UM-SJTU Joint Institute
Physics Laboratory
(Vp141)

Laboratory Report

Exercise 3
Simple Harmonic Motion:
Oscillations in Mechanical Systems

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1 Introduction

The main objective of this exercise is to study simple harmonic oscillation.

1.1 Hooke's Law

Within the elastic limit of deformation, the force applied on the spring to deform it by the distance x is:

$$F_x = kx$$

where k is the spring constant, and F_x is known as the restoring force. And we'll use a Jolly balance to test it.

1.2 Equation of Motion of the Simple Harmonic Oscillator

When using Hooke's Law to analyze simple harmonic motion, we apply the Newton's second law:

$$M\frac{d^2x}{dt^2} + (k_1 + k_2)x = 0 \quad \Rightarrow \quad x(t) = A\cos\omega_0 t + \phi_0 \tag{1}$$

where the natural frequency $\omega_0 = \sqrt{(k_1+k_2)/M}$, A is the amplitude and ϕ_0 is the initial phase. The natural period is calculated by:

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M}{k_1+k_2}} \quad \Rightarrow \quad \frac{T^2}{m} = \frac{4\pi^2}{k_1+k_2} \tag{2}$$

1.3 Mass of the Spring

We take into account the mass of the spring when it is non-negligible by using the effective mass $m_0 = 1/3 \times m_{spring}$, and then the angular frequency is:

$$\omega_0 = \sqrt{\frac{k_1+k_2}{M+m_0}} \tag{3}$$

1.4 Mechanical Energy in Harmonic Motion

The elastic potential energy for a spring-mass system is $U = kx^2/2$ and the kinetic energy of it is $K = mv^2/2$. And we can derive the equation to be:

$$k = \frac{mv_{max}^2}{A^2} \tag{4}$$

Other things needed: $\Delta x = (x_{in} + x_{out})/2$, $v_{max} = \Delta x/\Delta t$. Where x_{in} is the distance within the two legs of the U shape shutter and x_{out} is the outer distance. Δt is the time it takes to travel from one leg of the U shape shutter to the other, which is measured by the timer.

2 Experimental setup

The apparatus we used are 2 kinds of springs, Jolly balance, air track, electronic timer, electronic balance and masses. Figure 1 shows the image of Jolly balance.

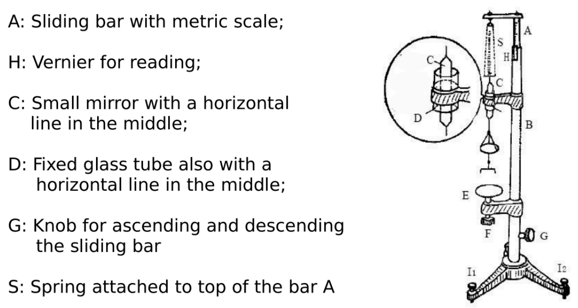

Figure 1

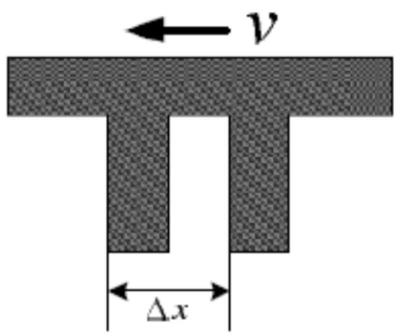

Figure 2

The Jolly balance measures the spring constant by reading the different scales L of the stretched spring with uncertainty of 0.02mm.

The air track is used to decrease friction when the object is moving. The I shape shutter and the T mode of the electronic timer is used for timing 10 times of oscillation, with uncertainty 0.0001s. The caliper measures the outer and inner distance of the U shape shutter (figure 2), x_{out} and x_{in} with uncertainty of 0.02mm, and calculate the Δx of travel. The U shape shutter and the S2 mode of the electronic timer measures the time Δt it needs for the object to travel the distance Δx with uncertainty of 0.00001s. The scale on the air track measures the amplitude A with an uncertainty of 0.1cm. The electronic balance weighs the masses, the springs, the object and U and I shape shutters with an uncertainty of 0.01g. And a collection of those uncertainties are shown in Table 1.

Apparatus		**Precision**
Jolly Balance		0.02[mm]
Caliper		0.02[mm]
Electronic balance		0.01[g]
Photoelectric measuring system	Mode T	0.0001[s]
	Mode S_2	0.00001[s]
Air track		0.1[cm]

Table 1: Apparatus Uncertainty

3 Measurement Procedure

3.1 Spring Constant

1. Adjust the Jolly balance to be vertical. The method is to add some preload first. Make sure the mirror on the jolly balance (figure 1) can move freely ,the spring is parallel to the Jolly balance and the mirror doesn't touch the edge of the tube.
2. Add adjust knob and make 3 lines coincide. Adjust the tube to set the initial position L_0 within 5.0   10.0 cm.
3. Record the reading L_0 on the scale, add mass m_1 and record L_1.
4. Keep adding masses and measure six positions.
5. Estimate the spring constant k_1
6. Replace spring 1 with spring 2, repeat the measurements.

