UM-SJTU Joint Institute
Physics Laboratory
(Vp141)

Laboratory Report

Exercise 5
Damped and Driven Oscillations
Mechanical Resonance

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1 Introduction

The objective of this experiment is to use the Pohl resonator to study damped and driven oscillations in mechanical systems. We will study the property of mechanical resonance by analyzing the figure based on our experimental data.

2 Theoratical background

If a force which varies periodically is exerted on a damped harmonic oscillator, the motion is called forced (or driven) oscillations. Suppose the form of the driving force is

$$F = F_0 cos\omega t \tag{1}$$

Where F_0 is the amplitude and ω is the angular frequency.

When the oscillator reaches the steady state, its angular frequency will be the same as ω.

The amplitude of the steady-state oscillation depends on the relationship between the natural angular frequency and the driving frequency, and the damping coefficient. When the amplitude becomes quite large, the phenomenon is called mechanical resonance.

There will be a phase lag between the driving force and the displacement from equilibrium of the oscillating particle. When ω equals the natural frequency, the phase lag is $\pi/2$.

In this experiment, we will use angular counterparts to replace the linear quantities.

When a driving torque $\tau_d r = \tau_0 cos\omega t$ and a damping torque $\tau_f = -b\frac{d\theta}{dt}$ and a restoring force acted upon the balance wheel, its equation of motion is

$$I\frac{d^2\theta}{dt^2} = -k\theta - b\frac{d\theta}{dt} + \tau_0 cos\omega t \tag{2}$$

where I is the moment of inertia of the balance wheel, τ_0 is the amplitude of the driving torque, and ω is angular frequency of the driving torque.

Introducing the following symbols ${}_0{}^2 = \frac{k}{I}, 2\beta = \frac{b}{I}, \mu = \frac{\tau_0}{I}$,Eq.2 can be written as:

$$I\frac{d^2\theta}{dt^2} + 2\beta\frac{d\theta}{dt} + \omega_0 = \mu cos\omega t \tag{3}$$

$$\theta(t) = \theta_{tr}(t) + \theta_(st)cos(\omega t + \phi) \tag{4}$$

where θ_{tr} denotes the transient solution and

$$\theta_{st} = \frac{\mu}{\sqrt{({\omega_0}^2 - \omega^2)^2 + 4\beta^2\omega^2}} \tag{5}$$

The phase shift ϕ satisfies the equation:

$$tan\phi = \frac{2\beta\omega}{\omega^2 - {\omega_0}^2} \tag{6}$$

where $-\pi \leq \phi \leq 0$, which is independent of the initial conditions.

By finding the maximum value of θ_{st}, we can find the resonance angular frequency $\omega = \omega_{res} = \sqrt{{\omega_0}^2 - 2\beta^2}$ (7), and the corresponding amplitude is $\theta_{res} = \theta_{st}(\omega_{res}) = \frac{\mu}{2\beta\sqrt{{\omega_0}^2-\beta^2}})$ (8).

The dependence of the amplitude and the phase on the driving angular frequency is shown in figure 3. When damping increases, the resonance frequency will move away from the natural frequency to a smaller value and the amplitude of the steady-state oscillation decreases.

3 Apparatus

The experimental setup is shown in figure 3 and 4.

The BG-2 Pohl resonator consists of two main parts: a vibrometer (Figure 3) and a control box(Figure 4). The scroll spring provides an elastic restoring torque so that the wheel can rotate about an equilibrium state. There are notches on the edge of the wheel, and one of them is deeper than the others. When this notch passes the photoelectric detector(1), the period of oscillations can be measured. The amplitude can also be measured by counting the notches and will be displayed. Another photoelectric detector(11) is set above the turntable and can measure the period of the driving force. When the Period Selection switch is at position "1", a single oscillation period will be displayed; when it is at position "2", the time of 10 period will be displayed.

The damping comes from the pair of coils at the bottom of the supporting frame. When the coils carry current, the electromagnetic force will serve as the damping torque. Its magnitude can be changed by changing the current. The Damping Selection knob can be used to change the current.

A motor and an electric wheel are used to drive the wheel.

As for the control box, a Period Selection switch and a Period of Driving Force can be used to control the speed of the motor precisely.

