The University Of British Columbia
Department of Statistics

STAT 404 Term Project Report

Mobile Phone Reception at UBC

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1 Abstract

The purpose of this project is that we aim to study the effect of location, position, level, and company, phone system on the mobile phone reception at UBC. This experiment can be described as an replicated $2^{(6-2)}$ factorial experiment in a mathematical way.

2 Introduction

Reception is considered as the most important factor when we are using mobile phones. In order to optimize the quality of communication and to find a better communication environment at UBC, the objective of this project is to study the effect of location, position, level, company and the system of mobile phones has on the mobile phone reception at UBC. This experiment is a $2^{(6-2)}$ factorial experiment with 4 treatments and 2 blocking factors, where we mainly focus on the main effect of each treatment and the interaction effect involving only treatment effects. The rest of the report is organized as follows. Details of the experimental design will be given in Section 3 followed by the empirical method we use and statistic results in Section 4. Lastly, a conclusion of this project will be given at Section 5.

3 Project Details

3.1 Data Description

Our experimental design is a $2^{(6-2)}$ fractional factorial experiment, with 4 treatment factors and 2 block factors. The four treatment factors are "different building locations at UBC", "Position of the building", "levels of floor in the building", "Phone company". Each factor has two levels, which are "IONA/CPSC", "inside/outside", "Upper/Lower", and "Fido/Chatr". The two blocking factors are "System of the phone (IOS and Android)" and "Time of the day (AM/PM)". We are interested in the two locations at UBC, which are considered as the south most and north most direction at UBC campus. In addition, in each building we think level 1(upper level) and the basement (lower level) would influence phone receptions the most. We will extract the runs where all factors are at either the -1 or +1 level. Details are shown in Table 1.
The response variable is the level of reception that appears on the phone. Since the reception scales are different for various phone systems, we will record the response variable in terms of percentage/ratio. We will rate the reception level with a range from 0 to 1. (e.g. 0 indicates no reception, 1 indicates full reception. if a reception level is $\frac{3}{5}$ shown on a phone, we will record as 0.6).

3.2 Methodology

We have 16 runs in total, and to estimate all main effects including the block factors, we use generators from table 7.6 in the course pack:

$$S = BPL; T = BPC;$$

With the above generators, we have the defining relations:

$$\text{I=SBPL=TBPC=STLC}$$

It clearly shows all aliases in the model. The resolution of the defining relation is III, which means all main effects are only aliased with interaction effects involving 3 or more factors. We assume interaction effects involving 3 or more factors are negligible in this project, so we will be able to estimate each main effects separately.
Theoretically, we should assign all treatments to each experimental unit and cell-phone randomly. However, it is not possible to make a cell-phone to a random system that we want. Instead, we randomly collected 40 phones, which 20 of them with IOS system, and the other 20 with Android system. Then, we randomly select 8 cell-phones from each of the 20-phone group with IOS and Andriod system corresponding. After that, assign all treatments to different system-phone (8 IOS and 8 Andriod) randomly. Therefore, we have $(\mathrm{C}_8^{20})^2$ ways of choosing 16 phones, and $(8!)^2$ ways of allocating treatment to experimental unit.

4 Statistical Analysis

4.1 Model and Model Check

With our experimental design, we are able to estimate all main effects including treatments and blocks main effect, and 2-factor interaction effect only involving the treatment effect:

- Six main effects (B, P, L, C, S, T);
- Six 2-factor interactions (B× P,B×L, B×C, P×L, P×C, L×C)

We omit the 2-factor interaction involving the blocking factors, 3-factor and 4-factor interactions, and assume that they are negligible. We left with three degrees of freedom for residual. Our assumed model is:

$$Y_i = \beta_0 + \beta_0 x_{iB} + \beta_P x_{iP} + \beta_L x_{iL} + \beta_C x_{iC} + \beta_S x_{iS} + \beta_T x_{iT} + \beta_{BP} x_{iB} x_{iP} + \beta_{BL} x_{iB} x_{iL} + \beta_{BC} x_{iB} x_{iC} + \beta_{PL} x_{iP} x_{iL} + \beta_{LC} x_{iL} x_{iC} + \beta_{PC} x_{iP} x_{iC} + E_i (i = 1, ..., 16)$$

