UM-SJTU Joint Institute
Physics Laboratory
(Vp141)

Laboratory Report

Exercise 3
Simple Harmonic Motion:
Oscillations in Mechanical Systems

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1 Introduction

The objective of this experiment is to study properties of a simple harmonic oscillation. We will first apply Hooke's law to find out the spring constant. Then, we will do several control experiments on the air track to study the relationship between the oscillation period and the mass, the period and the amplitude, and the maximum speed and the amplitude.

2 Theoratical background

2.1 Hooke's Law

Within the elastic limit of deformation, the restoring force of the spring has the direction opposite to the deformation and its magnitude is directly proportional to the distance. Hooke's Law can be expressed by:

$$F_x = kx \tag{1}$$

where k is the spring constant, F_x is known as the restoring force and can be found using the Jolly balance.

2.2 Equation of Motion of the Simple Harmonic Oscillator

An object with mass M is placed on an air track which serves to eliminate the frictional force, as shown in Figure.

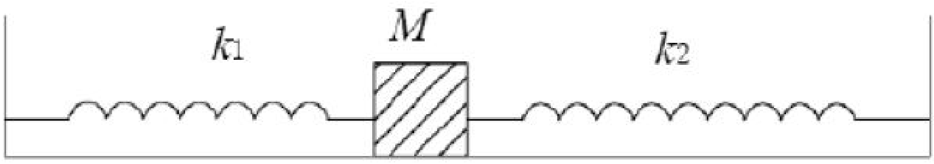

Figure 1: Mass-spring system

The two ends of the object are fixed to the air track using two springs whose spring constants are k1 and k2. Neglecting the masses of the springs and the damping and applying Newton's second law, the equation of motion of the object is:s

$$M\frac{d^2x}{dt^2} + (k_1 + k_2)x = 0 \quad \Rightarrow \quad x(t) = A\cos\omega_0 t + \phi_0 \tag{2}$$

where $\omega_0 = \sqrt{(k_1 + k_2)/M}$, which is the natural angular frequency of the oscillations and is determined by the parameters of the system itself. A is the amplitude and ϕ_0 is the initial phase which is determined by initial conditions.

The natural period of oscillation is:

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M}{k_1 + k_2}} \quad \Rightarrow \quad \frac{T^2}{m} = \frac{4\pi^2}{k_1 + k_2} \tag{3}$$

2.3 Mass of the Spring

When the mass of springs cannot be ignored, we consider the effective mass of the spring, which is 1/3 of the actual mass of the spring. The oscillator with object of mass M and spring of effective mass m_0 has the angular frequency:

$$\omega_0 = \sqrt{\frac{k_1 + k_2}{M + m_0}} \tag{4}$$

2.4 Mechanical Energy in Harmonic Motion

The elastic potential energy of the spring-mass system is $U = kx^2/2$ and the kinetic energy of it is $K = mv^2/2$.

The speed of the mass is maximized at the equilibrium position x=0, where the total mechanical energy equals to maximum kinetic energy. At maximum displacement, the mass has no speed and has maximum potential energy as the total mechanical energy. Therefore, as there are only conservative forces, Kmax = Umax, and we get:

$$k = \frac{mv_{max}^2}{A^2} \tag{5}$$

Other things needed: $\Delta x = (x_{in} + x_{out})/2$, $v_{max} = \Delta x/\Delta t$. Where x_{in} is the distance within the two legs of the U shape shutter and x_{out} is the outer distance. Δt is the time it takes to travel from one leg of the U shape shutter to the other, which is measured by the timer.

3 Experimental setup

The measurement equipment consists of: springs, Jolly balance, air track, electronic timer, electronic balance, and masses.

3.1 Jolly balance

The scale on the Jolly balance is used to measure the deformation of the spring, from which we can calculate the spring constant.

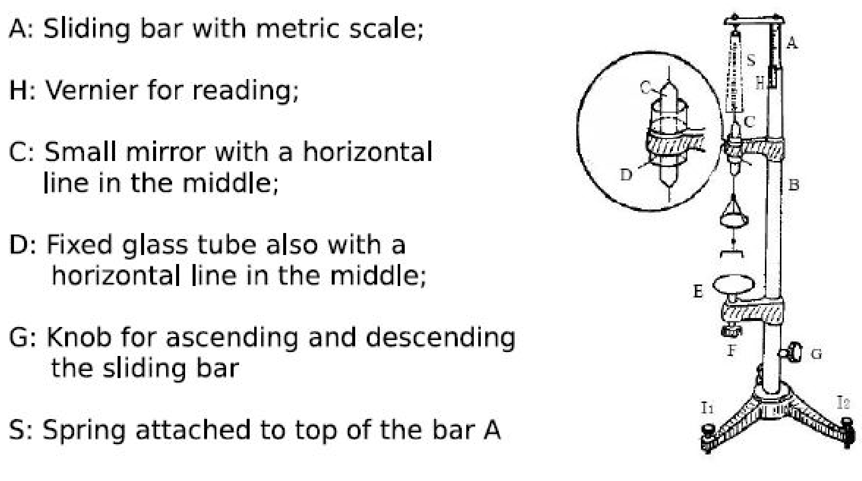

Figure 2: Jolly Balance

First, put the initial 20g mass on the bottom end of the spring, read the scale L1. Then add mass m to it and read L2. The spring constant can be found as:

$$k = \frac{mg}{L2 - L1}$$

Read the readings onlly if the three lines coincide: the line on the mirror, the line on the glass tude and its reflection in the mirror.

