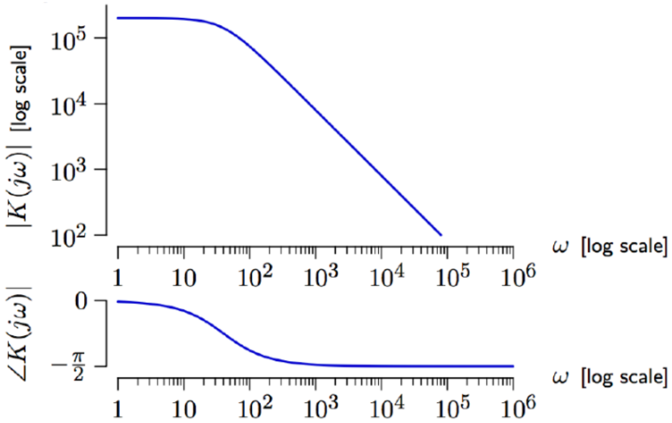
VE216 Lecture 12

> CT Feedback and Control

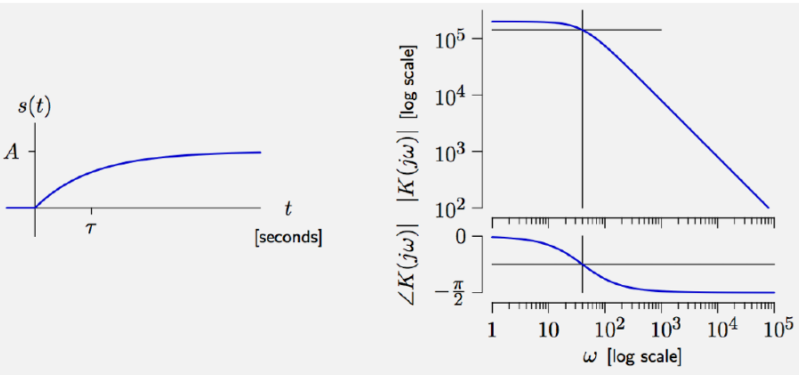
Op-Amp Data Introduction

The gain of an op-amp depends on frequency.



The **low-gain** at **high frequencies** limits applications.

Time Constant and System Function Control



In an increasing system, the time constant τ is the time for the step response to reach $1-\frac{1}{e}$ of the final value.

System Bode Plot to Frequency Response

We can see the plot of the frequency response looks like the bode plot of $\frac{1}{s+\alpha}$.

Then we do some improvements to make it real: $K(s)=\frac{\alpha K_0}{s+\alpha}$.

With inverse Laplace transform, we get $h(t)=\alpha K_0 e^{-\alpha t}u(t)$ as impulse response.

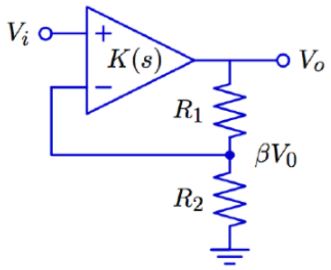
Then integrate from $-\infty$ to t, we get $s(t)=\int_{-\infty}^{t}h(\tau)d\tau=\int_0^t h(\tau)d\tau=K_0(1-e^{-\alpha t})u(t)$.

With the parameter $K_0=2\times10^5$ and $\alpha=40$, we get the result that $\tau=\frac{1}{\alpha}=\frac{1}{40}s$.

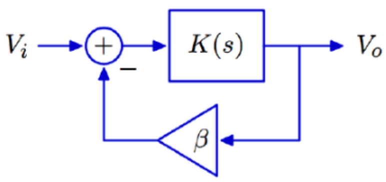
Op Amp Disadvantages

- Frequency Response: the high gain is only in low frequencies.
- Step Response: slow by electronic standards ($\tau = \frac{1}{40}s$).

