

Application of LP to Network Design - Project 1

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1 Description

Purpose of the project is to create software that estimates the optimal cost of designing a network using linear programming. We need to identify input, objective function, and constraints in order to create the network with capacity links.

1.1 Input

Number of nodes given is 30. Cost of using a link from node_i to node_j is given by a_{ij} . Source rate is given by b_{ij} from a node_i to node_j. Cost(a_{ij}) and source rate(b_{ij}) are obtained using the rules described.

1.2 Objective Function

Objective of the design is to minimize the cost of designing a network which satisfies the capacity requirements. Objective function is given by

$$Z_{OPT} = \sum_{k,l} b_{k,l}(a_{k,i} + a_{i,l}) \forall i \in k, l \quad (1)$$

1.3 Constraints

$$a_{ij} \geq 0 \quad (2)$$

$$b_{ij} \geq 0 \quad (3)$$

2 Algorithm

Algorithm to design the network takes matrix of costs(a_{ij}), matrix of source rate(b_{ij}) as inputs and generates the graph given by capacity matrix (z_{ij}). Description of the algorithm is given below.

- Create the shortest path connectivity matrix of the graph using Bellman ford algorithm and store it in a_{ij} .

```
for each vertex v in vertices:
    distance[v] := INT_MAX
    predecessor[v] := null
```

```
distance[source] := 0
```

```
// Step 2: relax edges repeatedly
for i from 1 to size(vertices)-1:
    for each edge (u, v) with weight w in edges:
        if distance[u] + w < distance[v]:
            distance[v] := distance[u] + w
            predecessor[v] := u
return distance[], predecessor[]
```

- Calculate the capacity matrix using $z_{ij} = b_{ij} * a_{ij}$

```
for each edge (u, v) in the edge matrix:
    z[i][j] = b[i][j](a[i][j]);
    z_opt += z[i][j];
return z, z_opt;
```

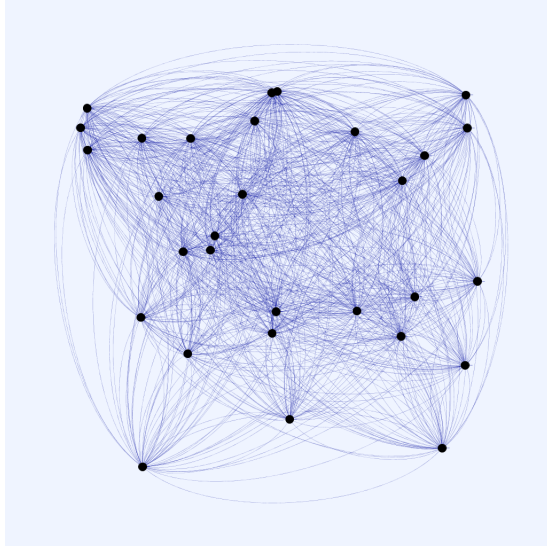


Figure 1: Graph obtained with $k = 3$

3 Output

The software package is run using 30 nodes, with K ranging from 3 to 15. Following is part of the output. Full output can be found in the ReadMe section.

k(3),	optz(6726)	density(0.889655)
k(4),	optz(5779)	density(0.889655)
k(5),	optz(5000)	density(0.889655)
k(6),	optz(4855)	density(0.889655)
k(7),	optz(4497)	density(0.889655)
k(8),	optz(4504)	density(0.889655)
k(9),	optz(4197)	density(0.889655)
k(10),	optz(3992)	density(0.889655)
k(11),	optz(3881)	density(0.889655)
k(12),	optz(3809)	density(0.889655)
k(13),	optz(3769)	density(0.889655)
k(14),	optz(3683)	density(0.889655)
k(15),	optz(3704)	density(0.889655)

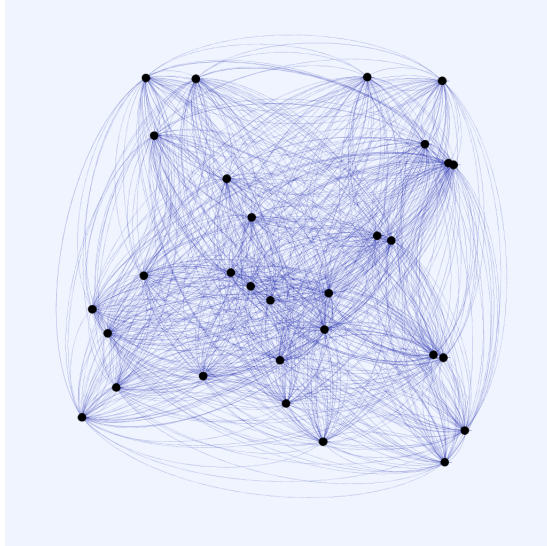


Figure 2: Graph obtained with $k = 9$

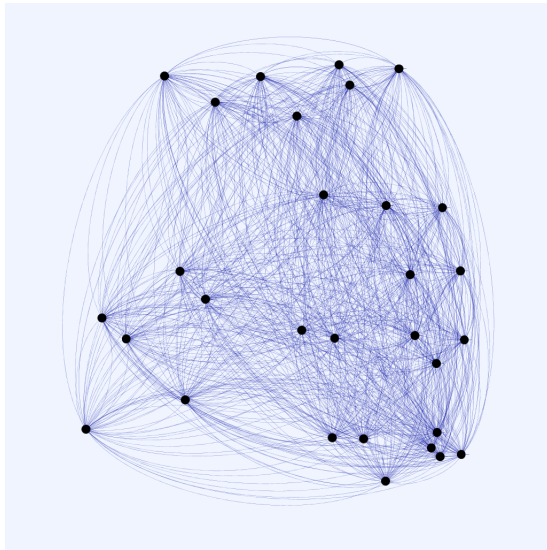


Figure 3: Graph obtained with $k = 15$

4 Analysis

As value k increases, cost of the links reduce and hence total cost of the network reduces. Density of the network is a constant since the number of zero capacity nodes seem to be constant and not changing with k . Minimum cost of designing the network is obtained by calculating the local minimum and using it in the final calculation(greedy approach).

5 ReadMe

Run Makefile using following command in the directory.

```
make -f Makefile
```

Makefile generates the executable(lp). Run lp in command window. Input number of nodes. Output will be printed on the screen. Output can be redirected to a file.

6 Appendix

File: lp.h

```
#ifndef _LP_
#define _LP_

#define KSTART 3
#define KEND 16

#ifndef DEBUG
#define printf(p, ...)
#else
#endif
#endif
```

File: LP.c

```
#include <stdio.h>
#include <stdlib.h>
#include <limits.h>
#include "lp.h"

void print(int nodes, int a[][nodes]) {
```

```

    int i = 0;
    int j = 0;
    for (i = 0; i < nodes; i++) {
        for (j = 0; j < nodes; j++) {
            printf("%d ", a[i][j]);
        }
        printf("\n");
    }
}

void initializeA(int nodes, int a[][nodes], int k) {
    int i = 0;
    int j = 0;
    int l = 0;

    for (i = 0; i < nodes; i++) {
        for (j = 0; j < nodes; j++) {
            a[i][j] = INT_MAX;
        }
    }

    // for all i
    //     get k random l
    //         set a[i][l] = 1
    //         if j is not in l, a[i][j] = 300
    for (i = 0; i < nodes; i++) {
        a[i][i] = 0;
        for (l = 0; l < k; l++) {
            int j_i = rand()%nodes;
            while (j_i == i) {
                j_i = rand()%nodes;
            }
            a[i][j_i] = 1;
        }
        for (j = 0; j < nodes; j++) {
            if (a[i][j] != 1 && i != j) {
                a[i][j] = 300;
            }
        }
    }
}

```

```

}

void initializeB(int nodes, int b[][nodes], char *id) {
    int i = 0;
    int j = 0;

    for (i = 0; i < nodes; i++) {
        for (j = 0; j < nodes; j++) {
            b[i][j] = 0;
            b[i][j] = abs((id[i] - '0') - (id[j] - '0'));
        }
    }
}

void get_capacities(int nodes, int z[][nodes],
                    int a[][nodes], int b[][nodes]) {
    int i = 0;
    int j = 0;
    int l = 0;
    for (i = 0; i < nodes; i++) {
        for (j = 0; j < nodes; j++) {
            if (i == j) {
                z[i][j] = 0;
            } else {
                z[i][j] = b[i][j]*a[i][j];
            }
        }
    }

    int m = 0;
    while (m < nodes) {
        for (i = 0; i < nodes; i++) {
            for (j = 0; j < nodes; j++) {
                if (i != j) {
                    int temp = INT_MAX;
                    for (l = 0; l < nodes; l++) {
                        temp = b[i][j]*(a[i][l] + a[l][j])
                        if (temp < z[i][j]) {
                            z[i][j] = temp;
                        }
                    }
                }
            }
        }
        m++;
    }
}

```

```

    }
    }
    }
    m++;
}
}

int get_opt_z(int nodes, int z[][nodes]) {
    int i = 0;
    int j = 0;
    int z_opt = 0;

    for (i = 0; i < nodes; i++) {
        for (j = 0; j < nodes; j++) {
            z_opt += z[i][j];
        }
    }

    return z_opt;
}

void form_topologies(int nodes, int a[][nodes]) {
    int i = 0;
    int j = 0;
    int k = 0;
    int l = 0;

    while (l < nodes) {
        for (i = 0; i < nodes; i++) {
            if (i != j) {
                for (j = 0; j < nodes; j++) {
                    int temp = INT_MAX;
                    for (k = 0; k < nodes; k++) {
                        temp = a[i][k] + a[k][j];
                        if (temp < a[i][j]) {
                            a[i][j] = temp;
                        }
                    }
                }
            }
        }
        l++;
    }
}

```



```

        }
    }
    l++;
}

float get_density(int nodes, int z[][nodes]) {
    int i = 0;
    int j = 0;
    float n = 0;
    float total = (float)(nodes * (nodes - 1));
    float density;

    for (i = 0; i < nodes; i++) {
        for (j = 0; j < nodes; j++) {
            if (z[i][j] != 0) {
                n++;
            }
        }
    }

    //printf("Number of non zero capacities: %f\n", n);
    density = n/total;
    return density;
}

void printGraph(int nodes, int z[][nodes]) {
    int i = 0;
    int j = 0;
    int k = 0;
    for (i = 0; i < nodes; i++) {
        for (j = 0; j < nodes; j++) {
            printf("%d\t%d\tDirected\n", i, j, k++, z[i][j]);
        }
    }
}

int main() {
    int nodes = 0;

```

```

int k = K_START;
int k_max = K_END;
char *id = "202134683520213468352021346835";
int index = k;

scanf("%d", &nodes);
for (index = k; index < k_max; index++) {
    int a[nodes][nodes];
    int b[nodes][nodes];

    initializeA(nodes, a, index);
    //printf("\nObtained Graph:\n");
    //print(nodes, a);

    initializeB(nodes, b, id);
    //printf("\nObtained B:\n");
    //print(nodes, b);
    form_topologies(nodes, a);
    printf("\nShortest Path Graph:\n");
    print(nodes, a);

    int z[nodes][nodes];
    get_capacities(nodes, z, a, b);
    printf("\nObtained Capacity Matrix:\n");
    print(nodes, z);

    int z_opt = get_opt_z(nodes, z);
    float density = get_density(nodes, z);
    printf("\nFor k(%d), z_opt(%d) density(%f)\n", index, z_opt, density);
}
return 1;
}

```

File: Makefile

```

lp: lp.o
    gcc -o lp lp.o
lp.o: LP.c lp.h
    gcc -c -Wall -DDEBUG LP.c -I.

clean:

```

rm -rf lp.o lp

6.1 Full Output with nodes 30

Shortest Path Graph:

```

0 2 3 4 3 3 4 1 3 2 3 4 4 2 4 3 5 2 3 1 4 3 5 1 4 3 2 4 2 3
3 0 1 2 4 3 4 2 1 2 1 5 5 3 4 3 5 3 3 3 4 4 5 2 2 3 2 5 3 3
3 2 0 2 3 3 3 3 1 2 2 4 4 2 4 2 5 3 3 3 4 3 4 1 1 2 3 4 2 4
1 3 3 0 4 1 5 2 4 3 4 4 5 2 4 3 6 3 4 1 5 3 6 2 4 4 2 5 3 3
3 2 1 3 0 4 4 4 2 2 3 5 1 3 5 3 5 4 3 2 4 4 5 2 2 3 3 1 3 4
2 2 3 4 3 0 4 1 3 2 3 4 4 2 4 3 5 2 3 1 4 3 5 1 4 3 2 4 2 3
2 3 2 1 3 2 0 2 3 3 1 5 4 3 4 3 4 3 2 2 3 4 1 3 3 4 2 2 3 3
2 3 3 4 2 3 3 0 4 1 4 4 3 2 4 3 4 1 2 1 3 3 4 2 3 2 2 3 2 3
2 4 2 1 3 2 3 2 0 1 1 5 4 3 4 3 4 3 2 2 3 4 4 3 3 2 2 4 3 3
4 3 3 3 2 2 2 3 3 0 3 4 3 3 3 2 3 3 1 3 2 3 3 2 2 1 1 3 2 2
3 3 1 3 3 2 4 1 2 2 0 4 4 3 3 2 5 2 3 2 4 3 5 2 2 3 1 4 3 2
2 4 4 1 3 2 4 1 5 2 5 0 4 3 5 4 2 2 3 2 1 4 2 3 3 3 3 3 3 4
2 4 3 4 3 3 3 3 4 1 4 4 0 2 4 3 4 4 2 1 3 3 4 3 3 2 2 4 3 3
3 3 1 3 3 3 3 1 2 2 3 2 4 0 2 3 4 2 3 2 3 1 4 2 2 3 3 4 3 4
3 3 2 2 4 1 1 2 3 3 2 3 5 1 0 4 5 3 3 2 4 2 2 2 3 4 3 3 3 4
3 1 2 2 4 1 4 2 1 2 2 5 5 3 5 0 5 3 3 2 4 4 5 2 3 3 3 5 3 4
3 3 3 2 3 3 3 1 3 2 4 1 4 2 4 2 0 2 3 2 2 3 3 3 1 2 3 4 3 4
4 2 2 4 1 2 5 3 3 3 3 4 2 2 4 2 6 0 4 3 5 3 6 1 3 4 4 2 1 5
4 3 3 4 3 2 3 3 3 2 4 3 4 2 4 2 2 2 0 3 1 3 2 3 1 2 3 3 1 4
1 3 2 4 4 2 4 2 3 3 4 3 5 1 3 2 5 3 3 0 4 2 5 2 3 4 1 5 3 2
4 3 4 3 3 3 3 2 4 1 4 2 4 3 4 3 1 3 2 3 0 4 1 3 2 2 2 2 3 3
3 4 2 2 4 2 2 2 3 3 3 1 5 1 1 4 3 3 4 3 2 0 3 3 3 4 4 4 4 5
5 2 3 4 2 3 4 4 3 2 3 4 3 3 5 3 3 3 1 4 2 4 0 4 2 3 3 1 2 4
4 1 2 3 3 2 4 2 2 3 2 3 4 1 3 2 5 2 4 3 4 2 5 0 3 4 3 4 1 4
4 2 2 3 2 2 2 2 2 3 3 3 3 1 3 1 5 3 4 3 4 2 3 2 0 1 4 3 3 5
3 2 2 2 1 3 1 3 3 3 2 4 2 2 4 3 5 3 3 3 4 3 2 1 3 0 3 2 2 4
3 2 3 3 4 1 3 2 2 3 3 3 5 3 2 1 4 3 2 2 3 2 4 2 3 4 0 5 3 1
4 1 2 3 1 3 3 3 2 1 2 5 2 4 4 3 4 4 2 3 3 4 4 3 3 2 2 0 3 3
3 2 3 3 2 1 5 2 2 3 3 5 3 3 5 1 6 1 4 2 5 4 6 2 4 4 3 3 0 4
4 4 3 3 4 2 2 3 4 3 3 2 5 2 1 3 3 3 1 3 2 1 3 3 2 3 4 4 2 0

