

FISF130020.01: Introduction to Computer Science

# Lecture 1: Numbers and Computation

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# Outline

- ❖ 1. Course Introduction
- ❖ 2. Binary System
- ❖ 3. Computation
- ❖ 4. In-class Practice

# 1. Course Introduction

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# Instruction Team

- Instructor: Xu, Hui
  - Ph. D. degree from CUHK
  - Research Interests: program analysis, software reliability
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  - Office: Room D6023, X2 Building, Jiangwan Campus
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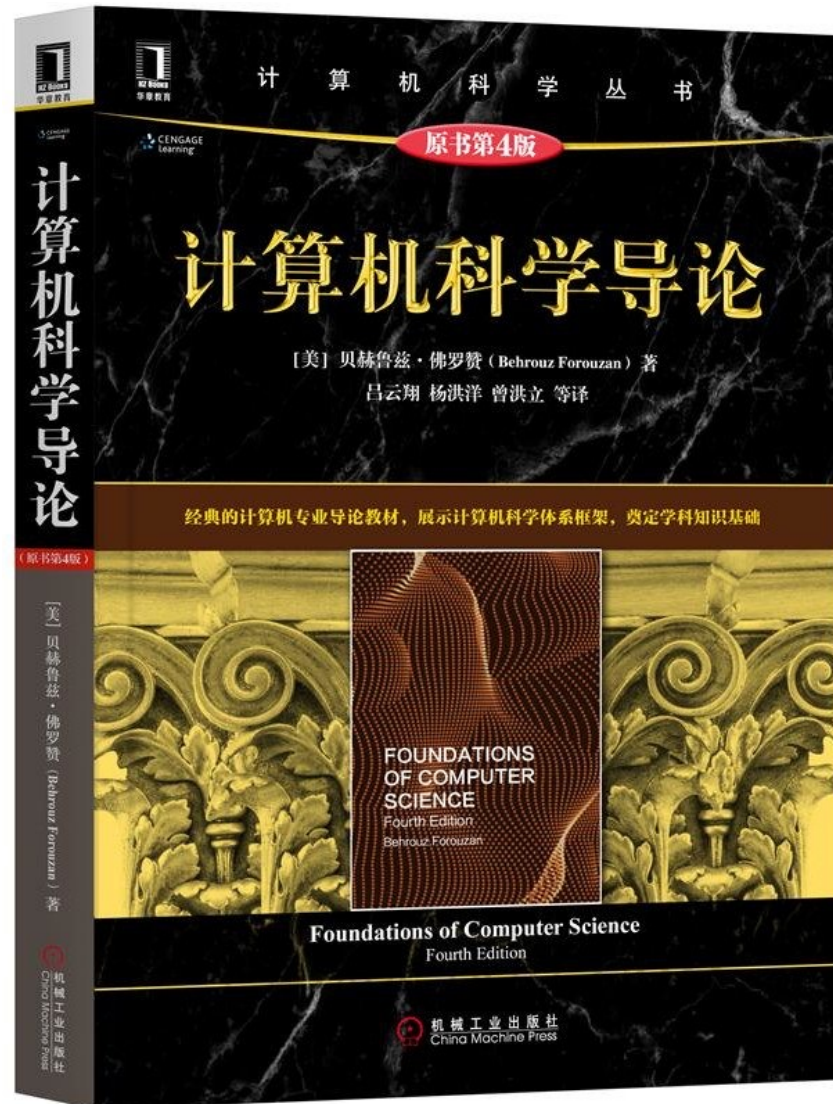
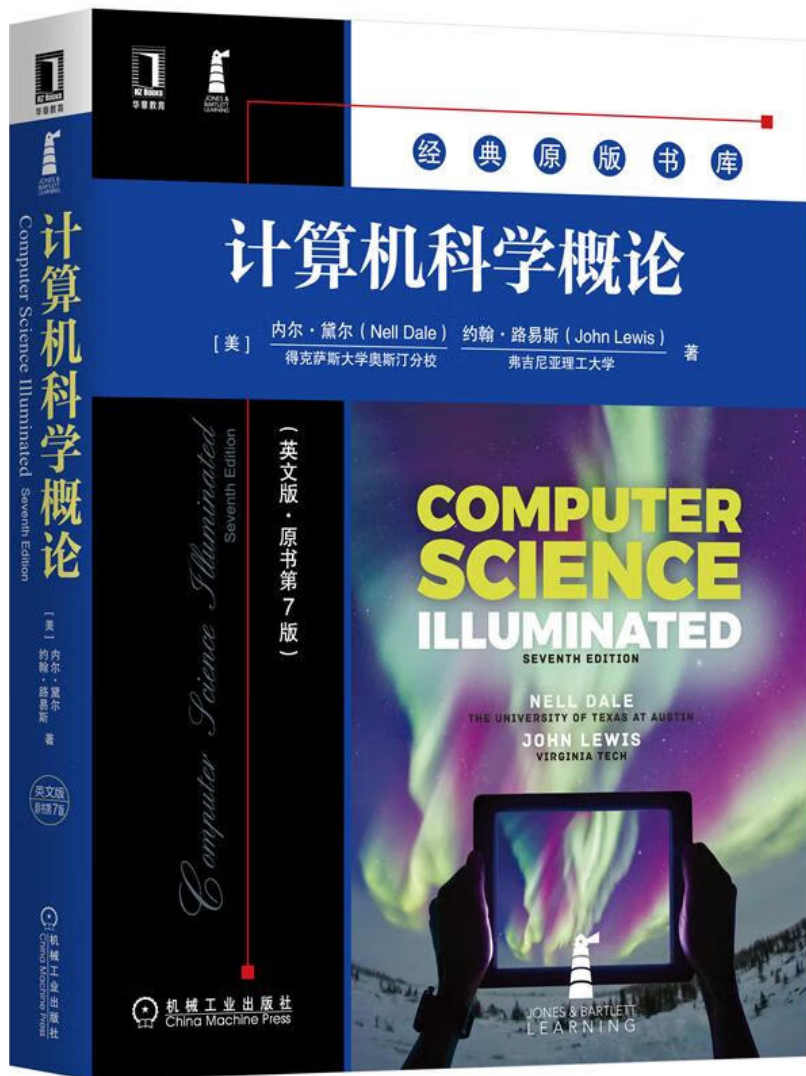
# Course Objective

