

CS188, Winter 2017
Problem Set 0: Math Prerequisites
Due Jan 19, 2017

1 Problem 1

Solution: Solution to problem 1a

$$y' = \sin(z)(e^{-x})(1-x)$$

2 Problem 2

(a) Problem 2a **Solution:** Solution to problem 2a

$$\begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2 + 9 = 11$$

(b) Problem 2b **Solution:** Solution to problem 2b

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 3 \times 4 \\ 1 \times 1 + 3 \times 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$$

(c) Problem 2c **Solution:** Solution to problem 2c

$$\frac{1}{2 * 3 - 4 * 1} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1.5 & -2 \\ -0.5 & 1 \end{bmatrix}$$

(d) Problem 2d **Solution:** Solution to problem 2d

Since the matrix is invertible, X is full rank and therefore has rank 2.

3 Problem 3

(a) Problem 3a **Solution:** Solution to problem 3a

$$\frac{1 + 1 + 0 + 1 + 0}{5} = \frac{3}{5}$$

(b) Problem 3b **Solution:** Solution to problem 3b

$$Var[X] = \frac{3}{5} - \left(\frac{3}{5}\right)^2 = \frac{6}{25}$$

(c) Problem 3c **Solution:** Solution to problem 3c

$$\frac{1}{2^5} = \frac{1}{32}$$

(d) Problem 3d **Solution:** Solution to problem 3d

Let p be the probability of getting heads and $f(p)$ be the probability of the occurrence of the sample set S. We use critical points to find the value p that maximizes $f(p)$.

$$f(p) = (p)(p)(1-p)(p)(1-p) = p^5 - 2p^4 + p^3$$

$$f'(p) = 5p^4 - 8p^3 + 3p^2$$

Critical points (given by factoring and quadratic formula): $0, 0, 3/5, 1$

$$f(0) = 0, f(3/5) = \frac{108}{3125} = 0.03456, f(1) = 0$$

The value that maximizes the probability is $\frac{3}{5}$.

(e) Problem 3e **Solution:** Solution to problem 3e

$$\frac{0.1}{0.1 + 0.15} = \frac{0.1}{0.25} = 0.4$$

4 Problem 4

(a) Problem 4a **Solution:** Solution to problem 4a

False

(b) Problem 4b **Solution:** Solution to problem 4b

True

(c) Problem 4c **Solution:** Solution to problem 4c

False

- (d) Problem 4d **Solution:** Solution to problem 4d
False
- (e) Problem 4e **Solution:** Solution to problem 4e
True

5 Problem 5

- (a) Problem 5a **Solution:** Solution to problem 5a
v
- (b) Problem 5b **Solution:** Solution to problem 5b
iv
- (c) Problem 5c **Solution:** Solution to problem 5c
ii
- (d) Problem 5d **Solution:** Solution to problem 5d
i
- (e) Problem 5e **Solution:** Solution to problem 5e
iii

