$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

例 6.4： $E(\hat{\theta}) = \theta$，不偏估計量

$E(\hat{\theta}) \neq \theta$，偏誤估計量

$\hat{\theta}_1 = \sum_{i=1}^{n}(x_i - \bar{x})^2$，$\hat{\theta}_2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1)$

$E(X_i) = \mu$，$V(X_i) = \sigma^2 = E(X_i^2) - \mu^2$

則 $E(\bar{x}) = \mu$，$V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}\right) = \frac{1}{n}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}\right) = \frac{1}{n-1}E\left(\sum_{i=1}^{n}x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \sigma^2$$

$\hat{\theta}_1 = \sum_{i=1}^{n}(x_i - \bar{x})^2/n \rightarrow$ 偏誤估計量

$\hat{\theta}_2 = \sum_{i=1}^{n}(x_i - \bar{x})^2/n-1 \rightarrow$ 不偏誤估計量 ※