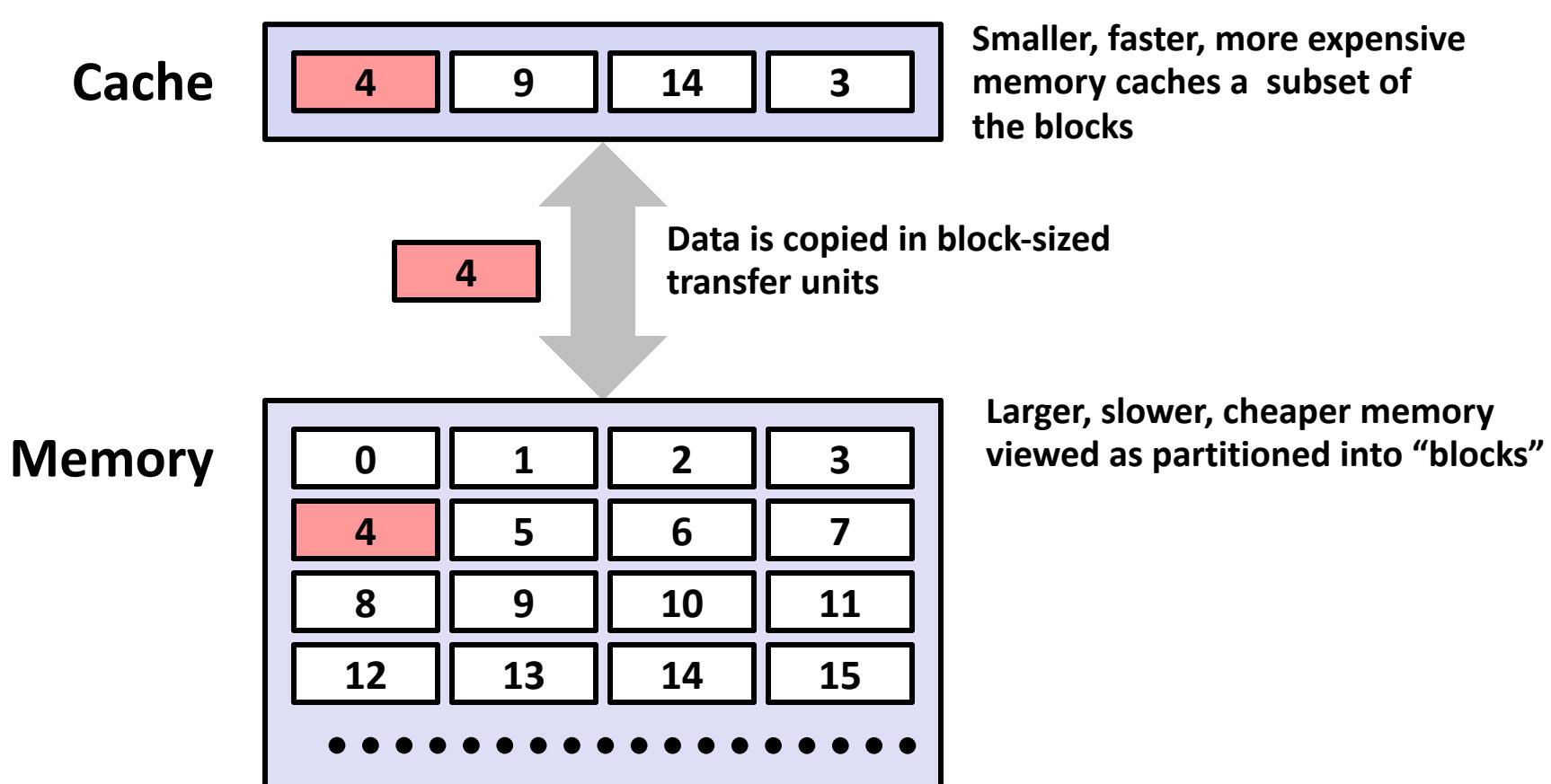
Larger,
slower,
and
cheaper
(per byte)
storage
devices

L6:

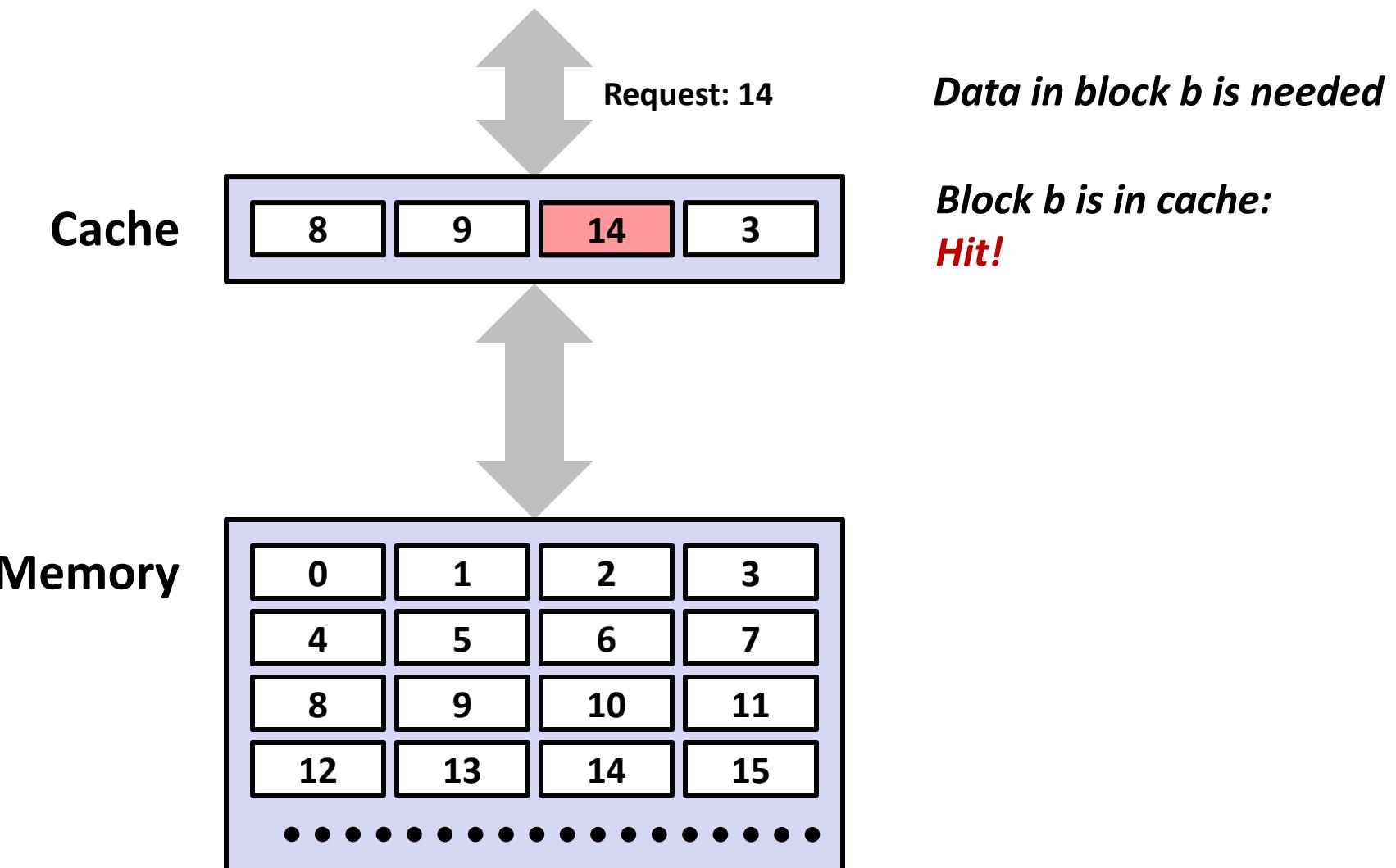
Remote secondary storage
(e.g., Web servers)



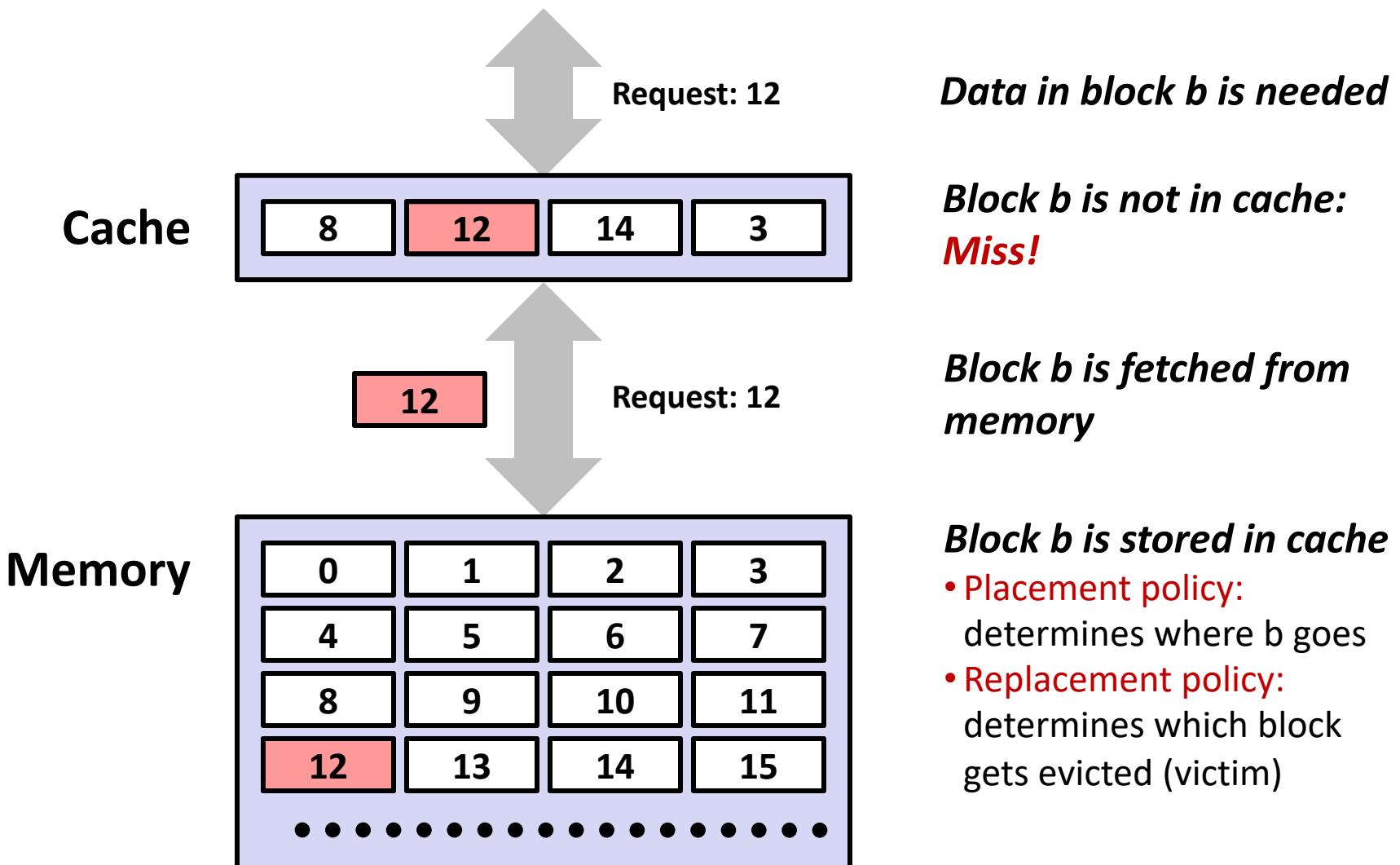
Recall: General Cache Concepts



General Cache Concepts: Hit



General Cache Concepts: Miss



Recall: General Caching Concepts:

3 Types of Cache Misses

■ Cold (compulsory) miss

- Cold misses occur because the cache starts empty and this is the first reference to the block.

■ Capacity miss

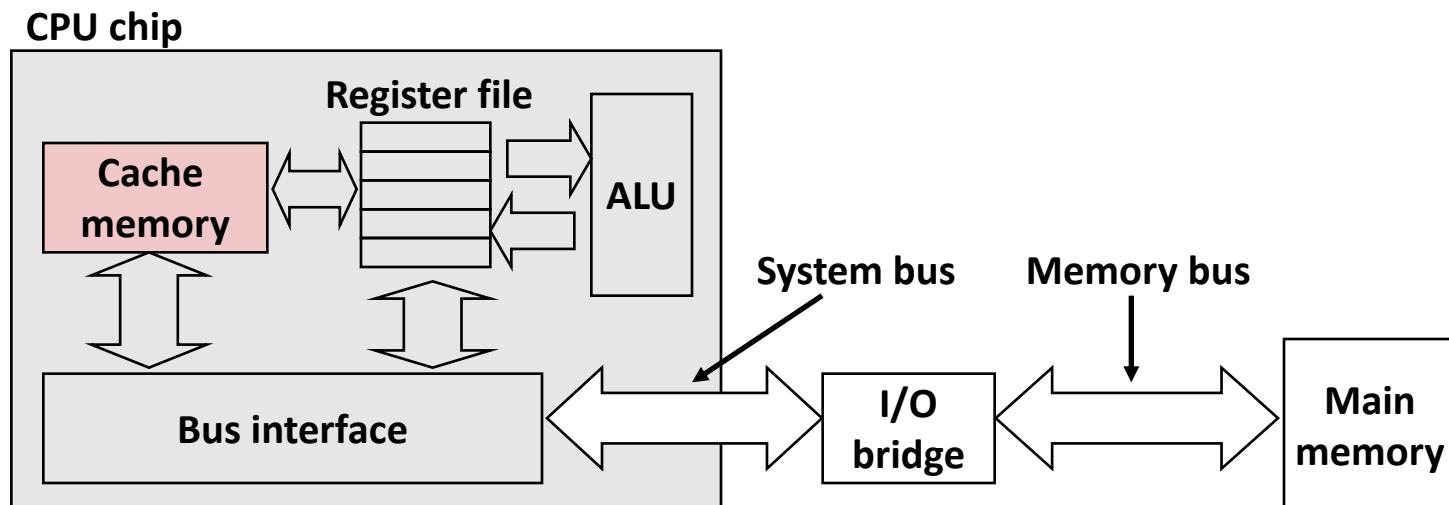
- Occurs when the set of active cache blocks (**working set**) is larger than the cache.

■ Conflict miss

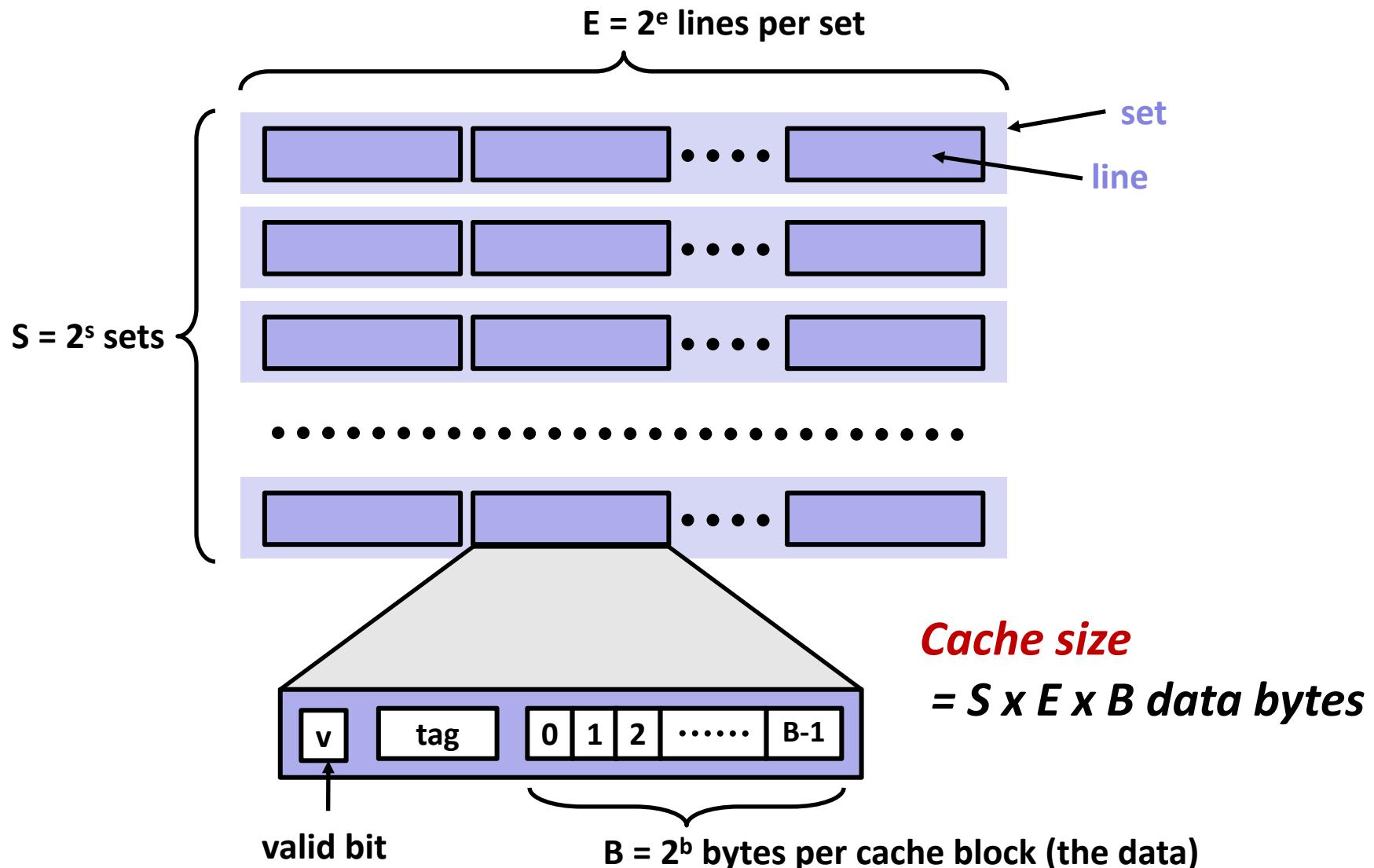
- Most caches limit blocks at level $k+1$ to a small subset (sometimes a singleton) of the block positions at level k .
 - E.g. Block i at level $k+1$ must be placed in block $(i \bmod 4)$ at level k .
- Conflict misses occur when the level k cache is large enough, but multiple data objects all map to the same level k block.
 - E.g. Referencing blocks $0, 8, 0, 8, 0, 8, \dots$ would miss every time.

