
UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP141)

LABORATORY REPORT

EXERCISE 1
MOMENT OF INERTIA

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Date: September 18, 2020

1 Introduction

1.1 Objectives

In this lab we need to use the digital multifunctional timer to measure time, to observe how the moment of inertia will vary as conditions change, and to verify the Parallel-Axis Theorem.

1.2 Theoretical Background

1.2.1 Rotational Motion of a Rigid Body

A rigid body will maintain the distance between all points of particle while applying any force on it. Pure rotational motion means each particle of a body moves in a circle. The center of all the circles lie on a straight line called the axis of rotation.

1.2.2 Moment of Inertia

The moment of inertia, I , describes the hardness to change an object's rotational motion, defined as the ratio of torque $\vec{\tau}$ to angular acceleration $\vec{\alpha}$.

$$\vec{\tau} = I\vec{\alpha}$$

The magnitude of the moment of inertia can be effected by shape, mass distribution and the position of the rotation axis. For a point of mass, the moment of inertia is defined by mass m and distance r

$$I = mr^2$$

1.2.3 Parallel-Axis Theorem

Parallel-Axis Theorem describes the relationship between the moment of inertia of a certain body I_{cm} about an axis through its center of mass with mass M and the moment of inertia I_P about any other axis parallel to the original axis with distance d

$$I_P = I_{cm} + Md^2$$

1.2.4 Experimental Determination of the Moment of Inertia

The platform will get a negative acceleration α_1 due to the frictional torque τ_μ . After adding the weight, the changed angular acceleration α_2 can be effected by mass m , distance R .

$$\begin{cases} \theta_m = km\pi = \omega_0 t_m + \frac{1}{2}\alpha_1 t_m^2 & \tau_\mu = I\alpha_1 \\ m(g - R\alpha_2)R - \tau_\mu = I\alpha_2 \end{cases} \quad (1)$$

Combining Eq.1, we can figure out the moment of inertia

$$I = \frac{mR(g - R\alpha_2)}{\alpha_2 - \alpha_1} \quad (2)$$

Superposition Principle allows us to obtain the moment of inertia for a combined object rotating together.

$$I_3 = I_1 + I_2 \quad (3)$$

2 Experimental Setup

The apparatus used in this experiment including a moment of inertia measurement combinational device, test samples (a disk, a cirque and two cylinders), a Vernier Caliper, an electronic scale, and a digital multifunctional timer.

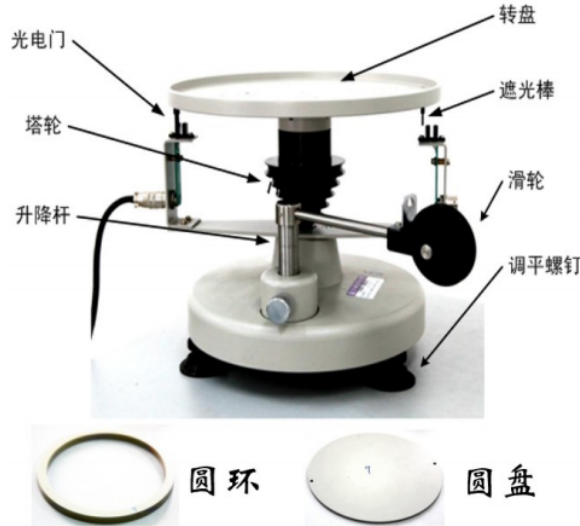


Figure 1: Experimental Setup

The multifunctional timer generates an impulse each time the stick passes through the gate of the sensor. It allows measurements with maximum uncertainty of $0.0001s$. It starts the measurement after the first impulse from the sensor, and recording the time used between each pass, thus measuring the time change of certain interval to calculate the angular acceleration.

The mass of test samples and the weight was measured by an electronic scale with maximum uncertainty $0.1g$.

The size of the test samples, the diameter of the cone pulley, and the distance of the changing rotation axis was determined by using a Vernier Caliper with maximum uncertainty 0.02 mm .

3 Measurement

3.1 Mass measurement

Measure the mass of the weight and test samples. For convenience, we measure the weight for cirque by measuring the weight of disk and the weight of the combination of disk and cirque, then use calculation to yield the certain mass.

3.2 Length Measurement

Use Vernier Caliper to measure the size of sample test and the cone pulley. Measure six times for each and take the average value.

3.3 Time measurement

Before starting measurements, the power supply was turned on.

- Through observing a bubble, change the height of the device feet to make the platform level
- Choose the multi-pulse mode and channel A to prepare for the recording
- Click start to begin recording the time measurement data
- Wait for the platform to go 8 round and get 16 data ($T_{01}-T_{16}$)
- Click pause to stop and read the recorded time through the screen
- Tie the heavy object on the pulley, make sure the rope is horizontal and won't touch the hole on the roll
- Click start immediately after letting the weight go
- Click pause when the weight about to hit the ground
- Record 8 data ($T_{01}-T_{08}$) from the screen

The above measurement of the rotating period was repeated four times (empty platform, platform with disk, platform with cirque and platform with cylinders). The obtained data is presented in Result.

4 Results

4.1 Mass and Length

The length was measured in the procedure described in 3.2. Because of the hardness of measuring the radius, I measure the diameter first. The diameter equals to

$$D = \frac{1}{6} \sum_{i=1}^6 D_i$$

and the radius equals to $\frac{D}{2}$. Followed is all the length data.

