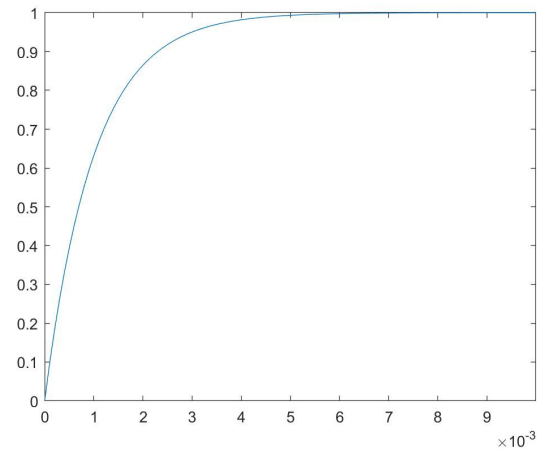


$$4.1) RC(V_{out}(t))' + V_{out}(t) = V_{in}(t) \quad (b)$$

$$\Rightarrow V_{out} = (1 - e^{-\frac{t}{RC}}) u(t)$$

$$\Rightarrow e^{-\frac{t}{RC}} u(t) + (1 - e^{-\frac{t}{RC}}) u(t) = V_{in}(t)$$

$$\Rightarrow V_{in}(t) = u(t)$$



$$4.2) h(t) = \frac{d}{dt} y_{step}(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$

$$4.3(a) u(t) \xrightarrow{h(t)} (1 - e^{-\frac{t}{RC}}) u(t)$$

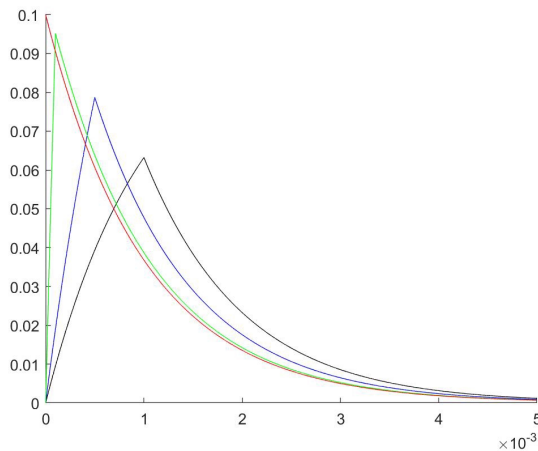
$$\Rightarrow u(t-\Delta) \xrightarrow{h(t)} (1 - e^{-\frac{t-\Delta}{RC}}) u(t-\Delta)$$

$$\Rightarrow V_{out} \xrightarrow{h(t)} \frac{b}{\Delta} [(1 - e^{-\frac{t}{RC}}) u(t) - (1 - e^{-\frac{t-\Delta}{RC}}) u(t-\Delta)]$$

$$(b) \lim_{\Delta \rightarrow 0, b=1} V_{out} = \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} [(1 - e^{-\frac{t}{RC}}) u(t) - (1 - e^{-\frac{t-\Delta}{RC}}) u(t-\Delta)]$$

$$= \lim_{\Delta \rightarrow 0} \frac{1}{RC} e^{-\frac{t-\Delta}{RC}} u(t-\Delta) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t) = 4.2 \text{ Answer}$$

(c) Plot



when $\Delta \uparrow$
the plot is more like $(1 - e^{-\frac{t}{RC}}) u(t)$

$$4.4. u(t) \xrightarrow{T} (1 - e^{-1000t}) u(t) \quad (1)$$

$$\bullet u(t-0.01) \xrightarrow{T} (1 - e^{-1000(t-0.01)}) u(t-0.01) \quad (2)$$

$$\bullet t u(t) \xrightarrow{T} \left[\frac{1}{1000} \cdot (e^{-1000t} - 1) + t \right] \cdot u(t)$$

$$(t-0.01) u(t-0.01) \xrightarrow{T} \left[\frac{1}{1000} \cdot (e^{-1000(t-0.01)} - 1) + (t-0.01) \right] u(t-0.01) \quad (3)$$

$$\bullet t u(t-0.005) \xrightarrow{T} \left[\frac{\frac{1}{1000} - 0.05}{e^{-1000 \times 0.05}} \cdot e^{-1000t} + t - \frac{1}{1000} \right] u(t-0.005)$$

$$(t-0.01) u(t-0.01) \xrightarrow{T} \left[\frac{\frac{1}{1000} - 0.05}{e^{-1000 \times 0.05}} \cdot e^{-1000(t-0.01)} + (t-0.01) \right] u(t-0.01) \quad (4)$$

$$\Rightarrow V_{out} = (1) + (2) + (3) + (4) \\ = (1 - e^{-1000t}) u(t) + (1 - e^{-1000(t-0.01)}) u(t-0.01) + \left[\frac{1}{1000} \cdot (e^{-1000(t-0.01)} - 1) + (t-0.01) \right] u(t-0.01) \\ + \left[\frac{\frac{1}{1000} - 0.05}{e^{-1000 \times 0.05}} \cdot e^{-1000(t-0.01)} + (t-0.01) \right] u(t-0.01)$$

4.5

$$RC(V_{out})' + V_{out} = V_{in}(t) \Rightarrow \text{let } v_m = e^{st} \Rightarrow V_{out} = H(s) e^{st}$$

$$RC \cdot H(s) \cdot s \cdot e^{st} + H(s) \cdot e^{st} = e^{st}$$

$$\Rightarrow H(s) = \frac{1}{RCs + 1} \Rightarrow H(j\omega) = \frac{1}{j\omega RC + 1}$$

4.6

$$f_c \quad H(j2\pi f_c) \quad \angle |H(j2\pi f_c)| / \text{deg} \quad T_{dm}$$

$$50 \quad 0.9540 \quad -17.481 \quad 0.9673$$

$$200 \quad 0.6227 \quad -51.488 \quad 0.7151$$

$$500 \quad 0.3023 \quad -71.343 \quad 0.4019$$

$$1k \quad 0.1572 \quad -80.957 \quad 0.2249$$

$$5k \quad 0.0318 \quad -88.177 \quad 0.0490$$