Cache Memories

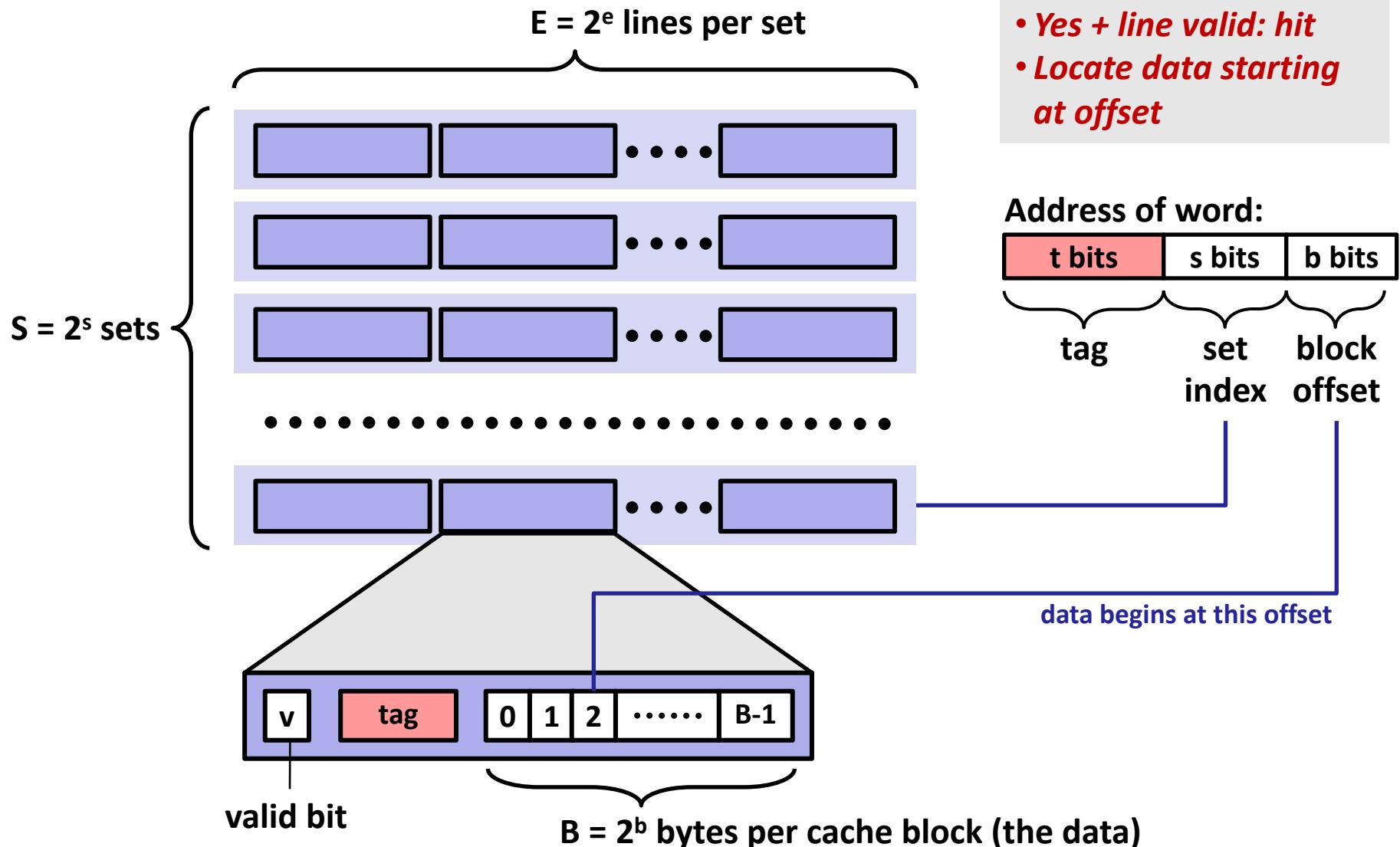
- Cache memories are small, fast SRAM-based memories managed automatically in hardware
 - Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



General Cache Organization (S, E, B)

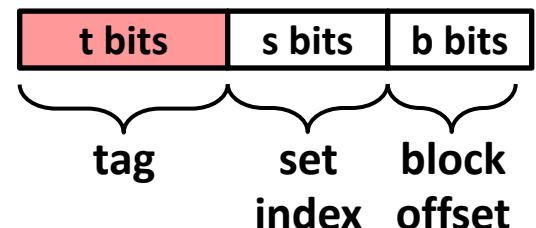


Cache Read



- Locate set
- Check if any line in set has matching tag
- Yes + line valid: hit
- Locate data starting at offset

Address of word:

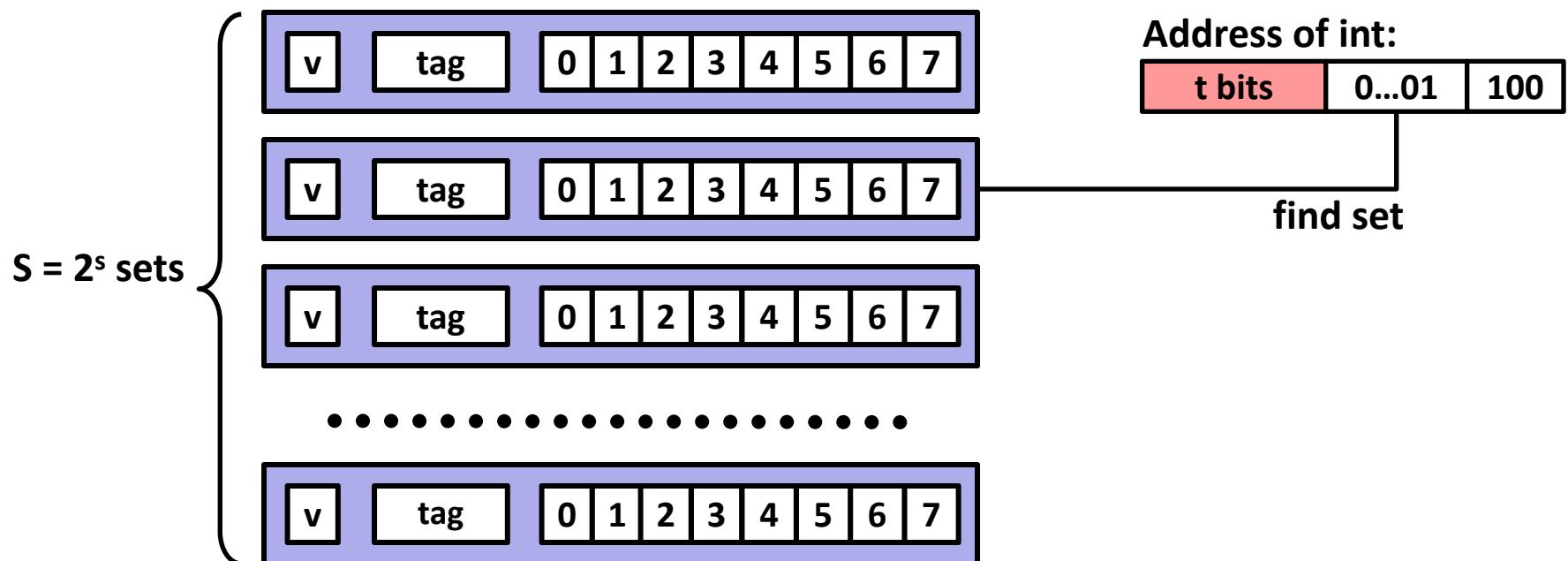


data begins at this offset

Example: Direct Mapped Cache ($E = 1$)

Direct mapped: One line per set

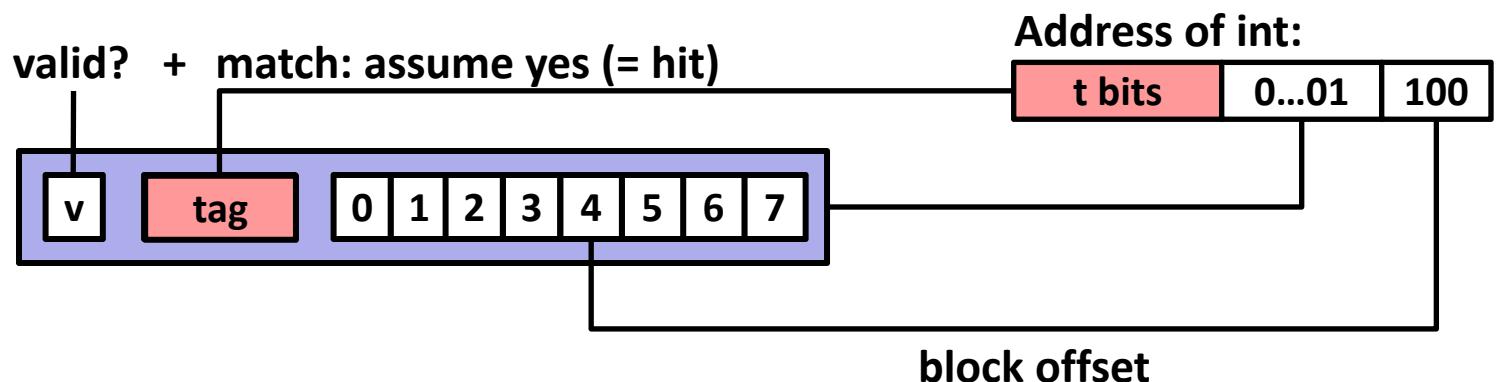
Assume: cache block size $B=8$ bytes



Example: Direct Mapped Cache ($E = 1$)

Direct mapped: One line per set

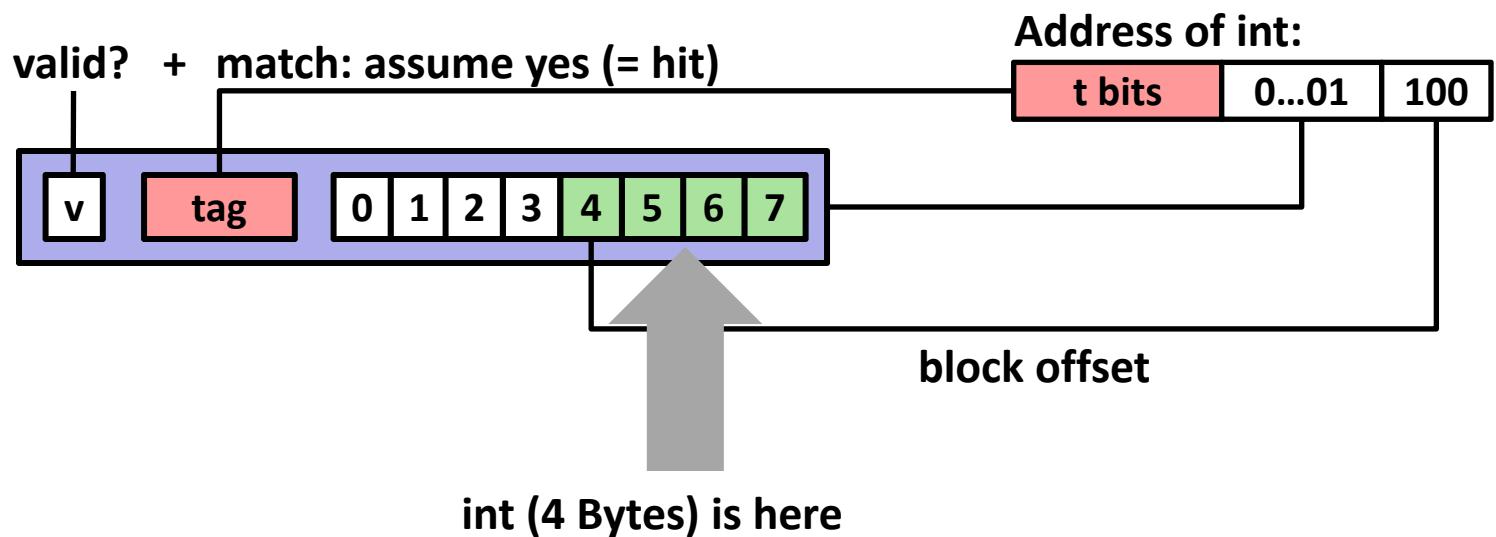
Assume: cache block size $B=8$ bytes



Example: Direct Mapped Cache ($E = 1$)

Direct mapped: One line per set

Assume: cache block size $B=8$ bytes



If tag doesn't match (= miss): old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1 s=2 b=1

4-bit addresses (address space size M=16 bytes)
 S=4 sets, E=1 Blocks/set, B=2 bytes/block

Address trace (reads, one byte per read):

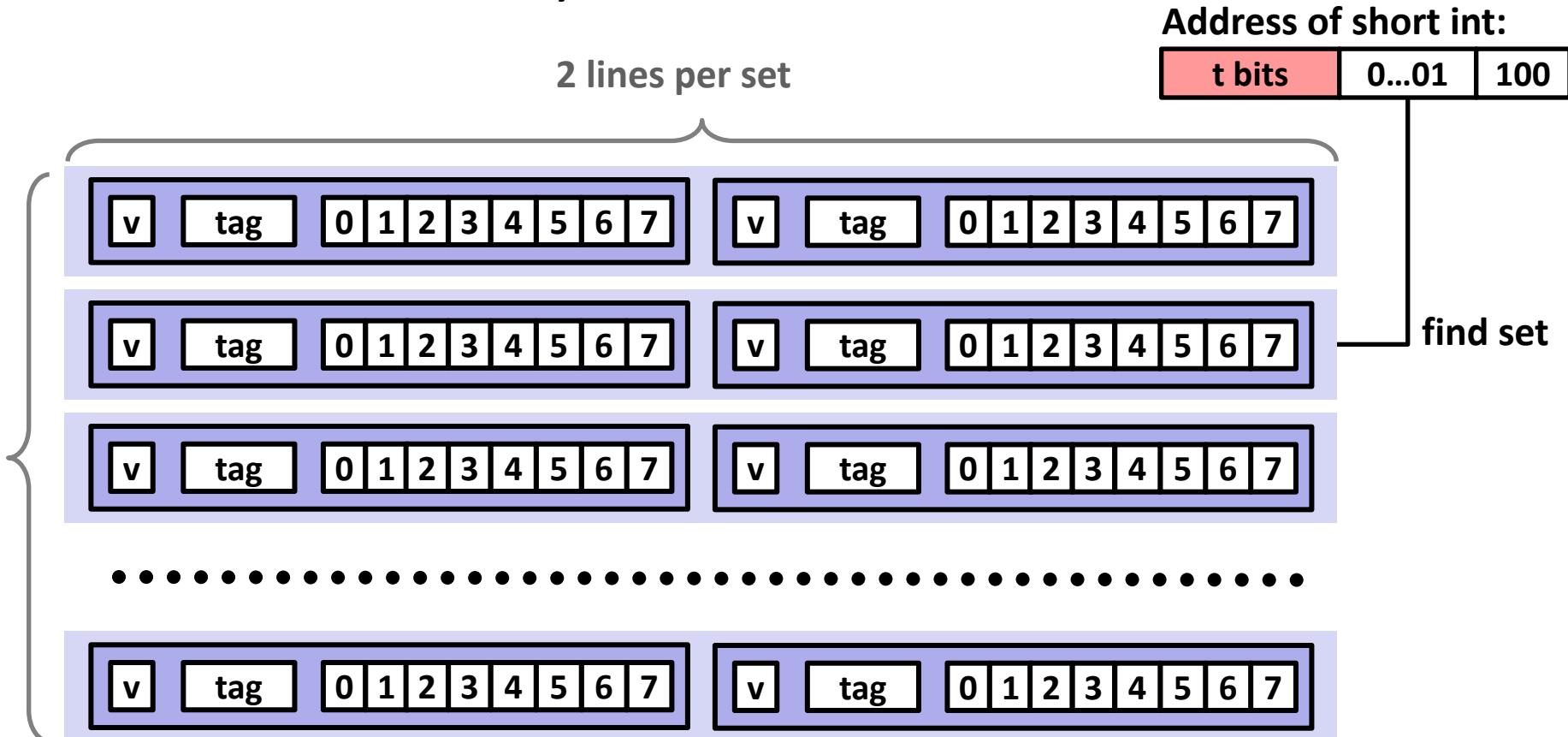
0	$[0\underline{0}0]_2$,	miss
1	$[0\underline{0}1]_2$,	hit
7	$[0\underline{1}1]_2$,	miss
8	$[1\underline{0}0]_2$,	miss
0	$[0\underline{0}0]_2$	miss

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1	0		
Set 2	0		
Set 3	1	0	M[6-7]

E-way Set Associative Cache (Here: E = 2)

