Homework 1

VE216 - Introduction to Signal and Systems, Qiao Heng, Spring 2021

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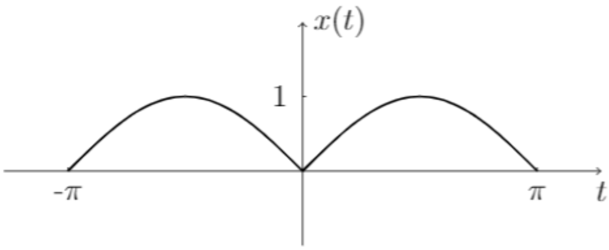
HW Notes:

- Problems where the number of points are followed by an exclamation are basic skill problems and will be graded without partial credit.
- $\boxed{\text{Box}}$ your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit. For full credit, ~~cross out~~ any incorrect intermediate step.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

Problems:

1. Consider the sinusoidal signal illustrated below.



(a) [5!] Find the mathematical representation for this signal, where x(t) is 0 outside the interval $[-\pi, \pi]$;
(b) [5!] Carefully sketch and find a mathematical expression for the output signal of an integrator system, i.e., $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, where $x(t)$ is under the interval $[-\pi, \pi]$.

Answer:

(a) $\boxed{x(t) = rect(\frac{t}{2\pi}) \times |sin(x)|}$

(b) $rect(\frac{t}{2\pi}) = u(t+\pi)u(-ket-\pi)$

$$f(x) = u(t+\pi)u(t-\pi)|sin(x)|$$

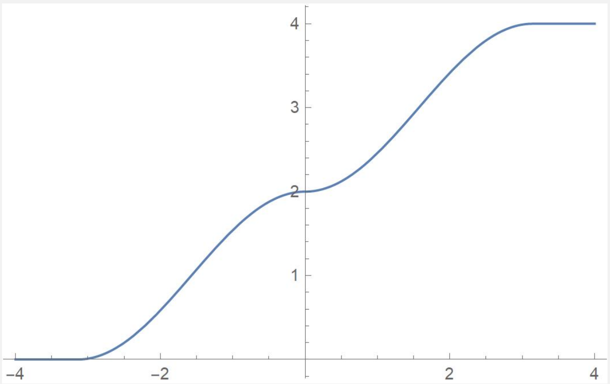
$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau =$$

for $t < -\pi$, $y(t) = 0$

for $-\pi < t < 0$, $x(t) = -sin(\tau)$, so $\boxed{y(t) = \int_{-\pi}^{t} -sin(\tau) = 1 + cos(t)}$

for $0 < t < \pi$, $x(t) = sin(\tau)$, so $\boxed{y(t) = \int_{0}^{t} sin(\tau) = 3 - cos(t)}$

for $t > \pi$, $y(t) = 4$



2. [10!] Calculate the average value, power and energy of signal f(n) =

$$\begin{cases} e^{-2t}, & t > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Is it an energy signal, power signal, or neither?

Answer:

$$\text{Average: } A \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)dt = \lim_{T\to\infty} \frac{1-e^{-2T}}{4T} = \boxed{0}$$

$$\text{Energy: } E \triangleq \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{0}^{\infty} e^{-4t} dt = \boxed{\tfrac{1}{4}}$$

$$\text{Power Signal: } P \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1-e^{-4T}}{8T} = \boxed{0}$$

It is an $\boxed{\text{energy signal}}$.

3. Suppose $x_1(t)$ and $x_2(t)$ are two periodic signals with fundamental periods $T_1 > 0$ and $T_2 > 0$ respectively.
(a) [5!] Show that if T_1/T_2 is rational, then $x(t) = x_1(t) + x_2(t)$ is periodic.
(b) [5!] Show that if T_1/T_2 is rational, then $x(t) = x_1(t)x_2(t)$ is periodic and the least common multiple of T_1 and T_2 is a period of $x(t)$.
(c) [5!] Determine whether the following signals are periodic. If so, find a period. If not, specify the reason.

$$x_1(t) = \sin(\pi t/3)\cos(\pi t/7) + \sin(\pi t/5)\sin(\pi t/11)$$

$$x_2(t) = \sin(\sqrt{3}\pi t/3) + \sin(\pi t/7)$$

Answer:
(a) Since $\frac{T_1}{T_2}$ is rational, there must exist certain $T_3 = aT_1 = bT_2$, while a and b are integers

$$x_1(t) = x_1(t+T_1)\ \forall t \quad x_1(t) = x_1(t+T_1)\ \forall t$$

$$\rightarrow x_1(t) + x_2(t) = x_1(t+T_3) + x_2(t+T_3)\ \forall t$$

so $x_1 + x_2$ is periodic.
(b) Since $\frac{T_1}{T_2}$ is rational, there must exist certain $T_3 = aT_1 = bT_2$, while a and b are integers

