VE216 Lecture 1

Function reformation notice!

$f(x) \to f(ax+b)$, how will the function change?

1. $f(x) \to f(ax+b)$, thus we know how to change:

- move the center of $f(x)$ according to b: if $b > 0$, then move to left side with length b; otherwise, move right side with length b.
- check the a, change the $x - axis$ $\frac{1}{a}$ times according to $y - axis$.

2. or we can see $f(x) \to f(a(x + \frac{b}{a}))$, thus get a result:

- move $f(x)$ according to the $\frac{b}{a}$, if positive left side, otherwise right side.
- change according to the center of $f(x)$ by the $x - axis$ with $\frac{1}{a}$ times.

These are two methods needed to be remembered.

VE216 Lecture 2

Multiple Representations of Discrete Time Systems

> Verbal description.

> Block diagram: try the step-by-step analysis to solve the problem.
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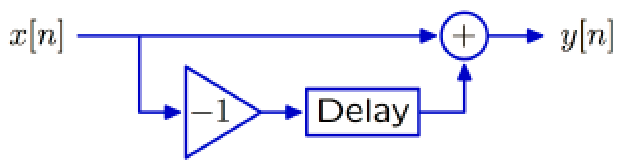
> 
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> Difference equation: mathematically precise and compact.
>
> $y[n] = x[n] - x[n-1]$

> Operator representation.
>
> $Y = (1 - R)X$

- **Delay**: make $x[n]$ into $x[n-1]$ or X into RX.
- **Value**: set it k in triangle symbol, let $x[n]$ into $k \cdot x[n]$ and X into $k \cdot X$.

Notice that on a subline, with a -1 and **Delay**, so these should be **multiplied by** -1 or R or something else, to get a $-RX$ or $-x[n-1]$ on that subline.

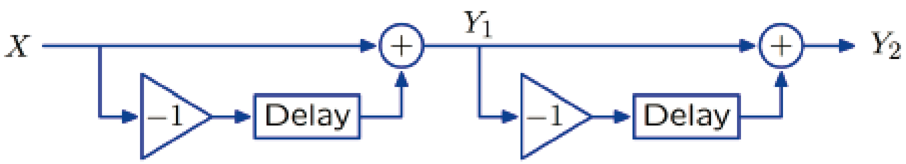
Also, the whole system are assumed commonly by start at rest.

Operators and Operator Notation

Delay: $R \rightarrow Y = R\{X\} \equiv RX \rightarrow (y[n] \rightarrow y[n-1])$

Value: $p \rightarrow Y = p \cdot X \rightarrow (y[n] \rightarrow p \cdot y[n])$

Operator of Cascaded System



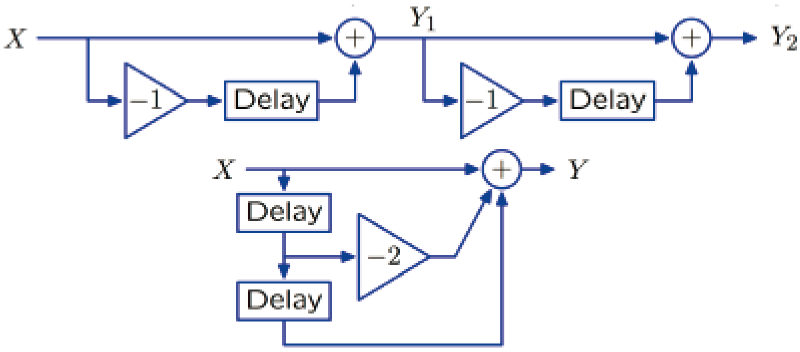
Then we see $Y_2 = (1-R)Y_1$ and $Y_1 = (1-R)X$.

So $Y = Y_2 = (1-R)(1-R)X = (1-R)^2X$.

Or so to say $y_2[n] = y_1[n] - y_1[n-1]$ and $y_1[n] = x[n] - x[n-1]$.

