Final Project – ECE1658 Geometric Nonlinear Control of Robotic Systems

VHC Design For Stable Walking Of the Acrobot

MAIN CONCEPTS COVERED IN THIS PROJECT

- VHCs and transversality
- Hybrid invariance and its relationship to VHC geometry
- Conditions for existence and stability of hybrid limit cycles

1 Preview of this project

This project has two parts. In the first part, you will design a VHC making the acrobot depicted in Figure 1 exhibit a stable walking gait. This document will guide you in detail through the design. In the second part of this project, you will perform a creative extension of this work, which will be the subject of a 15-minute presentation that you'll give at the end of the semester. Some ideas for creative extensions are given in the document.

2 Part 1: Stable Walking Gait Design for the Acrobot

In this project you will be working with the acrobot depicted in Figure 1, a robot that we've already modelled in class. The objective is to make the robot walk in a human-like fashion, with the swing leg rotating counterclockwise while the stance leg rotates clockwise. The formalization of the walking gait design problem presented in this document is due to Emily Vukovich, and it appears in [1]. In this project, however, you will not follow Emily's VHC design.

Since during the walking gait the acrobot's joint angles are confined within intervals of size less than 2π, in this project we will consider q_1 and q_2 to be real variables, so that the configuration manifold of the acrobot is $\mathcal{Q} = \mathbb{R}^2$. A preliminary and overarching restriction we impose is that the joint angle q_1 be confined within the interval $[0, \pi]$, since we want the stance leg to be above ground. Referring to

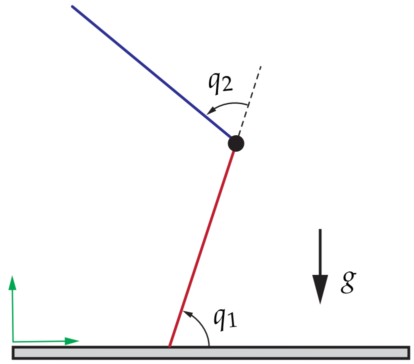

Figure 1: The walking acrobot. The stance leg is red and the swing leg is blue.

Table 1: Acrobot parameters

l	1 m
l_c	0.5 m
m	1 Kg
I_z	$(1/12)ml^2$ Kg m^2
g	9.81 m/sec^2

Figure 2, the impact surface is the segment $S = \{q \in \mathcal{Q} : q_1 \in [0, \pi],\ q_2 = -2q_1 + 2\pi\}$ on which the swing foot is on the ground. We subdivide S into two sub-segments, S^+ and S^-, with S^+ corresponding to configurations where the swing foot is *behind* the stance foot, while S^- corresponding to configurations where the swing foot is *ahead* of the stance foot. These sets are given by

$$S^- := \{q \in \mathcal{Q} : q_1 \in [0, \pi/2],\ q_2 = -2q_1 + 2\pi\},\ S^+ := \{q \in \mathcal{Q} : q_1 \in [\pi/2, \pi],\ q_2 = -2q_1 + 2\pi\},$$

and are depicted in Figure 2. The configuration $\bar{q} := [\pi/2; \pi]$ is the **scuffing point**, and it corresponds to the situation when the swing and stance leg overlap and are orthogonal to the ground.

For a gait to be symmetric, we need the pre- and post-impact configurations of the robot to be identical, which in the context of the acrobot means that in both cases we want an identical **leg aperture** β, displayed on the left-hand side of Figure 2. The angle β is a gait design parameter that you'll tune. Once β is set, the pre- and post-impact configurations $q^- \in S^-$ and $q^+ \in S^+$ are *uniquely* defined and given by

$$q^- = \begin{bmatrix} \frac{\pi-\beta}{2} \\ \pi + \beta \end{bmatrix},\ q^+ = \begin{bmatrix} \frac{\beta+\pi}{2} \\ \pi - \beta \end{bmatrix}. \tag{1}$$

You can verify that q^+ and q^- are related to one another via the relabelling map, i.e., $q^+ = T(q^-)$. The set of all robot configurations corresponding to having the swing foot above ground is

$$\mathcal{W} = \{q \in \mathcal{Q} : q_1 \in (0, \pi/2),\ -2q_1 + 2\pi < q_2 < 3\pi\} \cup \{q \in \mathcal{Q} : q_1 \in (\pi/2, \pi),\ -\pi < q_2 < -2q_1 + 2\pi\}.$$

We call $\mathcal{W}$ the **safe set**.

