1 Reference

Qin Tian, Zheng Huan, Li Yingyu, Li Tiantian, Mateusz Krzyzosiak, Vp141, Exercise 3, Simple Harmonic Motion: Oscillations in Mechanical Systems

A Uncertainty Analysis

A.1 Uncertainty in Measurement of Spring Constant

For the measurement of the spring constant, the uncertainty of mg is calculated as follow.

$$\mu_{mg} = \mu_m \cdot g = 0.01 \times 10^{-3} kg \times 9.794 m/s^2 = 9.794 \times 10^{-5} N$$

And the uncertainty for all L is:

$$\mu_L = 0.02 \times 10^{-3} m = 2 \times 10^{-5} m$$

The uncertainty for the spring constant k can be read from the slope of $mg - L$ line, which was calculated in the fitting procedure, which are respectively:

$$\mu_{k_1} =$$

$$\mu_{k_2} =$$

A.2 Uncertainty in Analyzing the Relation Between T and M

The uncertainty of 10T is given as 0.0001s, so that the uncertainty of T is 0.00001s. And then we can use that to calculate the uncertainty of T square (Table 1):

$$\mu_{T^2} = \left| \frac{dT^2}{dT} \right| \cdot \mu_T = 2T \cdot \mu_T$$

For example: for M1, horizontal, we have:

$$\mu_{T^2} = 2 \times 1.24349 s \times 0.00001 s = 0.00002 s^2$$

Uncertainty of $T^2 [s^2]$					
horizontal		incline 1		incline 2	
m_1	0.00002	m_1	0.00002	m_1	0.00002
m_2	0.00003	m_2	0.00003	m_2	0.00003
m_3	0.00003	m_3	0.00003	m_3	0.00003
m_4	0.00003	m_4	0.00003	m_4	0.00003
m_5	0.00003	m_5	0.00003	m_5	0.00003
m_6	0.00003	m_6	0.00003	m_6	0.00003

Table 1

And recall that the uncertainty of the theoretical T^2/m is:

$$\frac{T^2}{m} = \frac{4\pi^2}{k_1 + k_2}$$

Therefore the uncertainty of the theoretical T^2/m is:

$$\begin{aligned}\mu_{T^2/m} &= \sqrt{\left(\frac{\partial \frac{4\pi^2}{k_1+k_2}}{\partial k_1}\right)^2 \cdot \mu_{k_1}^2 + \left(\frac{\partial \frac{4\pi^2}{k_1+k_2}}{\partial k_2}\right)^2 \cdot \mu_{k_2}^2} \\ &= \sqrt{\left(\frac{-4\pi^2}{(k_1+k_2)^2}\right)^2 \cdot \mu_{k_1}^2 + \left(\frac{-4\pi^2}{(k_1+k_2)^2}\right)^2 \cdot \mu_{k_2}^2} \\ &= \\ &= \end{aligned}$$

A.3 Uncertainty of Analyzing the Relation Between Period T and Amplitude A

In this part, the amplitude has the uncertainty of 0.1cm and T_{10} has the uncertainty of 0.0001s, so the uncertainty of T is 0.00001s. The uncertainty of the relationship of T and A is given by the fitting.

A.4 Uncertainty of Analyzing the Relation Between Maximum Speed and Amplitude A

A.4.1 Δx : $\mathbf{x_{in}}$ and $\mathbf{x_{out}}$

The type-B uncertainty for x_{in} is $\Delta x_{in} = 0.02mm$. To find the type-A uncertainty, we first find the standard deviation.

$$s_{x_{in}} = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_{in_i} - \overline{x}_{in}\right)^2} = \sqrt{\frac{1}{2}((4.42-4.44)^2 + (4.44-4.44)^2 + (4.46-4.44)^2)} = 0.02[mm]$$

We have n=3, so the type-A uncertainty $\Delta_{x_{in_A}}$ is calculated as

$$\Delta_{x_{in_A}} = \frac{t_{0.95}}{\sqrt{n}} s_{x_{in}} = 2.48 \times 0.02 = 0.05[mm]$$

Hence the uncertainty for x_{in} is given by

$$\mu_{x_{in}} = \sqrt{\Delta_{x_{in_A}}^2 + \Delta_{x_{in_B}}^2} = 0.05[mm]$$

Hence x_{in} is given by

$$x_{in} = 4.44 \pm 0.05[mm]$$

Similarly, we can calculate the uncertainty for x_{out}:

The type-B uncertainty for x_{out} is $\Delta x_{out} = 0.02mm$. To find the type-A uncertainty, we first find the standard deviation.

$$s_{x_{out}} = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_{out_i} - \overline{x}_{out}\right)^2} = \sqrt{\frac{1}{2}((15.56-15.56)^2 + (15.56-15.54)^2 + (15.56-15.58)^2)} = 0.02[mm]$$

We have n=3, so the type-A uncertainty $\Delta_{x_{out_A}}$ is calculated as

$$\Delta_{x_{out_A}} = \frac{t_{0.95}}{\sqrt{n}} s_{x_{out}} = 2.48 \times 0.02 = 0.05[mm]$$

Hence the uncertainty for x_{out} is given by

$$\mu_{x_{out}} = \sqrt{\Delta_{x_{out_A}}^2 + \Delta_{x_{out_B}}^2} = 0.05[mm]$$

