2. [15]Consider the system in Figure 0903.

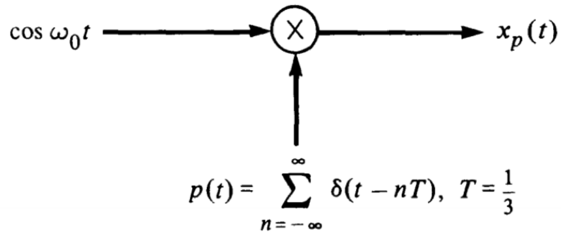


Figure 0903.

(a) [10]Sketch $X_p(\omega)$ for $-9\pi \le \omega \le 9\pi$ for the following values of ω_0.

i. $\omega_0 = \pi$

ii. $\omega_0 = 2\pi$

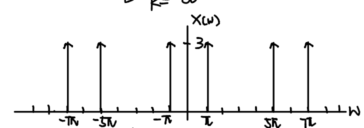
iii. $\omega_0 = 3\pi$

iv. $\omega_0 = 5\pi$

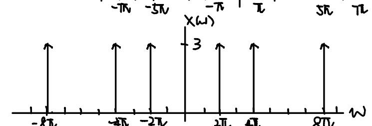
(1) $x(t) = \cos \omega_0 t \quad p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \frac{1}{3}n) \quad X_s(t) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s) \Rightarrow \omega_s = \frac{2\pi}{T_s} = 6\pi$

$x(t) = \cos \omega_0 t \xleftrightarrow{F} \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0) \quad \Rightarrow X_s(\omega) = 3 \sum_{k=-\infty}^{\infty} X(\omega - 6k\pi) = 3 \sum_{k=-\infty}^{\infty} [\pi\delta(\omega - \omega_0 - 6k\pi) + \pi\delta(\omega + \omega_0 - 6k\pi)]$

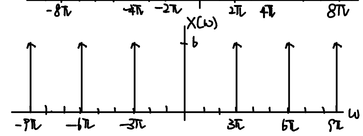
i) $\omega_0 = \pi \quad 3 \sum_{k=-\infty}^{\infty} [\pi\delta(\omega - \pi - 6k\pi) + \pi\delta(\omega + \pi - 6k\pi)] = X_s(\omega)$



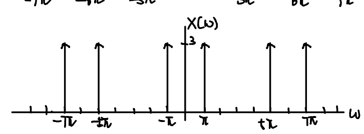
ii) $\omega_0 = 2\pi \quad X_p(\omega) = 3 \sum_{k=-\infty}^{\infty} [\pi\delta(\omega - 2\pi - 6k\pi) + \pi\delta(\omega + 2\pi - 6k\pi)]$



iii) $\omega_0 = 3\pi \quad X_p(\omega) = 3 \sum_{k=-\infty}^{\infty} [\pi\delta(\omega - 3\pi - 6k\pi) + \pi\delta(\omega + 3\pi - 6k\pi)]$



iv) $\omega_0 = 5\pi \quad X_p(\omega) = 3 \sum_{k=-\infty}^{\infty} [\pi\delta(\omega - 5\pi - 6k\pi) + \pi\delta(\omega + 5\pi - 6k\pi)]$



(b) [5]For which of the preceding values of ω_0 is $x_p(t)$ identical? For which of the preceding values of ω_0 will we NOT be able to recover the input sinusoidal signal after lowpass filtering $x_p(t)$?

(1) For $\omega_0 = 5\pi$ & π, they are identical

(2) $\omega_0 = 5\pi$ not able to be recover.

since $\omega_s = 6\pi < 2 \times 5\pi$

3. [10]Given the system in Figure 0905(a) and the Fourier transforms in Figure 0905(b), determine the maximum values for T and A in terms of W such that $y(t) = x(t)$ if $s(t)$ is the impulse train

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT).$$

State your reasoning.

