Problem Set 9

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Question 0

I followed the honor code on this problem set. Yes.

I spent 7hrs.

Question 1

A) The Nash Equilibrium if the firms set prices simultaneously is **($7,$7)**.
B) The SPNs for each subgame are ($6, $7), ($7, $7), ($8, $7), ($9, $8).
    a. But out of all, this game will end up at **($9, $8)**.
C)
    a. The first mover acts first. That means the first mover is going to produce some number of output or set a price that maximizes its profit with the consideration of how much the second mover is going to produce or what price it sets in response to the first mover's action.
    b. In other words, firm 1 is going to compare different SPNs from different subgames and choose the one that gives itself the highest utility. **Therefore, it can never be worse off then the simultaneous game.**
        i. This is just one example I made up. For example, if the utility of ($9, $7) is (28, 70), and all the other things are constant, then the sequential game won't end up at ($9, $8). Because the firm 1 knows that the firm 2 will choose ($9, $7) which doesn't give the firm 1 the highest utility. Therefore, this subgame is out of firm 1's picture. And now the firm 1 will choose from other subgames. In this scenario, the firm 1 will choose $7 and therefore the game ends up at ($7, $7).
    c. However, this won't be true for the second mover. The second mover has to choose from a subgame that the first mover has picked. And if its options are worse than the simultaneous game Nash equilibrium, it will still choose the option that gives itself the highest utility within the subgame, but it will be worse off than the simultaneous game Nash equilibrium. **So, in other words, there are possibilities that the second**

mover can be worse off in the sequential game than the simultaneous game based on how the utilities are structured.

i. For example, if ($9, $6), ($9, $7), ($9, $8), ($9, $9) are (12, 1), (28, 2), (36, 4), (40, 3) respectively, the sequential game will still end up at ($9, $8), but the second mover will be worse off than the simultaneous game Nash equilibrium.

Question 2

Question 2A: What are equilibrium quantities and profits for each firm if N = 2?

Inverse Demand Function:

$$P = 50 - \frac{1}{2} Q_T$$

$$P = 50 - \frac{1}{2}(Q_i + Q_{-i})$$

Profit Function:

$$\pi_i = [P(Q_T) - c]Q_i - F$$

FOC:

$$Q_i = 50 - c - \frac{1}{2} Q_{-i}$$

$$Q_{-i} = (\text{N-1})\, Q_i$$

Therefore,

$$\boldsymbol{Q_i = \frac{80}{(1+N)}}$$

When N = 2

$$\boldsymbol{Q_i = \frac{80}{3}}$$

$$\boldsymbol{\pi_i = 350..5}$$

Question 2B: What are equilibrium quantities and profits for each firm if N = 3?

$$\boldsymbol{Q_i = \frac{80}{4}}$$

$$\boldsymbol{\pi_i = 195}$$

Question 2C: What are equilibrium quantities and profits for each firm as a function of N?

$$\boldsymbol{Q_i = \frac{80}{(1+N)}}$$

$$\boldsymbol{\pi_i = 2\left(\frac{40}{N+1}\right)^2 - 5}$$

Question 2D:

$$\boldsymbol{\frac{1}{\alpha}\left(\frac{P_0 - c}{N+1}\right)^2 \geq F}$$

$$\boldsymbol{2\left(\frac{50-10}{N+1}\right)^2 \geq 5}$$

Solving for N, we get 24.29.

Question 2E:

The amount they are willing to pay is:

$$\boldsymbol{\left[\frac{1}{\alpha}\left(\frac{P_0 - c}{2+1}\right)^2 - F\right] - \left[\frac{1}{\alpha}\left(\frac{P_0 - c}{24.29+1}\right)^2 - F\right]}$$

Therefore, they are willing to pay $350