UM-SJTU Joint Institute

Physics Laboratory

(Vp141)

Laboratory Report

Exercise 4

Measurement of the Speed of Sound

Name: Wenxin He   ID: 518370910117 Group: 2

Name: Yuanchen Bai   ID:518370910183 Group:2

Date: 28 June 2019

1. Introduction

The objective of this experiment is to use three methods to measure the speed of the sound in air: the resonance method, the phase comparison method and the time difference method. The methods of operating a function generator and an oscilloscope are also studied.

2. Theoretical Background

2.1 Basic Quantitative Characteristics of Sound Waves

Sound is a longitudinal wave propagating through a compressible medium as the direction of the medium vibrations is the same as that of propagation. The frequency of sound perceptible to human ears is within the range of 20Hz to 20000Hz. In this experiment, we use ultrasonic wave because its wavelength is short enough to be measured in a teaching lab.

The formula of the phase speed v, the frequency f, and the wavelength λ is:

$$v = \lambda f \qquad (1)$$

For motion with constant speed v along a straight line with distance L over time t, we have:

$$v = \frac{L}{t} \qquad (2)$$

2.2 Measurement Method

2.2.1 Resonance Method

The distance between the wave source(s1) and the receiver(s2) is L. when arranged parallel to each other, the sound wave will be reflected. If L satisfies:

$$L = n\frac{\lambda}{2} \qquad (3)$$

Where n=1, 2,…. , standing waves will form as the distance is a multiple of half of the wavelength. Under this circumstance, a maximum output power will be observed in the oscillography. As

shown in figure 1, the interval between two maximum points is $\frac{\lambda}{2}$. Therefore, λ can be measured

from the oscillography. By reading the frequency f from the signal generator, we can calculate the wavelength using equation (1).
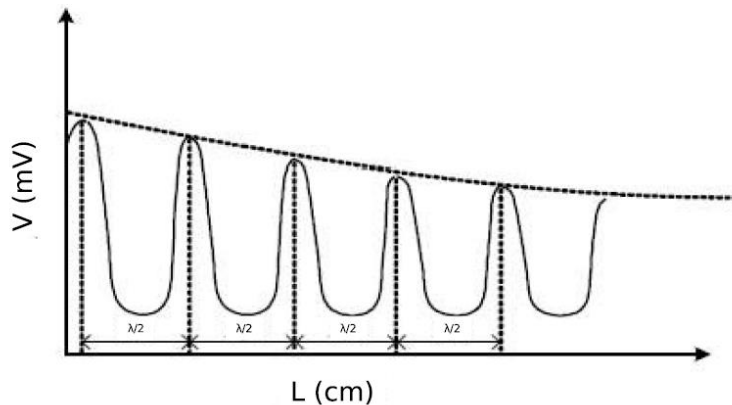

Figure 1 Relationship between the signal voltage and the distance between the transducers

2.2.2 Phase-comparison Method

Lissajous figures are used to display the trajectories of a particle moving simultaneously in two perpendicular directions. On x direction the wave function is $A_x \cos(\omega_x t + \phi_x)$ and on y direction the wave function is $A_y \cos(\omega_y t + \phi_y)$. Therefore, the trajectory of the particle is $r(t) = (A_x \cos(\omega_x t + \phi_x), A_y \cos(\omega_y t + \phi_y))$. When the two superimposed harmonic motions have identical frequency $\omega_x = \omega_y$ and $|\phi_x - \phi_y|$=nπ (n=0,1,2…), the Lissajous figure will show a straight line. For other phase difference values the figure will show corresponding elliptical shapes.

In this experiment, we find different values of L when a line of the same slope is displayed. When the distance increases by λ, ϕ increases by 2π, and the superimpose figure can be the same. Therefore, the slope of the linear fit of L v.s. i is λ.

2.2.3 Time-difference Method

When S2 receives an ultrasonic pulse signal emitted by S1, one can measure the time interval for the sound travelling from S1 to S2. When L and t is known, the speed of sound can be calculated from equation (2).