3.2 Relation Between the Oscillation Period T and the Mass of the Oscillator M

1. Adjust the air track so that it is horizontal.
   i Turn on the air pump and all the holes work well.
   ii Place the object on the track without initial velocity. Adjust the track until the object moves slowly back and forth in both directions.
2. Horizontal air track
   i Attach springs to the sides of the cart, and set up the I-shape shutter. Make sure that the photoelectric gate is at the equilibrium position.
   ii Add mass in an unchanged order for 6 times and make small oscillations with amplitude about 5cm. Set the timer to T mode to test the interval for the object to travel 10 periods of oscillations.
3. Inclined air track
   Place plastic plates under one side of the track to create inclined platform. First place 3 plates and then place 6 plates. Repeat steps in Horizontal air track.
4. Plot graphs to find the relationship between M and T^2.

3.3 Relation Between the Oscillation Period T and the Amplitude A

1. Keep the mass of the object unchanged and change the amplitude of the oscillations. The recommended amplitude is 5.0/10.0/15.0/20.0/25.0/30.0 cm.

2. Record the period and apply linear fit to find whether T and A are related.

3.4 Relation Between the Maximum Speed and the Amplitude

1. Measure the outer distance x_{out} and the inner distance x_{in} of the U-shape shutter by a caliper. Calculate the distance $\Delta x = (x_{in} + x_{out})/2$.

2. Change the shutter from I- to U-shape. Set the timer into the "S2" mode. Let the cart oscillate. Record the second readings of the time interval Δt only if the two subsequent readings show the same digits to the left of the decimal point.

3. Change the amplitude 5 times recommended to be 5.0/ 10.0/ 15.0/ ... /30.0 cm.

4. Measure the maximum speed v_{max} for different values of the amplitude A and obtain the spring constant. Compare this result to that of the first part.

3.5 Mass measurement

1. Adjust the electronic balance.

2. Add weights according to a fixed order. Weigh the cart with the I-shape shutter and with the U-shape shutter. Measure the mass of spring 1 and spring 2.

3. Record the data only after the circular symbol on the scales display disappears.

4 Results

4.1 Measurement of the spring constant

The procedure of measuring the spring constant is mentioned in part 3.1 and the measuring of masses is mentioned in part 3.5. Transfer the measured data to SI units and we can get two tables for L (Table 6) and mass (Table 8), and we further calculate Δ_L with the initial L_0, shown in Table 7 and the value of mg, taking $g = 9.794 m/s^2$, shown in Table 9. Recall that $k = mg/x$, so we do the linear fit of points $(\Delta L, m)$, and we get our k. The figures are shown in Figure 3

Read from the graph we can directly get the value of k (slope) and the uncertainty of k (CI Half-Width).

$$k_1 = 1.66 \pm 0.03[N/m] \quad k_2 = 1.66 \pm 0.03[N/m]$$

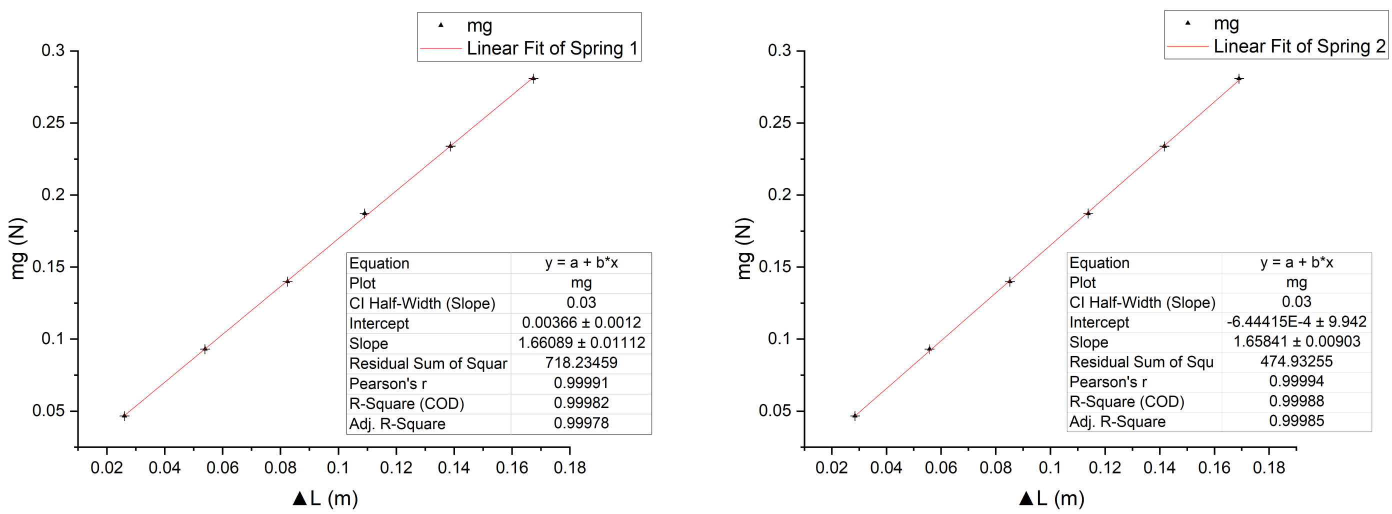

Figure 3: The linear fit figures of spring 1, 2

4.2 Analyzing the Relation Between T and Mass of the Oscillator M

The procedure of measuring T and M are shown respectively in 3.2 and 3.5. Here we use the mass of the sum of equivalent mass of I-shape and the masses, shown in Table 12. And for T, we first get T from T_{10} (see Table 10) and then we calculate the value of T^2, shown in Table 11, note that the uncertainty varies with T, so that the uncertainty of it is listed in another table 2.