$E = 2$: Two lines per set

Assume: cache block size $B=8$ bytes

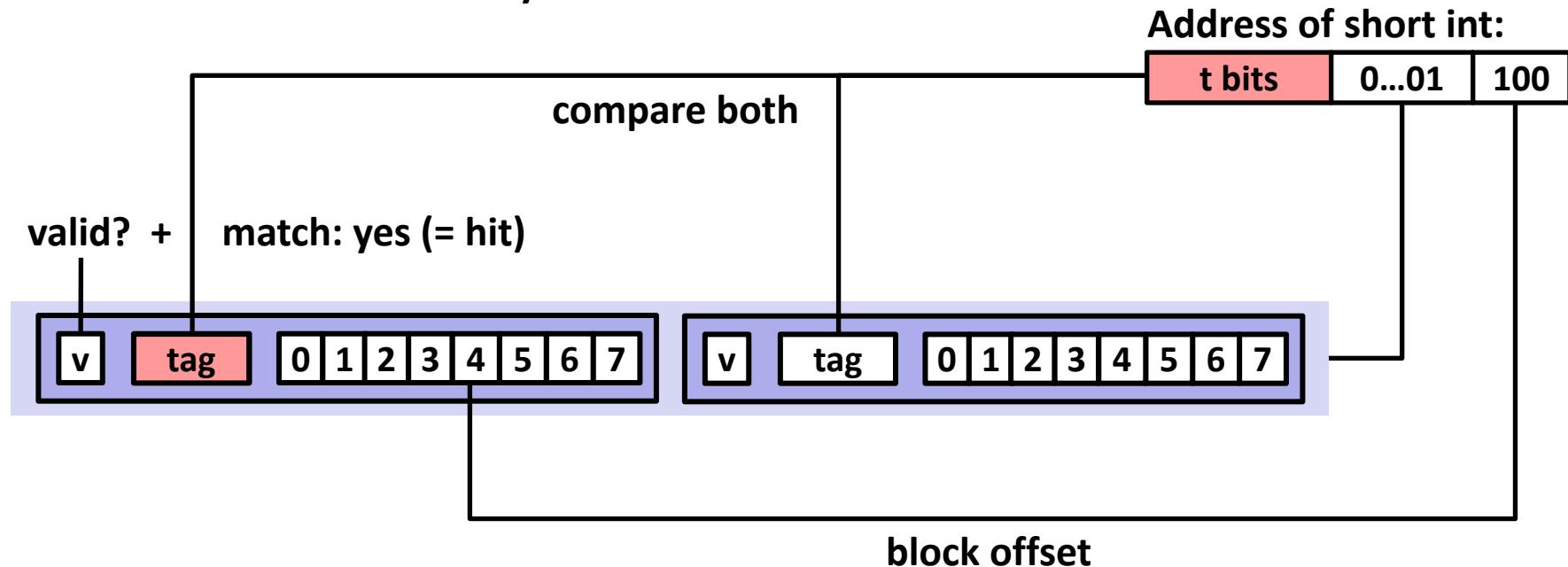


S sets

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

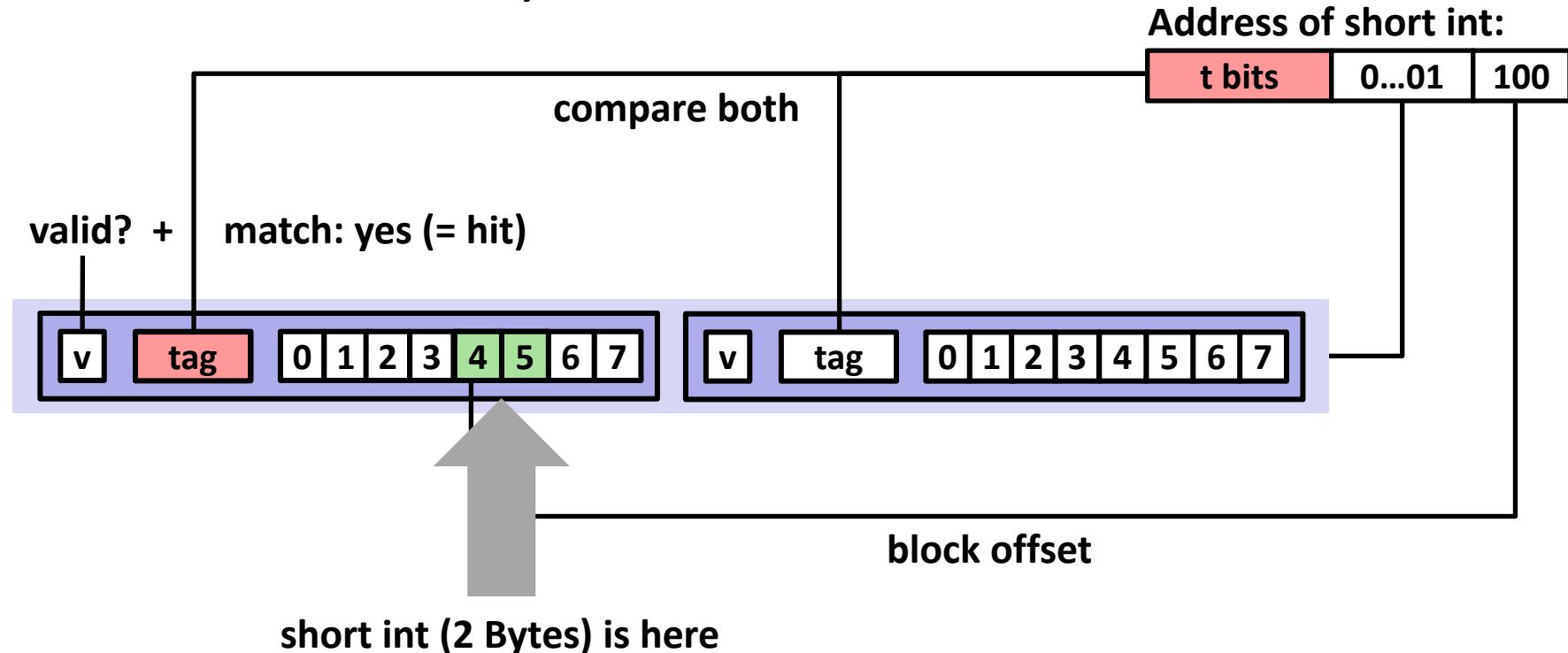
Assume: cache block size B=8 bytes



E-way Set Associative Cache (Here: E = 2)

$E = 2$: Two lines per set

Assume: cache block size $B=8$ bytes



No match or not valid (= miss):

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

2-Way Set Associative Cache Simulation

$t=2 \quad s=1 \quad b=1$



4-bit addresses ($M=16$ bytes)

$S=2$ sets, $E=2$ blocks/set, $B=2$ bytes/block

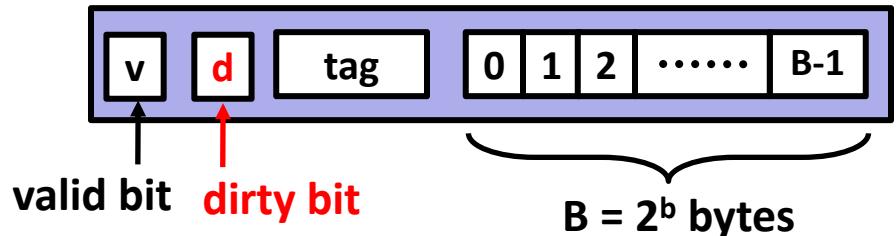
Address trace (reads, one byte per read):

0	$[0000_2]$	miss
1	$[0001_2]$	hit
7	$[0111_2]$	miss
8	$[1000_2]$	miss
0	$[0000_2]$	hit

	v	Tag	Block
Set 0	1	00	$M[0-1]$
	1	10	$M[8-9]$
Set 1	1	01	$M[6-7]$
	0		

What about writes?

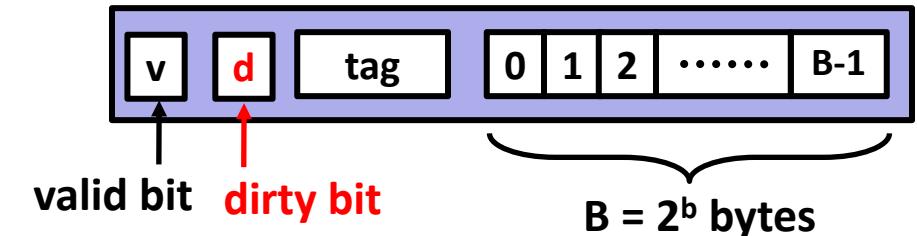
- **Multiple copies of data exist:**
 - L1, L2, L3, Main Memory, Disk
- **What to do on a write-hit?**
 - **Write-through** (write immediately to memory)
 - **Write-back** (defer write to memory until replacement of line)
 - Each cache line needs a dirty bit (set if data has been written to)
- **What to do on a write-miss?**
 - **Write-allocate** (load into cache, update line in cache)
 - Good if more writes to the location will follow
 - **No-write-allocate** (writes straight to memory, does not load into cache)
- **Typical**
 - Write-through + No-write-allocate
 - **Write-back + Write-allocate**



Practical Write-back Write-allocate

- A write to address X is issued

- If it is a hit
 - Update the contents of block
 - Set dirty bit to 1 (bit is sticky and only cleared on eviction)



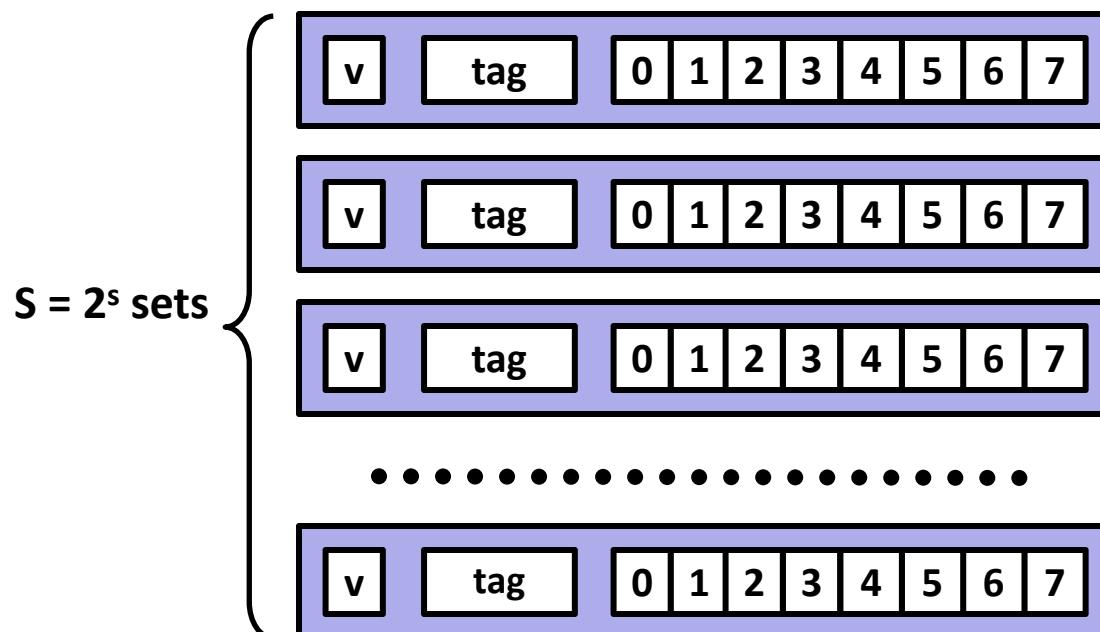
- If it is a miss
 - Fetch block from memory (per a read miss)
 - Then perform the write operations (per a write hit)

- If a line is evicted and dirty bit is set to 1
 - The entire block of 2^b bytes are written back to memory
 - Dirty bit is cleared (set to 0)
 - Line is replaced by new contents

Why Index Using Middle Bits?

Direct mapped: One line per set

Assume: cache block size 8 bytes



Standard Method: Middle bit indexing

Address of int:



find set

Alternative Method: High bit indexing

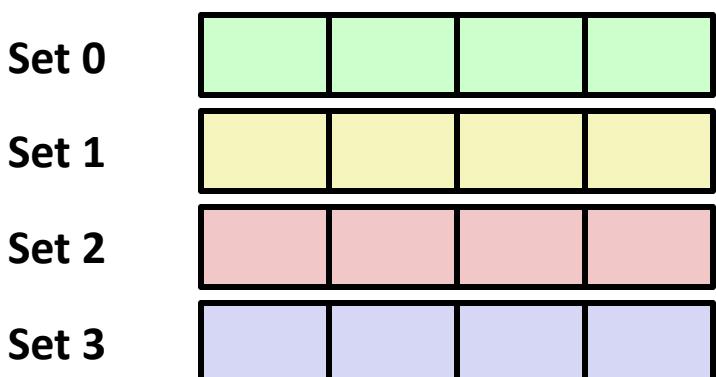
Address of int:



find set

Illustration of Indexing Approaches

- **64-byte memory**
 - 6-bit addresses
- **16 byte, direct-mapped cache**
- **Block size = 4. (Thus, 4 sets; why?)**
- **2 bits tag, 2 bits index, 2 bits offset**



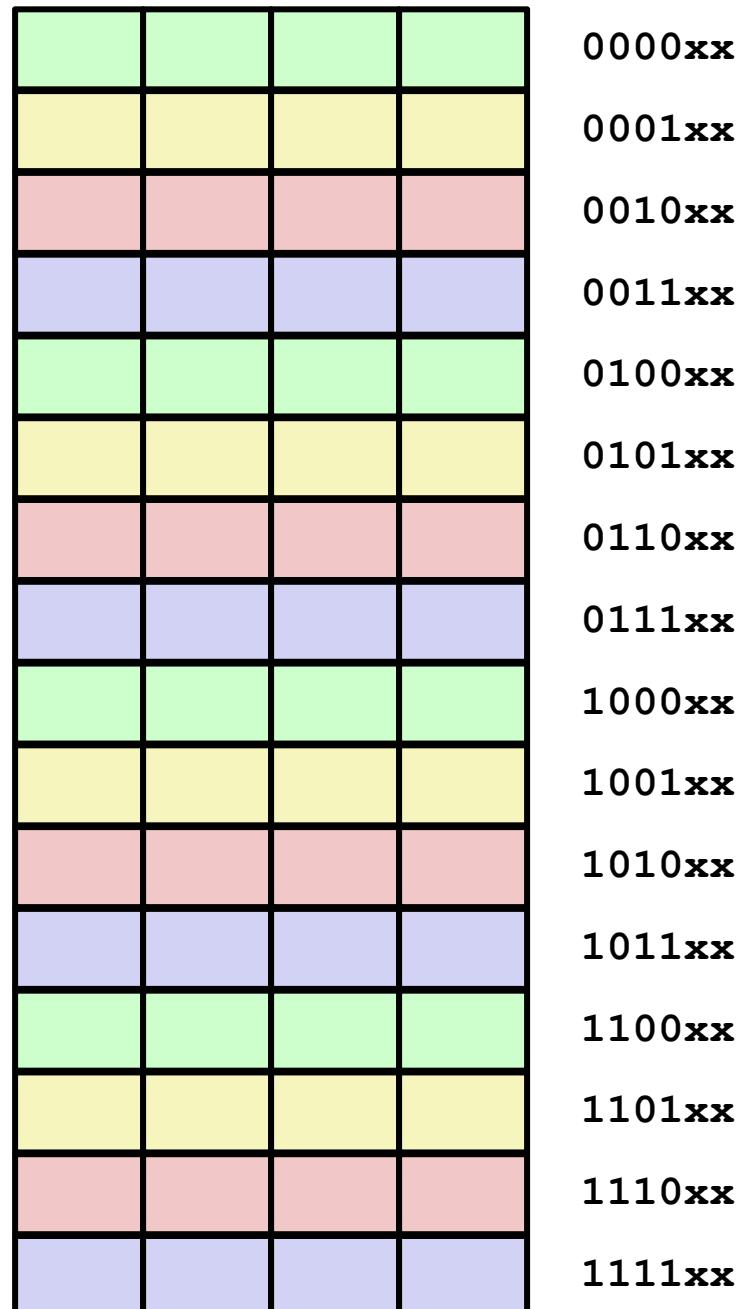
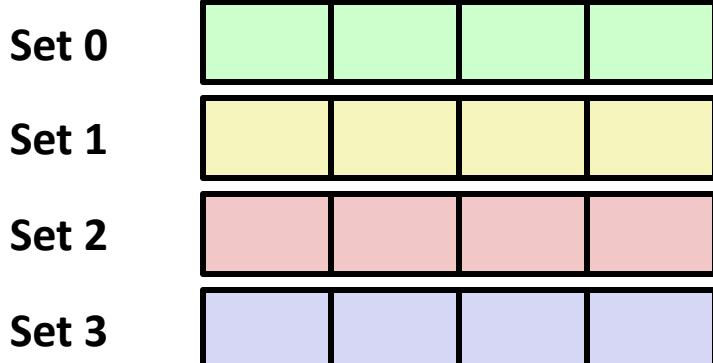
				0000xx
				0001xx
				0010xx
				0011xx
				0100xx
				0101xx
				0110xx
				0111xx
				1000xx
				1001xx
				1010xx
				1011xx
				1100xx
				1101xx
				1110xx
				1111xx

