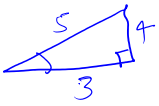
Q9

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

2. ii) $\sin^{-1}\frac{4}{5} + \cos^{-1}\frac{2}{\sqrt{5}} = \tan^{-1}\frac{11}{2}$

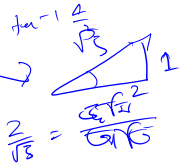
R.H.S =



$\frac{4}{5} = \sin\theta$

$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{\frac{4}{3}+\frac{1}{2}}{1-\frac{4}{3}\times\frac{1}{2}}$



$= \tan^{-1}\frac{11}{2}$

4. 2 $\tan(2\tan^{-1}x) = 2\tan(\tan^{-1}x + \tan^{-1}x^3)$

$= L.H.S = \tan(2\tan^{-1}x)$ $\quad \tan^{-1}x\frac{2x}{1-x^2}$

$= \tan\left(\tan^{-1}\frac{2x}{1-x^2}\right)$

$= \frac{2x}{1-x^2}$

R.H.S $= 2\tan(\tan^{-1}x + \tan^{-1}x^3)$

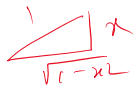
$= 2\tan\left(\tan^{-1}\frac{x+x^3}{1-x\cdot x^3}\right)$

$= 2\cdot\frac{x+x^3}{1-(x^2)^2}$

$= \frac{2x(1+x^2)}{(1-x^2)(1+x^2)}$

$= \frac{2x}{1-x^2}$

10. iii) $\cos\tan^{-1}\cot\sin^{-1}x = x$



L.H.S = $\cos\tan^{-1}\cot\sin^{-1}\frac{x}{1}$

$L.H.S = \cos\tan^{-1}\cot\sin^{-1}\frac{x}{1}$ $\quad \sqrt{1-x^2}$

$= \cos\tan^{-1}\cot\left(\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)$

$= \cos\tan^{-1}\frac{\sqrt{1-x^2}}{x} \rightarrow$

$= \cos\cos^{-1}\frac{x}{1}$

$= x$ $\qquad \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{(1-x^2)(1-y^2)}\right)$

13. v) $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \theta$

$\Rightarrow \cos^{-1}\left(\frac{x}{a}\cdot\frac{y}{b} - \sqrt{\left(1-\left(\frac{x}{a}\right)^2\right)\left(1-\left(\frac{y}{b}\right)^2\right)}\right) = \theta$

$\Rightarrow \frac{xy}{ab} - \sqrt{\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)} = \cos\theta$

$\Rightarrow \left(\frac{xy}{ab} - \cos\theta\right)^2 = \left(\sqrt{\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)}\right)^2$

$\Rightarrow \frac{x^2y^2}{a^2b^2} + \cos^2\theta - 2\frac{xy}{ab}\cos\theta = \left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$

$= 1 - \frac{x^2}{a^2} + \frac{x^2y^2}{a^2b^2} - \frac{y^2}{b^2}$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy\cos\theta}{ab} = 1 - \cos^2\theta$

$\downarrow$

$\sin^2\theta$

3. ii) $1 + \sqrt{3}\,i$ $\qquad x = 1$, $y = \sqrt{3}$

মডুলাস, $r = \sqrt{x^2 + (y)^2}$

$x^2 \rightarrow$ বাস্তব

$y \rightarrow$ অবাস্তব

প্রধান
আর্গুমেন্ট

$= \sqrt{1^2+(\sqrt{3})^2} = \sqrt{4} = 2$

আর্গুমেন্ট $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$

ω হলো 1 এর ঘনমূল (জটিল)

$\sqrt[3]{1} = 1, \omega, \omega^2$

$x = \sqrt[3]{1}$

$\Rightarrow x^3 = 1$

$\Rightarrow x^3 - 1 = 0$

$\Rightarrow x^3 - 1^3 = 0$

$\Rightarrow (x-1)(x^2 + x.1 + 1^2) = 0$

$x - 1 = 0$

$x = 1$

$x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{3}i}{2}$

$x = 1$

$x = \frac{-1 \pm \sqrt{3}i}{2}$

$= \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$

$\omega \quad , \quad \omega^2$

$1 + \omega + \omega^2 = 0$

$\omega^3 = 1$

$\omega^6 = \omega^3 \cdot \omega^3 = 1 \cdot 1$

$\omega^5 = \omega^3 \cdot \omega^2 = 1 \cdot \omega^2 = \omega^2$ (common 2)

$\omega^4 = \omega^3 \cdot \omega = \omega^1$ (common 1)

12. iv) $(1-\omega^2)(1-\omega^4)(1-\omega^8)(1-\omega^{10})$

$= (1-\omega^2)(1-\omega)(1-\omega^2)(1-\omega)$

$= (1-\omega)^2(1-\omega^2)^2$

$= ((1-\omega)(1-\omega^2))^2$

$= (1-\omega-\omega^2+\omega^3)^2$

$= (1-\omega^2+1)^2$

$= (1 - (\omega + \omega^2) + 1)^2$

$= (4 - (-1) + 1)^3$

$= (3)^2$

$= 9$

$1 + \omega + \omega^2 = 0$

$\omega + \omega^2 = -1$