Length[mm] ± 0.01 [mm]						Length[mm] ± 0.02 [mm]	
$R_{c,i}$	$R_{c,o}$	R_d	R_A	R_B	R	d_i	d_o
149.99	119.93	119.94	14.99	15.00	25.07	69.95	80.11

Table 1: Length Measurement Data

A simple measurement described in 3.1 yields that the mass is as followed

mass[g] ± 0.1 [g]			
m	54.5		
m_d	488.5	$m_c + m_d$	887.6
m_A	165.8	m_B	165.8

Table 2: Mass Measurement Data

4.2 Moment of Inertia

The moment of inertia can be calculated by Eq.1 mentioned in Introduction. The angular accelerations were calculated through making the figures for fitting quadratic curve. I choose Origin2020b to calculate the conic coefficient. Time measured as 3.3 mentioned is the x-axis and degree is the y-axis. The figures are in the appendix.

4.2.1 Empty Platform

The angular acceleration for the empty platform is $-0.0770 \pm 4 \times 10^{-4} \text{rad/s}^2$ (the angular acceleration is $2 \cdot B_2$ according to the Eq.1). The angular acceleration after hanging the weight is $1.6738 \pm 4.1 \times 10^{-3} \text{rad/s}^2$. According to Eq.2

$$I_p = \frac{mR(g - R\alpha_2)}{\alpha_2 - \alpha_1} = 7.6 \times 10^{-3} \pm 0.0148 \text{kg} \cdot \text{m}^2$$

4.2.2 Platform with Cirque

We can find that the platform with cirque at first have the angular acceleration $-0.0502 \pm 6 \times 10^{-4} \text{rad/s}^2$. The angular acceleration after hanging the weight is $1.1362 \pm 2.4 \times 10^{-3} \text{rad/s}^2$. According to Eq.1

$$I_p + I_c = \frac{mR(g - R\alpha_2)}{\alpha_2 - \alpha_1} = 11.2 \times 10^{-3} \pm 0.0218 \text{kg} \cdot \text{m}^2$$

Applying Eq.3, we can find the moment of inertia for the cirque is $3.6 \times 10^{-3} \pm 0.0263 \text{kg} \cdot \text{m}^2$

4.2.3 Platform with Disk

We can find that the platform with disk at first have the angular acceleration $-0.0567 \pm 2 \times 10^{-4} \text{rad/s}^2$. The angular acceleration after hanging the weight is $1.3247 \pm 2.7 \times 10^{-3} \text{rad/s}^2$. According to Eq.1

$$I_p + I_d = \frac{mR(g - R\alpha_2)}{\alpha_2 - \alpha_1} = 9.6 \times 10^{-3} \pm 0.0187 \text{kg} \cdot \text{m}^2$$

Applying Eq.3, we can find the moment of inertia for the disk is $2.0 \times 10^{-3} \pm 0.0238 \text{kg} \cdot \text{m}^2$

4.2.4 Platform with cylinders

We can find that the platform with cylinders at first have the angular acceleration $-0.0652 \pm 2 \times 10^{-4} \text{rad/s}^2$. The angular acceleration after hanging the weight is $1.5759 \pm 4.5 \times 10^{-3} \text{rad/s}^2$. According to Eq.1

$$I_p + I_{cy} = \frac{mR(g - R\alpha_2)}{\alpha_2 - \alpha_1} = 8.1 \times 10^{-3} \pm 0.0158 \text{kg} \cdot \text{m}^2$$

Applying Eq.3, we can find the moment of inertia for the cylinders is $0.5 \times 10^{-3} \pm 0.0216 \text{kg} \cdot \text{m}^2$.

5 Conclusions and Discussion

5.1 Moment of Inertia

According to 3.2, the inside and outside diameter for the cirque separately equals to $209.98 \pm 0.03mm$ and $239.92 \pm 0.06mm$. So, the radius equals to $104.99 \pm 0.01mm$ and $119.96 \pm 0.03mm$. A simple measurement described in 3.1 yields that the mass of the cirque equals to $399.1 \pm 0.1g$. By applying calculus to the mass point's moment of inertia formula, we can find the moment of inertia for the cirque.

$$I_c = \frac{1}{2}m(R_{c,i}^2 + R_{c,o}^2) = 5.071 \times 10^{-3} \pm 2.420 \times 10^{-3}kg \cdot m^2$$

Similarly, the radius of the disk equals to $\frac{D_{d,n}}{2} = 119.94 \pm 0.01mm$, and the mass of the disk equals to $488.5 \pm 0.1g$. The moment of inertia for the disk can be found.

$$I_d = \frac{1}{2}mR_d^2 = 3.513 \times 10^{-3} \pm 9.28 \times 10^{-4}kg \cdot m^2$$

The relative difference for the cirque is 28.2% and for the disk is 41.8%. Both of them are too large.

But when we turn to the absolute difference. They are similar. One is $1.43 \times 10^{-3}kg \cdot m^2$ and another is $1.47 \times 10^{-3}kg \cdot m^2$. This may show that there exists certain mistake in the measure of the moment inertia for the empty platform. After adding $1.45 \times 10^{-3}kg \cdot m^2(\frac{1}{2}(1.47 + 1.43))$ to each, the relative difference can be decreased to 0.4% and 0.6%. But we can not sure since there only exists two groups of data, we still need to turn to the cylinder data to further confirm this hypothesis.