Where Y_i is the random variable representing possible values of the reception y_i for observation i, 0 is an overall effect, β_B is a parameter for the main effect of factor B on mean reception, and x_{iB} is -1 or 1. if observation i has the factor Building in level of IONA or CPSC building, respectively (similarly $\beta_P, \ldots, \beta_T$ for factors P, ... , T), BP is a parameter for the interaction effect of factor B and factor P on mean reception (similarly for $\beta_{BL}, \ldots, \beta_{LC}$), an E_i is assumed to be independent N(0, σ^2) random variables.

Before building a model, we check the assumptions of the linear model and the errors are I.I.D. N(0, σ^2). We use Box-Cox transformation to see whether we need transformation for response variable y. According to Figure 1, lambda equaling to 1 falls in 95% confidence interval. Therefore, it is not necessary to have a transformation, so we keep the original y.

To use ANOVA table, we also need to check the assumption of normality and equal variance for residuals. The variance of residuals is fairly equal over the treatments, but there are more data in the two tails (see Figure 3). Also, residual is nearly normal distributed (see Figure 2). Because of the small sample size, the percentage of reception level, the assumption is not perfectly confirmed, but overall data is consistent with the assumption.

4.2 ANOVA and Linear Regression

Based on table 2, we can conclude that main effects of Building, Position, Level, System, and interaction effect of Position and Level are statistically significant in the model, at a significant level of 0.05. We are more interested in how the the mean reception of the cell-phone would change when we change factors from one level to another. It is straightforward if we can calculate these parameters, beta, and the contrasts of the effects. The estimate of the parameter for factor F is:

$$\hat{\beta} = \frac{(\bar{y}_{F=1} - \bar{y}_{F=-1})}{2}$$

For example, by looking at table 3, we can calculate the estimated effect of Building, which is

$$\frac{0.7250 - 0.8375}{2} = -0.056$$

The estimated interaction effect of Building and Position can be calculated as:

$$\frac{\frac{0.7250-0.8375}{2} - \frac{0.575+0.950}{2}}{2} = 0.019$$

Other estimated effects can be calculated similarly, as shown in table 4. The standard deviation for these estimated effects would all be the same. It is $\sqrt{\frac{\sigma^2}{n}}$. σ^2 is unknown, but we can use MS(residual) to estimate it, and give a standard error of $\sqrt{\frac{0.023}{16}}$=0.012. The absolute estimated effects have a half normal distribution of (β, $\sqrt{\frac{\sigma^2}{n}}$). We use the half-normal plot to identify null effects, and under the null hypothesis, =0, the points of absolute estimated effects of on Figure 4 should be on a straight line. However, in Figure 4, $|\hat{\beta}_B|, |\hat{\beta}_P|, |\hat{\beta}_L|, |\hat{\beta}_S| and |\hat{\beta}_{PL}|$ are off the line, which indicates that we have some evidence that the effects of Building, Position, Level and System, and interaction effect of Position and Level are not zero. The result is consistent with what we got in the analysis of variance. Because we use MS(residual) to estimate σ^2, a t-distribution with a degree of freedom equal to 3 would be suitable for computing the confidence interval for estimated effects: 95 percent confidence interval of estimated effect of factor F:

$$\hat{\beta}_F \pm (t_{3,0.975})se(\hat{\beta}_F)$$

For estimated effect of building, the 95 percent confidence interval would be:

$$\text{-0.056} \pm 3.18(0.012) = (\text{-0.094, -0.018})$$

Similarly, the confidence intervals of other estimated effects can be calculated by the same method. By looking at whether the confidence interval include zero or not, we could conclude whether the effects have positive, negative or no effect on mean reception. Alternatively, we can use one-sample t test:

$$t = \frac{\hat{\beta}_F}{se(\hat{\beta}_F)} = \frac{\hat{\beta}_F}{\sqrt{\frac{MS(Residual)}{n}}} = \frac{\hat{\beta}_F}{0.012}$$

The t-statistics and p-values are shown in Table 4.