The objective of this project is to design a VHC curve $q = \sigma(\theta)$ meeting the following specifications.

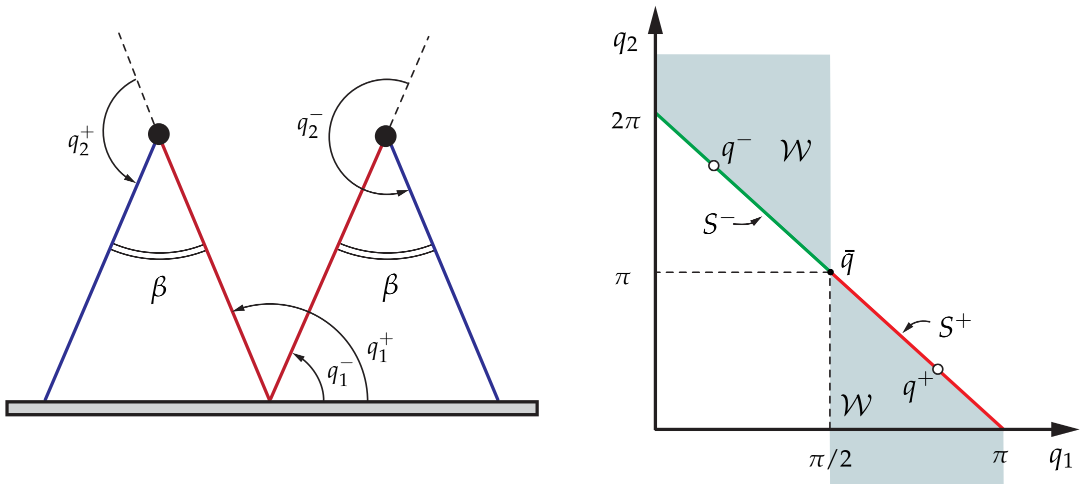

Figure 2: Configurations of the walking acrobot at impact. The configurations with a superscript '+' correspond to the swing foot lying on the ground behind the stance foot, while those with a '−' correspond to the swing foot lying ahead of the stance foot.

Specifications

SPEC 1 **VHC endpoints.** The VHC curve intersects S^+ and S^- at unique points q^+ and q^-, where q^+ and q^- are related to one another via the relabelling map, i.e., $q^+ = T(q^-)$.

SPEC 2 **Regularity.** The VHC should satisfy the transversality condition $B^\perp D\sigma'(\theta)\Big|_{q=\sigma(\theta)} \neq 0$ for all $\theta \in [\theta_a, \theta_b]$, where $\theta_a := \sigma^{-1}(q^+)$ and $\theta_b := \sigma^{-1}(q^-)$.

SPEC 3 **Orientation.** The VHC curve should be oriented in a way consistent with the gait evolution, i.e., such that $\theta_a < \theta_b$.

SPEC 4 **Hybrid invariance.** The VHC curve should satisfy the hybrid invariance condition

$$dT_{q^-}\Delta_{\dot{q}}(q^-)\sigma'(\theta_b) \in \operatorname{span}\{\sigma'(\theta_a)\}. \tag{2}$$

Note that the other requirement of hybrid invariance that $T(q^-) = q^+$ is built into SPEC 1.

SPEC 5 **Swing foot above ground.** The VHC curve should pass through the points $q^+, q^-, \bar{q}$, and elsewhere should be contained in the safe set $\mathcal{W}$.

SPEC 6 **Stable walking.** The hybrid constrained dynamics should possess a hybrid limit cycle corresponding to stable walking.

2.1 VHC design procedure

Based on the specifications above, we look for a VHC curve that roughly looks like the one depicted in Figure 3. This curve starts at q^+, crosses the scuffing point $\bar{q}$, and ends at q^-, and otherwise remains in the safe set $\mathcal{W}$.

