

# VE216 Lecture 8

## Convolution

### Convolution

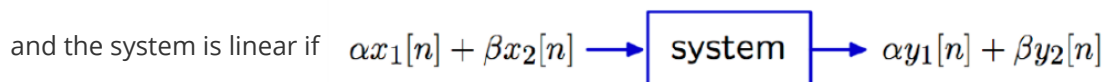
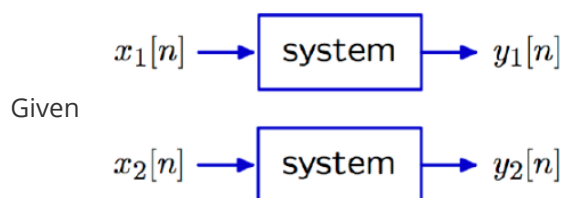
Hoping to represent a system by a single signal, while the input is more complicated.

### Superposition

Breaking input into additive parts and sum the responses to the parts.

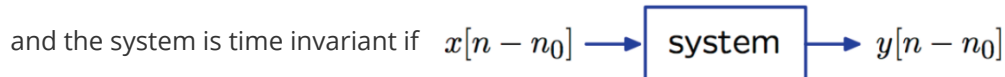
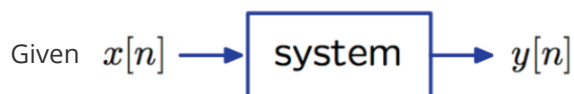
It is easy if system is **linear and time-invariant (LTI)**.

#### Linear



for all  $\alpha, \beta$ .

#### Time-Invariance



for all  $n_0$ .

### Structure of Superposition

$$\begin{aligned}\delta[n] &\rightarrow h[n] \\ \delta[n-k] &\rightarrow h[n-k] \\ x[k]\delta[n-k] &\rightarrow x[k]h[n-k] \\ x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] &\rightarrow y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]\end{aligned}$$

## Convolution Notation

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$x[n] \rightarrow \text{LTI System} \rightarrow y[n]$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = (x * h)[n]$$

## DT Convolution Remark

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Actually, unit sample response  $h[n]$  is a complete description of an **LTI system**.

## CT Convolution

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Similar form:

$$x(t) \rightarrow \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$
$$y(t) \rightarrow \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

## Convolution Summary

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DT:  $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = (x * h)[n]$

CT:  $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = (x * h)(t)$

## Summary

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The **impulse response** is a **complete description** of a **linear, time-invariant (LTI) system**.

One can find the output of such a system by **convolving the input signal with the impulse response**.