Middle Bit Indexing

- Addresses of form **TTSSBB**

- **TT** Tag bits
- **SS** Set index bits
- **BB** Offset bits

- Makes good use of spatial locality

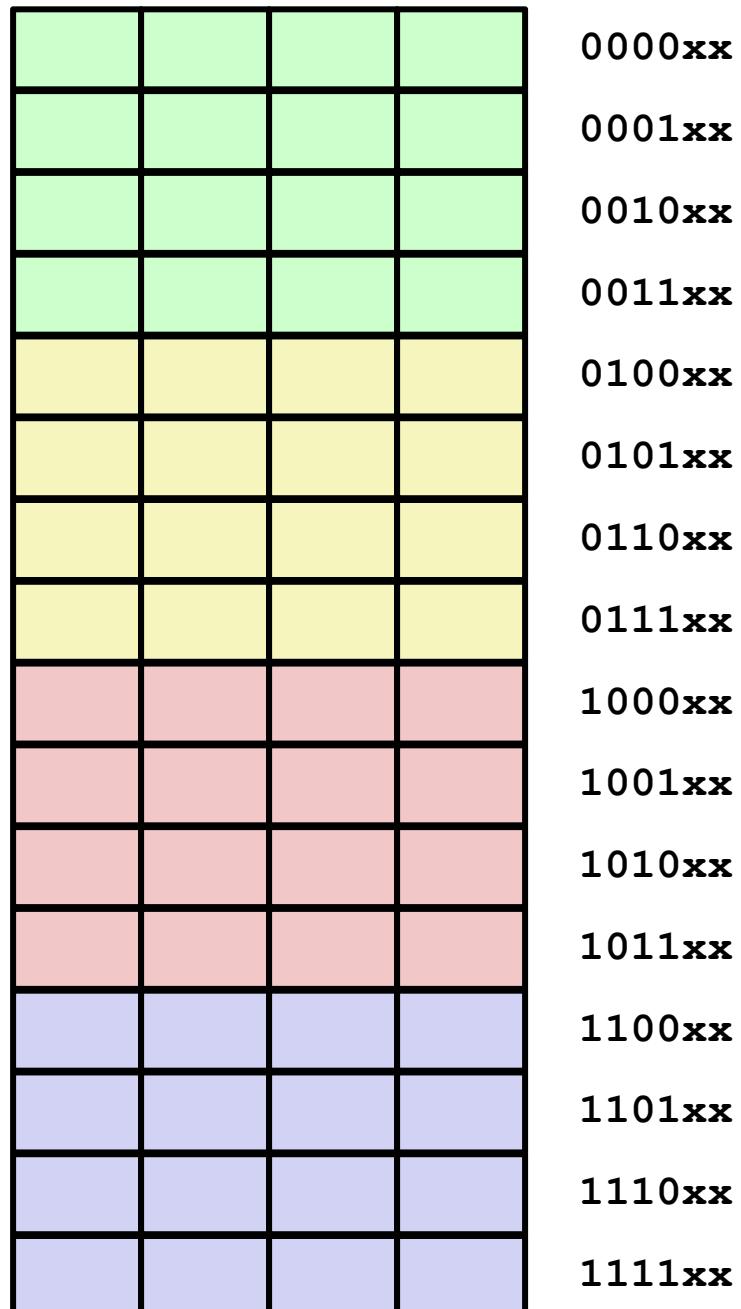
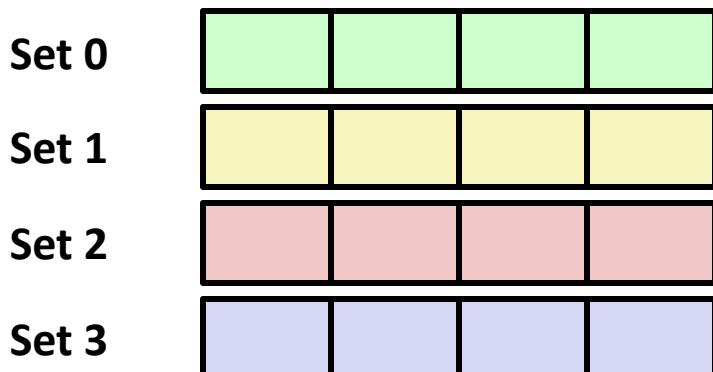


High Bit Indexing

■ Addresses of form **SSTTBB**

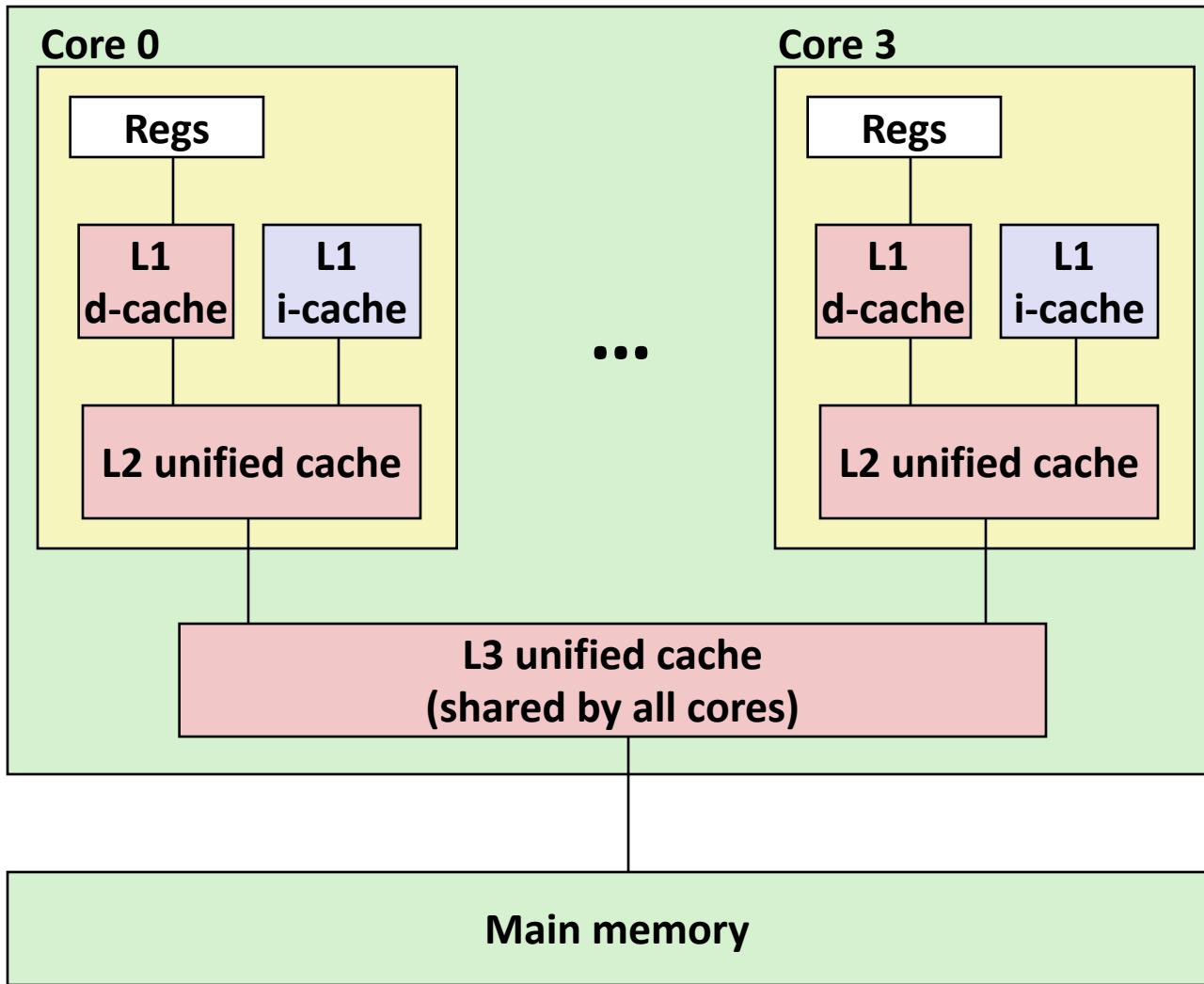
- **SS** Set index bits
- **TT** Tag bits
- **BB** Offset bits

■ Program with high spatial locality would generate lots of conflicts



Intel Core i7 Cache Hierarchy

Processor package



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for
all caches.

Cache Performance Metrics

■ Miss Rate

- Fraction of memory references not found in cache (misses / accesses)
= $1 - \text{hit rate}$
- Typical numbers (in percentages):
 - 3-10% for L1
 - can be quite small (e.g., < 1%) for L2, depending on size, etc.

■ Hit Time

- Time to deliver a line in the cache to the processor
 - includes time to determine whether the line is in the cache
- Typical numbers:
 - 4 clock cycle for L1
 - 10 clock cycles for L2

■ Miss Penalty

- Additional time required because of a miss
 - typically 50-200 cycles for main memory (Trend: increasing!)

Let's think about those numbers

- Huge difference between a hit and a miss
 - Could be 100x, if just L1 and main memory
- Would you believe 99% hits is twice as good as 97%?
 - Consider this simplified example:
cache hit time of 1 cycle
miss penalty of 100 cycles
 - Average access time:
97% hits: 1 cycle + 0.03×100 cycles = **4 cycles**
99% hits: 1 cycle + 0.01×100 cycles = **2 cycles**
- This is why “miss rate” is used instead of “hit rate”

Writing Cache Friendly Code

- **Make the common case go fast**
 - Focus on the inner loops of the core functions
- **Minimize the misses in the inner loops**
 - Repeated references to variables are good (**temporal locality**)
 - Stride-1 reference patterns are good (**spatial locality**)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories

Activity Time

- Do Activity 3-5

Today

- Cache organization and operation
- **Performance impact of caches**
 - The memory mountain
 - Rearranging loops to improve spatial locality
 - Using blocking to improve temporal locality

The Memory Mountain

- **Read throughput (read bandwidth)**
 - Number of bytes read from memory per second (MB/s)
- **Memory mountain: Measured read throughput as a function of spatial and temporal locality.**
 - Compact way to characterize memory system performance.

Memory Mountain Test Function

```

long data[MAXELEMS]; /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *         array "data" with stride of "stride",
 *         using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}

```

mountain/mountain.c

Call `test()` with many combinations of `elems` and `stride`.

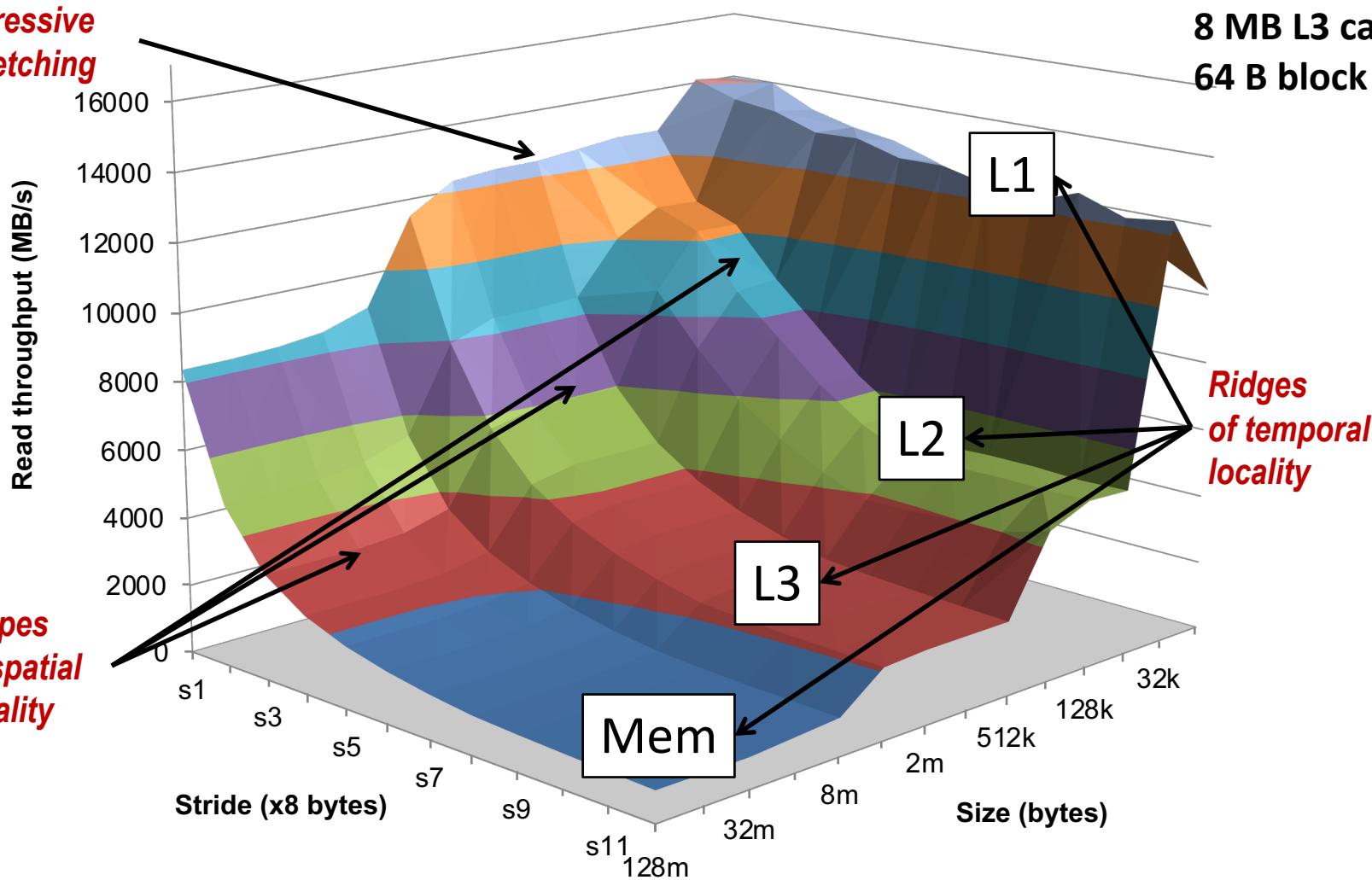
For each `elems` and `stride`:

1. Call `test()` once to warm up the caches.

2. Call `test()` again and measure the read throughput(MB/s)

The Memory Mountain

Aggressive prefetching

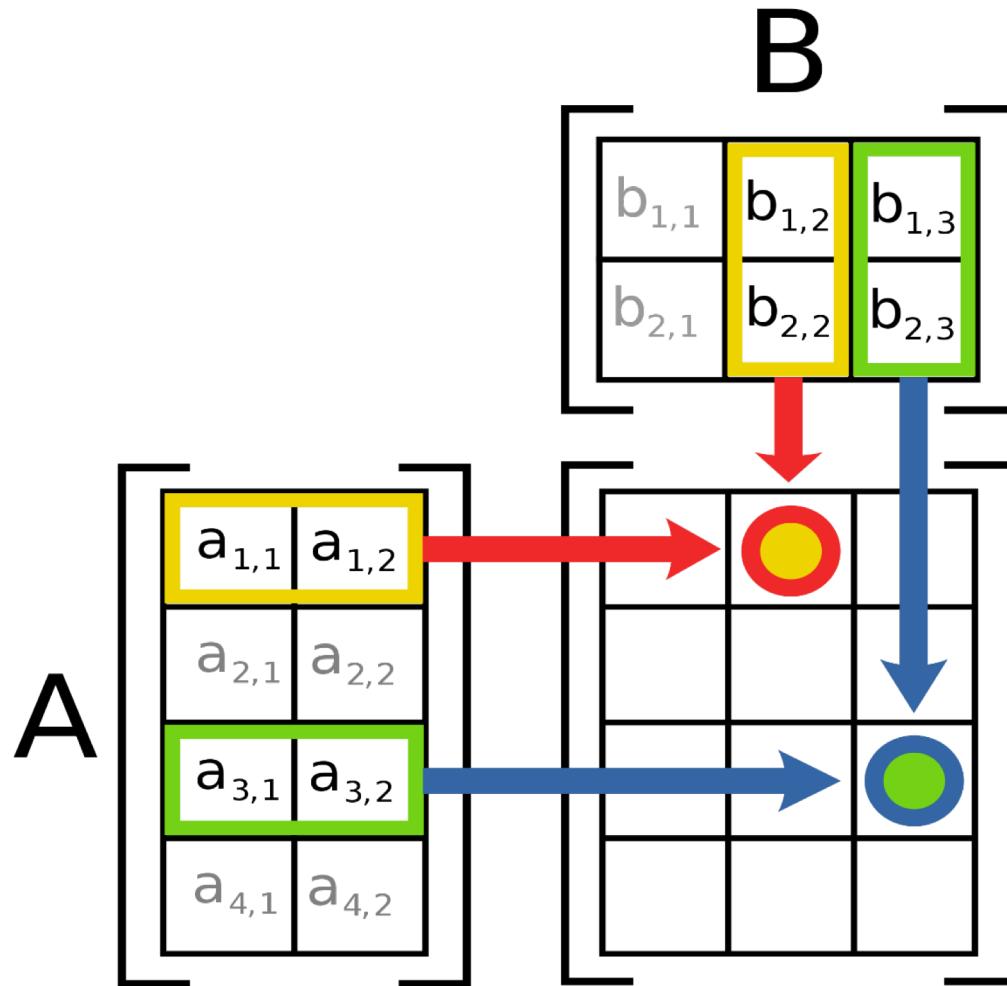


Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Today

- Cache organization and operation
- Performance impact of caches
 - The memory mountain
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Remember matrix multiplication



```
Out[i, j] =  
    dot product(A[i, ..], B[..,j])  
= sum (  
    a[i, 0] * b[0, j],  
    a[i, 1] * b[1, j]  
)
```

Matrix Multiplication Example

■ Description:

- Multiply $N \times N$ matrices
- Matrix elements are doubles (8 bytes)
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination
 - but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

Variable sum held in register

matmult/mm.c

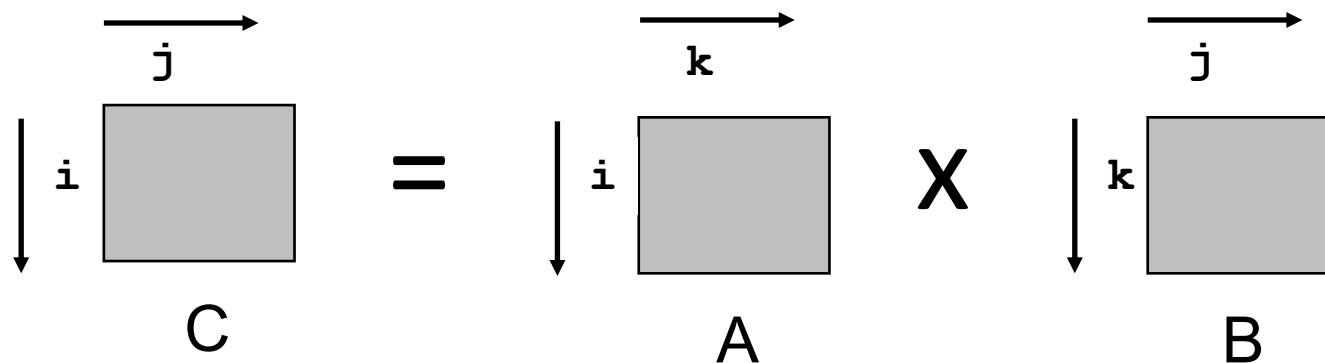
Miss Rate Analysis for Matrix Multiply

■ Assume:

- Block size = $32B$ (big enough for four doubles)
- Matrix dimension (N) is very large
 - Approximate $1/N$ as 0.0
- Cache is not even big enough to hold multiple rows

■ Analysis Method:

- Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

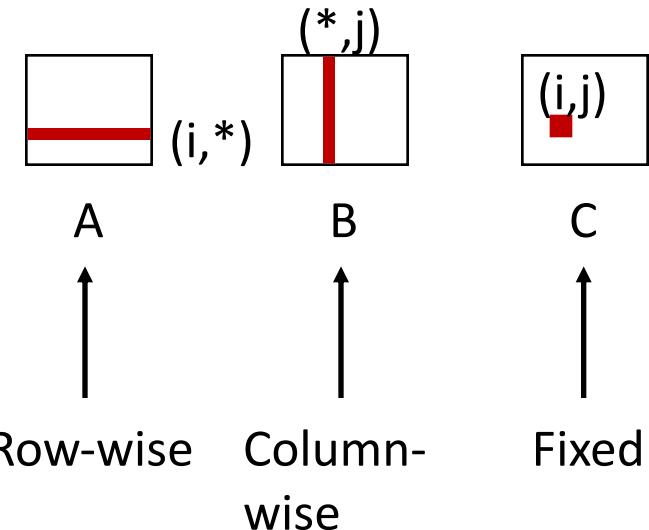
- C arrays allocated in row-major order
 - each row in contiguous memory locations
 - $a[i][j] = a[i*N + j]$ where N is the number of columns
- Stepping through columns in one row:
 - ```
for (i = 0; i < N; i++)
 sum += a[0][i];
```
  - accesses successive elements
  - if block size (B) > sizeof( $a_{ij}$ ) bytes, exploit spatial locality
    - miss rate =  $\text{sizeof}(a_{ij}) / B$
- Stepping through rows in one column:
  - ```
for (i = 0; i < n; i++)  
    sum += a[i][0];
```
 - accesses distant elements
 - no spatial locality!
 - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

matmult/mm.c

Inner loop:



Miss rate for inner loop iterations:

A

B

C

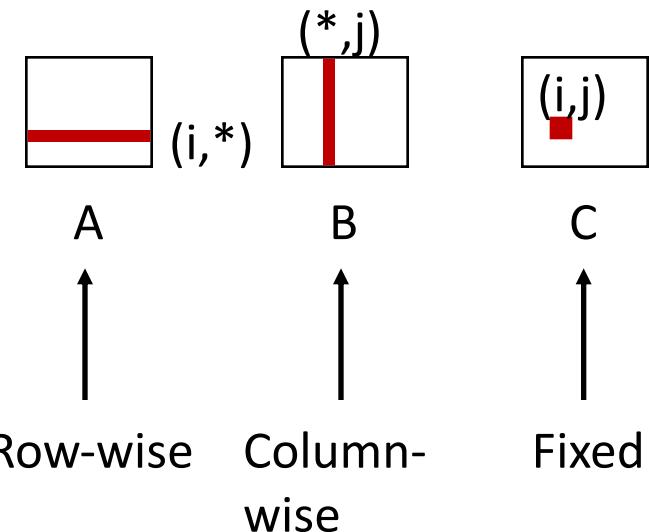
Block size = 32B (four doubles)

Matrix Multiplication (ijk)

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        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

matmult/mm.c

Inner loop:



Miss rate for inner loop iterations:

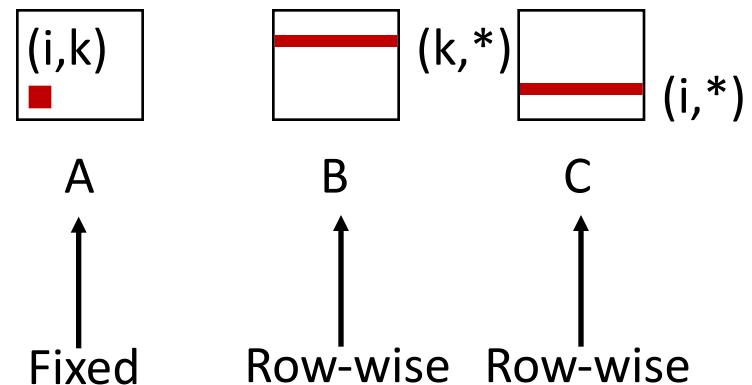
<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Block size = 32B (four doubles)

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

Inner loop:



Miss rate for inner loop iterations:

A

B

C

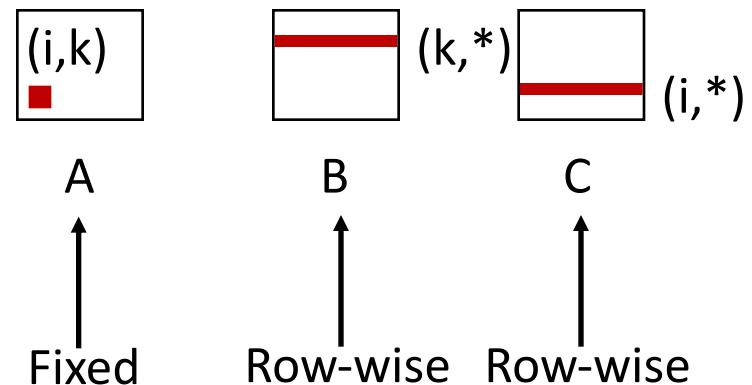
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Matrix Multiplication (kij)

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        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

matmult/mm.c

Inner loop:



Miss rate for inner loop iterations:

A
0.0

B
0.25

C
0.25

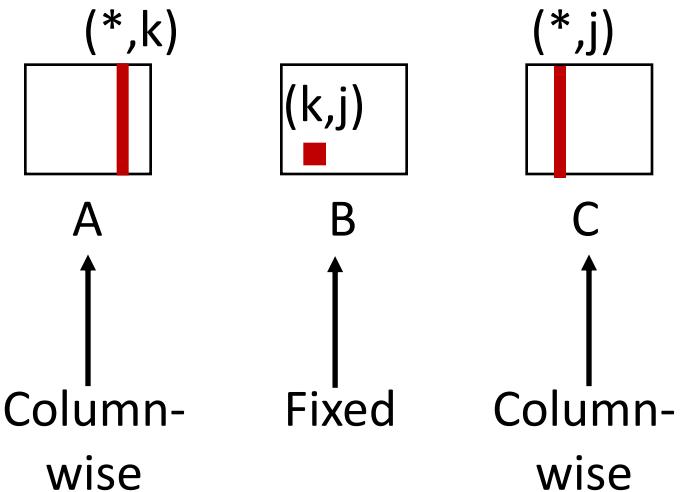
Block size = 32B (four doubles)

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

matmult/mm.c

Inner loop:



Miss rate for inner loop iterations:

A B C

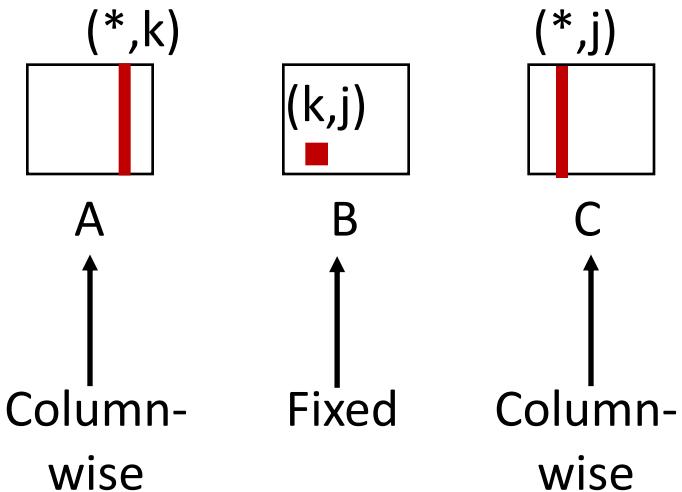
Block size = 32B (four doubles)

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

matmult/mm.c

Inner loop:



Miss rate for inner loop iterations:

A
1.0

B
0.0

C
1.0

Block size = 32B (four doubles)

Summary of Matrix Multiplication

```

for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}

```

```

for (k=0; k<n; k++) {
    for (i=0; i<n; i++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}

```

```

for (j=0; j<n; j++) {
    for (k=0; k<n; k++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}

```

ijk (& jik):

- 2 loads, 0 stores
- avg misses/iter = **1.25**

kij (& ikj):

- 2 loads, 1 store
- avg misses/iter = **0.5**

jki (& kji):

- 2 loads, 1 store
- avg misses/iter = **2.0**

Core i7 Matrix Multiply Performance

Cycles per inner loop iteration

100

jki / kji (2.0)

- jki
- kji
- ijk
- jik
- kij
- ikj

10

ijk / jik (1.25)

1

kij / ikj (0.5)

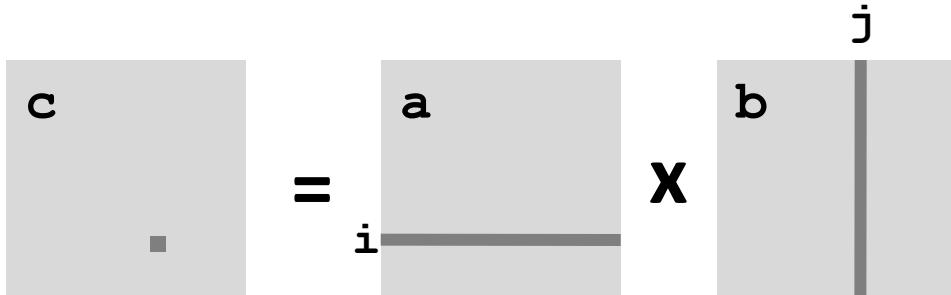


Today

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Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);  
  
/* Multiply n x n matrices a and b */  
void mmm(double *a, double *b, double *c, int n) {  
    int i, j, k;  
    for (i = 0; i < n; i++)  
        for (j = 0; j < n; j++)  
            for (k = 0; k < n; k++)  
                c[i*n + j] += a[i*n + k] * b[k*n + j];  
}
```



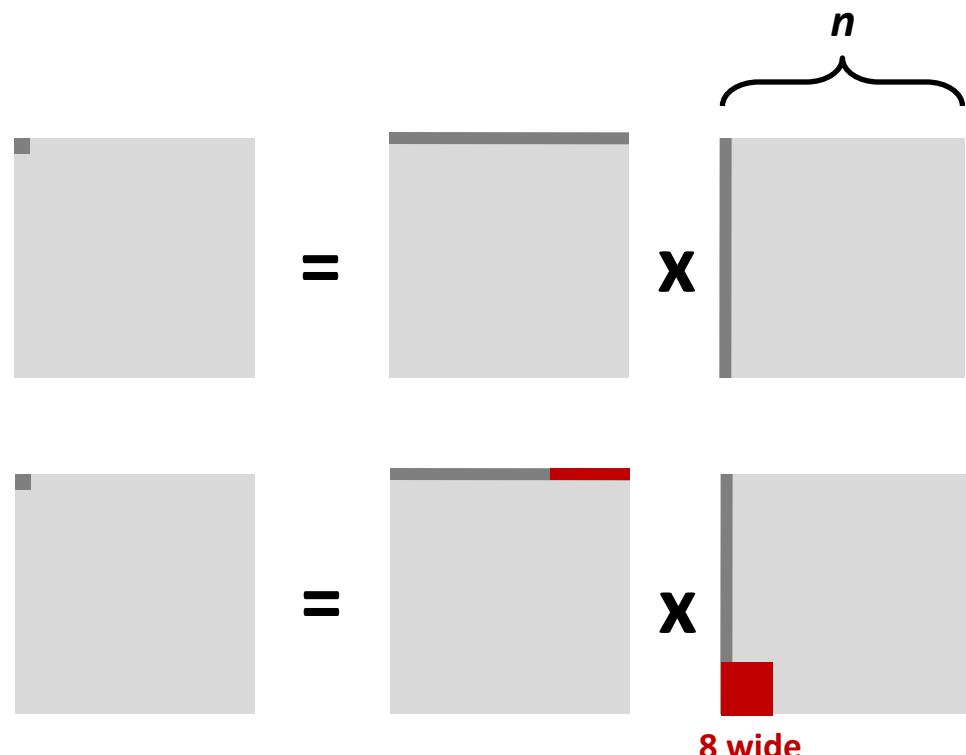
Cache Miss Analysis

■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)

■ First iteration:

- $n/8 + n = 9n/8$ misses



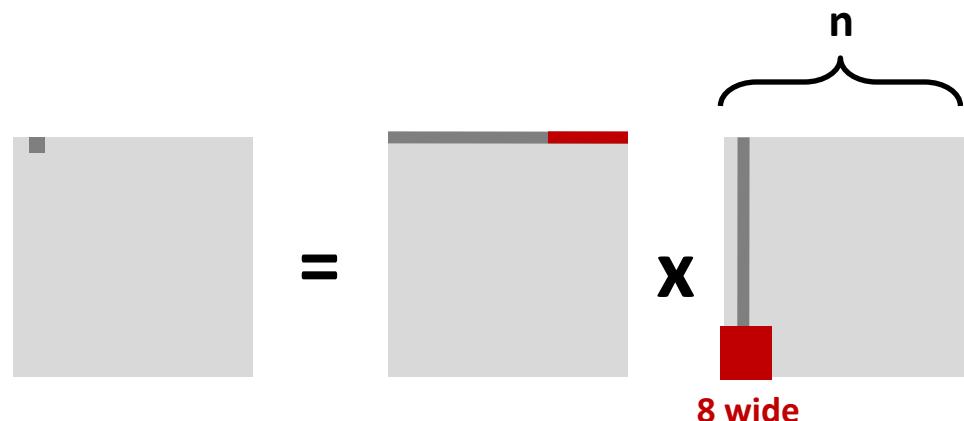
Cache Miss Analysis

■ Assume:

- Matrix elements are doubles
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)

■ Second iteration:

- Again:
 $n/8 + n = 9n/8$ misses



■ Total misses:

- $9n/8 n^2 = (9/8) n^3$

Blocked Matrix Multiplication

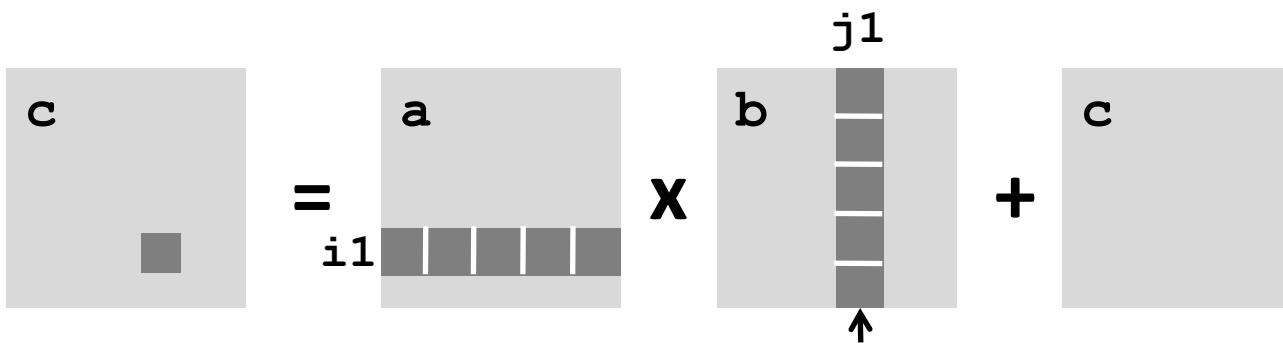
```