- Understand how computers work
- Learn to write simple programs
- Understand how real-world applications work
- Understand what AI can or cannot do
- Know common cybersecurity issues

# Tentative Schedule

Week	Date	Course
1	Sep-5	Numbers and Computation
2	Sep-12	Computer Architecture
3	Sep-19	Programming Language and Compiler
4	Sep-26	Data Structures
5	Oct-3	Holiday
6	Oct-10	Algorithm I
7	Oct-17	Algorithm II
8	Oct-24	Operating System
9	Oct-31	Network
10	Nov-7	Database
11	Nov-14	B/S Applications
12	Nov-21	C/S Applications
13	Nov-28	Cryptography
14	Dec-5	Cybersecurity
15	Dec-12	Artificial Intelligence I
16	Dec-19	Artificial Intelligence II

# Reference Book



# Course Information

- Time: Thursday 6:30pm - 9:10pm
- Classroom: H6304
- Platform:
  - Couse Webpage: eLearning
    - Lecture notes
    - Assignment submission
  - Webpage: <https://github.com/hxuhack/intro2cs>
  - Notification & Discussion: WeChat Group



# Grading

- In-Class Practice: 50%
  - Issued during the third class each week
  - Due one week later (before the class)
  - Any two practices can be waived (count n-2 best graded assignments)
- Final Exam: 50%
  - Closed book
  - Time: 2024-12-24 15:30~17:30

## 2. Binary System

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# All Data are Stored in Computers as Bits

- Natural Number
- Negative Integers
- Floating-point Numbers
- Text/Documents
- Multimedia
- Code
- Neural Networks
- ...

# Representing Natural Numbers as Bits

Decimal	Binary	Hex
0	0	0
1	01	1
2	10	2
3	11	3
4	100	4
5	101	5
6	110	6
7	111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F
16	10000	10
...	...	...

DES SCIENCES. 85

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*E X P L I C A T I O N*  
*D E L' A R I T H M E T I Q U E*  
*B I N A I R E,*

*Qui se sert des seuls caracteres 0 & 1 ; avec des Remarques sur son utilité, & sur ce qu'elle donne le sens des anciennes figures Chinoises de Fohy.*

PAR M. LEIBNITZ.

**L**E calcul ordinaire d'Arithmétique se fait suivant la progression de dix en dix. On se sert de dix caracteres, qui sont 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, qui signifient zero, un, & les nombres suivans jusqu'à neuf inclusivement. Et puis allant à dix, on recommence, & on écrit dix ; par 10 ; & dix fois dix, ou cent, par 100 ; & dix fois cent, ou mille, par 1000 ; & dix fois mille, par 10000. Et ainsi de suite.

1703.  
5. Mai

Mais au lieu de la progression de dix en dix, j'ai employé depuis plusieurs années la progression la plus simple de toutes, qui va de deux en deux ; ayant trouvé qu'elle sert à la perfection de la science des Nombres. Ainsi je n'y employe point d'autres caracteres que 0 & 1, & puis allant à deux, je recommence. C'est pourquoi deux s'écrit ici par 10, & deux fois deux ou quatre par 100 ; & deux fois quatre ou huit par 1000 ; & deux fois huit ou seize par 10000, & ainsi de suite. Voici la Table des Nombres de cette façon, qu'on peut continuer tant que l'on voudra.