If this hypothesis is true means that platform's theoretical moment of inertia is smaller than the practical measurement data. It may because of the string tied on the weight is not level or touch the hole at some time. These may cause the decrease of the force arm, thus lessening the angular acceleration. And finally the moment of inertia is bigger than the theoretical value.

As the fundamental of the measurement, I think that the moment of inertia for the empty platform should be measured several times to decrease the uncertainty.

In the lab period, I found that the cirque and the disk cannot fit the platform well, so I have to manually make the rotational axes at the same line often. If other students did not pay attention to it may cause the practical value larger than actual. So, the apparatus can somehow improved by providing a method to fix the position of test samples.

5.2 Parallel-Axis Theorem

According to 3.2, the diameter for cylinder A and B separately equal to $29.98 \pm 0.03mm$ and $30.01 \pm 0.02mm$. So, the radius equals to $14.99 \pm 0.02mm$ and $15.00 \pm 0.01mm$. A simple measurement described in 3.1 yields that the mass of two cylinders both equals to $165.8 \pm 0.1g$. By applying calculus to the mass point's moment of inertia formula, we can find that the moment of inertia for each cylinder equals

$$I_A = \frac{1}{2}mR_A^2 = 1.9 \times 10^{-5} \pm 4.4 \times 10^{-5}kg \cdot m^2$$

$$I_B = \frac{1}{2}mR_B^2 = 1.9 \times 10^{-5} \pm 3.2 \times 10^{-5} kg \cdot m^2$$

We could calculate distance required in theorem through measure both the inner distance d_i and outer distance d_o and find the average $\frac{d_i+d_o}{2}$. The distance we got is $75.03mm \pm 0.01mm$.

Applying the Parallel-Axis Theorem, we can find that the total moment of inertia for the cylinder system equals $1.9 \times 10^{-3} \pm 9.44 \times 10^{-4} kg \cdot m^2$. It has a huge relative difference that is 74.2%. But after adding $1.45 \times 10^{-3} kg \cdot m^2$, which was mentioned in the formal hypothesis, the relative difference is only 2.1%. The theoretical and practical value is almost the same. So the parallel-axis theorem can be somehow proved.

But I think only one group of data is not enough to prove the parallel-axis theorem. There exists 5 different distances to fix cylinders in 4 different directions, many other datas can be measured to further prove the parallel-axis theorem.

6 Reference

Xiner Shen, Neale Haugen, Exercise 1, Moment of Inertia

APPENDIX

A Measurement Uncertainty Analysis

A.1 Uncertainty of the mass

The uncertainty of the mass mainly caused by the apparatus. The electronic scale has the type-B uncertainty $0.1g$.

A.2 Uncertainty of the length

The uncertainty of the mass mainly caused by the apparatus. The electronic scale has the type-B uncertainty $0.02mm$.

In order to decrease the uncertainty caused by manual measurement, those length values were measured six times

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

length[mm] ± 0.02 [mm]						
	1	2	3	4	5	6
$D_{c,i}$	209.98	209.96	210.00	209.98	210.00	209.94
$D_{c,o}$	239.92	240.04	239.86	239.90	239.98	239.96
D_d	239.84	239.86	239.90	239.82	239.86	239.88
D_A	29.92	29.98	30.00	29.98	29.96	30.02
D_B	30.02	29.98	30.04	29.98	30.00	30.02
R	50.10	50.18	50.16	50.18	50.14	50.12
d_i	69.98	69.90	69.98	69.94	69.96	69.94
d_o	80.10	80.06	80.10	80.16	80.12	80.14

Table 3: Length Measurement Data

	Cirque Inner Diameter	Cirque Outer Diameter	Disk Diameter	Cylinder A Diameter
Mean(mm)	209.98	239.92	239.88	29.98
Uncertainty (mm)	0.03	0.06	0.03	0.03
	Cylinder B Diameter	Cone Pulley Diameter	Inner Axis Movement Distance	Outer Axis Movement Distance
Mean(mm)	30.00	50.15	69.95	80.11
Uncertainty (mm)	0.02	0.03	0.03	0.04

