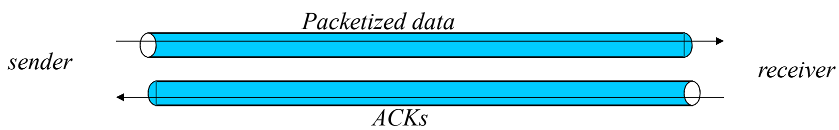
Analysis of a simple communication system

- Calculation of maximum system utilization



- Parameters:
  - D - number of data bits in a packet/frame
  - H - number of bits in a packet header
  - F - total number of bits in a packet/frame; $F=D+H$
  - C - channel capacity [bps]
  - A - number of bits in an ACK

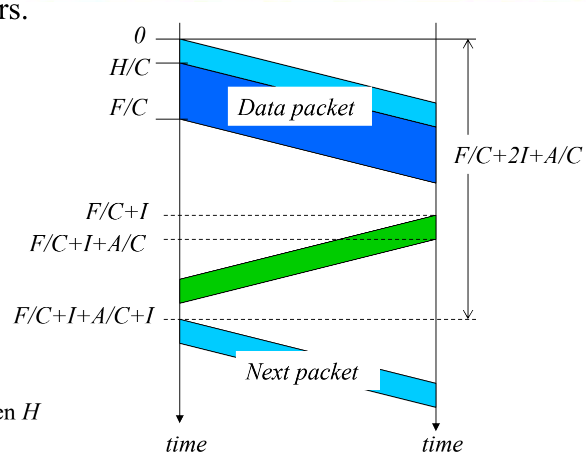
Analysis of a simple communication system (con't)

- Parameters (con't)
  - E - probability of a bit being in error
  - I - processing + queueing + propagation delays
  - L - probability that a packet or its ACK is lost
  - P_1 - probability that a data packet is lost
  - P_2 - probability that an ACK is lost
  - R - mean number of retransmission per data packet
  - T - Timeout interval
  - U - channel utilization
  - W - window size

Analysis of a simple communication system (con't)

- First assume no errors.

$$U_{no-errors} = \frac{D/C}{F/C + 2I + A/C} = \frac{D}{(H+D) + 2CI + A}.$$



High utilization is achieved when H and A are small and IC is small.

Analysis of a simple communication system (con't)

- Now assume transmission errors.
- The effect of errors: for each damaged frame, the sender times out T [sec] after the transmission was completed and retransmits the lost frame.
- Therefore, each unsuccessful transmission "uses" $F+CT$ [bits] of link capacity.
- If the number of retransmissions is R, the channel capacity used until successful transmission is: $R(F+CT)+(F+A+2CI)$
- To calculate the mean number of retransmissions, note that both, the data packet and its ACK, have to be correctly received; i.e., probability of success is: $(1- P_1)(1- P_2)$.

Analysis of a simple communication system (con't)

- The probability of failure is: $L=1-(1-P_1)(1-P_2)$ and the probability of k attempts is: $(1-L)L^{k-1}$.
- Expected number of transmissions per packet is: $\frac{1}{1-L}$ and the expected number of retransmissions is:

$$R=\frac{1}{1-L}-1=\frac{L}{1-L}.$$

- Thus,

$$U_{errors}=\frac{D}{\left(\frac{L}{1-L}\right)(F+CT)+(F+2CI+A)}.$$

- Assuming low variance in I, set $T=A/C+2I$, then

Analysis of a simple communication system (con't)

- Assuming low variance in

$$U_{errors}=\underbrace{\left(\frac{D}{D+H}\right)}_{\text{header overhead}}\underbrace{(1-P_1)}_{\text{loss due to packet errors}}\underbrace{(1-P_2)}_{\text{loss due to ACK errors}}\cdot\underbrace{\frac{1}{1+\frac{CT}{H+D}}}_{\text{loss due to partially filled pipe}}.$$

- If we call $a=2\frac{I}{F/C}$ and $A\approx 0$, then

$$\boxed{U_{errors}=\left(\frac{D}{F}\right)(1-P_1)(1-P_2)\cdot\frac{1}{1+a}.}$$

Analysis of a simple communication system (con't)

- Note that *a* is the *round trip time* in units of frame transmission time. It is a very important parameter in data link performance evaluation.
- Assuming independent errors,

$$U_{errors} = \left(\frac{D}{F}\right)(1-E)^{F+A} \cdot \frac{1}{1+a}.$$

- Short frames $\rightarrow$ low efficiency due to header overhead
- Large frames $\rightarrow$ large probability of a frame being in error (many retransmissions of large amount of data each)

Analysis of a simple communication system (con't)

- Optimal packet size:

$$\frac{dU_{errors}}{dD} = 0 \Rightarrow D^2. + D(H+CT) + \frac{H+CT}{\ln(1-E)} = 0;$$

$$D_{opt} = \frac{H+CT}{2}\left[\sqrt{1-\frac{4}{(H+CT)\ln(1-E)}} - 1\right];$$

$$E << 1 \Rightarrow \ln(1-E) \approx -E$$

$$E << 1 \Rightarrow \sqrt{1+x/E} - 1 \approx \sqrt{x/E};$$

$$\boxed{D_{opt} = \sqrt{\frac{H+CT}{E}} \approx \sqrt{\frac{CT}{E}}.}$$

Analysis of a simple communication system (con't)

- Note that $\lim_{E \to 0} D_{opt} = \infty$.
- Errors are not random. Experimentally, $P_1 = k(H+D)^{\alpha}$ provides a better fit than $P_1 = 1-(1-E)^{H+D}$.
- Example,

$$C = 1.5[Mbps]$$
$$T = 10[m\sec]$$
$$E = 10^{-6}$$
$$D_{opt} = \sqrt{\frac{1.5 \cdot 10^6 \cdot 10 \cdot 10^{-3}}{10^{-6}}} = 120[Kb] = 15[KByte].$$

The choice of packet size

- Reasons for large packet size:
  - lower "per bit" overhead
  - smaller packet processing rate
  - smaller assemble/reassemble time
- Reasons for large packet size:
  - smaller effect of errors
  - smaller delay jitter
  - smaller buffer size
  - smaller maximum delay
  - smaller acquisition delay (ATM; need for echo cancellers)
  - smaller "fill"