3. Experimental Setup

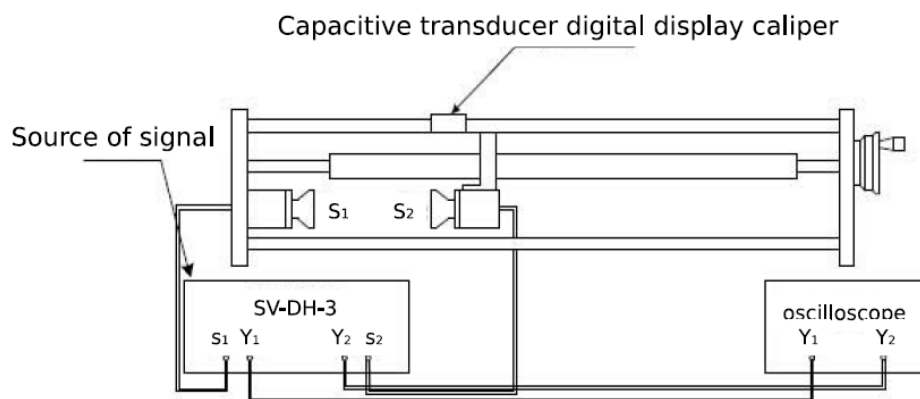Shown in figure 2 are the components of the transducer and oscilloscope.

Figure 2 Experimental Setup

The information of each measurement device is shown in Table 1.

apparatus	range	Minimum scale of value	Maximum uncertainty
Thermometer	-40℃~50℃	1℃	±1℃
Signal generator	/	0.001kHz	±0.001kHz
Scale on the transducer	0~300mm	0.001mm	±0.001mm
Cursor function	/	0.4μs	±0.4μs

Table 1 Information of Each Measurement Device

4. Measurements

4.1 Resonance Method

1. First set the distance between S1 and S2 at about 1cm.
2. Device Initialization: Turn on the signal source and the oscilloscope. Then set the *Method* on the signal source to be *continuous*. Adjust *Signal Strength* until a 10V peak is displayed on the oscilloscope. Adjust *signal frequency* between 34.5 kHz and 40 kHz until the peak-to-peak voltage reaches its maximum. Record the frequency.
3. Move S2 away from S1 to increase L and observe the figure shown on the oscilloscope. Record the position of S2 when the output voltage reaches its maximum.
4. Repeat step 3 to record 12 values of L and apply linear fit to get the value of v.

4.2 Phase-comparison Method

1. Set the device to corresponding mode.
2. Increase the distance and record the distance when there is a straight up-right line shown on the screen.
3. Repeat step 2 and collect 12 sets of data in total.
4. Apply linear fit to get the value of v.

4.3 Time difference Method

1. Set the device to corresponding mode. Set the initial distance to be larger than 100mm.
2. Set the distance to be different values and record the time the wave needs to propagate.
3. Collect 12 sets of data and apply linear fit to get the value of v.

5. Results

5.1 Initial settings

The temperature is measured at $21\pm1℃$
The frequency we choose is 39.118 ± 0.001kHz$=39118\pm1$Hz

5.2 Resonance Method

L_i[mm] ± 0.001[mm]			
1	77.298	7	103.776
2	81.796	8	108.520
3	85.926	9	112.746
4	90.302	10	116.840
5	94.704	11	121.820
6	99.612	12	126.123

Table 2 Resonance distances

According to equation(3), we want to know the value of λ, so we apply linear fit of L v.s. I, shown in figure3.
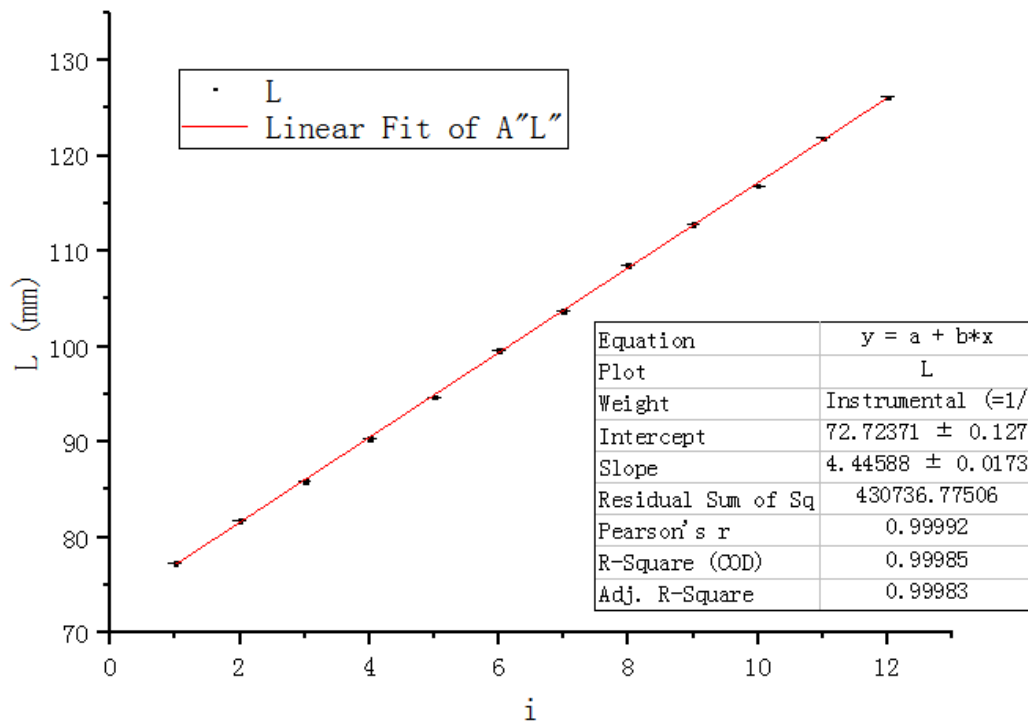
Figure 3 Linear fit of L v.s. i

