Learning curve models and applications: literature review and research directions. International Journal of Industrial Ergonomics, 41, 573-583

Article *in* International Journal of Industrial Ergonomics · September 2011

DOI: 10.1016/j.ergon.2011.05.001

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Volume 41
September 2011
Issue 5
ISSN 0169-8141
ELSEVIER
International Journal of
INDUSTRIAL
ERGONOMICS
In collaboration with IEA and ISOES

International Journal of Industrial Ergonomics 41 (2011) 573–583

Contents lists available at ScienceDirect

International Journal of Industrial Ergonomics

journal homepage: www.elsevier.com/locate/ergon

Learning curve models and applications: Literature review and research directions

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ARTICLE INFO

Article history:
Received 15 March 2010
Received in revised form
3 May 2011
Accepted 6 May 2011
Available online 31 May 2011

Keywords:
Learning curves
Forgetting curves
Production planning
Task allocation

ABSTRACT

Learning curves (LCs) are deemed effective tools for monitoring the performance of workers exposed to a new task. LCs provide a mathematical representation of the learning process that takes place as task repetition occurs. These curves were originally proposed by Wright in 1936 upon observing cost reduction due to repetitive procedures in production plants. Since then, LCs have been used to estimate the time required to complete production runs and the reduction in production costs as learning takes place, as well as to assign workers to tasks based on their performance profile. Further, effects of task interruption on workers' performance have also being modeled by modifications on the LCs. This wide variety of applications justifies the relevance of LCs in industrial applications. This paper presents the state of the art in the literature on learning and forgetting curves, describing the existing models, their limitations, and reported applications. Directions for future research on the subject are eventually proposed.

Relevance to industry: The Learning Curve (LC) models described here can be used in a wide variety of industrial applications where workers endeavor new tasks. LC modeling enables better assignment of tasks to workers and more efficient production planning, and reduces production costs.

1. Introduction

The way workers improve their performance as repetitions of a manual-based task take place has been studied in many industrial segments, such as electronic, automotive, construction, software, and chemical companies (Anderson, 1982; Nembhard and Osothsilp, 2002; Nembhard and Uzumeri, 2000a; Pananiswami and Bishop, 1991; Adler and Clark, 1991; Vits and Gelders, 2002; Hamade et al., 2007; Chen, 2009; Jarkas, 2010; Weber and Fayed, 2010). Several factors may impact the workers' learning process; namely: (*i*) structure of training programs (Terwiesch and Bohn, 2001; Vits and Gelders, 2002; Serel et al., 2003; Azizi et al., 2010); (*ii*) workers' motivations in performing the tasks (Kanfer, 1990; Eyring et al., 1993; Natter et al., 2001; Agrell et al., 2002); (*iii*) prior experience in the task (Nembhard and Uzumeri, 2000a, 2000b; Nembhard and Osothsilp, 2002); and (*iv*) task complexity (Pananiswami and Bishop, 1991; Nembhard and Osothsilp, 2002). Other studies have measured knowledge and dexterity retention after task interruption (Dar-El and Rubinovitz, 1991; Wickens et al., 1998; Nembhard and Uzumeri, 2000b; Jaber and Guiffrida, 2008). The way such factors impact workers' learning process can be analyzed by means of mathematical models named Learning Curves (LCs).

The LC has proven to be an efficient tool to monitor workers' performance in repetitive tasks, leading to reduced process loss due to workers' inability in the first production cycles (Argote, 1999; Dar-El, 2000; Salameh and Jaber, 2000; Jaber et al., 2008). LCs have been used to analyze and control productive operations (Chen et al., 2008; Jaber and Saadany, 2011; Janiak and Rudek, 2008; Lodree et al., 2009; Wahab and Jaber, 2010; Anzanello and Fogliatto, 2010), to allocate tasks to workers according to their learning profiles (Teplitz, 1991; Uzumeri and Nembhard, 1998; Nembhard and Uzumeri, 2000a; Anzanello and Fogliatto, 2007; Heimerl and Kolisch, 2010), to measure production costs as workers gain experience in a task (Wright, 1936; Teplitz, 1991; Sturm, 1999; Nadeau et al., 2010), and to estimate costs of consulting and technology implementation (Plaza and Rohlf, 2008; Plaza et al., 2010).

In view of its wide applicability in production systems and given the increasing number of publications on the subject, we present here a literature review on LCs covering the most relevant models and application scenarios. There are two main contributions in this paper. First, we discuss mathematical aspects of univariate and multivariate LCs, describing their applications, modifications to suit specific purposes, and limitations. Second, we propose directions for future research on LC that are aligned with current trends in production strategy such as Mass Customization.

This paper is organized as follows. Section 2 presents the main families of LC models and their mathematical aspects. Section 3 sets an agenda for future research on the subject. A conclusion is presented in Section 4.

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doi:10.1016/j.ergon.2011.05.001

2. Learning curves: Definition and models

A learning curve is a mathematical description of workers' performance in repetitive tasks (Wright, 1936; Teplitz, 1991; Badiru, 1992; Argote, 1999; Fioretti, 2007). As repetitions take place workers tend to demand less time to perform tasks due to familiarity with the operation and tools, and because shortcuts to task execution are found (Wright, 1936; Teplitz, 1991; Dar-El, 2000).

LCs were empirically developed by Wright (1936) after observing how assembly costs of airplanes decreased as repetitions were performed. Such reduction followed a constant rate as the number of assembled airplanes doubled, giving rise to a rule-of-thumb named "80% learning curve" that was widely applied in the aeronautical industry of the time. According to that rule cumulative assembly costs are reduced on average by 20% as the number of units is duplicated (Teplitz, 1991; Cook, 1991; Badiru, 1992; Argote, 1999; Askin and Goldberg, 2001).