3.2 Photoelectric Measuring System

For period measurement, we use the I-shape shutter on the moving object to block the light emitted from the photoelectric gate. Each time the light is blocked, half a period is counted.

For speed measurement, we use a U-shape shutter so that the light is blocked twice during a pass. The time interval t is measured by the timer, and we use a caliper to measure the distance x_{in} and x_{out} to get $\Delta_x = \frac{1}{2}(x_{in} + x_{out})$. Therefore, the instantaneous speed can be expressed by $v = \frac{\Delta_x}{\Delta_t}$.

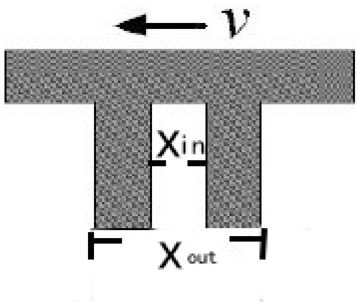

Figure 3: the U-shape shutter

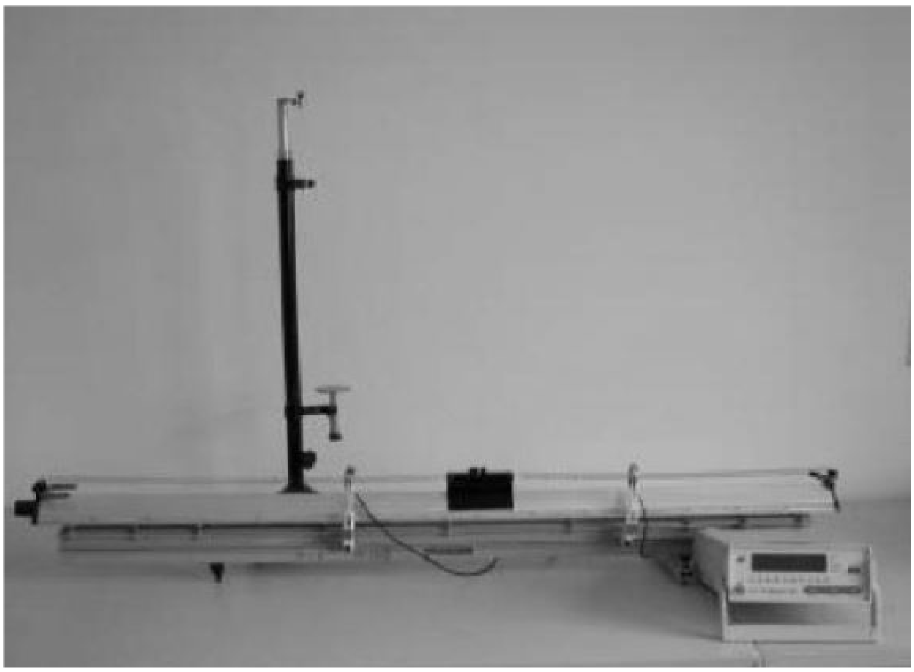

Figure 4: the experimental setup

3.3 Device information

Apparatus		Precision
Jolly Balance		0.02[mm]
Caliper		0.02[mm]
Electronic balance		0.01[g]
Photoelectric measuring system	Mode T	0.0001[s]
	Mode S_2	0.00001[s]
Air track		0.1[cm]

Table 1: Information of Each Measurement Device

4 Measurement

4.1 Spring Constant

1. Adjust the Jolly balance to be vertical, and add a 20g preload to the bottom end of the spring. (Fig.??j) Make sure the mirror can move freely.

2. Adjust the position of the tube to set the initial position L0 within 5.0-10.0cm and record.

3. Add mass m1 and record L1.

4. Keep adding masses in order and record 7 sets of positions in total.

5. Use the least square method to estimate the spring constant k1.

6. Replace spring1 with spring2 and repeat the above steps to get spring constant k2.

7. Remove the preload and repeat the measurement for springs1 and 2 connected in series and calculate k3. Compare k3 with theoretical value.

4.2 Relation Between the Oscillation Period T and the Mass of the Oscillator M

4.2.1 Adjustment of the air truck

1. Adjust the air track to be horizontal.

2. Turn on the air pump and check if there are any holes blocked.

3. Place the cart on the track without initial velocity and adjust the single knob until the object moves slowly back and forth in both directions.

