# VE216 Lecture 15

> Fourier Series

## Fourier Series

- analysis equation: $a_k = \frac{1}{T}\int_T x(t)e^{-j\omega_0 kt}dt = \frac{1}{T}\int_T x(t)e^{-j\frac{2\pi}{T}kt}dt$
- synthesis equation: $x(t) = x(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$

## Orthogonal Decompositions

> Integrate over period $T$, then we **sift** out the $k^{th}$ Fourier series component.

The sifting is a inner product:

$a_k = e^{j\frac{2\pi}{T}kt} \cdot x(t) = \frac{1}{T}\int_T x(t)e^{-j\frac{2\pi}{T}kt}dt$, or so to say $a(t) \cdot b(t) = \frac{1}{T}\int_T a^*(t)b(t)dt$.

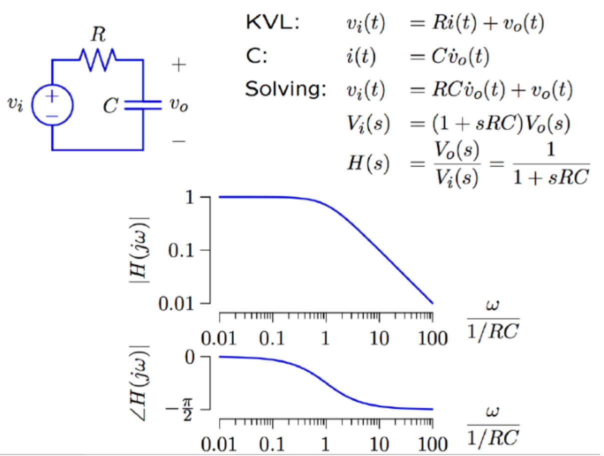
Then we see the inner product of $k^{th}$ and $m^{th}$ equal to 1 iff $k = m$.

## Filtering

It is trivial that we get the following input and output of a LTI system:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt} \to y(t) = \sum_{k=-\infty}^{\infty} a_k H(j\frac{2\pi}{T}k)e^{j\frac{2\pi}{T}kt}$$

## Low-Pass Filtering



Then we can see if the $\omega = k\omega_0$ under different frequency.