# SUPERCURVE®

# WHITE PAPER 白皮书

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# **Executive Summary**

## 摘要

# ➤ WHAT IS SuperCurve®? SuperCurve®定义

SuperCurve<sup>®</sup> is a software and data tool to help traders, market makers and brokers stay permanently updated on the fixed income securities market and assist them in their trading activity.

SuperCurve®是一款固定收益量化分析系统,可帮助交易员、做市商和经纪商实时更新固定收益市场的状态,获取利率债及利率衍生品的精准定价,并辅助他们进行报价或交易。

# ➤ WHY SuperCurve®? SuperCurve®初衷

Fixed income trading and information systems currently available are over-simplistic and don't allow the full market's proper monitoring in real-time. Based on each bond or IRS's yield-to-maturity, they are not adapted to handle an irregular data feed prone to errors and fake trades. SuperCurve® aims to make up for these shortcomings.

当前国内可用的固定收益交易系统及信息系统过于简单,且此类系统无法实时掌控市场的整体状态。仅仅基于每只债券或 IRS 的到期收益率,它们无法处理不规则的数据馈送,并容易被信息错漏或虚假交易误导。SuperCurve<sup>®</sup>旨在克服上述缺陷。

# ➤ HOW SuperCurve®? SuperCurve®构建

SuperCurve® collects quotes and transactions from available sources and uses this information flow in real-time to maintain a permanently up-to-date and faithful representation of each segment of the fixed income market: T-bonds, CDB bonds, EIBC & ADBC bonds, SHIBOR3M and FR007 swaps.

SuperCurve®从公开的数据来源中获取报价和成交数据,实时更新并忠实展示固定收益市场各类资产的全貌,其中主要包括:国债、国开债、非国开债(农发和口行债)、SHIBOR3M 利率互换和 FR007 利率互换。

# ➤ WHAT FOR SuperCurve®? SuperCurve®意义

Thanks to SuperCurve<sup>®</sup>: 通过使用 SuperCurve<sup>®</sup>:

• Market makers can maintain a bid and an offer price at any point in time and for any fixed income security, without taking the risk of quoting off the market.

做市商可实时对任意固定收益资产(债券、利率互换、国债期货,以及期权)提供买卖报价,而无需承担报价偏离市场水平的风险。

• Brokers can monitor market consistency and best serve their clients.

经纪商可实时掌握利率市场整体状态及一致性,更好地服务客户。

• Traders get a clear vision of the whole market and can identify mispricing and take strategic positions in the most efficient manner.

交易员可清晰监控整个市场,及时甄别错误定价,并高效地执行策略交易。

• Risk managers have an error-free historical record of all the fixed income market to compute statistics and risks.

风险控制经理可以获得固定收益市场上的所有债券及衍生品的历史记录和精准定价,并由此进行数据统计和风险评估。

# 1. Background and Introduction

# 背景介绍

While equity trading is widespread in China among fund managers and even day traders, trading fixed income securities (bonds, interest rate swaps and futures, and more recently options) is reserved for specialized teams in banks and insurance companies. Only a few types of fixed income instruments are actively traded and booked on a marked-to-market basis in accounting records. While this represents a large market in terms of capitalization, the trading volume, although on the rise, is still limited. Liquidity is significantly lacking in long-term instruments. Instruments not longer than 10 years (i.e., 1 year, 3 years, 5 years and 10 years) will typically trade several times a day, but long-term instruments (10 years beyond) may remain several weeks without a single transaction, even for the most liquid "on-the-run" bonds. Comparatively, in Western markets (US, major European countries, and even Japan), on-the-run Treasury bonds, long or short, trade many times per minute, while swaps and futures (bond, interest rate, Fed fund rate) trade virtually every second when the market is open.

国内市场上,尽管股票交易在基金经理甚至日内交易员中已普遍存在,目前却只有银行、券商等金融机构的专业团队进行固定收益资产(债券、利率互换、国债期货,以及期权)的交易,且仅有个别固定收益品种具有较高的流动性并于会计核算中以市价计算。这反映出即使目前市场交易量呈上升趋势,国内固定收益市场的活跃度仍有待增加。其中,长期债券尤其缺乏流动性。在国内,10年期及以下的债券具有较为可观的流动性,每天会有多笔成交;但对于10年期以上的债券,即使是"新券"亦常常出现连续几周没有成交的情况。然而,在西方国家(美国、欧洲主要国家,甚至日本)的债券市场中,任意期限的国债新券在每分钟内均有数次成交,而掉期和期货的成交更是以秒来计算。

Pricing accuracy comes with liquidity since any bias or discrepancy between securities is immediately arbitraged. In China, bid-offer spreads are significantly wider than those in Western markets because market makers cannot afford the risk of too narrow a spread if they cannot immediately hedge an order they fulfilled for a counterparty.

定价的准确性与流动性相关,因为证券之间的任何偏差或差异都会立即被套利。 如果做市商被单边点击成交而无法立即对冲交易,则他承受不起双边报价利差 太窄的风险。因此,国内的买卖价差明显大于西方市场,做市商不得不通过增 加价差来降低风险。

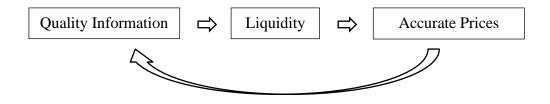
Newly developed by NM FinTech Shenzhen Co., Ltd, SuperCurve® is a system aimed at assisting fixed income traders and brokers for their quoting, pricing, and trading of liquid securities, such as government bonds, government-guaranteed bank bonds, and their derivatives. The goal is to maintain the most accurate and up-to-date image of the fixed income market at any moment in time. We use advanced mathematical and

statistical methods to permanently estimate the impact of any new information, from a quote or a transaction, onto all the other securities, and keep traders informed of the latest market movements and what they mean for their respective portfolio.

深圳九算数据科技有限公司自主研发的 SuperCurve<sup>®</sup>系统旨在辅助固定收益交易员和经纪商进行定价、报价和交易,使用户在任何时刻均能获取固定收益市场上最精准的实时信息和定价数据(如政府债券、政策性银行债券及其衍生产品)。我们采用国际领先的数学方法和统计学技术评估任何新信息(最新报价与成交)对债券市场上所有标的的影响,使得用户可实时追踪最新市场变化及这些变化对特定投资组合的影响。

We are convinced that, by easing access to market information for traders and market makers, we will help increase liquidity in a mutually beneficial feedback loop: more and better quality information means more transactions, hence more accurate pricing, leading to more and better quality information.

我们坚信,一旦交易员和做市商能够轻松获取实时信息和分析数据,则可以大 大增加债券市场的流动性,并由此形成一个良性循环:高质量的报价使市场拥 有更好的流动性并促成更多交易,交易员及做市商据此得到更精准的定价,并 进一步提升市场流动性和报价质量。



#### 1.1. Difficulties in Modeling Chinese Fixed Income Data

#### 1.1. 中国固定收益市场数据建模的难点

#### 1.1.1. Missing Data and Illiquidity

#### 1.1.1. 数据缺失和流动性不足

The liquidity of Chinese T-bond markets is comparable to that of investment-grade corporate bonds in Western markets. At some moments, quotes and transactions are relatively frequent while, at other moments, the market is "too calm" and nothing occurs. Quotes remain unchanged for days, and then, when the market wakes up, a sudden readjustment occurs with a jump that appears as a surprise to traders, while it could have been anticipated, would one had observed other more liquid instruments and assessed the consequence of their yield movements onto the less liquid instruments. This is why simple interpolation models, based on the latest known quote, often provide

lagged and erroneous information, subject to surprises when a new transaction occurs on a segment of the maturity range that hasn't traded for some time.