Therefore,we can study the relation between T and M(Table 12) by linear fit T^2 with M, as shown in figure 4. Directly from the figure, we can read the slope, which means the value and uncertainty of T^2/m, the three values are respectively:

$$11.795 \pm 0.015, \quad 11.826 \pm 0.030, \quad 11.82 \pm 0.04 \quad [s^2/kg]$$

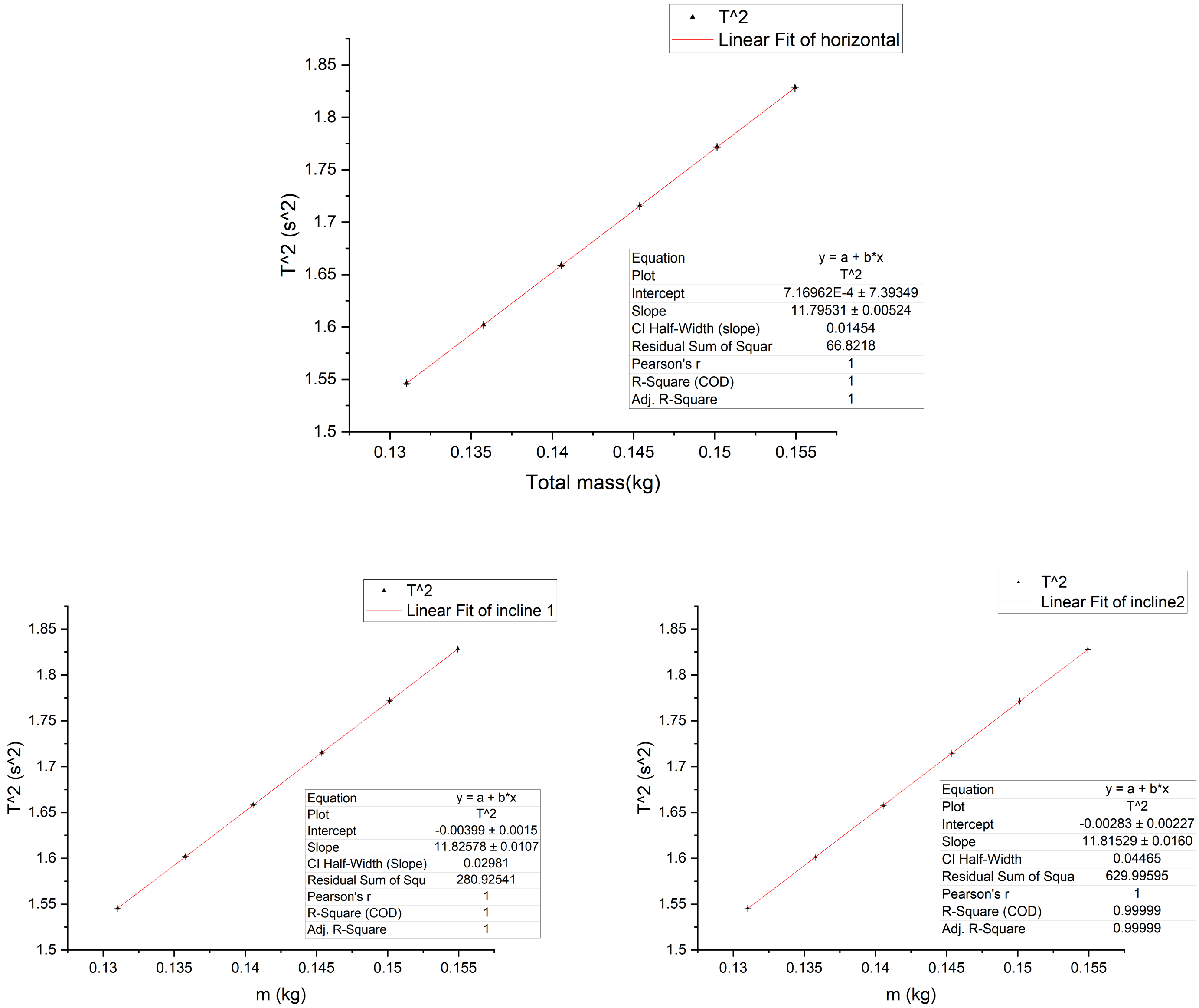

Figure 4: Linear fit of T^2/m for horizontal, incline 1, incline 2 air track.

From the graphs we can see that the three fitted k don't have obvious differences. Recall that $\frac{T^2}{M} = 4\pi^2 \cdot \frac{1}{k_1+k_2}$, so we can get the theoretical value of $\frac{T^2}{M}$ to be:

$$\frac{T^2}{M} = 4\pi^2 \cdot \frac{1}{1.66 + 1.66} = 11.9 \pm 0.15[s^2/kg]$$

So that the relative difference of the three fitted value is respectivly: 0.81%, 0.55%, 0.64%, so that the fitted values are close to the theoretical value.