```

Obtained Capacity Matrix:

```

0 4 0 4 3 6 16 6 3 6 0 8 0 2 4 6 20 12 3 3 0 6 0 1 4 6 8 24 2 9
6 0 2 2 12 12 24 16 3 10 2 0 10 3 12 12 30 24 9 15 8 0 10 2 6 12 12 40 9 15

```

```

0 4 0 2 3 6 12 18 1 6 0 8 0 2 4 4 20 18 3 9 0 6 0 1 1 4 12 24 2 12
1 3 3 0 8 3 25 14 8 12 4 4 5 0 8 9 30 21 8 4 5 3 6 0 8 12 10 35 6 12
3 6 1 6 0 4 12 20 0 4 3 15 1 6 0 3 15 20 0 4 4 12 5 4 0 3 9 5 0 8
4 8 6 12 3 0 8 4 3 2 6 16 8 6 4 0 10 8 3 1 8 12 10 3 4 0 4 16 2 3
8 18 8 5 9 4 0 4 9 3 4 30 16 15 12 6 0 6 6 2 12 24 4 15 9 8 0 4 9 3
12 24 18 28 10 12 6 0 20 3 24 32 18 14 20 12 8 0 10 3 18 24 24 14 15 8 4 0
2 12 2 2 0 2 9 10 0 2 1 15 4 6 0 3 12 15 0 4 3 12 4 6 0 2 6 20 0 6
12 15 9 12 4 2 2 9 6 0 9 20 9 12 6 2 3 9 2 0 6 15 9 8 4 1 1 9 4 0
0 6 0 3 3 4 16 6 2 6 0 8 0 3 3 4 20 12 3 6 0 6 0 2 2 6 4 24 3 6
4 0 8 1 9 8 24 8 15 10 10 0 8 3 15 16 12 16 9 10 2 0 4 3 9 12 18 24 9 20
0 8 0 4 3 6 12 18 4 3 0 8 0 2 4 6 16 24 2 3 0 6 0 3 3 4 8 24 3 9
3 3 1 0 6 9 15 7 4 8 3 2 4 0 4 9 20 14 6 8 3 1 4 0 4 9 15 28 6 16
3 9 2 4 0 1 3 10 0 6 2 9 5 2 0 4 15 15 0 4 4 6 2 4 0 4 9 15 0 8
6 4 4 6 4 0 8 8 1 2 4 20 10 9 5 0 10 12 3 2 8 16 10 6 3 0 6 20 3 4
12 18 12 10 9 6 0 2 9 2 16 6 16 10 12 4 0 4 9 2 8 18 12 15 3 4 0 8 9 4
24 16 12 28 5 8 10 0 15 9 18 32 12 14 20 8 12 0 20 9 30 24 36 7 15 16 8 0
4 9 3 8 0 2 9 15 0 4 4 9 4 4 0 2 6 10 0 6 1 9 2 6 0 2 9 15 0 8
3 15 6 16 8 2 4 6 6 0 12 15 15 4 6 2 5 9 6 0 12 10 15 8 6 4 1 15 6 0
0 6 0 3 3 6 12 12 4 3 0 4 0 3 4 6 4 18 2 9 0 8 0 3 2 4 8 12 3 9
6 0 4 2 12 8 12 16 9 15 6 0 10 1 3 16 18 24 12 15 4 0 6 3 9 16 24 32 12 25
0 4 0 4 2 6 16 24 3 6 0 8 0 3 5 6 12 18 1 12 0 8 0 4 2 6 12 6 2 12
4 1 2 0 6 6 20 14 4 12 2 3 4 0 6 6 25 14 8 12 4 2 5 0 6 12 15 28 2 16
4 6 2 6 0 2 6 10 0 6 3 9 3 2 0 1 15 15 0 6 4 6 3 4 0 1 12 15 0 10
6 8 4 6 1 0 2 12 3 3 4 16 4 6 4 0 10 12 3 3 8 12 4 3 3 0 6 8 2 4
12 12 12 15 12 2 0 4 6 3 12 18 20 15 6 2 0 6 6 2 12 12 16 10 9 8 0 10 9 1
24 8 12 21 5 12 6 0 10 3 12 40 12 28 20 12 8 0 10 9 18 32 24 21 15 8 4 0 1
3 6 3 6 0 1 15 10 0 6 3 15 3 6 0 1 18 5 0 4 5 12 6 4 0 4 9 15 0 8
12 20 9 12 8 2 2 9 8 0 9 10 15 8 2 3 3 9 2 0 6 5 9 12 4 3 4 12 4 0

```

For $k(3)$, $z_{\text{opt}}(6726)$ density(0.889655)

Shortest Path Graph:

```

0 4 2 3 2 1 3 3 3 2 2 2 3 2 2 2 2 2 1 3 3 3 1 1 3 2 3 3 3 2
4 0 4 2 3 4 2 3 2 2 3 3 1 3 3 2 1 3 3 3 2 2 3 3 2 2 1 1 4 5
3 4 0 4 3 3 4 2 1 3 2 3 2 2 2 2 3 4 2 3 3 1 3 2 1 2 2 1 3 4
3 5 3 0 3 4 4 3 1 3 2 1 3 3 3 2 3 1 2 3 2 2 2 2 3 2 2 3 4 5
2 3 2 4 0 3 4 1 2 3 3 3 2 4 2 2 3 4 3 3 4 3 3 3 1 2 2 1 2 3
3 4 3 2 3 0 4 2 3 2 3 1 3 3 3 1 3 3 3 2 2 2 2 2 2 2 3 3 3 1
3 5 2 4 2 3 0 3 1 2 2 3 3 2 3 2 3 2 2 1 3 3 1 2 2 2 2 3 4 4
1 2 1 3 3 2 4 0 2 3 3 3 3 3 2 2 3 3 2 3 3 2 2 2 1 2 3 2 1 2

```

2 4 3 3 2 3 3 2 0 3 1 2 4 2 2 2 2 3 1 3 3 2 2 1 3 1 4 3 3 4
 4 4 2 2 3 4 2 3 2 0 3 2 1 3 2 2 1 2 3 3 2 2 1 3 3 1 3 2 3 4
 2 3 2 4 4 3 4 1 3 2 0 3 3 2 3 2 3 2 3 3 3 1 1 3 2 2 3 2 2 3
 3 4 2 4 2 3 4 2 3 2 2 0 3 2 3 1 3 2 2 2 1 1 1 2 2 2 3 2 3 4
 3 4 3 2 3 3 2 2 1 3 2 3 0 2 3 2 1 3 2 3 2 1 3 2 3 2 2 1 3 4
 3 5 4 3 2 1 4 3 3 2 2 2 3 0 3 2 3 4 2 1 1 3 3 3 2 3 2 3 4 2
 3 2 1 2 3 3 2 2 2 2 3 3 3 3 0 1 1 3 3 2 2 2 3 2 2 2 3 2 1 2
 4 4 3 3 2 3 3 3 2 1 2 3 2 2 2 0 2 3 3 1 3 3 2 1 1 2 2 3 3 4
 3 4 3 1 2 4 1 3 2 2 2 2 3 3 2 2 0 2 2 2 1 3 2 3 3 1 3 3 3 4
 2 4 3 3 2 3 3 3 3 2 2 3 2 2 2 3 2 0 1 3 1 3 3 1 2 2 1 3 3 4
 1 3 2 2 1 2 2 2 3 3 3 3 3 3 1 2 1 3 0 3 2 3 2 2 2 2 3 2 2 3
 3 4 3 4 1 2 4 2 2 2 3 3 2 1 2 2 3 4 3 0 2 3 3 3 1 2 1 2 3 3
 1 3 2 2 1 2 2 2 3 3 3 3 3 3 1 2 1 3 0 3 2 3 2 2 2 2 3 2 2 3
 3 4 3 4 1 2 4 2 2 2 3 3 2 1 2 2 3 4 3 0 2 3 3 3 1 2 1 2 3 3
 2 4 3 3 1 3 3 2 3 1 1 3 2 3 2 3 2 3 1 4 0 2 2 3 2 2 3 2 3 4
 2 3 2 4 3 2 4 1 2 3 3 2 2 1 2 2 3 3 3 2 2 0 2 3 2 1 2 1 2 3
 3 5 1 3 3 4 3 3 2 1 3 3 2 3 3 1 2 1 2 2 2 2 0 2 2 2 2 2 4 5
 3 4 3 2 3 2 2 2 3 3 1 2 4 1 2 2 1 3 3 2 2 2 2 0 3 1 3 3 3 3
 3 3 2 3 3 4 3 3 1 2 2 2 3 3 1 1 2 3 2 2 3 3 2 2 0 1 3 3 2 3
 4 3 2 3 3 4 3 3 3 2 3 1 3 3 1 1 2 2 3 2 2 2 1 2 2 0 3 3 2 3
 4 4 3 3 4 4 3 3 2 1 3 2 1 3 2 2 2 3 3 3 3 2 2 3 1 1 0 2 3 4
 3 5 4 3 3 4 3 3 1 2 2 3 1 3 3 1 2 4 2 2 3 2 3 2 2 2 1 0 4 5
 2 1 2 2 3 2 3 1 3 3 2 3 2 2 3 3 2 3 3 3 2 3 3 1 2 2 2 2 0 1
 2 3 2 1 2 1 4 1 2 2 2 2 3 4 3 2 3 2 2 3 1 3 3 3 2 3 3 3 2 0

Obtained Capacity Matrix:

0 8 0 3 2 2 12 18 3 6 0 4 0 2 2 4 8 12 1 9 0 6 0 1 3 4 12 18 3 6
 8 0 8 2 9 16 12 24 6 10 6 0 2 3 9 8 6 24 9 15 4 0 6 3 6 8 6 8 12 25
 0 8 0 4 3 6 16 12 1 9 0 6 0 2 2 4 12 24 2 9 0 2 0 2 1 4 8 6 3 12
 3 5 3 0 6 12 20 21 2 12 2 1 3 0 6 6 15 7 4 12 2 2 2 0 6 6 10 21 8 20
 2 9 2 8 0 3 12 5 0 6 3 9 2 8 0 2 9 20 0 6 4 9 3 6 0 2 6 5 0 6
 6 16 6 6 3 0 8 8 3 2 6 4 6 9 3 0 6 12 3 2 4 8 4 6 2 0 6 12 3 1
 12 30 8 20 6 6 0 6 3 2 8 18 12 10 9 4 0 4 6 1 12 18 4 10 6 4 0 6 12 4
 6 16 6 21 15 8 8 0 10 9 18 24 18 21 10 8 6 0 10 9 18 16 12 14 5 8 6 0 5 6
 2 12 3 6 0 3 9 10 0 6 1 6 4 4 0 2 6 15 0 6 3 6 2 2 0 1 12 15 0 8
 12 20 6 8 6 4 2 9 4 0 9 10 3 12 4 2 1 6 6 0 6 10 3 12 6 1 3 6 6 0
 0 6 0 4 4 6 16 6 3 6 0 6 0 2 3 4 12 12 3 9 0 2 0 3 2 4 12 12 2 9
 6 0 4 4 6 12 24 16 9 10 4 0 6 2 9 4 18 16 6 10 2 0 2 2 6 8 18 16 9 20
 0 8 0 2 3 6 8 12 1 9 0 6 0 2 3 4 4 18 2 9 0 2 0 2 3 4 8 6 3 12
 3 5 4 0 4 3 20 21 6 8 2 2 3 0 6 6 15 28 4 4 1 3 3 0 4 9 10 21 8 8

```

3 6 1 4 0 3 6 10 0 4 3 9 3 6 0 1 3 15 0 4 2 6 3 4 0 2 9 10 0 4
8 16 6 9 2 0 6 12 2 1 4 12 4 6 2 0 4 12 3 1 6 12 4 3 1 0 4 12 3 4
12 24 12 5 6 8 0 6 6 2 8 12 12 15 6 4 0 4 6 2 4 18 8 15 9 2 0 6 9 4
12 32 18 21 10 12 6 0 15 6 12 24 12 14 10 12 4 0 5 9 6 24 18 7 10 8 2 0 15
1 9 2 4 0 2 6 10 0 6 3 9 3 6 0 2 3 15 0 6 2 9 2 4 0 2 9 10 0 6
9 20 9 16 2 2 4 6 4 0 9 15 6 4 4 2 3 12 6 0 6 15 9 12 2 2 1 6 6 0
0 8 0 3 1 6 12 12 3 3 0 6 0 3 2 6 8 18 1 12 0 4 0 3 2 4 12 12 3 12
4 0 4 4 9 8 24 8 6 15 6 0 4 1 6 8 18 24 9 10 4 0 4 3 6 4 12 8 6 15
0 10 0 3 3 8 12 18 2 3 0 6 0 3 3 2 8 6 2 6 0 4 0 2 2 4 8 12 4 15
3 4 3 0 6 6 10 14 6 12 1 2 4 0 4 6 5 21 6 8 2 2 2 0 6 3 15 21 6 12
3 9 2 6 0 4 9 15 0 4 2 6 3 6 0 1 6 15 0 4 3 9 2 4 0 1 9 15 0 6
8 12 4 9 3 0 6 12 3 2 6 4 6 9 1 0 4 8 3 2 4 8 2 6 2 0 6 12 2 3
16 24 12 15 12 8 0 6 6 1 12 12 4 15 6 4 0 6 9 3 12 12 8 15 3 2 0 4 9 4
18 40 24 21 15 16 6 0 5 6 12 24 6 21 15 4 4 0 10 6 18 16 18 14 10 8 2 0 20
2 3 2 4 0 2 9 5 0 6 2 9 2 4 0 3 6 15 0 6 2 9 3 2 0 2 6 10 0 2
6 15 6 4 4 1 4 3 4 0 6 10 9 16 6 2 3 6 4 0 3 15 9 12 4 3 3 9 4 0

```

For $k(4)$, $z_{\text{opt}}(5779)$ density(0.889655)

Shortest Path Graph:

```

0 1 1 3 2 2 2 1 2 2 2 2 4 3 3 2 2 2 1 2 2 1 3 2 2 2 3 2 3 3
3 0 3 2 3 1 1 2 1 2 2 2 3 2 3 1 3 2 3 2 2 2 2 3 3 1 2 3 2 2
3 3 0 2 1 2 1 2 2 2 2 2 4 2 2 2 3 2 3 1 2 1 2 2 2 3 2 1 2 3
3 1 3 0 1 2 2 2 2 2 2 3 2 3 2 1 2 3 2 3 3 1 2 1 3 2 2 3 3 2 2
2 3 2 2 0 1 2 1 2 1 3 3 3 1 2 1 2 2 2 2 2 2 3 3 2 2 3 2 3 2
3 2 3 2 3 0 1 2 2 2 2 3 2 2 3 1 3 2 3 2 1 3 2 3 2 1 2 2 2 1
2 2 3 1 2 3 0 3 2 1 3 3 3 2 2 2 2 2 3 3 2 2 1 2 3 2 1 2 1 2
2 3 3 2 2 3 3 0 1 2 2 2 4 3 3 2 2 1 1 3 2 1 3 2 1 2 2 2 3 3
2 2 3 1 2 3 3 3 0 1 2 1 3 3 2 2 3 3 3 3 2 1 2 2 3 2 2 2 3 2
1 2 2 3 1 2 2 2 2 0 2 3 2 2 3 2 3 3 2 2 3 2 3 3 3 1 3 2 3 1
3 3 2 2 1 2 3 2 2 2 0 3 4 1 2 1 2 2 3 1 2 3 3 2 2 3 2 1 2 3
2 2 3 2 3 3 2 3 2 2 3 0 3 2 1 1 2 2 3 2 1 3 3 2 2 2 1 1 2 2
1 2 2 3 2 3 2 2 2 3 3 3 0 3 2 2 2 2 1 3 1 2 3 2 1 2 3 1 2 3
3 3 1 3 1 2 2 2 2 2 3 2 3 0 1 2 1 3 3 2 2 2 3 2 2 3 3 1 2 2
3 3 3 2 3 4 3 2 1 2 3 1 2 2 0 1 3 2 3 2 2 2 3 2 3 2 2 2 1 1
2 2 2 1 2 3 3 3 2 1 2 3 3 1 2 0 2 1 3 1 2 3 2 2 3 2 2 2 3 2
2 3 1 2 2 3 1 3 3 2 3 3 4 2 2 1 0 2 2 2 2 2 2 1 2 3 2 1 2 3
1 2 2 2 1 2 3 2 3 2 1 3 4 2 3 2 2 0 2 2 3 2 3 1 3 2 1 2 3 3
3 2 2 2 2 3 2 3 1 2 3 2 4 2 3 1 1 2 0 2 1 2 3 2 1 2 3 2 3 3