c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i1++)
                    for (j1 = j; j1 < j+B; j1++)
                        for (k1 = k; k1 < k+B; k1++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}

```

matmult/bmm.c



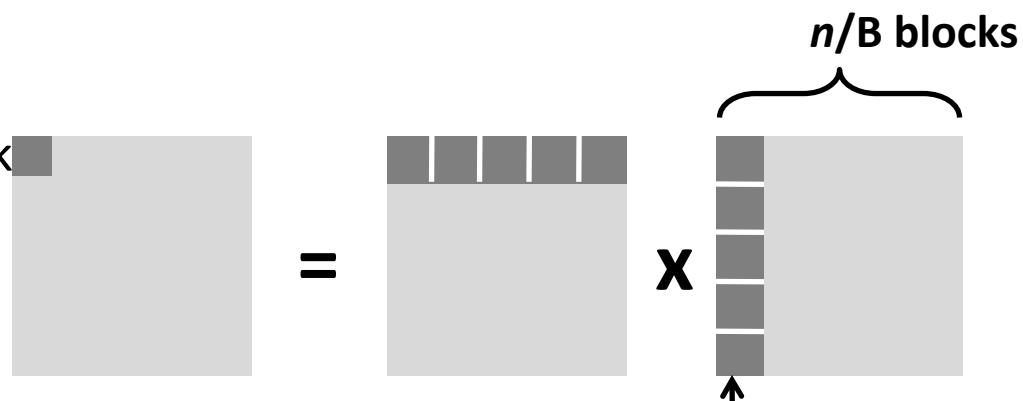
Cache Miss Analysis

■ Assume:

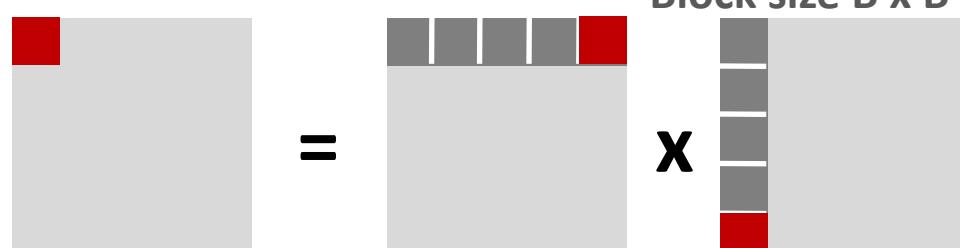
- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)
- Three blocks ■ fit into cache: $3B^2 < C$

■ First (block) iteration:

- $B^2/8$ misses for each block
- $2n/B \times B^2/8 = nB/4$
(omitting matrix c)



- Afterwards in cache
(schematic)



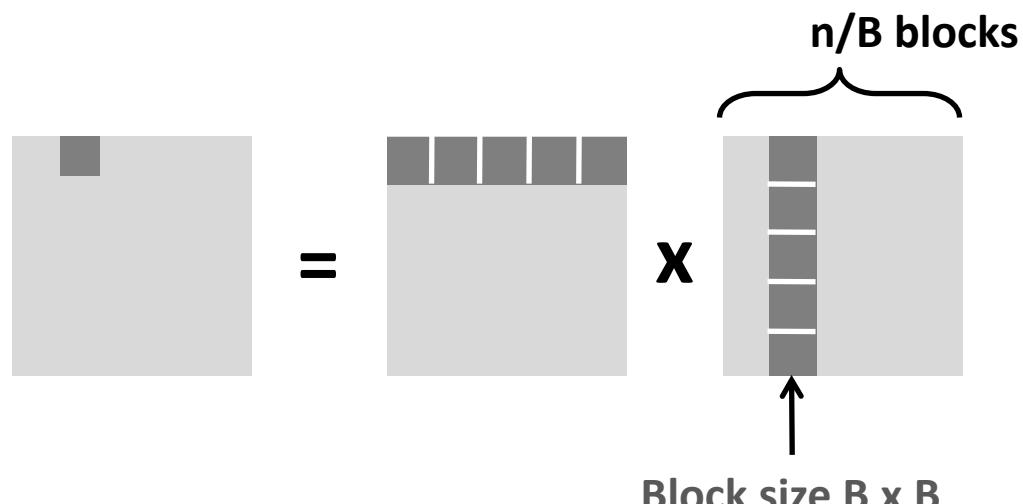
Cache Miss Analysis

■ Assume:

- Cache block = 8 doubles
- Cache size $C \ll n$ (much smaller than n)
- Three blocks ■ fit into cache: $3B^2 < C$

■ Second (block) iteration:

- Same as first iteration
- $2n/B \times B^2/8 = nB/4$



■ Total misses:

- $nB/4 * (n/B)^2 = n^3/(4B)$

Blocking Summary

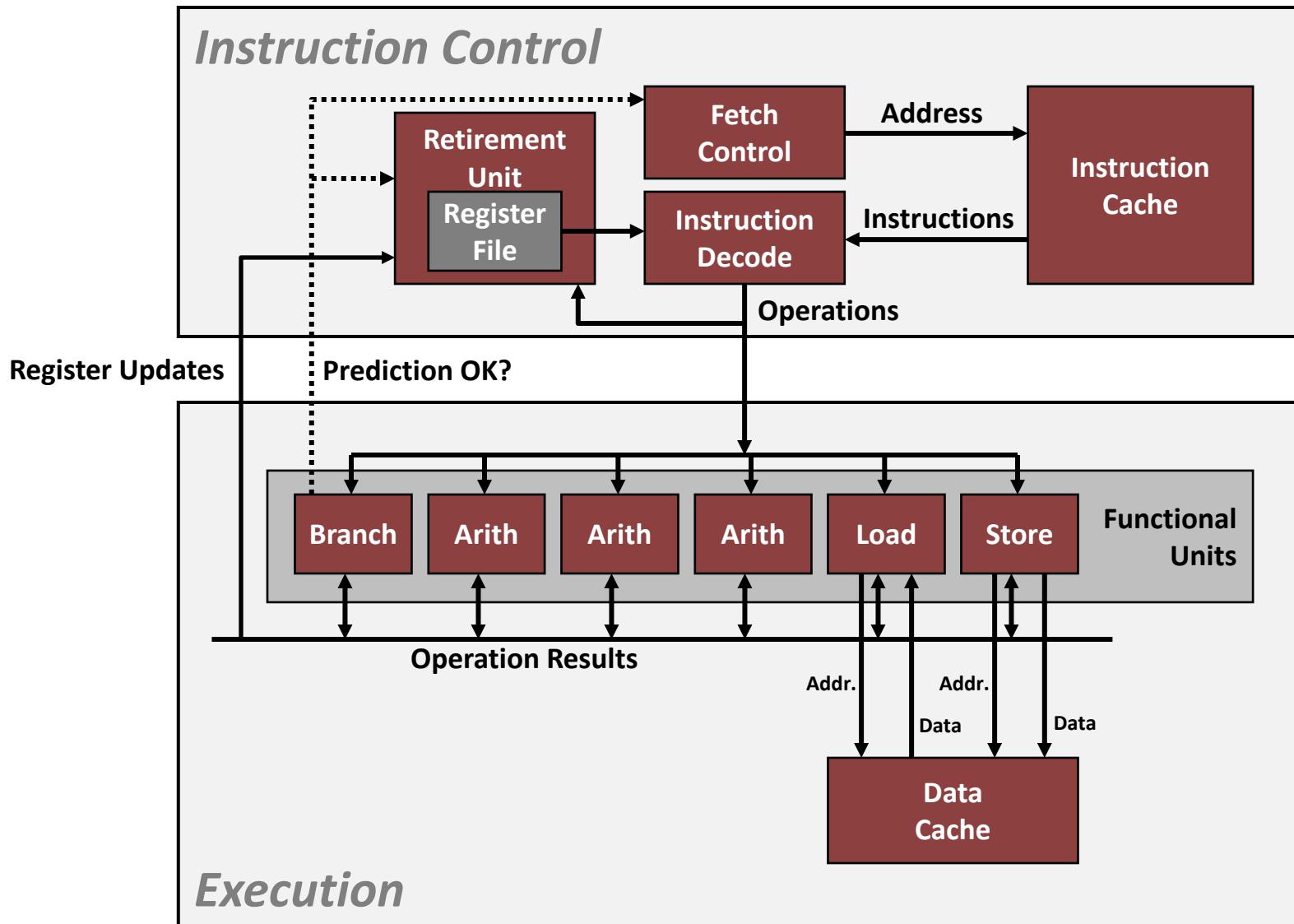
- No blocking: $(9/8) n^3$ misses
- Blocking: $(1/(4B)) n^3$ misses
- Use largest block size B , such that B satisfies $3B^2 < C$
 - Fit three blocks in cache! Two input, one output.
- Reason for dramatic difference:
 - Matrix multiplication has inherent temporal locality:
 - Input data: $3n^2$, computation $2n^3$
 - Every array elements used $O(n)$ times!
 - But program has to be written properly

Cache Summary

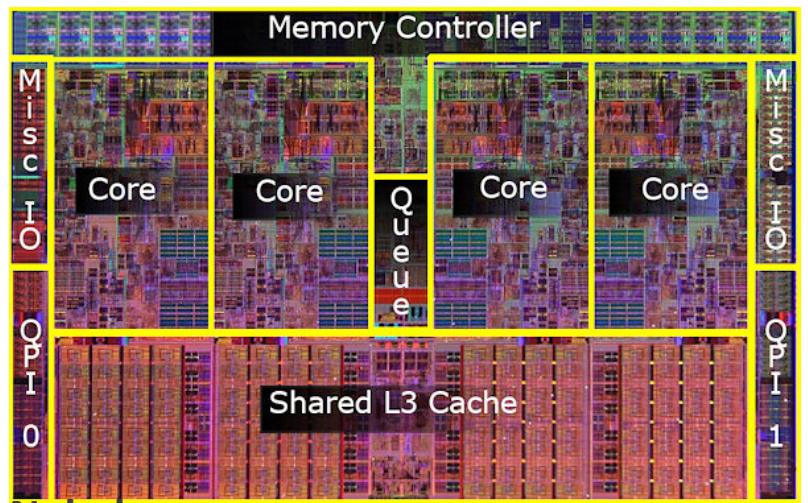
- Cache memories can have significant performance impact
- You can write your programs to exploit this!
 - Focus on the inner loops, where bulk of computations and memory accesses occur.
 - Try to maximize spatial locality by reading data objects sequentially with stride 1.
 - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

Supplemental slides

Recall: Modern CPU Design

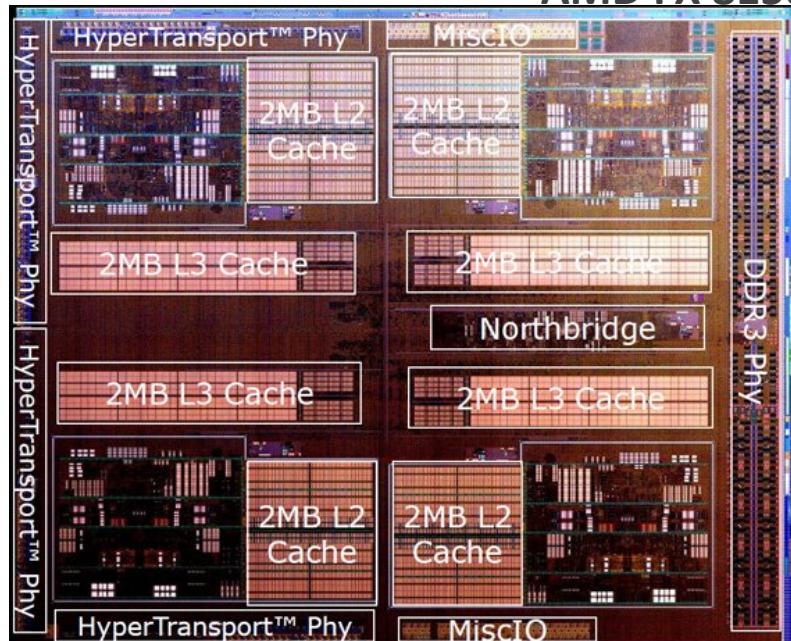


What it Really Looks Like

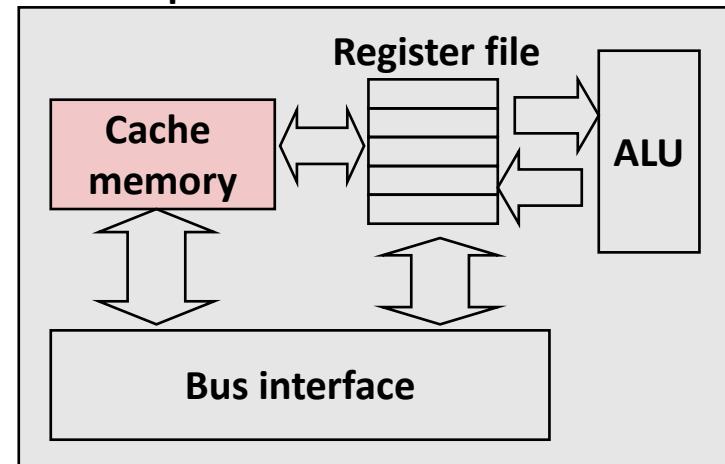


Nehalem

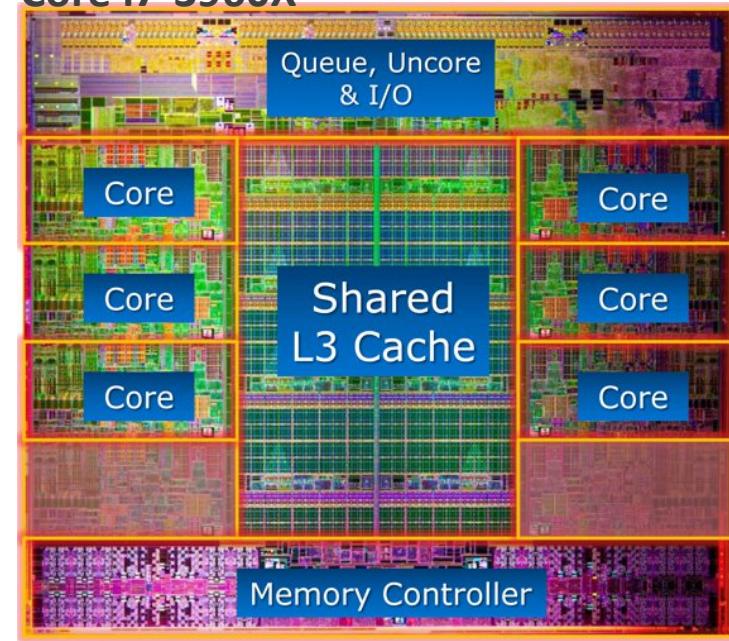
AMD FX 8150



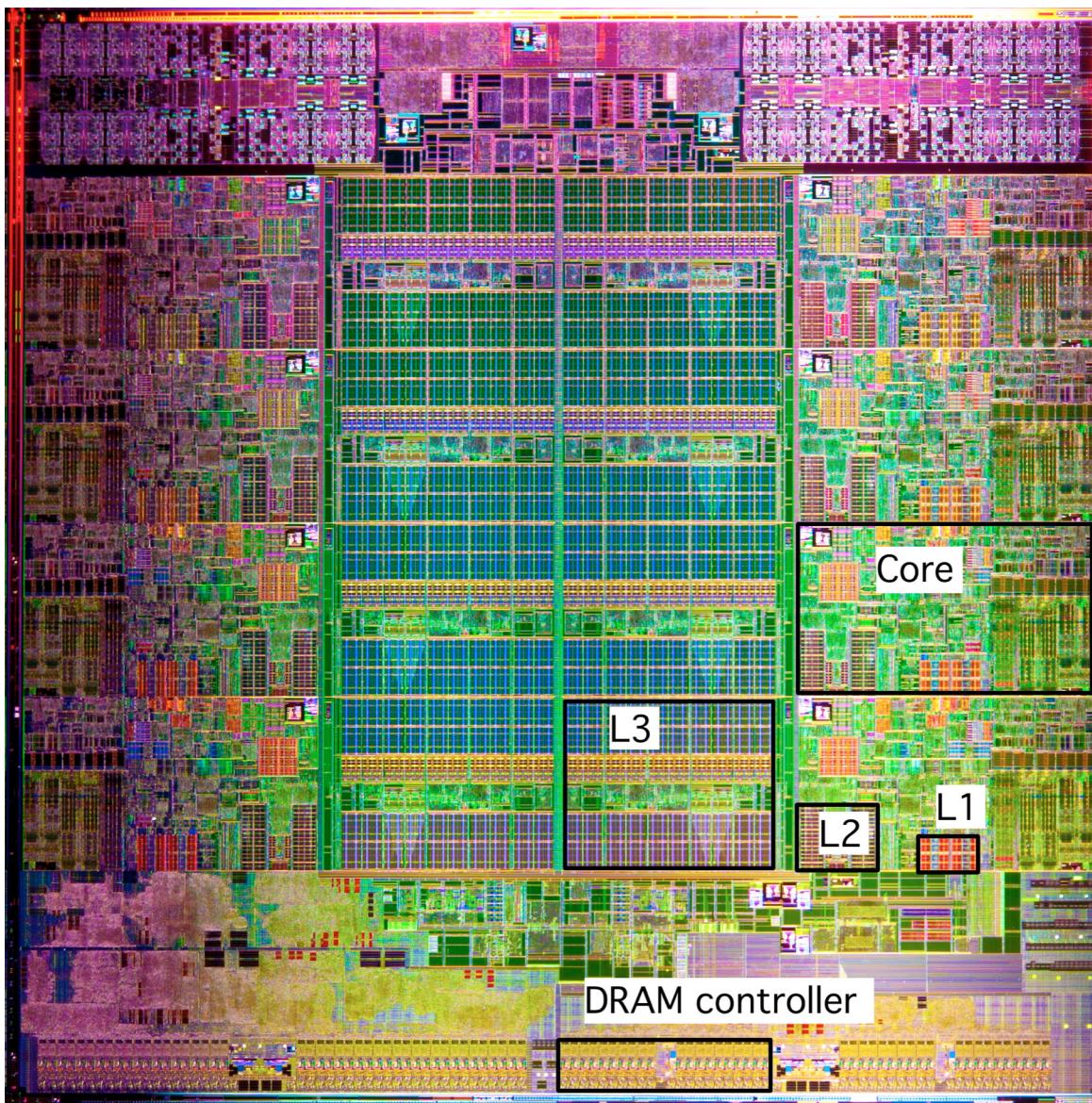
CPU chip



Core i7-3960X