Measures of workers' performance used as dependent variable in LC models include: (*i*) time to produce a single unit, (*ii*) number of units produced per time interval, (*iii*) costs to produce a single unit, and (*iv*) percent of non-conforming units (Teplitz, 1991; Franceschini and Galetto, 2002).

LC parameters may be estimated through a non-linear optimization routine aimed at minimizing the sum of squares error. If convergence is not achieved, which is a typical situation when dealing with non-linear regression, one might change the initial values for the parameters. The model's goodness-of-fit may be assessed through the coefficient of determination (R^2), the sum of squares error or the model adherence to a validation sample.

The wide range of applications conceived for LCs yielded univariate and multivariate models of varying complexity which enabled a mathematical representation of the learning process in several economical segments. Among the univariate models the log-linear, exponential and hyperbolic models are the best known. These are described in the following sections.

2.1. Log-linear model and modifications

Generally viewed as the first formal LC model, Wright's model is also referred to as the "Log-linear Model", with the following mathematical representation:

$$y = C_1 x^b \tag{1}$$

where y is the average time (or cost) per unit demanded to produce x units, and C_1 is the time (cost) to produce the first unit. Parameter b ($-1 < b < 0$) is the slope of the LC, which describes the workers' learning rate. Values of b close to -1 denote high learning rate and fast adaptation to task execution (Teplitz, 1991; Badiru, 1992; Argote, 1999; Dar-El, 2000).

Further mathematical modifications on Wright's model enable estimating the total time (cost) to produce x units, as in Eq. (2):

$$y_{1\to x} = C_1 x^{b+1} \tag{2}$$

and the time (cost) required to produce a specific unit i by means of Eq. (3):

$$y_i = C_1\left[i^{b+1} - (i-1)^{b+1}\right] \tag{3}$$

Numerical results from Eqs. (1) to (3) are summarized in tables for different learning rates (Wright, 1936; Teplitz, 1991), enabling prompt estimation of the time required to complete a task.

Due to its flexibility the log-linear model has been applied to estimate the time to task completion (Teplitz, 1991); to estimate a product's life cycle (Kortge et al., 1994); to evaluate the effect of interruptions in the production rate (Jaber and Bonney, 1996; Argote, 1999); and to assess production rate as product specifications are changed through the process (Towill, 1985). These applications are detailed in the paragraphs to follow.

Some industrial segments are well known for applying log-linear LCs and modifications to model the workers' learning process; namely, the semiconductor industry (Cook, 1991; Gruber, 1992, 1994, 1996, 1998), electronic and aerospace components manufacturers (Garvin, 2000), chemical industry (Lieberman, 1984), automotive parts manufacturers (Baloff, 1971; Dar-El, 2000), and truck assemblers (Argote, 1999). Use of the log-linear LC for cost monitoring is reported by Teplitz (1991), Rea and Kerzner (1997), Spence (1981), and Teng and Thompson (1996).

According to Blancett (2002), and Globerson and Gold (1997) the log-linear curve is the most used LC model for predicting production rate in repetitive operations. Both Globerson and Levin (1987) and Vits and Gelders (2002) state that this model describes most manual-based operations with acceptable precision while displaying a non-complex mathematical structure. Blancett (2002) applied the model in several sectors of a building company, evaluating workers' performance from product development to final manufacturing. With analogous purposes Terwiesch and Bohn (2001) assessed the effect of learning throughout the production process of a new product model. Finally, productivity in different cellular layouts was compared in Kannan and Palocsay (1999) using modifications of the log-linear LC.

Production planning activities may benefit from applications of the log-linear LC, as reported by Kopcso and Nemitz (1983), Muth and Spremann (1983), Salameh et al. (1993), Jaber and Bonney (1999, 2001, 2003), Rachamadugu and Tan (1997), Pratsini (2000), and Wahab and Jaber, 2010. These authors investigate the impact of workers' learning on inventory policies, Optimal Lot Size (Q) determination, and other production planning activities.

Studies reporting the integration of log-linear LCs to tools designed to assist production control have also been reported in the literature. Yelle (1980, 1983), Kortge (1993) and Kortge et al. (1994) combined LCs and Product Life Cycle Models – see Cox (1967) and Rink and Swan (1979), among others – aiming at improving production planning. Pramongkit et al. (2000) suggested the combination of LC and the Cobb-Douglas function to assess how specific factors (e.g. invested capital and expert workforce) affect workers' learning in Thai companies. Similarly, Pramongkit et al. (2002) used a log-linear LC associated to the Total Factor Productivity tool to assess workers' learning in large Thai companies. Finally, Karaoz and Albeni (2005) integrated LCs and indices describing technological aspects to evaluate workers' performance under production runs of long duration.

The combination of log-linear-based LCs and quality control techniques was suggested by Koulamas (1992) to evaluate the impacts of product redesign on process quality and cost, while Tapiero (1987) established an association between learning process and quality control in production plants. Teng and Thompson (1996) assessed the way workers' learning rate influences the quality and costs of new products in automotive companies. Further, Franceschini and Galetto (2002) used LCs to estimate the reduction of non-conformities in a juice production plant as workers increased their skills. Jaber and Guiffrida (2004) proposed modifications on Wright's LC for processes generating defects that require reworking, namely the quality learning curve (QLC). Jaber and Guiffrida (2008) investigated the QLC under the assumption that production is interrupted for quality maintenance aimed at bringing it back to an in-control state, while Yang et al. (2009) proposed a quality control approach integrating on-line Statistical Process Control and LCs. Finally, Jaber and Khan (2010) suggested modifications in the composite LC developed by Jaber and Guiffrida (2004) to address quality problems (e.g. product rework and scraps, among others) due to workers' learning process. In addition, Jaber and Khan (2010) performed a sensitivity analysis on the LC parameters aimed at describing the production and rework processes.

Log-linear LCs have also been applied in the service sector. Chambers and Johnston (2000) report the application of LC modeling in two service providers: a large air company and a small bank. Saraswat and Gorgone (1990) evaluated the performance of software installers in companies and private residences. In addition, Sturm (1999) verified a 15% cost reduction in the process of filling out clinical forms as the number of forms doubled.