Obtained from the figure, the slope is 4.44588mm and the standard error is 0.0173mm. The CI harf-width is 0.0173× $t_{10,0.95}$ =0.0173× 2.228 =0.0385mm, rounded to be 4×10^{-2}mm.

Therefore, the uncertainty of the slope is 4×10^{-2}mm. Since the slope equals $\frac{\lambda}{2}$, we obtain the rounded $\lambda = 2 \times 4.44588$ =8.89±8 × 10^{-2}mm=8.89× $10^{-3} \pm 8 \times 10^{-5}m$.

According to equation(1), $v = \lambda f = 8.89 \times 10^{-3} \times 39118 = 347.76 \pm 3$m/s.

5.3 Phase-comparison Method

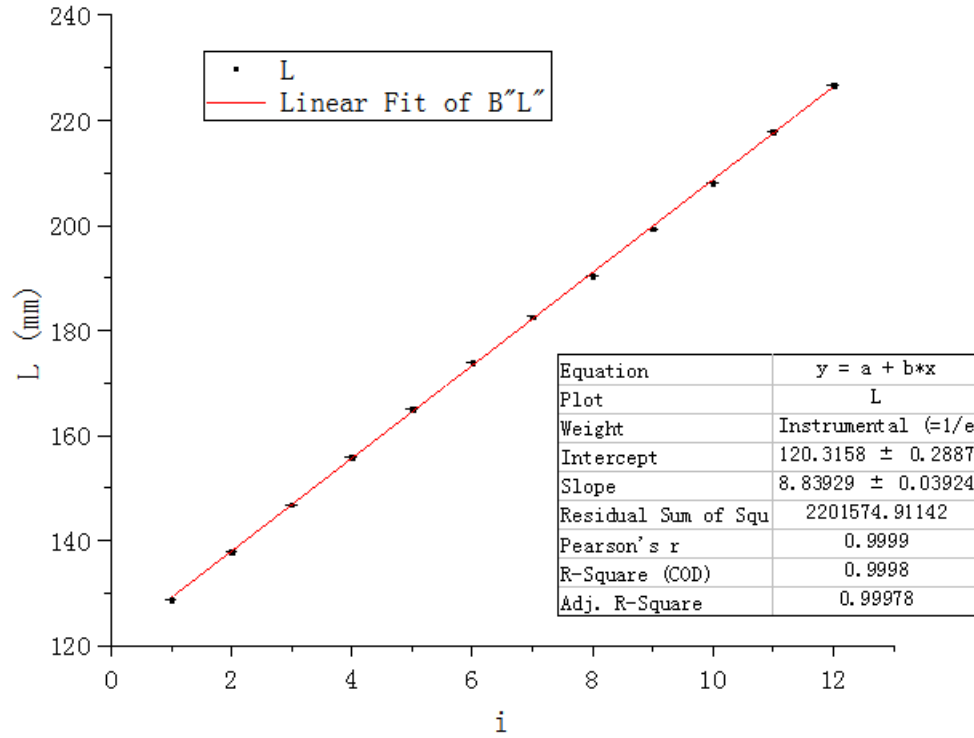| L_i[mm] $\pm$ 0.001[mm]			
1	128.780	7	182.630
2	137.823	8	190.430
3	146.896	9	199.310
4	155.902	10	208.061
5	164.966	11	217.849
6	173.876	12	226.731

Table 3 Distance of the same phase

Figure 4 Linear fit of L v.s. i

Obtained from the figure, the slope is 8.83929mm and the standard error is 0.03924mm. The CI harf-width is 0.03924× $t_{10,0.95}$ =0.03924× 2.228 =0.08743mm, rounded to be 9×10^{-2}mm. Therefore, the uncertainty of the slope is 9×10^{-2}mm. Since the slope equals λ, we obtain the rounded λ =8.84±9×10^{-2}mm=8.84× $10^{-3} \pm 9 \times 10^{-5}m$.