4.3 Analyzing the Relation Between Period T and Amplitude A

The procedure of measuring T and A are mentioned in part 3.3. The table of T and A in this part are listed together in Tabel 13. The graph of A and T is shown in figure 5: we can see from the graph that there's no obvious relationship of A and T, and the correlation coefficient($\gamma = 0.25$)

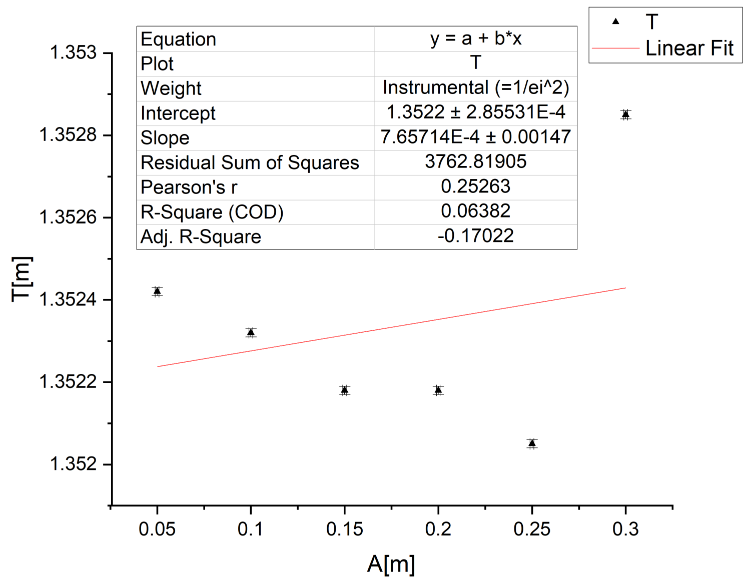

Figure 5: Relation between A and T

4.4 Analyzing the Relation Between Maximum Speed and Amplitude

To calculate speed, we get it from $v = \Delta x/\Delta t$, and $\Delta x = \frac{1}{2}(x_{in} + x_{out})$, and we get $\Delta x = 0.01000 \pm 0.00004[m]$. Hence we list the value of A, v in Table 14. Note that the uncertainty of v varies with the value of v, so that the uncertainty for v is listed in table 3.

Since we are to plot the figure of mv^2 vs. A^2, we then calculate the value of mv^2 and A^2 and their uncertainties, which are all listed in Table 16. The graph is shown in Figure 6

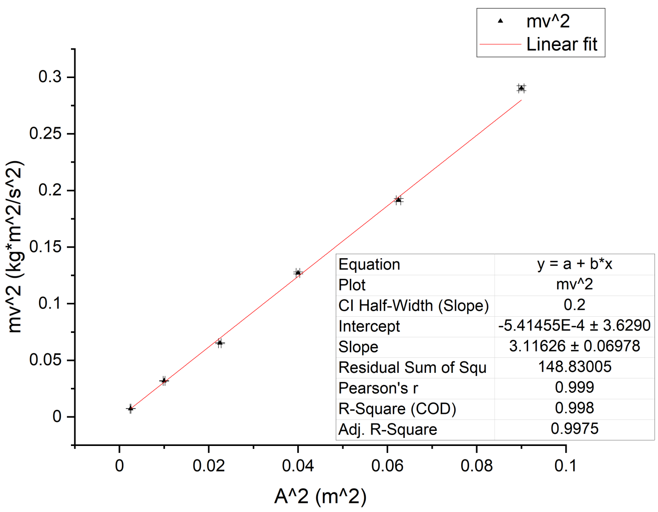

Figure 6: mv^2 vs. A^2

Read from the graph we can get the value of the slope to be $3.1 \pm 0.2[kg/s^2]$, Recall equation 4 so that the theoretical value of the slope should be $k_1 + k_2 = 3.32 \pm 0.04[kg/s^2]$. The relative error is 3.3%, so the two values are relatively close.

5 Conclusions and Discussion

5.1 Conclusions and Error Analyze

5.1.1 Measuring Spring Constant

In this experiment, we read two spring constants from the graph.

$$k_1 = 1.66 \pm 0.03[N/m], \text{ with relative uncertainty } 1.81\%$$
$$k_2 = 1.66 \pm 0.03[N/m], \text{ with relative uncertainty } 1.81\%$$

The relative uncertainty is very small and from the fitting stastic we can see that Adj. R-square are 0.99978 and 0.99985 respectively, which are very close to 1, these prove that the fitting is efficient. However there may be some error here because in this experiment, we read the caliper manually and the spring is always oscillating slightly so that the value we read isn't that exact.

5.1.2 Analyzing the Relation Between T and M

Before we do the experiment, we know that the theoretical relationship of T and M is

$$\frac{T^2}{M} = 4\pi^2 \cdot \frac{1}{k_1 + k_2}$$

we get 3 values from graphs which are 11.795 ± 0.015, 11.826 ± 0.030, 11.82 ± 0.04 $[s^2/kg]$ and calculate the theoretical value to be $11.9 \pm 0.15[s^2/kg]$, the three relative error are all smaller than 1%, and the R-squre of fitting is extremely close to 1, which means the fitting is very ideal, so that we can say that T and M follows the expected relation.