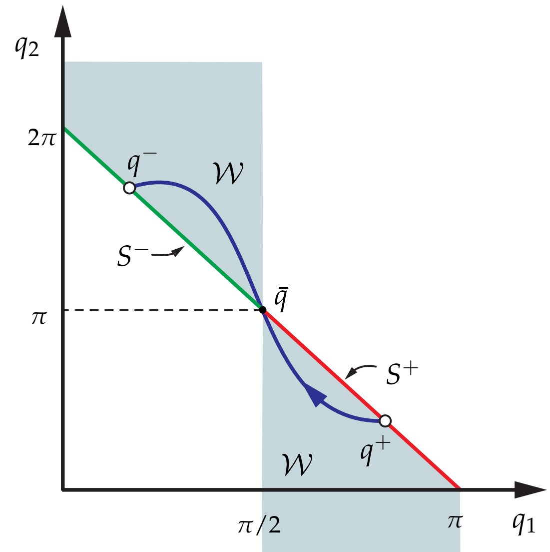

Figure 3: The kind of vhc curve we are looking for.

2.2 Polynomial vhcs

The approach we'll take to solving the gait design problem is to consider the class of polynomial vhcs $\sigma : [\theta_a, \theta_b] \to \mathbb{R}^2$ (with $\theta_a = 0$ and $\theta_b = 1$) defined as

$$q = \sigma(\theta) = \begin{bmatrix} q_1^+ - \theta \tilde{q}_1 \\ \phi_{\mathbf{a}}(\theta) \end{bmatrix}, \tag{3}$$

where $\tilde{q}_1 := q_1^+ - q_1^-$ and $\phi_a : [0, 1] \to \mathbb{R}$ is a polynomial with parameter vector $\mathbf{a} \in \mathbb{R}^k$,

$$\phi_{\mathbf{a}}(\theta) := a_1 + a_2\theta + \cdots + a_k\theta^{k-1},$$

where $a_i, i = 1, \ldots, k$, are the elements of $\mathbf{a}$. As mentioned above, for this parametrization we have $\theta_a = 0$ and $\theta_b = 1$. The first component of σ in (3) ensures that $\sigma_1(0) = q_1^+$ and $\sigma_1(1) = q_1^-$.

The design specifications presented earlier suggest six conditions that the polynomial $\phi_{\mathbf{a}}(\cdot)$ must satisfy:

$$\phi_{\mathbf{a}}(0) = q_2^+ \tag{4a}$$
$$\phi_{\mathbf{a}}(1) = q_2^- \tag{4b}$$
$$\phi_{\mathbf{a}}(0.5) = \pi \tag{4c}$$
$$\phi'_{\mathbf{a}}(0) = f(v_1) \tag{4d}$$
$$\phi'_{\mathbf{a}}(1) = v_1 \tag{4e}$$
$$\phi'_{\mathbf{a}}(0.5) = v_2, \tag{4f}$$

where $f : \mathbb{R} \to \mathbb{R}$ is a function guaranteeing hybrid invariance, see Section 2.3 below.

Noting that $\phi_{\mathbf{a}}$ is linear with respect to a and that the derivative $\phi'_{\mathbf{a}}$ is also a polynomial, the conditions in (2) amount to a *linear* system of equations in the unknown $\mathbf{a} \in \mathbb{R}^k$:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & 0.5 & 0.5^2 & \cdots & 0.5^{k-1} \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & k-1 \\ 0 & 1 & 2(0.5) & \cdots & (k-1)(0.5)^{k-2} \end{bmatrix} \mathbf{a} = \begin{bmatrix} q_2^+ \\ q_2^- \\ \pi \\ f(v_1) \\ v_1 \\ v_2 \end{bmatrix}. \tag{5}$$

The matrix in (5) has full row rank as long as $k \geq 6$, so you'll begin by setting $k = 6$ and later consider having higher degree polynomials.

Let's go over the conditions in (4):

- Conditions (4a) and (4b) and the way in which the first component of σ in (3) is defined guarantee that $\sigma(0) = q^+$ and $\sigma(1) = q^-$ and therefore meet SPEC 1.
- Condition (4c) and the way in which the first component of σ in (3) is defined ensure that $\sigma(0.5) = \bar{q}$, required by SPEC 5.
- By our choice of parametrization, we have $\theta_a = 0$ and $\theta_b = 1$, therefore our parametrization meets the orientation requirement of SPEC 3.
- As we shall see, the hybrid invariance requirement of SPEC 4 is met via the choice of function f appearing in (4d).
- Specifications SPEC 2, SPEC 6, and to some extent SPEC 5, will be checked after the polynomial $\phi_\mathbf{a}$ is found, and will be met by tuning the free parameters β (the leg aperture), v_1, and v_2.