On voit ici d'un coup d'œil la raison d'une propriété célèbre de la progression Géométrique double en Nombres entiers, qui porte que si on n'a qu'un de ces nombres de chaque degré, on en peut composer tous les autres nom-

L. iij

# Question

- What is the binary representation for:
  - 100
  - 1024 (also known as 1 kilo)
- What is the decimal value of:
  - 1111 1111
  - 1 0000 0000 0000 0000 0000 (1 mega)
- More quantifiers:
  - 1 giga:  $2^{30}$
  - 1 tera:  $2^{40}$
  - 1 peta:  $2^{50}$
  - ...

# How to Represent Negative Numbers?

$$-1 = ?$$

# Representing Negative Integers with Two's Complement

- Employ the leftmost bit as the sign bit
  - 0 for positive numbers
  - 1 for negative numbers
  - Padding 0 or 1 to a fixed size, e.g., 4 bits
- Two's complement for negative numbers
  - Invert all the bits of the positive number
  - Add one

$$-(0001)_2 \xrightarrow{\text{invert}} 1110 \xrightarrow{\text{add 1}} 1111$$

Decimal	Binary
-8	1000
-7	1001
-6	1010
-5	1011
-4	1100
-3	1101
-2	1110
-1	1111
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

# Question

- What is the largest signed integer that 32 bits can represent?
- What is the smallest signed integer that 32 bits can represent?



How to represent larger numbers?

How to represent decimals or fractions?

# Representing Floating-point Numbers with IEEE 754

- Meaning of bits for 32-bit single-precision floating-point numbers:
  - 32 (leftmost): sign bit
  - 24-31: exponent bits, to represent a wide range of values.
  - 1-23: mantissa bits
- Evaluation:  $2^{exp} * mantissa$

01000011010010000000000000000000

  
exponent (8 bits)      mantissa (23 bits)

$$\begin{array}{ll} 2^7 + 2^2 + 2^1 - 127 & 1 + 2^{-1} + 2^{-4} \\ = 7 & = 1.5625 \end{array}$$

$$2^7 * 1.5625 = 200$$

Why do we subtract a bias 127?

# Convert Numbers to Floating-point Representation

- E.g., 11.25 => 0**10000010**011010000000000000000000000000

Integer Part	Decimal Part
$11/2 = 5 + 1$	$0.25 * 2 = 0.5 + 0$
$5/2 = 2 + 1$	$0.50 * 2 = 0.0 + 1$
$2/2 = 1 + 0$	
$1/2 = 0 + 1$	