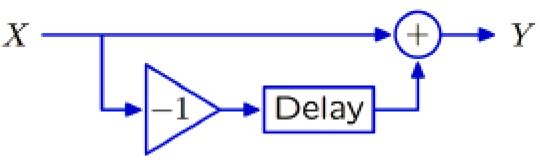
Thus $y_2[n] = x[n] - 2x[n-1] + x[n-2]$.

Operator Equivalence



Feedforward & Feedback

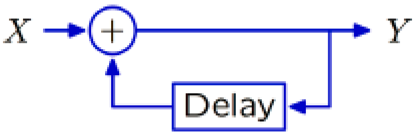
1. Feedforward



$Y = (1 - \mathcal{R})\, X$

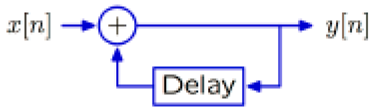
Subtract a right-shifted version of the input signal from a copy of the input signal.

2. Feedback



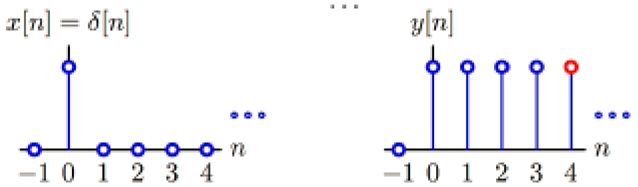
$$Y = \mathcal{R}Y + X$$
$$(1 - \mathcal{R})\,Y = X$$

Systems with signals that depend on **previous values** of the same signal are said to have feedback.



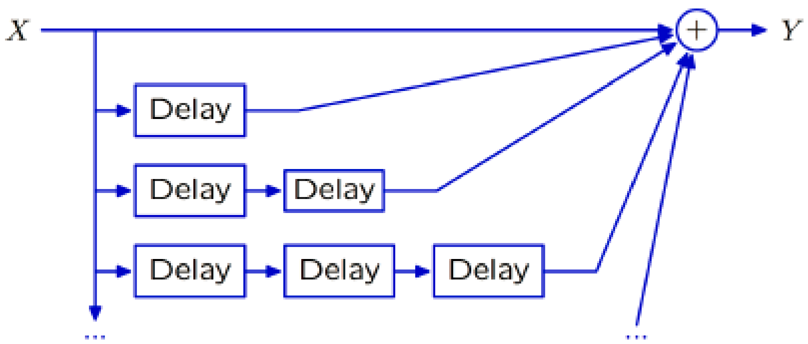
Find $y[n]$ given $x[n] = \delta[n]$: $\quad y[n] = x[n] + y[n-1]$

$$\begin{aligned} y[0] &= x[0] + y[-1] &&= 1 + 0 = 1 \\ y[1] &= x[1] + y[0] &&= 0 + 1 = 1 \\ y[2] &= x[2] + y[1] &&= 0 + 1 = 1 \\ &\cdots \end{aligned}$$



The feedback system change the unit sample into a **constant, persistent signal response**.

Convert Between Feedback and Feedforward

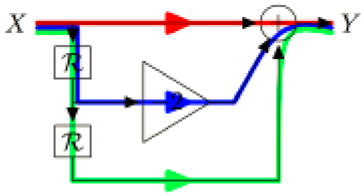


$Y = \left(\sum_{i \in \mathbb{N}} R^i\right) \cdot X \equiv \frac{X}{1-R}$

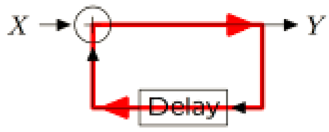
We can try to prove this by getting $\left(\sum_{i \in \mathbb{N}} R^i\right) \cdot (1 - R) \equiv 1$.