According to equation(1), $v = \lambda f = 8.84 \times 10^{-3} \times 39118 = 346.63 \pm 4m/s$.

5.4 Time Difference Method

	$t_i[\mu s] \pm 0.4[\mu s]$	$L_i[mm] \pm 0.001[mm]$
1	97.6	135.639
2	102.6	142.315
3	107.2	148.820
4	112.4	156.360
5	117.6	164.415
6	123.6	177.325
7	130.0	181.680
8	134.4	189.300
9	139.2	196.500
10	145.2	205.270
11	151.6	214.320
12	158.4	224.250

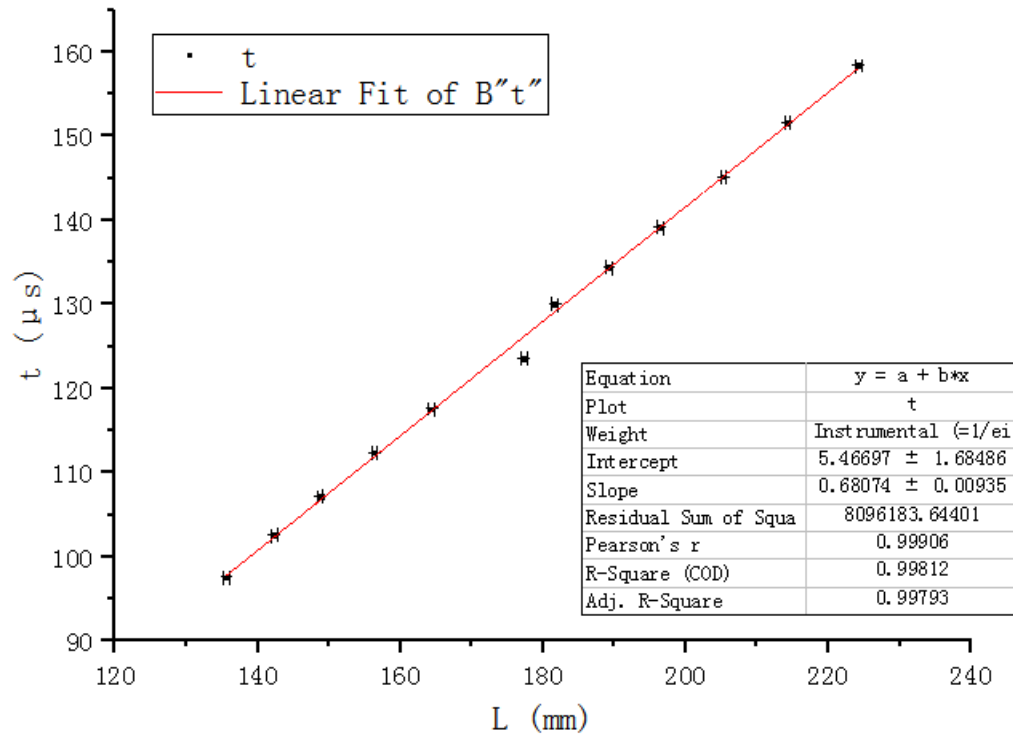Table 4 distance

As t depends on L, we apply linear fit to t v.s. L, shown in figure 5.

Figure 5 Linear fit of t v.s. L

Obtained from the figure, the slope is 0.68074μs/mm and the standard error is 0.00935μs/mm. The CI harf-width is 0.00935× $t_{10,0.95}$ =0.00935× 2.228 =0.02083μs/mm, rounded to be 2×10^{-2}μs/mm.

Therefore, the uncertainty of the slope is 2×10^{-2}μs /mm. Since the slope equals$\frac{1}{v}$, we obtain

the rounded $\frac{1}{v}$ =0.681±2×10^{-2}μs/mm = $6.81 \times 10^{-4} \pm 2 \times 10^{-5} s/m$.

Then we calculate v, which is $\frac{1}{6.81 \times 10^{-4}} = 1468.43 \pm 4 \times 10^{1} m/s$.

6. Conclusions and discussion

6.1 Conclusions

In this experiment, we calculate the speed of sound using three methods. Shown below are the experimental results.

- From the resonance method: $v_{air} = 347.76 \pm 3$m/s.
- From the phase − comparison method: $v_{air} = 346.63 \pm 4$m/s.
- From the time difference method: $v_{water} = 1468.43 \pm 4 \times 10^{1} m/s$.