中国国债市场的流动性相当于西方市场中投资级公司债券的流动性。国债市场的报价和成交有时比较频繁,有时则过于平淡。如果某些债券连续几天没有报价更新,新的报价会造成市场突然的"跳跃",虽然理论上交易员可以根据活跃券与非活跃券之间收益率的相关性预测到这些突然的变化,但实际上,交易员常常对此感到意外。在缺乏流动性的关键期限点,新的成交造成市场突然的"跳跃",使得那些仅仅基于最新成交价的简单插值模型计算得出不准确的定价,并向用户提供滞后和错误的信息。

Addressing this question has been the primary motivation supporting the choice of our mathematical models. We must, for this purpose, identify a set of variables that, on the one hand, is rich enough to fully represent the market, without omitting any sort of relative movement between instruments, while being as parsimonious as possible so that we can estimate the coefficients accurately at any point in time.

因此,在选取数学模型时,我们首先要考虑的因素便是如何解决上述问题。为 达此目标,我们需要确定一组能够充分表达当前市场的期限结构的变量,该变 量应尽可能精简,同时又不遗漏任何债券之间的相对变化,以确保我们在任何 时点都能够准确估算其系数。

#### 1.1.2. Data Errors and Fake Trades

#### 1.1.2. 数据错误和非真实成交

Quotes from brokers may occasionally contain errors or, instead, not be updated for too long, up to becoming off-market. SuperCurve® mathematical model immediately detects inconsistencies between the various quoted instruments and spots those not in line with the bulk. It uses robust statistical techniques to eliminate such data errors and replace them with a correctly estimated up-to-date quote.

经纪商的报价偶尔出现错漏或不及时更新,导致其偏离市场实际收益率水平。 SuperCurve®的数学模型可及时辨别各报价品种之间收益率水平的不一致之处, 发现与市场整体期限结构不符的报价数据,使用强大的统计学手段筛除错误数 据,并代之以准确估计的实时报价。

Data errors occur among actual transactions too. These are often "fake trades", whether pre-arranged or zero-volume trades which do not represent the market when they are entered into the system. Such fake transactions are either by chance in the market, in which case they have little consequences on the fitting, or are off-market, in which case they are ignored, exactly like off-market quotes.

实际成交中也会出现错误数据,即"虚假交易"。此种交易,无论其为提前安排的交易或是"零"成交量的交易,都不能反应当时市场的真实水平。若此类交易恰好符合市场正常水平,则此时对拟合结果影响不大;若此类虚假交易严重偏离市场水平,则其会被忽略。

## 1.1.3. Tax Adjustment

#### 1.1.3. 税收调整

Contrary to other bonds (e.g. corporate and bank bonds), the coupons of T-bonds and specific local government bonds are tax-exempt. This induces a particular appetite for bonds with a high coupon rate, which, consequently, trade at a higher price, that is to say, at a lower yield. SuperCurve® model for government bonds contains a tax correction to account for this difference when comparing securities and fitting the zero-coupon curve.

与其他债券(如信用债和政策性银行债)不同,国债或地方政府债券的息票可享有免税政策。这种政策导致市场对高票息免税品种存在偏好,从而推高其价格,即降低其到期收益率。SuperCurve®在拟合国债零息收益率曲线或为国债定价时,则会充分考虑此类税收因素,并对定价进行相应调整。

#### 1.2. The Status of Yield Curve Modeling in China before SuperCurve®

#### 1.2. SuperCurve®前的中国固定收益市场收益率曲线拟合状况

#### 1.2.1. Yield vs. Maturity Curve

#### 1.2.1. 到期收益率曲线

Although bonds are traded for cash, they are quoted in *yield-to-maturity* (YTM), that is, the implied interest rate at which future cash flows, principal and interest, must be discounted so that their total present value corresponds to the current price.

尽管债券是通过现金进行交易的,但却是以到期收益率(YTM)进行报价的。 到期收益率为可使未来现金流(本金和利息)贴现后的现值总和与当前价格相 等的收益率。

The YTM curve is what is usually shown in a fixed income trading system. Publicly available systems provide the YTM of a selection of the most liquid bonds at a predetermined sequence of maturities (typically ranging around 1m, 2m, 3m, 6m, 12m, 2y, 3y, 5y, 7y, 10y, 15y, 20y, 30y, 50y), then, unless a quote is available on other bonds, simply interpolate the YTM of the two encompassing selected liquid bonds.

通常在固定收益交易系统中显示的是 YTM 曲线。公开可用系统一般会提供在期限结构中流动性最好一系列债券的 YTM (通常为 1m、2m、3m、6m、12m、

2y、3y、5y、7y、10y、15y、20y、30y、50y), 然后系统将通过简单的插值法 计算出两个活跃期限之间无市场报价债券的 YTM。

This approach is overly simplistic because the relation between yield and price of a bond cannot be summarized with only the maturity. It also depends on the coupon rate. A simple yield interpolator, which doesn't account for the coupon rate and the exact payment schedule, will provide erroneous YTM and prices for bonds whose maturity is between two of the selected bonds, with a coupon which is higher or lower than them.

但此类方法过于简单,因为债券 YTM 与价格之间的关系不仅取决于剩余期限,还取决于债券的票面利率。简单的收益率插值法不考虑票面利率和债券付息日期,所以那些到期日在两个所选债券之间的、票面利率与之都不相等的债券将被给予错误的 YTM 和定价。

There is a need for a more accurate and sound yield curve modeling to ensure the permanent consistency of the fixed income market.

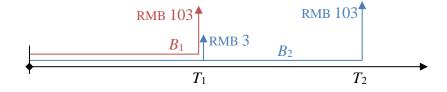
我们需要一个更准确、更合理的收益率曲线模型,以确保固定收益市场始终保持一致性。

#### 1.2.2. Bootstrap Method and Zero-Coupon Curve

#### 1.2.2. 自助抽样法与零息收益率曲线

Let us start with a simple example: we have a one-year bond  $B_1$  and a two-year bond  $B_2$ , both with a 3% coupon and annual payment frequency. The one-year bond  $B_1$  pays RMB 103, coupon plus principal, at maturity  $T_1$  in one year; while the two-year bond  $B_2$  pays a RMB 3 coupon at  $T_1$  in one year, and RMB 103, coupon plus principal, at maturity  $T_2$  in two years.

让我们先举个简单的例子:假设我们有一只一年期债券  $B_1$  和一只两年期债券  $B_2$ ,  $B_1$  和  $B_2$  的票面利率均为 3%并且每年付息一次。一年期债券  $B_1$  在  $T_1$  年到期时支付本息和 103 元;而两年期债券  $B_2$  在  $T_1$  年支付 3 元息票, $T_2$  年到期时支付 103 元的本息和。



The YTM of  $B_1$  applies to the payment at  $T_1$  while the YTM of  $B_2$  applies to both payments, at  $T_1$  and  $T_2$ . Now consider another two-year bond  $B_3$  with the same payment dates  $T_1$  and  $T_2$  but a higher coupon, say 4%. At coupon payment date  $T_1$ , the bond  $T_2$  delivers RMB 4, and at maturity  $T_2$ , it delivers RMB 104.

因此  $B_1$  的 YTM 适用于在  $T_1$  时刻的现金流,而  $B_2$  的 YTM 则被同时应用于  $T_1$  和  $T_2$  两个不同时刻的现金流。现在考虑另一只两年期债券  $B_3$ ,其现金流发生日期与债券  $B_1$  和  $B_2$  相同,但其票面利率为 4%,即债券  $B_3$  在付息日  $T_1$  支付人民币 4元,在到期日  $T_2$  兑付本息和 104 元。

Bond Time	$B_1$	$B_2$	$q_1B_1+q_2B_2$	<i>B</i> <sub>3</sub>
$T_1$	103	3	$103q_1 + 3q_2$	4
$T_2$		103	$103 \ q_2$	104

We can see that it is possible to exactly replicate the payments of  $B_3$  with a combination of the two bonds  $B_1$  and  $B_3$  with quantities  $q_1$  and  $q_2$  such that  $103q_1 + 3q_2 = 4$  and  $103q_2 = 104$ , in other words  $q_2 = \frac{104}{103} \approx 1.0097$  and  $q_1 = \frac{4-3q_2}{103} \approx 0.0094$ .