```

2 2 3 1 2 3 3 3 1 2 2 2 4 2 2 1 2 1 3 0 2 2 2 2 3 2 1 2 3 3
 2 1 3 2 2 2 1 3 1 2 2 2 4 3 3 2 3 1 3 3 0 2 2 2 1 2 2 2 2 3
 2 3 3 2 2 3 3 3 3 1 1 1 3 2 2 2 3 3 2 2 2 0 3 1 2 2 2 1 2 2
 2 3 2 2 2 3 3 3 1 1 2 2 3 1 2 3 2 1 3 3 3 1 0 2 3 2 2 2 3 2
 1 2 2 1 1 2 3 2 2 2 3 3 4 2 2 2 2 3 1 3 2 2 2 0 2 3 4 1 2 3
 3 2 3 3 1 2 2 2 2 2 2 3 4 2 2 2 3 2 3 3 1 3 3 2 0 1 3 1 2 3
 3 3 3 2 2 3 1 1 1 2 1 2 4 2 3 1 3 2 2 2 3 2 2 3 2 0 2 2 2 3
 1 2 2 2 2 3 2 2 2 3 2 3 4 3 2 2 1 3 2 3 2 2 3 1 2 1 0 1 2 3
 2 2 3 2 2 3 2 3 2 2 3 2 3 3 1 2 3 2 2 3 1 3 3 1 1 2 3 0 1 2
 2 3 3 2 2 3 2 2 2 1 2 3 3 2 2 1 3 2 2 2 2 3 3 1 2 1 3 1 0 2
 2 3 2 2 2 3 3 1 2 3 3 3 1 1 2 2 2 2 2 1 2 2 3 2 2 3 2 1 2 0

Obtained Capacity Matrix:

0 2 0 3 2 4 8 6 2 6 0 4 0 3 3 4 8 12 1 6 0 2 0 2 2 4 12 12 3 9
 6 0 6 2 9 4 6 16 3 10 4 0 6 2 9 4 18 16 9 10 4 0 4 3 9 4 12 24 6 10
 0 6 0 2 1 4 4 12 2 6 0 4 0 2 2 4 12 12 3 3 0 2 0 2 2 6 8 6 2 9
 3 1 3 0 2 6 10 14 4 8 3 2 3 0 2 6 15 14 6 12 1 2 1 0 4 6 15 21 4 8
 2 9 2 4 0 1 6 5 0 2 3 9 3 2 0 1 6 10 0 4 2 6 3 6 0 2 9 10 0 4
 6 8 6 6 3 0 2 8 2 2 4 12 4 6 3 0 6 8 3 2 2 12 4 9 2 0 4 8 2 1
 8 12 12 5 6 6 0 6 6 1 12 18 12 10 6 4 0 4 9 3 8 12 4 10 9 4 0 4 3 2
 6 8 6 6 3 0 2 8 2 2 4 12 4 6 3 0 6 8 3 2 2 12 4 9 2 0 4 8 2 1
 8 12 12 5 6 6 0 6 6 1 12 18 12 10 6 4 0 4 9 3 8 12 4 10 9 4 0 4 3 2
 12 24 18 14 10 12 6 0 5 6 12 16 24 21 15 8 4 0 5 9 12 8 18 14 5 8 4 0 15 9
 2 6 3 2 0 3 9 15 0 2 2 3 3 6 0 2 9 15 0 6 2 3 2 4 0 2 6 10 0 4
 3 10 6 12 2 2 2 6 4 0 6 15 6 8 6 2 3 9 4 0 9 10 9 12 6 1 3 6 6 0
 0 6 0 2 1 4 12 12 2 6 0 6 0 1 2 2 8 12 3 3 0 6 0 2 2 6 8 6 2 9
 4 0 6 2 9 12 12 24 6 10 6 0 6 2 3 4 12 16 9 10 2 0 6 2 6 8 6 8 6 10
 0 4 0 3 2 6 8 12 2 9 0 6 0 3 2 4 8 12 1 9 0 4 0 2 1 4 12 6 2 9
 3 3 1 0 2 6 10 14 4 8 3 2 3 0 2 6 5 21 6 8 2 2 3 0 4 9 15 7 4 8
 3 9 3 4 0 4 9 10 0 4 3 3 2 4 0 1 9 10 0 4 2 6 3 4 0 2 6 10 0 2
 4 8 4 3 2 0 6 12 2 1 4 12 6 3 2 0 4 4 3 1 4 12 4 6 3 0 4 8 3 2
 8 18 4 10 6 6 0 6 9 2 12 18 16 10 6 2 0 4 6 2 8 12 8 5 6 6 0 2 6 3
 6 16 12 14 5 8 6 0 15 6 6 24 24 14 15 8 4 0 10 6 18 16 18 7 15 8 2 0 15 9
 3 6 2 4 0 3 6 15 0 4 3 6 4 4 0 1 3 10 0 4 1 6 3 4 0 2 9 10 0 6
 6 10 9 4 4 3 3 9 2 0 6 10 12 8 4 1 2 3 6 0 6 10 6 8 6 2 1 6 6 0
 0 2 0 2 2 4 4 18 1 6 0 4 0 3 3 4 12 6 3 9 0 4 0 2 1 4 8 12 2 9
 4 0 6 2 6 12 18 24 9 5 2 0 6 2 6 8 18 24 6 10 4 0 6 1 6 8 12 8 6 10
 0 6 0 2 2 6 12 18 1 3 0 4 0 1 2 6 8 6 3 9 0 2 0 2 3 4 8 12 3 6
 1 2 2 0 2 6 15 14 4 8 3 3 4 0 4 6 10 21 2 12 2 2 2 0 4 9 20 7 4 12
 3 6 3 6 0 2 6 10 0 4 2 9 4 4 0 2 9 10 0 6 1 9 3 4 0 1 9 5 0 6

```

6 12 6 6 2 0 2 4 1 2 2 8 8 6 3 0 6 8 2 2 6 8 4 9 2 0 4 8 2 3
4 12 8 10 6 6 0 4 6 3 8 18 16 15 6 4 0 6 6 3 8 12 12 5 6 2 0 2 6 3
12 16 18 14 10 12 4 0 10 6 18 16 18 21 5 8 6 0 10 9 6 24 18 7 5 8 6 0 5 6
2 9 3 4 0 3 6 10 0 2 2 9 3 4 0 1 9 10 0 4 2 9 3 2 0 1 9 5 0 4
6 15 6 8 4 3 3 3 4 0 9 15 3 4 4 2 2 6 4 0 6 10 9 8 4 3 2 3 4 0

```

For $k(5)$, $z_opt(5000)$ density(0.889655)

Shortest Path Graph:

```

0 3 2 1 2 2 2 3 2 2 2 3 3 2 3 1 2 2 3 3 2 2 3 2 3 4 3 1 4 1
2 0 3 3 2 2 1 3 1 3 2 2 4 2 3 3 2 3 1 1 1 3 2 2 2 1 2 3 3 2
2 2 0 1 3 2 2 2 3 3 1 3 1 1 1 2 2 2 3 3 2 2 3 2 2 3 3 1 2 3
1 2 3 0 2 2 2 2 2 3 1 2 2 1 3 2 3 2 2 2 1 1 2 2 2 3 2 2 3 2
2 2 3 1 0 2 2 2 1 3 1 3 3 2 3 3 2 2 2 3 1 2 2 1 2 3 1 3 3 2
2 2 1 2 1 0 2 3 2 2 2 2 2 2 2 2 2 2 2 3 1 2 2 1 2 3 2 2 2 1
1 2 3 2 2 2 0 2 3 3 1 1 3 1 2 2 2 3 3 1 2 2 3 2 2 2 2 2 2 2
2 1 3 3 2 3 1 0 1 1 2 2 3 1 3 2 3 3 2 2 1 2 2 3 2 2 2 3 3 2
1 1 3 2 2 2 2 2 0 2 3 3 4 2 3 2 2 2 2 2 2 3 3 2 1 2 1 2 4 1
2 2 3 3 2 2 1 2 1 0 2 2 2 2 3 3 3 2 2 2 1 1 2 2 1 3 2 3 3 2
2 2 2 2 2 1 2 3 2 3 0 2 2 3 3 3 3 2 2 2 1 1 2 1 3 3 2 3 3 2
1 2 3 1 2 2 2 3 2 2 2 0 3 2 1 2 3 2 2 2 1 2 2 2 3 3 3 2 1 1
2 2 1 1 2 1 3 3 2 3 2 2 0 2 2 2 3 1 3 2 2 2 2 3 3 3 3 2 1 2
1 1 2 2 2 2 2 1 2 2 3 3 3 0 3 1 3 2 2 2 2 2 3 2 1 2 3 2 4 2
2 2 2 1 2 1 1 3 2 3 2 2 3 2 0 3 3 2 2 2 1 2 2 1 3 3 3 3 3 2
2 2 1 2 2 2 2 3 2 4 2 2 2 2 2 0 3 1 2 2 1 1 2 1 3 3 2 2 3 3
2 2 1 2 2 2 2 3 2 4 2 2 2 2 2 0 3 1 2 2 1 1 2 1 3 3 2 2 3 3
2 3 3 2 2 2 2 4 2 2 2 1 3 3 1 3 0 2 2 3 1 2 2 2 3 4 3 1 2 1
2 2 1 2 2 1 3 3 1 3 2 1 2 2 2 1 2 0 3 2 2 2 2 2 2 3 2 1 2 2
2 1 2 2 1 1 2 3 2 2 2 3 3 3 3 3 3 2 0 2 1 3 2 1 3 2 2 3 3 1
2 2 3 2 1 2 2 2 2 2 2 2 4 1 2 2 1 2 2 0 2 3 2 2 2 1 1 2 3 1
1 2 3 2 1 2 1 3 2 3 1 2 3 2 3 2 3 3 1 2 0 2 1 2 3 3 2 2 2 2
2 2 2 2 2 2 3 2 1 3 3 1 1 2 2 2 2 1 2 1 2 0 3 3 2 2 1 2 2 2
1 1 2 2 3 2 2 3 2 3 3 1 2 3 2 1 3 2 2 2 2 1 0 2 3 2 2 2 1 2
2 1 2 1 3 2 2 3 1 3 1 2 3 2 3 2 3 1 2 2 2 2 3 0 2 2 2 2 3 2
2 2 2 2 1 1 2 1 2 2 2 3 3 1 3 2 3 2 3 2 2 3 2 1 0 3 2 3 3 2
2 2 3 2 2 2 3 2 3 3 3 1 3 2 2 2 2 3 2 3 2 2 1 2 1 0 1 3 2 2
2 1 3 3 2 2 2 1 2 2 3 2 4 1 2 2 1 3 1 2 2 3 3 2 1 2 0 2 3 2
2 3 2 1 2 1 3 3 2 3 1 2 2 2 2 3 1 2 3 2 2 1 2 2 3 3 2 0 3 2
1 1 2 2 2 1 2 3 2 3 2 2 2 2 3 2 2 2 2 1 2 1 2 3 3 2 2 1 0 2
2 2 2 2 1 1 1 3 1 1 2 2 3 2 3 2 3 1 3 2 2 2 2 2 3 2 2 3 0

```


Obtained Capacity Matrix:

```

0 6 0 1 2 4 8 18 2 6 0 6 0 2 3 2 8 12 3 9 0 4 0 2 3 8 12 6 4 3
4 0 6 3 6 8 6 24 3 15 4 0 8 2 9 12 12 24 3 5 2 0 4 2 6 4 12 24 9 10
0 4 0 1 3 4 8 12 3 9 0 6 0 1 1 4 8 12 3 9 0 4 0 2 2 6 12 6 2 9
1 2 3 0 4 6 10 14 4 12 1 2 2 0 6 6 15 14 4 8 1 1 2 0 4 9 10 14 6 8
2 6 3 2 0 2 6 10 0 6 1 9 3 4 0 3 6 10 0 6 1 6 2 2 0 3 3 15 0 4
4 8 2 6 1 0 4 12 2 2 4 8 4 6 2 0 4 8 3 1 4 8 2 6 3 0 4 8 2 1
4 12 12 10 6 4 0 4 9 3 4 6 12 5 6 4 0 6 9 1 8 12 12 10 6 4 0 4 6 2
12 8 18 21 10 12 2 0 5 3 12 16 18 7 15 8 6 0 10 6 6 16 12 21 10 8 4 0 15 6
1 3 3 4 0 2 6 10 0 4 3 9 4 4 0 2 6 10 0 4 2 9 3 4 0 2 3 10 0 2
6 10 9 12 4 2 1 6 2 0 6 10 6 8 6 3 3 6 4 0 3 5 6 8 2 3 2 9 6 0
0 4 0 2 2 2 8 18 2 9 0 4 0 3 3 6 12 12 2 6 0 2 0 1 3 6 8 18 3 6
2 0 6 1 6 8 12 24 6 10 4 0 6 2 3 8 18 16 6 10 2 0 4 2 9 12 18 16 3 5
0 4 0 1 2 2 12 18 2 9 0 4 0 2 2 4 12 6 3 6 0 4 0 3 3 6 12 12 1 6
1 1 2 0 4 6 10 7 4 8 3 3 3 0 6 3 15 14 4 8 2 2 3 0 2 6 15 14 8 8
2 6 2 2 0 1 3 15 0 6 2 6 3 4 0 3 9 10 0 4 1 6 2 2 0 3 9 15 0 4
4 8 2 6 2 0 4 12 2 4 4 8 4 6 2 0 6 4 2 2 2 4 4 3 3 0 4 8 3 3
8 18 12 10 6 4 0 8 6 2 8 6 12 15 3 6 0 4 6 3 4 12 8 10 9 8 0 2 6 1
12 16 6 14 10 4 6 0 5 9 12 8 12 14 10 4 4 0 15 6 12 16 12 14 10 12 4 0 10 6
2 3 2 4 0 1 6 15 0 4 2 9 3 6 0 3 9 10 0 4 1 9 2 2 0 2 6 15 0 2
6 10 9 8 2 2 2 6 4 0 6 10 12 4 4 2 1 6 4 0 6 15 6 8 4 1 1 6 6 0
0 4 0 2 1 4 4 18 2 9 0 4 0 2 3 4 12 18 1 6 0 4 0 2 3 6 8 12 2 6
4 0 4 2 6 8 18 16 3 15 6 0 2 2 6 8 12 8 6 5 4 0 6 3 6 8 6 16 6 10
0 2 0 2 3 4 8 18 2 9 0 2 0 3 2 2 12 12 2 6 0 2 0 2 3 4 8 12 1 6
2 1 2 0 6 6 10 21 2 12 1 2 3 0 6 6 15 7 4 8 2 2 3 0 4 6 10 14 6 8
2 6 2 4 0 1 6 5 0 4 2 9 3 2 0 2 9 10 0 4 2 9 2 2 0 3 6 15 0 4
4 8 6 6 2 0 6 8 3 3 6 4 6 6 2 0 4 12 2 3 4 8 2 6 1 0 2 12 2 2
8 6 12 15 6 4 0 2 6 2 12 12 16 5 6 4 0 6 3 2 8 18 12 10 3 4 0 4 9 2
12 24 12 7 10 4 6 0 10 9 6 16 12 14 10 12 2 0 15 6 12 8 12 14 15 12 4 0 15
1 3 2 4 0 1 6 15 0 6 2 6 2 4 0 2 6 10 0 2 2 3 2 6 0 2 6 5 0 4
6 10 6 8 2 1 1 9 2 0 6 10 9 8 6 2 3 3 6 0 6 10 6 8 4 3 2 6 6 0

```

For $k(6)$, $z_{\text{opt}}(4855)$ density(0.889655)

