Ve 216: Introduction to Signals and Systems

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Based on Lecture Notes by Prof. Jeffrey A. Fessler

Outline

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Introduction

This chapter applied FT principles and tools to the analysis of communications systems (AM radios, FM radios, wireless phones, video, etc.)

All of it is based on the **modulation property** of the FT.

Overview

- Amplitude Modulation (AM radio, digital comm (modems))
- Synchronous demodulation
- Asynchronous demodulation
- Heterodyning tuner

We will only cover 8.1, 8.2, 8.3

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Definition

Modulation is the process used to shift the frequency of an information signal so that the resulting signal is the desired frequency band.

Importance of modulation

1. *Ear can hear from about 20Hz to 20kHz. For such audio signals, electrical transmission over copper wire works fine (and is used in all audio systems). So there is no need to modulate.*
2. *Human voice is dominated by frequency components of less than 1kHz. If we were attempt to transmit a human voice signal by the propagation of electromagnetic (radio) waves, we could encounter the several problems. Two of the more obvious problems are as follows.*

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Antenna length requirement (1)

1. Antenna length requirement.
For efficient radiation, antenna lengths should be longer than $\lambda/10$, where λ is the wavelength of the signal to be radiated, given by

$$\lambda = \frac{c}{f_c}$$

where c is the speed of the light and f_c is the frequency of the signal.

Example

For a 30Hz frequency component, the wavelength is

$$\lambda = \frac{c}{f_c} = \frac{3 \cdot 10^8 m/s}{30 s^{-1}} = 10,000\text{km},$$

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- If all radio stations transmitted baseband 20Hz-20kHz signals (over a cable for example), the receivers would receive all of those signals superimposed, and would have no way of separating them.
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FCC regulation

In the U.S., the frequency band allocation is controlled by the Federal Communications Commission (FCC).

- AM radio: 540-1600kHz
- FM radio: 88-108MHz

Note: lots of room for many non-overlapping channels each $\pm$20kHz wide.
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DSB/SC-AM

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- We have already seen that the **modulation property** of the FT provides a mechanism for this.
- There are many variations on how to do this (see Ve 353 Introduction to Communication Systems).
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Sinusoidal amplitude modulation (1)

Let $x(t)$ be the modulating signal (the audio signal) and let $c(t)$ be the carrier signal (a high-frequency cosinusoid). Specifically

$$c(t) = \cos(\omega_c t + \theta_c)$$

where ω_c is called the **carrier frequency**.

The transmitted signal is

$$y(t) = x(t)c(t) = x(t)\cos(\omega_c t + \theta_c)$$

$$\stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(\omega) = \frac{1}{2}[e^{j\theta_c} X(\omega - \omega_c) + e^{-j\theta_c} X(\omega + \omega_c)]$$

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Block diagram of modulation system:

$$\begin{array}{c} x(t) \to \bigotimes \to y(t) \to \text{ antenna} \\ \uparrow \\ \cos(\omega_c t + \theta_c) \end{array}$$

(Picture) in frequency domain (MIT, Lecture 13.9) (textbook, Figure 8.4, p. 586) This multiplication is also sometimes called **mixing**. Note that graphically it appears that all of the audio signal information is retained, it is just moved to a different part of the spectrum.

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40kHz would be a bare minimum, but that would require an ideal filter to separate the different channels. So somewhat wider separation desirable.

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The carriers of AM radio stations are spaced by 10kHz! How?

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Smaller bandwidth, so lower audio quality.

Outline

Synchronous Demodulation

- The signal must be restored to baseband for our ears to hear it.
- Conceptually, this can be done by another multiplication by a $\cos(\omega_c t + \theta_c)$ signal, followed by lowpass filtering.

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$$y(t) \rightarrow \bigotimes \rightarrow w(t) \rightarrow \boxed{h(t)} \rightarrow x(t)$$

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Frequency response and lowpass filtering

$$w(t) = y(t)\cos(\omega_c t + \theta_c)$$

$$W(\omega) = \frac{1}{2}[e^{j\theta_c}Y(\omega - \omega_c) + e^{-j\theta_c}Y(\omega + \omega_c)]$$

$$= \boxed{\frac{1}{4}e^{2j\theta_c}X(\omega - 2\omega_c) + \frac{1}{2}X(\omega) + \frac{1}{4}e^{-2j\theta_c}X(\omega + 2\omega_c)}$$

$$Y(\omega) = \frac{1}{2}[e^{j\theta_c}X(\omega - \omega_c) + e^{-j\theta_c}X(\omega + \omega_c)]$$

Now pass through a lowpass filter to extract the baseband signal. ***(Picture)***(MIT, Lecture 13.10 and 13.11)

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Yes! At your receiver you do not have the modulation signal $\cos(\omega_c t + \theta_c)$ *available. Specifically, the phase* θ_c *is not available.*

Loss of synchronization in phase (1)

Effect of loss of synchronization in phase. ***(Picture)****(MIT, Lecture 13.16) (textbook, Figure 8.9)*

1. *If the phase difference is $\pi/2$, the output will be zero.*
2. *If the amplitude is small and there exists noise in the system, the signal to noise ratio is small.*
3. *If the phase relation between θ_c and ϕ_c is not maintained over time, the amplitude factor $\cos(\theta_c - \phi_c)$ varies.*

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Synchronous demodulation requires a sophisticated phase-tracking receiver.

