

高 2 HL 数学 B 小テスト 夏期講習第 5 講

氏名 _____

①以下の連立漸化式を解け

$$(i) a_1 = 1, b_1 = -1, a_{n+1} = 5a_n - 2b_n \cdots (1) \quad b_{n+1} = a_n + 2b_n \cdots (2)$$

[解]

(1) - (2)より

$$\begin{aligned} a_{n+1} - b_{n+1} &= (5a_n - 2b_n) - (a_n + 2b_n) \\ &= 4(a_n - b_n) \end{aligned}$$

$\{a_n - b_n\}$ と 1 つ塊とみると等比数列型なので

$$a_n - b_n = (a_1 - b_1) \cdot 4^{n-1} = \{1 - (-1)\} \cdot 4^{n-1} = 2 \cdot 4^{n-1} \cdots (3)$$

(1) - 2 × (2)より

$$\begin{aligned} a_{n+1} - 2b_{n+1} &= (5a_n - 2b_n) - 2(a_n + 2b_n) \\ &= 3(a_n - 2b_n) \end{aligned}$$

$\{a_n - 2b_n\}$ と 1 つの塊とみると等比数列型なので

$$a_n - 2b_n = (a_1 - 2b_1) \cdot 3^{n-1} = \{1 - 2 \cdot (-1)\} \cdot 3^{n-1} = 3 \cdot 3^{n-1} = 3^n \cdots (4)$$

(3) - (4)より

$$\begin{aligned} (a_n - b_n) - (a_n - 2b_n) &= 2 \cdot 4^{n-1} - 3^n \\ b_n &= 2 \cdot 4^{n-1} - 3^n \end{aligned}$$

2 × (3) - (4)より

$$\begin{aligned} 2(a_n - b_n) - (a_n - 2b_n) &= 2 \cdot 2 \cdot 4^{n-1} - 3^n \\ a_n &= 4^n - 3^n \end{aligned}$$

よって

$$a_n = 4^n - 3^n, b_n = 2 \cdot 4^{n-1} - 3^n$$