Homework 4

HW Notes:

- $\boxed{\text{Box}}$ your final answer.
- If you need to make any additional assumptions, state them clearly.
- Simplify your result when possible.
- For the problems with [credit!], no partial credit will be given if the final answer is wrong.

Problems:

1. [10] Use the table of FT pairs and the table of properties to find the FT of each of the following signals (*DO NOT USE INTEGRATION*):
   (a) [5!] $x(t) = 2\text{rect}\left(\frac{t-2}{4}\right)$
   (b) [5!] $x(t) = e^{-3t}\text{rect}\left(\frac{t-2}{4}\right)$
   (c) [5!] $x(t) = t\text{rect}\left(\frac{t-2}{4}\right)$
   (d) [5!] $x(t) = \cos(4\pi t)\text{rect}\left(\frac{t-2}{4}\right)$
2. [5] Find a mathematical expression and sketch or plot the inverse FT of $F(\omega) = \text{sinc}^3(\omega/2)$.
   Hint: the inverse FT formula would probably be a hard way to do it.
3. [5] Find the FT of $t^2 e^{-(t/2)^2}$. *Hint*: see table of FT pairs.
4. [5] Show that if $f(t)$ is real and odd, then $F(\omega)$ is purely imaginary and odd.
5. [5] Consider a real signal $f(t)$ and let
   $$f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega), \quad F(\omega) = \text{real}\{F(\omega)\} + j\ \text{imag}\{F(\omega)\}$$
   and
   $$f(t) = f_e(t) + f_o(t)$$
   where $f_e(t)$ and $f_o(t)$ are the even and odd component of $f(t)$ respectively. Show that
   $$f_e(t) \overset{\mathcal{F}}{\longleftrightarrow} \text{real}\{F(\omega)\} \qquad f_o(t) \overset{\mathcal{F}}{\longleftrightarrow} j\ \text{imag}\{F(\omega)\}$$
6. [5] Find the energy of the signal $x(t) = t\text{sinc}^2(t)$ by Fourier methods.
7. [5] What percentage of the total energy in the energy signal $f(t) = e^{-t}u(t)$ is contained in the frequency band $-7\text{rad/s} \le \omega \le 7\text{rad/s}$.
8. [10] A LTI system has the following frequency response:
   $$H(j\omega) = \frac{-\omega^2 + j\omega + 1}{(-\omega^2 + 6j\omega + 25)(j\omega + 2)}.$$
   (a) [10] Find the impulse response of the LTI system.
   Hint: first find the partial differential equation.
   (b) [10] Find the differential equation corresponding to the LTI system.
   Hint: write $H(\omega) = Y(\omega)/X(\omega)$ and cross multiply.
9. [10] Determine the Fourier series representations for the following signals:

(a) $x(t)$ period with period 4 and

$$x(t) = \begin{cases} \sin \pi t & , 0 \le t \le 2 \\ 0 & , 2 < t \le 4 \end{cases}$$

(b) As illustrated in the following figure.

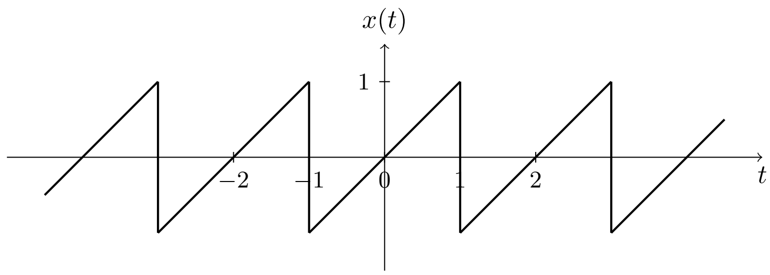

Figure: 0401

10. [5] Find the FT of the following signal: $x(t) = \sum_{n=-\infty}^{\infty} 2\delta(t-6n) - \delta(t-6n-2) - \delta(t-6n+2)$. sketch the magnitude of the spectrum.

11. [10] Compute the Fourier transform of each of the following signals

    (a) $[e^{-\alpha t} \cos \omega_0 t]u(t), \alpha > 0$

    (b) $e^{-3|t|} \sin 2t$

12. [10] Determine the continuous-time signal corresponding to the following transform.

    (a) $X(j\omega) = cos(4\omega + \pi/3)$

    (b) $X(j\omega)$ as given by magnitude and phase plots.