What may cause error here is 1. when we release the object, there may be an initial speed or resist the motion at the beginning 2. Although we use air track to eliminate air resistance, it does have some influence, which may affect the period.

5.1.3 Analyzing the Relation Between Period T and Amplitude A

From the figure, we can read that the correlation coefficient $\gamma = 0.25 << 1$ and the Adj. R-Square=-0.17<0, which shows that T and A don't have obvious relation. And the error is similar to the error in 5.2.

5.1.4 Analyzing the Relation Between Maximum Speed and Amplitude

From the figure we get

$$k = 3.1 \pm 0.2[kg/s^2]$$

. Our theoretical value of the spring constant here is

$$k_1 + k_2 = 3.32 \pm 0.04[kg/s^2]$$

and this gives the relative error to be 3.3%

What may cause error here except the reasons mentioned above in 5.2 and 5.3. It's also worth noticing that the value of v is calculated from several measured data. The complex calculation may also increase error.

5.2 Suggestions

For part 1 (measurement of spring constant), maybe we can use some electronic device to make the measurement more precise rather than read it manually. And for part 2,3,4, a release system can be used to ensure that when the object is released, no initial speed or resistance is forced on it.

6 Reference

Qin Tian, Zheng Huan, Li Yingyu, Li Tiantian, Mateusz Krzyzosiak, Vp141, Exercise 3, Simple Harmonic Motion: Oscillations in Mechanical Systems

A Uncertainty Analysis

A.1 Uncertainty in Measurement of Spring Constant

For the measurement of the spring constant, the uncertainty of mg is calculated as follow. Since m is get directly from measurement, $\mu_m = 0.01g = 0.01 \times 10^{-3}kg$

$$\mu_{mg} = \mu_m \cdot g = 0.01 \times 10^{-3}kg \times 9.794m/s^2 = 9.794 \times 10^{-5}N = 0.0001N$$

And the uncertainty for all L is:

$$\mu_L = 0.02 \times 10^{-3}m = 2 \times 10^{-5}m$$

Since we are using $\Delta L = L_x - L_0$, the uncertainty can be calculated from:

$$\mu_{\Delta_L} = \sqrt{\mu_L^2 \times 2} = \sqrt{0.00002^2 \times 2} = 0.00003m$$

The uncertainty for the spring constant k can be read from the CI Half-Wdith of $mg - L$ line, which was calculated in the fitting procedure, which are respectively:

$$\mu_{k_1} = 0.03[N/kg]$$
$$\mu_{k_2} = 0.03[N/kg]$$

A.2 Uncertainty in Analyzing the Relation Between T and M

The uncertainty of 10T is given as 0.0001s, so that the uncertainty of T is 0.00001s. And then we can use that to calculate the uncertainty of T square (Table 2):

$$\mu_{T^2} = \left|\frac{dT^2}{dT}\right| \cdot \mu_T = 2T \cdot \mu_T$$

For example: for M1, horizontal, we have:

$$\mu_{T^2} = 2 \times 1.24349s \times 0.00001s = 0.00002s^2$$

Uncertainty of $T^2[s^2]$					
horizontal		incline 1		incline 2	
m_1	0.00002	m_1	0.00002	m_1	0.00002
m_2	0.00003	m_2	0.00003	m_2	0.00003
m_3	0.00003	m_3	0.00003	m_3	0.00003
m_4	0.00003	m_4	0.00003	m_4	0.00003
m_5	0.00003	m_5	0.00003	m_5	0.00003
m_6	0.00003	m_6	0.00003	m_6	0.00003

Table 2

Also we need to calculate the uncertainty for m here since m is calculated by $m = M_0 + m_{mass} = m_{obj} + m_{mass} + \frac{1}{3}m_{spr1\&2}$

$$\begin{aligned}\mu_m &= \sqrt{\mu_{m_{obj}}^2 + \mu_{m_{spr1\&2}}^2 \times \frac{1}{3^2} + \mu_{m_{mass}}^2} \\ &= \sqrt{0.00001^2 \times (1 + 1 + \frac{1}{9})} \\ &= 1.5 \times 10^{-5}kg\end{aligned}$$

And recall that the uncertainty of the theoretical T^2/m is:

$$\frac{T^2}{m} = \frac{4\pi^2}{k_1 + k_2}$$

Therefore the uncertainty of the theoretical T^2/m is:

$$\begin{aligned}
\mu_{T^2/m} &= \sqrt{\left(\frac{\partial \frac{4\pi^2}{k_1+k_2}}{\partial k_1}\right)^2 \cdot \mu_{k_1}^2 + \left(\frac{\partial \frac{4\pi^2}{k_1+k_2}}{\partial k_2}\right)^2 \cdot \mu_{k_2}^2} \\
&= \sqrt{\left(\frac{-4\pi^2}{(k_1+k_2)^2}\right)^2 \cdot \mu_{k_1}^2 + \left(\frac{-4\pi^2}{(k_1+k_2)^2}\right)^2 \cdot \mu_{k_2}^2} \\
&= \sqrt{\left(\frac{-4\pi^2}{(1.66+1.66)^2}\right)^2 \cdot 0.03^2 + \left(\frac{-4\pi^2}{(1.66+1.66)^2}\right)^2 \cdot 0.03^2} \\
&= 0.15[s^2/kg]
\end{aligned}$$

A.3 Uncertainty of Analyzing the Relation Between Period T and Amplitude A

In this part, the amplitude has the uncertainty of 0.1cm and T_{10} has the uncertainty of 0.0001s, so the uncertainty of T is 0.00001s. The uncertainty of the relationship of T and A is given by the fitting.