我们发现,当  $103q_1 + 3q_2 = 4$  并且  $103q_2 = 104$ ,换而言之, $q_2 = \frac{104}{103} \approx 1.0097$ 并且 $q_1 = \frac{4-3q_2}{103} \approx 0.0094$ 时,可以用数量分别为 $q_1$ 与 $q_2$ 的 $g_1$ 与 $g_2$ 债券组合精确复制债券 $g_3$ 的现金流。

In this replication, the price of bond  $B_3$  should be precisely equal to that of the combination of  $B_1$  and  $B_2$  with quantities  $q_1$  and  $q_2$ . When pricing these bonds, we must discount all the payments at  $T_1$  with the same discounting rate for the three bonds, and all the payments at  $T_2$  with the same rate for the two bonds  $B_2$  and  $B_3$ . However, we can apply two different rates at  $T_1$  and  $T_2$ .

鉴于可进行上述复制,债券  $B_3$  的价格应正好等于数量分别为  $q_1$  和  $q_2$  的  $B_1$  和  $B_2$  债券组合的价格。这意味着,当为这些债券定价时,我们必须对在  $T_1$  时刻的三只债券的所有现金流用相同的折现率进行折现,并对在  $T_2$  时刻的两只债券  $B_2$  和  $B_3$  用相同的折现率进行折现,然而我们可以在  $T_1$  和  $T_2$  使用两个不同的折现率。

Would we use, for each bond, its own YTM, that is, one rate for  $B_1$  for the payment at  $T_1$ , and another rate for  $B_2$  for both payments at  $T_1$  and  $T_2$ , then we could not make it consistent with the YTM of  $B_3$ , which would be a third rate, possibly different from those of the first two bonds, and which would also apply to both payments at  $T_1$  and  $T_2$ .

如果将每只债券的到期收益率视作每次付息对应的贴现率,例如,将  $B_1$  的到期收益率视作其在  $T_1$  付息的贴现率,将  $B_2$  的到期收益率视作其在  $T_1$  和  $T_2$  现金流的贴现率, $B_3$  的到期收益率有可能不同于  $B_1$  和  $B_2$ ,而它们都可以用于  $T_1$  和  $T_2$  时刻现金流的贴现,这会造成收益率的不一致。

The bootstrapping method uses this logic to relate the bond prices and their yields to maturity. This technique, which is more accurate than the simple YTM interpolation, is a first attempt to produce a zero-coupon yield curve, a rate that must be applied to every single payment. One first computes the applicable rate for all the bonds having one single payment, either because these are short bonds with no coupon, or just one coupon paid jointly with the principal, or even coupon bonds getting close to maturity with only one single remaining coupon at maturity. This provides a list of rates to be applied at each maturity until some maturity  $T_1$  close to 1 year. Then we consider bonds with two coupons, one before maturity and one at maturity. We know the rate to apply to price the first coupon. If its payment date doesn't fall precisely on one of the short bonds, we may use some interpolation. By deducting the present value of this coupon from the bond price, we get the present value of the payment at maturity and the rate to apply for the second payment date. Repeating this operation provides the rate to use at each maturity until the maturity of the longest bond. This rate that applies to a single payment is called the zero-coupon yield because it corresponds to the YTM of a virtual bond with principal only and no coupon.

自助抽样法(bootstrapping)便是根据这种逻辑将债券价格与其到期收益率联系起来。这比简单的 YTM 插值法更为精确,也是对零息收益率曲线构造的首次尝试,零息收益率被用于每笔现金流的计算。对此,首先,我们需要计算所有处于最后付息周期债券的适用利率,这些债券包括:贴现债券、到期一次还本付息债券,或是处于最后一个付息周期的附息债券。从而获得一个贴现率列表,包含剩余期限接近 1 年的到期日  $T_1$  前每个剩余期限对应的贴现率。随后,需要计算发生两次现金流的债券,一次在到期前,另一次在到期日。此时因已知第一次付息所适用的利率,若它的支付日期与已知处于最后支付周期的短期债券不同,则可以使用下述插值法:从债券价格中减去该息票的现值,得到到期兑付金额的现值及到期日适用的贴现率。重复此操作便可得到每个到期日所适用的利率,直到最长的债券到期。这种适用于单笔现金流支付的利率便被称为零息收益率,因为其对应于虚拟零息债券的 YTM。

This method is efficient and commonly used when we have a limited number of bonds to handle, for instance, the selected most liquid bonds, or to produce a yield curve of Interest Rate Swaps (IRS). However, it has the drawback of propagating errors through the curve, or simple mispricing, due to taxes, liquidity or any other reason. It cannot be applied with a large number of bonds, which always contain small but irreducible inconsistencies. Even when such discrepancies are not explicitly appearing, the shape of the zero-coupon curve is often awkward, with strong, artificial oscillations between neighboring maturities to catch the exact price of each bond, whereas we know that this price cannot be more precise than a tick.

实际上,当我们处理较少个数的债券时(例如仅选择市场上流动性最好的几种债券或拟合利率互换的收益率曲线),这种方法较为常见且相对有效。但由于中国市场的税收、流动性等特殊因素,自助抽样法(bootstrapping)可能会存在

定价错误,并通过曲线传递这些误差。此外,由于无法消除的定价偏差和不一致性,它也不能适用于大量债券的定价。即使这种不一致性并非十分明显,但是为了捕捉每只债券的确切价格,相邻到期日之间的曲线常常会受此影响而呈现出强烈的、虚假的振荡,从而产生明显的误差,导致零息收益率曲线的形状呈现难以拟合的状态。

#### 1.2.3. Parametric and Semi-Parametric Representation

#### 1.2.3. 参数法和半参数法

A parametric representation of the yield curve is a function  $(\theta, t) \to f_{\theta}(t)$  which represents the zero-coupon rate at maturity t, where  $\theta = (\theta_1, ..., \theta_n)$  are the coefficients of the parametric representation, for instance, the coefficients of a polynomial or any other functional shape. In this approach, we use an optimizer to find, at any point in time, the coefficients that best fit the quoted yields and prices of a whole set of securities (bonds or IRS).

收益率曲线的参数法表达是指函数( $\theta$ , t)  $\rightarrow f\theta(t)$ ,它表示在到期日 t 的零息利率,其中 $\theta = (\theta_1, ..., \theta_n)$ 是参数法的系数,比如多项式或其他函数的系数。在此方法中,我们使用了优化器,以便在任何时间均能得到与所有资产(债券或 IRS)的报价收益率和价格最为吻合的系数。

The fitted yield of each bond does not necessarily exactly match the quoted one, which is only a *target* for the fitted curve. A small fitting error is unavoidable if the number of bonds to be fitted exceeds the number of parameters.

每个债券的拟合收益率不必与债券的报价完全匹配,完全匹配只是曲线拟合理 论上的一种目标。若拟合的债券数超过参数数量,这种情况下拟合误差是不可 避免的。

The parametric approach differs from the *bootstrap* because it takes all the set of bonds at once and tries to fit a curve within a specific class of possibilities. Modifying the yield of one long bond may, for instance, impact the whole fitting and, consequently, the fitted yield of shorter bonds, even if this impact is limited, if not negligible.

与自助抽样法不同,参数法将一次性获取所有债券,并尝试对各品种进行曲线 拟合。此时,修改某只长期债券的收益率会影响整个曲线的拟合,当然也会影 响较短期债券的收益率定价,即使这种不可忽略的影响极为有限。

The *semi-parametric approach* is an intermediary approach. In essence, it is a parametric approach but coefficients  $\theta_k$  can be grouped into small subsets where each subset of coefficients only monitors a given portion of the yield curve. This provides more flexibility to fit various shapes of curves, while still benefitting from the

advantages of a fully parametric approach. Cubic splines are a typical semi-parametric technique.

半参数法则是一种中间方法。本质上,这也是一种参数法,但系数 64 可分组为小子集,其中每个系数子集仅监控收益率曲线的给定部分。这就为拟合各种形状的曲线提供了更大的灵活性,同时也仍具有参数法的优点。三次样条就是一种典型的半参数法。

At present, up to our knowledge, no publicly available software in China uses a parametric or semi-parametric fixed income modeling method, the only approach, in our view, that is adapted to the complexity of this market.