Other interesting log-linear-based LCs approaches include Müller et al. (2010), who applied a simple logarithmic LC to assess the learning demands of replacing a computer mouse by a modified pen with similar functions. The LC demonstrated that the traditional mouse provides the user with a higher final performance when compared to the modified pen. Lee and Lee (2010) also applied a simple LC to evaluate the adaptation of users to a special mouse (namely, multiple mouse wheels) that provides multiple manipulation positions for scrolling. Finally, an interesting method for job rotation in manufacturing systems is proposed by Azizi et al., 2010; LCs enabled assessing workers' learning and forgetting processes during a specific production horizon.

Due to its simplicity and wide applicability, log-linear LCs have been thoroughly investigated regarding limitations and modifications for specific purposes (Baloff, 1966; Zangwill and Kantor, 1998, 2000; Waterworth, 2000). These modifications usually aim at eliminating inconsistencies in the mathematical structure of the log-linear LC.

Hurley (1996) and Eden et al. (1998) state that Wright's model yields execution times equal to zero under a high number of repetitions, which does not adhere to real applications. To overcome that, the authors propose the inclusion of a constant term in Wright's model. Further, Globerson et al. (1989) claim that Wright's model does not take into account workers' prior experience, which clearly impacts on production planning and workforce allocation.

Another limitation of Wright's LC is related to inconsistencies in definition and inferences regarding LC outputs. Towill (1985, 1990) and Waterworth (2000) claim that many applications consider the mean execution time until unit x and the specific execution time of unit i as analogous. For that, Smunt (1999) proposed an alternative definition of repetition based on the Continuous Learning Theory.

The variability in performance data collected from a process may also lead to poorly fitted LC models (Yelle, 1979). Globerson (1984) and Vigil and Sarper (1994) state that imprecise estimation of the learning parameter b jeopardizes the LC's predictive ability. They suggest using confidence intervals on the response estimates for predicting a process production rate. Globerson and Gold (1997) developed equations for estimating the LC's variance, coefficient of variation and probability density function. Finally, Smunt (1999) proposed modifications on Wright's model to embrace situations where parameter b changes as the process takes place, while Smunt and Watts (2003) proposed the use of data aggregation techniques to reduce variance of LC predicted values.

The use of cumulative units as an independent variable has also received attention in the LC literature. Fine (1986) argues that the number of produced units may hide learning deficiencies, since they do not take into account the quality of produced units. To overcome that, the author modified the LC to consider only data from conforming units. Li and Rajagopalan (1997) extended Fine's (1986) idea to include both conforming and non-conforming data in the LC model. Finally, Jaber and Guiffrida (2004) proposed modifications in Wright's model aimed at monitoring processes with high percent of non-conforming and reworked units.

Modifications in Wright's model were initially proposed to adapt the equation to specific applications, and then recognized as alternative models. One such model is the Stanford-B presented in Eq. (4), which incorporates worker's prior experience.

$$y = C_1(x+B)^b \tag{4}$$

Parameter B, corresponding to the number of units of prior experience, shifts the LC downwards with respect to the time/unit axis (Teplitz, 1991; Badiru, 1992; Nembhard and Uzumeri, 2000a). The model was tested in assembly stages of the Boeing 707, as well as in improvement activities performed thereafter in the product (Yelle, 1979; Badiru, 1992; Nembhard and Uzumeri, 2000a).

DeJong's model in Eq. (5) incorporates the influence of machinery in the learning process:

$$y = C_1\left[M + (1-M)x^b\right] \tag{5}$$

where $M\,(0 \le M \le 1)$ is the incompressibility factor that informs the fraction of the task executed by machines (Yelle, 1979; Badiru, 1992). When $M = 0$ there is no machinery involved in the task, while $M = 1$ denotes a task completely executed by machinery, where no learning takes place (Badiru, 1992).

The S-curve model aims at describing learning when machinery intervention occurs, and the first cycles of operation demand in-depth analysis. The model results from merging DeJong's and Stanford-B's models, as presented in Eq. (6). Parameters in the model maintain their original definitions (Badiru, 1992; Nembhard and Uzumeri, 2000a).

$$y = C_1\left[M + (1-M)(x+B)^b\right] \tag{6}$$

In the Plateau model, an additive constant C describing steady-state worker's performance is added to the log-linear model; see Eq. (7). The steady-state is reached after learning is concluded or when machinery limitations block workers' improvement (Yelle, 1979; Teplitz, 1991; Li and Rajagopalan, 1998).

$$y = C + C_1x^b \tag{7}$$

Fig. 1 gives a comparison of LC profiles generated by the models presented so far; in all models performance is measured in terms of

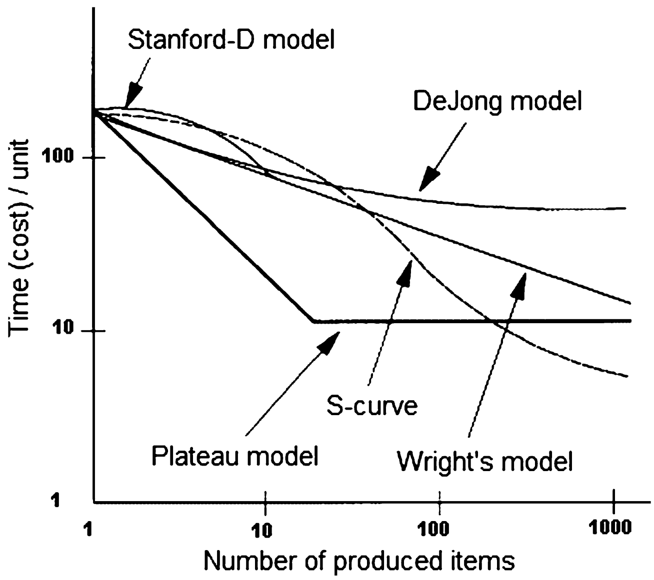


Fig. 1. Profiles from different LC models in logarithm scale (Adapted from Badiru, 1992).

time (cost) to produce each unit. A comprehensive comparison of several LCs discussed above is reported in Hackett (1983).