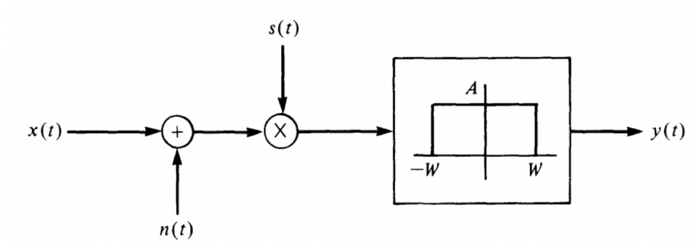


Figure 0905(a).

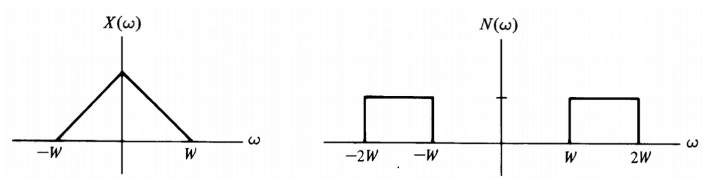
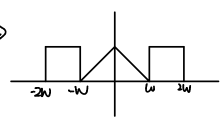


Figure 0905(b).

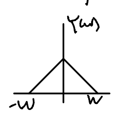
$X(t) + n(t) \Rightarrow X(\omega) + N(\omega) \Rightarrow$



$[X(\omega) + N(\omega)] * S(t)$

$\Rightarrow X(\omega) + N(\omega)$ follow the period of T

$\Rightarrow$ high frequency will be blocked $\Rightarrow$ we have to get



to let $x(t) = y(t)$

$\Rightarrow Y(\omega) = \frac{1}{2\pi} \{[X(\omega) + N(\omega)] * S(\omega)]\} \cdot A \operatorname{rect}(\frac{t}{2W}) = X(\omega)$

$S(t) \xleftrightarrow{F} \sum_{k=-\infty}^{+\infty} \frac{2\pi}{T} \delta(\omega - k\frac{2\pi}{T}) \quad \Rightarrow X(\omega) = \frac{A}{T} \operatorname{rect}(\frac{t}{2W}) \cdot \sum_{k=-\infty}^{+\infty} [X(\omega - k\frac{2\pi}{T}) + N(\omega - k\frac{2\pi}{T})]$

$\Rightarrow \frac{A}{T} = 1, \; 2W - \frac{2\pi}{T} \le -W \quad \Rightarrow T \le \frac{2\pi}{3W} \quad \Rightarrow A_{max} = T_{max} = \frac{2\pi}{3W}$

4. [12]Consider the system in Figure 0907.

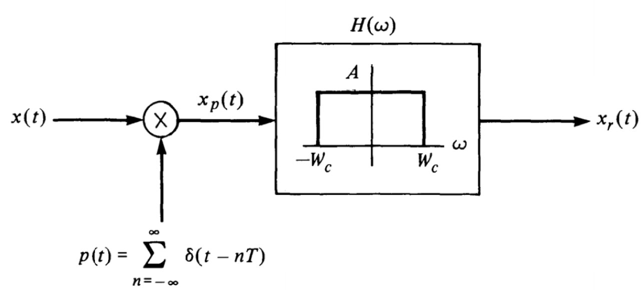


Figure 0907.

If $X_1(\omega) = 0$ for $|\omega| > 2W$ and $X_2(\omega) = 0$ for $|\omega| > W$. For the following inputs $x(t)$, find the ranges for the cutoff frequency W_c in terms of T and W and find the maximum values of T and A, such that $x_r(t) = x(t)$.

(a) $x(t) = x_1(t - \pi/2) + x_2(t)$

(b) $x(t) = x_1(t)x_2(t)$

(c) $x(t) = dx_2(t)/dt$

(d) $x(t) = x_2(t)\cos(2Wt)$

$X_s(t) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(w - k w_s) = X_p(w) \Rightarrow X_r(w) = X(w) = \frac{A}{T} \cdot \sum_{k=-\infty}^{+\infty} X(w - k\frac{2\pi}{T}) \cdot \mathrm{rect}\left(\frac{t}{2W_c}\right)$

(a) $X(w) = X_1(w) \cdot e^{-jw\frac{\pi}{2}} + X_2(w) \Rightarrow |X(w)| = 0$ when $|w| > 2\pi \Rightarrow 2W \le W_c \le \frac{2\pi}{T} - 2W \Rightarrow A_{max} = T_{max} = \frac{\pi}{2W}$