目前在中国,没有公开可用的软件使用参数或半参数固定收益建模方法,但在我们看来,此类方法却是应对中国这一复杂市场的唯一解决方案。

#### 2. Features of a "Good Curve"

## 优秀曲线的特征

#### 2.1. Accuracy and Absence of Bias

#### 2.1. 精确性和无偏差性

A sound pricing system should always provide a yield and a price, for every security that corresponds to the most up-to-date quotes. If such quotes are unavailable or outdated, the system is supposed to provide the best estimate of security prices and YTM, which can be used as a reference by traders and market makers to support their trading and quoting activity.

精准的定价系统应能始终为每个资产提供与最新报价最为吻合的到期收益率及价格。在无法得到报价或者报价过时的时候,系统则应能精准预估每只证券的价格和到期收益率(YTM),从而为交易员及做市商的交易、报价活动提供参考。

These estimates must be unbiased in the sense that surprises, when an actual transaction occurs, should happen either way. The system bias is determined as the average underpricing or overpricing of the security when a new quote or transaction occurs. It should be reduced as much as possible.

由于实际交易可能高于也可能低于理论估值,系统提供的估值相对于市场实际不应被整体高估或者低估。系统偏差是估值与实际交易之间的平均差别,这种偏差越小越好。

#### 2.2. Illiquidity, Missing and Erroneous Data

#### 2.2. 低流动性、数据缺失和错误

Compared to Western markets, the Chinese fixed income is nowhere close in terms of liquidity. A good yield curve model must be able to cope with a very irregular data flow. While some bonds, at certain times, may trade every minute, other bonds may stay still for several days, if not weeks, without a transaction. Quotes must be updated, which is not always the case. Missing data is the rule, not the exception, and erroneous or fake data frequently occur. Robust statistical techniques must be implemented to handle this aspect of the data input.

中国固定收益市场的流动性与西方大不相同。一个好的收益率曲线模型必须能够应对非常不规则的数据流。 因为有些债券在某些时候可能每分钟都有交易,但不排除有些债券可能会几天或几周都没有交易。虽然理论上报价应该是时刻更新的,可实践却不尽然,数据缺失是中国市场的一种常态,而非偶然现象。

此外,中国市场上的数据也常常是错误或虚假的,中国亟需引入有效的统计工具来解决这一问题。

#### 2.3. Real-Time and Reactivity

#### 2.3. 实时性和反应性

A real-time curve to handle incoming data is critical to trading, market making, and real-time risk management. Traders expect a system that reflects the current state of the market and not outdated past quotes. New quotes should be immediately incorporated into the system and their impact on the whole market estimated. For instance, if a short bond is newly quoted with a 10 bp increase, other short bonds can also expect the same 10 bp increase of their YTM. Long bonds will react too, but with a lesser movement, say 3 to 5 bp. A good system will accurately measure these dependencies, based on past statistics and events, to update the price of all the bonds each time a new quote arrives.

交易、做市和实时风险管理的关键是,曲线是否能够实时处理输入的数据。交易员期望定价系统能够反映当前市场状态,而非过时报价。同时,要求系统应能立即更新报价,并能预估其对整个市场的影响。例如,当某一短期债券的新报价增长了 10 个基点时,其他短期债券可能也会有 10 个基点的增长预期,长期债券的相应波动可能较小,如增加 3 到 5 个基点。此时,优秀的系统需要能够根据过去的统计数据和活动事件,准确地量化这些波动的相关性,并能在每次出现新报价时,更新所有相关债券的定价。

#### 2.4. Consistency

#### 2.4. 一致性

As explained in Section 1.2.2, the price of the various securities should bear some consistency, in the sense that the zero-coupon rate to apply at each coupon and principal payment should smoothly depend on their payment date. Discrepancies may occur, due to tax rules or other liquidity reasons. Aside from such explanations, one expects some regularity in the zero-coupon curve.

如 1.2.2 节所述,不同资产的价格应当具有一致性。这意味着将未来现金流折现为现值的过程中,用于贴现的即期利率取决于现金流发生的每个付息和兑付日期,并随着零息曲线平滑移动。即使在某些情况下,鉴于税收或流动性因素,有可能出现不一致,但除此之外,零息曲线应具有一定的规律性。

Oscillations and wide variations of the implied zero-coupon rates between neighboring maturities are not signs of a good curve modeling, since they don't reflect the reality of the market perception of interest rates. A good curve model should assign similar rates to similar maturities.

精准的曲线在相邻剩余期限的隐含零息利率不应该出现振荡和大幅波动,不然将无法真实反映市场的实际利率水平。一个准确的收益率曲线模型应能在相近的剩余期限拟合得出相似的利率。

Comparing the YTM of different securities of the same category (e.g. T-bonds) is only relevant when they result from the same zero-coupon curve. The portion of the yield discrepancy that is not explained by the zero-coupon curve is a market anomaly, and often a tradable arbitrage.

仅当同一资产类别(例如国债)不同标的的到期收益率是由同一条零息曲线计算所得时,比较这些到期收益率才有意义。零息曲线无法解释的收益率差异可视为市场异常现象,此时通常可进行套利交易。

#### 2.5. Error and Fake Trades Detection

#### 2.5. 错误和虚假交易检测

By permanently testing the market consistency, anomalies in the quotes and fake transactions are easily identified. Not only should they be eliminated from the curve calibration process, but if the anomaly is not a data error, then some arbitrage trade can be placed.

通过持续测试市场一致性,优秀的曲线可以轻松识别报价异常和虚假交易,并 从曲线的校准过程中将其剔除。若异常现象并非源于数据错误,则可将其视为 套利机会。

#### 2.6. Correlation between Bonds and IRS

#### 2.6. 债券和利率互换的相关性

A fixed income must model both bonds and IRS, and the two models should be consistent. Indeed, many traders use interest rate swaps (IRS) to hedge their bond positions. If the correlation between the two models does not reflect the market reality, they may have a biased perception of their residual risk. A good model will have correlations between rates correspond to the real correlation between the two related securities, whether two rates at different maturities in the same curve, or two rates in two different curves, including between a bond rate and an IRS rate.

固定收益定价中需要对债券和利率互换分别建模,同时这两个模型应具备相关性。实践中不少交易员会使用利率互换(IRS)对冲债券头寸,此时若这两个模型之间的相关系数无法反映市场现实,那交易员可能会对剩余风险的认识产生偏差。优秀的模型使利率之间的相关性与两个相应资产间的实际相关性相对应,包括同一条曲线上不同期限的两个利率,或两条不同曲线上的对应利率(包括债券利率和 IRS 利率)。

# 3. SuperCurve®

#### 3.1. Overview

#### 3.1. 概述

SuperCurve<sup>®</sup> uses the most up-to-date mathematical and statistical techniques to model and price every fixed income security – bond or interest rate swap (IRS) – using all the available information at any moment, both current and past. This involves:

SuperCurve®通过分析当前和历史数据,利用最新的数学和统计学方法,对各种债券或利率互换(IRS)进行定价。其中涉及的方法包括:

- A rigorous fixed-income modeling framework; 严谨的固定收益模型框架;
- Powerful statistical methods to handle irregular and sparse data, errors and fake trades

处理无规律的数据、错误数据和虚假交易的强大统计方法。

This erratic data environment, specific to Chinese markets, invalidates most Western fixed income technology, which assumes a very liquid market and an accurate and complete data feed, up to a minimal number of errors. SuperCurve<sup>®</sup> has been specially designed for the Chinese fixed income market and in all its complexity.

西方固定收益技术假定市场非常活跃、能准确且完整地提供数据,且错误信息十分有限。但是中国市场这种特定的不稳定的数据环境,使大多数西方固定收益技术失效。SuperCurve<sup>®</sup>是专门为中国固定收益市场及其复杂性而设计的。

To handle all the specificities of this market and address all the features required for a "good curve", we have opted for a semi-parametric model, which offers the best possible trade-off between the flexibility to match market data and smoothness in imposing consistency between the prices and yields across the entire maturity range.