What it Really Looks Like (Cont.)



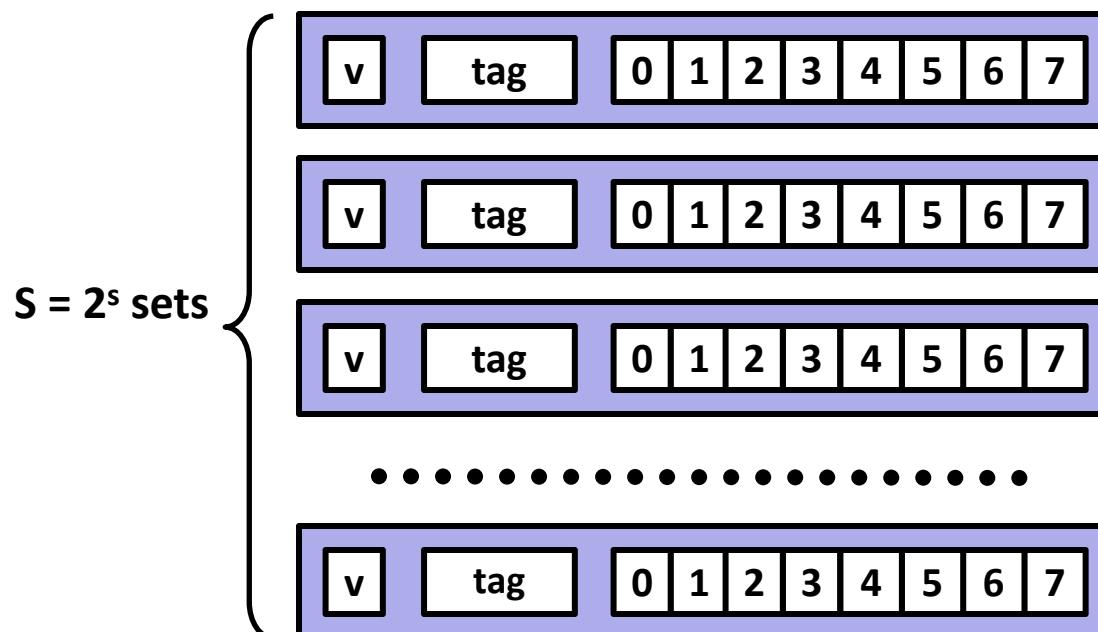
Intel Sandy Bridge
Processor Die

- L1: 32KB Instruction + 32KB Data
- L2: 256KB
- L3: 3–20MB

Why Index Using Middle Bits?

Direct mapped: One line per set

Assume: cache block size 8 bytes



Standard Method: Middle bit indexing

Address of int:



find set

Alternative Method: High bit indexing

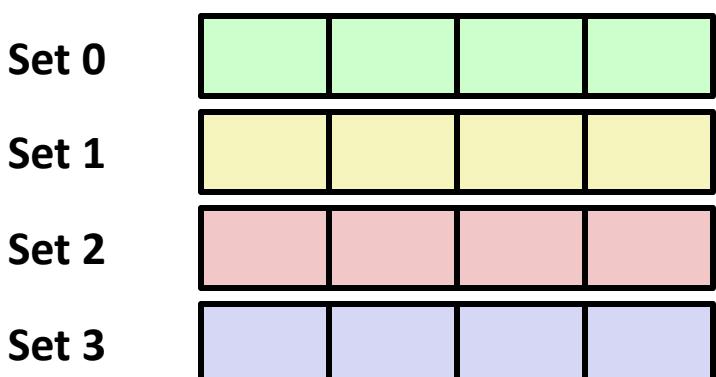
Address of int:



find set

Illustration of Indexing Approaches

- **64-byte memory**
 - 6-bit addresses
- **16 byte, direct-mapped cache**
- **Block size = 4. (Thus, 4 sets; why?)**
- **2 bits tag, 2 bits index, 2 bits offset**



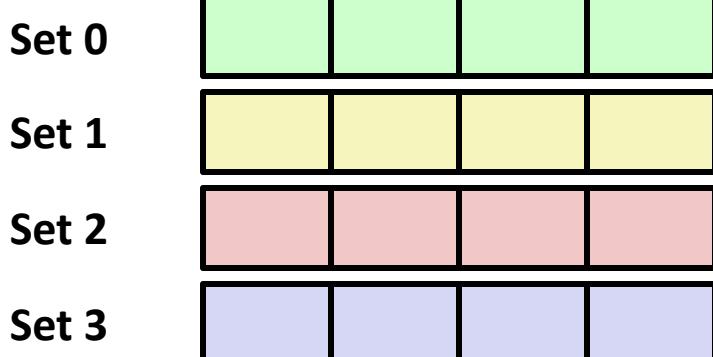
				0000xx
				0001xx
				0010xx
				0011xx
				0100xx
				0101xx
				0110xx
				0111xx
				1000xx
				1001xx
				1010xx
				1011xx
				1100xx
				1101xx
				1110xx
				1111xx

Middle Bit Indexing

■ Addresses of form **TTSSBB**

- **TT** Tag bits
- **SS** Set index bits
- **BB** Offset bits

■ Makes good use of spatial locality



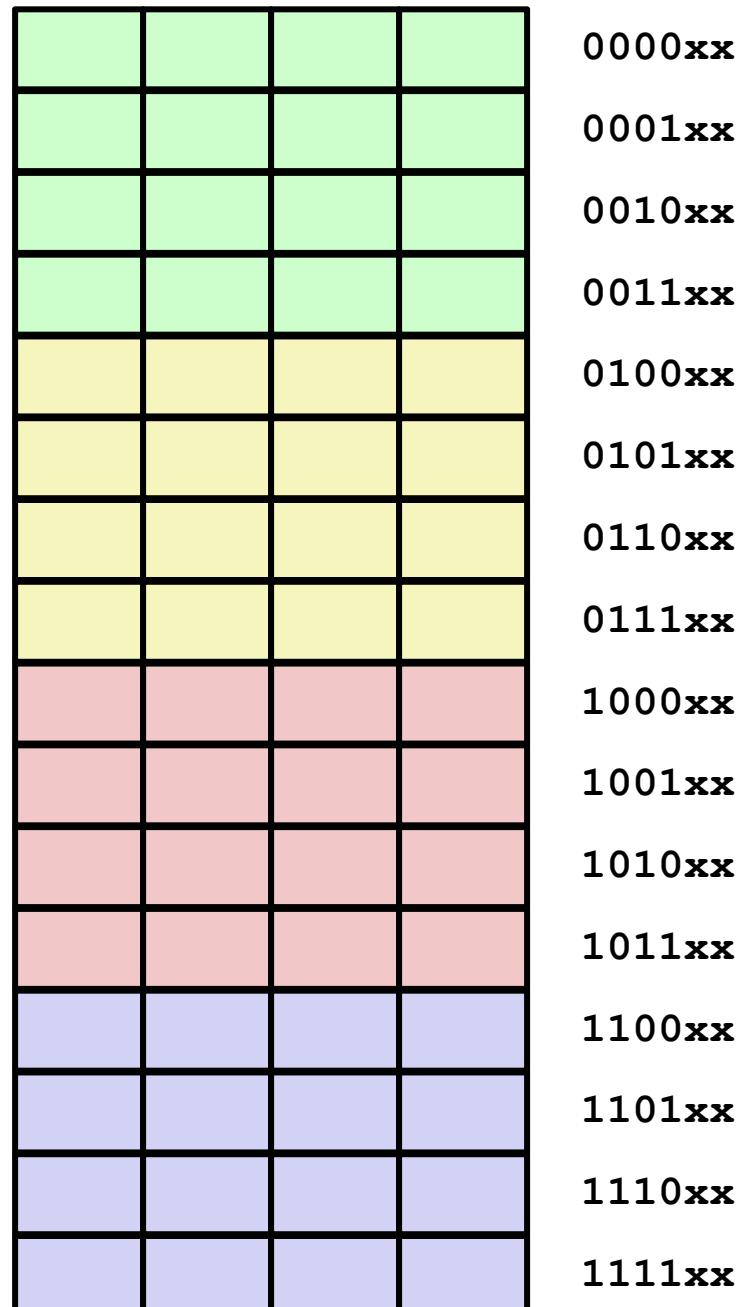
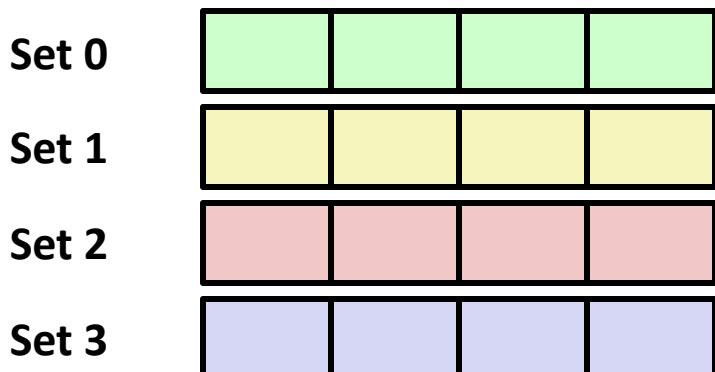
0000xx				
0001xx				
0010xx				
0011xx				
0100xx				
0101xx				
0110xx				
0111xx				
1000xx				
1001xx				
1010xx				
1011xx				
1100xx				
1101xx				
1110xx				
1111xx				

High Bit Indexing

■ Addresses of form **SSTTBB**

- **SS** Set index bits
- **TT** Tag bits
- **BB** Offset bits

■ Program with high spatial locality would generate lots of conflicts



Example: Core i7 L1 Data Cache

32 kB 8-way set associative

64 bytes/block

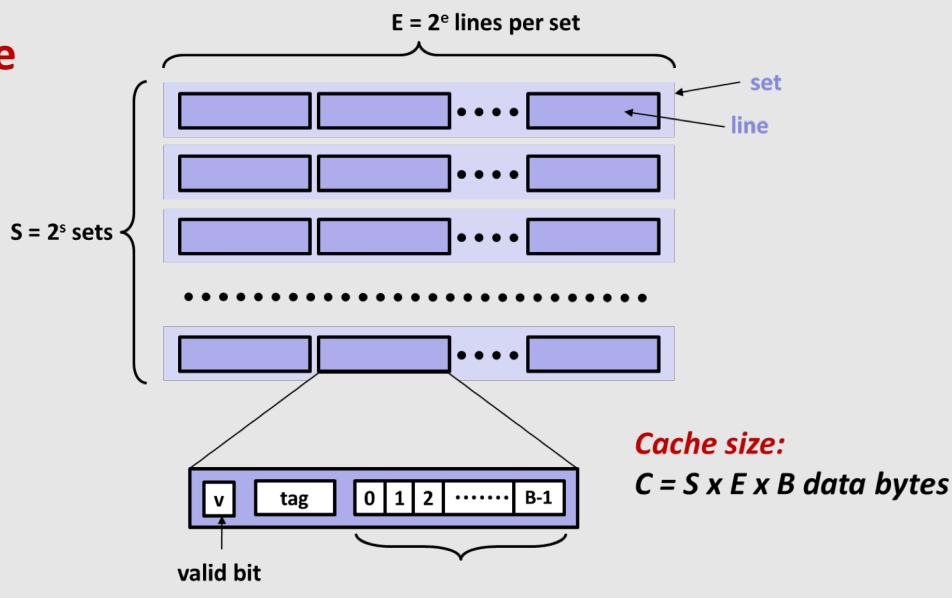
47 bit address range

B =

S = , s =

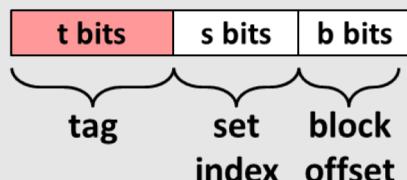
E = , e =

C =



Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Address of word:



Block offset: . bits

Set index: . bits

Tag: . bits

Stack Address:
0x00007f7262a1e010

Block offset: 0x??
Set index: 0x??
Tag: 0x??

Example: Core i7 L1 Data Cache

32 kB 8-way set associative

64 bytes/block

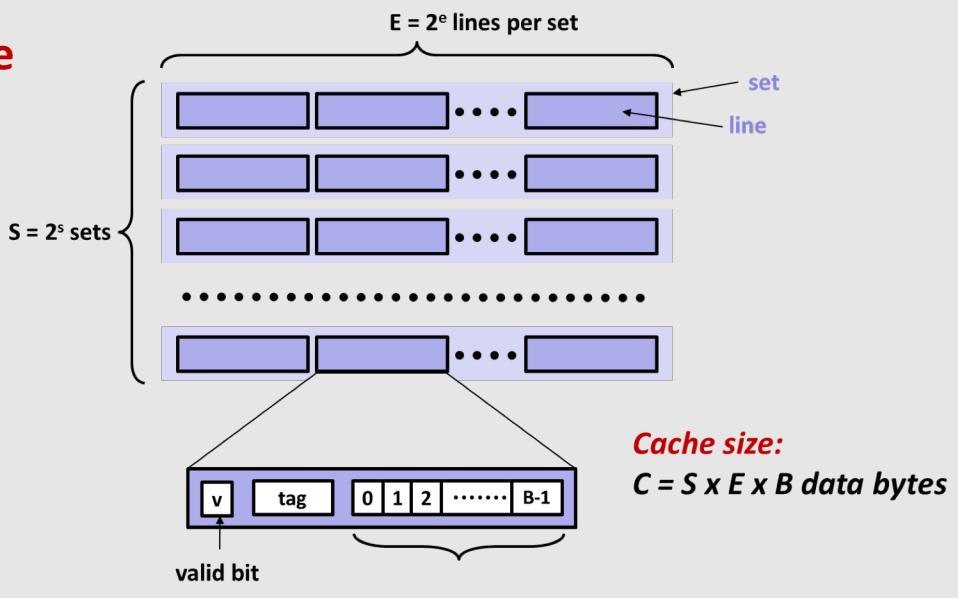
47 bit address range

$$B = 64$$