4.3 Contrast Effects and Interaction Effects

In addition, the estimated contrast effects of each factor are calculated as following:

$$\hat{k} = (\bar{y}_{F=1} - \bar{y}_{F=-1})$$

This gives twice of the estimated effect of each factor. Also, the standard error of estimated contrast effects is twice of $se(\beta_F)$.

We have 95 percent confidence intervals for the following:

The effect of change the factor of building from IONA to CPSC would decrease the mean reception by 2(-0.056)$\pm$ 3.182(0.012) = (0.036, 0.19).
The effect of changing factor of Position from inside to outside would increase the mean reception by 2(0.131)$\pm$3.182(0.012) = [0.186, 0.0.339].
The effect of changing factor of Level from lower to upper would increase the mean reception by 2(0.156)$\pm$3.182(0.012) = [0.236, 0.389].
The effect of changing factor of System from IOS to Android would increase the mean reception by 2(0.043)$\pm$3.182(0.012) = [0.011, 0.164].

To analyze the significant interaction effect of the Position and Level, we make an interaction plot (see Figure 5). From Figure 5, we can see that the effects of changing level of Position from inside to outside are positive on mean reception for both levels of Level. However, the effect seems to be greater for "lower" level. Mathematically, we can calculate how the estimated effects depends on the level of factor Level.

- Estimated effect of Position given that Level is "lower":

$$\frac{(\bar{y}_{P=1,L=-1} - \bar{y}_{P=-1,L=1})}{2} = \frac{0.825 - 0.425}{2} = 0.2$$

- Estimated effect of position given that Level is "upper":

$$\frac{(\bar{y}_{P=1,L=1} - \bar{y}_{P=-1,L=1})}{2} = \frac{1 - 0.875}{2} = 0.0625$$

The effect of changing level of Position from inside to outside increases the mean reception by 0.2 on average when factor Level is "lower", and the effect of changing level of Position from inside to outside only increases the mean reception by 0.0625 when factor Level is upper. Also, the optimal combination of the two factors is at Position of outside and Level of upper, which gives the highest sample mean in Figure 5 and Table 3. How the other estimated effect of one factor depends on the level of another can be calculated through the way, and more information about averages are given in Table 3.

4.4 Optimal levels and Prediction

Throughout the analysis, we find some levels of factors that will increase the mean reception. If we want to maximize the reception, we need to choose Building IONA, Position of outside, Level of upper, and System of Android. For the other factors, Company and Time, we will choose the optimal level based on the estimated effect. The optimal level is:

$$x_B = -1, x_P = 1, x_L = 1, x_C = 1, x_S = 1, x_T = -1$$

The prediction of optimal mean reception is:

$$\hat{y} = 0.781-0.056(-1)+0.131(1)+0.156(1)+0.019(1)+0.043(1)-0.006(-1)$$
$$+0.019(-1)(1)-0.006(-1)(1)-0.019(-1)(1)-0.069(1)(1)+0.006(1)(1)-0.019(1)(1) = 1.119$$

The 95 percent confidence interval for the mean reception:

$$\hat{y} \pm t_{3,0.975}(\sqrt{MS(Residual)x^T(X^TX)^{-1}x}$$

$$1.119 \pm (3.182(\sqrt{\tfrac{0.023\times 13}{16}}))=[0.98,\ 1.25]$$

The 95 percent prediction interval for the individual observed reception:

$$\hat{y} \pm t_{3,0.975}(\sqrt{MS(Residual)(1+x^T(X^TX))^{-1}x}$$

$$1.119 \pm 3.182(\sqrt{\tfrac{1+0.023\times 13}{16}})=[0.91,\ 1.32]$$

Our response variable, reception, is a percentage of how strong the reception is shown in the cellphone system. It has a maximum of 1 (means 100 percent reception). At our optimal levels, both the confidence interval for the mean reception and prediction interval for the individual mean reception go over 1. Also, the lower bounds of both intervals are close to 1 as well, which means it guarantees a good reception at UBC at our optimal levels in most of the cases (95 percent confidence).