1011.01



1+.01101 with exp = 3

0**10000010**011010000000000000000000000000



3+127

# Question

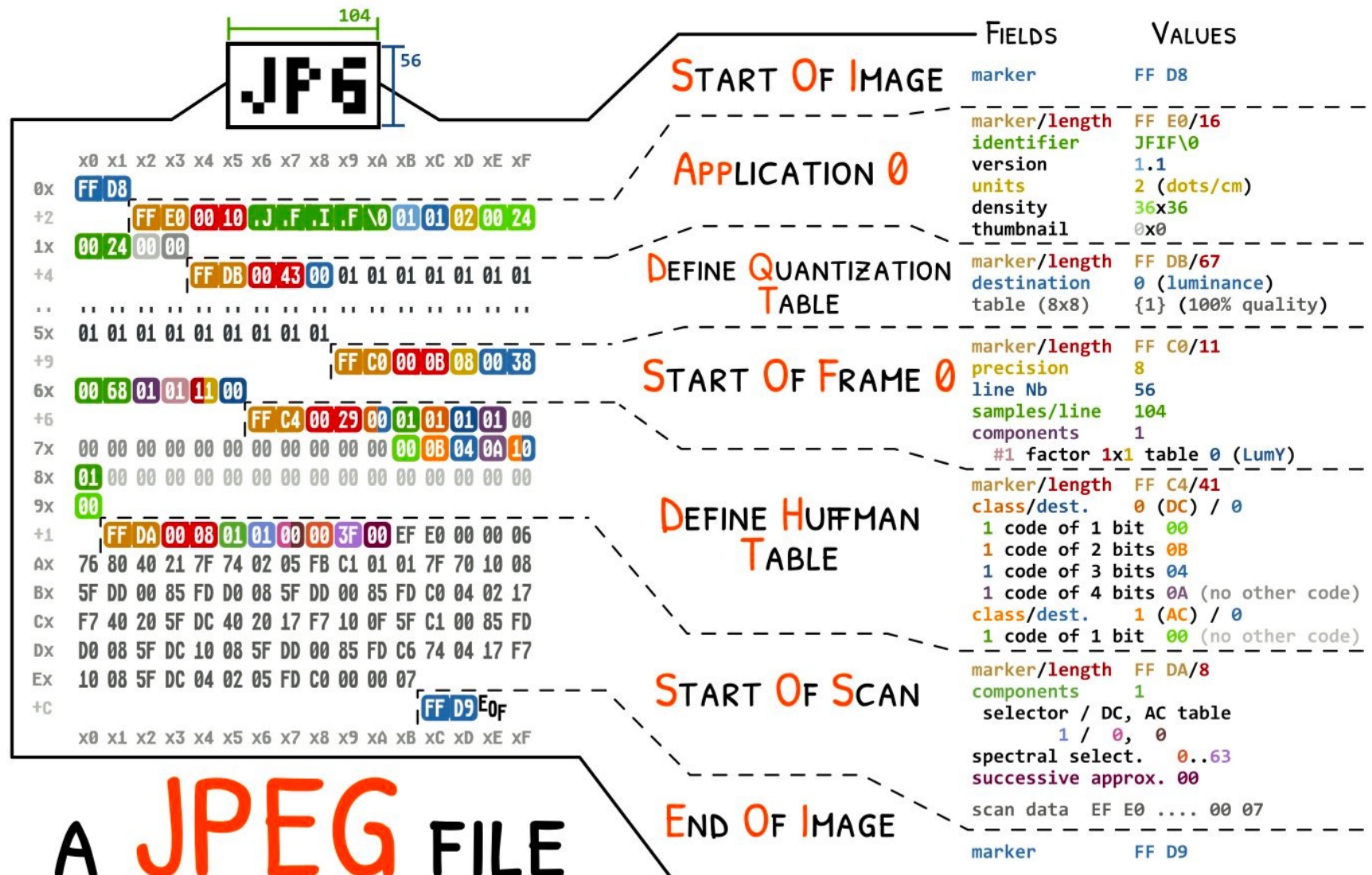
- Which of the following numbers can be represented by floating-point numbers without precision loss?
  - 0.1
  - 0.2
  - 0.3
  - 0.4
  - 0.5

# Encoding Characters in Bits (as Bytes)

- ASCII (American Standard Code for Information Interchange)
  - Using 8 bits (7 useful bits) to represent a character
  - *e.g.*, 0 can be represented as 0011 0000
- 1 byte = 8 bits, or 1 B = 8 b

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0x00	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
0x10	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
0x20	SP	!	"	#	\$	%	&	'	(	)	*	+	,	-	.	/
0x30	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
0x40	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0x50	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
0x60	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
0x70	p	q	r	s	t	u	v	w	x	y	z	{		}	~	DEL

# Images



A **JPEG** FILE

# 3. Computation

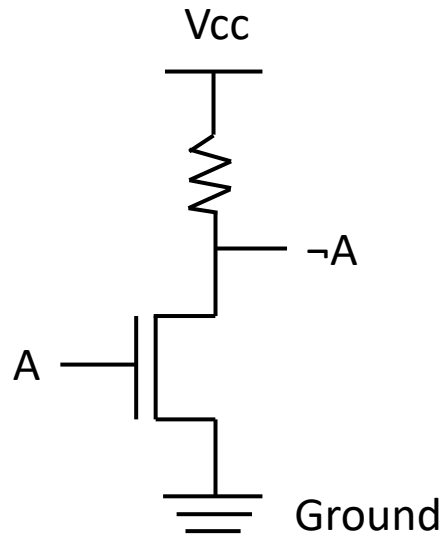
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# High-level Idea

- Compose **logic gates** based on **transistors**
- Compose **arithmetic computation units** based on logic gates
- Manufacture **CPUs** with ALU, control unit, *etc.*
- Build **computers** with CPU, memory, input, and output