A.4 Uncertainty of Analyzing the Relation Between Maximum Speed and Amplitude A

A.4.1 $\Delta \mathbf{x}$: $\mathbf{x_{in}}$ and $\mathbf{x_{out}}$

The type-B uncertainty for x_{in} is $\Delta x_{in} = 0.02mm$. To find the type-A uncertainty, we first find the standard deviation.

$$s_{x_{in}} = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_{in_i} - \overline{x}_{in}\right)^2} = \sqrt{\frac{1}{2}\left((4.42-4.44)^2 + (4.44-4.44)^2 + (4.46-4.44)^2\right)} = 0.02[mm]$$

We have n=3, so the type-A uncertainty $\Delta_{x_{in_A}}$ is calculated as

$$\Delta_{x_{in_A}} = \frac{t_{0.95}}{\sqrt{n}} s_{x_{in}} = 2.48 \times 0.02 = 0.05[mm]$$

Hence the uncertainty for x_{in} is given by

$$\mu_{x_{in}} = \sqrt{\Delta_{x_{in_A}}^2 + \Delta_{x_{in_B}}^2} = 0.05[mm]$$

Hence x_{in} is given by

$$x_{in} = 4.44 \pm 0.05[mm]$$

Similarly, we can calculate the uncertainty for x_{out}:

The type-B uncertainty for x_{out} is $\Delta x_{out} = 0.02mm$. To find the type-A uncertainty, we first find the standard deviation.

$$s_{x_{out}} = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_{out_i} - \overline{x}_{out}\right)^2} = \sqrt{\frac{1}{2}\left((15.56-15.56)^2 + (15.56-15.54)^2 + (15.56-15.58)^2\right)} = 0.02[mm]$$

We have n=3, so the type-A uncertainty $\Delta_{x_{out_A}}$ is calculated as

$$\Delta_{x_{out_A}} = \frac{t_{0.95}}{\sqrt{n}} s_{x_{out}} = 2.48 \times 0.02 = 0.05[mm]$$

Hence the uncertainty for x_{out} is given by

$$\mu_{x_{out}} = \sqrt{\Delta_{x_{out_A}}^2 + \Delta_{x_{out_B}}^2} = 0.05[mm]$$

Hence x_{out} is given by

$$x_{out} = 15.56 \pm 0.05[mm]$$

Then we proceed to calculate the uncertainty for Δx, Recall that:

$$\Delta x = \frac{\overline{x}_{in} + \overline{x}_{out}}{2}$$

So the uncertainty of Δx can be calculated by:

$$\begin{aligned}\mu_{\Delta_x} &= \sqrt{\frac{1}{2^2} \cdot \mu_{x_{in}}^2 + \frac{1}{2^2} \cdot \mu_{x_{out}}^2} \\ &= \sqrt{\frac{1}{4} \times 0.05^2 + \frac{1}{4} \times 0.05^2} \\ &= 0.0350725 = 0.04[mm]\end{aligned}$$

Hence Δx is given by

$$\Delta x = 10.0 \pm 0.04[mm] = 0.01 \pm 0.00004[m]$$

A.4.2 v : Δx and Δt

Recall that

$$v = \frac{\Delta x}{\Delta t}$$

So that we can calculate the uncertainty for v by:

$$\begin{aligned}\mu_v &= \sqrt{\left(\frac{\partial v}{\partial \Delta t}\right)^2 \cdot (\mu_{\Delta t})^2 + \left(\frac{\partial v}{\partial \Delta x}\right)^2 \cdot (\mu_{\Delta x})^2} \\ &= \sqrt{\left(\frac{1}{\Delta t}\right)^2 \cdot (\mu_{\Delta x})^2 + \left(\frac{\Delta x}{\Delta t^2}\right)^2 \cdot (\mu_{\Delta t})^2}\end{aligned}$$

For example, for the set of data: $A = 0.050 \pm 0.001[m]$ $\Delta t = 0.04771 \pm 0.00001[s]$

$$\begin{aligned}\mu_v &= \sqrt{\left(\frac{1}{\Delta t}\right)^2 \cdot (\mu_{\Delta x})^2 + \left(\frac{\Delta x}{\Delta t^2}\right)^2 \cdot (\mu_{\Delta t})^2} \\ &= \sqrt{\frac{1}{0.04771^2} \times 0.0000350725^2 + \left(\frac{0.01}{0.04771^2}\right)^2 \times 0.00001^2} \\ &= 0.0007[m/s]\end{aligned}$$