Cyclic Signal Path

- **Acyclic**: all the paths through system flow from input to output with no cycles.
- **Cyclic**: at least one cycle.



acyclic

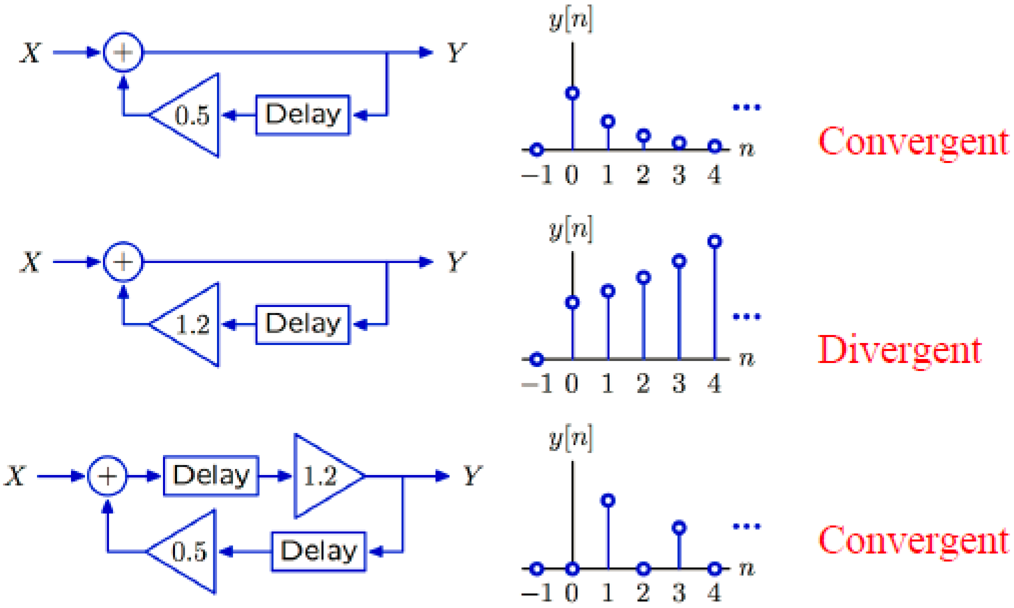


cyclic

- Feedback and Cyclic Paths are related, **response will persist even though the input is transient**.

- The impulse response of an **acyclic system** has **finite duration**, while **cyclic system** has **infinite duration**.

Fundamental Modes



We can get **geometric sequence**: $y[n] = (0.5)^n$ or $y[n] = (1.2)^n$ ($\forall n \in \mathbb{N}$).

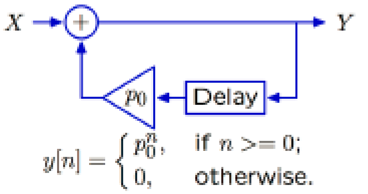
The **geometric sequences** are called **fundamental modes**.

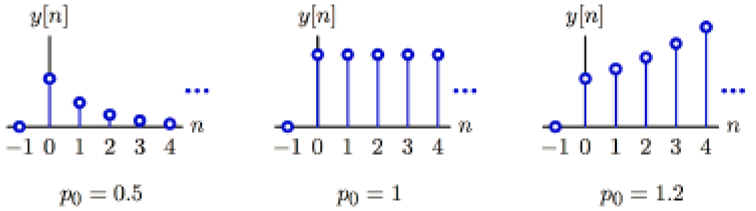
VE216 Lecture 3

Poles

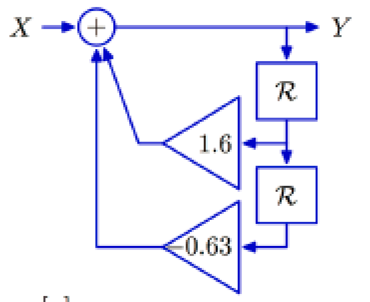
Pole is the base of the geometric sequence.

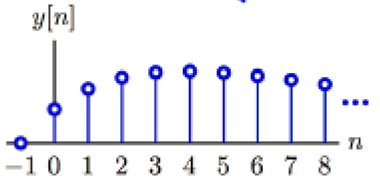
It can be used to characterize a **unit-sample/impulse** response system.