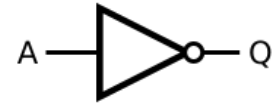


# Transistor => Logical Gate: NOT



**NMOS NOT**

**Simplified Representation:**



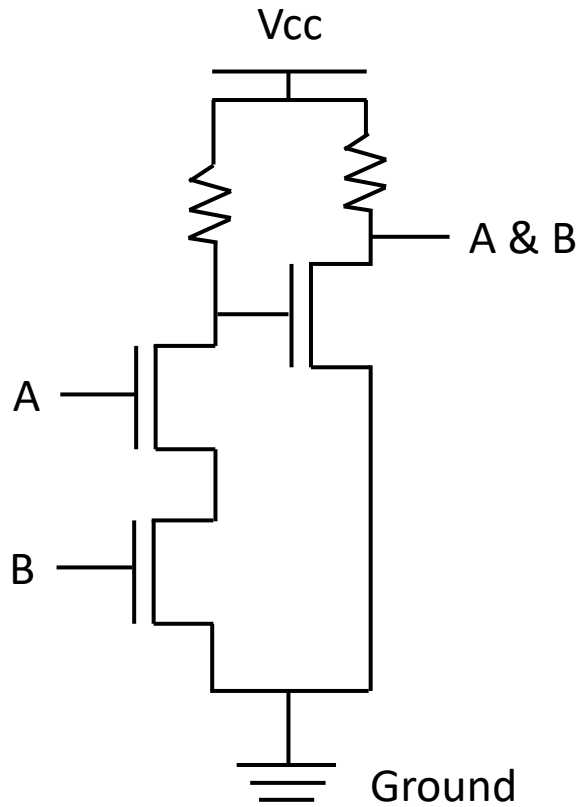
**Operator Symbol:**

$\neg$

**Truth Table:**

A	$\neg A$
0	1
1	0

# Transistor => Logical Gate: AND



**NMOS AND**

**Simplified Representation:**

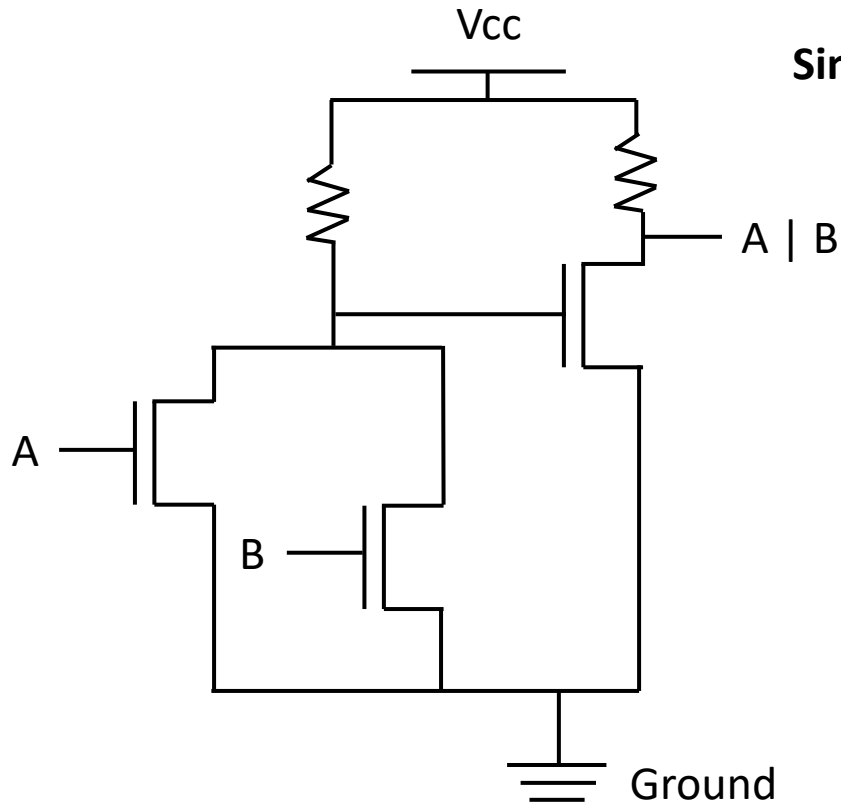


**Operator Symbol:** &

**Truth Table:**

A	B	A & B
0	0	0
0	1	0
1	0	0
1	1	1

# Transistor => Logical Gate: OR



**NMOS OR**

**Simplified Representation:**



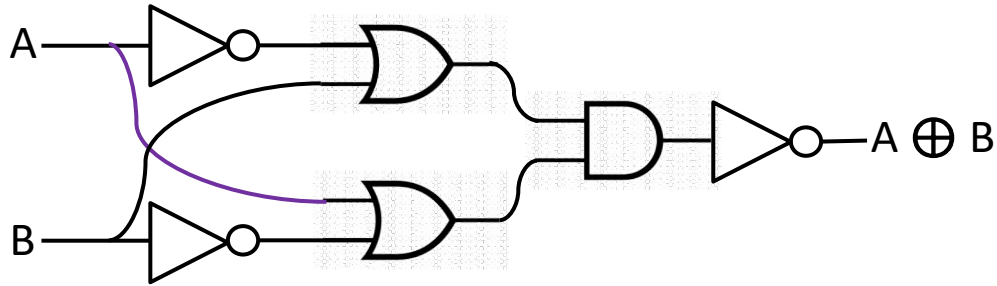
