Risk Management Capstone Project
Written Report
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INTRODUCTION
Risk management constitute an important part of asset pricing (sell side) and arbitrage opportunity identification (buy side). This project explore the implementation of various risk measurement and hedging techniques and their implication, such as Value at Risk, Expected Shortfall, MRC, Z-test and Kupiec Test under different return distribution assumption such as Gaussian and non-parametric approach. Following this, this report embarks on Principal Component Analysis (PCA) to further test a bond's sensitivity to level, slop, and curvature factors. To conclude the discussion, the final part looks at the risk management form perspective of a portfolio. Individual asset's Contribution VaR is examined and the optima hedge is provided as well.

For details of the technique implementation including the execution of the Monte Carlo Simulation, please refer to the attached Excel sheets for reference.

MAIN CONTENT
1. *The Square Root Rule*
(Excel: RA_Introduction; Worksheet: Square Root Rule)

In order to simulate the price volatility as a function of trading horizon, the square root rule is applied. For example, assuming 250 trading days per year, when historical annualised return is 5.00% (lognormal return) and volatility is 0.3. We have the daily return is 0.05/250 = 0.0002, while the daily standard deviation is set to be $0.3/\sqrt{250}$. The standard deviation at t=100 is thus: $0.3*\sqrt{100/250}$.

Days	1	5	10	20	50	100	250
Mean	0.0002	0.001	0.002	0.004	0.01	0.02	0.05
Std. Dev.	0.018974	0.042426	0.06	0.084853	0.134164	0.189737	0.3
Variance	0.00036	0.0018	0.0036	0.0072	0.018	0.036	0.09

Table 1. Square Root Rule

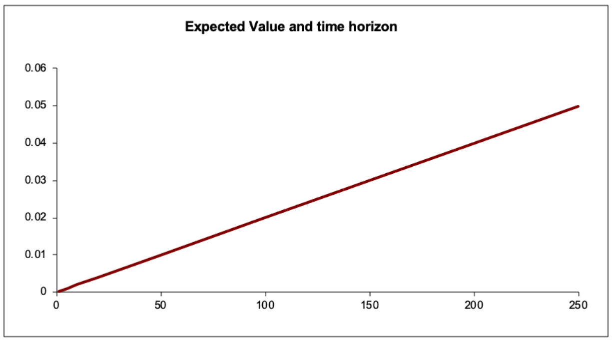


Figure 1. Expected Value and the Time Horizon

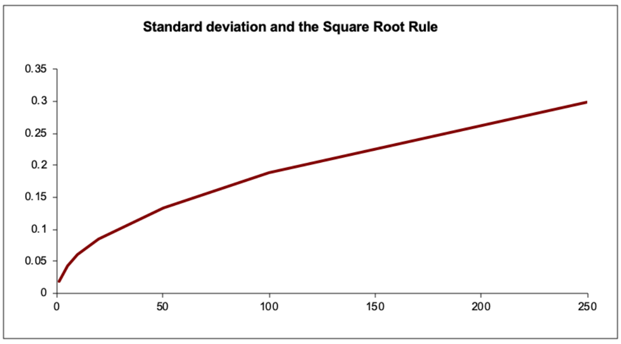


Figure 2. Standard Deviation and the Square Root Rule

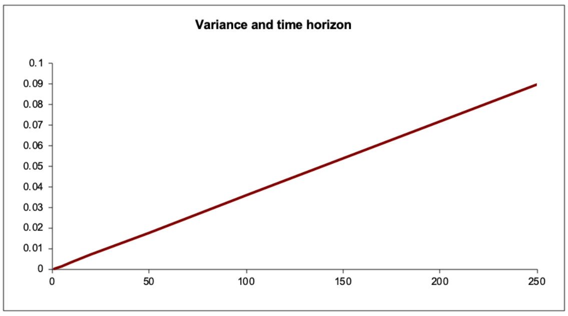


Figure 3. Variance and Time Horizon

2. *Log Return*
(Excel RA_Introduction; Worksheet: Pick Up Index)

Log return is defined as $\ln\left(\frac{P_t}{P_{t-1}}\right)$.