Shortest Path Graph:

```

0 2 2 2 1 3 1 2 2 3 2 2 2 2 2 1 2 3 2 1 2 2 2 3 2 1 1 3 2
1 0 2 2 2 2 2 2 1 2 2 2 3 3 1 3 2 2 3 2 2 1 3 2 3 2 2 2 1 2
2 2 0 2 1 2 2 2 1 1 2 2 2 2 3 1 2 1 3 2 2 3 2 2 3 1 2 2 2 2
1 2 2 0 1 2 2 2 2 2 2 2 2 3 2 2 2 1 1 2 2 2 2 2 1 3 2 2 1 1

```

2 1 2 1 0 2 2 3 1 2 1 2 2 2 2 1 2 2 2 2 2 2 3 2 2 3 1 2 2 2
 1 2 2 2 2 0 2 1 3 2 2 2 1 2 3 1 1 2 3 2 2 2 2 3 1 1 2 2 2 2
 2 2 1 3 2 3 0 2 2 2 2 3 2 2 3 1 3 1 4 1 1 3 2 3 2 2 2 1 3 2
 2 2 2 1 1 3 2 0 2 1 2 3 2 2 2 1 2 2 2 1 2 1 3 2 2 2 2 3 2 2
 1 2 2 2 2 1 2 2 0 1 2 2 2 2 2 2 2 2 2 3 3 1 2 2 1 2 2 2 2 1 1
 2 1 3 2 2 2 2 1 1 0 2 2 3 3 2 2 3 3 3 2 2 2 3 1 3 2 1 3 2 2
 2 2 2 2 2 2 2 2 2 2 0 2 2 2 2 1 1 1 3 1 2 2 2 3 2 2 2 1 1 1
 2 2 1 3 2 3 2 2 2 1 3 0 2 2 1 1 3 2 4 2 2 3 2 2 1 2 2 1 3 2
 2 2 2 2 3 1 1 2 3 3 2 2 0 3 2 2 2 2 3 1 2 1 2 2 2 1 2 2 2 1
 2 2 2 1 2 2 3 2 3 2 2 1 2 0 1 2 3 2 2 3 3 2 2 3 2 2 3 2 1 1
 2 2 2 2 1 2 2 3 2 2 2 1 2 3 0 2 2 1 3 2 3 3 2 3 2 1 2 2 2 1
 2 1 1 2 1 2 1 3 2 2 2 2 1 1 2 0 3 2 3 2 1 2 3 3 3 2 2 2 2 2
 2 2 2 2 2 2 2 1 2 2 1 2 1 3 2 2 0 1 3 2 3 2 2 3 3 1 3 2 2 1
 2 2 2 2 2 2 1 2 1 1 2 2 2 3 2 2 2 0 3 1 2 3 1 2 2 2 2 1 2 1
 2 3 1 2 2 2 2 2 1 2 2 2 2 3 2 2 1 1 0 2 2 1 2 1 3 2 3 2 2 2
 2 1 3 2 2 3 2 1 2 2 1 2 3 3 2 2 2 2 3 0 2 2 2 2 1 1 1 2 2 2
 2 1 1 2 1 2 2 3 1 2 2 2 1 1 2 2 3 1 3 2 0 2 2 2 3 2 2 2 2 2
 3 2 1 2 2 2 3 2 2 2 1 2 2 3 1 2 1 2 3 1 2 0 2 1 2 2 2 2 2 1
 2 1 3 1 2 1 2 2 2 1 2 3 1 3 2 2 1 2 2 2 3 2 0 2 2 2 2 3 2 2
 2 2 2 1 2 2 3 3 2 2 1 1 2 2 2 2 2 2 2 2 1 3 2 0 2 1 3 2 2 1
 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 3 2 2 3 1 1 2 1 2 0 2 1 3 2 1
 1 2 2 1 2 3 2 2 2 2 2 1 2 3 2 2 1 1 2 2 2 2 2 3 2 0 2 1 1 2
 3 2 2 2 1 2 1 3 2 2 2 1 2 3 1 2 2 2 3 2 2 1 2 1 2 2 0 2 3 1
 1 1 2 2 1 2 1 3 2 2 2 3 2 2 2 2 2 2 3 2 1 2 1 3 3 3 2 0 2 3
 2 2 2 1 2 3 2 1 2 1 1 2 3 3 1 2 2 1 2 2 3 1 2 2 2 2 2 2 0 2
 2 1 1 2 2 1 2 2 2 2 3 1 1 3 1 2 2 2 3 2 3 2 1 3 2 2 3 2 2 0

Obtained Capacity Matrix:

0 4 0 2 1 6 4 12 2 9 0 4 0 2 2 4 4 12 3 6 0 4 0 2 3 4 4 6 3 6
 2 0 4 2 6 8 12 16 3 10 4 0 6 3 3 12 12 16 9 10 4 0 6 2 9 8 12 16 3 10
 0 4 0 2 1 4 8 12 1 3 0 4 0 2 3 2 8 6 3 6 0 6 0 2 3 2 8 12 2 6
 1 2 2 0 2 6 10 14 4 8 2 2 2 0 4 6 10 7 2 8 2 2 2 0 2 9 10 14 2 4
 2 3 2 2 0 2 6 15 0 4 1 6 2 4 0 1 6 10 0 4 2 6 3 4 0 3 3 10 0 4
 2 8 4 6 2 0 4 4 3 2 4 8 2 6 3 0 2 8 3 2 4 8 4 9 1 0 4 8 2 2
 8 12 4 15 6 6 0 4 6 2 8 18 8 10 9 2 0 2 12 1 4 18 8 15 6 4 0 2 9 2
 12 16 12 7 5 12 4 0 10 3 12 24 12 14 10 4 4 0 10 3 12 8 18 14 10 8 4 0 10
 1 6 2 4 0 1 6 10 0 2 2 6 2 4 0 2 6 10 0 6 1 6 2 2 0 2 6 10 0 2
 6 5 9 8 4 2 2 3 2 0 6 10 9 12 4 2 3 9 6 0 6 10 9 4 6 2 1 9 4 0
 0 4 0 2 2 4 8 12 2 6 0 4 0 2 2 2 4 6 3 3 0 4 0 3 2 4 8 6 1 3
 4 0 2 3 6 12 12 16 6 5 6 0 4 2 3 4 18 16 12 10 4 0 4 2 3 8 12 8 9 10

```

0 4 0 2 3 2 4 12 3 9 0 4 0 3 2 4 8 12 3 3 0 2 0 2 2 2 8 12 2 3
2 2 2 0 4 6 15 14 6 8 2 1 2 0 2 6 15 14 4 12 3 2 2 0 4 6 15 14 2 4
2 6 2 4 0 2 6 15 0 4 2 3 2 6 0 2 6 5 0 4 3 9 2 6 0 1 6 10 0 2
4 4 2 6 1 0 2 12 2 2 4 8 2 3 2 0 6 8 3 2 2 8 6 9 3 0 4 8 2 2
8 12 8 10 6 4 0 2 6 2 4 12 4 15 6 4 0 2 9 2 12 12 8 15 9 2 0 4 6 1
12 16 12 14 10 8 2 0 5 3 12 16 12 21 10 8 4 0 15 3 12 24 6 14 10 8 4 0 10 3
2 9 1 4 0 2 6 10 0 4 2 6 2 6 0 2 3 5 0 4 2 3 2 2 0 2 9 10 0 4
6 5 9 8 4 3 2 3 4 0 3 10 9 12 4 2 2 6 6 0 6 10 6 8 2 1 1 6 4 0
0 2 0 2 1 4 8 18 1 6 0 4 0 1 2 4 12 6 3 6 0 4 0 2 3 4 8 12 2 6
6 0 2 2 6 8 18 16 6 10 2 0 4 3 3 8 6 16 9 5 4 0 4 1 6 8 12 16 6 5
0 2 0 1 2 2 8 12 2 3 0 6 0 3 2 4 4 12 2 6 0 4 0 2 2 4 8 18 2 6
2 2 2 0 4 6 15 21 4 8 1 1 2 0 4 6 10 14 4 8 1 3 2 0 4 3 15 14 4 4
2 3 2 4 0 2 6 10 0 4 2 6 2 4 0 3 6 10 0 2 1 6 1 4 0 2 3 15 0 2
2 8 4 3 2 0 4 8 2 2 4 4 4 9 2 0 2 4 2 2 4 8 4 9 2 0 4 4 1 2
12 12 8 10 3 4 0 6 6 2 8 6 8 15 3 4 0 4 9 2 8 6 8 5 6 4 0 4 9 1
6 8 12 14 5 8 2 0 10 6 12 24 12 14 10 8 4 0 15 6 6 16 6 21 15 12 4 0 10 9
2 6 2 2 0 3 6 5 0 2 1 6 3 6 0 2 6 5 0 4 3 3 2 4 0 2 6 10 0 4
6 5 3 8 4 1 2 6 4 0 9 5 3 12 2 2 2 6 6 0 9 10 3 12 4 2 3 6 4 0

```

For $k(7)$, $z_{\text{opt}}(4497)$ density(0.889655)

Shortest Path Graph:

```

0 2 1 2 2 2 2 2 3 2 3 1 2 2 2 3 3 3 2 1 1 2 2 2 2 3 2 1 2 3
1 0 1 2 2 2 2 3 3 2 1 2 2 2 3 2 2 3 2 1 2 2 1 3 2 3 1 2 2 1
1 3 0 2 1 1 2 2 2 2 2 1 3 2 2 3 3 2 1 2 2 1 2 2 2 3 2 2 2 2
2 2 3 0 2 3 2 2 2 2 2 1 2 2 2 1 3 2 1 2 1 1 2 2 2 3 2 1 2 3
2 2 2 1 0 1 3 1 1 2 2 2 2 1 2 2 2 2 2 2 2 1 2 2 2 2 1 2 2 1
1 3 2 1 1 0 3 2 2 1 2 2 3 1 3 2 3 3 2 2 2 2 1 3 2 2 2 2 2 2
2 2 1 1 2 1 0 3 2 2 3 2 3 2 1 1 3 3 1 3 2 2 1 3 2 3 2 2 2 2
1 2 2 2 1 2 2 0 2 3 1 2 2 2 2 3 1 2 1 2 1 1 2 2 2 2 2 1 2 2
2 3 2 2 1 1 3 2 0 2 2 2 2 1 2 3 1 1 2 2 2 1 2 2 2 2 2 3 1 2
3 2 3 1 2 3 2 3 3 0 2 2 2 3 2 2 3 2 2 1 1 1 3 2 1 4 2 2 2 2
2 3 2 2 1 1 3 2 2 2 0 2 1 1 3 3 3 3 1 2 2 2 1 3 2 2 2 3 2 2
1 2 2 1 2 2 1 1 3 3 2 0 2 1 2 2 2 3 2 2 1 2 2 2 2 2 3 1 2 2
1 3 2 1 2 2 2 1 2 2 1 1 0 2 3 2 2 3 1 2 2 2 2 3 1 3 2 2 1 2
2 2 1 2 2 2 2 2 3 2 1 2 2 0 2 3 2 2 2 1 1 2 2 3 1 1 2 2 1 1
2 1 2 1 1 1 3 2 2 2 2 2 3 2 0 1 3 3 2 2 2 2 2 3 3 3 2 2 2 2
2 2 2 2 2 2 2 2 3 1 2 1 3 1 1 0 2 3 2 2 2 2 2 2 2 2 1 1 1 2
2 2 3 2 2 2 3 1 1 3 2 2 1 2 1 2 0 2 2 2 1 2 3 3 2 1 3 2 2 2
1 2 2 2 2 3 3 3 3 2 2 2 2 1 3 2 2 0 2 2 2 2 1 3 2 2 1 2 1 1

```

```

1 3 2 1 2 2 3 3 1 3 2 2 2 2 2 2 2 2 0 2 1 1 3 2 1 3 2 2 2 2
2 1 2 2 1 2 1 2 2 1 2 1 3 2 2 2 2 3 2 0 2 2 2 2 2 3 1 1 2 2
2 2 3 2 1 2 2 2 2 2 2 1 1 2 1 2 3 3 2 1 0 2 3 3 1 3 2 2 1 2
2 2 3 2 2 2 2 2 3 2 1 1 2 2 1 2 3 1 2 2 2 0 2 1 1 3 2 2 1 2
2 2 1 1 2 2 2 2 2 1 2 1 2 2 3 2 2 3 1 2 2 2 0 3 2 3 1 2 2 1
1 3 2 2 1 2 1 1 2 2 2 2 2 1 2 2 2 3 2 2 2 2 2 0 2 2 2 2 1 1
3 2 2 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 3 1 2 1 2 2 0 3 1 1 2 1
2 3 2 2 2 1 2 1 2 2 1 1 2 1 2 3 1 1 2 2 2 2 2 3 2 0 2 2 2 1
2 1 1 2 1 2 3 2 2 2 2 2 2 2 2 1 1 3 2 2 1 2 2 3 2 2 0 2 1 2
2 1 2 2 2 2 1 2 3 2 2 2 2 2 2 2 3 4 2 1 1 3 1 1 2 3 2 0 2 2
1 3 2 1 2 2 3 3 2 1 1 2 2 2 2 2 3 2 1 2 1 1 2 2 2 3 3 2 0 3
2 2 1 2 2 2 2 2 3 2 1 2 1 2 3 3 3 3 2 2 2 2 1 2 1 3 2 1 2 0

```

Obtained Capacity Matrix:

```

0 4 0 2 2 4 8 12 3 6 0 2 0 2 2 6 12 18 2 3 0 4 0 2 2 6 8 6 2 9
2 0 2 2 6 8 12 24 9 10 2 0 4 2 9 8 12 24 6 5 4 0 2 3 6 12 6 16 6 5
0 6 0 2 1 2 8 12 2 6 0 2 0 2 2 6 12 12 1 6 0 2 0 2 2 6 8 12 2 6
2 2 3 0 4 9 10 14 4 8 2 1 2 0 4 3 15 14 2 8 1 1 2 0 4 9 10 7 4 12
2 6 2 2 0 1 9 5 0 4 2 6 2 2 0 2 6 10 0 4 2 3 2 4 0 2 3 10 0 2
2 12 4 3 1 0 6 8 2 1 4 8 6 3 3 0 6 12 2 2 4 8 2 9 2 0 4 8 2 2
8 12 4 5 6 2 0 6 6 2 12 12 12 10 3 2 0 6 3 3 8 12 4 15 6 6 0 4 6 2
6 16 12 14 5 8 4 0 10 9 6 16 12 14 10 12 2 0 5 6 6 8 12 14 10 8 4 0 10 6
2 9 2 4 0 1 9 10 0 4 2 6 2 2 0 3 3 5 0 4 2 3 2 4 0 2 6 15 0 4
9 10 9 4 4 3 2 9 6 0 6 10 6 12 4 2 3 6 4 0 3 5 9 8 2 4 2 6 4 0
0 6 0 2 1 2 12 12 2 6 0 4 0 1 3 6 12 18 1 6 0 4 0 3 2 4 8 18 2 6
2 0 4 1 6 8 6 8 9 15 4 0 4 1 6 8 12 24 6 10 2 0 4 2 6 8 18 8 6 10
0 6 0 1 2 4 8 6 2 6 0 2 0 2 3 4 8 18 1 6 0 4 0 3 1 6 8 12 1 6
2 2 1 0 4 6 10 14 6 8 1 2 2 0 4 9 10 14 4 4 1 2 2 0 2 3 10 14 2 4
2 3 2 2 0 1 9 10 0 4 2 6 3 4 0 1 9 15 0 4 2 6 2 6 0 3 6 10 0 4
4 8 4 6 2 0 4 8 3 1 4 4 6 3 1 0 4 12 2 2 4 8 4 6 2 0 2 4 1 2
8 12 12 10 6 4 0 2 3 3 8 12 4 10 3 4 0 4 6 2 4 12 12 15 6 2 0 4 6 2
6 16 12 14 10 12 6 0 15 6 12 16 12 7 15 8 4 0 10 6 12 16 6 21 10 8 2 0 5 3
1 9 2 2 0 2 9 15 0 6 2 6 2 4 0 2 6 10 0 4 1 3 3 4 0 3 6 10 0 4
6 5 6 8 2 2 1 6 4 0 6 5 9 8 4 2 2 9 4 0 6 10 6 8 4 3 1 3 4 0
0 4 0 2 1 4 8 12 2 6 0 2 0 2 1 4 12 18 2 3 0 4 0 3 1 6 8 12 1 6
4 0 6 2 6 8 12 16 9 10 2 0 4 2 3 8 18 8 6 10 4 0 4 1 3 12 12 16 3 10
0 4 0 1 2 4 8 12 2 3 0 2 0 2 3 4 8 18 1 6 0 4 0 3 2 6 4 12 2 3
1 3 2 0 2 6 5 7 4 8 2 2 2 0 4 6 10 21 4 8 2 2 2 0 4 6 10 14 2 4
3 6 2 4 0 2 6 10 0 4 2 6 2 4 0 2 6 10 0 2 2 3 2 4 0 3 3 5 0 2
4 12 4 6 2 0 4 4 2 2 2 4 4 3 2 0 2 4 2 2 4 8 4 9 2 0 4 8 2 1