Table 4: The Mean and Uncertainty of Length Measurement Data

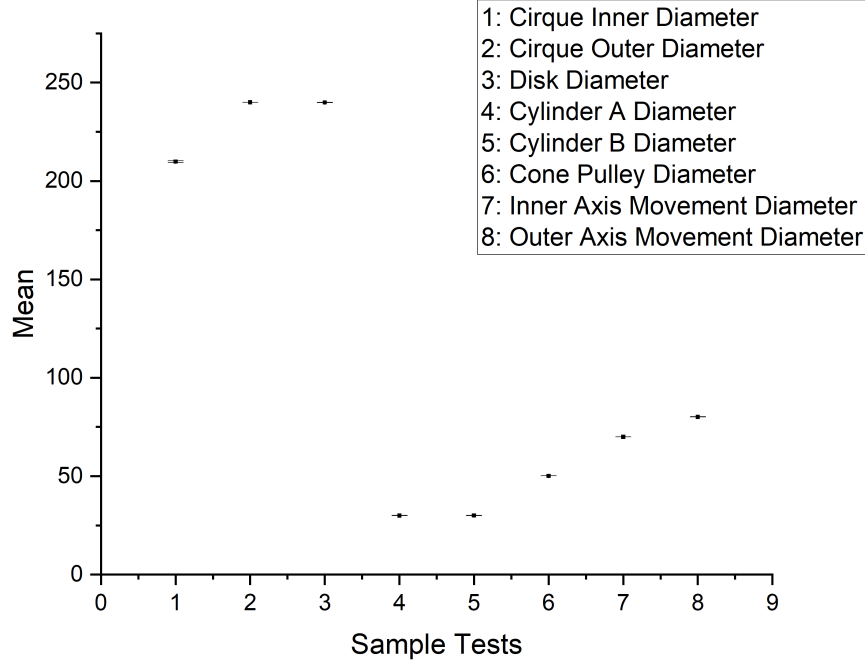


Figure 2: Uncertainty of Length

A.3 The Calculation Uncertainty

A.3.1 The Angular Acceleration Uncertainty

The formation to descript the relationship between degree and angular acceleration is $\theta_m = km\pi = \omega_0 t_m + \frac{1}{2}\alpha t_m^2$. So

$$\sigma_\alpha = \sqrt{\left(\frac{\partial \theta_m}{\partial \alpha}\right)^2 \sigma_{B_2}^2} = 2\sigma_{B_2}$$

σ_{B_2} is calculated automatically by Origin2020b.

A.3.2 the Superposition Theorem Uncertainty

The formation to descript the relationship between the moment of inertia for the system and for separte objects, $I_3 = I_1 + I_2$.

$$\sigma_{I_3} = \sqrt{\left(\frac{\partial I_3}{\partial I_1}\right)^2 \sigma_{I_1}^2 + \left(\frac{\partial I_3}{\partial I_2}\right)^2 \sigma_{I_2}^2} = \sqrt{\sigma_{I_1}^2 + \sigma_{I_2}^2}$$

A.3.3 the Practical Moment of Inertia Uncertainty

The formation to calculate the moment of inertia is $I = \frac{mR(g-R\alpha_2)}{\alpha_2-\alpha_1}$. So

$$\begin{cases} \frac{\partial I}{\partial m} = \frac{R(g-R\alpha_2)}{\alpha_2-\alpha_1} \\ \frac{\partial I}{\partial R} = \frac{m(g-2R\alpha_2)}{\alpha_2-\alpha_1} \\ \frac{\partial I}{\partial \alpha_1} = \frac{mR(g-R\alpha_2)}{(\alpha_2-\alpha_1)^2} \\ \frac{\partial I}{\partial \alpha_2} = \frac{mR(R\alpha_1-g)}{(\alpha_2-\alpha_1)^2} \end{cases}$$

$$\begin{aligned} \sigma_I &= \sqrt{\left(\frac{\partial I}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial I}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial I}{\partial \alpha_1}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{\partial I}{\partial \alpha_2}\right)^2 \sigma_{\alpha_2}^2} \\ &= \sqrt{\left(\frac{R(g-R\alpha_2)}{\alpha_2-\alpha_1}\right)^2 \sigma_m^2 + \left(\frac{m(g-2R\alpha_2)}{\alpha_2-\alpha_1}\right)^2 \sigma_R^2 + \left(\frac{mR(g-R\alpha_2)}{(\alpha_2-\alpha_1)^2}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{mR(R\alpha_1-g)}{(\alpha_2-\alpha_1)^2}\right)^2 \sigma_{\alpha_2}^2} \end{aligned}$$

Through putting the data calculated separately for empty platform, platform with cirque, platform with disk and platform with cylinders, we can get all the uncertainty for the practical moment of inertia.

$$\begin{aligned} \sigma_{I_p}^2 &= \left(\frac{\frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.6738)}{1.6738 - (-0.0770)}\right)^2 \times 0.1^2 \\ &\quad + \left(\frac{\frac{54.5}{1000} \times (9.794 - 2 \times \frac{25.07}{1000} \times 1.6738)}{1.6738 - (-0.0770)}\right)^2 \times 0.016^2 \\ &\quad + \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.6738)}{(1.6738 - (-0.0770))}\right)^2 \times 0.00035362^2 \\ &\quad + \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} (\frac{25.07}{1000} \times (-0.0770) - 9.794)}{(1.6378 - (-0.0770))^2}\right)^2 \times 0.00410^2 \end{aligned}$$

$$\sigma_{I_p} = 0.0148 \text{ kg} \cdot \text{m}^2$$

$$\begin{aligned} \sigma_{I_p}^2 &= \left(\frac{\frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.1362)}{1.1362 - (-0.0502)}\right)^2 \times 0.1^2 \\ &\quad + \left(\frac{\frac{54.5}{1000} \times (9.794 - 2 \times \frac{25.07}{1000} \times 1.1362)}{1.1362 - (-0.0502)}\right)^2 \times 0.016^2 \\ &\quad + \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.1362)}{(1.1362 - (-0.0502))}\right)^2 \times 0.00057596^2 \\ &\quad + \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} (\frac{25.07}{1000} \times (-0.0502) - 9.794)}{(1.1362 - (-0.0502))^2}\right)^2 \times 0.00236^2 \end{aligned}$$