2.3 Hybrid invariance and safe set considerations

In this section we discuss the slope conditions (4d), (4e), and (4f), and in particular the choices of design parameters v_1, v_2 that appear in them. We'll also determine the function $f(v_1)$ appearing in (4d).

For hybrid invariance, we need to ensure that condition (2) is met. This condition relates the tangent vector to the VHC curve at the beginning of the curve, $\sigma'(0)$ to that at the end of the curve[1], $\sigma'(1)$. In (4), we're assigning the slopes of these tangent vectors by setting $\phi'_\mathbf{a}(1) = v_1$ (which determines the slope of $\sigma'(1)$), and $\phi'_\mathbf{a}(0) = f(v_1)$ (which sets the slope of $\sigma'(0)$ in terms of the slope of $\sigma'(1)$). A judicious choice of function $f(v_1)$ will guarantee hybrid invariance. Next, we find this function.

Using the definition of σ in (3), we rewrite (2) as

$$dT_{q^-}\Delta_{\dot{q}}(q^-) \cdot \begin{bmatrix} -\tilde{q}_1 \\ \phi'_\mathbf{a}(1) \end{bmatrix} \in \operatorname{span}\left\{ \begin{bmatrix} -\tilde{q}_1 \\ \phi'_\mathbf{a}(0) \end{bmatrix} \right\}.$$

[1]Recall that, for our parametrization $\sigma(\cdot)$, we have $\theta_a = 0$ and $\theta_b = 1$.

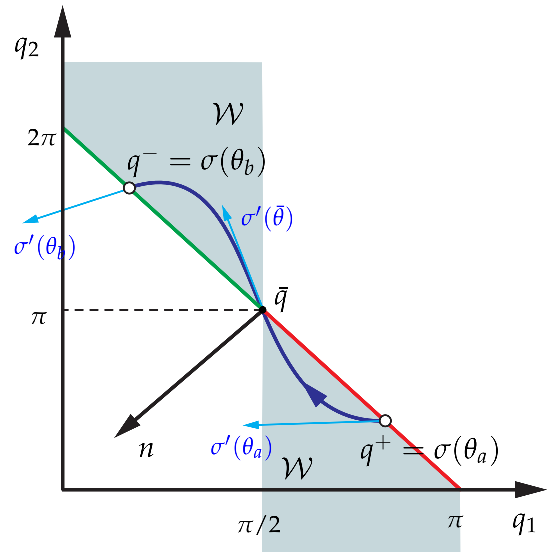

Figure 4: Tangent vectors to the VHC curve at its endpoints and middle point.

Using identities (4d) and (4e), we get

$$\underbrace{dT_{q^-}\Delta_{\dot{q}}(q^-)}_{\mathbf{I}} \cdot \begin{bmatrix} -\tilde{q}_1 \\ v_1 \end{bmatrix} \in \operatorname{span}\left\{ \begin{bmatrix} -\tilde{q}_1 \\ f(v_1) \end{bmatrix} \right\}.$$

The 2×2 matrix $\mathbf{I} := dT_{q^-}\Delta_{\dot{q}}(q^-)$ and the scalar $\tilde{q}_1$ are fixed once β (and hence q^+, q^- through (1)) is fixed, and we need to find $f(v_1)$ satisfying the condition above. The condition above can be equivalently expressed as

$$\begin{bmatrix} f(v_1) & \tilde{q}_1 \end{bmatrix} \mathbf{I} \begin{bmatrix} -\tilde{q}_1 \\ v_1 \end{bmatrix} = 0.$$

A solution $f(v_1)$ exists if and only if

$$\begin{bmatrix} \mathbf{I}_{11} & \mathbf{I}_{12} \end{bmatrix} \begin{bmatrix} -\tilde{q}_1 \\ v_1 \end{bmatrix} \neq 0, \tag{6}$$

and it is given by

$$f(v_1) = -\frac{\tilde{q}_1(-\mathbf{I}_{21}\tilde{q}_1 + \mathbf{I}_{22}v_1)}{-\mathbf{I}_{11}\tilde{q}_1 + \mathbf{I}_{12}v_1}. \tag{7}$$

We have thus obtained a function $f : \mathbb{R} \setminus \{\mathbf{I}_{11}\tilde{q}_1/\mathbf{I}_{12}\} \to \mathbb{R}$ to be used in (4d) to guarantee hybrid invariance.