Performance Improvement by Feedback



$$V_- = \beta V_o = \left(\frac{R_2}{R_1 + R_2}\right) V_o$$



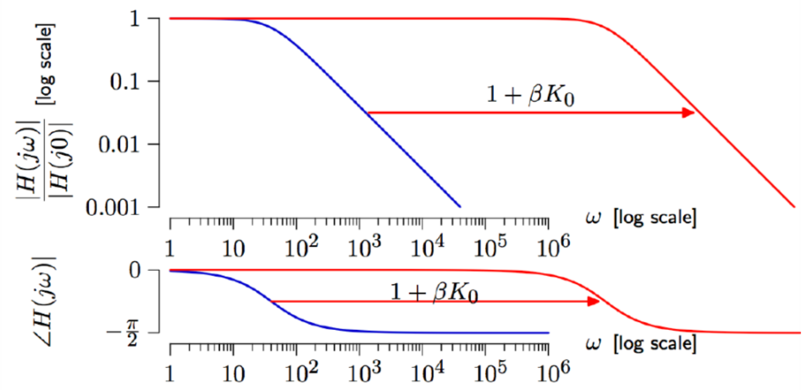
$$\begin{aligned} \frac{V_o}{V_i} &= \frac{K(s)}{1 + \beta K(s)} \\ &= \frac{\frac{\alpha K_0}{s+\alpha}}{1 + \beta \frac{\alpha K_0}{s+\alpha}} \\ &= \frac{\alpha K_0}{s + \alpha + \alpha\beta K_0} \end{aligned}$$

So the origin pole is $s = -\alpha$.

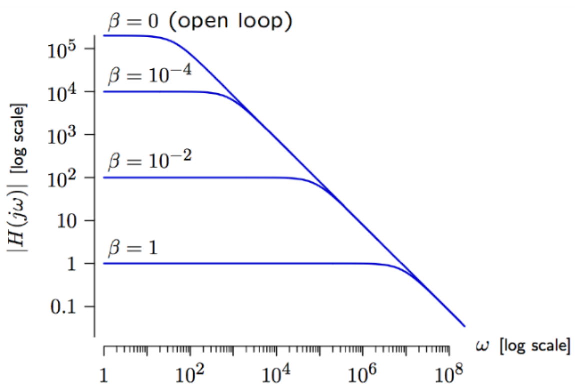
The new pole is $s = -\alpha(1 + \beta K_0)$, even negative then 0.

So consider τ will be smaller, the response is faster.

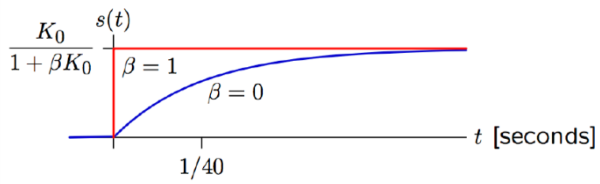
Detailed Performance Improvement

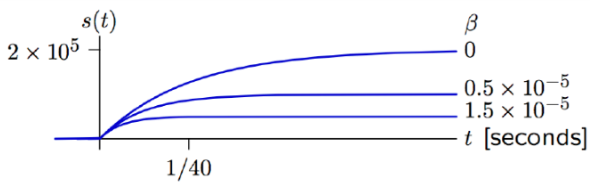


- Feedback trades gain for band width (the width of frequency to maintain the gain unchanged):



- Step response: $s(t) = \frac{K_0}{1+\beta K_0}(1 - e^{-\alpha(1+\beta K_0)t})u(t)$

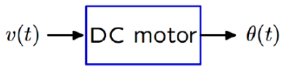




with the same $\frac{d}{dt}s(t)\Big|_{t=0^+}$

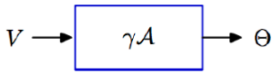
Motor Controller

We build a robot arm with input $v(t)$ and output as $\theta(t)$.

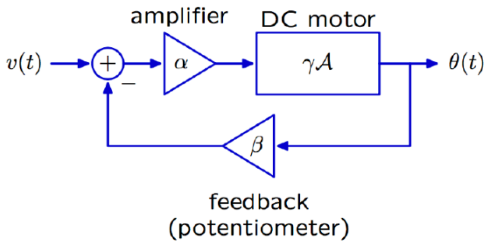


The **rotational speed** is proportional to the **input voltage**.

So the angle is the **integral** of rotation speed.