Second-Order System

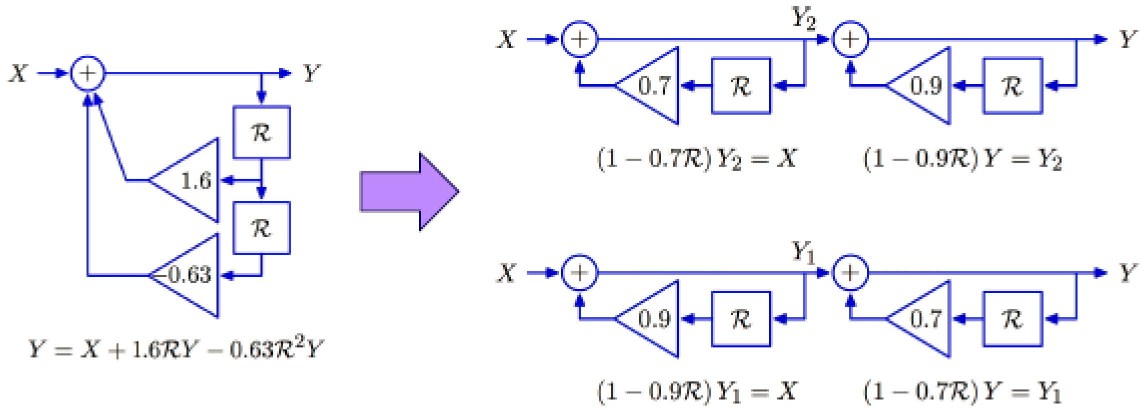




We can **break the system into two simpler systems**.

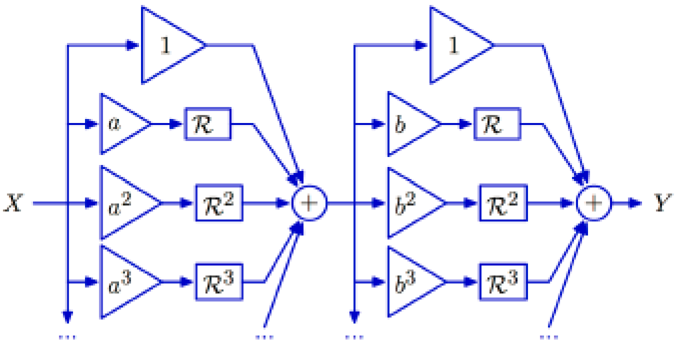
$\{Y = X + 1.6RY - 0.63R^2Y\} \rightarrow \{(1 - 0.7R)(1 - 0.9R)Y = X\}$

Then we can change the block diagram into simpler form:



Thus we get $\frac{Y}{X} = \frac{1}{(1-0.7R)\times(1-0.9R)} = \frac{1}{1-0.7R} \times \frac{1}{1-0.9R} \equiv (\sum_{i\in\mathbb{N}}(0.7R)^i) \times (\sum_{i\in\mathbb{N}}(0.9R)^i)$.

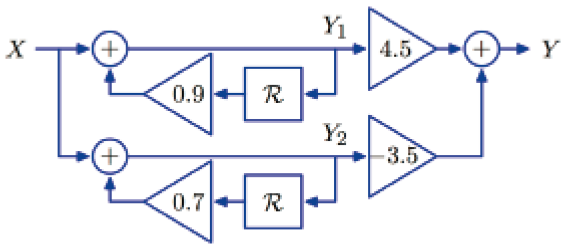
Change the **Feedback** to **Feedforward**:



	1	$b\mathcal{R}$	$b^2\mathcal{R}^2$	$b^3\mathcal{R}^3$	$\cdots$
1	1	$b\mathcal{R}$	$b^2\mathcal{R}^2$	$b^3\mathcal{R}^3$	$\cdots$
$a\mathcal{R}$	$a\mathcal{R}$	$ab\mathcal{R}^2$	$ab^2\mathcal{R}^3$	$ab^3\mathcal{R}^4$	$\cdots$
$a^2\mathcal{R}^2$	$a^2\mathcal{R}^2$	$a^2b\mathcal{R}^3$	$a^2b^2\mathcal{R}^4$	$a^2b^3\mathcal{R}^5$	$\cdots$
$a^3\mathcal{R}^3$	$a^3\mathcal{R}^3$	$a^3b\mathcal{R}^4$	$a^3b^2\mathcal{R}^5$	$a^3b^3\mathcal{R}^6$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Then we can change $\frac{Y}{X} = \frac{1}{1-0.7R} \times \frac{1}{1-0.9R} = \frac{4.5}{1-0.9R} - \frac{3.5}{1-0.7R}$.