Hence x_{out} is given by

$$x_{out} = 15.56 \pm 0.05[mm]$$

Then we proceed to calculate the uncertainty for Δx, Recall that:

$$\Delta x = \frac{\overline{x}_{in} + \overline{x}_{out}}{2}$$

So the uncertainty of Δx can be calculated by:

$$\begin{aligned}\mu_{\Delta_x} &= \sqrt{\frac{1}{2^2} \cdot \mu_{x_{in}}^2 + \frac{1}{2^2} \cdot \mu_{x_{out}}^2} \\ &= \sqrt{\frac{1}{4} \times 0.05^2 + \frac{1}{4} \times 0.05^2} \\ &= 0.0350725 = 0.04[mm]\end{aligned}$$

Hence Δx is given by

$$\Delta x = 10.0 \pm 0.04[mm] = 0.01 \pm 0.00004[m]$$

A.4.2 v : Δx and Δt

Recall that

$$v = \frac{\Delta x}{\Delta t}$$

So that we can calculate the uncertainty for v by:

$$\begin{aligned}\mu_v &= \sqrt{\left(\frac{\partial v}{\partial \Delta t}\right)^2 \cdot (\mu_{\Delta t})^2 + \left(\frac{\partial v}{\partial \Delta x}\right)^2 \cdot (\mu_{\Delta x})^2} \\ &= \sqrt{\left(\frac{1}{\Delta t}\right)^2 \cdot (\mu_{\Delta x})^2 + \left(\frac{\Delta x}{\Delta t^2}\right)^2 \cdot (\mu_{\Delta t})^2}\end{aligned}$$

For example, for the set of data: $A = 0.050 \pm 0.001[m]$ $\Delta t = 0.04771 \pm 0.00001[s]$

$$\begin{aligned}\mu_v &= \sqrt{\left(\frac{1}{\Delta t}\right)^2 \cdot (\mu_{\Delta x})^2 + \left(\frac{\Delta x}{\Delta t^2}\right)^2 \cdot (\mu_{\Delta t})^2} \\ &= \sqrt{\frac{1}{0.04771^2} \times 0.0000350725^2 + \left(\frac{0.01}{0.04771^2}\right)^2 \times 0.00001^2} \\ &= 0.0007[m/s]\end{aligned}$$

Similarly, the uncertainty of other sets of data are presented in Table 2

$A[m] \pm 0.001[m]$	$\mu_v[m/s]$
0.050	0.0007
0.100	0.0016
0.150	0.002
0.200	0.003
0.250	0.004
0.300	0.005

Table 2: Uncertainty for v

A.4.3 $\mathbf{v_{max}}$, A and m

Recall that

$$k = \frac{mv_{max}^2}{A^2}$$

so that for plotting we need to calculate the uncertainty for mv_{max}^2 and the uncertainty for A^2

To start with, we first calculate the uncertainty for m since in the experiment, we use $m = M_0 + m_6$, so we need to calculate the uncertainty.

$$\mu_m = \sqrt{\mu_{M_0}^2 + \mu_{m6}^2} = \sqrt{0.01^2 \times 2} = 0.01[g] = 0.00001[kg]$$

a. $\mu_{\mathbf{mv_{max}^2}}$

To be simplified, we denote v_{max} by v in the following calculation.

$$\begin{aligned}\mu_{mv^2} &= \sqrt{\left(\frac{\partial mv^2}{\partial m}\right)^2 \cdot \mu_m^2 + \left(\frac{\partial mv^2}{\partial v}\right)^2 \cdot \mu_v^2} \\ &= \sqrt{\left(v^2\right)^2 \cdot \mu_m^2 + \left(2mv\right)^2 \cdot \mu_v^2}\end{aligned}$$

For example for the set of data :

v	μ_v	m	μ_m
0.21	0.0007	0.16407	0.00001

We have:

$$\begin{aligned}\mu_{mv^2} &= \sqrt{\left(v^2\right)^2 \cdot \mu_m^2 + \left(2mv\right)^2 \cdot \mu_v^2} \\ &= \sqrt{(0.21^2)^2 \times 0.00001^2 + (2 \times 0.16407 \times 0.21)^2 \times 0.0007^2} \\ &= 0.00005[kg \cdot m^2/s^2]\end{aligned}$$

Similarly, the uncertainty of all data sets are shown in table 3:

v	μ_v	m	μ_m	μ_{mv^2}
0.21	0.0007	0.16407	0.00001	0.00005
0.44	0.0016	0.16407	0.00001	0.0002
0.63	0.0023	0.16407	0.00001	0.0005
0.88	0.0032	0.16407	0.00001	0.0009
1.08	0.0040	0.16407	0.00001	0.0014
1.33	0.0050	0.16407	0.00001	0.002

Table 3: Uncertainty for mv^2

b. $\mu_{\mathbf{A^2}}$

$$\mu_{A^2} = \left|\frac{dA^2}{dA}\right| \cdot \mu_A = 2A \cdot \mu_A$$

For example: for A=0.05m we have: $\mu_A = 2 \times 0.05 \times 0.001 = 0.0001[m^2]$ And similarly the uncertainty for all A^2 are shown in table following:

A[m]	$\mu_{A^2}[m^2]$
0.05	0.0001
0.1	0.0002
0.15	0.0003
0.2	0.0004
0.25	0.0005
0.3	0.0006