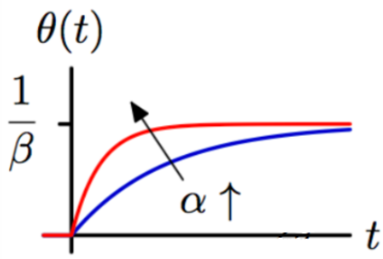
So we apply the proportional **feedback** to control the angle of the motor's shift.



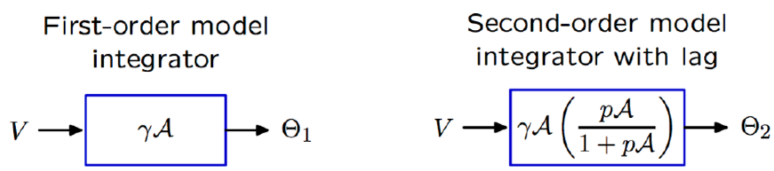
the feedback function is $\frac{\Theta}{V}=\frac{\alpha\gamma A}{1+\alpha\beta\gamma A}=\frac{\alpha\gamma}{s+\alpha\beta\gamma}$ (substitute A with $\frac{1}{s}$) with single pole $-\alpha\beta\gamma$.

Then the impulse response: $h(t)=\alpha\gamma e^{-\alpha\beta\gamma t}u(t)$.

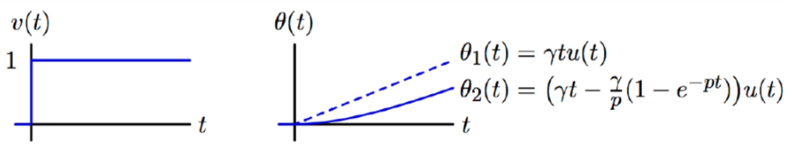
Then the step response: $s(t)=\frac{1}{\beta}(1-e^{-\alpha\beta\gamma t})u(t)$.



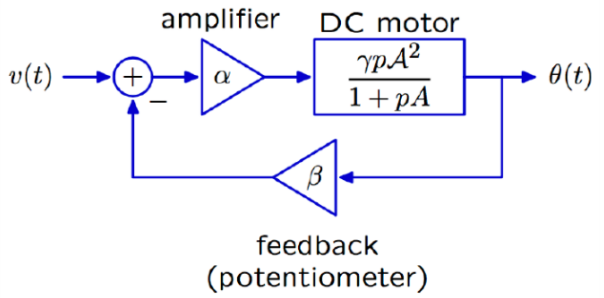
In real life, the motor integrator has lag, so we change it:



with the step response:



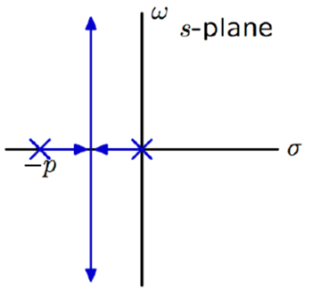
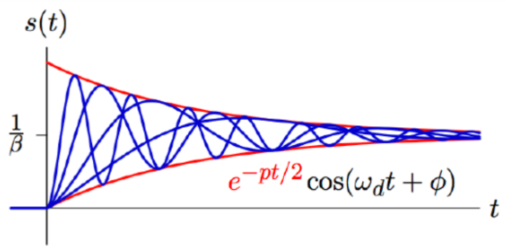
So the second-order model is:



with feedback frequency response: $\frac{\frac{\alpha\gamma pA^2}{1+pA}}{1+\frac{\alpha\beta\gamma pA^2}{1+pA}} = \frac{\alpha\beta pA^2}{1+pA+\alpha\beta\gamma pA^2} = \frac{\alpha\gamma p}{s^2+ps+\alpha\beta\gamma p}$

thus the poles are $s = -\frac{p}{2} \pm \sqrt{(\frac{p}{2})^2 - \alpha\beta\gamma p}$

When we increase the β, then we get the two poles collide from 0 and $-p$ to imaginary part.

poles	diagrams
	

Summary

CT feedback is useful to

- Increase speed and bandwidth
- Control position instead of speed