5 Conclusion

According to our ANOVA analysis and LM models in our analysis, there are some evidence that the treatment factors that may affect the mean level of mobile phone receptions are Building, Position, Level, and System. In particular, very strong evidence for the effect of Level. There is also evidence on the interaction effect of Position and Level on the mean reception. It is estimated from our experiment that in order to obtain maximum reception at UBC, one needs to have an Android phone, go to IONA building, and stand outside of the building on the first floor.

The design of the experiment (2^{6-2} factorial) was considered to be very cost-effective, not only did it save time to conduct but it also gave insightful results to pinpoint important factors that affect cell reception.

However, there are a few assumptions that need to be clarified and evaluated. It was assumed that Y is a random variable in our experiment, but actually in real life, it is a discrete variable because the system of the cell phone and how the reception shown on the phone, therefore, if we have a uniform and accurate way to measure reception might be more helpful. We also assumed that the higher-level interaction effects (more than 2-factor interaction effects), and we ignore the two-factor interaction effect involving the block factors. We consider these effects are negligible because we want to keep the DFs for the residuals. We might have alised pair of effects that ends up being the confounding effect (eg. the significant interaction effect of PL is aliased with SB). Also, the main effects are aliased with 3-factor interactions which could be problematic if we do decide to include these 3-factor interaction effects.

A replicated 2^{6-2} factorial design or a full factorial design would be more time consuming and expensive to run, but might give more insights on the higher level interactions effects. Also, we could include some random effect for the buildings at UBC instead of using just the two buildings(IONA and CPSC). Therefore, this experiment can be used as an initial experiment to gain insights to screen factors, and a follow-up experiment is desired.

6 Tables and Figures

Here are some figures and tables referred to previous sections.

