Homework 6

HW Notes:

- $\boxed{\text{Box}}$ your final answer.
- If you need to make any additional assumptions, state them clearly.
- Simplify your result when possible.
- For the problems with [credit!], no partial credit will be given if the final answer is wrong.

Problems:

1. [5] Is this system stable? Explain. (Note: the system is causal.)

$$2 \cdot 10^6 y(t) + 10^5 \frac{d}{dt} y(t) + 60 \frac{d^2}{dt^2} y(t) + \frac{d^3}{dt^3} y(t) = 8 \cdot 10^6 x(t) - 10^4 \frac{d}{dt} x(t)$$

2. [5] How many signals have a Laplace transform that may be experessed as

$$\frac{(s-1)}{(s+2)(s+3)(s^2+s+1)}$$

in its region of convergence?

3. [5] Use geometric evaluation from the pole-zero plot to determine the magnitude of the Fourier transform of the signal whose Laplace transform is specified as

$$X(s) = \frac{s^2 - s + 1}{s^2 + s + 1}, Re\{s\} > -\frac{1}{2}$$

4. [5] Consider two right-sided signals x(t) and y(t) related through the differential equations

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t)$$

and

$$\frac{dy(t)}{dt} = 2x(t)$$

Determine Y(S) and X(S), along with their regions of convergence.

5. [10] A causal LTI system S with impluse response h(t) has its input x(t) and output y(t) related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

   (a) If

$$g(t) = \frac{dh(t)}{dt} + h(t)$$

   ,
   how many poles does G(s) have?

   (b) For what real values of the parameter α is S guaranteed to be stable?

6. [10] Draw a direct-form representation for the causal LTI systems with the following system fuctions:

(a)

$$H_1(s) = \frac{s+1}{s^2+5s+6}$$

(b)

$$H_2(s) = \frac{s^2-5s+6}{s^2+7s+10}$$

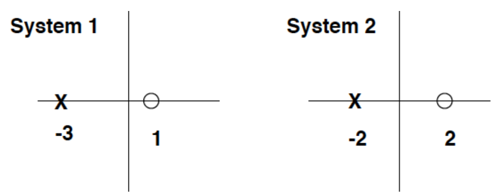
(c)

$$H_3(s) = \frac{s}{(s+2)^2}$$

7. [10] A causal LTI system with impulse response h(t) has the following properties:1.When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = \frac{1}{6}e^{2t}$ for all t. 2.The impulse respose h(t) satisfies the differential equation $\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t)$, where b is an unkhown constant.

   Determine the system function H(s) of the system, consistent with information above. There should be no unkhowm constants in your answer, that is, the constant b should not appear in the answer.

8. [10] A unit step signal is applied to a system consisting of two LTI system connected in parallel. The pole-zero plots of each of the system are shown below. Determine the output signal. Assume that each of the system has unit gain at DC.



   Hint: first find the Laplace transform $Y(s)$ of the output signal using the convolution and linearity properties of the Laplace transform, Then take the inverse Laplace transform to get $y(t)$ using PFE. The "unit gain at DC" specifies $H_1(0)$ and $H_2(0)$, which you can use to determine the scaling factor.

9. [10] Consider an LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

   (a) Determine the Laplace transform of $x(t)$ and $h(t)$.

   (b) Using the convolution property, determine the Laplace transform $Y(s)$ of the output $y(t)$.

   (c) From the Laplace transform of $y(t)$ as obtained in part(b), determine $y(t)$.

   (d) Verify your result in part (c) by explicitly convolving $x(t)$ and $h(t)$.

10. [10] The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}$$

   Determine and sketch the response $y(t)$ when the input is

$$e^{-|t|}, \quad -\infty < t < \infty$$

11. [20] In this problem, we consider the construction of various type of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$, and system function

$$H(s) = \frac{2s^2+4s-6}{s^2+3s+2}$$

   To derive the direct-diagram representation of S, we first consider a causal LTI system S_1 that has the same input $x(t)$ as S, but whose system function is

$$H_1(s) = \frac{1}{s^2 + 3s + 2}$$

With the output of S_1 denoted by $y_1(t)$, the direct-form diagram representation of S_1 is shown in Figure 1. The signals $e(t)$ and $f(t)$ indicates in the figure represent respective inputs into the two integrators.

(a) Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $dy_1(t)/dt$, and $d^2y_1(t)/dt^2$.

(b) How is $dy_1(t)/dt$ related to $f(t)$.

(c) How is $d^2y_1(t)/dt^2$ related to $e(t)$.

(d) Express $y(t)$ as a linear combination of $e(t)$, $f(t)$, $y_1(t)$.

(e) Use the result from the previous part to extend the direct-form block diagram representation of S_1 and create a block diagram representation of S.

(f) Observing that

$$H(s) = \left(\frac{2(s-1)}{s+2}\right)\left(\frac{s+3}{s+1}\right)$$

draw a block diagram representation for S as a cascade combination of two subsystems.

(g) Observing that

$$H(S) = 2 + \frac{6}{s+2} - \frac{8}{s+1}$$

draw a block-diagram representation for S as parallel combination of three subsystems.

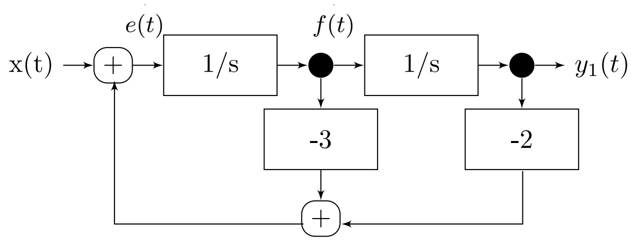


Figure 1