$$S = 64, s = 6$$

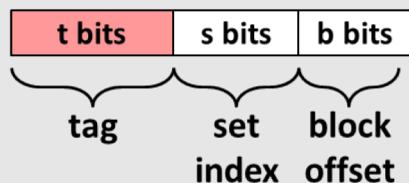
$$E = 8, e = 3$$

$$C = 64 \times 64 \times 8 = 32,768$$



	Hex	Decimal	Binary
0	0	0000	
1	1	0001	
2	2	0010	
3	3	0011	
4	4	0100	
5	5	0101	
6	6	0110	
7	7	0111	
8	8	1000	
9	9	1001	
A	10	1010	
B	11	1011	
C	12	1100	
D	13	1101	
E	14	1110	
F	15	1111	

Address of word:



Block offset: 6 bits

Set index: 6 bits

Tag: 35 bits

Stack Address:
0x00007f7262a1e010

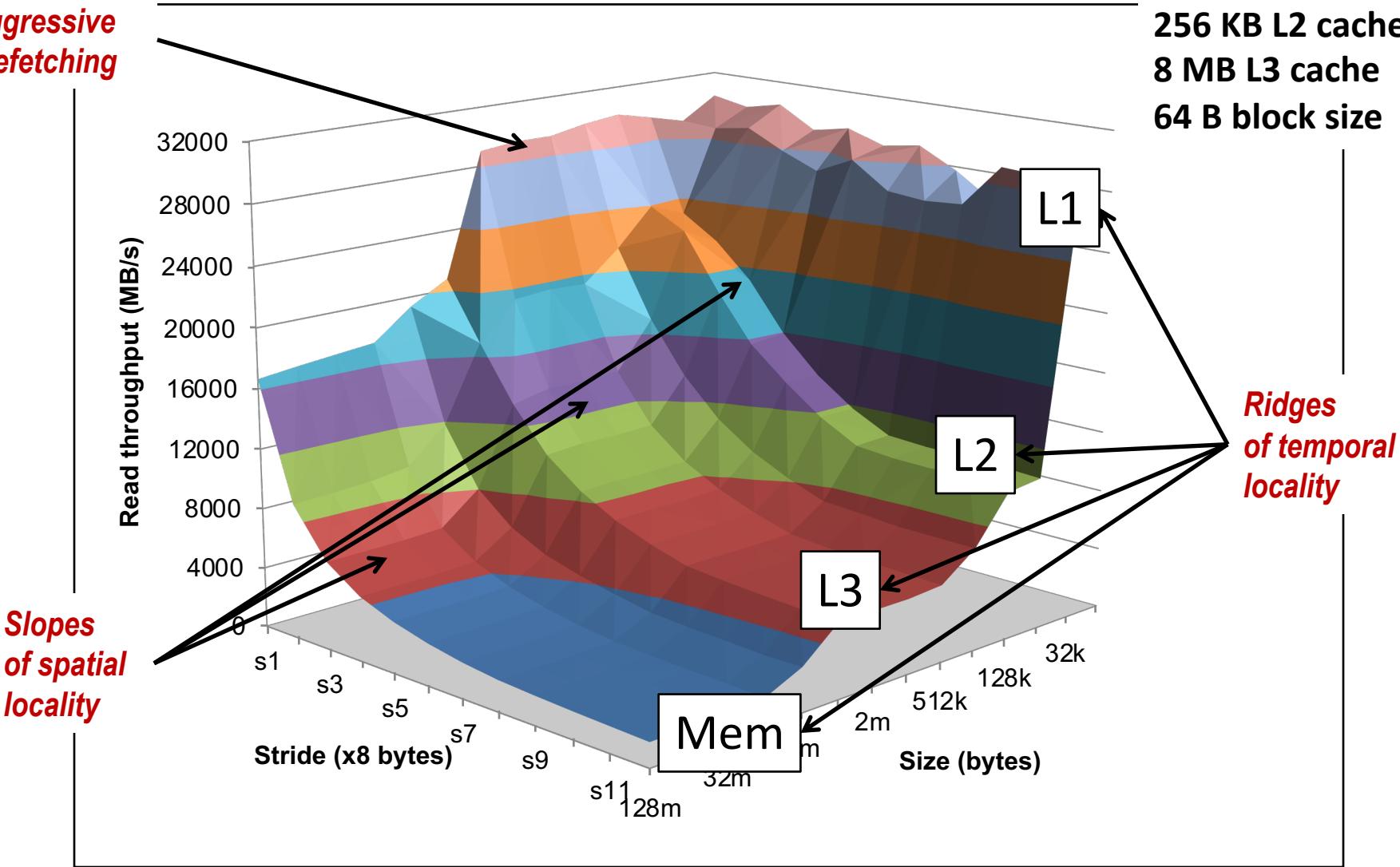
0000 0001 0000

Block offset: **0x10**
Set index: **0x0**
Tag: **0x7f7262a1e**

The Memory Mountain

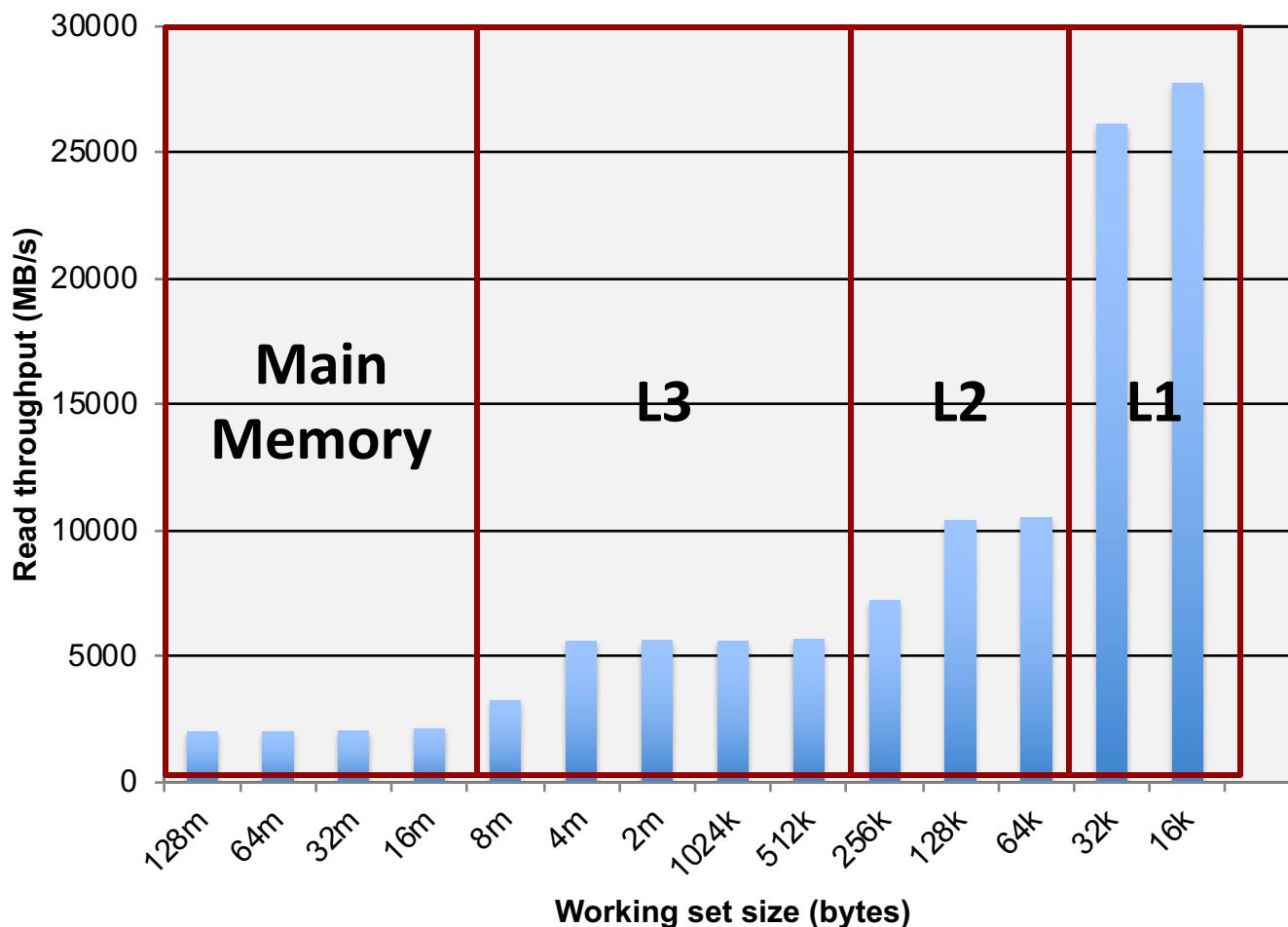
Core i5 Haswell
3.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Aggressive
prefetching



Cache Capacity Effects from Memory Mountain

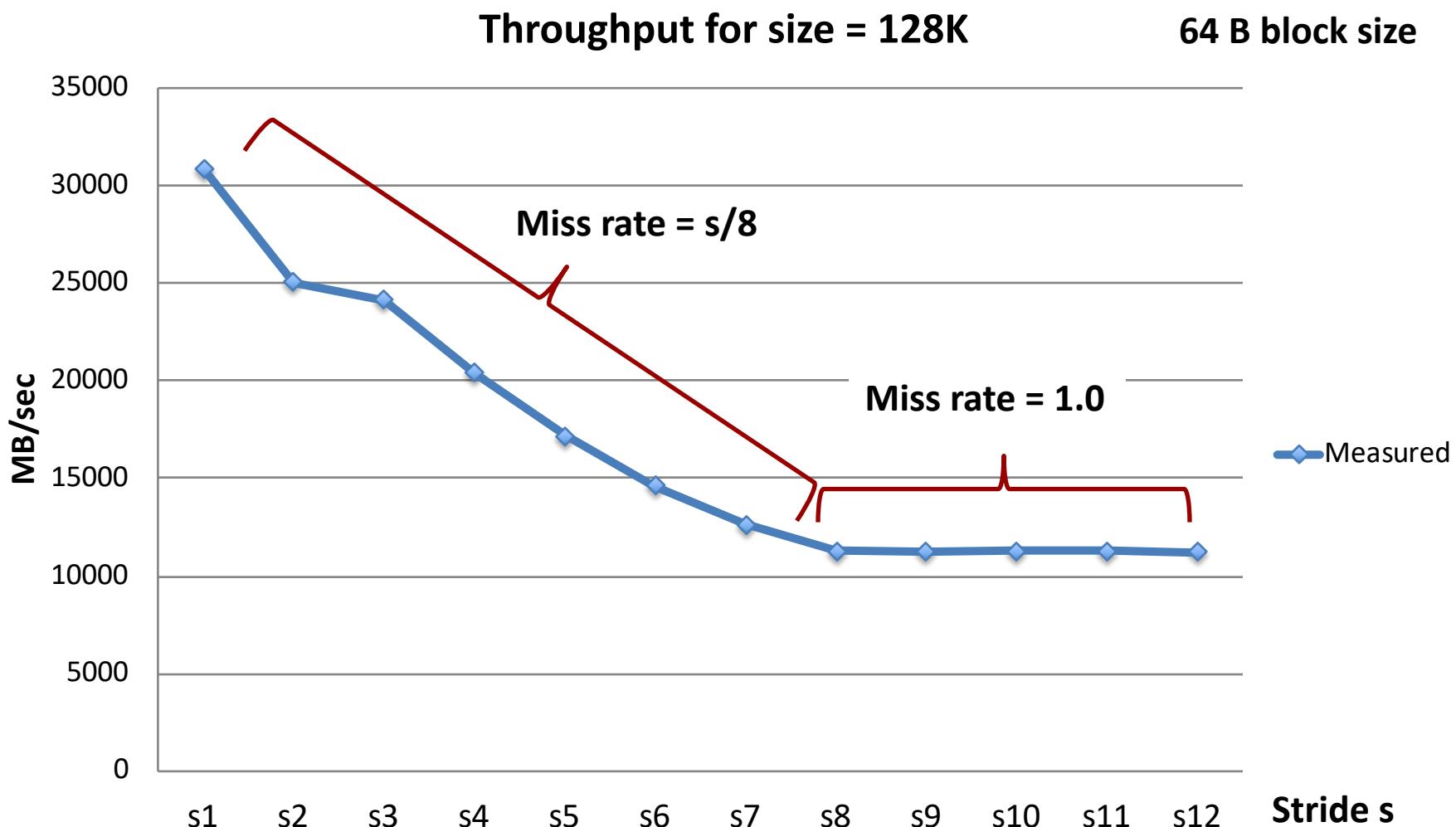
Core i7 Haswell
3.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



Slice through
memory
mountain with
stride=8

Cache Block Size Effects from Memory Mountain

Core i7 Haswell
2.26 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

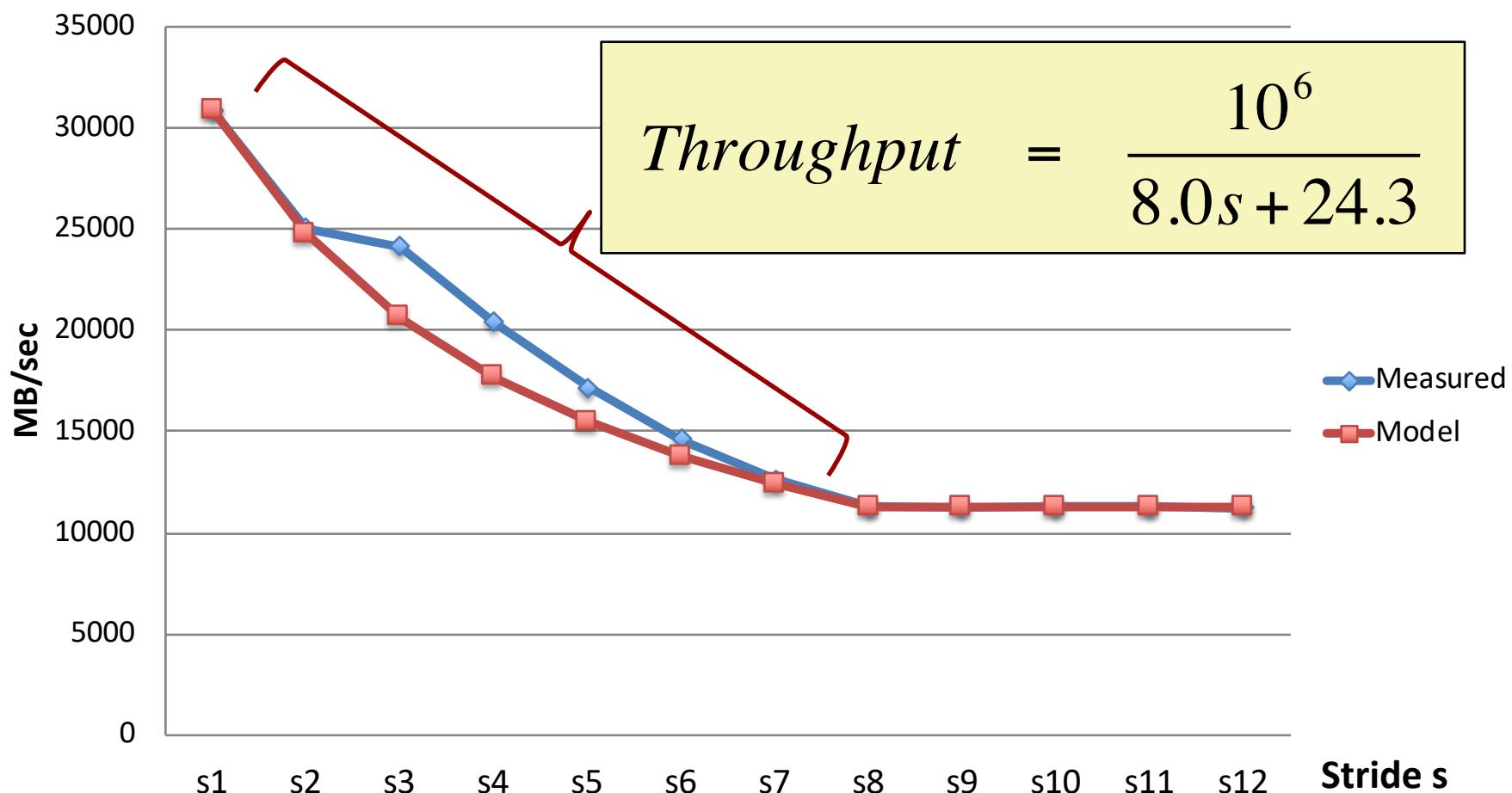


Modeling Block Size Effects from Memory Mountain

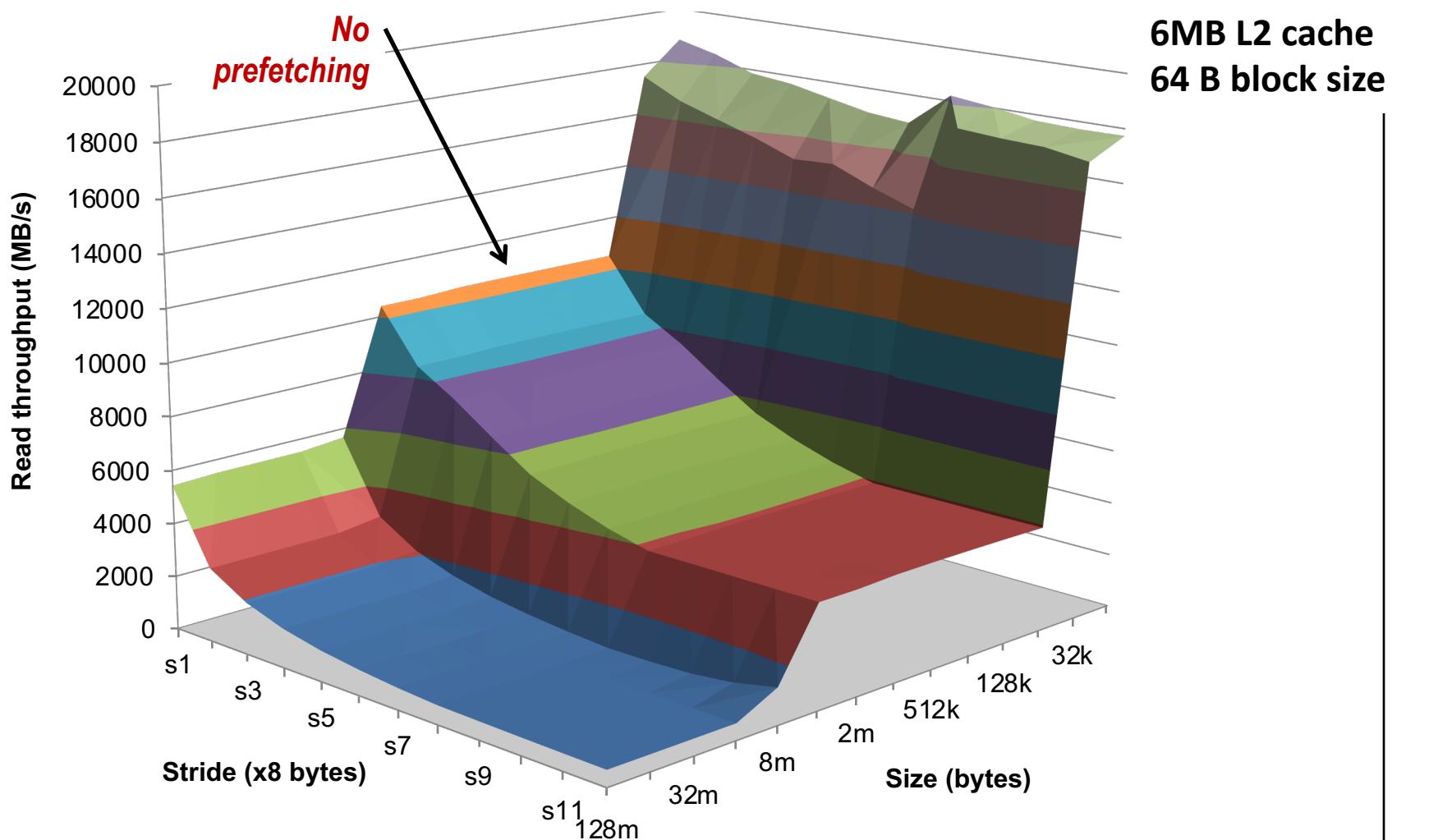
Core i7 Haswell
2.26 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Throughput for size = 128K

$$\text{Throughput} = \frac{10^6}{8.0s + 24.3}$$



2008 Memory Mountain

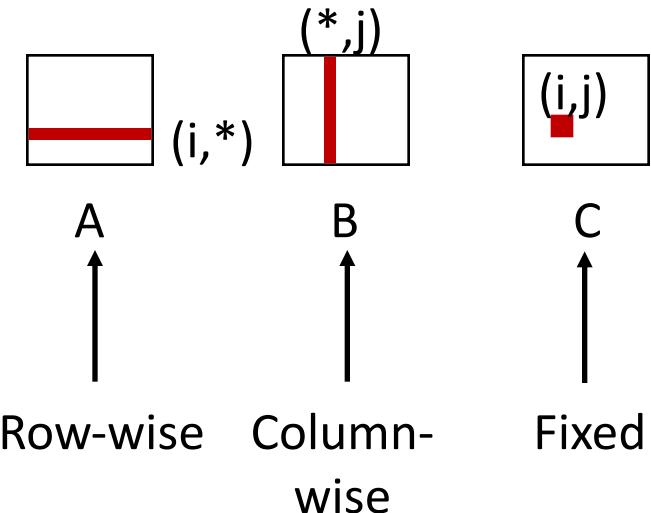


Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
    for (i=0; i<n; i++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum
    }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

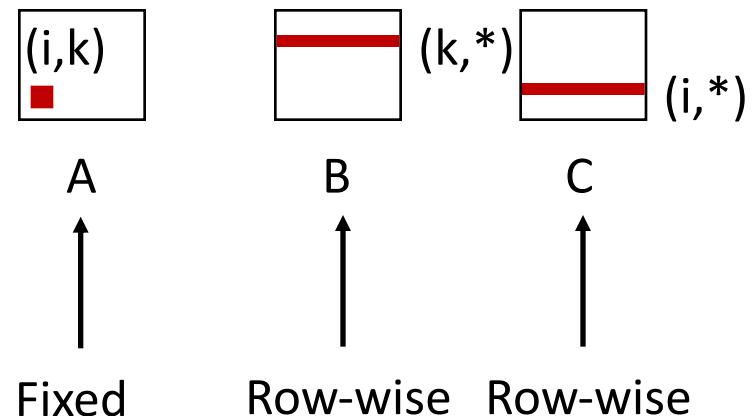
Block size = 32B (four doubles)

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

A
0.0

B
0.25

C
0.25

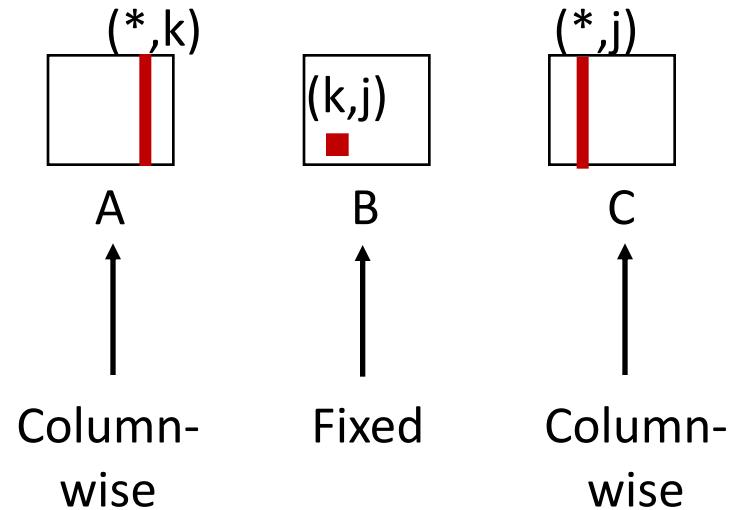
Block size = 32B (four doubles)

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
    for (j=0; j<n; j++) {
        r = b[k][j];
        for (i=0; i<n; i++)
            c[i][j] += a[i][k] * r;
    }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

A
1.0

B
0.0

C
1.0

Block size = 32B (four doubles)