Ve216 Introduction to Signals and Systems: Summary
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(Based on Lecture Notes by Prof. Jeffrey A. Fessler)

Signals and Systems: Summary 1

- Circuits: $v(t) = Ri(t)$, $v(t) = L\frac{\mathrm{d}}{\mathrm{d}t}i(t)$, $i(t) = C\frac{\mathrm{d}}{\mathrm{d}t}v(t)$
- Notation: $x(t) = \begin{cases} e^{-t}, & t > 2, \\ 0, & \text{otherwise} \end{cases} = e^{-t}u(t-2)$
- Time transformation:
  - $x\left(\frac{t-t_0}{w}\right)$. First scale according to w, then shift according to t_0.
  - $x(at - b)$. First time-delay by b, then time-scale by a
- Integrator system $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = x(t) * u(t)$
- Even symmetry: $x(-t) = x(t)$, Odd symmetry: $x(-t) = -x(t)$
- $\mathrm{Ev}\{x(t)\} = \frac{1}{2}(x(t) + x(-t))$, $\mathrm{Od}\{x(t)\} = \frac{1}{2}(x(t) - x(-t))$, $x(t) = \mathrm{Ev}\{x(t)\} + \mathrm{Od}\{x(t)\}$.

- Average value: $A \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\, dt$
- Energy: $E \triangleq \int_{-\infty}^{\infty} |x(t)|^2\, dt$
- Average power: $P \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt$
- Energy signal: $E < \infty$, $P = 0$.
- Power signal: $E = \infty$, $0 < P < \infty$.
- Power of periodic signal: $P = \frac{1}{T_0}\int_{0}^{T_0} |x(t)|^2\, dt$

- Step function: $u(t) = 1$ for $t > 0$.
- Rect function: $\mathrm{rect}(t) = 1$ for $-1/2 < t < 1/2$, $\mathrm{rect}(t) = u(t + 1/2) - u(t - 1/2) = u(t + 1/2)u(1/2 - t)$
- Impulse functions
  - Sifting property: $\int_{-\infty}^{\infty} x(t)\delta(t - t_0)\, dt = x(t_0)$ if $x(t)$ is continuous at t_0
  - Sampling property: $x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$ if $x(t)$ is continuous at t_0
  - unit area property: $\int_{-\infty}^{\infty} \delta(t - t_0)\, dt = 1$ for any t_0
  - scaling property: $\delta(at + b) = \frac{1}{|a|}\delta(t + b/a)$ for $a \neq 0$.
  - symmetry property: $\delta(t) = \delta(-t)$
  - support property: $\delta(t - t_0) = 0$ for $t \neq t_0$
  - relationships with unit step function: $\delta(t) = \frac{\mathrm{d}}{\mathrm{d}t}u(t)$, $u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$

Continuous-time system properties
- Stability (BIBO): all bounded input signals produce bounded output signals
- Invertibility: each output signal is the response to only one input signal
- Causal: output signal value $y(t)$ at any time t depends only on present and past input signal values.
- Static (memoryless): output at any time only depends on input signal at the same time. (otherwise dynamic)
- Time invariant:

- $x(t) \xrightarrow{\mathcal{T}} y(t)$ implies that $x(t-t_0) \xrightarrow{\mathcal{T}} y(t-t_0)$
- Recipe for showing time-invariance.
  - Determine output signal $y(t)$ due to a generic input signal $x(t)$.
  - Determine the delayed output signal $y(t-t_0)$, by replacing t with $t-t_0$ in $y(t)$ expression.
  - Determine output signal $y_d(t)$ due to a delayed input signal $x_d(t) = x(t-t_0)$.
  - If $y_d(t) = y(t-t_0)$, then system is time-invariant.
- Linear systems:
  - superposition property: $\mathcal{T}[\sum_k a_k x_k(t)] = \sum_k a_k \mathcal{T}[x_k(t)]$
  - additivity property: $\mathcal{T}[x_1(t) + x_2(t)] = \mathcal{T}[x_1(t)] + \mathcal{T}[x_2(t)]$
  - scaling property: $\mathcal{T}[ax(t)] = a\mathcal{T}[x(t)]$

LTI systems

input-output relationship described by convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)\,d\tau$$

Properties:
- Commutative property: $x(t) * h(t) = h(t) * x(t)$
- Associative property: $[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)]$
- Distributive property: $x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$
- The order of serial connection of LTI systems does not affect the overall impulse response.
- $x(t) * \delta(t) = x(t)$
- Delay property: $x(t) * \delta(t-t_0) = x(t-t_0)$
- $\delta(t-t_0) * \delta(t-t_1) = \delta(t-t_0-t_1)$
- Time-invariance: If $y(t) = x(t) * h(t)$, then $x(t-t_0) * h(t-t_1) = y(t-t_0-t_1)$

LTI system properties
- causal: $h(t) = 0$ for all $t < 0$
- static: $h(t) = k\delta(t)$, otherwise dynamic
- stable: $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$
- invertible: $h(t) * h_i(t) = \delta(t)$ for some $h_i(t)$
  If $h(t) * x(t) = 0$ for some nonzero signal $x(t)$, then not invertible
- step response: $h(t) = \frac{\mathrm{d}}{\mathrm{d}t}s(t)$, where $u(t) \xrightarrow{\text{LTI}} s(t)$

Linear, constant coefficient, differential equation systems
- LTI and causal if initially at rest
- dynamic unless $N = M = 0$
- homogenous solution, natural response:
  $y_h(t) = \sum_l C_l e^{s_l t}$, where s_l's are the N roots of the characteristic polynomial $\sum_{k=0}^{N} a_k s^k = 0$.
- particular solution, forced response: $y_p(t) = P_0 x(t) + P_1 \frac{\mathrm{d}}{\mathrm{d}t} x(t) + \cdots$

Topics covered in Lecture 1-7

Chap. 1

- signal classes
- signal notation
- * time transformations
- amplitude transformations
- signal operations
- integrator system (running integral operation)
- * even/odd signals
- * energy and power signals
- periodicity
- * unit step / rect signals
- * unit impulse function
- * impulse function properties (sifting, sampling)
- CT systems
- block diagrams
- system classes
- * amplitude properties: linearity, stability, invertibility
- * time properties: causality, memory, time-invariance

Chap. 2

- impulse response $h(t)$
- convolution for CT LTI systems
- * graphical convolution
- * properties of convolution and LTI systems
- impulse response vs step response
- * properties of convolution and impulse response
- LTI system properties characterized by $h(t)$(causality, memory, stability, invertibility)
- diffeq systems
- solutions of diffeq

The items with a * are virtually guaranteed to be on the exam.

Tips for Exam Preparation

- **Review quizzes.** Some quiz problems are selected from previous exams. Make sure that you completely understand the answers to all problems on previous quizzes.
- **Study lecture slides.** Read through lecture slides and the summary notes carefully. Make sure that you fully understand all the lecture materials.
- **Study homework solutions.** Review your HW sets and the posted solutions on SAKAI.
- **Attend exam recitation classes.** TAs will posted times on SAKAI.