Descriptive Analysis			
Indice: FTSE FTSE 100 London			
	P&L	Log Ret	
Nobs	980	980	=COUNT(F12:F991)
Mean	-0.814	0.000	=AVERAGE(F12:F991)
Std. Dev.	83.322	0.017	=STDEV(F12:F991)
Skewness	-0.053	-0.008	=SKEW(F12:F991)
Kurtosis	3.318	5.994	=KURT(F12:F991)
Min	-391.1	-0.09265	=MIN(F12:F991)
Max	431.3	0.096404	=MAX(F12:F991)
JB	449.94	1467.19	=K14*(K17^2+K18^2/4)/6
Test Normality	NO	NO	=IF(K21<5.99146,"SI","NO")

Table 2. Descriptive Analysis of FTSE 100 Stock Log Return

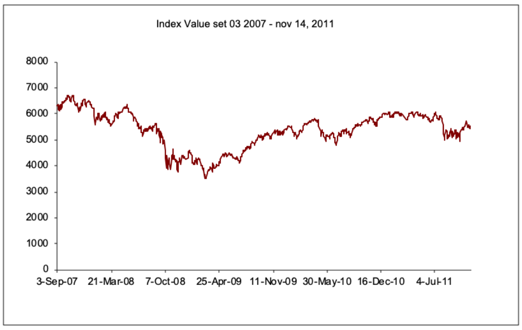


Figure 4. FTSE 100 Index Value

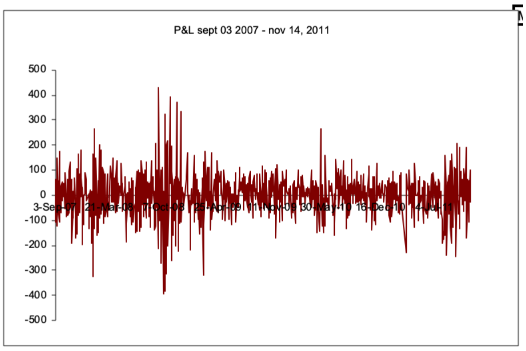


Figure 8. FTSE 100 P&L

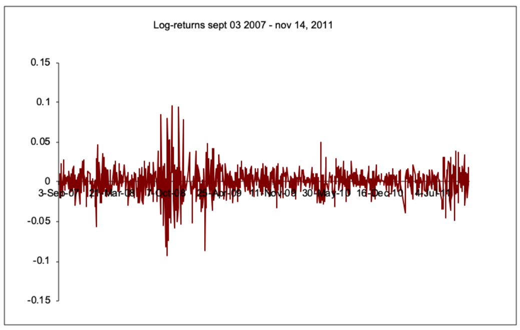


Figure 9. FTSE 100 Log-returns

3. *Simulating Paths of Returns*

(Excel RA_Introduction; Worksheet: Simulating Returns)

Assuming stock return follows Brownian motion, simulation of stock returns can be obtained through

$$R_t^i = \mu_0 dt + \sigma_0 \sqrt{T-t} W_t dt$$

Where μ_0 is the expected return, σ_0 is the historical volatility, and W_t indicates Brownian motion. i indicates the number of simulations[1]. In this case simulation time is set to 5, daily returns are simulated as follow (in a period of 250 trading days):

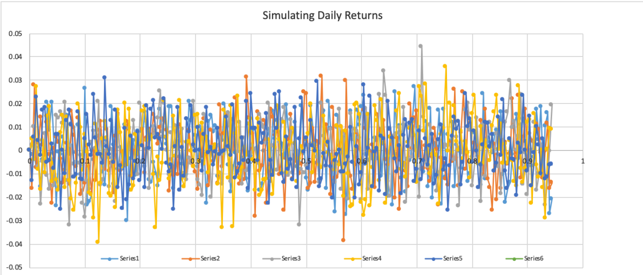


Figure 10. Daily Return Simulation

Following lognormal distribution, the stock price can be simulated as:

$$P_t^i = P_{t-1}^i * e^{R_i^t}$$

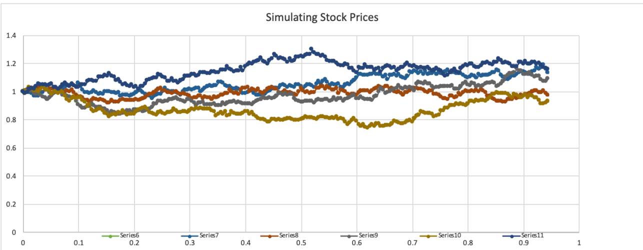


Figure 11. Stock Price Simulation

4. *Return Autocorrelation Simulation*

(Excel: RA_Introduction; Worksheet: Simulating AutoCorr Returns)

