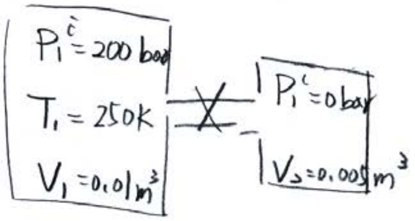
3.25



Find P_1^f. T_1^f. P_2^f. T_2^f

for ideal gas system.

$C_P^* = 30$ J/mol K

$Q = 0$

(Sol).

From mass balance

$$N_1^i + N_2^i = N_1^f + N_2^f$$

$$\Rightarrow \frac{P_1^i V_1}{R T_1^i} + 0 = \frac{P_1^f V_1}{R T_1^f} + \frac{P_2^f V_2}{R T_2^f} \qquad \because V_1 = 2 V_2$$

$$\Rightarrow \frac{2 P_1^i}{T_1^i} = \frac{2 P_1^f}{T_1^f} + \frac{P_2^f}{T_2^f} \quad -①$$

From energy balance

$$N_1^i \underline{U}_1^i + N_2^i \underline{U}_2^i = N_1^f \underline{U}_1^f + N_2^f \underline{U}_2^f \quad -②$$

for ideal gas $\underline{U}^{IG}_{(T)} = C_V^* T - C_P^* T_R \quad -③$

substitute ③ into ②

$$\Rightarrow \frac{P_1^i V_1}{\cancel{R} T_1^i}(C_V^* T_1 - C_P^* T_R) + 0 = \frac{P_1^f V_1}{\cancel{R} T_1^f}(C_V^* T_1^f - C_P^* T_R) + \frac{P_2^f V_2}{\cancel{R} T_2^f}(C_V^* T_2^f - C_P^* T_R)$$

$$\Rightarrow \left(-\frac{2P_1^i}{T_1^i} + \frac{2P_1^f}{T_1^f} + \frac{P_2^f}{T_2^f}\right) C_p^* T_R + \left(2P_1^i - 2P_1^f - P_2^f\right) C_v^* = 0$$

0 " from eq'n ①

since $C_v^* = 0$

$$\Rightarrow 2P_1^i = 2P_1^f + P_2^f$$

Since $P_1^f = P_2^f$

$$\Rightarrow P_1^f = \frac{2}{3} P_1^i = P_2^f = \frac{2}{3} \times 200 \text{ bar} = 133 \text{ bar}$$

#

substitute into eq'n ①

$$\frac{2 \cdot 200 \text{ bar}}{250 K} = \frac{2 \cdot 133 \text{ bar}}{T_1^f \, K} + \frac{133 \text{ bar}}{T_2^f \, K}$$

$$\Rightarrow \frac{400}{250} = \frac{266}{T_1^f} + \frac{133}{T_2^f} \quad — ④$$

Take cylinder 1 as the system

From mass balance.

$$\frac{dN_1}{dt} = \dot{N} \quad — ⑤$$

From energy balance. (neglect $\frac{1}{2}Mv^2$. $M\phi$)

$$\frac{d(N_1 U_1)}{dt} = \dot{N} H_1 + \cancel{\dot{Q}}^{\,0} + \cancel{\dot{W}}^{\,0}$$ substitute

$$\Rightarrow N_1 \frac{d\underline{U}_1}{dt} + \underline{U}_1 \frac{dN_1}{dt} = \frac{dN_1}{dt} \underline{H}_1$$

$$\Rightarrow N_1 \frac{d\underline{U}_1}{dt} = (\underline{H}_1 - \underline{U}_1)\frac{dN_1}{dt}$$

From previous eqn $\underline{U}^{IG} = C_v^* T - C_p^* T_R$

$\underline{H}^{IG} = C_p^*(T - T_R)$

$\frac{PV}{RT} = N$

$$\Rightarrow \frac{P_1 V_1}{\cancel{R} T_1} \frac{d(C_v^* T_1 - \cancel{C_p^* T_R})}{dt} = (C_p^* T_1 - \cancel{C_p^* T_R} - C_v^* T_1 + \cancel{C_p T_R}) \frac{d\frac{P_1 V_1}{\cancel{R} T_1}}{dt}$$

not function of t.

V_1 & R are constants

$$\Rightarrow \frac{P_1}{T_1} C_v^* \frac{dT_1}{dt} = RT_1 \frac{d\frac{P_1}{T_1}}{dt}$$

$$\Rightarrow \frac{C_v^*}{R} \frac{1}{T_1} \frac{dT_1}{dt} = \frac{T_1}{P} \frac{d\left(\frac{P_1}{T_1}\right)}{dt}$$

$$\Rightarrow \left(\frac{T_1^f}{T_1^i}\right)^{\frac{C_p^*}{R}} = \frac{P_1^f}{P_1^i} \Rightarrow \left(\frac{T_1^f}{250}\right)^{\frac{30}{8.31}} = \frac{133}{200}$$

$$\Rightarrow T_1^f = 250 \cdot (0.665)^{0.277} = 223 \text{ K} \#$$

Substitute into eqn 4

$$\frac{400}{250} = \frac{266}{223} + \frac{133}{T_2^f} \Rightarrow T_2^f = \frac{133}{0.41} = 324 \text{ K} \#$$