Similarly, the uncertainty of other sets of data are presented in Table 3

$A[m] \pm 0.001[m]$	$\mu_v[m/s]$
0.050	0.0007
0.100	0.0016
0.150	0.002
0.200	0.003
0.250	0.004
0.300	0.005

Table 3: Uncertainty for v

A.4.3 $\mathbf{v_{max}}$, A and m

Recall that

$$k = \frac{mv_{max}^2}{A^2}$$

so that for plotting we need to calculate the uncertainty for mv_{max}^2 and the uncertainty for A^2
To start with, we first calculate the uncertainty for m which is the same as the uncertainty of m in A.2

$$\mu_m = 0.000015[kg]$$

a. $\mu_{\mathbf{mv_{max}^2}}$

To be simplified, we denote v_{max} by v in the following calculation.

$$\begin{aligned}\mu_{mv^2} &= \sqrt{\left(\frac{\partial mv^2}{\partial m}\right)^2 \cdot \mu_m^2 + \left(\frac{\partial mv^2}{\partial v}\right)^2 \cdot \mu_v^2} \\ &= \sqrt{(v^2)^2 \cdot \mu_m^2 + (2mv)^2 \cdot \mu_v^2}\end{aligned}$$

For example for the set of data :

v	μ_v	m	μ_m
0.21	0.0007	0.16407	0.000015

We have:

$$\begin{aligned}\mu_{mv^2} &= \sqrt{(v^2)^2 \cdot \mu_m^2 + (2mv)^2 \cdot \mu_v^2} \\ &= \sqrt{(0.21^2)^2 \times 0.000015^2 + (2 \times 0.16407 \times 0.21)^2 \times 0.0007^2} \\ &= 0.00005[kg \cdot m^2/s^2]\end{aligned}$$

Similarly, the uncertainty of all data sets are shown in table 4:

v	μ_v	m	μ_m	μ_{mv^2}
0.21	0.0007	0.16407	0.00001	0.00005
0.44	0.0016	0.16407	0.00001	0.0002
0.63	0.0023	0.16407	0.00001	0.0005
0.88	0.0032	0.16407	0.00001	0.0009
1.08	0.0040	0.16407	0.00001	0.0014
1.33	0.0050	0.16407	0.00001	0.002

Table 4: Uncertainty for mv^2

b. $\mu_{\mathbf{A^2}}$

$$\mu_{A^2} = \left|\frac{dA^2}{dA}\right| \cdot \mu_A = 2A \cdot \mu_A$$

For example: for A=0.05m we have: $\mu_A = 2 \times 0.05 \times 0.001 = 0.0001[m^2]$ And similarly the uncertainty for all A^2 are shown in table following (Table 5):

A[m]	$\mu_{A^2}[m^2]$
0.0500	0.0001
0.1000	0.0002
0.1500	0.0003
0.2000	0.0004
0.2500	0.0005
0.3000	0.0006

Table 5: Uncertainty for A^2

A.4.4 k

From the figure of $mv_{max}^2\ vs.\ A^2$, we can directly read the uncertainty of k.
And for the theoretical value of k, we get it from $k_1 + k_2$, so the uncertainty for this is

$$\begin{aligned}\mu_k &= \sqrt{\mu_{k_1}^2 + \mu_{k_2}^2} \\ &= \sqrt{0.03^2 + 0.03^2} \\ &= 0.04[kg/s^2]\end{aligned}$$

B Data Sheet

UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY
DATA SHEET (EXERCISE 3)

Name: Yifei Zhang Student ID: 519370910103.

Name: Yuqi Gu. Student ID: 519370910104

Group: 10 Date: 9.19.

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with a pencil or modified with a correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used. You are required to hand in the original data with your lab report, so please keep the data sheet properly.

spring 1 [cm] ± 0.02 [mm]		spring 2 [cm] ± 0.02 [mm]		series [ ]± ____ [ ]	
L_0	36.040	L_0	36.992	L_0	
L_1	38.648	L_1	39.842	L_1	
L_2	41.432	L_2	42.570	L_2	
L_3	44.282	L_3	45.510	L_3	
L_4	46.948	L_4	48.376	L_4	
L_5	49.910	L_5	51.162	L_5	
L_6	52.778	L_6	53.894	L_6	

Table 1. Spring constant measurement data.

Instructor's signature: Li

ten periods [s] $\pm$ 0.0001 [s]					
horizontal		incline 1		incline 2	
m_1	12.4349	m_1	12.4316	m_1	12.4319
m_2	12.6571	m_2	12.6565	m_2	12.6535
m_3	12.8792	m_3	12.8773	m_3	12.8744
m_4	13.0976	m_4	13.0956	m_4	13.0939
m_5	13.3103	m_5	13.3107	m_5	13.3100
m_6	13.5214	m_6	13.5211	m_6	13.5199

Table 2. Measurement data for the T vs. M relation.

m_6

A [cm] $\pm$ 0.1 [cm]		ten periods [s] $\pm$ 0.0001 [s]
1	5.0	13.5242
2	10.0	~~13.5207~~ 13.5232
3	15.0	13.5218
4	20.0	13.5218
5	25.0	13.5205
6	30.0	~~13.52~~ 13.5285

Table 3. Data for the T vs. A relation.

m_6

A [cm] $\pm$ 0.1 [cm]		Δt [s] $\pm$ 0.0001 [s]
1	5.0	~~0.47710~~ 0.04771
2	12.0	0.02272
3	15.0	0.01580
4	20.0	0.01131
5	25.0	0.00924
6	30.0	0.00750
x_{in} [mm] $\pm$ 0.02 [mm]		x_{out} [mm] $\pm$ 0.02 [mm]
	~~2.46~~ 4.42	15.56
	~~2.44~~ 4.44	15.54
	~~2.50~~ 4.46	15.58

Table 4. Data for the v_{max}^2 vs. A^2 relation.