6 Problem 6

- (a) Problem 6a **Solution:** Solution to problem 6a

$$\text{Mean : } E[X] = (0)(0.5) + (1)(0.5) = 0.5$$

$$\text{Variance : } Var[X] = 0.5 - 0.5^2 = 0.25$$

- (b) Problem 6b **Solution:** Solution to problem 6b

$$Var[2X] = 4\sigma^2$$

$$Var[X + 2] = E[X^2 + 4X + 4] - E[X + 2]^2$$

$$Var[X + 2] = E[X^2] + E[4X] + E[4] - (E[X]^2 + 2E[X]E[2] + E[2]^2)$$

$$Var[X + 2] = E[X^2] - E[X]^2 + 4E[X] - 4E[X] + 4 - 4$$

$$Var[X + 2] = E[X^2] - E[X]^2 = \sigma^2$$

7 Problem 7

(a) Problem 7a **Solution:** Solution to problem 7a

- i. We know that $\ln(2)\lg(n) = \ln(n)$, and $\ln(2)$ is a constant factor between $\ln(n)$ and $\lg(n)$, which means that both $f(n) = O(g(n))$ and $g(n) = O(f(n))$ are valid.
- ii. $g(n) = O(f(n))$, since $f(n)$ is an exponential function and grows faster than $g(n)$, which is a polynomial function.
- iii. $g(n) = O(f(n))$, since the $f(n)$ grows more quickly than $g(n)$ by a factor of $(\frac{3}{2})^n$, which is not a constant factor, meaning that $f(n)$ will always be an upper bound of $g(n)$ (for sufficiently large n) and not vice-versa.

(b) Problem 7b **Solution:** Solution to problem 7b

Algorithm description: Building upon a binary search algorithm, start at the middle of the list jump left-wards when encountering a 1 and rightwards when encountering a 0. Return the final index if the item is 0 or (index - 1) if the item is +1.

Proof of Correctness: In order to find the transition point, we want to eliminate all elements of the array that could not be the transition point: namely, all indices on the left of 0's and all indices on the right of 1's. This is achieved in the binary search fashion. Eventually, the search space is reduced to a list of a single item (whether this is a 0 or +1 will be implementation based), which will indicate the transition point.

Runtime Complexity Justification: Binary search is $O(\log N)$, and since this algorithm does trivial (constant time) operations on top of the classic binary search, the runtime complexity is the same.

8 Problem 8

(a) Problem 8a **Solution:** Solution to problem 8a

i.

$$E[XY] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{XY}(x, y) dy dx$$
$$E[XY] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_X(x) f_Y(y) dy dx$$

$$E[XY] = \int_{-\infty}^{\infty} xf_X(x)dx \int_{-\infty}^{\infty} yf_Y(y)dy$$

$$E[XY] = E[X]E[Y]$$

(b) Problem 8b **Solution:** Solution to problem 8b

- i. By the law of large numbers, the frequency of a particular event occurring will converge with the expected value given by the probability of it occurring, which in this case is $\frac{1}{6}$ of 6000, which works out to be 1000.
- ii. By the central limit theorem, as the size of the sample approaches infinity, the random variable will become normally distributed, and since the mean of the coin toss should approach $\frac{1}{2}$, the shift on the left side allows the distribution to be centered at 0.

9 Problem 9

Solution: Solution to problem 9 See following page for solutions.

10 Problem 10

Solution: Solution to problem 10 See following page for code and graphs.

