

## Problem Set 5

3<sup>rd</sup>-Degree Price Discrimination

Due: 11PM Eastern Time on Sunday, October 6th

Econ 316: Industrial Organization

To turn in this problem set: email to [submit.io.psets@gmail.com](mailto:submit.io.psets@gmail.com)

### Honor code

*I am allowed to discuss the problem sets with others. However, I will write everything I submit, such as code, mathematical derivations, and final answers. I will not copy others. When I receive advice from others, I will cite them in my problem set.*

For example, if student named Juana Diaz gave you advice on a line of code, you should write “(Received advice from Juana Diaz)” on that line. Receiving advice does not affect your grade or how the grader thinks of you.

### Question 0

0A: I followed the honor code on this problem set. (Answer Yes or No.)

0B: How much time did you spend on this problem set?

### Question 1: Online third-degree price discrimination

The Wall Street Journal recently published an article suggesting that Staples.com price discriminates. Click [here](#) to see the article if you are interested. It is not publicly known how Staples decides what prices to charge. This question guides you through the sort of process that Staples should be using.

Consider the stapler market. When consumers (indexed by  $i$ ) visit the Staples stapler webpage, Staples observes a vector of  $k=4$  observable covariates  $X_i$ : whether the customer's IP address is near a competing store, whether the customer's IP address is in a high-income zip code, whether it is a weekday or weekend, and a cookie that reveals whether the customer is an existing Staples customer.

Assume that a consumer with covariates  $X_i$  purchases the stapler with probability

$$\Pr(\text{Purchase} / P_i, X_i) = \beta_0 + \beta_1 \cdot X_i + \beta_2 P_i + \beta_3 \cdot X_i \cdot P_i,$$

where  $P_i$  is the price that the website offers to consumer  $i$ . (Note that  $\beta_1$  and  $\beta_3$  are vectors of length  $k=4$ .) This means that Staples's expected profit earned from consumer  $i$  is

$$\pi(P_i / X_i) = \Pr(\text{Purchase} / P_i, X_i) \cdot (P_i - c),$$

where  $c = \$10$  is the marginal cost of producing and delivering staplers.

Imagine further that Staples has run an experiment where they randomly assigned  $P_i$  across 40,000 different consumers and recorded whether each consumer bought the stapler. In the dropbox folder for this problem set, you will find StaplesData.csv, a csv file containing (simulated) data from that

experiment. You will also find a python notebook called pset5template.ipynb. Download both, and modify the notebook as needed to answer the below questions.

1A: Derive the formula for the profit-maximizing price  $P_i^{monop}$  as a function of  $(\beta_0, \beta_1, \beta_2, \beta_3, c, X_i)$ .

1B: Use OLS to estimate the demand function parameters  $(\beta_0, \beta_1, \beta_2, \beta_3)$  using StaplesData.csv. Report the coefficient estimates whose p-values are below 10%.

1C: Interpret the signs and magnitudes of the  $\beta_3$  coefficient estimates whose p-values are below 10%. (For example, write that “conditional on the other  $X$  variables, being in a wealthy zip code reduces price responsiveness by [ ] percentage points per dollar of price increase.”) Are the signs and magnitudes consistent with what you expected?

1D: Using the formula you derived in 1A and demand function parameters  $(\beta_0, \beta_1, \beta_2, \beta_3)$ , construct a vector of the profit-maximizing price  $P_i^{monop}$  to offer each individual consumer. Report the minimum, maximum, and mean.

1E: Construct a vector of the expected profit earned from each consumer. (To do this, plug price  $P_i^{monop}$ , characteristics  $X_i$ , and demand function parameters  $(\beta_0, \beta_1, \beta_2, \beta_3)$  into the profit function.) Report the minimum, maximum, and mean.

Your manager is concerned that price discrimination would be perceived negatively by customers and in the media. Let's now estimate the profit that the company would forego if it set the optimal uniform price instead of optimal customer-specific prices. For the rest of question 1, ignore covariates  $X$  characteristics and assume that the average demand function is  $Pr(Purchase | P) = \alpha_0 + \alpha_2 P$ .

The profit function is now

$$\pi(P) = Pr(Purchase | P) \cdot (P - c).$$

1F: Write the formula for the profit-maximizing uniform price  $P^{unif}$  as a function of  $(\alpha_0, \alpha_2, c)$ . (Hint: You just need to modify your answer from 1A.)

1G: Use OLS to estimate the average demand function parameters  $(\alpha_0, \alpha_2)$  using StaplesData.csv. On your problem set, report the  $(\alpha_0, \alpha_2)$  coefficient estimates.

1H: Interpret the  $\alpha_2$  coefficient in words.

1I: Using the formula you derived in 1F and demand function parameters  $(\alpha_0, \alpha_2)$ , calculate and report the profit-maximizing uniform price  $P^{unif}$ .

1J: Calculate and report the expected profit earned from each consumer.

1K: Compare 1J to 1E and report the expected profit loss from uniform pricing instead of price discrimination.

1L: What data other than the variables in StaplesData.csv might better predict purchase probability and thus be useful for price discrimination?

1L: Concisely discuss two or three caveats to this analysis that you (as an economist at Staples) would want your manager to understand.