To measure the phase shift, the glass turntable with an angle scale and a strobe light are used. When the deep notch passes the detector, a line on the scale will be highlighted and can be read.

The information of each measuring scale is shown in Table 1.

apparatus	range	resolution	uncertainty
Vibrometer-period	/	0.001s	±0.001s
Vibrometer-angle	0-180°	1°	±1°
Vibrometer-amplitude	/	1°	±1°

Table 1: Information of Each Measuring scale

4 Measurement

4.1 Natural Angular Frequency

a. Rotate the wheel to a degree of about 90°and release it, and record the time of 1 period.

b. Repeat step a and record 4 sets of data and calculate the natural angular frequency.

4.2 Damping Coefficient

a. Turn the Damping mode to 2 and do not change it.

b. Rotate the wheel to a degree of about 90°and release it, and record the value of amplitude each period and the time of 10 intervals.

c. The damping coefficient can be calculated using the equation $\beta = \frac{1}{5T} ln \frac{\theta_i}{\theta_{i+5}}$ where T is the average period

4.3 θ_{st} vs. ω and ϕ vs. ω Characteristics of Forced Oscillations

a. Choose damping mode 2 and set the speed of the motor.

b. Wait until the oscillation reaches its steady state where the amplitude doesn't change. Record the amplitude θ_{st}, the period T and the phase shift ϕ.

c. Repeat step 2 for 15-20 times. Change the speed of motor slowly and collect more data when phase shift is near $\pi/2$.

d. Choose Damping Selection 1 or 3. Repeat the above steps.

e. Plot the θ_{st} characteristics, set ω/ω_0 the horizontal axis and θ_{st} the vertical axis. Plot the data of two different modes on the same graph.

f. Plot the ϕ characteristics, set ω/ω_0 the horizontal axis and ϕ the vertical axis. Plot the data of two different modes on the same graph.

5 Result

5.1 Natural Angular Frequency

	T[s]±0.001[s]
1	1.556
2	1.556
3	1.556
4	1.556

Table 2: Measurement of the natural frequency

From table 2 we get the average value of the time of one period:

$$\overline{T} = \frac{1}{4}\sum_{n=1}^{4} T_n = \frac{1.556 + 1.556 + 1.556 + 1.556}{4} = 1.556 \pm 0.001s$$

The natural frequency can be calculated as $\omega_0 = \frac{2\pi}{T} = 4.038 \pm 0.003 rad/s$

5.2 Damping Coefficient

The measured time of a period 10T=15.550s±0.001s, so T=1.5550±0.0001s

The last data in table 3 is calculated by

$$\overline{ln(\theta_i/\theta_{i+5})} = \frac{1}{5}\sum_{i=0}^{4} ln(\theta_i/\theta_{i+5}) = \frac{0.470 + 0.470 + 0.462 + 0.452 + 0.457}{5} = 0.46 \pm 0.03$$

Use the equation $\beta = \frac{1}{5T} ln(\theta_i/\theta_{i+5}) = \frac{1}{5\times1.5550} \times 0.462 = 0.059 \pm 0.004 s^{-1}$

Amplitude[°]± 1[°]		Amplitude[°]± 1[°]		$\ln(\theta_i/\theta_{i+5})$
θ_0	88	θ_5	55	0.470
θ_1	80	θ_6	50	0.470
θ_2	73	θ_7	46	0.462
θ_3	66	θ_8	42	0.452
θ_4	60	θ_9	38	0.457
The average value of $\ln(\theta_i/\theta_{i+5})$				0.462

Table 3: Measurement of the damping coefficient

5.3 θ_{st} vs. ω and ϕ vs. ω Characteristics of Forced Oscillations

5.3.1 Damping Selection 2

We can found that $\frac{\omega}{\omega_0} = \frac{T_{natural}}{T_{driven}}$ and $T_{natural} = 1.556 \pm 0.001s$