11 Problem 11

Solution: Solution to problem 11

$$\begin{bmatrix} 0 \\ 0.89442719 \end{bmatrix}$$

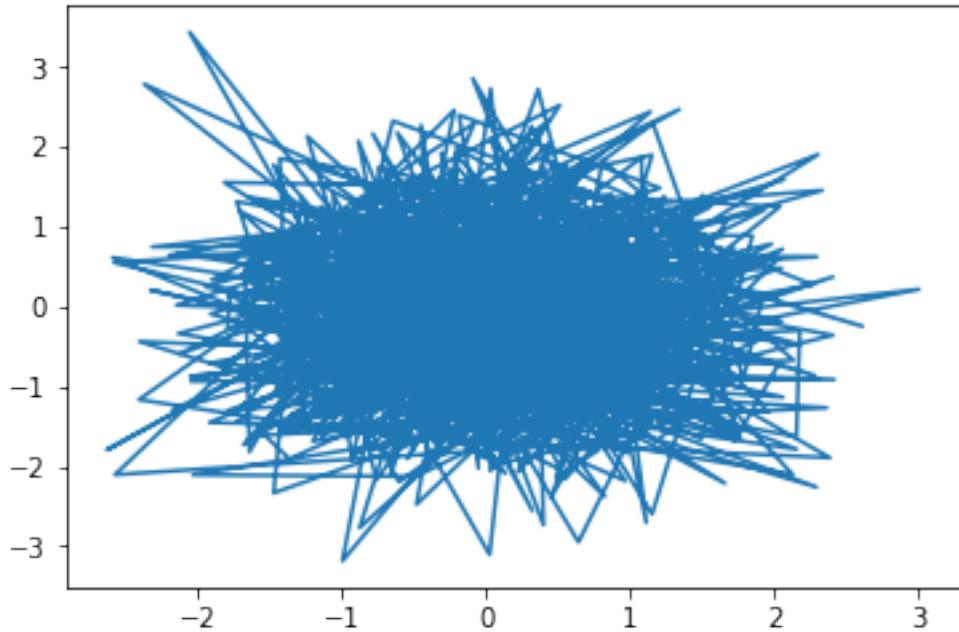
pset0

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```
In [1]: %matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
```

```
In [3]: mean_0 = (0, 0)
cov_identity = [[1,0],[0,1]]
x1, x2 = np.random.multivariate_normal(mean_0, cov_identity, 1000).T
plt.plot(x1, x2)
```

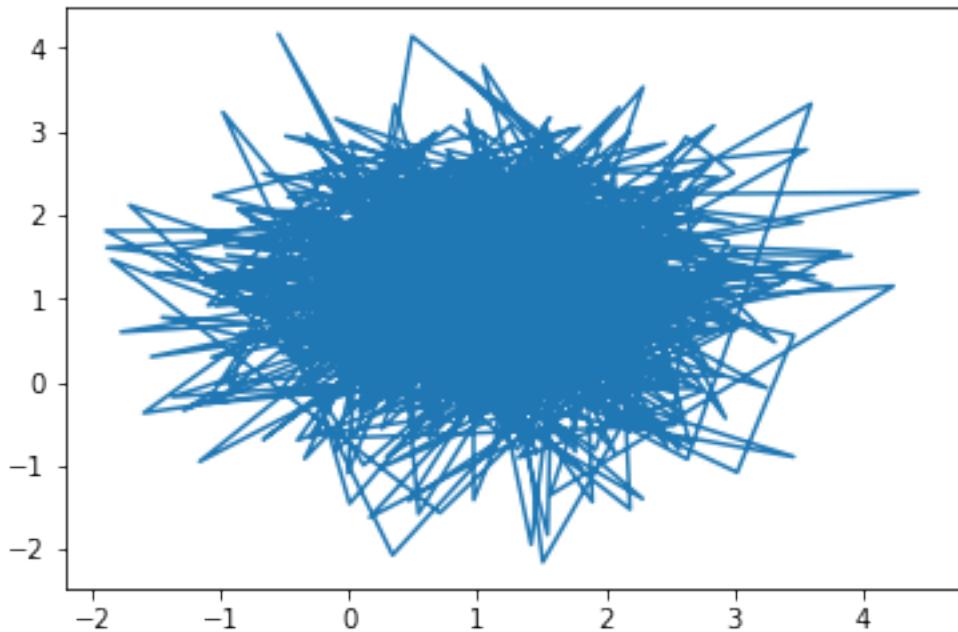
```
Out[3]: [<matplotlib.lines.Line2D at 0x1107d9c10>]
```



