VE216 Lecture 8

> Convolution

Convolution

Hoping to represent a system by a single signal, while the input is more complicated.

Superposition

Breaking input into additive parts and sum the responses to the parts.

It is easy if system is **linear and time-invariant (LTI)**.

Linear

Given

$$x_1[n] \longrightarrow \boxed{\text{system}} \longrightarrow y_1[n]$$

$$x_2[n] \longrightarrow \boxed{\text{system}} \longrightarrow y_2[n]$$

and the system is linear if $\alpha x_1[n] + \beta x_2[n] \longrightarrow \boxed{\text{system}} \longrightarrow \alpha y_1[n] + \beta y_2[n]$

for all α, β.

Time-Invariance

Given $x[n] \longrightarrow \boxed{\text{system}} \longrightarrow y[n]$

and the system is time invariant if $x[n-n_0] \longrightarrow \boxed{\text{system}} \longrightarrow y[n-n_0]$

for all n_0.

Structure of Superposition

$$\begin{aligned}
\delta[n] &\rightarrow h[n] \\
\delta[n-k] &\rightarrow h[n-k] \\
x[k]\delta[n-k] &\rightarrow x[k]h[n-k] \\
x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] &\rightarrow y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]
\end{aligned}$$

Convolution Notation

$x[n] \rightarrow \text{LTI System} \rightarrow y[n]$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = (x * h)[n]$$

DT Convolution Remark

Actually, unit sample response $h[n]$ is a complete description of an **LTI system**.

CT Convolution

Similar form: $\begin{aligned} x(t) &\rightarrow \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau \\ y(t) &\rightarrow \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \end{aligned}$

Convolution Summary

DT: $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = (x * h)[n]$

CT: $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = (x * h)(t)$

Summary

The **impulse response** is a **complete description** of a **linear, time-invariant (LTI) system**.

One can find the output of such a system by **convolving the input signal with the impulse response**.