(b) $X(w) = \frac{1}{2\pi} X_1(w) * X_2(w) \Rightarrow |X(w)| = 0$ when $|w| \ge 3W \Rightarrow 3W \le W_c \le \frac{2\pi}{T} - 3W \Rightarrow A_{max} = T_{max} = \frac{\pi}{3W}$

(c) $X(w) = jw \cdot X_2(w) \Rightarrow X_2(w) = 0$ when $|w| \ge W \Rightarrow W \le W \le \frac{2\pi}{T} - W \Rightarrow A_{max} = T_{max} = \frac{\pi}{W}$

(d) $X(w) = \frac{1}{2\pi} \cdot X(w) * \{\pi[\delta(w-2W) + \delta(w+2W)]\} = \frac{1}{2}[X_2(w-2W) + X_2(w+2W)]$

$\Rightarrow |X(w)| = 0$ when $|w| > 3W \Rightarrow A_{max} = T_{max} = \frac{\pi}{3W}$

5. [18]Suppose we have the system in Figure 0908(a) and 0908(b), in which $x(t)$ is sampled with an impulse train. Sketch $x_p(t)$, $y(t)$ and $w(t)$. State your reasoning.

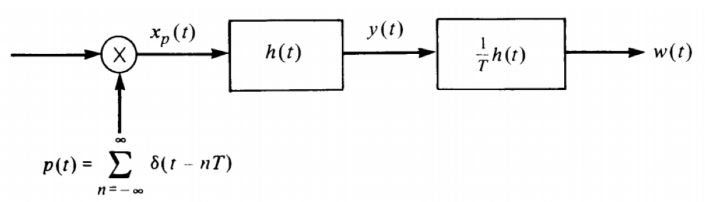
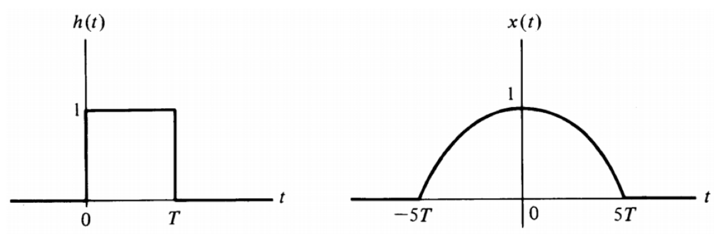
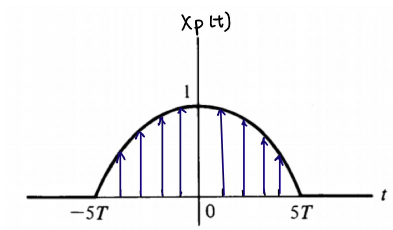
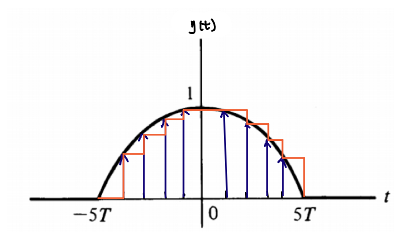
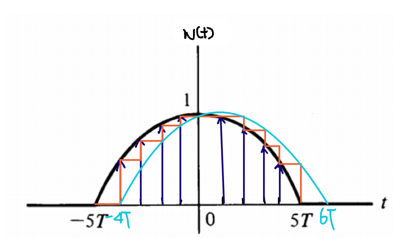


Figure 0908(a).