Caution: Don't place anything on the track when it's off.

4.2.2 Horizontal air track

1. Attach the I-shape shutter to the cart and attach springs to the sides of the cart to connect it to the air track. Make sure the photoelectric gate is at the equilibrium position.

2. Add weight m1 and let the cart oscillate about the photoelectric gate. Use a pen to release the cart and ensure that the amplitude is about 5cm. Set the timer into 'T' mode and it will automatically record the time of 10 oscillation periods. Record both the total time and the mass of cart.

3. Add weights to the cart. Repeat step 2 and take the measurements for 5 times.

4. Plot a graph to analyze the relation between T and M.

4.2.3 inclined aire track

1. Place three plastic plates under one side of the air track. Repeat steps in 4.2.2.

2. Add three plastic plates under that side and repeat steps in 4.2.2.

3. Plot a graph to analyze the relation between T and M.

4.3 Relation Between the Oscillation Period T and the Amplitude A

1. Fix the mass of the cart and measure the period under 6 different values of the amplitude. The recommended amplitude is 5.0/10.0/.../30.0cm.

2. linear fit to the data and analyze the relation between T and A based on the correlation coefficient γ.

4.4 Relation Between the Maximum Speed and the Amplitude

1. Use a caliper to measure x_{out} and x_{in} of the U-shape shutter. Calculate the distance $\Delta x = (x_{in} + x_{out})/2$.

2. Replace the I-shape shutter with the U-shape shutter. Set the timer into 'S2'mode.

3. Measure the maximum speed of the cart under 6 different values of amplitude. The recommended amplitude is 5.0/10.0/.../30.0cm. Record the second readings of the time interval only if the two subsequent readings show the same digits to the left of the decimal point.

4. Apply the data to calculate the spring constant k in Eq.6 and compare this result to that in 4.1.

4.5 Mass measurement

1. Adjust the electronic balance every time before you use it.
2. First weigh the cart with the I-shape shutter, then the cart with the U-shape shutter. Then measure the mass of spring1 together with spring2.
3. Record the data only after the circular symbol on the scales display disappears.

5 Results

5.1 Measurement of the spring constant

The acceleration due to gravity given by instructor is $9.794 m/s^2$, we use the formula w=mg to calculate the weight: Take 1 as example, $w = mg = 4.88 \times 10^{(}-3) \times 9.794 = 0.0478 \pm 9.794 \times 10^{(}-5) N$.

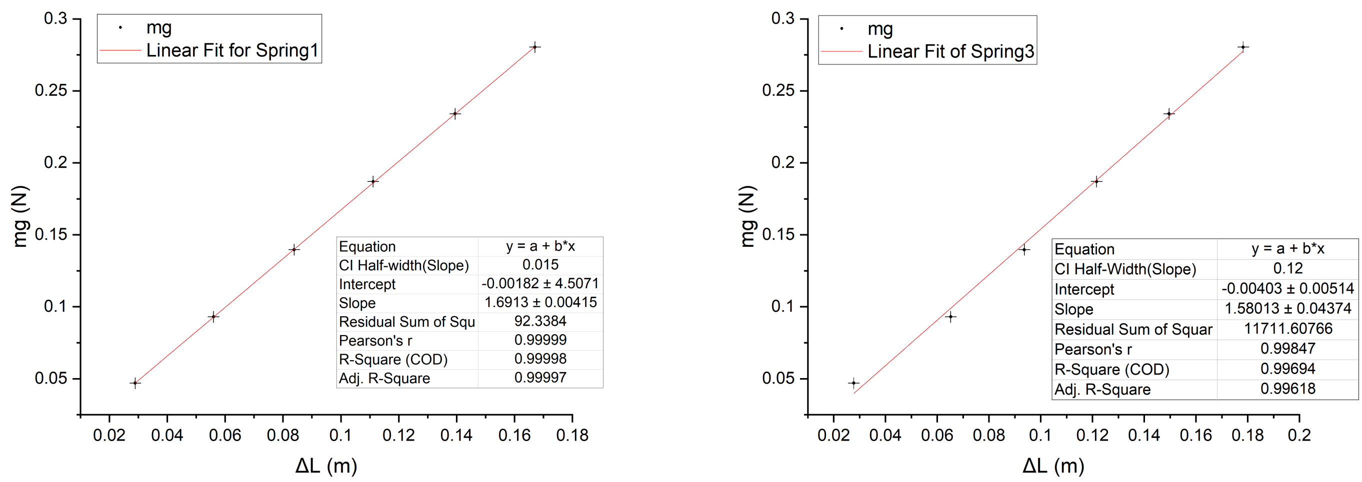

Figure 5: The linear fit figures of spring 1, 2

6 Reference

Qin Tian, Zheng Huan, Li Yingyu, Li Tiantian, Mateusz Krzyzosiak, Vp141, Exercise 3, Simple Harmonic Motion: Oscillations in Mechanical Systems