T[s]±0.001[s]	ω/ω_0	uncertainty for ω/ω_0	ϕ[°]±1[°]	θ[°]±1[°]
1.474	1.0556	0.0013	-164	34
1.486	1.0471	0.0012	-161	40
1.501	1.0366	0.0012	-157	49
1.511	1.0298	0.0012	-152	59
1.524	1.0210	0.0012	-145	76
1.533	1.0150	0.0012	-135	90
1.541	1.0097	0.0012	-123	109
1.553	1.0019	0.0012	-100	128
1.556	1.0000	0.0012	-92	130
1.558	0.9987	0.0012	-89	130
1.560	0.9974	0.0012	-84	129
1.564	0.9949	0.0012	-74	126
1.573	0.9892	0.0012	-58	112
1.583	0.9829	0.0012	-47	93
1.592	0.9774	0.0012	-38	78
1.605	0.9695	0.0012	-30	63
1.619	0.9611	0.0011	-24	51
1.628	0.9558	0.0011	-21	46
1.642	0.9476	0.0011	-18	39

Table 4: θ_{st} vs. ω and ϕ vs. ω Characteristics of 5.3.1

5.3.2 Damping Selection 3

T[s]$\pm$0.001[s]	ω/ω_0	uncertainty for ω/ω_0	ϕ[°]$\pm$1[°]	θ[°]$\pm$1[°]
1.476	1.0542	0.0013	-164	35
1.487	1.0464	0.0012	-162	40
1.501	1.0366	0.0012	-158	50
1.508	1.0318	0.0012	-155	56
1.521	1.0230	0.0012	-149	72
1.529	1.0177	0.0012	-141	88
1.541	1.0097	0.0012	-125	114
1.552	1.0026	0.0012	-101	133
1.553	1.0019	0.0012	-99	137
1.555	1.0006	0.0012	-93	137
1.558	0.9987	0.0012	-88	138
1.559	0.9981	0.0012	-84	137
1.564	0.9949	0.0012	-74	130
1.571	0.9905	0.0012	-60	122
1.581	0.9842	0.0012	-48	99
1.594	0.9762	0.0012	-36	76
1.606	0.9689	0.0012	-28	62
1.618	0.9617	0.0011	-23	52
1.629	0.9552	0.0011	-20	45
1.643	0.9470	0.0011	-17	39

Table 5: θ_{st} vs. ω and ϕ vs. ω Characteristics of 5.3.2

5.4 Comparing Figures

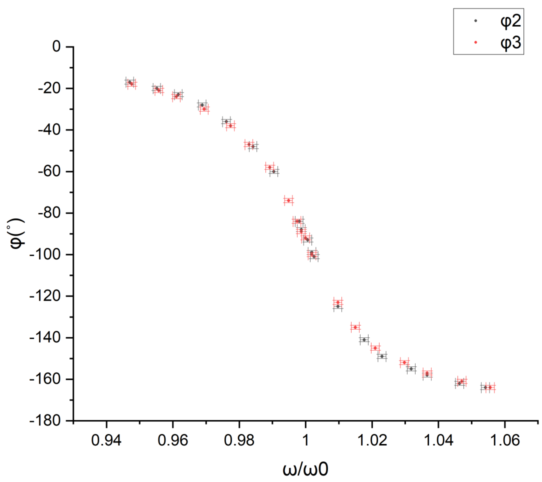

Figure 1: ϕ v.s. ω/ω_0 characteristics

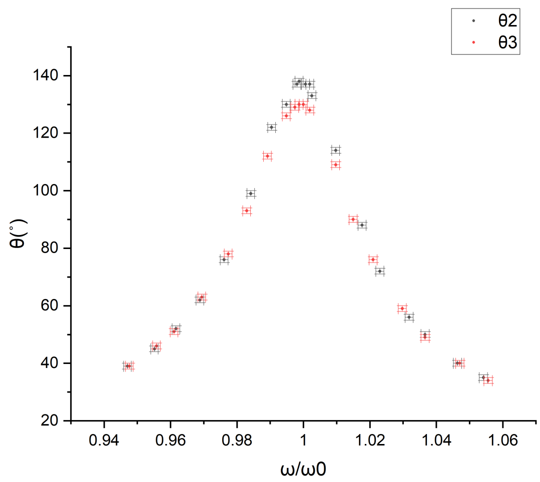

Figure 2: θ_{st} v.s. ω/ω_0 characteristics

From the graph of θ_{st} we can see when ω/ω_0 is close to 1, the amplitude reaches its maximum, which shows that the wheel is at mechanical resonance. Also, from the higher points, we can find that $\beta_2 < \beta_3$.