6. [10]We discussed the effect of a loss of synchronization in phase between the carrier signals in the modulator and demodulator in sinusoidal amplitude modulation. We showed that the output of the demodulation is attenuated by the cosine of the phase difference, and in particular, when the modulator and demodulator have a phase difference of $\pi/2$, the demodulator output is zero. As we demonstrate in this problem, it is also important to have frequency synchronization between the modulator and demodulator.
Consider the amplitude modulation and demodulation systems with $\theta_c = 0$ and with a change in the frequency of the modulator carrier so that

$$w(t) = y(t)\cos\omega_d t$$

where

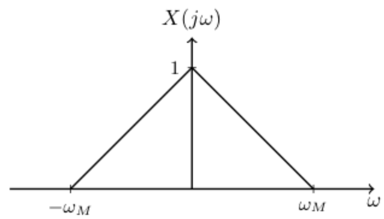
$$y(t) = x(t)\cos\omega_c t$$

Let us denote the difference in frequency between the modulator and demodulator as $\Delta\omega$ (i.e., $\omega_d - \omega_c = \Delta\omega$). Also assume that $x(t)$ is band limited with $X(j\omega) = 0$ for $|\omega| \geq \omega_M$, and assume that the cufoff frequency ω_{co} of the lowpass filter in the demodulator satisfies the inequality

$$\omega_M + \Delta\omega < \omega_{co} < 2\omega_c + \Delta\omega - \omega_M$$

(a) [5] Show that the output of the lowpass filter in the demodulator is proportional to $x(t)\cos(\Delta\omega t)$.

(b) [5] If the spectrum of $x(t)$ is that shown in figure below, sketch the spectrum of the output of the demodulator.



(a) $W(\omega) = X(\omega) * \{\frac{1}{2\pi}\cdot\pi[\delta(\omega+\omega_c)+\delta(\omega-\omega_c)]\} * \{\frac{1}{2\pi}\cdot\pi[\delta(\omega+\omega_d)+\delta(\omega-\omega_d)]\}$

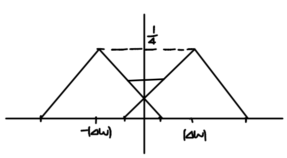
$= \frac{1}{4}[X(\omega+\omega_c+\omega_d)+X(\omega+\omega_c-\omega_d)+X(\omega-\omega_c+\omega_d)+X(\omega-\omega_c-\omega_d)]$

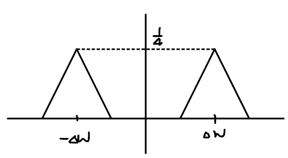
We know $|W(\omega)|=0$ when $|\omega|>\omega_{max}$ $\Rightarrow$ $\omega_{max}+\Delta\omega<\omega_{co}<2\omega_c+\Delta\omega-\omega_{max}$

$\Rightarrow X(\omega+\omega_c+\omega_d) = X(\omega-\omega_c-\omega_d)=0 \Rightarrow W(\omega) = \frac{1}{4}[X(\omega-\Delta\omega)+X(\omega+\Delta\omega)]$

$\Rightarrow w(t) = \frac{1}{2}x(t)\cos(\Delta\omega t)$ $\Rightarrow$ proportional to $x(t)\cos(\Delta\omega t)$

(b) $|\Delta\omega| < \omega_{max}$ $\quad$ $|\Delta\omega| > \omega_{max}$



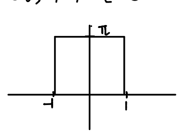


7. [10] (This problem is related to how AM radios work.) The signal $x(t) = \mathrm{sinc}(t/\pi)$ is modulated to form $y(t) = \cos(\omega_0 t)x(t)$, and then (almost?) demodulated by forming $z(t) = cos(\omega_0 t)y(t)$. Find and sketch $Z(\omega)$, assuming ω_0 is large. Check: $Z(0) = \pi/2$.

$W(\omega) = X(\omega) * \{\frac{1}{2\pi}\cdot\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]\} * \{\frac{1}{2\pi}\cdot\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]\}$

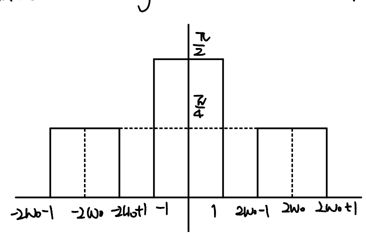
$= \frac{1}{4}[2X(\omega)+X(\omega-2\omega_0)+X(\omega+2\omega_0)]$

$x(t) = \mathrm{sinc}(\frac{t}{\pi}) \xleftrightarrow{F} \pi\,\mathrm{rect}(\frac{\omega}{2})$