Assuming autocorrelation in the stock price, namely, last period stock price is likely to have an impact on the price movement in the future:

$$\rho R_{t-1} + \sigma \sqrt{\frac{T-t}{T}} W_t dt$$

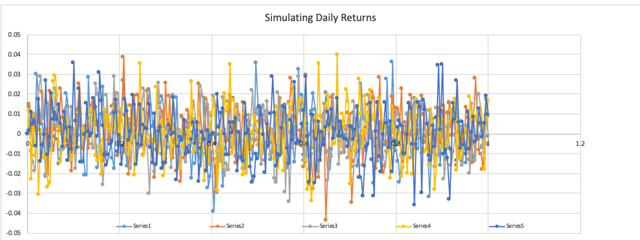


Figure 12. Daily Return Simulation with Autocorrelation

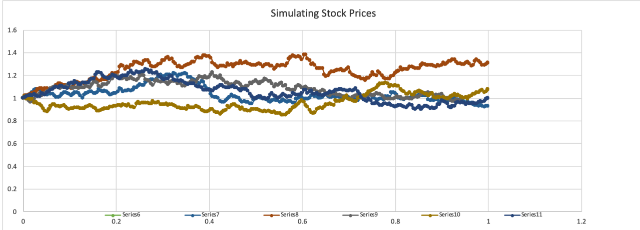


Figure 13. Stock Price Simulation with Autocorrelation

5. *Non-parametric VaR with Different Sample Sizes*

(Excel: RA_Introduction; Worksheet: Non-parametric VaR Sample Size)

It is worth noticing that in real life trading, stock returns may not strictly follow Gaussian or Log-normal distribution. Non-parametric distribution and calculation of VaR is as follow[2]:

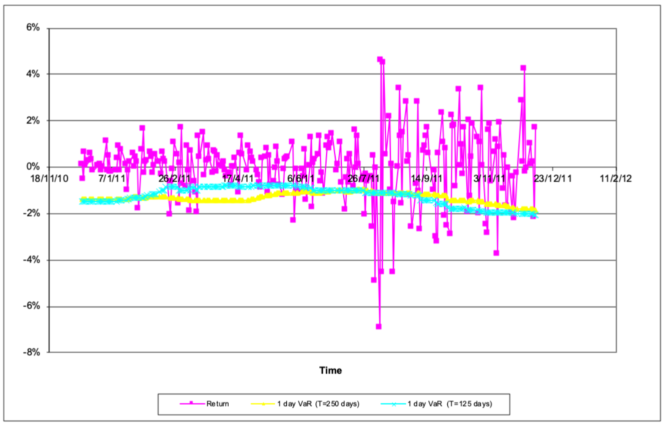


Figure 14. Non-parametric VaR

The time horizon used for VaR calculation is either 250 days or 125 days in the above example.

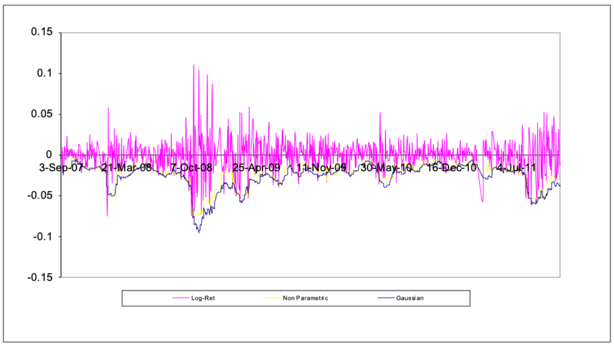


Figure 15. VaR Back Test under Gaussian and Non-parametric Approach

When compared with Gaussian distribution, empirical distribution does show deviations. To compare the two distributions, the maximum and minimum log returns of the observation period are identified. While the number of bins is set to be 20, the bin size is thus (Max – Min)/No. of bins. In this case, the bin size is 0.006067.

Then out of the 503 observations, the frequency of each bin is calculated[3]. Relative frequency is simply the Frequency obtained divide by 503 (total number of observations). The empirical density is calculated using Relative frequency divided by the bin size.

[1] In Excel Implementation, Brownian motion is implemented using function NORM.S.INV(RAND())

[2] In Excel implementation, the 10% VaR is located using function =PERCENTILE(E8:E260,0.1)

[3] In Excel implementation, the frequency for each bin is calculated with FREQUENCY(E7:E509,H14:H33), where the 1st argument is the population (log-returns of the entire period), the 2nd argument is the bins)

Lastly, with the bin interval, sample expected return and standard deviation known, Gaussian distribution is plotted as well:

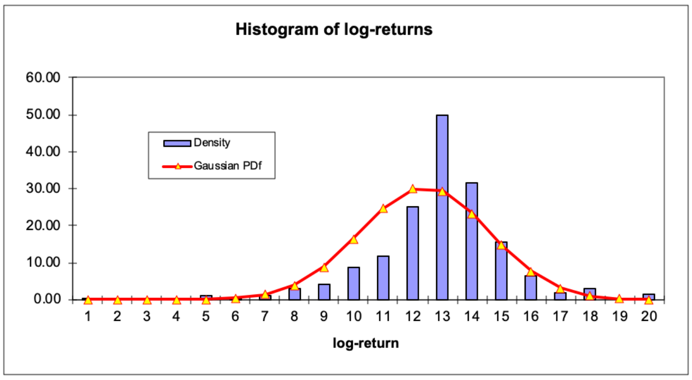


Figure 16. Empirical density vs. Gaussian density

6. *Market Required Capital*

(Excel: RA_Introduction; Worksheet: MRC)

At 99% confidence level and 10-day horizon (n=10), the 10-day standard deviation of stock returns is calculated as: $\sigma_{10}\sqrt{n}$. Then 10-day 99% VaR is $\sigma_{10}\sqrt{n} * Z_{0.99}$.

Violation is identified when the current day's loss exceed the 1-day VaR set predominantly. Counting all violations, the number is then compared to the MRC table, multiplicative factor is obtained.

Finally, MRC is obtained by selecting the bigger value of the 10-day standard deviation and the result of multiplicative factor times the average VaR throughout the entire period.

α	0.99
1−α	0.01
σ	0.014446
n	10
10 days st. dev	0.045682
z	-2.32635
10-days 99% VaR	0.106273
Average VaR	0.098793
Violations	10
S(t)	4
MRC	0.395173
MRC (Capital)	**496.0178**

Table 3. MRC

7. *VaR and Expected Shortfall*

(Excel: RA_Introduction; Worksheet: Parametric Gaussian ES)

Expected Shortfall (ES) is a more coherent measure compared to VaR as it does not subject to the sub-additivity bias.

$$ES_{\alpha}(t, t+dt) = -(\mu_{\Delta}n - \frac{\sigma_{\Delta}\sqrt{n}}{1-\alpha}\phi(Z_{i-\alpha}))$$

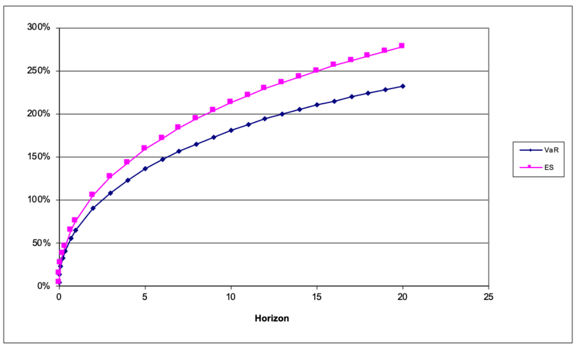


Figure 17. VaR vs. ES

VaR can be defined as number of standard deviation away from the expected return, hence VaR is a function of time horizon as well: (confidence level: 99%)

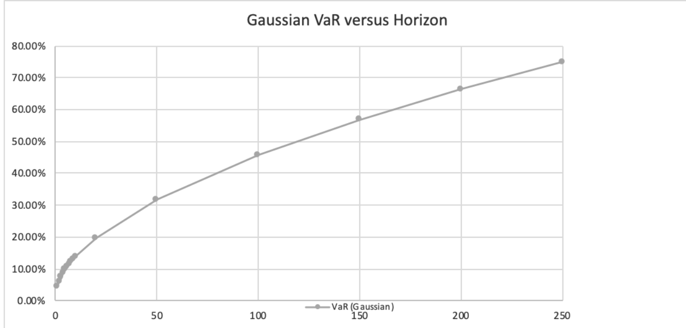


Figure 18. Gaussian VaR as a function of time horizon

8. *Volatility Estimation via EWMA and VaR*

(Excel: Parametric Gaussian; Worksheet: VaR Risk Metrics)

Under EWMA (Exponentially Weighted Moving Average) model, the volatility is the near future is given larger weights (importance) compared to historical volatilities. The smoothing parameter is set to be 0.98. The formula is given by:

$$\sigma_n^2(EWMA) = \lambda\sigma_n^2 + (1-\lambda)u_{n-1}^2$$

Where λ is the smoothing parameter.