Also, from the graph of ϕ, we could find that the slope reaches its maximum when ω/ω_0 approximately equals 1 and $\beta_2 < \beta_3$.

6 Conclusion and Discussion

6.1 Conclusions

The natural frequency is $4.038 \pm 0.003 rad/s$.

The damping coefficient is $0.059 \pm 0.04 s^-1$.

The relative uncertainties are 0.06% and 0.07% respectively, which are small enough to prove the accuracy of our measurements.

From the $\theta - \omega/\omega_0$ figure we can conclude:

- When the driving frequency is near the natural frequency, the steady-state amplitude will peak
- When damping increases, the resonance frequency will move away from the natural frequency to a smaller value and the amplitude of the steady-state oscillation decreases.

From the $\phi - \omega/\omega_0$ figure we can conclude:

- The phase lag becomes larger when the driving frequency increases.
- When the driving frequency is near the natural frequency, the phase lag is near $\pi/2$, and there's a sharp change in the phase lag within this range

6.2 Discussions

The errors might exist because:

- The air drag and friction inside the wheel will disturb the oscillation.
- It's hard to tell whether the oscillation has reached its steady state because the amplitude might be changing even if we think that it stays still.
- When reading the phase lag, the instantaneous flash light is hard to catch because it disappears too fast. Also, we observed that the phase lag was not stable, switching between two adjacent values. Therefore, the reading of phase lag might be inaccurate.

6.3 Improvements

- The accuracy of the device can be improved. As can be seen from table 4 and 5, some value of θ are the same due to the limitation of the precision of the vibrometer. This will result in a flat slope on the figure. It will be better if the minimum scale of value can be small enough to distinguish these data.
- Use another method to read the phase lag since reading when the light flashes is not optimal.

APPENDIX

A Implementary Figures

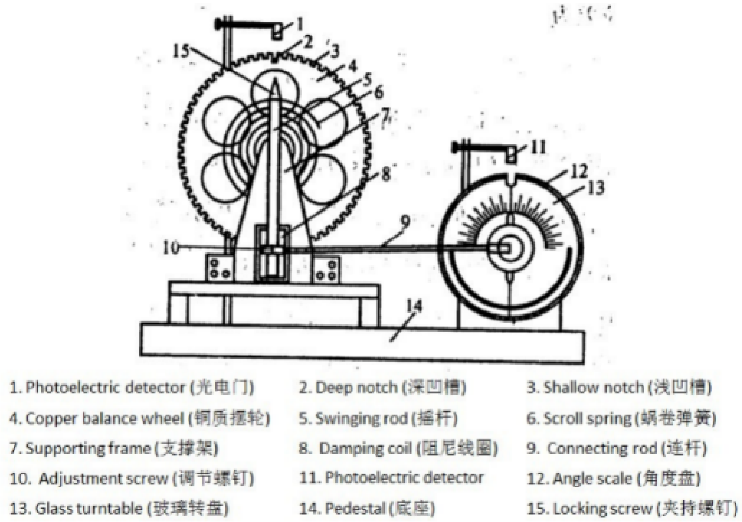

Figure 3: The vibrometer

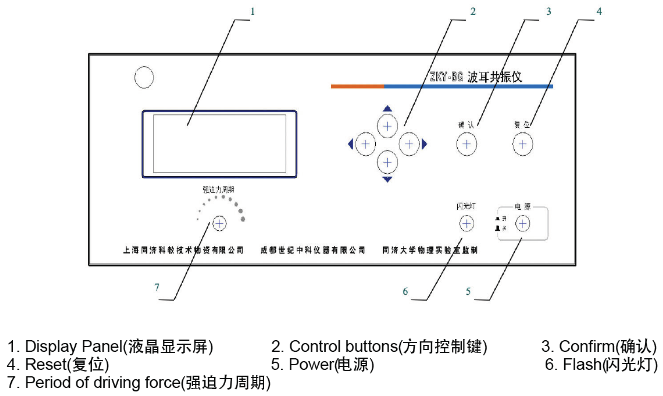

Figure 4: The front panel of the control box

B Reference

Qin Tian, Wang Yin, Tianyi Li, Mateusz Krzyzosiak, Physics Laboratory VP141 Exercise 5 Damped and Driven Oscillations. Mechanical Resonance

C Data Sheet and Uncertainty Data Sheet