Since ω_0 is large there is no overlap

$\Rightarrow$



$Z_0 = \frac{\pi}{4}\cdot 2\cdot 1 = \frac{\pi}{2}$

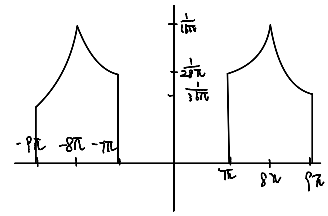
8. [5] A signal $x(t)$ with spectrum $X(\omega) = \left(1 - \left|\frac{\omega}{2\pi}\right|\right) \text{rect}\left(\frac{\omega}{4\pi}\right)$ is filtered by a system with frequency response $H(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$. The resulting signal is then multiplied by $\cos(8\pi t)$. Finally, that signal is passed through an integrator system, yielding a signal $y(t)$. Find and sketch $Y(\omega)$.

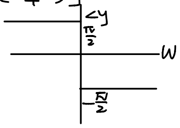
Integrator: $u(t) \xleftrightarrow{F} \pi\delta(\omega) + \frac{1}{j\omega}$

$Z(\omega) = [X(\omega) \cdot H(\omega)] * \{\frac{1}{2\pi}[\pi\delta(\omega-8\pi) + \pi\delta(\omega+8\pi)]\}(\pi\delta(\omega) + \frac{1}{j\omega})$

$= (1-|\frac{\omega}{2\pi}|)\text{rect}(\frac{\omega}{2\pi}) * \{\frac{1}{2}[\delta(\omega-8\pi) + \delta(\omega+8\pi)]\}(\pi\delta(\omega) + \frac{1}{j\omega})$

$= \frac{1}{2j\omega}[(1-|\frac{\omega-8\pi}{4\pi}|)\cdot\text{rect}(\frac{\omega-8\pi}{4}) + (1-|\frac{\omega+8\pi}{4\pi}|)\cdot\text{rect}(\frac{\omega+8\pi}{4})]$





9. [10]Asynchronous modulation-demodulation requires the injection of the carrier signal so that the modulated signal is of the form

$$y(t) = [A + x(t)]\cos(\omega_c t + \theta_c)$$

where $A + x(t) > 0$ for all t. The presence of the carrier means that more transmitter power is required, representing an inefficiency.

(a) [5] Let $x(t) = \cos\omega_M t$ with $\omega_M < \omega_c$ and $A + x(t) > 0$. For a periodic signal $y(t)$ with period T, the average power over time is defined as $P_y = (1/T)\int_T y^2(t)dt$. Determine P_y for $y(t)$ defined above. Express your answer as a function of the modulation index m, defined as the maximum absolute value of $x(t)$ divided by A.

(b) [5] The efficiency of transmission of an amplitude-modulated signal is defined to be the ratio of the power in the sidebands (i.e., power of modulated signal that does not come from DC input) of the signal to the total power in the signal. With $x(t) = \cos\omega_M t$, and with $\omega_M < \omega_c$ and $A + x(t) > 0$, determine and sketch the efficiency of the modulated signal as a function of the modulation index m.

(a) $x(t) \xleftrightarrow{F} \pi[\delta(\omega+\omega_M) + \delta(\omega-\omega_M)]$

$y(t) = [A + \cos\omega_M t]\cdot\cos(\omega_c t + \theta_c) = (A + \frac{1}{2}e^{-j\omega_M t} + \frac{1}{2}e^{j\omega_M t})(\frac{1}{2}e^{j\omega_c t + j\theta_c} + \frac{1}{2}e^{-j\omega_c t - j\theta_c})$

$\Rightarrow$ we can get all the C_k: $\frac{A}{2}, \frac{A}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$ $\Rightarrow P = 2\times\frac{A^2}{4} + 4\times\frac{1}{16} = \frac{A^2}{2} + \frac{1}{4} = \frac{1}{2m^2} + \frac{1}{4}$

(b) $P_{effect} = 4\times\frac{1}{16}$ $\Rightarrow$ Efficiency: $\frac{\frac{1}{4}}{\frac{1}{2m^2}+\frac{1}{4}} = \frac{m^2}{2+m^2}$