Thus the equivalent form is:



$y[n] = 4.5(0.9)^n - 3.5(0.7)^n, n \in \mathbb{N}$

Some tricks

Substitute $R = z^{-1}$ we can find the root easier: $\frac{Y}{X} = \frac{1}{(1-0.7R)(1-0.9R)} = \frac{z^2}{(z-0.7)(z-0.9)}$.

If the **denominator** is **second-ordered**, then **2** poles.

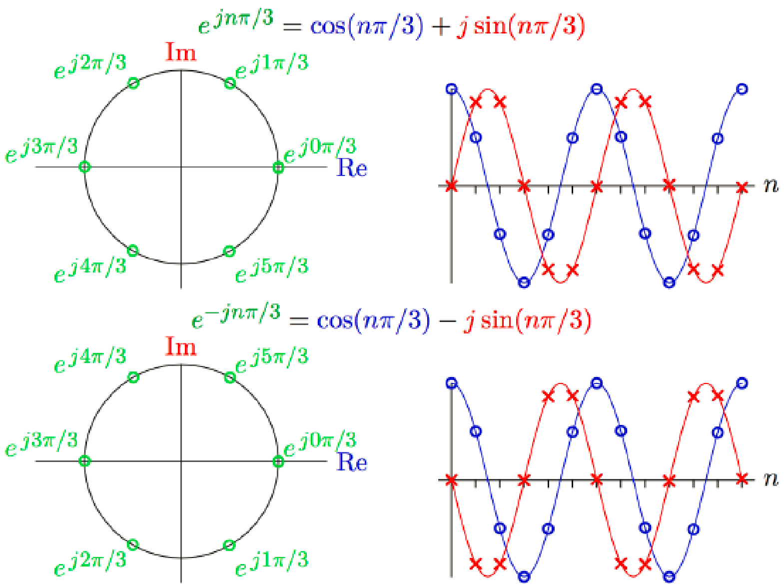
Unit-sample Response of **second-order system** can be written as **weighted sum of fundamental modes**.

Complex Poles

$\frac{Y}{X} = \frac{1}{1-R+R^2} = \frac{z^2}{z^2-z+1}$, and the corresponding $z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}j = e^{\pm\frac{\pi}{3}j}$.

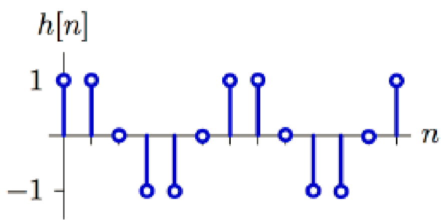
$\frac{Y}{X} = \frac{1}{1-e^{\frac{\pi}{3}j}R} \times \frac{1}{1-e^{-\frac{\pi}{3}j}R}$, so the **fundamental modes**:

- $e^{\frac{n\pi}{3}j} = cos(\frac{n\pi}{3}) + j \cdot sin(\frac{n\pi}{3})$
- $e^{-\frac{n\pi}{3}j} = cos(\frac{n\pi}{3}) - j \cdot sin(\frac{n\pi}{3})$



So $H = \frac{Y}{X} = \frac{1}{j\sqrt{3}} \cdot (\frac{e^{\frac{\pi}{3}j}}{1-e^{\frac{\pi}{3}j}R} - \frac{e^{-\frac{\pi}{3}j}}{1-e^{-\frac{\pi}{3}j}R})$.

So $h[n] = \frac{1}{j\sqrt{3}} \cdot (e^{\frac{(n+1)\pi}{3}j} - e^{-\frac{(n+1)\pi}{3}j}) = \frac{2}{\sqrt{3}} \cdot sin\frac{(n+1)\pi}{3}$.



The output of a "real" system has real values.