In addition to the LC models presented above, other not so often cited log-linear-based LCs were proposed in the literature. Given their complexity, those models find very specific applications. Levy's (1965) Adapted Function is one such model:

$$My = \left[\frac{1}{\beta} - \left(\frac{1}{\beta} - \frac{x^b}{C_1}\right)k^{-kx}\right]^{-1} \tag{8}$$

where β is a task-defined production coefficient for the first unit, and k is the workers' performance in steady-state. Remaining parameters are as previously defined.

Focused on production runs of long duration, Knecht (1974) proposed an alternative Adapted Function model that allows evaluating the production rate as parameter b changes during the production run; see Eq. (9). Parameters are as previously defined.

$$y = \frac{C_1 x^{b+1}}{(1+b)} \tag{9}$$

A summation of LCs characterized by n different learning parameters b, proposed by Yelle (1976), is given in Eq. (10). The resulting model could be applied to production processes comprised of n different tasks. However, according to Howell (1980) the model in Eq. (10) leads to imprecise production rate estimates.

$$y = C_1 x_1^{b1} + C_2 x_2^{b2} + \ldots C_n x_n^{bn} \tag{10}$$

Alternative LC models were developed following the log-linear model's principles, although relying on more elaborate mathematical structures to describe complex production scenarios. We only explain the purpose of such models in that the demonstration of all mathematical details would overload the exposition. Klenow (1998) proposed an LC model to support decisions on updating production technology. Demeester and Qi (2005) developed an LC customized to situations in which two generations of the same product are under production (i.e. old and new models). Their LC helps identifying the best moment to allocate learning resources (e.g. training programs and incentive policies) to produce the new model.

Mazzola et al. (1998) developed an LC-based algorithm to synchronize multiproduct manufacturing in environments characterized by workers' learning and forgetting processes. Gavious and Rabinowitz (2003) proposed an approach using LC to evaluate training efficiency of internal resources in comparison with that of outsourced resources. Similarly, Fioretti (2007) suggested a disaggregated LC model to analyze complex production environments in terms of time reduction for task completion. Park et al. (2003) proposed a multiresponse LC aimed at evaluating knowledge transference at distinct production stages in an LCD (Liquid Crystal Display) factory, while Badiru and Ijaduola (2009) integrated the concept of Half-life traditionally used in nuclear applications to LC models aimed at predicting workers' performance.

The integration of LC and scheduling techniques was introduced by Biskup (1999), analyzing the effect of learning on the position of jobs in a single machine. Mosheiov and Sidney (2003) extended that approach by combining job dependent LCs (i.e. LCs with a different parameter for each job) to programming formulations aimed at minimizing flow-time and makespan in a single machine, as well as flow-time in unrelated parallel machines. Finally, Okolowski and Gawiejnowicz (2010) proposed a novel heuristic for scheduling jobs in a parallel-machine scenario where job processing times were estimated through the general DeJong's LC. Other applications relating learning effects to scheduling scenarios can be found in Janiak et al. (2009), Wu et al. (2011), and Cheng et al. (2011).

2.2. Exponential models

Exponential LCs rely on a more complete set of parameters as compared to log-linear models. Such parameters enable extracting additional information on the workers' learning process, yielding more precise estimates of production rate than those provided by log-linear models (Nembhard and Uzumeri, 2000a).

Seminal studies on exponential LCs are due to Knecht (1974), who integrated exponential and log-linear functions to improve predictions in long duration production runs. The proposed model is given in Eq. (11).

$$y = C_1 x^b e^{cx} \tag{11}$$

where c is a second constant; other parameters are as previously defined.

Three exponential LC models are frequently discussed in the literature; they are: the 3-parameter exponential, the 2-parameter exponential, and the Constant Time models. The 3-parameter exponential LC model is as follows:

$$y = k\left(1 - e^{-(x+p)/r}\right) \tag{12}$$

where y describes the worker's performance in terms of number of items produced after x units of operation time ($y \geq 0$ and$x \geq 0$). There are three parameters in Eq. (12)'s LC: (i) k, which is the maximum worker's performance when the learning process is concluded, given in number of items produced per operation time ($k \geq 0$); (ii) p, which corresponds to worker's prior experience evaluated in time units ($p \geq 0$); and (iii) the learning rate r, also given in time units.

Mazur and Hastie (1978) state that the LC in Eq. (12) provides a poor fit to processes characterized by workers submitted to complex and demanding new tasks. On the other hand the model fits well when workers have prior experience.

In the 2-parameter exponential LC parameter p is not present, offering poorer fit to performance data if compared to the 3-parameter exponential LC (Mazur and Hastie, 1978).

The Constant Time model proposed by Towill (1990) is structurally similar to the 3-parameter exponential LC, as shown next:

$$y = y_c + y_f\left(1 - e^{-t/\tau}\right) \tag{13}$$

In Eq. (13), y_c corresponds to the initial worker's performance (in terms of number of items produced per time), and y_f is the maximum performance when worker's learning is completed, given in the same units. Variable t is the cumulative operation time (analogous to x in the previous models) which enables easier estimation of the time required to achieve a certain performance level. Towill (1990) recommends the Constant Time model for processes where performance data collection starts after a short adaptation of workers to the task.

Naim and Towill (1990) added trigonometric functions to the model in Eq. (13) to better describe cases where cyclical variations in performance are verified. Further, Howell (1990) evaluated the impact of inaccurate parameter inputs in the model's predictive ability, and proposed approaches to achieve convergence in complex modeling situations. Finally, Dardan et al. (2006) applied the Time Constant model in a hardware company. The objective was to evaluate the relationship between workers' learning process and duration of technological investments.

