Lab1 prelab

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4.1

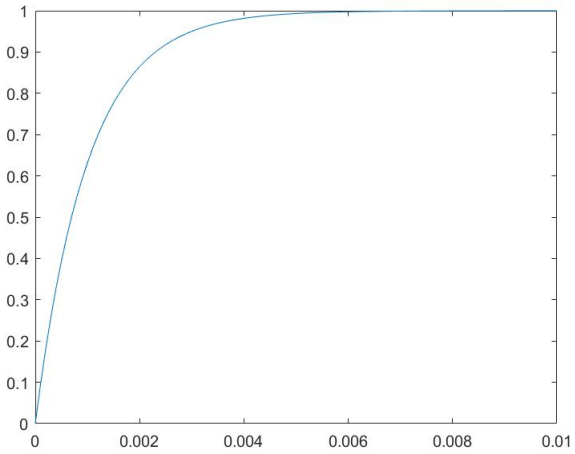
a) when $t \geq 0$

$$V_{in}(t) = RC\frac{dV_{out}(t)}{dt} + V_{out} = RC(\frac{1}{RC}e^{-\frac{1}{RC}}) + 1 - e^{-\frac{t}{RC}} = 1 = u(t)$$

when $t < 0$ $V_{out} = \frac{dV_{out}(t)}{dt} = 0$

$$V_{in}(t) = RC\frac{dV_{out}(t)}{dt} + V_{out} = 0 = u(t)$$

Therefore, $V_{in} = u(t)$.
b)



4.2

$$y_{step}(t) = u(t) * h(t) = (1 - e^{-\frac{t}{RC}})u(t)$$

$$h(t) = \delta(t) * h(t) = \frac{d}{dt}y_{step}(t) = (1 - e^{-\frac{t}{RC}})\delta(t) + (\frac{1}{RC}e^{-\frac{t}{RC}})u(t) = (\frac{1}{RC}e^{-\frac{t}{RC}})u(t)$$

4.3

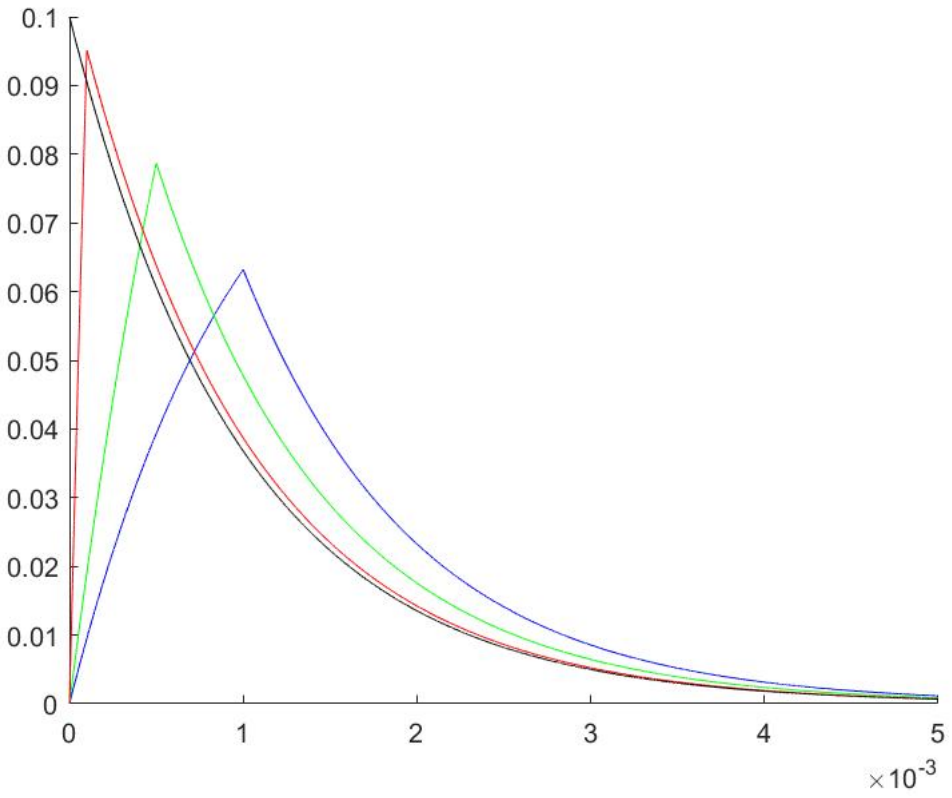
a)

$$y_{b,\Delta}(t) = \frac{b}{\Delta}(u(t) - u(t-\Delta)) * h(t) = \frac{b}{\Delta}(y_{step}(t) - y_{step}(t-\Delta))$$
$$= \frac{b}{\Delta}[(1 - e^{-\frac{t}{RC}})u(t) - (1 - e^{-\frac{t-\Delta}{RC}})u(t-\Delta)]$$

b) When $b = 1$

$$\lim_{\Delta\to 0} y_{b,\Delta}(t) = \lim_{\Delta\to 0} \frac{1}{\Delta} u(t)[-e^{-\frac{t}{RC}}(1 - e^{\frac{\Delta}{RC}})] = u(t)e^{-\frac{t}{RC}}[\lim_{\Delta\to 0} \frac{1}{\Delta}(e^{\frac{\Delta}{RC}} - 1)]$$
$$= u(t)e^{-\frac{t}{RC}} \frac{e^0}{RC} = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$

c)



4.4

Since $s(t) = (1 - e^{-\frac{t}{RC}})u(t)$

$$tu(t) = u(t) * u(t) \leftrightarrow (t + RC(e^{-\frac{t}{RC}} - 1))u(t)$$

$$\implies (t - 0.011)u(t - 0.011) \leftrightarrow (t - 0.011 + RC(e^{-\frac{t-0.011}{RC}} - 1))u(t - 0.011)$$

$(t-0.011)u(t-0.016) \leftrightarrow (t-0.016+RC(e^{-\frac{t-0.016}{RC}}-1))u(t-0.016)+0.005u(t-0.016)(1-e^{-\frac{t-0.016}{RC}})$

Therefore,

$$\begin{aligned} V_{out}(t) &= (1 - e^{-1000t})u(t) + (1 - e^{-1000t+10})u(t - 0.01) \\ &+ 200[(t - 0.012 + \frac{e^{-1000t+11}}{1000})u(t - 0.011) + (t - 0.017 \\ &+ \frac{e^{-1000t+16}}{1000})u(t - 0.016) - 0.005u(t - 0.016)(1 - e^{-1000t+16})] \end{aligned}$$

4.5

Since $Z_C = \frac{1}{j\omega C}$, $Z_R = R$

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_C + Z_R} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega CR}$$

4.6

f_c	$\lvert H\left(j2\pi f_c\right)\rvert$	$\angle \lvert H\left(j2\pi f_c\right)\rvert \deg$	$\tau_d ms$
50	0.9540	−17.4406	0.2422
200	0.6227	−51.4881	0.7151
500	0.3033	−72.3432	0.4019
1k	0.1572	−80.9569	0.2249
5k	0.0318	−88.1768	0.0490