Judging from the uncertainty, the phase-comparison method provides a more accurate result. According to authorities, the speed of sound in air at 20°C = 344m/s, and the speed of sound in water at 20°C = 1482m/s.[1] Our results are all larger than theory, which is reasonable because in both gas and liquid, the higher the temperature, the larger the speed of sound. But our data still have errors, which will be discussed in 6.2.

6.2 Error Analysis

- In resonance method, there exist errors in the measurement of L. It is hard to control when to read the readings because we have to take the readings at the exact moment that the amplitude of the V function starts to decrease. There are too many objective judgements in this step.
- In phase-comparison method, we also need to judge when the figure is a straight line by ourselves. Since the figure is changing slightly all the time, the readings are inaccurate.
- In time difference method, due to the use of devices, the temperature of water might increase, which results in change of speed of sound in water.
- There are too many factors that will disturb the sound wave. The shown figure on the oscilloscope might not be the actual one.
- The wave will lose energy when propagating from S1 to S2, so S2 might receive a different wave from the one that S1 emits.

6.3 Discussions

- The temperature T must be measured because it is a factor which affects the speed of sound. Take liquid for example, v=$\sqrt{\frac{B}{\rho}}$, where B is the bulk modulus of fluid and ρ is the fluid density, and of course the density is subject to temperature.
- In phase-comparison method, we should use 'straight line' Lissajous figures as reference because it is the easiest to judge when the same line appears again. For other elliptical figures, it is hard to find out the exact same pattern as the previous time.
- In time difference method, we should set the initial distance to be larger than 100mm. Since the speed of sound in water is expected to be around 1500m/s, the time needed for sound to propagate 100mm is 6.67×10^{-5}s. If the distance is too small, the device is not precise enough to measure the time interval.
- We have to measure the speed of sound under the resonance frequency. Only under this circumstance can a standing wave be formed so that the transducer can receive the strongest voltage signal and has the highest sensitivity.

6.4 Improvements

- Let the machine detect the distance where V reaches its maximum rather than humans.
- Do more experiments, apply more linear fits and take the average value as the speed of sound.
- Use ultrasonic wave of different frequencies in the resonance frequency range.

7. Reference

[1]Young, Hugh.A and Freedman, Roger. A. *University Physics*.p.535.

A. Measurement uncertainty analysis

A.1 Uncertainty in Analysis of the Resonance Method

A.1.1 Uncertainty of λ

Ths slope k is $\frac{\lambda}{2} = 4.43539 \pm 0.06mm$

$$\lambda = 2k$$
$$u_\lambda = 2u_k = 2 \times 0.04 = 0.08mm = 8 \times 10^{-5}m$$

A.1.2 Uncertainty of v

V= λf

$$\frac{\partial v}{\partial \lambda} = f = 1Hz$$

$$\frac{\partial v}{\partial f} = \lambda = 8.89 \times 10^{-3}m$$

$$u_v = \sqrt{(\frac{\partial v}{\partial \lambda})^2 u_\lambda{}^2 + (\frac{\partial v}{\partial f})^2 u_f{}^2} = \sqrt{39118^2 \times (8 \times 10^{-5})^2 + (8.89 \times 10^{-3})^2 \times 1} = 3m/s$$

A.2 Uncertainty in Analysis of the Phase-comparison Method

A.2.1 Uncertainty of λ

From 5.3 we know the slope k is λ=8.84$\times 10^{-3} \pm 9 \times 10^{-5}m$.
$$u_\lambda = 9 \times 10^{-5}m$$

A.2.2 Uncertainty of v

V= λf

$$\frac{\partial v}{\partial \lambda} = f = 39118Hz$$

$$\frac{\partial v}{\partial f} = \lambda = 8.84 \times 10^{-3}m$$

$$u_v = \sqrt{(\frac{\partial v}{\partial \lambda})^2 u_\lambda{}^2 + (\frac{\partial v}{\partial f})^2 u_f{}^2} = \sqrt{39118^2 \times (9 \times 10^{-5})^2 + (8.84 \times 10^{-3})^2 \times 1} = 4m/s$$

A.2 Uncertainty in Analysis of the Time Difference Method

$$\frac{1}{v} = 6.8 \times 10^{-4} \pm 2 \times 10^{-5}s/m$$

$$\frac{\partial \frac{1}{v}}{\partial v} = \frac{-1}{v^2}$$

$$u_v = \sqrt{(\frac{\partial \frac{1}{v}}{\partial v})^2 u_{\frac{1}{v}}{}^2} = \sqrt{(\frac{-1}{(6.81 \times 10^{-4})^2})^2 \times (2 \times 10^{-5})^2} = 4 \times 10^1 m/s$$

B. Data sheet