Figure 1: Box-Cox Plot

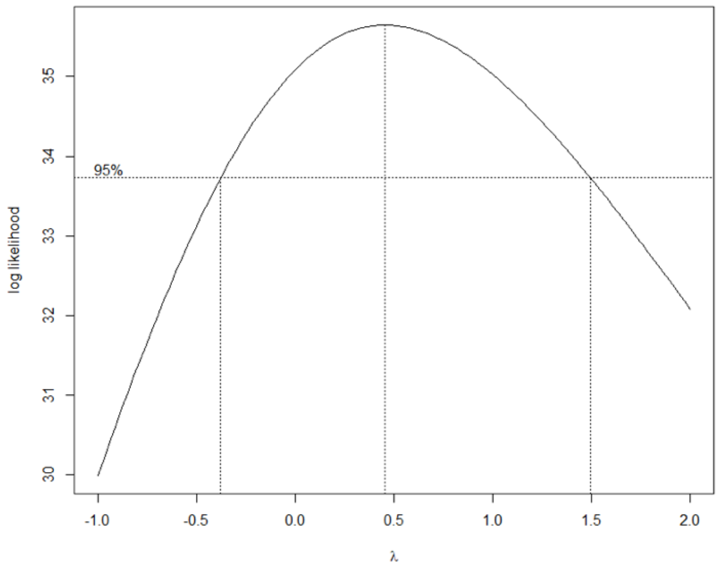

Figure 2: QQ Normality Plot

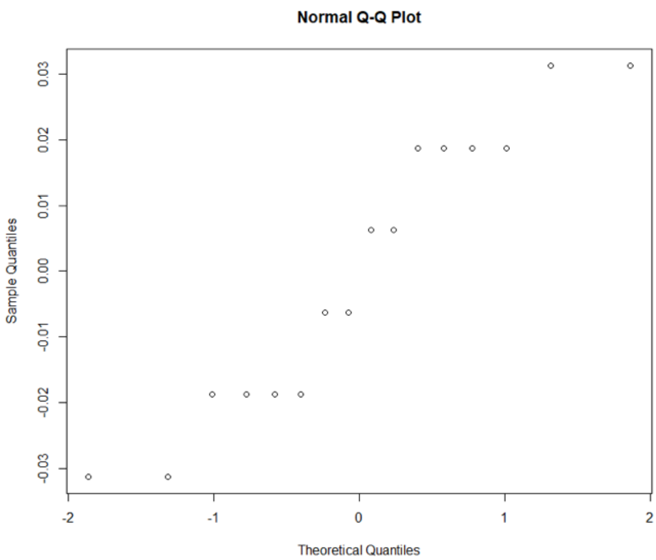

Figure 3: Residual VS Fitted Values

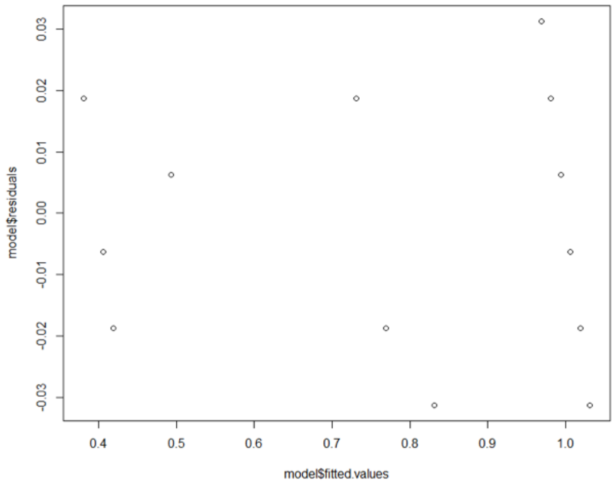

Figure 4: Half-Normal plot for absolute estimated effects

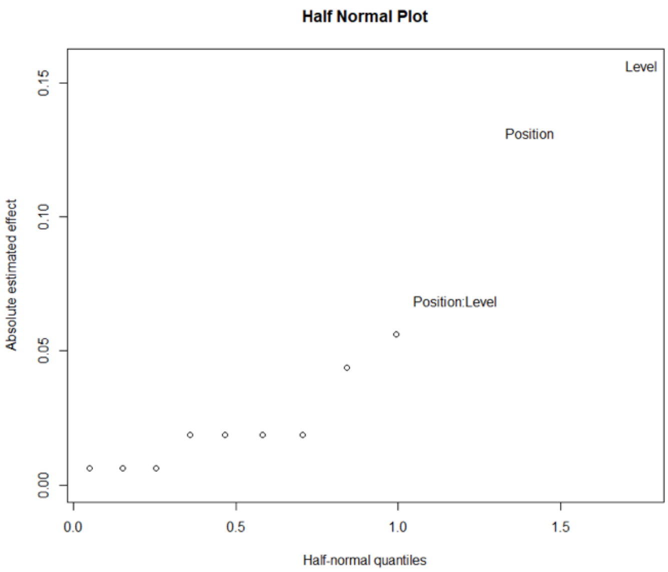

Figure 5: changing plot

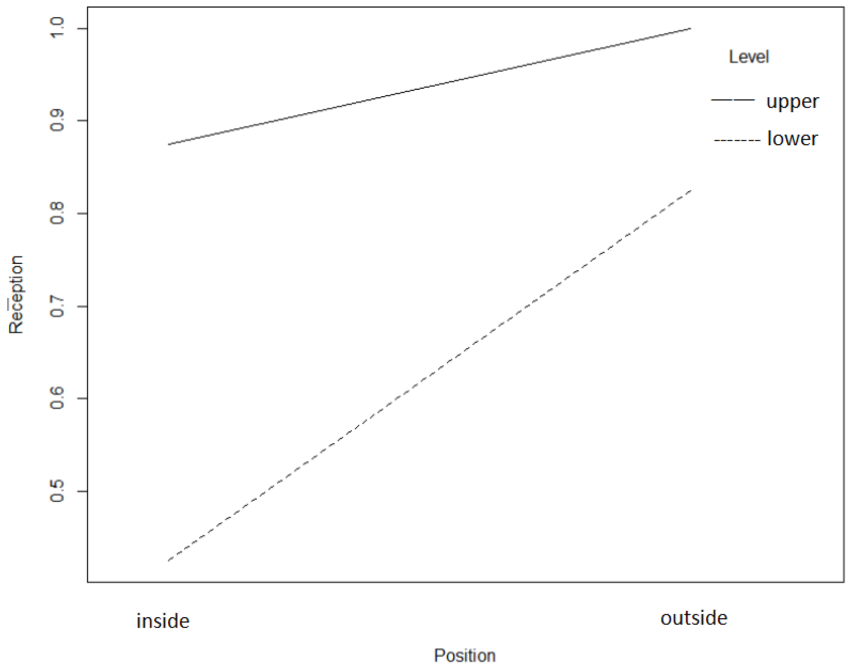

Figure 6: Description of Factors

Experimental factors and Factor levels:

Treatment Factor	Description	Levels
B	Building in different location at UBC	IONA(-1), CPSC(1)
P	Position (inside/outside the building)	Inside(-1), Outside(1)
L	Level of floor	Lower(-1), Upper(1)
C	Company of the mobile plan	Fido(-1), Chatr (1)

Block Factor	Description	Levels
S	System of the mobile phone	IOS(-1), Android(1)
T	Time of the day	10AM(-1), 3PM(1)

Figure 7: ANOVA table

```
                  Df Sum Sq Mean Sq F value   Pr(>F)
Building           1 0.0506  0.0506  22.091 0.01822 *
Position           1 0.2756  0.2756 120.273 0.00162 **
Level              1 0.3906  0.3906 170.455 0.00097 ***
Company            1 0.0056  0.0056   2.455 0.21517
System             1 0.0306  0.0306  13.364 0.03535 *
Time               1 0.0006  0.0006   0.273 0.63762
Building:Position  1 0.0056  0.0056   2.455 0.21517
Building:Level     1 0.0006  0.0006   0.273 0.63762
Building:Company   1 0.0056  0.0056   2.455 0.21517
Position:Level     1 0.0756  0.0756  33.000 0.01048 *
Position:Company   1 0.0006  0.0006   0.273 0.63762
Level:Company      1 0.0056  0.0056   2.455 0.21517
Residuals          3 0.0069  0.0023