- *An expensive receiver is reasonable in some applications like satellite communications where transmit power is limited, so an expensive base station is acceptable.*
- *But in applications like commercial AM radio, one would like the receivers to be inexpensive, even if that means a more complicated and power-inefficient transmitter.*

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A simple system and associated waveform for an **asynchronous demodulation** system. ***(Picture)***(MIT, Lecture 13.17)

- The envelop of $y(t)$ (a smooth curve connecting the peaks) is a reasonable approximation of $x(t)$.
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Two basic assumptions are required, so that the envelop is easily tracked.

1. $x(t)$ be positive. Solution: $x(t) + A > 0$.
2. $x(t)$ vary slowly compared to ω_c.

Example

- $x(t)$: 15-20 kHz
- $\omega_c/2\pi \in [500K, 2M]$Hz.

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DSB/WC-AM

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Transmitting some of the modulation signal too (uses power) is called **double sideband**, **with carrier**, **amplitude modulation** or **DSB/WC-AM**.

Now the modulated signal (transmitted) is:

$$y(t) = (A + x(t))\cos(\omega_c t)$$

(Picture)of block diagram (MIT, Lecture 13.18) (textbook, Figure 8.12) Spectrum of the DSB/WC signal:

$$Y(\omega) = A\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{1}{2}[X(\omega - \omega_c) + X(\omega + \omega_c)]$$

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As A is increased

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Single crystal receiver

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$$m(t) = A + \hat{x}(t)$$

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Frequency-division multiplexing (1)

- Different stations are allocated different carrier frequencies, separated by (at least) the allowed bandwidth of each station.
- Commercial AM: 10KHz spacing between carriers.
- The transmitted spectrum of the *k*th station occupies $\omega_k \pm \omega_B$.

Example

- ***(Picture)*** of block diagram multiple transmitters (MIT, Lecture 13.12)
- ***(Picture)*** of spectra illustrating (MIT, Lecture 13.13)
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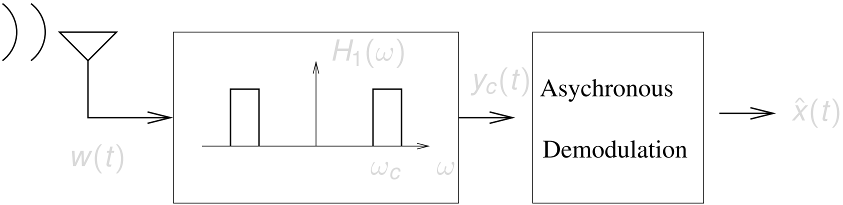
Tunable bandpass filter (1)

Question

How can we “tune in” to our favorite station?

Design 1: a tunable bandpass filter.

Tuning (Design #1)



Bandpass (tunable)

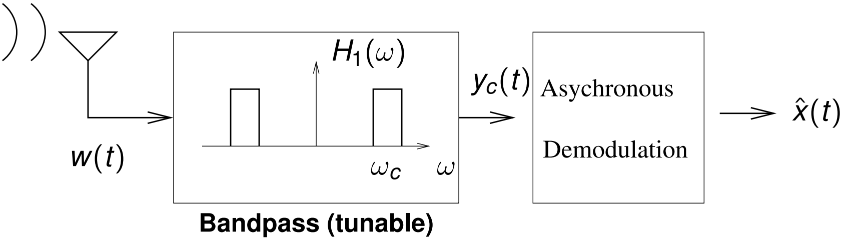
Tunable bandpass filter (1)

Question

How can we "tune in" to our favorite station?

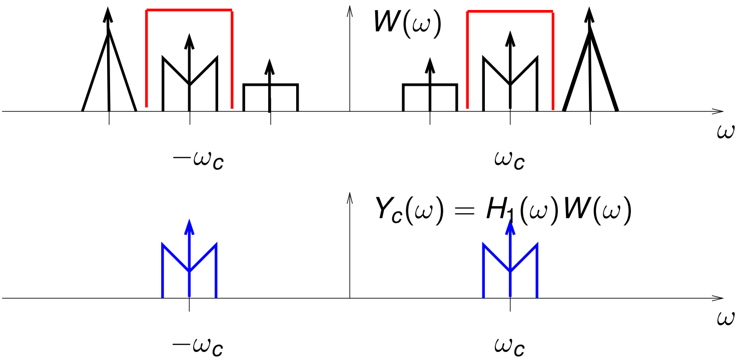
Design 1: a tunable bandpass filter.