2.3. Hyperbolic models

Mazur and Hastie (1978) proposed an LC model relating the number of conforming units to the total number of units produced, which is represented by the 2-parameter hyperbolic curve given in Eq. (14). In the proposed model, x describes the number of conforming units, and r is the number of non-conforming units; thus, y corresponds to the fraction of conforming units multiplied by a constant k.

$$y = k\left(\frac{x}{x+r}\right) \quad (14)$$

For learning modeling purposes, parameters in Eq. (14) resemble those in the 2-parameter exponential model, with y as the number of items produced in x units of operation time, k as the maximum performance level, and r as the learning rate (Nembhard and Uzumeri, 2000a).

Mazur and Hastie (1978) also proposed the inclusion of a parameter p in Eq. (14) to allow considering worker's prior experience in executing the task. That leads to the 3-parameter hyperbolic LC, given in Eq. (15).

$$y = k\left(\frac{x+p}{x+p+r}\right)$$

Parameters in Eq. (15) are defined as those in the 3-parameter exponential model in Eq. (12). Uzumeri and Nembhard (1998), and Nembhard and Uzumeri (2000a) improved the definition of parameter r associating it to the time required to achieve production level $k/2$, which is half of the maximum performance level k, as in Fig. 2. A worker presenting high values of r requires long practicing to achieve k, displaying slow learning.

Uzumeri and Nembhard (1998), and Nembhard and Uzumeri (2000a) state that r acts as shape factor in the hyperbolic model, leading to three possible learning profiles: (i) $r > 0$ – the curve presents an increasing profile until k, representing the typical behavior of workers performing new tasks; (ii) $r \rightarrow 0$ – the curve follows a horizontal pattern, denoting absence of workers' improvement; and (iii) $r < 0$ – the curve follows a decreasing performance pattern, usually associated to fatigue or forgetting. Fig. 2 depicts profiles generated by the 3-parameter hyperbolic model according to parameter r.

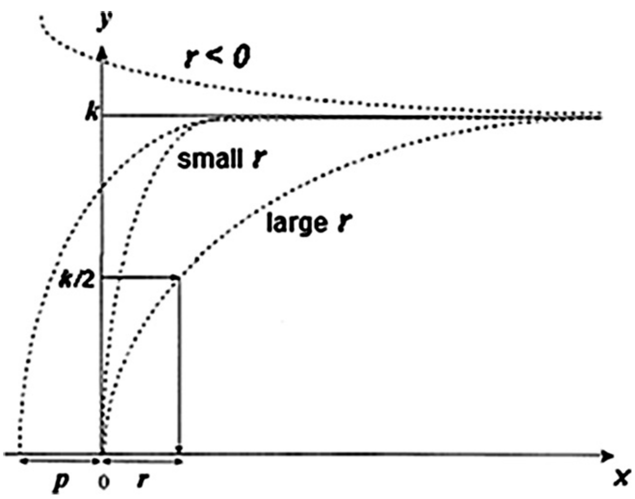


Fig. 2. Profiles generated by the 3-parameter hyperbolic model (Adapted from Uzumeri and Nembhard, 1998).

Mazur and Hastie (1978) exhaustively compared the 3-parameter hyperbolic and exponential models using process data. They found that parameters p and r assume similar values, while parameter k is generally underestimated by the exponential model. In addition, the 3-parameter hyperbolic model presented better fit to the data analyzed, based on the coefficient of determination (R^2).

The 3-parameter hyperbolic model also outperformed 10 LC models in the analysis presented by Nembhard and Uzumeri (2000a). LC models were evaluated in terms of efficiency, stability, parsimony and ability to model scenarios with negative learning (i.e. forgetting). Further, Anzanello and Fogliatto (2007) found the hyperbolic model to be more robust in comparison with the 3-parameter exponential and Time Constant models.

The 3-parameter hyperbolic LC has enabled efficient allocation of tasks to workers aimed at improving production systems. Uzumeri and Nembhard (1998), and Shafer et al. (2001) applied the model to data from a population of workers submitted to new tasks. The authors concluded that fast learners (workers presenting low values of r) tend to achieve lower maximum performance k if compared to slow learners (workers presenting high values of r). Therefore, they propose allocating fast learners to short duration tasks, while slow learners should perform tasks of long duration, given their final performance. With similar objectives, Nembhard and Osothsilp (2002) evaluated the effects of task complexity on the allocation of tasks to workers, while Nembhard and Uzumeri (2000b) evaluated distinct workers' profiles in terms of dexterity gaining and retention under different tasks. Anzanello and Fogliatto (2007) used the 3-parameter hyperbolic LC to allocate tasks to workers according to the duration of production runs in a shoe manufacturing process. Finally, Nembhard and Uzumeri (2000a) stated that the 3-parameter hyperbolic LC can also be used to evaluate the efficiency of training programs.

Other applications of the 3-parameter hyperbolic include Anzanello and Fogliatto (2010), who estimated processing times required by workers to complete a production lot through LC integration. Such processing times were then combined to novel scheduling heuristics aimed at minimizing the sum of earliness and tardiness in manufacturing processes. That LC model has also been used by Anzanello and Fogliatto (2011) to improve clustering performance in a shoe manufacturing process. The original set of clustering variables consisted of two groups of variables: (i) a set of process experts' variables subjectively assessing product complexity, and (ii) a set of learning related variables represented by the parameters of the LC. Variables were selected using a leave-one-variable-out-at-a-time approach and the performance of the clustering procedure evaluated by means of the Silhouette Index. Clustering performance was significantly improved by using a reduced combination of the two sets of variables.