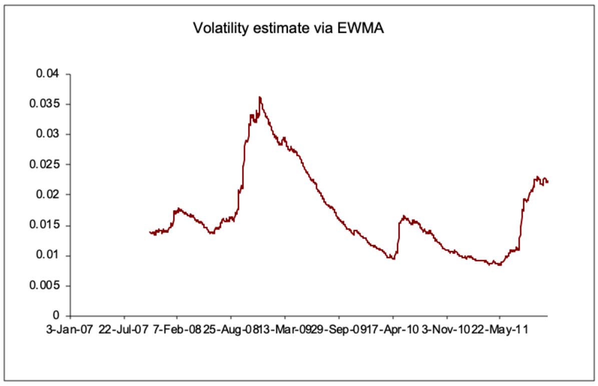


Figure 17. Volatility Estimation via EWMA

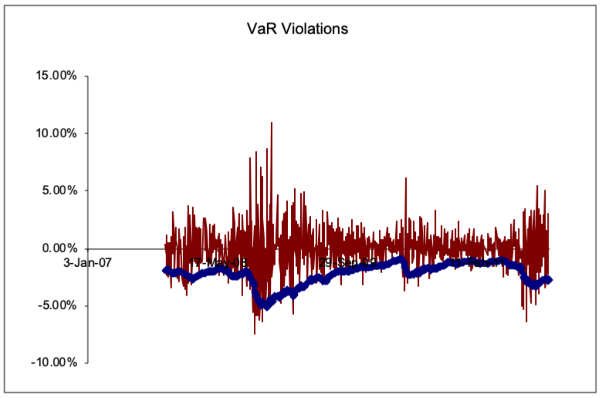


Figure 19. VaR Violations

With a confidence level of 10%, we calculated the VaR for each period. (shown in blue) Violations are identified as the returns exceed the VaR bound.

9. *Hit Sequence Approach in Violation Measurement*

(Excel: RA_Backtesting; Worksheet: Violations)

Z-test and Kupiec Test are performed to determine whether one should accept the hypothesis that the number of violations are under prescribed acceptance range.

Under Z-test, the expected number of violation is calculated as (1 – confidence level)/no. of observations. Z-score is computed as (observed no. of violations – expected no. of violations)/standard deviation of violations. As significant level of 0.95, we reject the null hypothesis:

Z-Test	
E(Sum Viol)	149.8
Var(Sum Viol)	134.82
z(j)	2.945
Significance Level	0.95
Accept H0?	NO

Kupiec Test	
Estimated a	0.877
LR UC	8.149
Sig. Level	0.95
CV	3.841
Accept H0?	NO

Table 4. Z-test and Kupiec Test Results

The log-likelihood ratio is calculated as:

$$LR_{uc} = -2\ln\left(\left(\frac{1-\alpha}{1-\hat{\alpha}}\right)^{j} \times \left(\frac{\alpha}{\hat{\alpha}}\right)^{n-j}\right) \sim \chi_1^2$$

At significant level of 0.95, we reject the null hypothesis as well.

10. *Principal Component Analysis of the US Term Structure*

(Excel: RA_PCA; Worksheet: PCA)

Main driver factors of the US treasury bond can be obtained with principal component analysis (PCA). The covariance matrix of changes in US spot rates for the period June 1 1997 till June 30 1997 is as below:

Term	Covariance matrix of changes in US spot rates for the period 1/6/1997-6/30/1999									
	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y
1Y	1.18E-07	1.46E-07	1.49E-07	1.46E-07	1.43E-07	1.40E-07	1.41E-07	1.38E-07	1.34E-07	1.27E-07
2Y	1.46E-07	1.94E-07	2.00E-07	1.99E-07	1.96E-07	1.93E-07	1.94E-07	1.91E-07	1.87E-07	1.79E-07
3Y	1.49E-07	2.00E-07	2.17E-07	2.14E-07	2.13E-07	2.10E-07	2.12E-07	2.10E-07	2.06E-07	2.00E-07
4Y	1.46E-07	1.99E-07	2.14E-07	2.21E-07	2.19E-07	2.17E-07	2.19E-07	2.18E-07	2.15E-07	2.09E-07
5Y	1.43E-07	1.96E-07	2.13E-07	2.19E-07	2.28E-07	2.25E-07	2.28E-07	2.27E-07	2.24E-07	2.20E-07
6Y	1.40E-07	1.93E-07	2.10E-07	2.17E-07	2.25E-07	2.34E-07	2.31E-07	2.32E-07	2.29E-07	2.24E-07
7Y	1.41E-07	1.94E-07	2.12E-07	2.19E-07	2.28E-07	2.31E-07	2.40E-07	2.39E-07	2.36E-07	2.31E-07
8Y	1.38E-07	1.91E-07	2.10E-07	2.18E-07	2.27E-07	2.32E-07	2.39E-07	2.47E-07	2.42E-07	2.35E-07
9Y	1.34E-07	1.87E-07	2.06E-07	2.15E-07	2.24E-07	2.29E-07	2.36E-07	2.42E-07	2.45E-07	2.37E-07
10Y	1.27E-07	1.79E-07	2.00E-07	2.09E-07	2.20E-07	2.24E-07	2.31E-07	2.35E-07	2.37E-07	2.39E-07