$$\sigma_{I_p+I_c} = 0.218 \text{ kg} \cdot \text{m}^2$$

$$\begin{aligned}
\sigma_{I_p+I_d}^2 &= \left(\frac{\frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.3247)}{1.3247 - (-0.0567)} \right)^2 \times 0.1^2 \\
&+ \left(\frac{\frac{54.5}{1000} \times (9.794 - 2 \times \frac{25.07}{1000} \times 1.3247)}{1.3247 - (-0.0567)} \right)^2 \times 0.016^2 \\
&+ \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.3247)}{(1.3247 - (-0.0567))} \right)^2 \times 0.00023631^2 \\
&+ \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} (\frac{25.07}{1000} \times (-0.0567) - 9.794)}{(1.3742 - (-0.0567))^2} \right)^2 \times 0.00272^2 \\
\sigma_{I_p+I_d} &= 0.0187 kg \cdot m^2 \\
\sigma_{I_p+I_{cy}}^2 &= \left(\frac{\frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.6738)}{1.6738 - (-0.0770)} \right)^2 \times 0.1^2 \\
&+ \left(\frac{\frac{54.5}{1000} \times (9.794 - 2 \times \frac{25.07}{1000} \times 1.6738)}{1.6738 - (-0.0770)} \right)^2 \times 0.016^2 \\
&+ \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} \times (9.794 - \frac{25.07}{1000} \times 1.5759)}{(1.5759 - (-0.0652))} \right)^2 \times 0.00024561^2 \\
&+ \left(\frac{\frac{54.5}{1000} \times \frac{25.07}{1000} (\frac{25.07}{1000} \times (-0.0652) - 9.794)}{(1.5759 - (-0.0652))^2} \right)^2 \times 0.00478^2 \\
\sigma_{I_p+I_{cy}} &= 0.0158 kg \cdot m^2
\end{aligned}$$

A.3.4 the Theoretical Moment of Inertia Uncertainty

Through applying the calculus on $I = mr^2$. We could find that for disks and cylinders $I_d = I_{cy} = \frac{1}{2}mR^2$, and for cirques $I_c = \frac{1}{2}m(R_i^2 + R_o^2)$.

For cirques, we can find that

$$\begin{aligned}
\sigma_{I_c} &= \sqrt{\left(\frac{\partial I_c}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial I_c}{\partial R_{c,i}}\right)^2 \sigma_{R_{c,i}}^2 + \left(\frac{\partial I_c}{\partial R_{c,o}}\right)^2 \sigma_{R_{c,o}}^2} \\
&= \sqrt{\left(\frac{1}{2}(R_{c,i}^2 + R_{c,o}^2)\right)^2 \sigma_m^2 + (mR_{c,i})^2 \sigma_{R_{c,i}}^2 + (mR_{c,o})^2 \sigma_{R_{c,o}}^2} \\
&= \sqrt{\left(\frac{1}{2}\left(\frac{104.99^2}{1000} + \frac{119.96^2}{1000}\right)\right)^2 \times 0.1^2 + \left(\frac{399.1}{1000} \times \frac{104.99}{1000}\right)^2 \times 0.01^2 + \left(\frac{399.1}{1000} \times \frac{119.96}{1000}\right)^2 \times 0.03^2} \\
&= 2.420 \times 10^{-3} kg \cdot m^2
\end{aligned}$$

For disks and cylinders, we can find that

$$\begin{aligned}
\sigma_{I_d} &= \sqrt{\left(\frac{\partial I_d}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial I_d}{\partial R}\right)^2 \sigma_R^2} = \sqrt{\left(\frac{R^2}{2}\right)^2 \sigma_m^2 + (mR)^2 \sigma_R^2} \\
&= \sqrt{\left(\frac{(\frac{119.94}{1000})^2}{2}\right)^2 \times 0.1^2 + \left(\frac{488.5}{1000} \times \frac{119.94}{1000}\right)^2 \times 0.014^2} = 9.28 \times 10^{-4} kg \cdot m^2
\end{aligned}$$

$$\begin{aligned}
\sigma_{I_{cyA}} &= \sqrt{\left(\frac{\partial I_{cy}}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial I_{cy}}{\partial R}\right)^2 \sigma_R^2} = \sqrt{\left(\frac{R^2}{2}\right)^2 \sigma_m^2 + (mR)^2 \sigma_R^2} \\
&= \sqrt{\left(\frac{(\frac{14.99}{1000})^2}{2}\right)^2 \times 0.1^2 + \left(\frac{165.8}{1000} \times \frac{14.99}{1000}\right)^2 \times 0.017^2} = 4.4 \times 10^{-5} kg \cdot m^2 \\
\sigma_{I_{cyB}} &= \sqrt{\left(\frac{\partial I_{cy}}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial I_{cy}}{\partial R}\right)^2 \sigma_R^2} = \sqrt{\left(\frac{R^2}{2}\right)^2 \sigma_m^2 + (mR)^2 \sigma_R^2} \\
&= \sqrt{\left(\frac{(\frac{15.00}{1000})^2}{2}\right)^2 \times 0.1^2 + \left(\frac{165.8}{1000} \times \frac{15.00}{1000}\right)^2 \times 0.012^2} = 3.2 \times 10^{-5} kg \cdot m^2
\end{aligned}$$

A.3.5 The Parallel-Axis Theorem Uncertainty

The parallel-axis theorem is $I_p = I_{cm} + Md^2$.