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m [g] $\pm$ 0.01 [g]	
1	4.76
2	9.50
3	14.28
4	19.11
5	23.87
6	28.67

Table 5. Weight measurement data.

object with I-shape m_{obj} [g] $\pm$ 0.01 [g]	
117.21	
object with U-shape m_{obj} [g] $\pm$ 0.01 [g]	
126.34	
mass of springs 1 & 2 $m_{\text{spr1\&2}}$ [g] $\pm$ 0.01 [g]	
27.19	
equivalent mass $M_0 = m_{\text{obj}} + \frac{1}{3}m_{\text{spr1\&2}}$ [g]	
I-shape	126.27
U-shape	135.40

Table 6. Mass measurement data.

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C Supporting Information

spring 1[m]±0.00002[m]		spring 2[m]±0.00002[m]	
L_0	0.36040	L_0	0.36992
L_1	0.38648	L_1	0.39842
L_2	0.41432	L_2	0.42570
L_3	0.44282	L_3	0.45510
L_4	0.46948	L_4	0.48376
L_5	0.49910	L_5	0.51162
L_6	0.52778	L_6	0.53894

Table 6: measurement data for spring constant

spring 1[m]±0.00002[m]		spring 2[m]±0.00002[m]	
ΔL_1	0.02608	ΔL_1	0.02850
ΔL_2	0.02784	ΔL_2	0.02728
ΔL_3	0.02850	ΔL_3	0.02940
ΔL_4	0.02666	ΔL_4	0.02866
ΔL_5	0.02962	ΔL_5	0.02786
ΔL_6	0.02868	ΔL_6	0.02732

Table 7: ΔL

$m[kg] \pm 0.00001[kg]$	
1	0.00476
2	0.00950
3	0.01428
4	0.01911
5	0.02387
6	0.02867

Table 8: mass of the masses

$mg[N] \pm 0.0001[N]$	
1	0.047
2	0.093
3	0.140
4	0.187
5	0.234
6	0.281

Table 9: value of mg

$T \pm 0.00001[s]$					
horizontal		incline 1		incline 2	
m_1	1.24349	m_1	1.24316	m_1	1.24319
m_2	1.26571	m_2	1.26565	m_2	1.26535
m_3	1.28792	m_3	1.28773	m_3	1.28744
m_4	1.30976	m_4	1.30956	m_4	1.30939
m_5	1.33103	m_5	1.33107	m_5	1.33100
m_6	1.35214	m_6	1.35211	m_6	1.35199

Table 10: Value for T in $T\ vs.\ M$ relation

$T^2[s^2]$					
horizontal		incline 1		incline 2	
m_1	1.54627	m_1	1.54545	m_1	1.54552
m_2	1.60202	m_2	1.60187	m_2	1.60111
m_3	1.65874	m_3	1.65825	m_3	1.65750
m_4	1.71547	m_4	1.71495	m_4	1.71450
m_5	1.77164	m_5	1.77175	m_5	1.77156
m_6	1.82828	m_6	1.82820	m_6	1.82788

Table 11: Value for T^2 in $T\ vs.\ M$ relation

$m_{all}[kg] \pm 0.000015[kg]$	
1	0.13103
2	0.13577
3	0.14055
4	0.14538
5	0.15014
6	0.15494

Table 12: equivalent mass of different masses with I-shape

A[m]$\pm 0.001[m]$	$T_{10} \pm 0.0001[s]$	$T \pm 0.00001[s]$
0.050	13.5242	1.35242
0.100	13.5232	1.35232
0.150	13.5218	1.35218
0.200	13.5218	1.35218
0.250	13.5205	1.35205
0.300	13.5185	1.35185

Table 13: Data for T vs. A

A$\pm 0.001[m]$	**v[m/**s^2**]**
0.050	0.21
0.100	0.44
0.150	0.63
0.200	0.88
0.250	1.08
0.300	1.33

Table 14: A and V

$x_{in} \pm 0.02[mm]$	$x_{out} \pm 0.02[mm]$
4.42	15.56
4.44	15.54
4.46	15.58

Table 15: x_{in} and x_{out}

$mv^2[kg \cdot m^2/s^2]$	$\mu_{mv^2}[kg \cdot m^2/s^2]$	$A^2[m^2]$	$\mu_{A^2}[s^2]$
0.0072	0.00005	0.0025	0.0001
0.0318	0.0002	0.0100	0.0002
0.0651	0.0005	0.0225	0.0003
0.1271	0.0009	0.0400	0.0004
0.1914	0.0014	0.0625	0.0005
0.2902	0.002	0.0900	0.0006

Table 16: mv^2 and A^2