```

```

8 6 4 10 3 4 0 4 6 2 8 12 8 10 6 2 0 6 6 2 4 12 8 15 6 4 0 4 3 2
12 8 12 14 10 8 2 0 15 6 12 16 12 14 10 8 6 0 10 3 6 24 6 7 10 12 4 0 10 6
1 9 2 2 0 2 9 15 0 2 1 6 2 4 0 2 9 10 0 4 1 3 2 4 0 3 9 10 0 6
6 10 3 8 4 2 2 6 6 0 3 10 3 8 6 3 3 9 4 0 6 10 3 8 2 3 2 3 4 0

```

For $k(8)$, $z_{\text{opt}}(4504)$ density(0.889655)

Shortest Path Graph:

```

0 1 1 1 2 2 2 2 2 2 1 1 2 1 2 2 2 2 1 2 2 2 2 2 3 2 1 3 2 2
2 0 2 2 1 1 1 2 2 1 1 3 2 2 2 2 1 3 3 2 1 2 2 2 2 2 2 2 2 1
2 2 0 2 2 2 2 2 1 1 1 3 2 2 2 3 2 2 1 2 1 2 1 2 2 2 2 3 2 2
2 2 2 0 2 2 2 2 2 2 2 1 1 1 2 1 2 2 1 1 2 2 2 2 2 1 2 2 2 1
1 2 2 2 0 1 2 2 2 2 1 2 2 2 2 3 2 3 2 2 2 1 2 2 1 2 2 2 1 1
1 2 2 2 2 0 2 2 2 1 1 2 1 2 2 2 2 3 2 2 2 1 2 1 3 1 2 2 2 2
1 2 1 2 1 2 0 2 2 2 1 2 2 2 1 2 2 3 2 1 2 1 2 2 1 3 1 2 2 2
1 2 1 2 2 1 2 0 2 1 2 1 2 2 2 3 2 3 2 1 2 2 1 2 1 2 2 2 2 2
1 2 1 2 2 3 1 2 0 2 2 2 1 1 2 2 2 3 2 2 2 1 2 1 2 2 2 2 1 1
2 1 2 1 1 2 1 1 1 0 2 2 2 2 2 2 1 3 2 2 2 1 2 2 2 1 2 2 2 2
1 2 2 2 2 2 1 1 1 2 0 2 2 2 2 3 3 3 2 2 2 1 1 2 2 2 2 2 2 1
2 2 1 1 2 1 1 1 2 2 2 0 1 2 2 2 1 3 2 2 2 2 2 1 2 2 2 2 2 2
2 1 3 1 2 2 2 2 2 1 1 2 0 2 3 1 2 2 2 2 2 2 2 3 3 2 2 1 1 2
1 2 2 2 1 2 2 2 1 2 2 2 2 0 2 3 2 2 1 2 2 2 1 1 2 1 2 2 2 1
3 2 3 1 2 2 2 2 2 1 2 2 2 2 0 2 1 3 2 2 2 2 1 2 3 2 1 2 1 1
2 2 2 2 2 2 2 1 2 1 3 1 2 2 0 2 2 1 2 1 2 1 2 2 2 1 1 2 2
2 1 3 2 2 2 2 2 2 1 2 1 1 3 1 0 3 2 3 2 2 1 1 3 2 2 2 1 2
2 1 3 2 2 2 2 2 1 2 3 2 1 2 2 1 0 2 1 2 2 2 2 2 2 1 1 2 1
2 2 3 2 1 2 3 3 2 2 2 3 2 2 1 2 1 1 0 2 3 2 2 2 2 1 2 2 2 2
2 1 2 2 2 2 2 1 3 2 2 2 1 3 1 2 2 3 2 0 1 1 2 1 1 3 2 2 2 2
2 2 2 1 1 2 2 1 2 1 2 2 2 2 1 2 2 3 2 1 0 1 2 2 1 2 2 2 2 1
2 2 1 1 2 3 2 2 2 1 1 2 2 2 1 2 2 2 2 2 1 0 2 1 2 2 2 1 2 1
2 1 3 1 2 1 2 3 2 2 2 2 2 2 3 2 2 3 2 2 2 2 0 2 3 2 1 2 1 1
2 1 2 1 2 2 2 1 2 2 2 2 2 2 2 2 1 2 1 2 1 2 1 0 2 2 2 2 2 1
1 2 1 2 2 1 2 3 2 2 2 2 1 2 1 2 1 2 2 3 2 1 2 2 0 2 1 1 2 2
2 1 2 2 1 2 2 2 1 1 2 2 2 1 2 3 2 2 2 1 2 2 2 2 2 0 2 1 1 2
2 2 2 1 2 3 1 2 1 1 2 2 2 2 2 1 2 3 2 2 2 2 2 2 2 2 0 2 1 1
2 2 2 2 1 2 1 2 1 1 2 2 2 2 2 2 2 1 3 2 2 1 3 2 2 2 1 0 1 2
2 1 2 1 2 2 2 2 1 2 2 1 2 1 2 2 1 2 2 2 1 2 2 2 2 2 2 1 0 2
2 2 2 2 2 3 2 2 1 2 2 2 1 1 2 2 2 2 2 1 2 2 2 1 2 1 1 1 0

```

Obtained Capacity Matrix:

```

0 2 0 1 2 4 8 12 2 6 0 2 0 1 2 4 8 12 1 6 0 4 0 2 3 4 4 18 2 6
4 0 4 2 3 4 6 16 6 5 2 0 4 2 6 8 6 24 9 10 2 0 4 2 6 8 12 16 6 5
0 4 0 2 2 4 8 12 1 3 0 6 0 2 2 6 8 12 1 6 0 4 0 2 2 4 8 18 2 6
2 2 2 0 4 6 10 14 4 8 2 1 1 0 4 3 10 14 2 4 2 2 2 0 4 3 10 14 4 4
1 6 2 4 0 1 6 10 0 4 1 6 2 4 0 3 6 15 0 4 2 3 2 4 0 2 6 10 0 2
2 8 4 6 2 0 4 8 2 1 2 8 2 6 2 0 4 12 2 2 4 4 4 3 3 0 4 8 2 2
4 12 4 10 3 4 0 4 6 2 4 12 8 10 3 4 0 6 6 1 8 6 8 10 3 6 0 4 6 2
6 16 6 14 10 4 4 0 10 3 12 8 12 14 10 12 4 0 10 3 12 16 6 14 5 8 4 0 10 6
1 6 1 4 0 3 3 10 0 4 2 6 1 2 0 2 6 15 0 4 2 3 2 2 0 2 6 10 0 2
6 5 6 4 2 2 1 3 2 0 6 10 6 8 4 2 1 9 4 0 6 5 6 8 4 1 2 6 4 0
0 4 0 2 2 4 4 6 1 6 0 4 0 2 2 6 12 18 2 6 0 2 0 2 2 4 8 12 2 3
4 0 2 1 6 4 6 8 6 10 4 0 2 2 6 8 6 24 6 10 4 0 4 1 6 8 12 16 6 10
0 2 0 1 2 4 8 12 2 3 0 4 0 2 3 2 8 12 2 6 0 4 0 3 3 4 8 6 1 6
1 2 2 0 2 6 10 14 2 8 2 2 2 0 4 9 10 14 2 8 2 2 1 0 4 3 10 14 4 4
3 6 3 2 0 2 6 10 0 2 2 6 2 4 0 2 3 15 0 4 2 6 1 4 0 2 3 10 0 2
4 8 4 6 2 0 4 8 1 2 2 12 2 6 2 0 4 8 1 2 2 8 2 6 2 0 2 4 2 2
8 6 12 10 6 4 0 4 6 2 4 12 4 5 9 2 0 6 6 3 8 12 4 5 9 4 0 4 3 2
12 8 18 14 10 8 4 0 10 3 12 24 12 7 10 8 2 0 10 3 12 16 12 14 10 8 2 0 10 3
2 6 3 4 0 2 9 15 0 4 2 9 2 4 0 2 3 5 0 4 3 6 2 4 0 1 6 10 0 4
6 5 6 8 4 2 2 3 6 0 6 10 3 12 2 2 2 9 4 0 3 5 6 4 2 3 2 6 4 0
0 4 0 1 1 4 8 6 2 3 0 4 0 2 1 4 8 18 2 3 0 2 0 2 1 4 8 12 2 3
4 0 2 1 6 12 12 16 6 5 2 0 4 2 3 8 12 16 6 10 2 0 4 1 6 8 12 8 6 5
0 2 0 1 2 2 8 18 2 6 0 4 0 2 3 4 8 18 2 6 0 4 0 2 3 4 4 12 1 3
2 1 2 0 4 6 10 7 4 8 2 2 2 0 4 6 5 14 2 8 1 2 1 0 4 6 10 14 4 4
1 6 1 4 0 1 6 15 0 4 2 6 1 4 0 2 3 10 0 6 2 3 2 4 0 2 3 5 0 4
4 4 4 6 1 0 4 8 1 1 4 8 4 3 2 0 4 8 2 1 4 8 4 6 2 0 4 4 1 2
8 12 8 5 6 6 0 4 3 1 8 12 8 10 6 2 0 6 6 2 8 12 8 10 6 4 0 4 3 1
12 16 12 14 5 8 2 0 5 3 12 16 12 14 10 8 4 0 15 6 12 8 18 14 10 8 2 0 5 6
2 3 2 2 0 2 6 10 0 4 2 3 2 2 0 2 3 10 0 4 1 6 2 4 0 2 6 5 0 4
6 10 6 8 4 3 2 6 2 0 6 10 3 4 4 2 2 6 4 0 6 10 6 4 4 1 1 3 2 0

```

For $k(9)$, $z_{\text{opt}}(4197)$ density(0.889655)

Shortest Path Graph:

```

0 2 1 2 2 2 3 2 2 2 2 1 2 1 1 1 2 2 2 1 2 1 1 2 2 1 2 2 1 2
2 0 3 2 2 1 2 2 1 2 1 2 2 2 2 1 2 2 1 2 1 2 2 2 1 2 2 2 2 1
2 2 0 2 1 2 2 1 2 2 2 2 2 1 2 1 1 3 2 2 2 2 2 1 1 1 2 1 2 2
2 2 2 0 1 2 1 2 2 1 2 2 2 1 1 2 2 2 2 2 2 2 2 1 2 1 1 1 2 2
2 1 1 2 0 2 2 1 2 2 1 2 2 2 2 1 2 2 2 2 1 2 1 2 1 1 2 2 2 2
2 2 2 2 3 0 2 1 2 2 1 2 1 2 2 2 2 2 2 2 1 1 2 1 2 1 1 2 2 2

```

```

1 2 2 2 1 2 0 2 1 2 1 2 2 1 2 1 1 2 1 2 2 2 1 2 2 2 2 2 2 1
2 2 1 2 2 2 1 0 1 1 2 2 2 2 2 1 2 3 2 2 2 2 1 2 2 1 2 1 2 1
2 1 2 2 1 2 1 2 0 2 2 2 2 1 1 2 2 2 2 1 2 1 1 2 2 2 1 2 2 2
2 1 1 2 2 2 3 2 2 0 2 2 2 2 2 2 2 3 2 2 1 1 2 2 1 1 1 2 1 2
3 2 2 2 2 2 2 1 2 2 0 2 2 1 1 1 1 1 2 1 2 1 2 2 2 2 2 1 2
2 2 2 2 2 2 3 2 2 2 2 0 1 2 1 2 1 2 2 2 1 1 1 1 2 2 1 2 2 1
1 1 2 2 2 2 2 2 1 2 2 1 0 1 2 2 2 1 2 2 2 2 2 2 2 2 1 2 2 1 2
3 1 1 2 1 1 2 1 1 2 2 3 2 0 2 1 2 3 2 2 2 2 2 1 2 2 1 2 2 1
2 2 2 2 1 2 2 1 2 1 2 2 2 1 0 2 1 1 1 2 2 1 2 1 2 2 2 1 2 2
2 2 2 2 2 2 3 2 2 2 2 2 2 1 2 0 2 2 3 1 1 2 2 2 1 2 1 2 1 1
2 1 1 1 2 1 2 2 1 2 1 1 1 2 2 2 0 2 2 2 2 2 1 2 1 2 2 2 2 2
2 1 2 2 2 2 1 1 2 1 2 2 2 2 2 2 0 2 1 1 2 1 2 1 1 2 2 2 2
1 2 2 2 2 2 1 2 2 1 2 1 2 2 2 2 2 0 1 1 2 2 2 2 2 1 1 1
2 2 1 2 2 1 2 2 2 2 2 2 2 2 2 2 1 1 2 0 1 1 1 2 2 1 2 2 2
1 2 2 1 2 2 2 2 2 2 2 2 1 2 2 1 2 2 2 2 0 1 1 2 2 1 1 1 2
2 1 1 2 2 2 3 2 2 2 2 1 2 2 2 1 2 3 2 2 1 0 1 2 1 1 2 2 1 2
2 2 1 1 2 2 2 2 1 1 3 1 1 1 2 2 2 2 3 2 1 1 0 2 2 2 2 2 1
2 1 2 1 2 2 2 2 2 2 2 2 1 2 1 1 2 2 2 2 2 2 1 0 1 1 2 2 1 2
1 1 2 2 1 2 3 2 2 2 2 1 2 1 1 2 2 2 2 2 2 1 1 2 0 1 1 2 2 2
2 2 2 1 2 2 2 2 2 1 2 2 1 1 1 2 1 2 1 2 1 2 2 1 2 0 2 2 1 2
2 2 1 3 2 2 2 1 2 1 1 2 1 1 2 2 2 2 3 1 2 2 2 2 2 0 1 2 1
1 2 2 2 3 1 3 2 2 2 2 1 1 2 2 1 2 2 3 2 1 2 1 1 2 2 2 0 1 2
2 2 2 2 2 2 3 2 2 1 1 2 2 1 2 1 2 2 3 1 2 1 1 2 2 2 2 1 0 2
2 2 2 2 1 3 2 1 2 2 2 2 1 2 1 2 2 2 2 2 2 2 1 1 1 1 1 2 2 0

```

Obtained Capacity Matrix:

```

0 4 0 2 2 4 12 12 2 6 0 2 0 1 1 2 8 12 2 3 0 2 0 2 2 2 8 12 1 6
4 0 6 2 6 4 12 16 3 10 2 0 4 2 6 4 12 16 3 10 2 0 4 2 3 8 12 16 6 5
0 4 0 2 1 4 8 6 2 6 0 4 0 1 2 2 4 18 2 6 0 4 0 1 1 2 8 6 2 6
2 2 2 0 2 6 5 14 4 4 2 2 2 0 2 6 10 14 4 8 2 2 2 0 4 3 5 7 4 8
2 3 1 4 0 2 6 5 0 4 1 6 2 4 0 1 6 10 0 4 1 6 1 4 0 1 6 10 0 4
4 8 4 6 3 0 4 4 2 2 2 8 2 6 2 0 4 8 2 2 2 4 4 3 2 0 2 8 2 2
4 12 8 10 3 4 0 4 3 2 4 12 8 5 6 2 0 4 3 2 8 12 4 10 6 4 0 4 6 1
12 16 6 14 10 8 2 0 5 3 12 16 12 14 10 4 4 0 10 6 12 16 6 14 10 4 4 0 10 3
2 3 2 4 0 2 3 10 0 4 2 6 2 2 0 2 6 10 0 2 2 3 1 4 0 2 3 10 0 4
6 5 3 8 4 2 3 6 4 0 6 10 6 8 4 2 2 9 4 0 3 5 6 8 2 1 1 6 2 0
0 4 0 2 2 4 8 6 2 6 0 4 0 1 1 2 4 6 2 3 0 2 0 2 2 4 8 12 1 6
4 0 4 2 6 8 18 16 6 10 4 0 2 2 3 8 6 16 6 10 2 0 2 1 6 8 6 16 6 5
0 2 0 2 2 4 8 12 1 6 0 2 0 1 2 4 8 6 2 6 0 4 0 2 2 2 8 12 1 6
3 1 1 0 2 3 10 7 2 8 2 3 2 0 4 3 10 21 4 8 2 2 2 0 4 6 5 14 4 4