Table 5. Covariance matrix of changes in US spot rates (1997.06.01 – 1999.06.30)

The Eigenvalue vector is obtained as follow[4]:

Eigenvalues		Sorted Eigenvalues
10	2.95E-09	2.05E-06
9	3.29E-09	8.63E-08
8	3.51E-09	1.30E-08
7	4.4E-09	9.98E-09
6	4.72E-09	6.90E-09
5	6.9E-09	4.72E-09
4	9.98E-09	4.40E-09
3	1.3E-08	3.51E-09
2	8.63E-08	3.29E-09
1	2.05E-06	2.95E-09

Table 6. Eigenvalues

The cumulative variance is the sum of all eigenvalues, which is valued at 2.18e^{-06}. Dividing each eigenvalue by the cumulative variance, the %Variance is obtained, which explains how much percent of the spot rate movement is explained by each factor:

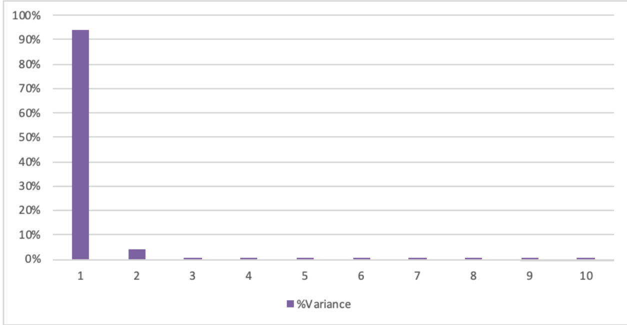


Figure 20. %Variance

With input of the eigenvalue of each maturity and the covariance matrix of the US spot rates, loading sensitivity for each factor is obtained:

Maturity	Loading Factors									
	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
1	0.2125	0.4768	0.5129	0.2873	0.3631	0.3663	0.0352	0.0651	0.1378	0.3024
2	0.2902	0.4701	0.2005	-0.0454	-0.0214	-0.3293	-0.1150	-0.1755	-0.1842	-0.6824
3	0.3145	0.3466	-0.1270	-0.3064	-0.1982	-0.4506	0.1321	0.1709	-0.2280	0.5745
4	0.3220	0.2122	-0.3174	-0.3308	-0.2489	0.3262	-0.2406	-0.0463	0.6398	-0.0580
5	0.3297	0.0280	-0.4025	-0.0510	0.1108	0.5505	0.2835	0.1254	-0.5376	-0.1595
6	0.3318	-0.0947	-0.4335	0.5274	0.3259	-0.2350	-0.4835	-0.0736	-0.0078	0.1191
7	0.3377	-0.1637	-0.0499	0.3054	-0.0753	-0.1768	0.7140	-0.3368	0.3192	-0.0363
8	0.3394	-0.2699	0.2712	0.2953	-0.4807	0.0131	-0.0918	0.6296	0.0121	-0.1225
9	0.3357	-0.3440	0.3473	-0.1486	-0.2372	0.1777	-0.2765	-0.5874	-0.2699	0.2099
10	0.3275	-0.3986	0.1825	-0.4783	0.5958	-0.1636	0.0356	0.2400	0.1478	-0.0973

Table 7. Loading Factors

As the first 3 factors explained most of the spot rate movement, the loading sensitivities of these 3 factors (corresponds to level, slop, and curvature) are plotted against maturity (10-year):

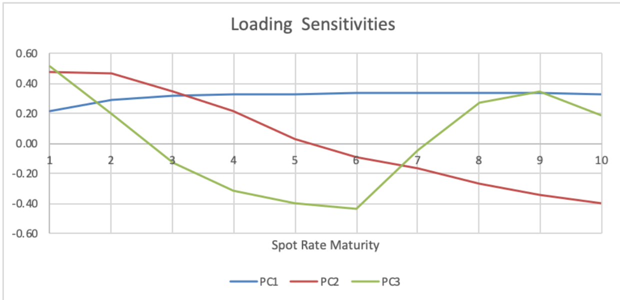
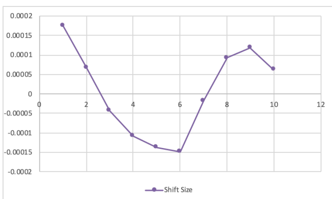


Figure 21. Loading Sensitivities

11. *Interest Exposure via PCA*

(Excel: RA_PCA; Worksheet: Interest Exposure via PCA)

Following the above example, the swap rates, discount factors with different tenors (up to 10 years) are pasted below. The shifted PC is obtained by timing eigenvalues calculated above with the square root of the standard deviation (3).