$$\begin{aligned}
\sigma_{I_p} &= \sqrt{\left(\frac{\partial I_p}{\partial M}\right)^2 \sigma_M^2 + \left(\frac{\partial I_p}{\partial d}\right)^2 \sigma_d^2 + \left(\frac{\partial I_p}{\partial I_{cm}}\right)^2 \sigma_{I_{cm}}^2} = \sqrt{(d^2)^2 \sigma_M^2 + (2Md)^2 \sigma_d^2 + (1)^2 \sigma_{I_{cm}}^2} \\
&= \sqrt{\left(\left(\frac{75.03}{1000}\right)^2 \times 0.014^2 + \left(2 \times \frac{331.6}{1000} \times \frac{75.03}{1000}\right)^2 \times 0.0104^2 + 5.44 \times 10^{-5}\right)} \\
&= 9.44 \times 10^{-4} kg \cdot m^2
\end{aligned}$$

B Data Sheet

UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY DATA
SHEET (EXERCISE 1)

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Student ID: 519370910123

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Date: 2020.9.12

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with a pencil or modified with a correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used. You are required to hand in the original data with your lab report, so please keep the data sheet properly.

mass [g] \pm 0.1 [g]			
m	54.5		
m_d	488.5	$m_c + m_d$	887.6
m_A	165.8	m_B	165.8

Table 1: Mass Measurement Data.

length [mm] \pm 0.02 [mm]						
	1	2	3	4	5	6
$D_{c,i}$	209.98	209.96	210.02	209.98	208.00	209.94
$D_{c,o}$	239.92	238.04 240.04	239.86	239.90	239.98	239.96
D_d	239.84	239.86	239.90	239.82	239.86	239.88
D_A	29.92	29.98	28.00 30.00	29.98	29.96	28.02 30.02
D_B	28.02 30.02	29.98	28.04 30.04	29.98	28.00 30.00	28.02 30.02
R	50.40 50.10	50.38	50.36	50.38 50.18	50.44 50.14	50.42
d_i	69.98	69.90	69.98	69.94	69.96	69.94
d_o	79.79 80.10	80.06	80.10	80.16	80.12	80.14

Table 2: Length Measurement Data.

Instructor's signature: SXL

	time [s] ± 0.001 [s]		time [s] ± 0.001 [s]
1	0.6385	9	6.0112
2	1.2839	10	6.7191
3	1.9364	11	7.4356
4	2.5961	12	8.1610
5	3.2634	13	8.8956
6	3.9382	14	9.6394
7	4.6209	15	10.3929
8	5.3116	16	11.1564

Table 3: Time Measurement Data (Empty Platform without Weight).

	time [s] ± 0.001 [s]
1	0.9671
2	1.6433
3	2.1959
4	2.6751
5	3.1046
6	3.4973
7	3.8614
8	4.2024

Table 4: Time Measurement Data (Empty Platform with Weight).

Instructor's signature: sxe

	time [s] ± 0.0001 [s]		time [s] ± 0.0001 [s]
1	0.5230	9	4.7974
2	1.0486	10	5.3431
3	1.5767	11	5.8914
4	2.1072	12	6.4423
5	2.6402	13	6.9969
6	3.1758	14	7.5523
7	3.7138	15	8.1113
8	4.2543	16	8.6732

Table 5: Time Measurement Data (Platform with Cirque but without Weight).

	time [s] ± 0.0001 [s]
1	1.1159
2	1.9178
3	2.5780
4	3.1534
5	3.6688
6	4.1428
7	4.5816
8	4.9930

Table 6: Time Measurement Data (Platform with Cirque and with Weight).

Instructor's signature: SKL

	time	s	± 0.0001	s		time	s	± 0.0001	s
1	0.5614				9	5.1749			
2	1.1260				10	5.7676			
3	1.6940				11	6.3643			
4	2.2653				12	6.9646			
5	2.8402				13	7.5688			
6	3.4184				14	8.1769			
7	4.0003				15	8.7891			
8	4.5857				16	9.4052			

Table 7: Time Measurement Data (Platform with Disk but without Weight).

	time	s	± 0.0001	s
1	1.1050			
2	1.8720			
3	2.4969			
4	3.0285			
5	3.5230			
6	3.9659			
7	4.3761			
8	4.7603			

Table 8: Time Measurement Data (Platform with Disk and with Weight).

Instructor's signature: Sxe

	time	$\frac{s}{\pm 0.0001}$	$\frac{s}{\pm 0.0001}$		time	$\frac{s}{\pm 0.0001}$	$\frac{s}{\pm 0.0001}$
1	0.5846			9	5.4256		
2	1.1735			10	6.0523		
3	1.7671			11	6.6844		
4	2.3650			12	7.3214		
5	2.9616			13	7.9639		
6	3.5748			14	8.6117		
7	4.1869			15	9.2651		
8	4.8037			16	9.9240		

Table 9: Time Measurement Data (Platform with Cylinders but without Weight).

	time	$\frac{s}{\pm 0.0001}$	$\frac{s}{\pm 0.0001}$
1	0.8543		
2	1.4973		
3	2.0360		
4	2.6096		
5	3.1312		
6	3.6306		
7	4.1966		
8	4.7406		

Table 10: Time Measurement Data (Platform with Cylinders and with Weight).