```
In [4]: mean_1 = (1, 1)
```

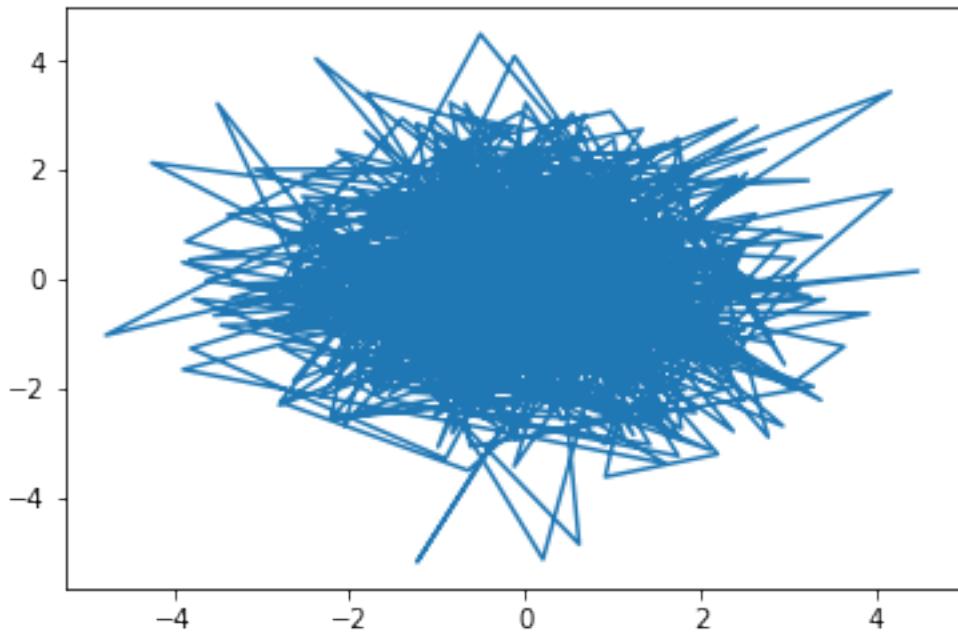
```
In [5]: x1, x2 = np.random.multivariate_normal(mean_1, cov_identity, 1000).T
plt.plot(x1, x2)
```

Out [5]: [<matplotlib.lines.Line2D at 0x110c02090>]



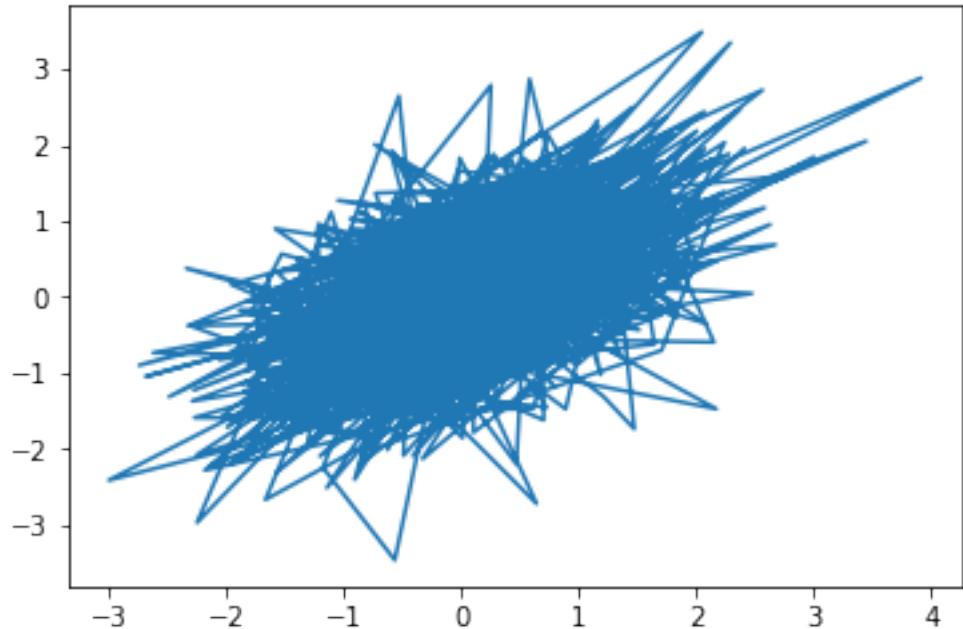
```
In [6]: cov_doubled = [[2,0],[0,2]]  
x1, x2 = np.random.multivariate_normal(mean_0, cov_doubled, 1000).T  
plt.plot(x1, x2)
```

Out [6]: [<matplotlib.lines.Line2D at 0x110e7d390>]



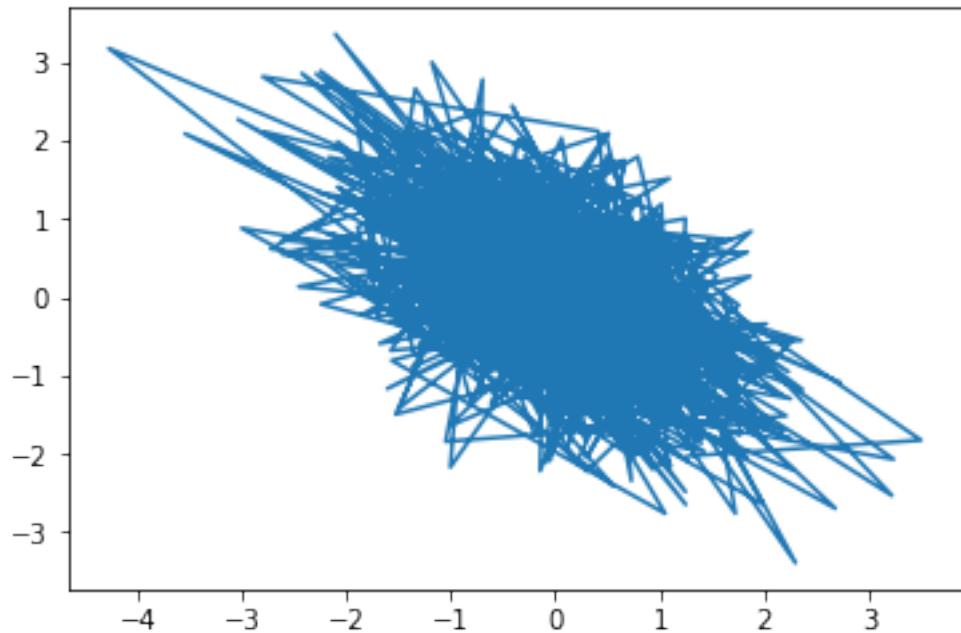
```
In [7]: cov_d = [[1, 0.5], [0.5, 1]]  
x1, x2 = np.random.multivariate_normal(mean_0, cov_d, 1000).T  
plt.plot(x1, x2)
```