为处理中国市场的特殊性,并实现"优秀曲线"的所有特征,我们选择半参数法构建模型。该模型可以在匹配市场数据的灵活性和平滑度之间提供最佳平衡,并在整个到期范围内增强价格与收益率的一致性。

#### 3.2. Features

#### 3.2. 特点

Users of SuperCurve® – brokers, market makers, traders, asset managers, risk managers – benefit from:

SuperCurve<sup>®</sup>的使用者,如经纪商,做市商,交易员,资产管理经理,风险管理 经理等,将从以下方面受益:  Permanently updated accurate prices and yields of all commonly traded fixed income securities.

持续更新所有固定收益资产的准确价格和收益率。

• Unbiased price and yield suggestions, bid and offer, for all securities that haven't received a quote otherwise.

提供所有无报价资产的无偏价格、收益率,及买入和卖出报价建议。

- Historical series (daily and intraday) of quotes and transactions, a historical comparison of model yields with quoted or transacted yields. 提供日间和日内的报价成交时间序列,以及估值收益率与市场实际报价或交易数据的历史对比。
- Identification of market anomalies: yield spreads and butterflies. 识别市场异常,包括: 利差和蝶式交易。
- Suggestions of trades: TB vs. CDB and other types of bonds, on-the-run vs. off-the-run bonds, bond vs. IRS, inter-maturity spreads and butterflies, etc. 给予交易建议,包括: 国债 vs.国开债或其他类型债券; 新券 vs.老券; 期限利差和蝶式利差,等等。

#### 3.3. Fixed Income Market Coverage and Data Source

#### 3.3. 固定收益市场覆盖范围和数据源

The following securities are (currently) handled in the system:

当前的系统可处理以下固定收益资产品种:

- Treasury Bonds (TB) 国债(TB)
- China Development Bank Bonds (CDB) 国家开发银行债券 (CDB)
- Export-Import Bank of China and Agricultural Development Bank of China Bonds (EIBC & ADBC)

中国进出口银行和中国农业发展银行债券(EIBC & ADBC)

- SHIBOR3M Swaps (SHI3M-IRS) SHIBOR3M 利率互换(SHI3M-IRS)
- FR007 Swaps (FR007-IRS) FR007 利率互换(FR007-IRS)

Data, provided by Shanghai CFETS-NEX International Money Broking, contain instruments that range from one day to 50 years, depending on the type of security (TB are regularly issued with 50 years maturity, CDB until 30 years, while IRS are only quoted until 10 years, as of now).

上海国际货币经纪公司提供的数据包含从 1 天到 50 年不等的证券。具体数据取决于资产的类型(就目前而言,国债的常规发行期限最长为 50 年,国开债最长为 30 年,IRS 为 10 年)。

In its current version, SuperCurve® only handles "bullet bonds" of the inter-bank market, that is, non-callable bonds with fixed coupon, non-amortizable principal reimbursed in fine.

在当前版本中,SuperCurve®仅用于处理银行间市场的"子弹债券",即固定利率、不可分期偿还本金的非含权债券。

Bullet bond characteristics are the inception and maturity date, coupon rate and frequency, and for the short bonds, the price at inception. Data provided are their yield-to-maturity (YTM), clean and dirty price and accrued interest.

子弹债券的基本要素主要包括起息日、到期日、票面利率和付息频率,对于短期债券来说,还包括发行价格。可获取的数据包括此类债券的到期收益率(YTM)、净价、全价和应计利息。

IRS exchange a fixed payment against a variable payment. This variable payment is indexed either on the SHIBOR3M, quarterly set in advance, paid in arrears, or on the weekly cumulation of the FR007 rate computed between the quarterly payment dates.

利率互换将固定端和浮动端进行交换。当浮动端基准利率为 SHIBOR3M 时,支付频率和重置频率都为季度,且季前重置季后支付; 当基准利率为 FR007 时,每周重置每季支付,季度内的应计利息按每周重置利率计算并累计。

For both types of swaps, the system receives *par swap rates*, that is, for each maturity, the level of fixed payment one should set for the swap to be "at par" for this maturity.

对于这两种类型的利率互换,系统可获得的数据为"票面"互换利率,即对于每个互换期限,固定利率支付方发生的现金流现值应与互换的票面价值相等。

For each class of security, a zero-coupon yield curve is built, which allows for pricing any security at any time and getting its risk characteristics (e.g. duration, convexity, etc.)

对于每种资产,SuperCurve<sup>®</sup>都会建立零息收益率曲线,实时进行资产定价并计算风险指标(例如久期,凸性等)。

#### 3.4. SuperCurve® Construction

#### 3.4. SuperCurve®的构建

The consistency between the various instruments is ensured by rigorous interest rate modeling for each class of instruments. A "zero-coupon" yield curve is built in each case. Given the irregularity and sparsity of the data feed, a semi-parametric

representation is used instead of a fully non-parametric representation, such as bootstrapping, which would be unable to reflect the mutual influence of various areas of the market onto one another. A "best consistent fit" is provided at each point in time, which may occasionally slightly differ from existing quotes. These small discrepancies are indications of likely market corrections to come, leading to potential trade suggestions in case they exceed the bid-offer spread.

我们对每个债券类型都进行了严谨的利率建模,确保了各种债券类型之间的一致性,同时我们针对各种情况都建立了零息收益率曲线。此外,考虑到数据馈送的不规则性和稀疏性,我们决定使用半参数法,而不使用完全非参数表示法(如自助抽样法),因为后者将无法反映出市场各个期限结构之间的相互影响。这样一来,即使定价偶尔会与报价存在细微偏差,SuperCurve®的曲线也能在每个时间点都确保"拟合的一致性"。实际上,曲线出现的细小差异是在预示市场可能会出现调整,进而向用户提出潜在的交易建议,以免超出买卖价差。

#### 3.4.1. Semi-Parametric Representation of the Zero-Coupon Yield Curve

#### 3.4.1. 零息票收益率曲线的半参数表示

To handle the complexities of the Chinese fixed income market, SuperCurve<sup>®</sup> has chosen a *semi-parametric* approach to model the zero-coupon curve. The spline representation, the number of splines and the fitting algorithm have been carefully chosen to provide consistent and stable prices that permanently reflect the current state of the market.

为了适应中国固定收益市场的复杂情况,SuperCurve®选择使用半参数法来拟合零息收益率曲线。我们审慎选择样条曲线表示方法、样条数量和拟合算法,使得系统能够提供符合一致性的稳定定价,并持续反映当前市场状况。

This semi-parametric representation divides the maturity range into 6 different portions with a smooth connection between the portions, thanks to *cubic splines*. These portions are not equally spaced in the maturity range but equally divided in incoming information flow. This information flow, that is, the amount of new prices or yields per unit of time in each portion of the curve, turns out to be logarithmically distributed across maturities. In other words, there are, on average per day, as many new bond quotes between 1 month and 2 months, between 1 year and 2 years and between 10 years and 20 years. The same can be observed with correlations between YTM at different maturities. For this reason, the cubic splines are not directly built as a function of the maturity T, but as a function of a modified variable  $x = \ln(\mu + T)$ , where  $\mu$  is a maturity shift parameter 1 to avoid the singularity of the ln function in 0.