```

```

2 6 2 4 0 2 6 5 0 2 2 6 2 2 0 2 3 5 0 4 2 3 2 2 0 2 6 5 0 4
4 8 4 6 2 0 6 8 2 2 4 8 4 3 2 0 4 8 3 1 2 8 4 6 1 0 2 8 1 1
8 6 4 5 6 2 0 4 3 2 4 6 4 10 6 4 0 4 6 2 8 12 4 10 3 4 0 4 6 2
12 8 12 14 10 8 2 0 10 3 12 16 12 14 10 8 4 0 10 3 6 16 6 14 5 4 4 0 10 6
1 6 2 4 0 2 3 10 0 2 2 3 2 4 0 2 6 10 0 2 1 6 2 4 0 2 6 5 0 2
6 10 3 8 4 1 2 6 4 0 6 10 6 8 4 2 1 3 4 0 3 5 3 8 4 1 2 6 4 0
0 4 0 1 2 4 8 12 2 6 0 2 0 2 1 4 8 12 2 6 0 2 0 2 2 2 4 6 2 6
4 0 2 2 6 8 18 16 6 10 4 0 4 2 6 4 12 24 6 10 2 0 2 2 3 4 12 16 3 10
0 4 0 1 2 4 8 12 1 3 0 2 0 1 2 4 8 12 3 6 0 2 0 2 2 4 8 12 2 3
2 1 2 0 4 6 10 14 4 8 2 2 1 0 2 3 10 14 4 8 2 2 1 0 2 3 10 14 2 8
1 3 2 4 0 2 9 10 0 4 2 3 2 2 0 2 6 10 0 4 2 3 1 4 0 1 3 10 0 4
4 8 4 3 2 0 4 8 2 1 4 8 2 3 1 0 2 8 1 2 2 8 4 3 2 0 4 8 1 2
8 12 4 15 6 4 0 2 6 1 4 12 4 5 6 4 0 4 9 1 8 12 8 10 6 4 0 2 6 1
6 16 12 14 15 4 6 0 10 6 12 8 6 14 10 4 4 0 15 6 6 16 6 7 10 8 4 0 5 6
2 6 2 4 0 2 9 10 0 2 1 6 2 2 0 1 6 10 0 2 2 3 1 4 0 2 6 5 0 4
6 10 6 8 2 3 2 3 4 0 6 10 3 8 2 2 2 6 4 0 6 10 3 4 2 1 1 6 4 0

```

For $k(10)$, $z_{\text{-opt}}(3992)$ density(0.889655)

Shortest Path Graph:

```

0 2 1 2 2 2 2 3 2 2 2 2 2 2 1 1 2 2 1 2 2 1 2 2 2 2 1 2 1
2 0 2 2 2 1 2 1 1 2 1 1 2 1 2 2 2 1 2 2 2 2 2 1 2 2 1 2 1 2
2 1 0 1 1 2 2 2 2 2 2 1 1 1 2 2 1 2 2 2 2 2 1 1 2 2 2 2 2 2
1 2 2 0 2 2 2 1 2 2 1 1 2 1 1 1 2 2 1 2 1 2 2 1 2 2 1 2 2 2
1 1 1 2 0 2 1 2 2 2 2 2 2 2 2 2 1 2 1 1 2 2 1 2 1 1 2 2 2 2
2 1 1 2 2 0 2 1 2 2 1 2 1 2 2 2 2 1 2 2 2 1 2 2 1 2 2 1 1 2
2 1 2 2 2 2 0 1 1 1 2 2 3 2 1 1 2 1 2 2 2 2 2 1 2 1 2 2 2 2
2 1 2 2 1 1 2 0 2 1 2 2 2 2 1 2 2 1 2 1 1 2 1 2 2 2 1 1 2 2
2 2 1 1 2 2 1 2 0 2 2 1 2 2 2 2 1 2 2 1 2 1 2 1 2 1 1 2 2 1
1 1 2 2 2 2 2 1 2 0 2 2 2 2 1 2 2 2 1 2 2 1 2 2 2 1 2 1 2 1
2 2 1 1 1 1 2 2 2 1 0 2 2 1 2 1 2 1 2 2 1 2 2 2 2 2 1 2 2 2
1 1 2 2 1 2 2 2 2 2 1 0 1 1 1 2 2 1 1 2 2 2 2 2 2 2 1 2 1 2
2 2 2 1 2 2 1 2 2 1 1 2 0 1 1 2 2 2 2 3 2 2 2 2 2 2 1 2 1 1
2 2 2 2 1 1 2 2 2 1 2 2 2 0 2 1 2 1 2 2 2 2 2 2 2 2 1 2 1 1
2 2 1 2 2 2 1 2 1 1 2 2 2 1 0 1 1 2 2 2 2 2 2 2 2 2 2 2 1 1
2 2 2 2 1 1 2 2 2 1 1 2 2 1 2 0 1 2 2 2 2 1 1 1 1 2 2 2 2 1
1 2 2 2 2 2 2 1 2 2 1 2 2 2 2 0 1 2 2 2 2 1 1 2 2 1 1 1 2
2 2 2 2 2 2 1 1 2 2 1 2 2 1 1 2 2 0 1 2 1 2 2 2 2 2 1 1 1 2
2 2 2 2 2 2 2 2 2 1 2 2 1 1 1 2 1 2 0 2 1 2 1 1 2 2 2 2 2 2
2 2 2 1 2 2 2 2 2 1 1 1 2 2 1 2 2 1 2 0 2 1 2 1 1 2 2 2 2 2

```


2 1 1 2 2 2 2 2 2 2 1 2 2 1 2 2 2 1 1 3 0 2 2 2 1 2 1 2 1 2
 2 1 2 2 1 2 2 2 2 2 1 1 2 1 2 2 2 1 2 1 1 0 2 2 2 1 2 2 1 2
 2 2 1 2 1 1 1 2 2 2 1 1 2 2 1 2 2 1 2 2 2 1 0 2 2 2 2 2 2 2
 2 1 2 2 2 2 2 2 2 1 1 2 3 2 1 1 2 2 2 1 2 2 2 0 1 2 2 2 2 2
 2 2 1 2 2 2 2 1 2 1 2 2 2 2 1 2 2 2 2 2 2 2 2 1 1 0 1 2 1 2 1
 2 2 2 2 2 2 1 2 1 2 2 2 2 2 2 2 1 1 2 2 1 1 1 1 1 0 1 2 2 2
 1 2 2 2 2 1 1 2 1 2 2 2 1 2 2 1 2 1 2 2 2 1 2 2 1 2 0 1 2 2
 2 2 1 2 2 2 1 2 1 1 2 2 2 1 2 2 1 2 1 2 2 2 1 1 2 2 1 0 2 2
 1 1 1 2 2 2 1 2 2 2 2 1 2 1 2 2 2 2 2 2 2 2 1 2 1 2 2 1 0 1
 2 1 1 1 2 2 2 2 2 2 2 2 1 2 2 1 2 1 2 2 1 1 2 2 2 1 1 2 2 0

Obtained Capacity Matrix:

0 4 0 2 2 4 8 18 2 6 0 4 0 2 2 2 4 12 2 3 0 4 0 2 2 4 8 6 2 3
 4 0 4 2 6 4 12 8 3 10 2 0 4 1 6 8 12 8 6 10 4 0 4 1 6 8 6 16 3 10
 0 2 0 1 1 4 8 12 2 6 0 2 0 1 2 4 4 12 2 6 0 4 0 1 2 4 8 12 2 6
 1 2 2 0 4 6 10 7 4 8 1 1 2 0 2 3 10 14 2 8 1 2 2 0 4 6 5 14 4 8
 1 3 1 4 0 2 3 10 0 4 2 6 2 4 0 2 3 10 0 2 2 6 1 4 0 1 6 10 0 4
 4 4 2 6 2 0 4 4 2 2 2 8 2 6 2 0 4 4 2 2 4 4 4 6 1 0 4 4 1 2
 8 6 8 10 6 4 0 2 3 1 8 12 12 10 3 2 0 2 6 2 8 12 8 5 6 2 0 4 6 2
 12 8 12 14 5 4 4 0 10 3 12 16 12 14 5 8 4 0 10 3 6 16 6 14 10 8 2 0 10 6
 2 6 1 2 0 2 3 10 0 4 2 3 2 4 0 2 3 10 0 2 2 3 2 2 0 1 3 10 0 2
 3 5 6 8 4 2 2 3 4 0 6 10 6 8 2 2 2 6 2 0 6 5 6 8 4 1 2 3 4 0
 0 4 0 1 1 2 8 12 2 3 0 4 0 1 2 2 8 6 2 6 0 4 0 2 2 4 4 12 2 6
 2 0 4 2 3 8 12 16 6 10 2 0 2 1 3 8 12 8 3 10 4 0 4 2 6 8 6 16 3 10
 0 4 0 1 2 4 4 12 2 3 0 4 0 1 1 4 8 12 2 9 0 4 0 2 2 2 8 6 1 3
 2 2 2 0 2 3 10 14 4 4 2 2 2 0 4 3 10 7 4 8 2 2 2 0 4 3 10 7 2 4
 2 6 1 4 0 2 3 10 0 2 2 6 2 2 0 1 3 10 0 4 2 6 2 4 0 2 6 10 0 2
 4 8 4 6 1 0 4 8 2 1 2 8 4 3 2 0 2 8 2 2 4 4 2 3 1 0 4 8 2 1
 4 12 8 10 6 4 0 4 3 2 8 6 8 10 6 4 0 2 6 2 8 12 4 5 6 4 0 2 3 2
 12 16 12 14 10 8 2 0 10 6 6 16 12 7 5 8 4 0 5 6 6 16 12 14 10 8 2 0 5 6
 2 6 2 4 0 2 6 10 0 2 2 6 1 2 0 2 3 10 0 4 1 6 1 2 0 2 6 10 0 4
 6 10 6 4 4 2 2 6 4 0 3 5 6 8 2 2 2 3 4 0 6 5 6 4 2 2 2 6 4 0
 0 2 0 2 2 4 8 12 2 6 0 4 0 1 2 4 8 6 1 9 0 4 0 2 1 4 4 12 1 6
 4 0 4 2 3 8 12 16 6 10 2 0 4 1 6 8 12 8 6 5 2 0 4 2 6 4 12 16 3 10
 0 4 0 2 1 2 4 12 2 6 0 2 0 2 1 4 8 6 2 6 0 2 0 2 2 4 8 12 2 6
 2 1 2 0 4 6 10 14 4 4 1 2 3 0 2 3 10 14 4 4 2 2 2 0 2 6 10 14 4 8
 2 6 1 4 0 2 6 5 0 2 2 6 2 4 0 2 6 10 0 4 2 6 1 2 0 1 6 5 0 2
 4 8 4 6 2 0 2 8 1 2 4 8 4 6 2 0 2 4 2 2 2 4 2 3 1 0 2 8 2 2
 4 12 8 10 6 2 0 4 3 2 8 12 4 10 6 2 0 2 6 2 8 6 8 10 3 4 0 2 6 2
 12 16 6 14 10 8 2 0 5 3 12 16 12 7 10 8 2 0 5 6 12 16 6 7 10 8 2 0 10 6

1 3 1 4 0 2 3 10 0 4 2 3 2 2 0 2 6 10 0 4 2 6 1 4 0 2 6 5 0 2
6 5 3 4 4 2 2 6 4 0 6 10 3 8 4 1 2 3 4 0 3 5 6 8 4 1 1 6 4 0

For $k(11)$, $z_{\text{opt}}(3881)$ density(0.889655)

Shortest Path Graph:

0 2 2 2 2 2 2 1 2 1 2 2 2 1 2 1 2 2 2 1 2 1 2 2 2 1 1 1 2 1
1 0 1 2 1 2 1 1 1 1 2 2 2 2 2 2 2 2 1 1 2 1 2 1 2 2 2 1 2 2
2 2 0 2 1 2 2 1 1 1 2 2 2 2 2 2 2 2 1 1 1 1 2 2 2 2 1 2 2
1 1 2 0 2 1 2 2 2 2 2 2 2 2 1 2 2 1 2 1 2 2 2 2 3 1 2 2 1 2
2 2 2 2 0 2 1 1 2 2 2 2 2 1 2 1 2 1 2 3 2 2 1 1 1 2 2 1 1 2
1 2 2 2 2 0 2 2 3 2 1 2 1 2 2 2 3 2 2 2 2 1 1 2 2 1 1 1 1 1
1 1 1 2 2 2 0 2 2 2 2 1 2 1 2 1 2 2 2 2 1 2 2 2 1 1 2 1 3 1
2 1 1 1 1 2 1 0 1 2 2 2 2 2 2 2 2 2 2 2 2 1 1 2 1 2 1 1 1
1 2 2 2 1 2 1 1 0 1 2 2 1 2 1 1 1 2 2 2 2 1 2 2 2 2 2 2 2
2 1 2 2 1 1 1 1 2 0 2 1 2 2 1 1 1 2 2 2 1 2 2 2 2 2 2 2 1
2 2 2 1 2 2 1 1 2 2 0 2 1 1 2 2 2 2 2 1 2 2 1 2 2 1 1 2 1 2
1 2 2 1 1 2 2 1 2 2 2 0 2 2 2 2 2 2 1 2 2 2 2 1 2 1 1 2 2 1
1 2 2 1 2 1 1 2 2 2 2 1 0 2 1 2 2 2 1 2 2 1 2 1 2 1 2 2 2 1
1 1 2 2 2 2 2 2 2 1 1 2 1 0 2 1 2 1 1 2 1 2 1 1 2 2 2 2 2
1 1 1 2 2 2 1 2 2 2 2 2 1 2 0 2 2 2 2 1 2 1 2 2 2 2 1 2 2 1
2 1 2 2 2 2 2 2 2 2 1 2 2 1 1 0 1 2 1 2 2 2 2 2 2 1 2 1 2 2
1 1 1 2 2 2 2 2 1 2 2 2 2 2 2 2 0 2 2 1 2 1 2 2 2 2 1 1 2 1
2 2 1 1 1 2 1 2 1 2 2 2 1 2 2 2 2 0 2 2 2 2 2 1 2 2 1 1 1 1
2 2 1 2 1 1 1 2 2 2 2 1 2 2 2 2 1 1 0 1 2 2 2 2 1 2 2 1 2 1
1 2 2 2 2 2 1 2 2 2 2 2 2 1 1 2 1 1 2 0 1 2 2 1 2 2 1 1 2 2
2 2 1 1 2 1 1 1 2 2 1 2 2 2 1 2 2 2 1 2 0 2 2 2 2 2 1 1 2 2
2 2 2 1 1 2 1 1 2 1 2 1 2 2 2 1 2 2 1 2 2 0 2 2 1 2 2 2 1 1
1 2 1 2 2 1 2 2 2 1 1 1 2 1 1 2 2 2 2 2 1 0 2 2 2 2 1 2 2
2 1 2 2 2 1 2 2 1 2 1 2 2 1 1 2 2 2 1 2 1 2 2 0 2 1 2 2 1 2
1 2 1 2 1 2 2 1 2 2 1 2 2 1 2 2 2 2 1 2 1 2 1 2 0 1 1 2 2 2
2 2 1 2 1 2 2 1 2 2 2 2 2 2 1 1 2 2 1 2 1 2 1 2 2 0 2 1 2 2
2 1 2 1 2 1 1 2 2 2 1 2 2 2 2 1 2 2 2 2 2 1 2 2 1 0 2 1 2
2 2 2 1 2 1 2 1 2 1 2 2 2 1 2 2 2 1 2 2 2 1 1 2 2 1 1 0 2 2
1 2 2 2 2 2 1 1 2 2 2 1 2 2 2 1 2 3 2 2 1 1 2 2 2 1 1 2 0 1
2 2 2 1 2 2 2 1 2 2 1 2 1 2 1 3 3 2 2 2 2 2 1 1 1 2 2 1 2 0