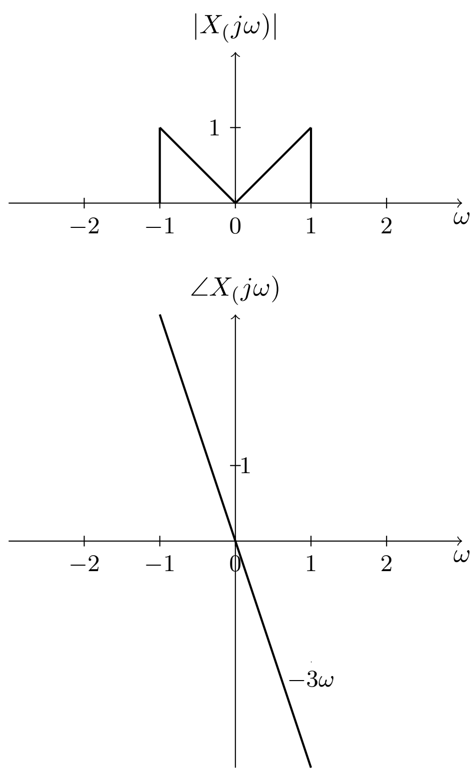

Figure: 0402

13. [10] Shown in the figure 0403 is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For each of the input signals x(t) below, determine the filter output signal $y(t)$.

    (a) $x(t) = \cos(2\pi t + \theta)$

    (b) $x(t) = \cos(4\pi t + \theta)$

    (c) $x(t)$ is a half-wave rectified sine wave of period 1, as sketched in figure 0404.

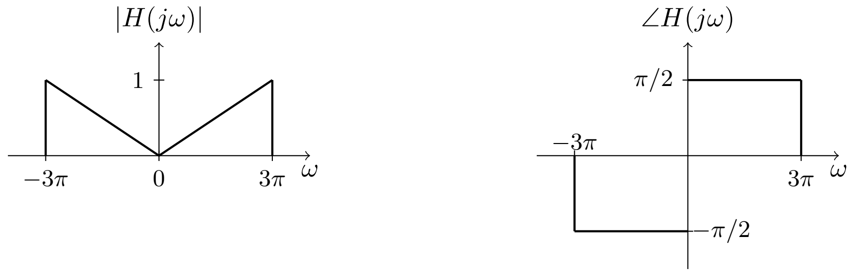

Figure: 0403

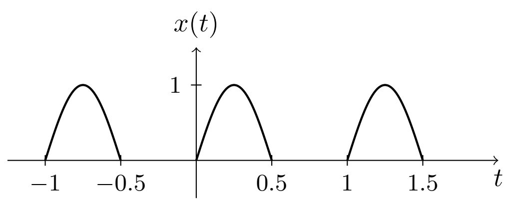

Figure: 0404

$$x(t) = \begin{cases} \sin(2\pi t) & , m \le t \le m + \frac{1}{2} \\ 0 & , (m + \frac{1}{2}) \le t \le m \end{cases}$$

14. [5] A power signal with the power spectral density shown in figure 0405 is the input of a linear system with the frequency response shown in figure 0406. Calculate and sketch the power spectral density of the system's output signal.

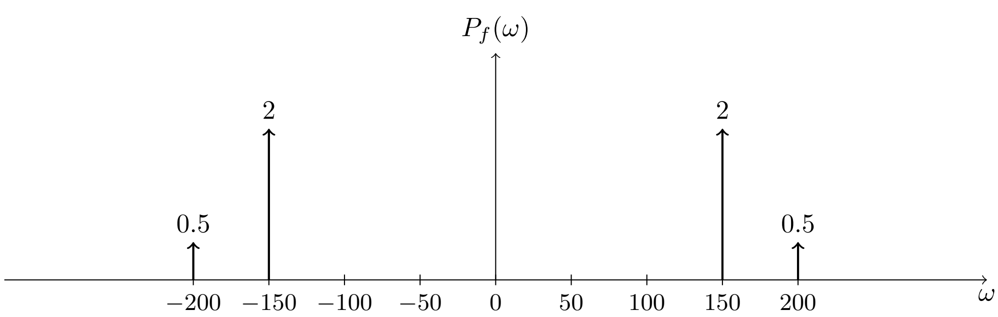

Figure: 0405

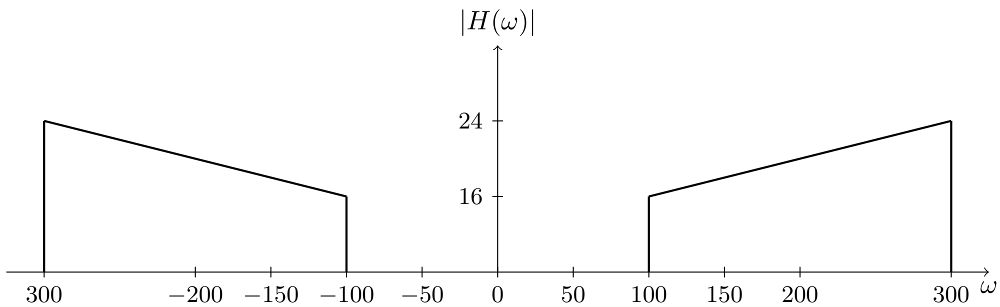

Figure: 0406