Geometrically, the situation is this. The parameter v_1 is related to the slope of the tangent vector $\sigma'(\theta_b)$ at the *end* of the VHC curve. The requirement (6) means that the velocity vector $dT_{q^-}\Delta_{\dot{q}}(q^-)\sigma'(\theta_b)$ obtained by applying the impact map to $\sigma'(\theta_b)$ should *not* be orthogonal to the ground, i.e., proportional to $[0; 1]$. Indeed, if said vector were orthogonal to the ground, there wouldn't be any choice of polynomial $\phi_{\mathbf{a}}(\theta)$ making $\sigma'(\theta_a)$ parallel to $dT_{q^-}\Delta_{\dot{q}}(q^-)\sigma'(\theta_b)$ (required by hybrid invariance) because the first component of σ' is $-\tilde{q}_1$, a nonzero number.

There are further restrictions on v_1. In order to meet SPEC 5, we need the vectors $\sigma'(\theta_a)$ and $\sigma'(\theta_b)$ to point to the left of the impact line S, as shown in Figure 3. In other words, letting $n := [-2; -1]$ be the normal vector to the impact line showed in the figure, we need $\langle \sigma'(\theta_b), n\rangle \geq 0$ and $\langle \sigma'(\theta_a), n\rangle \geq 0$, or

$$v_1 \leq 2\tilde{q}_1 \tag{8a}$$
$$f(v_1) \leq 2\tilde{q}_1. \tag{8b}$$

Using the definition of $f(v_1)$ in (7), we can break down condition (8b) into two inequalities for v_1, as follows

$$\text{Either } \left(v_1 > \frac{\mathbf{I}_{11}}{\mathbf{I}_{12}}\tilde{q}_1 \text{ and } v_1 \geq \frac{\mathbf{I}_{21} + 2\mathbf{I}_{11}}{2\mathbf{I}_{12} + \mathbf{I}_{22}}\tilde{q}_1 \right) \tag{9a}$$
$$\text{Or } \left(v_1 < \frac{\mathbf{I}_{11}}{\mathbf{I}_{12}}\tilde{q}_1 \text{ and } v_1 \leq \frac{\mathbf{I}_{21} + 2\mathbf{I}_{11}}{2\mathbf{I}_{12} + \mathbf{I}_{22}}\tilde{q}_1 \right). \tag{9b}$$

Finally, we turn to the parameter v_2 appearing in (4f) and related to the slope of the tangent vector $\sigma'(0.5)$ at the scuffing point $\sigma(0.5) = \bar{q}$. The tangent vector $\sigma'(0.5)$ must point to the interior of the safe set $\mathcal{W}$ in the manner displayed in Figure 3. Since $\sigma'(0.5) = \left[-\tilde{q}_1\,;\, \phi'_{\mathbf{a}}(0.5) \right]$, one way to express this requirement is to impose that the slope of $\phi'_{\mathbf{a}}(0.5)/(-\tilde{q}_1)$ be less than -2, or

$$v_2 > 2\tilde{q}_1. \tag{10}$$

Summary of conditions for hybrid invariance

The parameters v_1 and v_2 are subject to the following inequality constraints

$$v_1 \leq 2\tilde{q}_1 \tag{11a}$$
$$v_2 > 2\tilde{q}_1 \tag{11b}$$
$$\begin{cases} \text{Either } \left(v_1 > \frac{\mathbf{I}_{11}}{\mathbf{I}_{12}}\tilde{q}_1 \text{ and } v_1 \geq \frac{\mathbf{I}_{21} + 2\mathbf{I}_{11}}{2\mathbf{I}_{12} + \mathbf{I}_{22}}\tilde{q}_1 \right) \\ \text{Or } \left(v_1 < \frac{\mathbf{I}_{11}}{\mathbf{I}_{12}}\tilde{q}_1 \text{ and } v_1 \leq \frac{\mathbf{I}_{21} + 2\mathbf{I}_{11}}{2\mathbf{I}_{12} + \mathbf{I}_{22}}\tilde{q}_1 \right) \end{cases} \tag{11c}$$