Instructor's signature: SXe

C Supporting Information

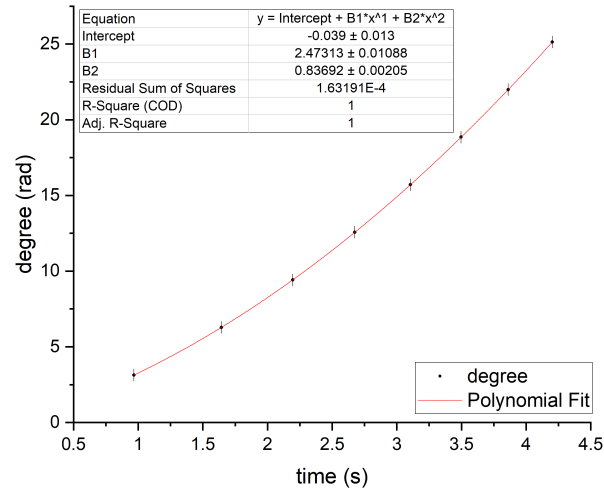


Figure 3: Time Measurement Data for Empty Platform without weight

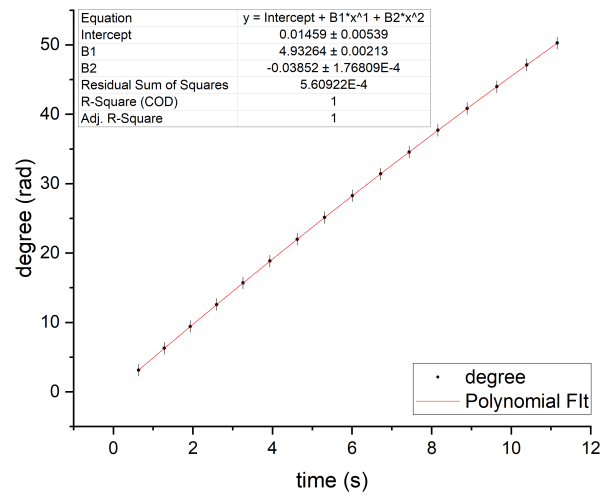


Figure 4: Time Measurement Data for Empty Platform with weight

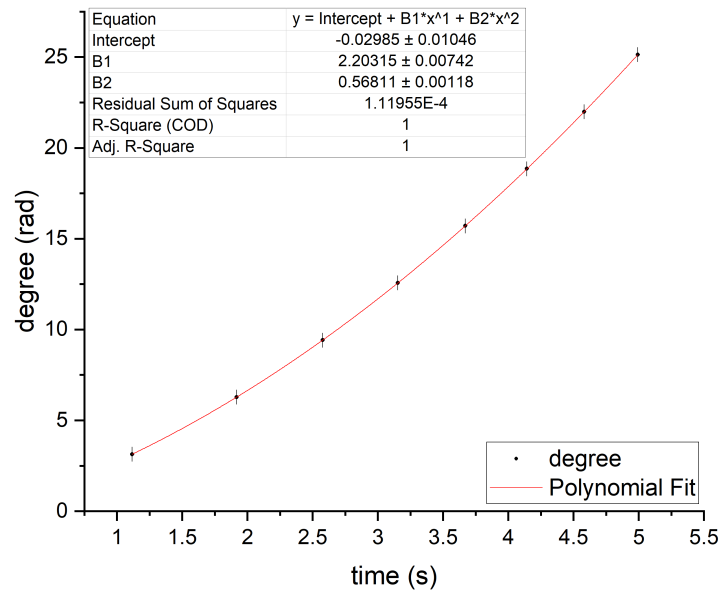


Figure 5: Time Measurement Data for Platform with Cirque but without weight

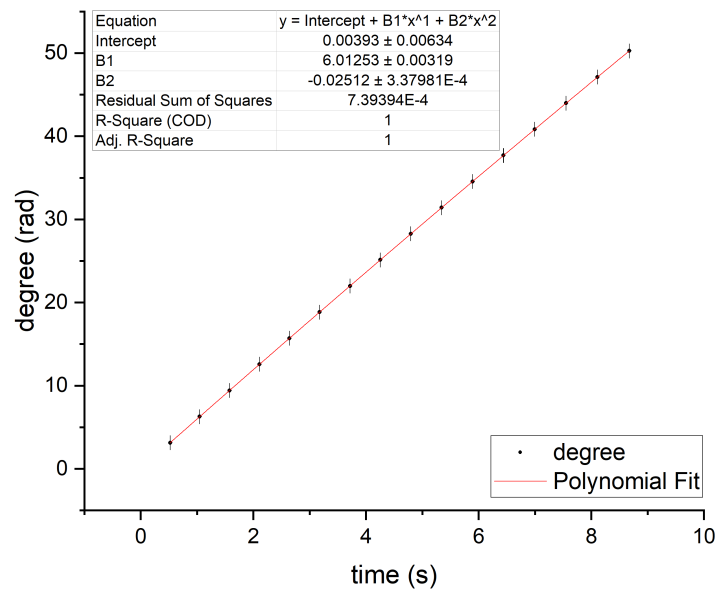


Figure 6: Time Measurement Data for Platform with Cirque and with weight

