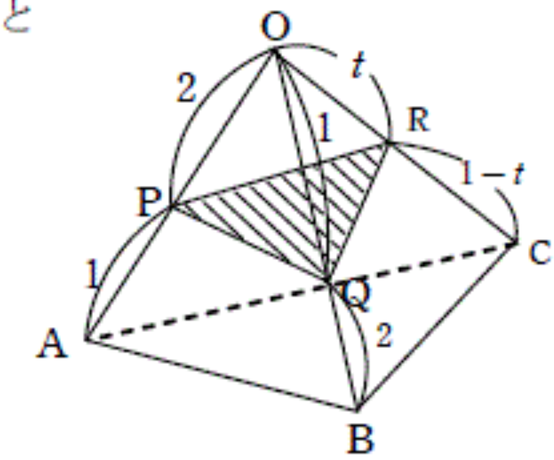
一橋大学

法，商，社会，経済

数学 - 解答

$\boxed{4}$

(1) $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, $\overrightarrow{OC}=\vec{c}$ とおくと



$$\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|=1$$

$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=\frac{1}{2}$$

$$\overrightarrow{OP}=\frac{2}{3}\vec{a},\quad \overrightarrow{OQ}=\frac{1}{3}\vec{b}$$

$$\overrightarrow{PQ}=-\frac{1}{3}\left(2\vec{a}-\vec{b}\right)$$

$$\left|\overrightarrow{PQ}\right|^2=\frac{1}{9}\left|2\vec{a}-\vec{b}\right|^2=\frac{1}{9}\left\{4\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-4\vec{a}\cdot\vec{b}\right\}$$

$$=\frac{1}{9}\{4+1-2\}=\frac{1}{3}\qquad \therefore \left|\overrightarrow{PQ}\right|=\frac{1}{\sqrt{3}}\ \cdots(答)$$

(2) $\overrightarrow{OR}=t\vec{c}$

$$\overrightarrow{PR}=\overrightarrow{OR}-\overrightarrow{OP}=-\frac{1}{3}(2\vec{a}-3t\vec{c})$$

$$\left|\overrightarrow{PR}\right|^2=\frac{1}{9}\left|2\vec{a}-3t\vec{c}\right|^2=\frac{1}{9}\left\{4\left|\vec{a}\right|^2+9t^2\left|\vec{c}\right|^2-12t\vec{c}\cdot\vec{a}\right\}$$

$$=\frac{1}{9}\left\{4+9t^2-6t\right\}=\frac{1}{9}(9t^2-6t+4)$$

$$\overrightarrow{PQ}\cdot\overrightarrow{PR}=\frac{1}{9}\left(2\vec{a}-\vec{b}\right)\left(2\vec{a}-3t\vec{c}\right)$$

$$=\frac{1}{9}\left\{4\left|\vec{a}\right|^2-6t\vec{c}\cdot\vec{a}-2\vec{a}\cdot\vec{b}+3t\vec{b}\cdot\vec{c}\right\}$$

$$=\frac{1}{9}\left\{4-3t-1+\frac{3}{2}t\right\}=\frac{-1}{6}(t-2)$$

$\triangle$PQR の面積を $S(t)$ とおくと,

$$S=\frac{1}{2}\sqrt{\left|\overrightarrow{PQ}\right|^2\left|\overrightarrow{PR}\right|^2-\left(\overrightarrow{PQ}\cdot\overrightarrow{PR}\right)^2}$$

$$=\frac{1}{2}\sqrt{\frac{1}{3}\cdot\frac{1}{9}(9t^2-6t+4)-\frac{1}{36}(t-2)^2}$$

$$=\frac{1}{2}\sqrt{\frac{11}{36}t^2-\frac{4}{36}t+\frac{1}{27}}=\frac{1}{2}\sqrt{\frac{11}{36}\left(t-\frac{2}{11}\right)^2+\frac{1}{27}-\frac{4}{36\cdot 11}}$$

S は $t=\dfrac{2}{11}$ のとき最小となる. ……(答)

このウインドウを閉じる