

Problem Set 8

Hotelling and Logit

Due: 11PM Eastern Time on Sunday, October 27

Econ 316: Industrial Organization

Honor code

I am allowed to discuss the problem sets with others. However, I will write everything I submit, such as code, mathematical derivations, and final answers. I will not copy others. When I receive advice from others, I will cite them in my problem set.

For example, if student named “Juana Diaz” gave me advice on a line of code, I will write “(Received advice from Juana Diaz)” on that line. Receiving advice does not affect your grade or how the grader thinks of you.

Question 0

I followed the honor code on this problem set. (Answer Yes or No.)

OB: How much time did you spend on this problem set?

Question 1: Estimating cereal demand

CerealData.csv contains the manufacturer name, average transaction prices, market shares, and characteristics of the top 50 cereal brands from a particular recent year. (These are real data derived from a UPC scanner dataset provided by [IRI](#).) The outside option is all other brands not listed, and the utility from the outside option is normalized to zero. We will use these data to estimate cereal demand using a logit model.

Assume that consumer i 's utility from purchasing brand j is:

$$U_{ij} = \beta X_j + \xi_j + \varepsilon_{ij},$$

where X_j is a vector of characteristics (Price, Calories, Fat grams, Sugar grams, and segment indicators (Adult and Kids; Family is the omitted segment)), ξ_j is an unobserved attribute, and ε_{ij} is consumer i 's idiosyncratic taste for brand j . Here a “consumer” i is really a “choice occasion,” i.e. a particular consumer buying one box of cereal.

Recall the following things from lecture:

Assuming that ε_{ij} is distributed type I extreme value, the choice probabilities are:

$$P_{ij} = \frac{e^{\beta X_j + \xi_j}}{\sum_k e^{\beta X_k + \xi_k}}$$

If we assume that all consumers have the same β and ξ parameters, then:

- A shortcut to $e^{\beta X_j + \xi_j}$ for any product is

$$e^{V_j} = \frac{S_j}{S_0},$$

where S_j is the market share of brand j and S_0 is the market share of the outside option.

- The following equation characterizes market shares and can be used to estimate β 's:

$$\ln(S_j/S_0) = \beta X_j + \xi_j.$$

- Use python to estimate the market share equation. Report the β parameter estimates. Note: you should include an intercept term in the regression as part of the X characteristics.
- Remember that “correlation doesn't imply causality.” For the β parameter estimates to be unbiased, the X_j characteristics (including price) must be uncorrelated with the unobserved attribute ξ_j . Although you will use this assumption below, give two reasons why it might not be true.
- Write the formula for own-price elasticity of demand, i.e. the elasticity of demand for good j with respect to its own price. (This should be a function of β_{price} , P_j , and P_{ij} . It should be a negative number.)
- Compute the vector of own-price demand elasticities. Report the mean. Does it seem elastic or inelastic?

For questions E through I, imagine that Kellogg's decides to stop selling Raisin Bran.

- Use the formula for choice probabilities to re-simulate market shares without Kellogg's Raisin Bran. Then, for each product, find the ratio of (market share without Kellogg's Raisin Bran) to (original market share). Report the minimum, maximum, and mean of these ratios.
- What is the ratio from part E for Post Raisin Bran?
- What is the ratio from part E for Kellogg's Corn Pops?
- Compare your answers in G and H. Is this realistic?

The dataset you were given did not include marginal cost. However, we can infer what marginal cost must have been to rationalize the observed prices.

- Assume that every cereal brand j sets prices to maximize its own profits. Write the profit maximization problem for brand j , assuming constant marginal cost C_j .

- J) Using the first-order condition, find the equation for the marginal cost c_j that would make the observed prices profit-maximizing. This c_j should be a function of β_{price} , market share s_j , and price p_j .
- K) Using R, construct the vector of marginal costs for each brand j . Report the minimum, maximum, and mean.
- L) Explain why your answers to K are negative. Hint: this is driven by your answer to question D.
- M) In reality, every cereal manufacturer should set the prices of all of its brands jointly to maximize total profits. For example, Kellogg's should set the prices of Corn Flakes, Frosted Flakes, Raisin Bran, Special K, and all other Kellogg's brands together to maximize the sum of profits. Write the profit function and briefly discuss the intuition for what each manufacturer needs to consider when it set prices for all of its brands.
- N) Will prices be higher or lower when manufacturers set prices for all their brands jointly (as in part M) compared to when manufacturers set prices for each brand separately (as in J)? Why?