2.4. Comparison of univariate models

A graphical representation of the LC models presented so far is given in Fig. 3. The curves are generated using the same LC parameters for comparison purposes. We also compare the models' capability to describe specific situations; e.g. machinery dependent scenarios and workers' forgetting. We group curves according to the nature of the dependent variable y, which may be described in terms of "number of units/operation time" or "time/unit".

2.5. Multivariate models

Extensions of the traditional LC models are required when modelingmodeling learning scenarios characterized by quantitative

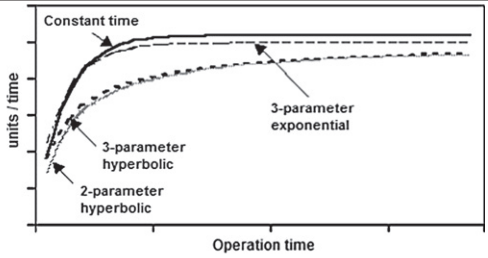
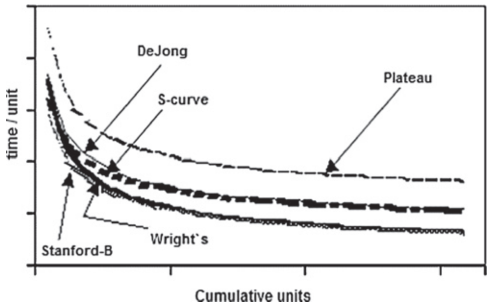
MODEL	MATHEMATICAL REPRESENTATION	NUMBER OF PARAMETERS	FORGETTING MODELING	PRIOR EXPERIENCE	MACHINERY MODELING	LC PROFILES
2-PARAMETER HYPERBOLIC	$y=k\,[x/(x+r)]$	2	√			
3-PARAMETER HYPERBOLIC	$y=k\,[(x+p)/(x+p+r)]$	3	√	√		
3-PARAMETER EXPONENTIAL	$y=k\,(1-e^{-(x+p)/r})$	3		√		
CONSTANT TIME	$y=y_c+y_f(1-e^{-x/t})$	3		√		
WRIGHT`S	$y=Cx^{-b}$	2				
PLATEAU	$y=B+Cx^{-b}$	3				
STANFORD-B	$y=C(x+B)^{-b}$	3		√		
DEJONG	$y=C[M+(1-M)x^{-b}]$	3	√		√	
S-CURVE	$y=C[M+(1-M)(x+B)^{-b}]$	4	√		√	

Fig. 3. Comparative analysis of univariate LCs.

and qualitative factors (Badiru, 1992). Multivariate LCs are based on two or more independent variables, and often display the following generic structure:

$$C_x = K \prod_{i=1}^{n} c_i x_i^{b_i} \tag{15}$$

where K is the performance (cost) to produce the first unit, and c_i is the coefficient for the independent variable i; other parameters are as previously defined. Note that multivariate LC models designate LCs with more than one independent variable, and are not related to multivariate statistical techniques.

LC models relying on two independent variables are analyzed using surface graphs, as depicted in Fig. 4; x_1 and x_2 may represent operation and training times, for example.

Gold (1981) and Camm et al. (1987) used relations similar to that in Eq. (15) to monitor production costs as a function of the following independent variables: number of items produced, production rate, duration and cost of training programs, and task complexity. McIntyre (1977) and Womer (1979) proposed alternative multivariate LCs based on integration procedures, which present limited application due to their complexity. Recently, Hamade et al. (2009) used univariate and multivariate LCs to analyze CAD (computer-aided design) procedural and cognitive data describing the performance of trainees.

Badiru (1992) compared the predictive ability of univariate and multivariate LCs. The author states that multivariate models provide relevant information on variables' interactions, but the addition of non-significant variables jeopardizes the model's quality. Estimation of parameters for multivariate models can become numerically unstable and be affected by multicollinearity among variables, thereby making for low quality models. The use of univariate models should thus be preferred when the importance of additional independent variables is not clear in describing the learning process.

Once the multivariate model is defined, a goodness-of-fit analysis is performed using traditional criteria; e.g. R^2 and the sum of squares error. Finally, we emphasize that reports on multivariate LC models and applications are very limited in the literature.

2.6. Forgetting models

Frequent interruptions in the production process and modifications in product specifications impose extra challenges to workers in terms of resuming activities (Hewitt et al, 1992; Jaber, 2006). Forgetting is a major consequence of interruptions, and becomes evident by (*i*) reduction in production rate after an inactive period, and (*ii*) manufacturing of lower quality products compared to those produced during continuous operation, especially in the first cycles after the process is resumed. Globerson et al. (1989), Argote (1999), Dar-El and Rubinovitz (1991), Dar-El (2000)

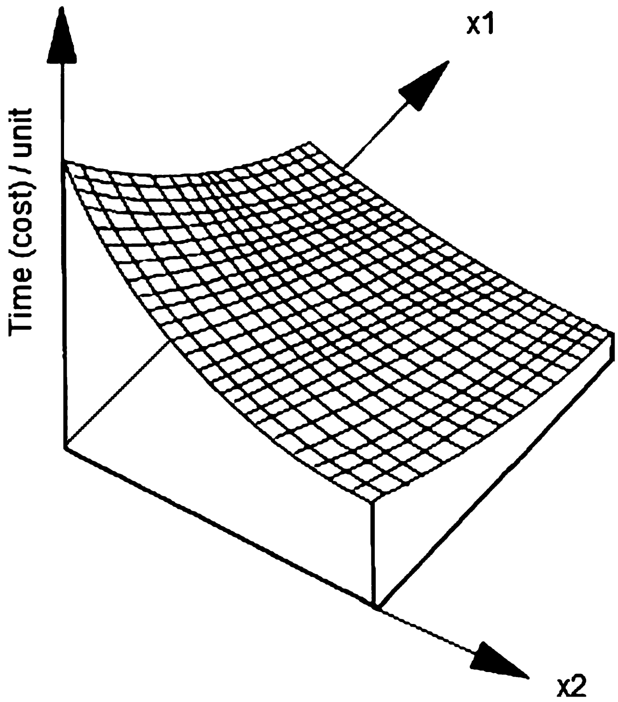


Fig. 4. A bivariate LC surface plot (Adapted from Badiru, 1992).

and Bailey and Mcintyre (1997, 2003) state that predicting workers' production rate at resuming tasks enables better production planning and precise resource allocation.