**Operator Symbol:**

|

**Truth Table:**

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

# Transistor => Logical Gate: XOR



**Simplified Representation:**

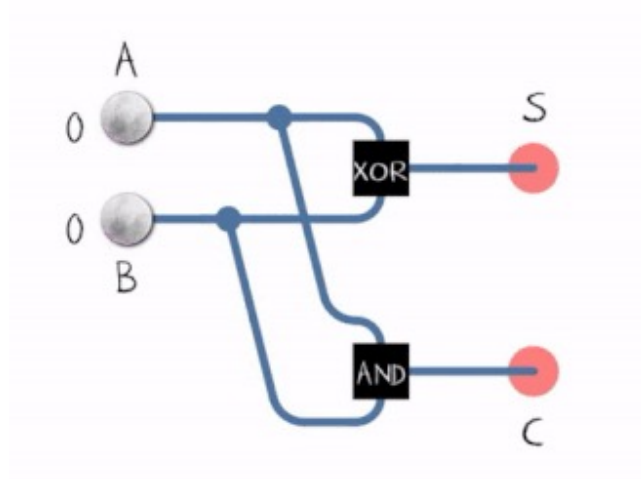


**Operator Symbol:**  $\oplus$

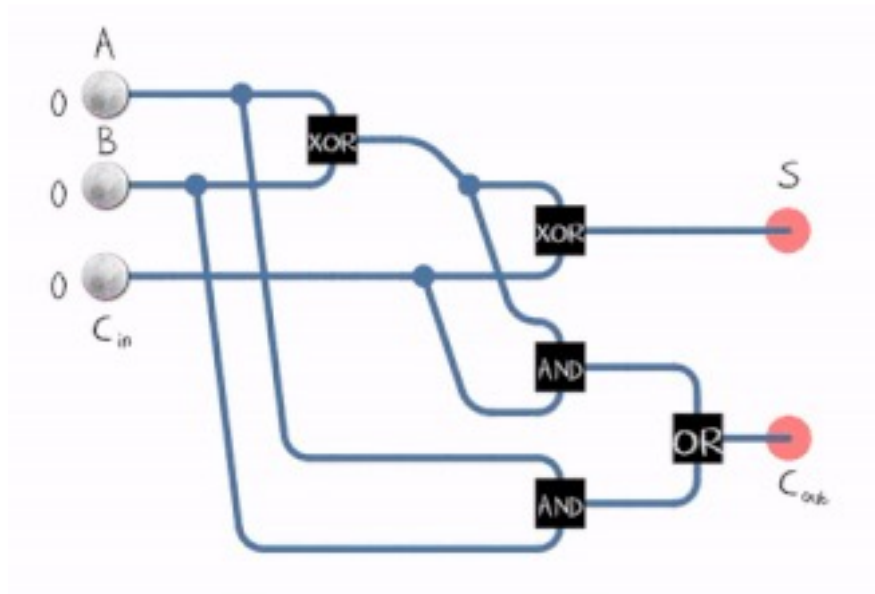
**Truth Table:**

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

# Compose an Adder with Logical Gates



Half Adder

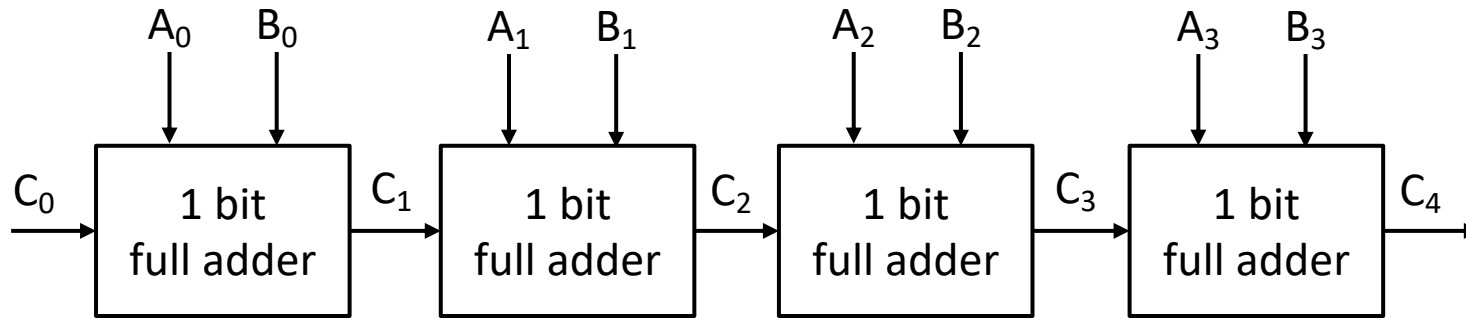


Full Adder (with carrier input)