Obtained Capacity Matrix:

0 4 0 2 2 4 8 6 2 3 0 4 0 1 2 2 8 12 2 3 0 2 0 2 2 2 4 6 2 3
2 0 2 2 3 8 6 8 3 5 4 0 4 2 6 8 12 16 3 5 4 0 4 1 6 8 12 8 6 10

```

0 4 0 2 1 4 8 6 1 3 0 4 0 2 2 4 8 12 2 3 0 2 0 2 2 4 8 6 2 6
1 1 2 0 4 3 10 14 4 8 2 2 2 0 2 6 10 7 4 4 2 2 2 0 6 3 10 14 2 8
2 6 2 4 0 2 3 5 0 4 2 6 2 2 0 1 6 5 0 6 2 6 1 2 0 2 6 5 0 4
2 8 4 6 2 0 4 8 3 2 2 8 2 6 2 0 6 8 2 2 4 4 2 6 2 0 2 4 1 1
4 6 4 10 6 4 0 4 6 2 8 6 8 5 6 2 0 4 6 2 4 12 8 10 3 2 0 2 9 1
12 8 6 7 5 8 2 0 5 6 12 16 12 14 10 8 4 0 10 6 12 16 6 7 10 4 4 0 5 3
1 6 2 4 0 2 3 5 0 2 2 6 1 4 0 1 3 10 0 4 2 3 2 4 0 2 6 10 0 4
6 5 6 8 2 1 1 3 4 0 6 5 6 8 2 1 1 6 4 0 3 10 6 8 4 2 2 6 4 0
0 4 0 1 2 4 4 6 2 6 0 4 0 1 2 4 8 12 2 3 0 4 0 2 2 2 4 12 1 6
2 0 4 1 3 8 12 8 6 10 4 0 4 2 6 8 12 16 3 10 4 0 4 1 6 4 6 16 6 5
0 4 0 1 2 2 4 12 2 6 0 2 0 2 1 4 8 12 1 6 0 2 0 1 2 2 8 12 2 3
1 1 2 0 4 6 10 14 4 4 1 2 1 0 4 3 10 7 2 8 1 2 1 0 4 6 10 14 4 8
1 3 1 4 0 2 3 10 0 4 2 6 1 4 0 2 6 10 0 2 2 3 2 4 0 2 3 10 0 2
4 4 4 6 2 0 4 8 2 2 2 8 4 3 1 0 2 8 1 2 4 8 4 6 2 0 4 4 2 2
4 6 4 10 6 4 0 4 3 2 8 12 8 10 6 4 0 4 6 1 8 6 8 10 6 4 0 2 6 1
12 16 6 7 5 8 2 0 5 6 12 16 6 14 10 8 4 0 10 6 12 16 12 7 10 8 2 0 5 3
2 6 1 4 0 1 3 10 0 4 2 3 2 4 0 2 3 5 0 2 2 6 2 4 0 2 6 5 0 2
3 10 6 8 4 2 1 6 4 0 6 10 6 4 2 2 1 3 4 0 3 10 6 4 4 2 1 3 4 0
0 4 0 1 2 2 4 6 2 6 0 4 0 2 1 4 8 12 1 6 0 4 0 2 2 4 4 6 2 6
4 0 4 1 3 8 6 8 6 5 4 0 4 2 6 4 12 16 3 10 4 0 4 2 3 8 12 16 3 5
0 4 0 2 2 2 8 12 2 3 0 2 0 1 1 4 8 12 2 6 0 2 0 2 2 4 8 6 2 6
2 1 2 0 4 3 10 14 2 8 1 2 2 0 2 6 10 14 2 8 1 2 2 0 4 3 10 14 2 8
1 6 1 4 0 2 6 5 0 4 1 6 2 2 0 2 6 10 0 4 1 6 1 4 0 1 3 10 0 4
4 8 2 6 1 0 4 4 2 2 4 8 4 6 1 0 4 8 1 2 2 8 2 6 2 0 4 4 2 2
8 6 8 5 6 2 0 4 6 2 4 12 8 10 6 2 0 4 6 2 8 12 4 10 6 2 0 4 3 2
12 16 12 7 10 4 4 0 10 3 12 16 12 7 10 8 4 0 10 6 12 8 6 14 10 4 2 0 10 6
1 6 2 4 0 2 3 5 0 4 2 3 2 4 0 1 6 15 0 4 1 3 2 4 0 1 3 10 0 2
6 10 6 4 4 2 2 3 4 0 3 10 3 8 2 3 3 6 4 0 6 10 3 4 2 2 2 3 4 0

```

For $k(12)$, $z_{\text{opt}}(3809)$ density(0.889655)

Shortest Path Graph:

```

0 2 1 2 2 2 2 2 2 2 1 1 2 2 1 2 1 1 2 1 1 1 2 2 2 2 1 2 2 1
2 0 2 2 2 1 2 2 2 1 2 2 1 1 2 2 2 1 1 1 2 2 2 1 1 2 2 1 2 1
1 1 0 2 2 1 2 1 1 1 1 2 2 1 2 2 2 1 2 1 1 2 2 2 2 2 1 1 2
1 1 1 0 2 2 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 1 2 1 2 2 2 1 1
2 2 2 1 0 1 2 2 2 1 2 2 2 2 1 1 3 1 1 2 2 2 1 2 1 1 1 2 2 2
1 1 2 1 2 0 2 1 1 1 2 2 2 2 2 1 2 2 2 1 2 2 2 2 1 1 2 2 1 2 1
1 2 2 2 2 2 0 2 1 2 2 2 1 1 2 1 2 2 2 2 1 2 2 1 1 2 1 1 2 2
1 1 1 1 2 1 2 0 2 2 2 2 2 2 2 1 2 2 2 1 2 2 2 2 1 1 2 2 2 1 2

```

2 2 2 2 1 2 2 2 0 1 2 2 2 2 2 1 2 2 1 2 2 1 1 1 1 2 1 2 2 2
 2 2 2 1 2 2 1 1 2 0 3 2 1 2 2 2 3 1 2 2 2 2 1 2 2 1 2 1 1 2
 2 2 2 1 1 1 1 2 2 2 0 2 2 1 2 1 2 1 1 2 2 1 2 1 1 2 2 2 2 2
 1 2 1 2 2 1 2 1 2 2 2 0 2 2 2 2 1 1 1 1 1 1 2 2 1 2 1 2 2 2
 2 1 2 2 2 1 1 2 1 1 2 1 0 2 3 1 2 1 2 2 2 1 2 2 2 1 2 2 2 1
 2 2 2 1 1 2 2 2 2 1 1 1 1 0 1 2 2 2 2 1 1 2 1 1 2 2 2 2 2 1
 1 2 2 2 1 2 2 2 1 2 2 2 2 2 0 2 2 2 1 1 2 1 1 2 1 2 1 2 2 2
 2 1 2 2 2 1 2 1 1 1 2 1 1 2 2 0 2 1 2 1 2 1 1 2 2 1 2 2 2 2
 2 2 2 2 2 2 2 2 2 1 2 1 2 1 1 2 0 2 1 2 2 1 2 1 1 2 1 2 2 2
 2 2 1 2 2 1 2 2 2 2 2 2 1 1 2 2 2 0 2 1 1 2 2 1 1 1 3 2 2 1
 2 2 2 2 2 1 2 2 2 1 1 2 2 1 2 1 2 1 0 2 2 1 2 1 2 2 1 2 1 2
 1 2 2 1 2 2 2 1 2 2 2 2 1 1 1 1 2 2 1 0 2 1 2 1 2 2 2 2 2 2
 2 1 1 1 2 2 1 1 2 1 1 2 2 2 1 1 2 2 2 1 0 1 2 2 2 2 2 2 2 1
 2 1 1 2 2 2 1 1 2 2 1 2 2 1 2 2 1 2 2 2 1 0 1 2 2 1 2 2 1 2
 2 2 1 1 2 1 2 1 1 1 2 2 1 2 3 2 2 2 1 2 2 1 0 1 2 1 2 2 2 2
 2 2 2 2 3 2 1 1 2 2 2 1 2 2 2 2 1 2 2 1 1 1 1 0 1 2 2 1 2 1
 1 2 2 1 3 2 2 2 2 1 2 2 2 2 2 1 2 1 1 1 2 2 1 2 0 2 2 1 2 2
 2 2 3 2 2 1 3 2 1 2 2 2 2 2 2 1 3 2 1 1 2 2 2 2 1 0 2 1 2 1
 1 2 2 2 1 1 1 2 1 1 2 1 2 2 1 2 2 2 2 2 2 2 1 2 2 2 0 1 2 2
 1 1 2 2 2 2 2 2 2 2 2 2 1 2 2 1 2 2 1 2 1 1 2 2 2 2 2 0 1 2
 1 2 2 2 1 2 1 2 2 2 2 1 2 1 1 2 2 2 2 2 1 2 2 2 2 1 1 1 0 1
 2 1 2 1 2 1 2 2 2 2 2 2 2 1 2 2 3 2 1 2 2 2 1 2 1 2 2 1 1 0

Obtained Capacity Matrix:

0 4 0 2 2 4 8 12 2 6 0 2 0 2 1 4 4 6 2 3 0 2 0 2 2 4 4 12 2 3
 4 0 4 2 6 4 12 16 6 5 4 0 2 1 6 8 12 8 3 5 4 0 4 1 3 8 12 8 6 5
 0 2 0 2 2 2 8 6 1 3 0 4 0 1 2 4 8 6 2 3 0 4 0 2 2 4 8 6 1 6
 1 1 1 0 4 6 5 7 2 4 2 2 2 0 4 6 10 14 4 8 2 2 1 0 2 6 10 14 2 4
 2 6 2 2 0 1 6 10 0 2 2 6 2 4 0 1 9 5 0 4 2 6 1 4 0 1 3 10 0 4
 2 4 4 3 2 0 4 4 1 1 4 8 4 6 2 0 4 8 1 2 4 8 4 3 1 0 4 4 2 1
 4 12 8 10 6 4 0 4 3 2 8 12 4 5 6 2 0 4 6 2 4 12 8 5 3 4 0 2 6 2
 6 8 6 7 10 4 4 0 10 6 12 16 12 14 10 4 4 0 5 6 12 16 12 7 5 8 4 0 5 6
 2 6 2 4 0 2 6 10 0 2 2 6 2 4 0 1 6 10 0 4 2 3 1 2 0 2 3 10 0 4
 6 10 6 4 4 2 1 3 4 0 9 10 3 8 4 2 3 3 4 0 6 10 3 8 4 1 2 3 2 0
 0 4 0 1 1 2 4 12 2 6 0 4 0 1 2 2 8 6 1 6 0 2 0 1 1 4 8 12 2 6
 2 0 2 2 6 4 12 8 6 10 4 0 4 2 6 8 6 8 3 5 2 0 4 2 3 8 6 16 6 10
 0 2 0 2 2 2 4 12 1 3 0 2 0 2 3 2 8 6 2 6 0 2 0 2 2 2 8 12 2 3
 2 2 2 0 2 6 10 14 4 4 1 1 1 0 2 6 10 14 4 4 1 2 1 0 4 6 10 14 4 4
 1 6 2 4 0 2 6 10 0 4 2 6 2 4 0 2 6 10 0 2 2 3 1 4 0 2 3 10 0 4
 4 4 4 6 2 0 4 4 1 1 4 4 2 6 2 0 4 4 2 1 4 4 2 6 2 0 4 8 2 2

```

8 12 8 10 6 4 0 4 6 1 8 6 8 5 3 4 0 4 3 2 8 6 8 5 3 4 0 4 6 2
12 16 6 14 10 4 4 0 10 6 12 16 6 7 10 8 4 0 10 3 6 16 12 7 5 4 6 0 10 3
2 6 2 4 0 1 6 10 0 2 1 6 2 2 0 1 6 5 0 4 2 3 2 2 0 2 3 10 0 4
3 10 6 4 4 2 2 3 4 0 6 10 3 4 2 1 2 6 2 0 6 5 6 4 4 2 2 6 4 0
0 2 0 1 2 4 4 6 2 3 0 4 0 2 1 2 8 12 2 3 0 2 0 2 2 4 8 12 2 3
4 0 2 2 6 8 6 8 6 10 2 0 4 1 6 8 6 16 6 10 2 0 2 2 6 4 12 16 3 10
0 4 0 1 2 2 8 6 1 3 0 4 0 2 3 4 8 12 1 6 0 2 0 1 2 2 8 12 2 6
2 2 2 0 6 6 5 7 4 8 2 1 2 0 4 6 5 14 4 4 1 1 1 0 2 6 10 7 4 4
1 6 2 2 0 2 6 10 0 2 2 6 2 4 0 1 6 5 0 2 2 6 1 4 0 2 6 5 0 4
4 8 6 6 2 0 6 8 1 2 4 8 4 6 2 0 6 8 1 1 4 8 4 6 1 0 4 4 2 1
4 12 8 10 3 2 0 4 3 1 8 6 8 10 3 4 0 4 6 2 8 12 4 10 6 4 0 2 6 2
6 8 12 14 10 8 4 0 10 6 12 16 6 14 10 4 4 0 5 6 6 8 12 14 10 8 4 0 5 6
1 6 2 4 0 2 3 10 0 4 2 3 2 2 0 2 6 10 0 4 1 6 2 4 0 1 3 5 0 2
6 5 6 4 4 1 2 6 4 0 6 10 6 4 4 2 3 6 2 0 6 10 3 8 2 2 2 3 2 0

```

For $k(13)$, $z_{\text{opt}}(3769)$ density(0.889655)

Shortest Path Graph:

```

0 2 2 2 2 2 2 2 2 2 1 1 2 1 1 2 2 2 2 1 2 1 2 2 2 1 1 1 1 2
2 0 2 2 2 2 2 2 1 2 1 1 2 1 1 1 1 2 2 2 2 2 2 2 1 1 2 1 2 1
2 1 0 1 1 2 2 2 2 1 1 2 1 2 1 2 2 1 1 2 2 2 2 2 1 2 2 2 2 2
1 2 1 0 2 2 3 1 2 2 2 2 2 2 2 1 1 1 1 1 2 1 2 2 2 2 2 1 1 2
2 2 2 2 0 2 1 2 2 1 1 1 2 2 1 1 2 1 2 1 2 2 2 2 2 1 2 2 2 2
2 1 1 2 1 0 1 2 2 2 1 2 2 2 2 2 2 1 2 2 1 1 1 1 1 2 2 1 2 1
1 2 1 2 2 1 0 1 1 2 2 2 2 2 1 2 2 2 1 2 1 2 1 2 2 2 2 2 2 1
1 2 2 1 2 2 3 0 1 2 2 1 2 2 1 2 2 1 2 1 1 2 1 1 2 1 2 1 1 2
2 2 2 1 1 2 2 2 0 1 2 2 1 2 2 2 2 1 2 1 2 2 2 2 1 1 2 1 1 2
1 1 1 2 2 2 2 2 1 0 2 2 2 1 2 2 1 1 2 1 1 2 2 2 2 2 1 1 2 2
2 2 1 1 2 1 2 2 1 2 0 2 2 2 2 2 2 2 2 1 1 1 1 1 2 2 2 1 1 2
2 2 2 1 2 1 2 1 1 2 2 0 2 1 1 2 1 2 1 2 1 1 2 2 2 2 1 2 1 2
2 2 1 2 2 2 1 1 2 2 2 1 0 1 1 2 2 1 2 2 1 1 1 2 2 1 2 1 2 1
1 2 2 2 2 2 1 2 1 2 2 2 1 0 2 2 1 1 1 2 2 2 1 2 2 1 2 1 1 2
1 1 1 2 1 2 2 2 2 1 2 2 1 1 0 2 1 2 1 2 2 1 2 2 2 1 2 1 1 2
2 1 1 2 2 1 2 1 2 2 2 2 2 1 1 0 2 2 1 2 2 1 2 1 2 1 2 1 2 1
2 1 2 1 1 2 2 1 1 2 2 1 2 1 2 2 0 2 1 1 2 1 1 2 2 2 2 2 2 1
2 2 2 1 2 2 2 1 1 2 1 1 2 2 1 1 1 0 1 1 2 2 2 2 2 2 1 2 2 1
2 1 1 2 1 2 2 1 2 2 2 2 2 2 2 2 2 1 1 0 1 1 2 1 2 2 2 1 1 1 2
2 1 2 2 2 2 2 2 2 2 2 1 1 2 1 2 1 2 2 1 0 2 2 1 1 1 2 1 2 2 1
1 2 2 1 2 2 3 1 2 2 2 2 2 2 2 1 2 2 1 2 1 0 1 1 2 2 1 2 1 1 2
2 1 2 2 1 2 2 2 1 2 2 2 1 1 1 2 1 1 2 2 2 0 2 2 2 2 2 1 2 1

