# PHL245 Lecture Notes

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## Contents

# §1 Day 1: Course Details, Arguments, and Validity (Jan. 6, 2025)

## §1.1 About this Course

1. Course email
2. Quercus
3. Lectures on Mondays and Wednesdays
4. Office hours on Thursdays!

First half of the course will be sentential logic, while the second half will be about predicate logic. This course is actually hybrid! For this reason, **sometimes the lecture on Wednesday won't happen**. When sending emails to the course email, include LEC 5101 and your Student Number. Assignments are due at 11:59PM on Sundays. Refer to the syllabus for the latest details.

## §1.2 Arguments and Validity

Arguments are sequences of statements, which are either true (T) or false (F). Such statements consists of 2 parts:

1. Premise
2. Conclusion

If the premise makes it more likely that the conclusion is true, it is an *inductive* argument. If the premise is best explained by the conclusion, it is an *abductive* argument. If the premise necessitates the conclusion, it is a *deductive* argument, which logically guarantees the conclusion. Such an argument is 'good' if and only if it is **valid** and **sound**.

**Definition 1.1** (Validity)**.** It's not possible for all of the premises $P$ to be true and for the conclusion $Q$ to be false.

$$\neg(P \land \neg Q) \iff (\neg P \lor Q) \iff (P \implies Q)$$

**Definition 1.2** (Soundness)**.** An argument is sound if and only if the argument is valid and all of its premises are "actually" true

For the sake of argument, we suppose (or assume by hypothesis) that the premises are true to analyze the statement's validity.

**Definition 1.3** (Logical OR (Disjunction))**.** An infix binary logical connective[1] denoted by $\lor$. The inputs are called **disjuncts**.

| $P$ | $Q$ | $P \lor Q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $T$ |
| $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ |

[1] joins things together (in this case binary indicates 2, unary indicates 1...)

**Definition 1.4** (Logical AND (Conjunction))**.** An infix binary logical connective denoted by $\wedge$. The inputs are called **conjuncts**.

| $P$ | $Q$ | $P \wedge Q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $F$ |

**Definition 1.5** (Negation (NOT))**.** An infix unary logical connective denoted by $\sim$ or $\neg$. The input is called the **negand**.

| $P$ | $\neg P$ |
|---|---|
| $T$ | $F$ |
| $F$ | $T$ |

**Problem 1.6**

Are the following arguments **valid** and **sound**? When isn't the argument sound?

1. If the ground is raining, then the ground is wet.
2. If all humans are mortal, then I (a human) am immortal.
3. If $P \wedge \neg P$, then $Q$.
4. If $(P \vee Q) \wedge (Q \vee R)$, then $P \vee R$,

**Bonus**: Recall the definition of a 'good' argument. What do you notice about the premise(s) and the conclusion(s) of a 'good' argument?

# §2 Day 2: Truth-Functional Connectives & Characteristic Truth-Tables (Jan. 13, 2025)

One of the 0-point simplification questions in Logic 2010 will appear on the test. For the homework problems you have unlimited attempts, and your most recent score is submitted. This is not an invitation to solve by infinite monkey. There are 1 or 2 truth table questions on test 1, which give a statement/argument and asks you to complete the truth table. The table and combinations will be provided.

An argument is a set of premises that support the conclusion deductively. By the way the logical connectives are called **truth-functional**, because they're functions over the set {True, False}.

**Proposition 2.1**

The word *either* won't play a large role in this course. When you see "either-or", it is equivalent to $\vee$.
Note that either $P \wedge Q$ or $R$ is interpreted as $(P \wedge Q) \vee R$. Similarly, $P \wedge$ either $P$ or $Q$ can be interpreted as $P \wedge (Q \vee R)$.
*But* and $\wedge$ are logically indistinguishable.

$$A \text{ but } B \iff A \wedge B$$

To prove that $(P \wedge Q) \vee R$ and $P \wedge (Q \vee R)$ are logically distinct, we only need to provide a single **counterexample**. If $P = F, Q = F, R = T$, the first statement is true, yet the second is false. Another counterexample is $FTT$.

**Theorem 2.2** (De Morgan's Laws)

Steve didn't eat any noodles nor fries ≡ Steve didn't eat noodles **and** Steve didn't eat fries.
Steve didn't eat noodles **and** fries (together) ≡ Steve didn't eat noodles **or** Steve didn't eat fries

$$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$$
$$\neg(P \wedge Q) \iff (\neg P \vee \neg Q)$$

**Definition 2.3** (Implication)**.** A statement of the form $P \implies Q$ is called a conditional, where $P$ is the antecedent and $Q$ is called consequent.

The professor spoke about the implication being transitive, meaning

$$(A \implies B \wedge B \implies C) \implies (A \implies C)$$

**Definition 2.4** (Biconditional)**.** A statement of the form $P \iff Q$ is called a biconditional. $P$ and $Q$ are called the left and right constituents. We say $P$ if and only if $Q$.

Breaking down "$P$ if and only if $Q$", we have ($P$ if $Q \vee P$ only if $Q$), equivalent to:

$$(Q \implies P) \wedge (P \implies Q)$$
$$(\neg Q \vee P) \wedge (\neg P \vee Q)$$
$$((\neg Q \vee P) \wedge \neg P) \vee ((\neg Q \vee P) \wedge \neg Q)$$

The biconditional has the following truth table:

| $P$ | $Q$ | $P \iff Q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $T$ |

Beware of stylistic english variants! There won't be any surprises on the test however.

## §2.1 The "Formal" Language

### The Components

1. Sentence Letters: $P, Q, R, P_1 \ldots$
2. Connectives: $\wedge, \vee, \implies, \neg, \iff$
3. Parentheses: ()

### The Rules

1. Every sentence letter is a symbolic sentence.
2. If '$\square$' is a symbolic sentence, so is '$\neg\square$'.
3. If '$\square$' and '$\triangle$' are symbolic sentences, so is '$(\square \wedge \triangle)$'.
4. $\ldots$ every connective $\vee, \implies, \neg, \iff$
5. Nothing else is a symbolic sentence (in official notation).

You **do not** put parentheses over a unary connective. $(\neg P)$ is a nono.
The rule of thumb is that every binary connective must be contained entirely within parentheses, to ensure that the order of operations isn't ambiguous.

### Informal Conventions

1. Outermost parentheses may be omitted.
2. Parentheses always go around $\wedge$ and $\vee$ before $\implies$ and $\iff$.
3. Parentheses are not needed for associative connectives.

The last point is because if $\odot$ is associative, then by definition

$$(A \odot B) \odot C \iff A \odot (B \odot C)$$

Although including the parentheses does differentiate between the two, it doesn't add any new information as the two of them are logically equivalent.

## §2.2 Truth Tables

Truth tables show the conditions that make a statement True or False. Such conditions are called Truth Value Assignments, and a particular TVA is called a **valuation**. They enable us to analyze the **semantic properties** of statements, sets of statements, and arguments.

**Theorem 2.5**

(For fun) Let $n$ be the number of "sentence letters". The associated truth table for that expression will contain $2^n$ rows.

There are $2^n$ distinct True/False sequences of length $n$.

**Definition 2.6** (Tautology)**.** A statement that is $T$ for every truth value assignment.

In the truth table, the entire column for that statement must be $T$.