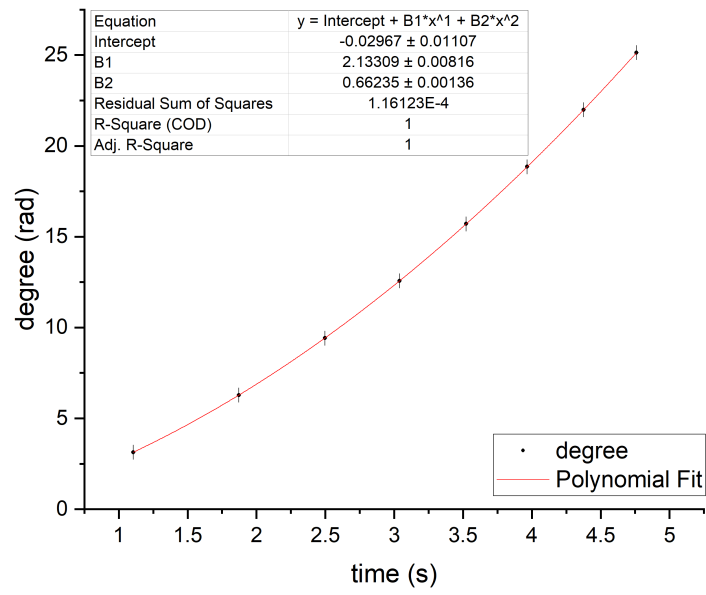


Figure 7: Time Measurement Data for Platform with Disk but without weight

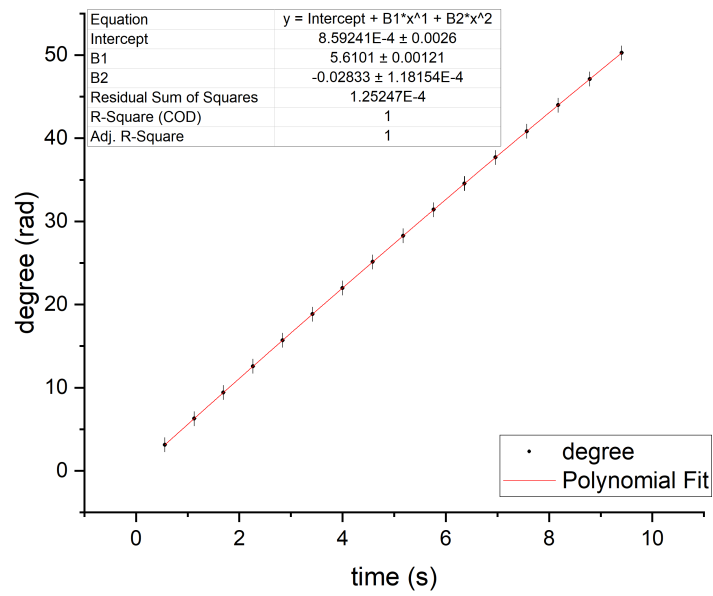


Figure 8: Time Measurement Data for Platform with Disk and with weight

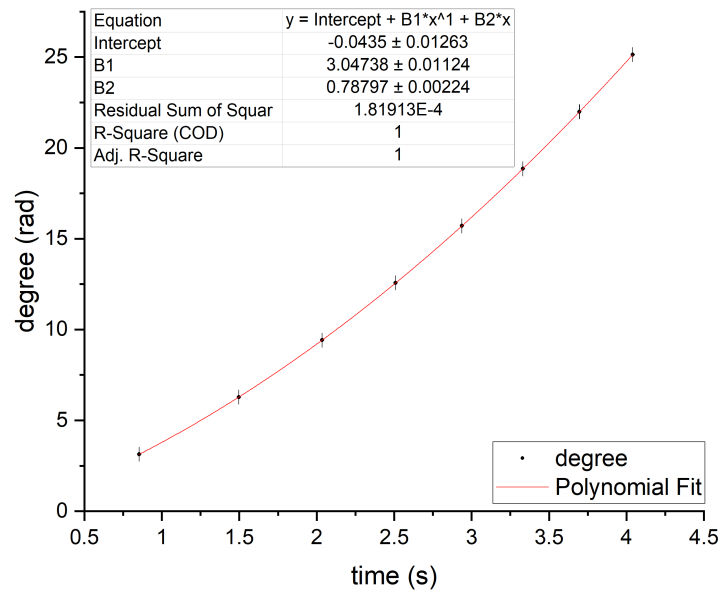


Figure 9: Time Measurement Data for Platform with Cylinders but without weight

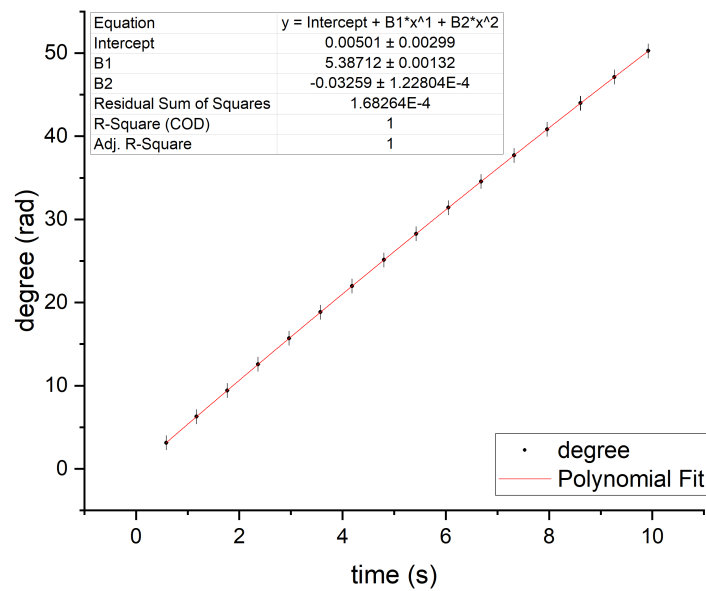


Figure 10: Time Measurement Data for Platform with Cylinders and with weight