 $<sup>^1</sup>$  Our research has found that  $\mu=0.25$  (i.e. 3 months) is optimal. 我们研究发现  $\mu=0.25$  (即 3 个月)为最优解。

通过半参数法,将收益率曲线的期限范围划分为 6 个不同部分,并用三次样条将各部分平滑连接。不过,这些部分并非以剩余期限为基础进行等距划分,而是基于输入的信息流。该信息流(即在单位时间内曲线每个部分出现新的价格或收益率的数量)在剩余期限上呈对数分布。换言之,平均每天都会出现同等数量的债券报价,剩余期限分别在 1 个月到 2 个月、1 年到 2 年、10 年到 20 年之间。此类现象也体现在不同剩余期限债券的 YTM 相关性中。基于此,我们不将三次样条函数直接建立在剩余期限 T上,而将其建立于修正变量  $x = \ln(\mu + T)$ 上,其中  $\mu$ 为到期期限偏移参数,用于避免  $\ln$  函数在 0 点的奇异性。

#### 3.4.2. Logarithmic Scale for the Maturity and the Rates

#### 3.4.2. 剩余期限和利率的对数标度

Another aspect of fixed income markets, which has been observed across many countries over various periods and economic conditions, is that the volatility of interest rates is more or less proportional to the level of the rates when the inflation and the rates are high, but independent of their level when the inflation is low and rates are low as well, if not negative, as we often see today. We need a second change of variable in the rate level. The zero-coupon rate Y at maturity T is represented by a modified variable  $y = \ln(\delta + Y)$ , where  $\delta$  is a *rate shift* parameter<sup>2</sup> to allow rates to become negative, up to  $-\delta$ .

固定收益市场的另外一个现象是,当通货膨胀和利率都很高时,利率的波动或多或少都会与利率水平成正比;但在通胀率和利率都很低而又不至于为负数时,如当下我们常见之情景,利率波动又会独立于利率水平,这在许多国家的不同时期和经济形势下都被证实。因此,我们需要对收益率变量进行二次调整,将剩余期限 T 对应的零息利率 Y 表示为调整后的变量  $y = \ln(\delta + Y)$ 。此处  $\delta$  是偏移参数,它使得利率可取负数,最小可以取到- $\delta$ 。

Finally, our cubic splines represent the curve y = f(x), where the range between the minimum and the maximum value of x is equally divided into 6 portions. The final formula for the zero-coupon yield as a function of the maturity becomes:

最终,此三次样条表示曲线 y = f(x),在 x 的最小值和最大值之间被平均地划分为 6 个部分。由此,得到零息收益率和剩余期限的函数关系式:

$$Y(T) = -\delta + e^{f(\ln(\mu + T))}$$

The function f is represented by a combination of basic cubic splines  $\varphi_i(x)$  and  $\psi_i(x)$ :

<sup>&</sup>lt;sup>2</sup> Currently set at  $\delta = 1.5\%$ , we may revise this parameter to a higher value, would rates become more negative in Western markets.

目前设置为 $\delta=1.5\%$ 。若西方市场利率负值增加,此系数的值或可被调高。

函数f通过基础的三次样条 $\varphi_i(x)$  和 $\psi_i(x)$ 组合表示:

$$f(x) = \sum_{i=0}^{6} \alpha_i \varphi_i(x) + \beta_i \psi_i(x)$$

#### 3.4.3. Ridge Least Square Criterion and Oscillations Control

#### 3.4.3. 脊最小二乘准则和振荡控制

Given the coefficients  $\alpha_i$  and  $\beta_i$ , we can compute the theoretical price of each bond  $B_k$  in the market for which we have a quote. Say  $P_k$  is the real quote and  $\hat{P}_k(\alpha, \beta)$  the theoretical price, which depends on the coefficients. We want these prices to be as close as possible for each bond. For this to happen, we minimize the *Least Square Criterion*:

通过给定系数 $\alpha_i$ 和 $\beta_i$ ,我们可以计算出市场上每一只已有报价的债券 $B_k$ 的理论价格。此时, $P_k$ 是一个真实报价, $\hat{P}_k(\alpha,\beta)$ 是由系数决定的理论价格。此外,为使这些价格尽可能接近每一只债券的实际价格,我们最小化最小二乘标准:

$$L(\alpha, \beta) = \sum_{k} \omega_{k} (P_{k} - \hat{P}_{k}(\alpha, \beta))^{2} + \varepsilon H(\alpha, \beta)$$

The first term of this objective function aims at fitting the theoretical price of each bond to its real price. The weights  $\omega_k$  ensure a uniform weighting of the error in yield, rather than in price. They are inversely proportional to the square of the duration of each bond.

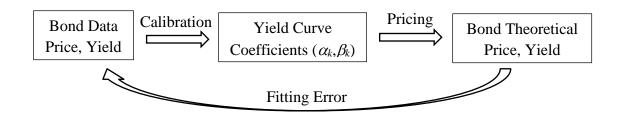
目标函数中,第一项用于确保每只债券的理论价格可与其真实价格相吻合。权重 $\omega_k$ 对收益率偏差而非价格偏差进行平均加权,其与每只债券久期的平方成反比。

The second term is a *ridge regularization*. Its goal is to ensure the regularity of the yield curve. Should we need large oscillations of the curve to fit actual market prices, a small error on the fitting would then be accepted in exchange for a more regular curve. The small regularization parameter  $\varepsilon$  monitors the trade-off between the acceptable fitting error (not more than the bid-offer spread) and the control of oscillations.

第二项是一个脊正则化,目标是确保收益率曲线的规律性。若我们需要用曲线的大幅振荡来表达真实的市场价格,就要接受拟合误差,以获取一条具有规律性的曲线。而此正则化参数  $\varepsilon$  将会平衡可接受的拟合误差(不超过买卖价差)与振荡控制。

This approach is very robust to an irregular input data flow. The set of bonds on which the curve is to be fit may completely change at each re-run of the fitting procedure, provided some global market consistency holds.

此外,每次重新拟合曲线时,参与拟合的债券都可能会全然不同,并无规律可寻。而我们所使用的半参数法能有效处理这种不规则的输入数据流,并有利于维护市场一致性。



#### 3.4.4. Robust Statistics and Error Elimination

#### 3.4.4. 有效统计和误差消除

As mentioned earlier several times, price and yield data in the Chinese fixed-income market are often away from the actual market situation, either because quotes have not been updated for a long time or because these are fake or pre-arranged trades at prices that are not in line with the current market situation. This results in specific prices that cannot be fitted by a parametric or semi-parametric formula. Their impact on the global value of the objective function  $L(\alpha,\beta)$  prevents other bonds from being fitted.

如前所述,中国固定收益市场的价格和收益率数据往往远离实际市场情况,要么是因为报价长期没有更新,要么是因为这些是虚假的或预先安排的交易,它们的价格不符合当前的市场情况,这会使得一些价格无法由参数法或者半参数法拟合,从而影响到目标函数 $L(\alpha,\beta)$ 的全局拟合,最终会影响其他债券的定价。

To address this issue, robust techniques are used. First, replacing the simple square in L by an M-estimator reduces the influence of poorly fitted prices $^3$ . Then by filtering out bonds whose price cannot be correctly matched and re-running the fitting procedure without these bonds. These well-known yet efficient techniques ensure a curve that always correctly fits the price and YTM of bonds in line with the bulk of the market.

为了解决这个问题,我们采取了一种先进技术。首先,我们用 *M* 估计替换 *L* 中的简单平方,以减少估值偏差的影响;之后,系统将过滤掉价格不匹配的债券,并且用其他债券重新拟合曲线。这一技术使得曲线被有效拟合,同时使被定价债券的到期收益率水平与市场实际利率水平保持一致。

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<sup>&</sup>lt;sup>3</sup> The  $M_{\eta}(x) = x^2$  if  $-\eta \le x \le \eta$ , then outside this interval, it is extended by a straight line that smoothly extends the square function. Precisely  $M(x) = 2\eta |x| - \eta^2$  when  $|x| \ge \eta$ .

对于  $M_{\eta}(x) = x^2$ , $-\eta \le x \le \eta$ ,在此区间之外,它被平滑延长平方函数的一条直线延展。准确地说,当 $|x| \ge \eta$ 时, $M(x) = 2\eta |x| - \eta^2$ 。

#### 3.4.5. Real-Time Fitting

#### 3.4.5. 实时拟合

Real-time curve fitting is a more significant challenge. Information on various parts of the maturity range arrives in a disorganized manner. It needs specific techniques to be combined to merge new prices and yields coherently with past information. Based on Bayesian statistics, Kalman filtering is a method to optimally update the whole zero-coupon curve each time one or several new quotes are received. The algorithm finds the most probable change in the coefficients that allow matching the new quotes, given the estimated covariance of the set of curve coefficients ( $\alpha_k, \beta_k$ ), k = 0,...,6. The covariance of coefficients is then updated given the estimated change to remain aligned with actual market statistics.