```

2 2 2 1 1 1 2 1 1 2 1 2 2 1 1 1 2 2 2 2 2 2 0 2 1 2 1 2 1 2
 2 2 2 2 1 1 2 2 2 2 1 2 1 2 1 2 2 2 2 1 1 1 2 0 1 2 1 2 2 1
 2 1 1 2 2 2 2 2 1 1 1 2 1 1 1 2 2 1 2 2 2 2 2 2 0 2 2 2 2 2
 2 2 2 2 1 2 2 2 1 2 2 1 1 2 2 1 2 2 2 2 1 2 2 1 2 0 2 1 1 1
 2 1 1 1 2 2 2 1 2 2 2 1 1 2 2 2 2 2 1 2 2 2 1 1 2 2 0 1 2 2
 1 1 2 2 2 1 2 1 1 1 1 2 2 1 1 2 2 2 2 2 1 2 2 1 2 2 2 0 2 1
 2 2 1 2 2 2 2 1 1 1 1 2 1 1 2 1 2 2 1 2 1 2 1 2 2 2 1 2 0 1
 2 1 2 2 2 2 1 2 1 2 2 2 2 1 2 2 2 1 1 2 2 1 1 1 2 2 2 1 2 0

Obtained Capacity Matrix:

0 4 0 2 2 4 8 12 2 6 0 2 0 1 1 4 8 12 2 3 0 2 0 2 2 2 4 6 1 6
 4 0 4 2 6 8 12 16 3 10 2 0 4 1 3 4 6 16 6 10 4 0 4 2 3 4 12 8 6 5
 0 2 0 1 1 4 8 12 2 3 0 4 0 2 1 4 8 6 1 6 0 4 0 2 1 4 8 12 2 6
 1 2 1 0 4 6 15 7 4 8 2 2 2 0 4 3 5 7 2 4 2 1 2 0 4 6 10 7 2 8
 2 6 2 4 0 2 3 10 0 2 1 3 2 4 0 1 6 5 0 2 2 6 2 4 0 1 6 10 0 4
 4 4 2 6 1 0 2 8 2 2 2 8 4 6 2 0 4 4 2 2 2 4 2 3 1 0 4 4 2 1
 4 12 4 10 6 2 0 2 3 2 8 12 8 10 3 4 0 4 3 2 4 12 4 10 6 4 0 4 6 1
 6 16 12 7 10 8 6 0 5 6 12 8 12 14 5 8 4 0 10 3 6 16 6 7 10 4 4 0 5 6
 2 6 2 2 0 2 6 10 0 2 2 6 1 4 0 2 6 5 0 2 2 6 2 4 0 1 6 5 0 4
 3 5 3 8 4 2 2 6 2 0 6 10 6 4 4 2 1 3 4 0 3 10 6 8 4 2 1 3 4 0
 0 4 0 1 2 2 8 12 1 6 0 4 0 2 2 4 8 12 1 3 0 2 0 1 2 4 8 6 1 6
 4 0 4 1 6 4 12 8 3 10 4 0 4 1 3 8 6 16 3 10 2 0 4 2 6 8 6 16 3 10
 0 4 0 2 2 4 4 6 2 6 0 2 0 1 1 4 8 6 2 6 0 2 0 2 2 2 8 6 2 3
 1 2 2 0 4 6 5 14 2 8 2 2 1 0 4 6 5 7 2 8 2 2 1 0 4 3 10 7 2 8
 1 3 1 4 0 2 6 10 0 2 2 6 1 2 0 2 3 10 0 4 2 3 2 4 0 1 6 5 0 4
 4 4 2 6 2 0 4 4 2 2 4 8 4 3 1 0 4 8 1 2 4 4 4 3 2 0 4 4 2 1
 8 6 8 5 3 4 0 2 3 2 8 6 8 5 6 4 0 4 3 1 8 6 4 10 6 4 0 4 6 1
 12 16 12 7 10 8 4 0 5 6 6 8 12 14 5 4 2 0 5 3 12 16 12 14 10 8 2 0 10 3
 2 3 1 4 0 2 6 5 0 4 2 6 2 4 0 2 3 5 0 2 1 6 1 4 0 2 3 5 0 4
 6 5 6 8 4 2 2 6 4 0 3 5 6 4 4 1 2 6 2 0 6 10 3 4 2 2 1 6 4 0
 0 4 0 1 2 4 12 6 2 6 0 4 0 2 1 4 8 6 2 3 0 2 0 2 2 2 8 6 1 6
 4 0 4 2 3 8 12 16 3 10 4 0 2 1 3 8 6 8 6 10 4 0 4 2 6 8 12 8 6 5
 0 4 0 1 1 2 8 6 1 6 0 4 0 1 1 2 8 12 2 6 0 4 0 2 1 4 4 12 1 6
 2 2 2 0 2 3 10 14 4 8 1 2 1 0 2 6 10 14 4 4 1 1 2 0 2 6 5 14 4 4
 2 3 1 4 0 2 6 10 0 2 1 6 1 2 0 2 6 5 0 4 2 6 2 4 0 2 6 10 0 4
 4 8 4 6 1 0 4 8 1 2 4 4 2 6 2 0 4 8 2 2 2 8 4 3 2 0 4 4 1 1
 8 6 4 5 6 4 0 2 6 2 8 6 4 10 6 4 0 4 3 2 8 12 4 5 6 4 0 2 6 2
 6 8 12 14 10 4 4 0 5 3 6 16 12 7 5 8 4 0 10 6 6 16 12 7 10 8 4 0 10 3
 2 6 1 4 0 2 6 5 0 2 1 6 1 2 0 1 6 10 0 4 1 6 1 4 0 2 3 10 0 2
 6 5 6 8 4 2 1 6 2 0 6 10 6 4 4 2 2 3 2 0 6 5 3 4 4 2 2 3 4 0

For $k(14)$, $z_{\text{opt}}(3683)$ density(0.889655)

Shortest Path Graph:

```

0 2 2 1 1 2 2 1 1 2 1 2 2 1 1 2 2 2 2 2 1 2 1 1 1 1 2 1 2 1
1 0 2 2 1 2 1 2 2 2 2 1 2 1 2 2 2 2 1 2 1 2 1 2 1 1 2 2 1 1
1 2 0 2 1 2 2 1 1 2 2 1 2 2 1 2 2 2 2 1 2 1 1 2 1 2 2 2 2 1
1 2 1 0 2 1 1 1 2 1 1 1 2 1 2 2 2 2 1 2 2 1 2 1 2 2 2 2 1 2
2 1 2 2 0 1 1 2 1 1 2 2 2 1 2 2 1 1 2 2 1 3 2 1 2 1 2 2 1 2
1 1 1 1 1 0 1 1 2 2 2 2 2 2 2 2 1 2 2 2 2 2 1 2 2 1 2 1 2 2
1 1 1 2 2 1 0 2 2 1 2 2 1 1 2 2 1 2 1 2 2 2 2 1 2 2 1 1 2 2
2 2 2 1 2 1 2 0 2 2 2 2 2 2 2 1 1 1 2 2 2 2 2 1 1 1 2 1 2 1
2 2 1 1 1 2 1 1 0 2 1 1 2 1 2 2 2 2 2 2 1 2 2 2 2 1 2 1 2 1
2 2 2 2 2 2 1 2 2 0 1 1 1 2 2 2 2 1 1 1 1 2 1 2 1 2 1 2 1 1
1 1 2 1 2 2 1 2 2 2 0 1 2 2 2 2 1 2 1 1 2 2 2 2 1 2 1 1 1 2
2 2 2 2 2 2 1 2 2 2 1 0 2 2 2 2 1 1 2 1 1 1 1 2 2 2 1 2 1 1
2 2 2 2 1 2 2 1 2 2 2 1 0 1 1 1 2 2 1 2 1 2 2 2 1 2 1 1 1 1
1 2 2 1 1 1 2 2 2 2 1 1 3 0 2 1 2 2 2 2 2 2 2 1 2 2 2 1 2 1
1 2 2 1 2 2 1 2 1 1 2 1 2 1 0 2 1 2 2 1 2 1 2 1 2 2 1 2 2 2
2 1 1 1 2 1 2 2 2 1 2 1 2 1 2 0 1 2 1 2 1 2 2 1 2 2 2 2 2 2
2 2 1 2 1 1 1 2 2 2 3 2 1 2 1 2 0 2 2 2 1 2 1 2 2 2 1 2 2 1
2 2 2 1 1 1 2 2 2 2 1 1 1 2 1 2 1 0 1 2 2 2 1 2 1 2 2 1 2 2
1 1 2 2 2 1 1 1 2 2 2 2 1 2 1 1 1 2 0 1 1 2 2 1 2 2 2 2 2 2
2 1 1 2 1 2 1 1 1 2 2 2 1 2 2 1 2 1 1 0 1 2 1 2 2 2 2 2 2 2
1 1 2 1 2 1 2 2 2 2 2 2 2 2 2 1 1 2 1 1 0 2 2 2 1 2 2 2 1 2
1 1 2 2 2 2 2 2 2 2 2 2 1 2 1 2 2 2 1 1 1 0 1 2 2 2 2 1 1 1
2 2 1 2 2 2 1 2 2 2 2 2 2 2 2 1 2 1 1 1 1 2 0 1 1 2 1 1 2 2
1 1 2 1 2 2 1 1 2 1 2 2 1 2 1 2 2 2 2 2 2 2 0 2 1 1 1 2 1
2 1 2 1 1 2 2 2 2 1 2 1 2 2 2 2 1 1 1 2 2 2 2 0 2 2 2 1 1
1 2 2 1 2 2 2 1 2 1 1 1 2 1 2 2 2 1 2 2 1 2 2 2 1 0 2 1 1 1
2 1 2 1 1 1 1 2 2 2 2 2 2 2 2 2 1 2 2 1 2 1 2 2 1 2 0 1 2 1
1 1 2 2 2 2 2 2 1 1 2 2 2 2 2 2 1 2 2 1 1 3 1 1 1 1 2 0 2 2
2 1 2 2 1 2 2 2 2 1 1 1 2 2 2 1 2 2 1 1 2 2 2 1 2 2 1 1 0 1
2 2 1 2 1 1 2 1 1 1 2 2 2 1 2 1 2 2 2 2 1 2 2 2 2 1 1 2 1 0

```

Obtained Capacity Matrix:

```

0 4 0 1 1 4 8 6 1 6 0 4 0 1 1 4 8 12 2 6 0 4 0 1 1 2 8 6 2 3
2 0 4 2 3 8 6 16 6 10 4 0 4 1 6 8 12 16 3 10 2 0 2 2 3 4 12 16 3 5
0 4 0 2 1 4 8 6 1 6 0 2 0 2 1 4 8 12 2 3 0 2 0 2 1 4 8 12 2 3
1 2 1 0 4 3 5 7 4 4 1 1 2 0 4 6 10 14 2 8 2 1 2 0 4 6 10 14 2 8

```

2 3 2 4 0 1 3 10 0 2 2 6 2 2 0 2 3 5 0 4 1 9 2 2 0 1 6 10 0 4
 2 4 2 3 1 0 2 4 2 2 4 8 4 6 2 0 2 8 2 2 4 8 2 6 2 0 4 4 2 2
 4 6 4 10 6 2 0 4 6 1 8 12 4 5 6 4 0 4 3 2 8 12 8 5 6 4 0 2 6 2
 12 16 12 7 10 4 4 0 10 6 12 16 12 14 5 4 2 0 10 6 12 16 6 7 5 8 2 0 5 3
 2 6 1 2 0 2 3 5 0 4 1 3 2 2 0 2 6 10 0 4 1 6 2 4 0 1 6 5 0 2
 6 10 6 8 4 2 1 6 4 0 3 5 3 8 4 2 2 3 2 0 3 10 3 8 2 2 1 6 2 0
 0 2 0 1 2 4 4 12 2 6 0 2 0 2 2 4 4 12 1 3 0 4 0 2 1 4 4 6 1 6
 4 0 4 2 6 8 6 16 6 10 2 0 4 2 6 8 6 8 6 5 2 0 2 2 6 8 6 16 3 5
 0 4 0 2 1 4 8 6 2 6 0 2 0 1 1 2 8 12 1 6 0 4 0 2 1 4 4 6 1 3
 1 2 2 0 2 3 10 14 4 8 1 1 3 0 4 3 10 14 4 8 2 2 1 0 4 6 5 14 2 4
 1 6 2 2 0 2 3 10 0 2 2 3 2 2 0 2 3 10 0 2 2 3 2 2 0 2 3 10 0 4
 4 4 2 3 2 0 4 8 2 1 4 4 4 3 2 0 2 8 1 2 2 8 4 3 2 0 4 8 2 2
 8 12 4 10 3 2 0 4 6 2 12 12 4 10 3 4 0 4 6 2 4 12 4 10 6 4 0 4 6 1
 12 16 12 7 5 4 4 0 10 6 6 8 6 14 5 8 2 0 5 6 12 16 6 14 5 8 4 0 10 6
 1 3 2 4 0 1 3 5 0 4 2 6 1 4 0 1 3 10 0 2 1 6 2 2 0 2 6 10 0 4
 6 5 3 8 2 2 1 3 2 0 6 10 3 8 4 1 2 3 2 0 3 10 3 8 4 2 2 6 4 0
 0 2 0 1 2 2 8 12 2 6 0 4 0 2 2 2 4 12 1 3 0 4 0 2 1 4 8 12 1 6
 2 0 4 2 6 8 12 16 6 10 4 0 4 1 6 8 12 8 3 5 2 0 2 2 6 8 12 8 3 5
 0 4 0 2 2 4 4 12 2 6 0 4 0 2 1 4 4 6 1 3 0 4 0 1 1 4 4 6 2 6
 1 1 2 0 4 6 5 7 4 4 2 2 1 0 2 6 10 14 4 8 2 2 2 0 4 3 5 7 4 4
 2 3 2 2 0 2 6 10 0 2 2 3 2 4 0 2 3 5 0 4 2 6 2 4 0 2 6 10 0 2
 2 8 4 3 2 0 4 4 2 1 2 4 4 3 2 0 4 4 2 2 2 8 4 6 1 0 4 4 1 1
 8 6 8 5 3 2 0 4 6 2 8 12 8 10 6 4 0 4 6 1 8 6 8 10 3 4 0 2 6 1
 6 8 12 14 10 8 4 0 5 3 12 16 12 14 10 8 2 0 10 3 6 24 6 7 5 4 4 0 10 6
 2 3 2 4 0 2 6 10 0 2 1 3 2 4 0 1 6 10 0 2 2 6 2 2 0 2 3 5 0 2
 6 10 3 8 2 1 2 3 2 0 6 10 6 4 4 1 2 6 4 0 3 10 6 8 4 1 1 6 2 0

For $k(15)$, $z_{\text{opt}}(3704)$ density(0.889655)