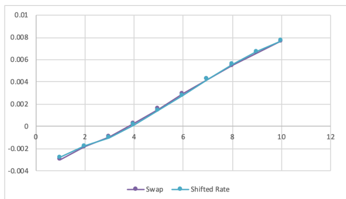


Figure 22. Shift Size

Figure 23. Swap Rate Before and After the Shift

Shift size is obtained as the product of the Shift PC and the loading factor. Shifted rate is thus the original swap rates plus the shift size calculated.

[4] In Excel Implementation, Eigenvalue is obtained using function MEigenvalueQR(C8:L17)

AF	Annuity	Shifted Annuity
1	1.0030	1.0028
1	2.0067	2.0063
1	3.0095	3.0093
1	4.0084	4.0087
1	5.0004	5.0014
1	5.9828	5.9846
1	6.9532	6.9552
1	7.9099	7.9111
1	8.8513	8.8514
1	9.7764	9.7846

Table 8. Annuity and Shifted Annuity

Suppose the bond in question has a tenor of 10 years, face value of \$100, and coupon fixed at 5.00%. The Present value of the bond before and after the swap rate shift is present below:

Tenor	Swap	DF	Shift PC (3 s.d.)	Loading	Shift Size	Shifted Rate	Shifted DF	CF	PV CF	PV CF (Shift)
		1					1			
1	-0.296%	1.003	0.03%	0.513	0.02%	-0.278%	1.003	5	5.01	5.01
2	-0.184%	1.004	0.03%	0.201	0.01%	-0.177%	1.004	5	5.02	5.02
3	-0.095%	1.003	0.03%	-0.127	0.00%	-0.099%	1.003	5	5.01	5.01
4	0.027%	0.999	0.03%	-0.317	-0.01%	0.016%	0.999	5	4.99	5.00
5	0.160%	0.992	0.03%	-0.403	-0.01%	0.146%	0.993	5	4.96	4.96
6	0.295%	0.982	0.03%	-0.433	-0.01%	0.280%	0.983	5	4.91	4.92
7	0.425%	0.970	0.03%	-0.050	0.00%	0.423%	0.971	5	4.85	4.85
8	0.548%	0.957	0.03%	0.271	0.01%	0.557%	0.956	5	4.78	4.78
9	0.662%	0.941	0.03%	0.347	0.01%	0.674%	0.940	5	4.71	4.70
10	0.766%	0.925	0.03%	0.183	0.01%	0.772%	0.933	105	97.14	97.98
								Bond Price	**141.39**	**142.24**

Table 9. Bond PV Before and After the Shift

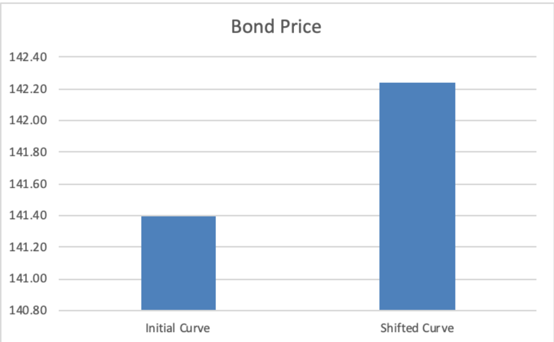


Figure 24. Bond Price Before and After the Shift

After the shift, the bond becomes more exposure to the first factor (level) and less to the second and third factor (slope and curvature), which provides guidance on interest risk exposure and hedging measures.

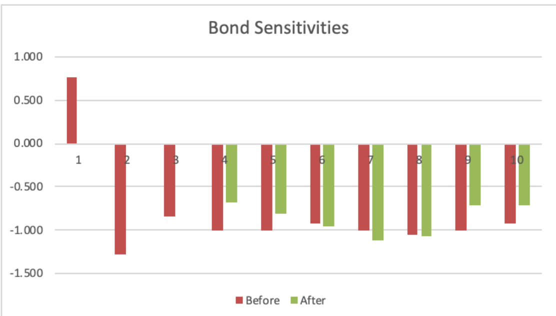


Figure 25. Bond Sensitivity Before and After the Shift

12. *Portfolio Risk Management*

(Excel: RA_Portfolio; Worksheet: Risk Contribution)

Suppose a portfolio with 10 assets. The sample covariance matrix (Σ), and default portfolio weights ($\vec{w}$)are given by:

Portfolio Weights									
1%	12%	18%	4%	10%	19%	5%	18%	10%	2%

Table 10. Portfolio Weights

Sample Covariance Matrix (figures have to be multiplied by 10^{-3}									
62.97%	14.96%	33.36%	35.08%	18.44%	30.78%	21.81%	13.47%	22.04%	37.97%
14.96%	32.62%	17.58%	16.04%	14.30%	14.23%	11.38%	9.42%	17.87%	16.46%
33.36%	17.58%	86.77%	36.16%	21.66%	32.54%	23.58%	15.35%	26.64%	43.84%
35.08%	16.04%	36.16%	82.59%	19.98%	40.30%	32.38%	14.72%	26.57%	40.86%
18.44%	14.30%	21.66%	19.98%	48.82%	14.44%	16.09%	13.70%	21.00%	30.06%
30.78%	14.23%	32.54%	40.30%	14.44%	102.74%	22.09%	14.37%	16.56%	31.80%
21.81%	11.38%	23.58%	32.38%	16.09%	22.09%	52.82%	11.01%	18.46%	28.69%
13.47%	9.42%	15.35%	14.72%	13.70%	14.37%	11.01%	29.78%	15.48%	23.24%
22.04%	17.87%	26.64%	26.57%	21.00%	16.56%	18.46%	15.48%	95.49%	38.17%
37.97%	16.46%	43.84%	40.86%	30.06%	31.80%	28.69%	23.24%	38.17%	142.79%

Table 11. Sample Covariance Matrix

The standard deviation of each asset is the diagonal values applied square root:

Sdev
0.7935
0.5712
0.9315
0.9088
0.6987
1.0136
0.7268
0.5457
0.9772
1.1949

Table 12. Standard Deviation of Each Asset

Portfolio variance is obtained by $Cov.\Sigma\overrightarrow{w}$[5], which is valued at 0.2680. Given that:

$$CVaR = MVaR * w$$

$$Optimal\ Hedge\ (H) = \frac{\sigma_p^2}{\sigma_i^2}$$

$$IVaR = MVaR * H$$

Marginal VaR, Contribution CaR and Investment VaR are computed in the table below, assuming confidence level of 0.95, n = 10/250 = 0.04.

Weights	Individual VaR	Cov. Σ **w**	**MVaR**	**CVaR**	**CVaR%**	Best Hedge	IVaR	VaR at the best Hedge
1%	0.28%	0.24379	0.1549	0.0017	1%	-38.72%	-6.00%	11.03%
12%	2.27%	0.16573	0.1053	0.0127	7%	-50.80%	-5.35%	11.68%
18%	5.65%	0.35930	0.2283	0.0421	25%	-41.41%	-9.45%	7.58%
4%	1.30%	0.30070	0.1911	0.0083	5%	-36.41%	-6.96%	10.07%
10%	2.24%	0.20304	0.1290	0.0126	7%	-41.59%	-5.37%	11.66%
19%	6.44%	0.37027	0.2353	0.0454	27%	-36.04%	-8.48%	8.55%
5%	1.26%	0.20336	0.1292	0.0068	4%	-38.50%	-4.98%	12.05%
18%	3.18%	0.16725	0.1063	0.0189	11%	-56.17%	-5.97%	11.06%
10%	3.25%	0.27785	0.1766	0.0178	10%	-29.10%	-5.14%	11.89%
2%	0.73%	0.33480	0.2128	0.0040	2%	-23.45%	-4.99%	12.04%
Undiv.	26.61%							
Diversified				0.1703	1			

Table 13. Component VaR Analysis

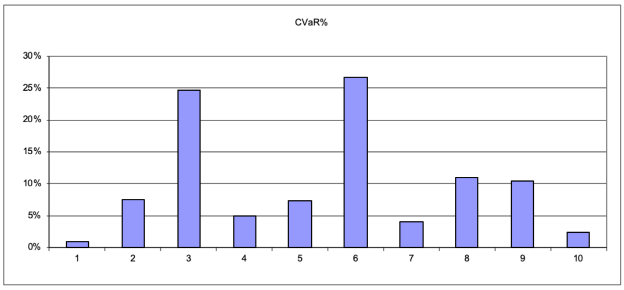


Table 14. CVaR%

Asset #3 and #6 have the highest CVaR%, valued at 25% and 27%, indicating the fact that half of the portfolio risks are attributed by the two.

CONCLUSION

This report presented the main results of the simulations and back tests introduced in the project. The test robustness should be under closer examination when introduced real-time market data. This will be left for further research endeavours.

[5] In Excel Implementation, the function is =MMULT(MMULT(TRANSPOSE(B36:B45),B12:K21),B36:B45)