2.4 Initial VHC design procedure (inputs: β, v_1, v_2)

1. Select a leg aperture angle $\beta \in (0, \pi)$ and find q^+, q^- according to (1). Extract from the impact map[2] the 2×2 matrix $\mathbf{I} = dT_{q^-}\Delta_{\dot{q}}(q^-)$ and find its numerical value.
2. Consider a vhc curve of the form (3), where initially we set $k = 6$ so that $\phi_{\mathbf{a}}(\theta)$ is a polynomial of degree 5.
3. Choose design parameters v_1, v_2 satisfying the inequalities (11).
4. Solve the linear system of equations (5) to find the polynomial parameter vector $\mathbf{a} \in \mathbb{R}^6$.

Write a script with input (β, v_1, v_2) implementing the procedure above. Now, verify your design by performing the following steps.

[2]In other words, from the impact map $\Delta(q, \dot{q})$, $\mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^4$, extract the last two components. These will be a linear function of $\dot{q}$, and $\mathbf{I}$ is the matrix of this function. Programmatically, you can extract $\mathbf{I}$ by computing the Jacobian with respect to $\dot{q}$ of the last two components of Δ.

(a) Set $\beta = 0.316637$, $v_1 = -0.894373$, $v_2 = 1.9$ and implement the procedure above.

(b) Plot the resulting VHC $q = \sigma(\theta)$ with $\theta \in [0,1]$, and verify that it passes through $q^+, q^-, \bar{q}$ and that its tangent vectors at these three points have the correct slope.

(c) Verify that the curve is contained in $\mathcal{W}$ for all $\theta \in (0, 0.5) \cup (0.5, 1)$.

(d) Check whether or not the curve is a regular VHC, i.e., whether $B^\perp D\sigma'(\theta)\big|_{q=\sigma(\theta)}$ is not zero for all $\theta \in [0,1]$. In Matlab this can be done quite efficiently by computing $B^\perp D\sigma'(\theta)\big|_{q=\sigma(\theta)}$ over a fine grid of θ values and checking if the resulting array has any zero crossings.

(e) Finally, verify whether the constraint induces a stable hybrid limit cycle. For this, you need to form expressions for $\Psi_1(\theta), \Psi_2(\theta)$, and use them to find the virtual mass $M(\theta)$ and virtual potential $V(\theta)$ (see the posted `pendubot_vhc` code for an example of how to do that). The existence and stability conditions are

$$
\begin{aligned}
&0 < \frac{\delta^2}{M^-} < 1 \\
&\frac{V^- \delta^2}{M^- - \delta^2} + V_{\max} < 0,
\end{aligned}
\tag{12}
$$

where $M^- = M(1)$, $V^- = V(1)$, $V_{\max} = \max_{[0,1]} V(\theta)$, and δ is given by

$$
\delta = \frac{\langle \sigma'(0), \mathbf{I}\,\sigma'(1)\rangle}{\sigma'(0)^\top \sigma'(0)}.
$$

With the given values of (β, v_1, v_2), your VHC should pass the tests above. If it doesn't, there's an error in your code.

(f) Having verified that the VHC passes all tests, simulate the closed-loop system over several steps and animate the result using the provided Matlab script. Use a feedback linearizing controller that enforces the VHC. For that, use proportional and derivative gains $K_p = 20^2$ and $K_d = 40$, and note that the output you want to stabilize is $y = q_2 - \phi_{\mathbf{a}}((q_1^+ - q_1)/\tilde{q}_1)$. In your controller, use the term $K_p \sin(y)$ in place of $K_p\, y$, as you did in Assignment 1.

(g) Initialize the acrobot on the constraint manifold at an initial condition on the hybrid limit cycle: $(q(0), \dot{q}(0)) = (\sigma(\theta_a), \sigma'(\theta_a)\dot{\theta}_a)$, where

$$
\dot{\theta}_a = \delta \sqrt{\frac{-2V^-}{M^- - \delta^2}}
$$

is the fixed point of the Poincarè map corresponding to the limit cycle, presented in lecture.

Produce an animation of the robot walking for 25 steps.

