矢量代数

矢量微分

$$\frac{d\vec{A}}{dt} = \hat{A}\frac{dA}{dt} + A\frac{d\hat{A}}{dt}$$

这里记得要考虑参考系的问题,如果矢量A的方向在参考系中始终与坐标轴相同,那么后一项的值为0,反之,如果A的方向在参考系中处于变化状态,那么后一项的值要考虑。

$$rac{d(ec{A} imesec{B})}{dt}=ec{A} imesrac{dec{B}}{dt}+rac{dec{A}}{dt} imesec{B}$$

注意叉乘的顺序不能颠倒

张量初步

并矢:

 $ec{A}ec{B}$

单纯放在一起,不做任何运算

通常

$$ec{A}ec{B}
eq ec{B}ec{A}$$

张量:

$$\stackrel{\leftrightarrow}{T}=ec{A}ec{B}=\sum_{i,j=1}^{3}A_{i}B_{j}\overrightarrow{e_{i}}\overrightarrow{e_{j}}=\sum_{i,j=1}^{3}T_{ij}\overrightarrow{e_{i}}\overrightarrow{e_{j}}$$
 $T=\left\{egin{array}{ccc}T_{11}&T_{12}&T_{13}\T_{21}&T_{22}&T_{23}\T_{31}&T_{32}&T_{33}\end{array}
ight\}$

从这个矩阵中可以看出来如果 $ec{A}ec{B}=ec{B}ec{A}$ 那么 $T_{ij}=T_{ji}$,T矩阵就是对称矩阵

张量的运算

加法:

$$\stackrel{\longleftrightarrow}{T} + \stackrel{\longleftrightarrow}{V} = \sum_{i,j} (T_{ij} + V_{ij}) \overrightarrow{e_i} \overrightarrow{e_j}$$

并矢与矢量点乘:

$$\begin{split} \vec{A}\vec{B}\cdot\vec{C} &= \vec{A}(\vec{B}\cdot\vec{C}) = \vec{A}(\vec{C}\cdot\vec{B}) = \vec{A}\vec{C}\cdot\vec{B} \\ \vec{C}\vec{A}\cdot\vec{B} &= \vec{C}(\vec{A}\cdot\vec{B}) = \vec{B}(\vec{C}\cdot\vec{A}) = \vec{B}\vec{A}\cdot\vec{C} \\ (\vec{A}\vec{B})\cdot\vec{C} &\neq \vec{C}\cdot(\vec{A}\vec{B}) \end{split}$$

并矢与矢量叉乘:

$$\left\{ egin{aligned} ec{A}ec{B} imesec{C} &= ec{A}(ec{B} imesec{C}) \ ec{C} imesec{A}ec{B} &= (ec{C} imesec{A})ec{B}) \end{aligned}
ight.$$

两并矢的双点乘:

$$ec{A}ec{B}:ec{C}ec{D}=(ec{B}\cdotec{C})(ec{A}\cdotec{D})$$

单位张量与矢量的点乘

$$\overleftrightarrow{I} \cdot \vec{f} = \vec{f} \cdot \overleftrightarrow{I} = \vec{f}$$

单位张量:

$$\stackrel{\longleftrightarrow}{I} = \overrightarrow{e_i} \overrightarrow{e_i} + \overrightarrow{e_j} \overrightarrow{e_j} + \overrightarrow{e_k} \overrightarrow{e_k}$$

场论初步

矢量微分算符

梯度:

$$abla = \overrightarrow{e_x} rac{\partial}{\partial x} + \overrightarrow{e_y} rac{\partial}{\partial y} + \overrightarrow{e_z} rac{\partial}{\partial z}$$

散度:

$$abla \cdot ec{A} = rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$$

旋度:

$$abla imes ec{A} = egin{array}{ccc} e_x & e_y & e_z \ rac{\partial}{\partial x} & rac{\partial}{\partial y} & rac{\partial}{\partial z} \ A_x & A_y & A_z \ \end{array}$$

高斯公式:

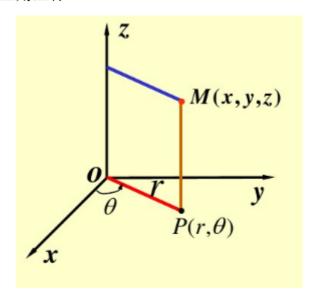
$$\iiint_{\Omega} (rac{\partial P}{\partial x} + rac{\partial Q}{\partial y} + rac{\partial R}{\partial z}) dV = \oiint_{\partial\Omega} (P\cos lpha + Q\cos eta + R\cos \gamma) dS$$

斯托克斯公式:

$$\left| egin{array}{ll} \int \!\!\! \int_S \left| egin{array}{ccc} \!\!\! dy dz & dz dx & dx dy \ rac{\partial}{\partial x} & rac{\partial}{\partial y} & rac{\partial}{\partial z} \ P & Q & R \end{array}
ight| = iggtacture \int_L P dx + Q dy + R dz$$

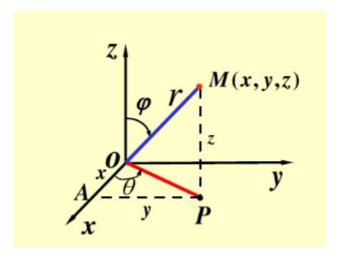
三类坐标转换

柱坐标与直角坐标



$$\left\{egin{aligned} x = r\cos heta,\ y = r\sin heta,\ z = z. \end{aligned}
ight.$$

球坐标与直角坐标



$$\left\{egin{aligned} x = r\sin\phi\cos heta,\ y = r\sin\phi\sin heta,\ z = r\cos\phi. \end{aligned}
ight.$$