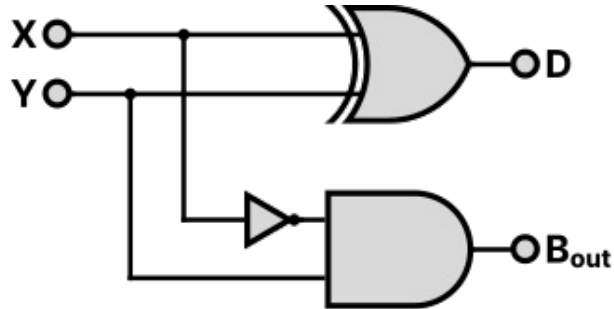
Inputs		Outputs	
A	B	C <sub>out</sub>	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Inputs			Outputs	
A	B	C <sub>in</sub>	C <sub>out</sub>	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

# Add 4-bits with Full Adder

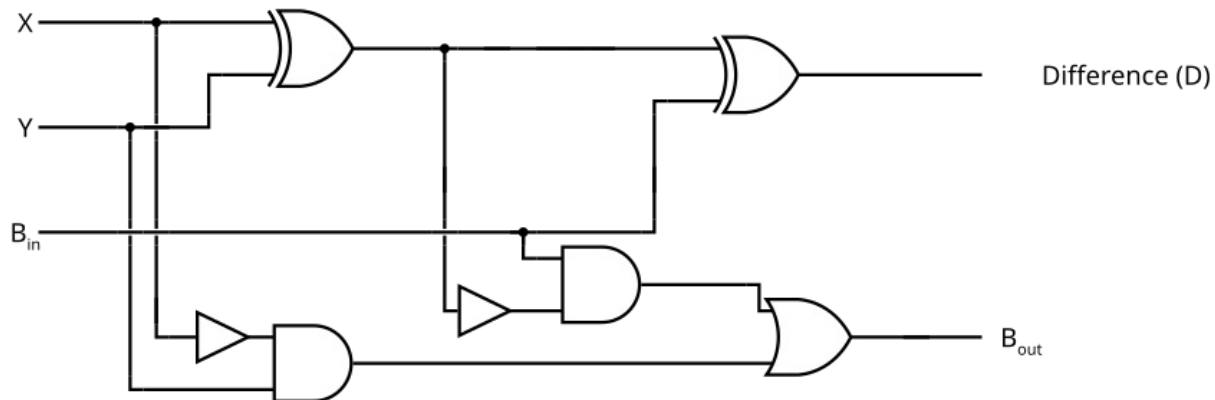


# Combinational Circuits: Subtractor



**Half Subtractor**

Inputs		Outputs	
X	Y	D	B <sub>out</sub>
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

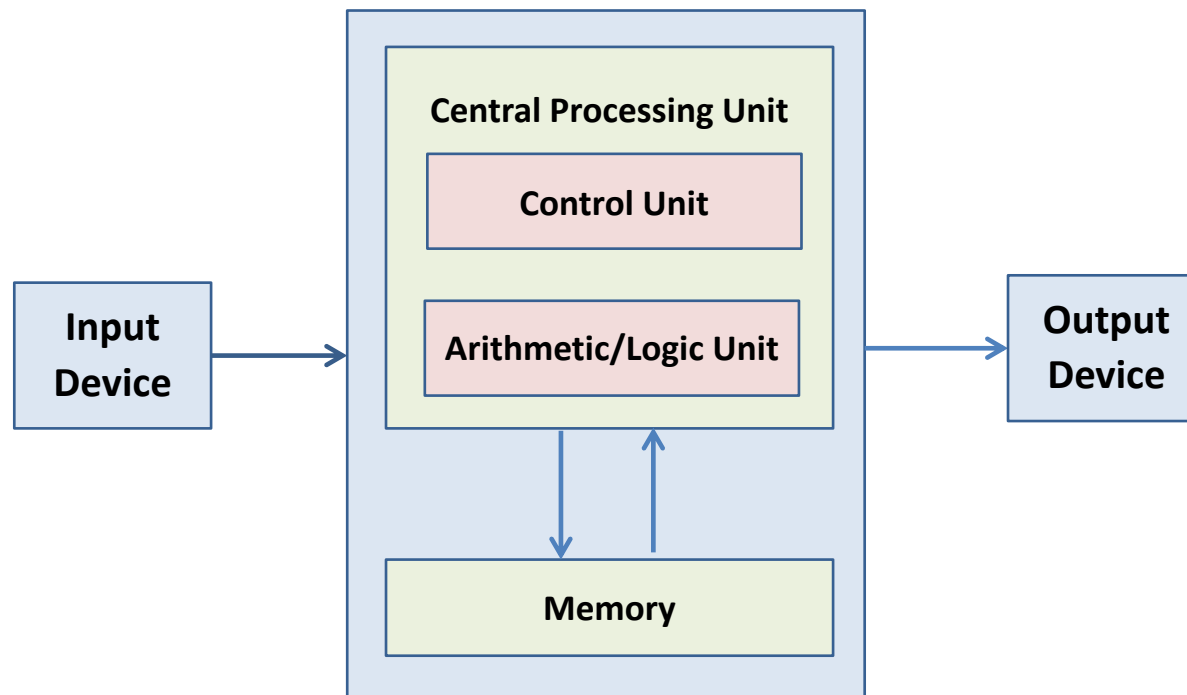


**Full Subtractor (with borrow input)**

Inputs			Outputs	
X	Y	B <sub>in</sub>	D	B <sub>out</sub>
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