实时拟合曲线更加具有挑战性。不同剩余期限的信息输入是无序的,因而需要使用特定技术将其进行组合,以使新的价格及收益率与之前的信息保持一致。基于贝叶斯统计,每次在收到一个或多个新报价时,卡尔曼滤波能够高效地更新整条零息曲线,并实时调整系数以匹配新的报价。只要给定曲线系数( $\alpha_{k}$ , $\beta_{k}$ ), k=0,...,6 的协方差估值,系数的协方差就会根据预计的改变进行更新,并与真实市场数据保持一致。

Given the nonlinear relation between coefficients and bond prices, an adaptation of Kalman filtering can handle this nonlinearity. It consists of an extra term added to the objective function:

鉴于相关系数和债券价格之间并非线性关系,我们需要引入卡尔曼滤波这一自适应的方法,于是我们在目标函数中添加了附加项:

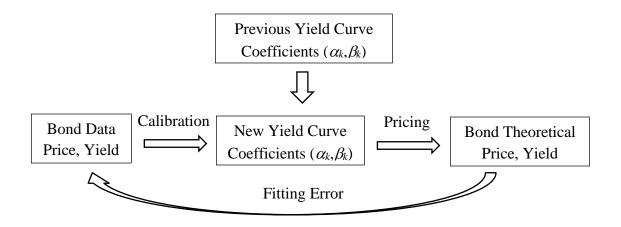
$$L(\alpha,\beta) = \sum_{k} \omega_{k} (P_{k} - \hat{P}_{k}(\alpha,\beta))^{2} + \varepsilon H(\alpha,\beta) + \gamma (d\alpha,d\beta)^{\mathrm{T}} \Gamma^{-1}(d\alpha,d\beta)$$

where  $d\alpha$  and  $d\beta$  are the change in coefficients  $\alpha$  and  $\beta$  with respect to the previous fit,  $\Gamma$  is their covariance matrix and  $\gamma$  is the *gain* parameter.

其中 $d\alpha$ 和 $d\beta$ 是系数 $\alpha$ 和 $\beta$ 相对于之前拟合的变化量, $\Gamma$ 是它们的协方差矩阵, $\gamma$ 是增益参数。

The estimation of covariance was the result of research that could not only rely on Chinese market history. Indeed, despite all precautions described above, some instabilities in fitted coefficients could not be avoided. The covariance matrix is estimated using Bayesian methods with a prior that corresponds to our experience in fixed-income markets in general, and not specifically Chinese markets. The inherited stability is visible in the regularity of the fitted curve, and will benefit a market that needs better information to safely increase its liquidity.

协方差估计不能仅仅依靠中国市场的研究而得出结论。事实上,尽管有上述所有的预先步骤,拟合系数的一些不稳定性仍是无法避免的。协方差矩阵是用贝叶斯方法估计的,利用了在固定收益市场的普遍经验,而非仅根据中国的市场经验。其天然的稳定性不仅体现在拟合曲线的规律性中,而且这种稳定性将安全有效地促进市场流动性,从而令市场更加完善。



# 3.4.6. Fitting the IRS Curve

#### 3.4.6. 拟合 IRS 曲线

Interest rate Swaps are financial instruments that exchange a stream of fixed payments, at a certain frequency (usually quarterly in China), over a given period of time, against another stream of payments that is indexed on some public rate, such as the SHIBOR3M or the FR007.

利率互换是一种金融工具,交易双方在确定期限内以固定的频率交换固定利率和浮动利率(国内支付频率通常为季度),浮动利率以 SHIBOR3M、FR007 等公开市场利率为基准。

Each variable payment corresponds to the interest due on a loan of the principal between the previous payment date or the swap inception and the date of this payment. When cumulating these variable payments, we get the interest due for the loan of the principal over the full life of the swap. That makes the IRS pricing relatively simple, since it is like a bond, with a stream of coupons corresponding to the stream of fixed payments and a principal paid at inception and received at maturity together with the last coupon. Finally, the IRS is equivalent to a virtual bond with the same maturity and a coupon equal to the fixed payment, minus the payment of the principal on the inception date.

每笔浮动利率支付都等于上一付款日(或起息日)至本次付款日之间的应计利息。累计每个浮动利率支付周期的应计利息,可得本金在整个利率互换期限对应的固定利率。这使 IRS 的定价变得相当简单,浮动利率支付方与固定利率支付方发生现金流的时间是一致的,起息前确定名义本金并在到期时收到最后一

次利息(类似债券)。因此,利率互换可视为一只具有相同期限的虚拟债券, 其票面利率等同于固定利率,并减去本金。

We use this equivalence to calibrate the IRS zero-coupon curves in the same way we calibrate the bond curves. Thanks to the fact that we must only fit a small number of swap rates at fixed maturities, we can adjust the yield curve coefficients and ensure that these reference swap rates match their theoretical values from the model.

鉴于此种等价性,我们可以按照债券曲线的拟合方式来拟合 IRS 零息曲线。由于我们只拟合为数不多的期限对应的利率互换收益率,我们可以调整收益率曲线系数,以确保这些参考互换收益率与通过定价模型计算的理论值完全匹配。

#### 3.4.7. Fitting Curves Other Than T-Bonds

#### 3.4.7. 非国债曲线的拟合

Treasury bonds and CDB bonds are the most liquid ones, but CDB bonds are only issued until 30 years maturity, whereas T-bonds exist up to 50 years maturity. The market practice for bonds other than T-bonds, or for IRS, is to estimate their yield, relative to that of an equivalent T-bond, by monitoring the "spread" between the two yields, that is, their difference. Any fluctuation on the T-bond yield would tend to be immediately reflected on the other bond or IRS.

国债与国开债是流动性最高的债券,但近年来国开债的发行期限最长为 30 年,而国债最长有 50 年期。市场交易实践中,常常基于相同剩余期限的国开债(或利率互换)与国债之间的利差对它们进行估值。国债收益率的任何波动都会即时影响到其他类型债券或利率互换的收益率水平。

To take into account this market practice, and ensure that the various yield curves are fitted in a consistent way, SuperCurve<sup>®</sup> first proceeds for each update cycle, with the fitting of the T-bond curve, using the method described in Section 3.4.5. Then the  $(\alpha_{\text{CDB}}, \beta_{\text{CDB}})$  coefficients of the CDB curve are chosen to be those that allow the best match of the newly updated yields of available CDB bonds, with the least possible difference with the newly fitted coefficients of the T-bond curve  $(\alpha_{\text{TB}}, \beta_{\text{TB}})$ , other than a constant spread. This doesn't mean that we impose a constant spread across the maturity range, since every single new CDB bond must match the updated market yield, but the shift in coefficients between  $(\alpha_{\text{CDB}}, \beta_{\text{CDB}})$  and  $(\alpha_{\text{TB}}, \beta_{\text{TB}})$  is made as parsimonious as possible to achieve this fit.

考虑到上述市场惯例,并确保不同收益率曲线的拟合保持一致性,SuperCurve®首先用 3.4.5 节中描述的方法拟合国债收益率曲线。接下来,选择国开债收益率曲线的系数( $\alpha_{CDB}$ , $\beta_{CDB}$ ),使其最大化匹配国开债的最新市场收益率,并且与最新拟合的国债收益率曲线系数( $\alpha_{TB}$ , $\beta_{TB}$ )的差异最小(而不是在国债收益率上加减恒定利差)。这并不意味着我们在整个期限范围内强加一个恒定的利差(因

为每个新的国开债收益率都必须与更新后的市场收益率相匹配),而意味着为了达到拟合目标,我们运用了( $\alpha_{\text{CDB}}, \beta_{\text{CDB}}$ )和( $\alpha_{\text{TB}}, \beta_{\text{TB}}$ )之间的偏移系数,并使之尽可能简化。

This procedure is applied for all curves other than the T-bonds: CDB, EIBC & ADBC and IRS (both SHIBOR3M and FR007).