```
Out[7]: []
```



```
In [8]: cov_e = [[1, -0.5], [-0.5, 1]]  
x1, x2 = np.random.multivariate_normal(mean_0, cov_e, 1000).T  
plt.plot(x1, x2)
```

```
Out[8]: []
```



```
In [9]: prob_11 = [[1, 0], [1, 3]]
eig_vals, eig_vects = np.linalg.eig(prob_11)
eig_vals; eig_vects
```

```
Out[9]: array([[ 0.          ,  0.89442719],
               [ 1.          , -0.4472136 ]])
```

```
In [10]: eig_vals
```

```
Out[10]: array([ 3.,  1.])
```

```
In [11]: eig_vects
```

```
Out[11]: array([[ 0.          ,  0.89442719],
               [ 1.          , -0.4472136 ]])
```

```
In [ ]:
```

12 Problem 12

- (a) Problem 12a **Solution:** Solution to problem 12a
Book crossing dataset.
- (b) Problem 12b **Solution:** Solution to problem 12b
<http://www2.informatik.uni-freiburg.de/~cziegler/BX/>
- (c) Problem 12c **Solution:** Solution to problem 12c
Although the dataset is given as a couple tables, they can be joined to have each rating as a sample. The features for each sample include the rating, the geographic location of the rater, the age of the rater, and the book itself, which could allow us to predict what kinds of books a certain person of a certain age/geographic location would enjoy.
- (d) Problem 12d **Solution:** Solution to problem 12d
1,149,780 ratings.
- (e) Problem 12e **Solution:** Solution to problem 12e
There are 4 features for each sample.