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 8: Table of Means

Building IONA CPSC 0.8375 0.7250	Position inside outside 0.6500 0.9125	Level lower upper 0.6250 0.9375
Company fido chatr 0.7625 0.8000	System IOS Android 0.7375 0.8250	Time AM PM 0.7875 0.7750
Building: Position P B inside outside IONA 0.725 0.950 CPSC 0.575 0.875	Building: Level L B lower upper IONA 0.675 1.000 CPSC 0.575 0.875	Building: Company C B fido chatr IONA 0.800 0.875 CPSC 0.725 0.725
Position: Company C P fido chatr inside 0.6375 0.6625 outside 0.8875 0.9375	Position: Level L P lower upper inside 0.425 0.875 outside 0.825 1.000	Level: Company C L fido chatr lower 0.5875 0.6625 upper 0.9375 0.9375

Figure 9: Estimated Effects

```
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.78125    0.01197  65.279 7.92e-06 ***
Building          -0.05625    0.01197  -4.700  0.01822 *
Position           0.13125    0.01197  10.967  0.00162 **
Level              0.15625    0.01197  13.056  0.00097 ***
Company            0.01875    0.01197   1.567  0.21517
System             0.04375    0.01197   3.656  0.03535 *
Time              -0.00625    0.01197  -0.522  0.63762
Building:Position  0.01875    0.01197   1.567  0.21517
Building:Level    -0.00625    0.01197  -0.522  0.63762
Building:Company  -0.01875    0.01197  -1.567  0.21517
Position:Level    -0.06875    0.01197  -5.745  0.01048 *
Position:Company   0.00625    0.01197   0.522  0.63762
Level:Company     -0.01875    0.01197  -1.567  0.21517
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```