# CPU and Von Neumann Model

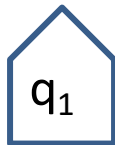
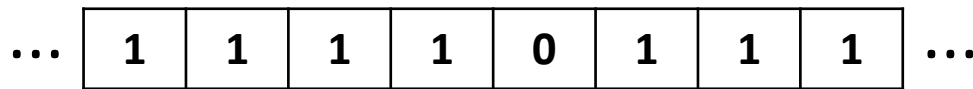
- Control unit: fetch instructions from memory at the address specified by the program counter.
- ALU: perform the operation specified by the instruction, write the results to memory or registers.





# General Turing Machine

- Infinite tape: composed on 0/1s
- Head: read/write symbols
- State register: finite number of values
- Table of instructions



	0	1
q <sub>1</sub>	1 R q <sub>2</sub>	1 R q <sub>1</sub>
q <sub>2</sub>	0 L q <sub>3</sub>	1 R q <sub>2</sub>
q <sub>3</sub>	0 H q <sub>3</sub>	0 H q <sub>3</sub>

## ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

The “computable” numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth. The fundamental problems involved are, however, the same in each case, and I have chosen the computable numbers for explicit treatment as involving the least cumbrous technique. I hope shortly to give an account of the relations of the computable numbers, functions, and so forth to one another. This will include a development of the theory of functions of a real variable expressed in terms of computable numbers. According to my definition, a number is computable if its decimal can be written down by a machine.

In §§ 9, 10 I give some arguments with the intention of showing that the computable numbers include all numbers which could naturally be regarded as computable. In particular, I show that certain large classes of numbers are computable. They include, for instance, the real parts of all algebraic numbers, the real parts of the zeros of the Bessel functions, the numbers  $\pi$ ,  $e$ , etc. The computable numbers do not, however, include all definable numbers, and an example is given of a definable number which is not computable.

Although the class of computable numbers is so great, and in many ways similar to the class of real numbers, it is nevertheless enumerable. In § 8 I examine certain arguments which would seem to prove the contrary. By the correct application of one of these arguments, conclusions are reached which are superficially similar to those of Gödel†. These results

† Gödel, “Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I”, *Monatshefte Math. Phys.*, 38 (1931), 173–198.

# Summary

- All data are stored in computers as bits
  - Integers => negative integers => floating-point numbers => files
- Computers are made of transistors that accept bit inputs
  - Transistors => logic gates => arithmetic units => CPU => computer

## 4. In-class Practice

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# Setup Rust Programming Environment

## 1) Install Rust

- <https://www.rust-lang.org/tools/install>

## 2) Or use the web version

- <https://play.rust-lang.org/>

# Option 1: Analyze Floating-point Numbers

- 1) Write a program that output the bits of a floating-point number
- 2) Explain the bit representation

```
fn main() {  
    let number: f32 = 200.1;  
    let bits = number.to_bits();  
    println!("Bit representation of {}: {:032b}", number, bits);  
}
```

# Option 1: Analyze Floating-point Numbers

- 3) Write a program that evaluate the bits of a floating-point number
- 4) Manually verify the correctness

```
use std::num::ParseIntError;

fn bits_to_f32(bits: &str) -> Result<f32, ParseIntError> {
    let int_value = u32::from_str_radix(bits, 2)?;
    let float_value = f32::from_bits(int_value);
    Ok(float_value)
}

fn main() {
    let bit_sequence = "01000011010010000000000000000000";
    match bits_to_f32(bit_sequence) {
        Ok(float) => println!("The f32 value is: {}", float),
        Err(e) => println!("Failed to parse bit sequence: {}", e),
    }
}
```

## Option 2: Analyze the Bit Representation of Files

- 1) Change the file path of the following program and execute it;
- 2) Try to interpret the output;
- 3) Modify the content of the output and create a new valid jpg file;
  - *e.g.*, via hex editor tools like <https://hexed.it/>
- 4) Discuss whether your changes to the image are visible?

```
use std::fs::File;
use std::io::{self, Read};

fn main() -> io::Result<()> {
    let mut file = File::open("/home/aisr/john.jpg")?;
    let mut buffer = Vec::new();
    file.read_to_end(&mut buffer)?;
    for byte in buffer {
        print!("{:02x} ", byte); // Prints in hexadecimal format
    }
    Ok(())
}
```