Tuning (Design #1)



Tunable bandpass filter (2)

(Frequency Division Multiplexing)



Tunable bandpass filter (3)

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1. *narrowband ($\pm$ 5 kHz)*
2. *high center frequency (540 to 1600 kHz)*
3. *tunable*
4. *stable*
5. *sharp cutoff (to avoid interference from neighboring channels)*

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4. *stable*
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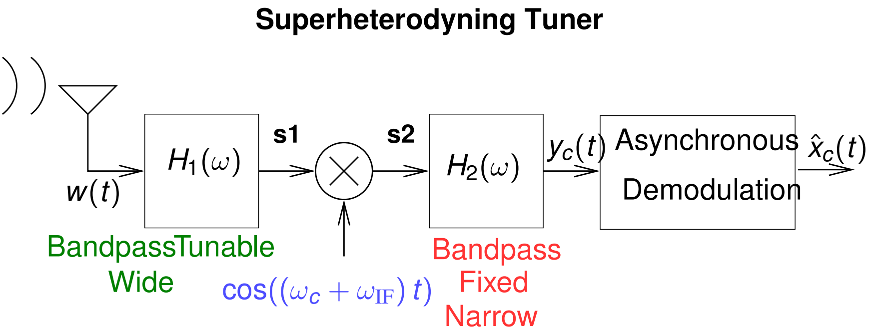
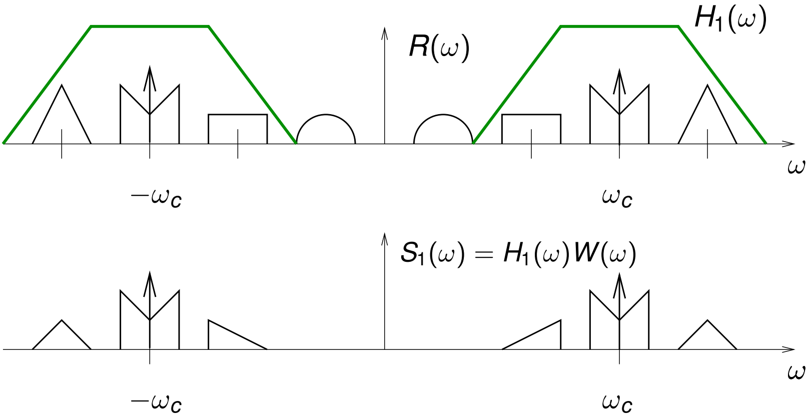
These concurrent requirements are difficult to achieve in practice, so commercial AM radios are not built this way.

Superheterodyning receiver

Design 2: **superheterodyning receiver** (avoids a tunable narrow bandpass filter)



Bandpass tunable wide



IF filter and mixing frequency

Definition

The fixed frequency bandpass filter called the **IF filter** for "intermediate frequency."

In commercial AM this is $\omega_{\mathrm{IF}}/2\pi = 455\mathrm{kHz}$.

Modulate the received RF signal by $\omega_0 = \omega_c + \omega_{\mathrm{IF}}$ to move the spectrum of the desired signal to be centered at ω_{IF}.

$$\begin{array}{c} s_1(t) \rightarrow \bigotimes \rightarrow s_2(t) \\ \uparrow \\ \cos(\omega_c + \omega_{\mathrm{IF}})\,t) \end{array}$$

When you "tune in" you are actually controlling the **mixing frequency** ω_0.

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Mixing frequency: example

Example

If we want to tune in to the station at

$$\omega_c/2\pi = 1060\text{kHz}$$

then use

$$\omega_0/2\pi = (\omega_c + \omega_{\mathrm{IF}})/2\pi = 1060 + 455 = 1515\text{kHz}.$$

Before mixing, centered at $\pm$ 1060kHz.
After mixing by 1515kHz, centers at

$$\pm\omega_c \pm\omega_0 \Longrightarrow \pm\omega_{\mathrm{IF}} \text{ and } \pm(2\omega_c + \omega_{\mathrm{IF}})$$

$$\pm 1060 \pm 1515\text{kHz} \Longrightarrow \pm 455\text{kHz and } \pm 2575\text{kHz}$$

