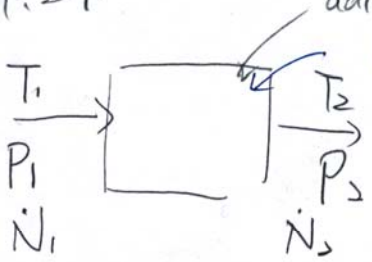
4.24 ①



a. show minimum T_2 occurs when $S_{gen} = 0$

From mass balance

$$\frac{dN}{dt} = \dot{N}_1 + \dot{N}_2 = 0 \Rightarrow \dot{N}_2 = -\dot{N}_1 \quad -①$$

From energy balance

$$\frac{dU}{dt}^{0\ \text{s.s.}} = \dot{N}_1 \underline{H}_1 + \dot{N}_2 \underline{H}_2 + \dot{Q}^{0\ \text{adiabatic}} + \dot{W}_s - P\frac{dV}{dt}^{0\ \text{constant vol}}$$

$$\Rightarrow \dot{N}_1 \underline{H}_1 + \dot{N}_2 \underline{H}_2 + \dot{W}_s = 0 \quad -②$$

① substitute into ②

$$\Rightarrow \dot{N}_1 \underline{H}_1 - \dot{N}_1 \underline{H}_2 + \dot{W}_s = 0$$

$$\Rightarrow \dot{W}_s = \dot{N}_1 C_p^* (T_2 - T_1)$$

∵ ideal gas $\underline{H} = C_p^* T$

From entropy balance

$$\frac{dS}{dt}^{0\ \text{s.s}} = \dot{N}_1 \underline{S}_1 + \dot{N}_2 \underline{S}_2 + \frac{\dot{Q}^{0\ \text{adiabatic}}}{T} + \dot{S}_{gen}$$

(2)

$$\Rightarrow \dot{S}_{gen} = \dot{N}_1 (\underline{S}_2 - \underline{S}_1) \quad -③$$

$\because d\underline{H} = T d\underline{S} + \underline{V} dP$ ideal gas $\frac{RT}{P}$

$$\Rightarrow d\underline{S} = \frac{d\underline{H}}{T} - \frac{\underline{V}}{T} dP$$

$$\Rightarrow \Delta \underline{S} = C_p^* \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad -④$$

④ substitute into ③

$$\Rightarrow \dot{S}_{gen} = \dot{N}_1 \left(C_p^* \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right)$$

$$\Rightarrow \frac{\frac{\dot{S}_{gen}}{\dot{N}_1} + R \ln \frac{P_2}{P_1}}{C_p^*} = \ln \frac{T_2}{T_1}$$

$$\Rightarrow T_2 = T_1 \exp \left[\frac{\frac{\dot{S}_{gen}}{\dot{N}_1} + R \ln \frac{P_2}{P_1}}{C_p^*} \right]$$

$\because \dot{S}_{gen} \geq 0.$

$\Rightarrow$ minimum T_2 occurs when $\dot{S}_{gen} = 0$

(3)

$\dot{W}_s = \dot{N}_1 C_p^* (T_3 - T_1)$, since $T_1 > T_2$

$\Rightarrow$ minimum T_2 $\Rightarrow$ minimum work

$\Rightarrow$ maximum work can be extracted when $S_{gen} = 0$