上述过程被用于除国债外其他品种收益率曲线的拟合,包括:国开债、农发和口行债,以及利率互换(基准利率为SHIBOR3M和FR007)。

Maintaining a strong consistency between the various fitted curves provides extra safety when traders use the SuperCurve<sup>®</sup> system to analyze relative trades between different classes of bonds or when hedging a bond position with IRS.

在交易员用 SuperCurve®系统分析不同品种债券之间的利差或用利率互换对冲债券头寸时,保持不同收益率曲线拟合的强一致性为其提供了额外的安全性。

#### 3.4.8. Tax Adjustment

#### 3.4.8. 税收调整

Contrary to other bonds, the coupons paid by Treasury Bonds (TB) are not taxable. This induces a price difference between these bonds and other taxable bonds, the higher the coupon, the larger the difference. It can be measured by comparing the after-tax outcome of one and the other kind of bonds.

与其他债券不同,国债息票免税。这就导致这些免税债券和其他应税债券之间 会存在价差。且当息票越大时,价差就越大。对此,我们通过比较债券间的结 果来衡量这一价差。

For a bond investor, the remuneration of capital comprises the coupons *C*, taxable or not, depending on the type of bond, and the return on capital. If not invested in the bond, the capital is used for other purposes, and its remuneration is deemed taxable. So, the benchmark is a taxed remuneration of capital. In other words, we have a "pre-tax" and "after-tax" remuneration of capital.

对于债券投资者来说,资本收益由息票 *C* 和资本利得构成,其中息票是否应税则取决于债券的种类。若其资本不投资于债券而用于其他用途,那么其收益是应税的。所以,其基准便是应纳税的资本收益。换句话说,存在税前和税后收益。

When investing in a taxable bond, the "pre-tax" remuneration is the bond yield-to-maturity Y (that is, the carry, plus the short rate, since it is not tax-deductible).

当投资应税债券时,税前收益是债券到期收益率 Y (因其无减税优惠,所以此为套利加短期利率)。

If we consider the combined taxes on the coupons on the one hand and of the principal on the other hand, the combined estimated tax is  $\tau Y$  (and not  $\tau C$  which is just that on the coupons), where  $\tau$  is the applicable tax rate (currently  $\tau$ =25%). Hence the "after-tax" remuneration is  $Y'' = (1 - \tau) Y$ .

若我们考虑息票和本金的综合税,则其综合估计税为 $\tau Y$ (而不是 $\tau C$ ,因为 $\tau C$  只为息票上的税),其中 $\tau$ 是适用税率(目前 $\tau$  =25%)。因此,"税后"收益为Y'' =  $(1-\tau)Y$ 。

There is no special preference towards a high or a low coupon bond, since the "after-tax" remuneration doesn't depend on the coupon. Even a zero-coupon bond has an expected taxable profit on the principal, equal to its yield.

由于税后收益并不取决于息票,因此其不存在对高息票或低息票债券的特别偏好。即使是零息债券,本金的预期应税利润也等于其收益率。

When the bond is not taxable, we consider a virtual taxable bond with an augmented coupon  $C' = \frac{C}{1-\tau}$  which, after tax, delivers the same remuneration. This virtual bond has a "pre-tax" yield-to-maturity Y, which is close to  $\frac{Y}{1-\tau}$  if Y and C are close to one another but may be higher if C > Y. The tax-adjusted YTM of the bond is the after-tax remuneration of the virtual bond, that is  $Y'' = (1-\tau)Y'$ . It is very close to the original YTM if the coupon C is close to Y but higher if the coupon is larger.

当债券不征税时,我们将使用虚拟应税债券,其增加息票为 $C' = \frac{C}{1-\tau}$ ,税后也会产生相同的收益。该虚拟债券的税前到期收益率为Y',若Y = C接近,则税前到期收益率Y'接近 $\frac{Y}{1-\tau}$ ,但若C > Y,则税前到期收益率Y'可能高于 $\frac{Y}{1-\tau}$ 。债券的税收调整YTM为虚拟债券的税后收益,即 $Y'' = (1-\tau)Y'$ 。如果息票C接近Y,则Y''将接近于原始YTM,但若息票C更大,则Y''便会高于原始YTM。

Market consistency aligns the tax-adjusted yield Y'' of the various non-taxable bonds. Bonds with a large coupon will, naturally, have a lower non-adjusted yield Y, hence a higher price, for their tax-adjusted yield Y'' be aligned with that of other bonds.

市场一致性使各种免税债券的纳税调整收益率Y"保持一致。对于高息票债券,因为其税收调整收益率Y"与其他债券一致,则其会将有一个较低的未调整的到期收益率Y和一个较高的价格。

#### 4. Conclusion

# 总结

The Chinese fixed-income market is in rapid evolution, with a substantial increase in its liquidity and the appearance of new derivative instruments, such as futures, options and swaptions. SuperCurve® is the *premier* system that provides everything needed to accompany this evolution: data, software, financial analysis and suggestions of trades and hedges.

中国固定收益市场发展迅速,其流动性增长强劲,期货、期权、互换期权等衍生工具不断涌现。SuperCurve<sup>®</sup>是首个伴随这一演变,为中国固收市场提供所需数据、软件、财务分析以及交易和对冲建议的系统。

Specific challenges of the Chinese markets: illiquidity, irregular input data flow, erroneous data, fake trades, etc. create a barrier to entry that prevents the most classical tools available in Western markets to address this market.

因中国市场上存在其特有困境,例如流动性不足、输入数据流不规则、数据错误、虚假交易等,我们难以应用西方固定收益市场上的经典工具来应对当前中国市场的问题。

SuperCurve® is unique in that it uses the most advanced financial mathematics and statistical methods to tackle each of these challenges:

SuperCurve®的独特之处在于它运用了先进的数学及统计方法,以解决这些挑战。数学方法包括:

- Rigorous yield curve model based on a zero-coupon curve model 基于零息曲线模型的严谨收益率曲线模型
- Semi-parametric curve fitting to handle the irregular data flow 处理不规则数据流的半参数曲线
- Kalman filtering for real-time adjustment with partial information 利用部分信息进行实时调整的卡尔曼滤波
- Robust statistics to eliminate errors and fake trades 用于消除错误和虚假交易的可靠统计方法

This complete suite is:

系统包含以下所有特征:

• **Real-time:** permanently in line with the latest evolutions of the market.

实时:始终与市场最新发展趋势保持一致。

 Comprehensive: covers all liquid fixed income markets – T-bonds, CDB, EIBC & ADBC bonds, interest rate swaps (IRS: fixed-rate vs. SHIBOR3M and FR007).

全面:涵盖所有流动性固定收益市场——国债、国开债、农发和口行债、利率互换(固定端 vs. SHIBOR3M 以及固定端 vs. FR007)。

• **Exhaustive and accurate:** provides a precise quote estimate of every single fixed income security in its coverage, updated in real-time to account for the latest quotes and transactions.

**详尽而准确:**根据实时报价和成交信息,对其覆盖范围内的每一种固定收益资产提供精准的报价估计。

• **Smart:** detects market anomalies and provides trading signals to take advantage of them.

智能: 检测市场异常并以此提供交易信号。

• **Safe:** provides risk characteristics (duration and convexity) of each security – bond or IRS – and proposes hedge ratios to keep yield curve risks under control.

**安全:** 提供每种资产(债券和利率互换)的风险特征(久期和凸性),并提供对冲比率以控制收益率曲线风险。

SuperCurve<sup>®</sup> is an ever-evolving system, designed and supported by a team of highly trained programmers, statisticians, mathematicians, and experienced market professionals. Market innovations, data availability, new products, are continuously monitored and incorporated into this state-of-the-art system.

SuperCurve®是一个持续更新的固定收益量化分析系统,由一群经验丰富的工程师、统计学家、数学家及业界专业人士负责设计和维护,并将不断跟进市场创新、优化数据有效性、进行新产品研发。



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