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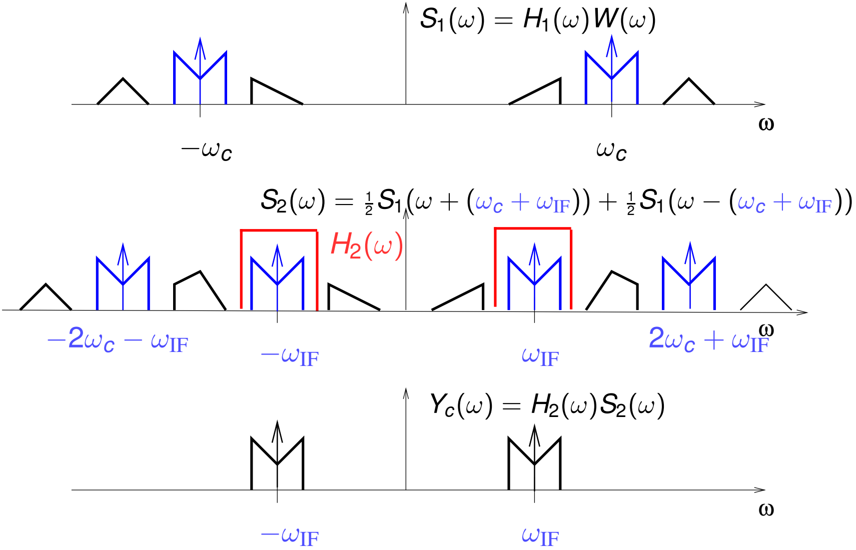
Modulation and IF filter



Recovering the original signal

- After mixing with $\cos((\omega_c + \omega_{\mathrm{IF}})\,t)$, the desired station is centered in the passband of the IF filter.
- After the IF filter, asynchronous demodulation can be used to recover the original audio signal.

Question

One point we have omitted though. In the above example, what happens to the part of the received signal spectrum centered at

$$\omega_0/2\pi + \omega_{\mathrm{IF}}/2\pi = 1515 + 455 = 1970kHz?$$

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Interfering spectrum components

After mixing by 1515kHz, this part gets moved to

$$\pm 1970 \pm 1515kHz \Longrightarrow \pm 455 \ and \ \pm 3485kHz.$$

So that signal also would leak through the IF filter and interfere with the desired signal.

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How to solve this problem?

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Tunable bandpass filter

The desired signal, centered at 1060kHz, and the interfering signal, centered at 1970kHz, are separated by

$$1970kHz - 1060kHz = 910kHz = 455kHz + 455kHz = 2\omega_{\mathrm{IF}}/2\pi.$$

We just need a tunable bandpass filter that is centered at ω_c, but with a passband that is about 900kHz wide. It is fairly easy to build such a tunable bandpass filter.

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Superheterodyning receiver

$$w(t) \rightarrow \boxed{\text{tunable bandpass at } \omega_c\text{, 900kHz wide}}$$

$$\rightarrow \boxed{\text{mix at } \omega_0 = \omega_c + \omega_{\mathrm{IF}}} \rightarrow \boxed{\text{bandpass at } \omega_{\mathrm{IF}}/2\pi\text{=455kHz} \pm \text{5kHz}}$$

$$\rightarrow \boxed{\text{asynchronous demodulate}} \rightarrow \hat{x}(t) \text{ audio}$$

A better approach?

Question

Is the following a better approach?

$$w(t) \rightarrow \boxed{\textit{mix at } \omega_0 = \omega_c + \omega_{\mathrm{IF}} \textit{ with } \omega_{\mathrm{IF}} = 0}$$

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Consider the effects of loss of synchronization in phase between the carrier signal and the mixing signal.

A better approach? (2)

To simplify the analysis, let's consider one station only. The modulated (transmitted) signal is

$$y(t) = (A + x(t))\cos(\omega_c t + \theta_c)$$

After mixing at the receiver

$$\begin{aligned} s_2(t) &= y(t)\cos((\omega_c + \omega_{\mathrm{IF}})\,t + \phi_c) \\ &= \tfrac{1}{2}(A + x(t))\cos((2\omega_c + \omega_{\mathrm{IF}})\,t + \theta_c + \phi_c) \\ &\quad + \tfrac{1}{2}(A + x(t))\cos(\omega_{\mathrm{IF}} t + \theta_c - \phi_c) \end{aligned}$$

If $\omega_{\mathrm{IF}} = 0$, then after passing a lowpass filter,

$$\hat{x}_c(t) = \tfrac{1}{2}(A + x(t))\cos(\theta_c - \phi_c)$$

See the effects of the additional term $\cos(\theta_c - \phi_c)$ on Chap. 8, p.49.

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Summary

- double sideband, suppressed carrier, amplitude modulation (DSB/SC-AM)
- double sideband, with carrier, amplitude modulation (DSB/WC-AM)
- synchronous demodulation
- asynchronous demodulation
- Frequency-division multiplexing (superheterodyning receiver)
- Systems-level analysis of communication system