Globerson et al. (1989) found that a log-linear model properly described the forgetting process, indicating that learning and forgetting occur in similar ways. Jaber and Bonney (1996), and Bailey and Mcintyre (2003) also modeled the forgetting process using log-linear-based models. Jaber and Kher (2004) extended the studies above to evaluate the variability on workers' time to achieve a complete forgetting status. Their models also enable estimation of the time required to produce the first unit, once the task is resumed. Further, Jaber and Bonney (1997) proposed three mathematical models for describing both learning and forgetting processes, while Bailey and Mcintyre (1997) introduced a Relearning Curve based on the log-linear model, recommending it to model processes where interruptions in production are frequent.

Jaber et al. (2003) described factors that influence workers' forgetting in industrial environments and assessed the efficiency of existing mathematical structures in modeling the forgetting process. Other studies of forgetting and learning were carried by Jaber and Guiffrida (2007), Jaber and Sikström (2004a,b), and Zamiska et al. (2007). Nembhard and Uzumeri (2000a), Shafer et al. (2001) and Nembhard and Osothsilp (2002) also modeled forgetting processes by means of a 3-parameter hyperbolic LC, while the efficiency of the learning–forgetting–relearning process was analyzed using simulation by Davidovitch et al. (2008). The authors evaluated how the length of interruptions in production affects the performance once the activity is resumed.

The impact of forgetting on production planning activities, such as Optimal Lot Size definition and inventory level decisions, was analyzed by Salameh et al. (1993), Bailey (1989), Jaber and Bonney (1996), and Jaber et al. (2009). Jaber and Bonney (1996, 2003) found that the Optimal Lot Size decreases as workers continuously perform the task without major interruptions. In addition, Alamri and Balkhi (2007) assessed the effects of learning and forgetting on lot size problems considering an infinite planning horizon. Other studies relating learning and forgetting, and the lot size determination problem, are reported by Jaber and Bonney (2007).

Similarly to interruptions in production, product redesign may disrupt workers' performance (Yelle, 1979). Such modifications are usually related to customers' requirements or product customization to new markets, and demand workers' adaptation as they are submitted to wholly new tasks – see Eden et al. (1998). Finally, Lam et al. (2001) applied LC to evaluate the effects of forgetting and product modifications in the construction segment, and estimated their impact on productivity indices.

3. Research agenda

Learning curves reemerges as an important research topic in Industrial Engineering and Operations Management, mostly due to two motivating events. The first one is the increasing popularity of Mass Customization (MC) as a production strategy in manufacturing and service industries. Mass-customized products and services are tailored to the individual needs and expectations of customers. Such personalization conflicts with the inflexibility that characterizes labor intensive production systems. LC modeling emerges as an optimization tool for the assignment of tasks to individuals or teams, enabling economically feasible production of small lots. The second event is the greater availability of automated data collection devices. Learning data may be easily collected and stored, and the use of more complex LC models becomes possible. That opens an opportunity to propose and explore multivariate LC models, expanding the scarce literature on the subject. Such models may allow a better description and understanding of the learning mechanism in task performing.

The future research directions on learning curves we propose in the remainder of this paper investigate issues related to the two motivating events above. The first two sections convey research propositions focused on the use of univariate LCs as a tool to support and enable MC applications. The third section explores a topic related to Production Control: the impacts of job rotation on learning, and related open research opportunities. The last section presents a proposal on which multivariate LCs are used to tackle a problem related to mass-customized product design.

3.1. Choice menu configuration

In highly customized production systems a large variety of items is produced with constant changes in lot size. To attain that, resources must be flexible and production agents (machinery and labor) must interact to quickly and efficiently adapt to changes imposed on the system (Da Silveira et al., 2001). Therefore estimates of workers' adaptation to new, customized models prior to process setup are highly desirable if production is to be satisfactorily managed.

Choice menus are producer–user interfaces that enable customers to select product and service attributes and features in a consistent and economical way (Oliva, 2002). Although increasingly present in business and consumer industries as reported by Fogliatto and Da Silveira (2008), a limited number of authors has so far dealt with problems associated to their design and configuration in the literature. In particular, the design of choice menus must balance a trade-off between flexibility and value, specifying a set of options that are both meaningful to customers and feasible in terms of production.

Keeping such aims in mind, Ben-Akiva and Gershenfeld (1998), and Liechty et al. (2001) carried out studies that modeled conjoint data on preferences on multiple items from menus. However, their primarily focused on forecasting demand and revenue based on customer preferences rather than on designing choice menus. More recently, Fogliatto and Da Silveira (2008) proposed a choice menu design method based on stated customer preferences. Their work, as those of the authors mentioned above, does not consider workers' flexibility when selecting attributes (and therefore models) to be included in the menu.

A relevant contribution in the LC literature would be a method for choice menu design aimed at optimizing the trade-off between workers' adaptability to new models, as estimated by their learning parameters, and the value perceived by customers from offering such models. That would update the literature on LC applications in production management through integration with a highly desirable production strategy.

3.2. Job scheduling

Labor intensive production systems such as assembly lines are usually not challenging in terms of job scheduling. That is explained by the fact that linear flow layouts were conceived to optimize the production of items with similar processing requirements, and medium to large lot sizes (Tompkins et al., 1996). However, as production systems adapt to supply customized items, job scheduling becomes a primary determinant of the efficiency of linear layouts.