(h) Perturb the initial conditions, first by choosing them on the constraint manifold but not on the limit cycle, then by picking them outside of the constraint manifold. Gain a sense of the amount of perturbation your controller can accept without compromising stable walking.

2.5 Optimization-based design

Having obtained an initial VHC design, you'll now develop an optimization-based procedure to find the polynomial parameters in the vector $a \in \mathbb{R}^k$. In this approach, instead of finding the parameter vector $\mathbf{a}$ by solving the linear system of equations in (5), you will pose a constrained optimization problem. In this section, you'll let $k \geq 6$ (your choice) and define the augmented parameter vector $X := \text{col}(\mathbf{a}, \beta, v_1, v_2)$. Choose a cost function $J(X)$ and consider the nonlinear program (to be solved using Matlab's `fmincon`)

$$\begin{aligned} &\text{minimize } J(X) \\ &\text{subject to } h(X) = 0 \\ &g(X) \leq 0 \end{aligned}$$

where the equality constraint $h(X) = 0$ is the linear system (5), and the inequality constraint $g(X) \leq 0$ contains the inequalities (11) as well as the following additional requirements:

- For a given X, the resulting VHC should satisfy the transversality condition

$$\left(B^{\perp} D\sigma'(\theta_i) \Big|_{q=\sigma(\theta_i)} \right)^2 \geq \varepsilon > 0,$$

  where ε is a design parameters and $\theta_i, i = 1, \ldots, N$, are uniform samples over the interval $[0, 1]$. (Try $N = 10^3$).

- For a given X, the resulting VHC should give a virtual mass $M(\theta)$ and virtual potential $V(\theta)$ satisfying the existence and stability conditions in (12), where the scalar $V_{\max}$ is calculated by taking the maximum of V over the samples $\theta_i, i = 1, \ldots, N$ used earlier. The samples $M(\theta_i)$ and $V(\theta_i)$ will be obtained numerically.

- You can add more constraints, depending on where your creative process guides you.

You can initialize the optimization using the parameters values $\mathbf{a}, \beta, v_1, v_2$ from the previous section (padding $\mathbf{a}$ with zeros in the appropriate location to embed the sixth order polynomial of the previous section into the higher order polynomial of this section).

Experiment with different cost functions. A couple of ideas:

- You could try minimizing the length of the VHC curve, $\int_0^1 \|\sigma'(\theta)\| d\theta$.
- You could try minimizing v_1^2 to give a constraint that makes the curve as flat as possible at its endpoint.

Experiment also with the degree of the polynomial.

If the optimization ends successfully at a local minimum satisfying all constraints, you will still need to verify the feasibility of the VHC that you've obtained by repeating the checks described in Section 2.4, in particular items (b), (c), (f), (g), (h) in that section.

3 Part 2: Creative Project

Now creatively extend what you've done in part 1 in a direction that appeals to you. You need to be mindful of the difficulty of the project and the fact that there is limited time available. You will give a 15-minute presentation showcasing your results. You will also describe your results in this part in the final report (see the next section).

Some ideas you might consider exploring:

- Changing the acrobot walking gait by changing the formulation of the optimization problem.
- Explore different gaits for the acrobot, such as the walkover gait explored in [1].
- Adapting the VHC design technique presented in part I to design stable walking gaits for the five-link robot that you modelled in Assignment 1.

4 Submission

Submit your code, your videos, and a report meeting the following specifications.

- **Code.** You should submit two folders with code.

  One folder should contain the code to make the acrobot walk that you developed following the procedure in part 1 (Section 2). Your code in this folder should produce two animations, one for the basic VHC of Section 2.4 and one for the improved VHC that you've come up with in Section 2.5.

  The second folder should contain the code for the creative part of the project in part 2 (Section 3), and produce a video (or more, if necessary).

  Include ample comments in your code to make it readable.

- **Videos.** Please submit the two videos from part 1 and any additional videos from part 2.
- **Report.** The report should be brief and to the point. Go over the results you've obtained in parts 1 and 2, and what ideas you've explored. Give a sense of your thought process and how successful or unsuccessful your work has been.

References

[1] E. Kao-Vukovich and M. Maggiore, "On the synthesis of stable walkover gaits for the acrobot," *IEEE Transactions on Control Systems Technology*, vol. 31, pp. 1379–1394, 2023.