Job scheduling in manufacturing and in the service industry has been an important research topic in the literature (Pinedo, 2008). The objective is to allocate jobs to resources such that an objective is optimized. However, it seems that understanding the impacts of workers' learning on the scheduling problem has only recently become a research topic.

Biskup (1999) was the first to analyze the effects of learning on the position of jobs in a single machine. Mosheiov (2001) extended Biskup's (1999) results for applications comprised of identical parallel machines using an LC model in which parameters were the same, independent of the job considered. Mosheiov and Sidney (2003) integrated job dependent LCs (*i.e.* LCs presenting different parameters for each job) to scheduling formulations where the objective was to minimize flow-time and makespan in a single machine, as well as flow-time in unrelated parallel machines. More recently, Anzanello and Fogliatto (2010) proposed an approach to scheduling problems where job completion times are dependent on workers' learning process, and where a total weighted earliness and tardiness objective function is to be optimized.

Learning curves may be used to model the impact of assigning a new model on workers' performance in assembling lines given the model currently under production, as well as previous models already produced in the line. More realistic job scheduling should arise from taking into account such information; however, some research problems related to the subject remain open in the literature. A couple of them are outlined next.

Estimates of job completion times are affected by the LC goodness-of-fit. Imprecise estimates lead to unreliable scheduling results, particularly when the time to complete a batch of jobs is considered. LC goodness-of-fit may be assessed by different performance indices, which could be used to reorganize jobs in the scheduling procedure such that those with early due dates and completion times obtained from well-fitted LCs are prioritized. In addition, multivariate LC models that take into account the effect of other independent variables in addition to time should also be investigated. A potentially relevant independent variable could be the training procedure.

Finally, more complex scheduling environments where workers' learning takes place such as pure mass-customized job shops remain to be investigated in the literature. In those production systems LC modeling becomes quite challenging due to the high variety of products involved and their small lot sizes. The planning and optimization of LC data collection in environments of potential data scarcity such as those is a key research problem.

3.3. Job rotation

Job rotation makes it possible to observe workers performing different tasks, and determine which task is best suited for each worker. Although usually dominant, the productivity criterion is not the only one motivating job rotation. Rotation may benefit workers ergonomically by reducing exposure to work-related musculoskeletal disorders due to repetitive postures, as investigated by Kuijer et al. (1999), and Frazer et al. (2003). In addition, production managers may benefit from the flexibility provided by multitask workers, which is a natural outcome of job rotation.

A few authors have worked on the association of job rotation and learning. Jovanovic (1979) was one of the first to formally investigate job rotation associated with workers' learning. Although not explicitly using LC modeling, the author proposes a regression model in which a worker's contribution to the total company output is dependent on the amount of time the worker is functional and on independent parameters related to the worker-job match. Ortega (2001) extends the propositions in Jovanovic (1979) to investigate whether job rotation policies are an efficient way to learn about workers within a company. To accomplish that, data are collected from different, predetermined worker-job matches. Two rotation policies are investigated: specialization and job rotation.

None of the works above make direct use of LC modeling to determine job rotation strategies, leaving the topic open for future investigation. A promising way to address the problem would be through the introduction of sequence-dependent learning models, in which workers' performance on a new model depends on the model previously assigned and its characteristics. A job rotation scheme that minimizes productivity losses would be sought. The model clustering strategy proposed by Anzanello and Fogliatto (2007) could be thought of as a starting point in the development of such an approach.

3.4. Product development

In product development efforts are usually concentrated on the optimization of product and process parameters. The objective is to determine parameter settings that yield outputs in compliance with specifications, while being robust to environmental interferences. Product development is a major issue in Mass Customization, and considered one of its main enablers. Focus has been placed on the platform formation problem in which proper configuration of platforms allows several products designed to be simultaneously optimal to share the same system structure. The platform formation problem is reviewed by Chowdhury and Siddique (2010), among others.

While most efforts are concentrated on maximizing parts commonality when designing a family of products, product structure plays a major role when items are to be manufactured. Therefore, in addition to designing product models that share components, models that require the same skills for manufacturing should also be favored. Fujita et al. (1998) were the first to incorporate learning in the platform formation problem. The authors propose a total profit model for product variety design optimization in which the objective function is a sum of costs to be minimized. A learning curve parameter is used to compute the production cost, such that models that demand complex workers' setup are likely to be dismissed from the platform. The final result is a set of models that are similar with respect to the production skills required and share a large number of parts. Their achievements are illustrated in a case study from the aircraft manufacturing industry.

We propose expanding their approach using multivariate learning models. Such models tend to better represent the learning effect of products that are similar in terms of configuration if a commonality index is considered in the modeling. Alizon et al. (2008) proposed two indices based on shape and functional similarity to achieve differentiation within a family of products. Such indices could be used as starting point in our proposition.

4. Conclusion

Research on LC has received academic and industrial attention for more than seven decades. LCs have been widely used in several segments to estimate lot conclusion time, to evaluate production cost reduction, to optimize the allocation of tasks to workers based on their learning profile, and to mitigate production losses after task interruption. LCs have also been integrated to tools aimed at controlling production and quality of products and processes. Such variety of applications explains the large number of publications dealing with the subject.

In this paper we presented a comprehensive review of univariate and multivariate LC models, focusing on their structural aspects, modifications to suit specific applications, and limitations in modeling workers' learning process. We also compared models in terms of their ability to describe specific episodes, e.g. those affected by workers' forgetting and need of machinery.

We end up by proposing an agenda for future research on LC based on recent trends and demands in the Industrial Engineering and Operations Management field. Most of our propositions are motivated by the increasing popularity of Mass Customization in manufacturing and services, and the benefits it may draw from LC modeling. We also envision research opportunities that explore uses of multivariate LCs which are to this date poorly investigated in the literature. The increasing availability of devices to measure and store performance data as well as correlated process information mitigates one of the main obstacles to a better use of multivariate LC models. We thus propose investigating such models in the context of product development for Mass Customization.

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