$$x_1(t) = x_1(t+T_1)\ \forall t \quad x_1(t) = x_1(t+T_1)\ \forall t$$

$$\rightarrow x_1(t) \times x_2(t) = x_1(t+T_3) \times x_2(t+T_3)\ \forall t$$

so $x_1 \times x_2$ is periodic.
Also, with the condition that $aT_1 = bT_2$, T must be the least common multiple of T_1 and T_2 (c) As a, b have determined, 1 is periodic and the smallest period is lcm(T_1, T_2, T_3, T_4)=lcm(6, 14, 10, 22)=$\boxed{2310}$.
2 is $\boxed{\text{not periodic}}$ since the divide product of period of $sin(\sqrt{3}\pi t/3)$ and $sin\pi t/7$ is not rational

4. [20!]Indicate whether the following systems are Memoryless, Time Invariant, Linear, Causal, Stable. Justify your answers.
(a) $y(t) = x(t-3) + x(3-t)$
(b) $y(t) = \cos(4x(t))$
(c) $y(t) = \int_{-\infty}^{t/3} x(\tau)d\tau$

Answer:
(a) $\boxed{\text{Not memoryless, not time invariant, linear, not causal, stable}}$

Memoryless: The system is NOT memoryless because the output at time t depends on input values at times other than t

Time invariance: This system is time invariant. We let y(t) be the output corresponding to the input x(t) and $x_a(t) = x(t-a)$. Then

$$\begin{aligned} y_a(t) &= x_a(t-3) + x_a(3-t) \\ &= x(t-a-3) + x(3-t-a) \\ &\neq x((t-a)-3) + x(3-(t-a)) = y(t-t_0) \end{aligned}$$

Linearity: The system is linear because

$$\begin{aligned} y_1(t) =& x_1(t-3) + x_1(3-t) \\ y_2(t) =& x_2(t-3) + x_2(3-t) \text{ and} \\ x(t) =& \alpha x_1(t) + \beta x_2(t) \end{aligned}$$

then the output y(t) corresponding to the input x(t) is

$$\begin{aligned} y(t) &= x(t-3) + x(3-t) \\ &= (\alpha x_1 + \beta x_2)(t-3) + (\alpha x_1 + \beta x_2)(3-t) \\ &= \alpha(x_1(t-3) + x_1(3-t)) + \beta(x_2(t-3) + x_2(3-t)) \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Causality: The system is not causal, for example: y(0)=y(-3)+y(3), so it depends on future input

Stability: $y(t) = x(t-3) + x(3-t) < x_{Max} + x_{Max} = 2x_{Max}$, let $y_{Max} = 2x_{Max} + 1$, $|y(t)| < y_{Max}$

Answer:
(b) $\boxed{\text{Memoryless, time invariant, not linear, causal, stable}}$
Memoryless: The system is memoryless because the output at time t depends on input values at time t

Time invariance: This system is time invariant. We let y(t) be the output corresponding to the input x(t) and $x_a(t) = x(t-a)$, let $x(t) = \delta(t)$. Then

$$\begin{aligned} y_a(t) &= cos(4x_a(t)) \\ &= cos(4x(t-a)) \\ &= y(t-a) \end{aligned}$$

Linearity: The system is not linear because

$$\begin{aligned} y_1(t) =& cos(4x_1(t)) \\ y_2(t) =& cos(4x_2(t)) \text{ and} \\ x(t) =& \alpha x_1(t) + \beta x_2(t) \end{aligned}$$

then the output y(t) corresponding to the input x(t) is

$$\begin{aligned} y(t) &= cos(4x(t)) \\ &= cos(4()\alpha x_1(t) + \beta x_2(t))) \\ &\neq \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Causality: The system is causal because the output only depend on the current t
Stability: $|y(t)| < 1$, the output is bounded

Answer:
(c) $\boxed{\text{Not memoryless, not time invariant, linear, not causal, not stable}}$
Memoryless: The system depends on t in past, future or current depends on $x(\tau)$. So we cannot tell whether it is memoryless or not.

Time invariance: This system is NOT time invariant. We let y(t) be the output corresponding to the input x(t) and $x_a(t) = x(t-a)$. Then

$$\begin{aligned} y_a(t) &= \int_{-\infty}^{t/3} x(\tau - \tau_0) d\tau \\ &= \int_{-\infty}^{t/3-\tau_0} x(\tau) d\tau \\ &\neq \int_{-\infty}^{(t-\tau_0)/3} x(\tau) d\tau = y(t-a) Legible \end{aligned}$$

Linearity: The system is linear because

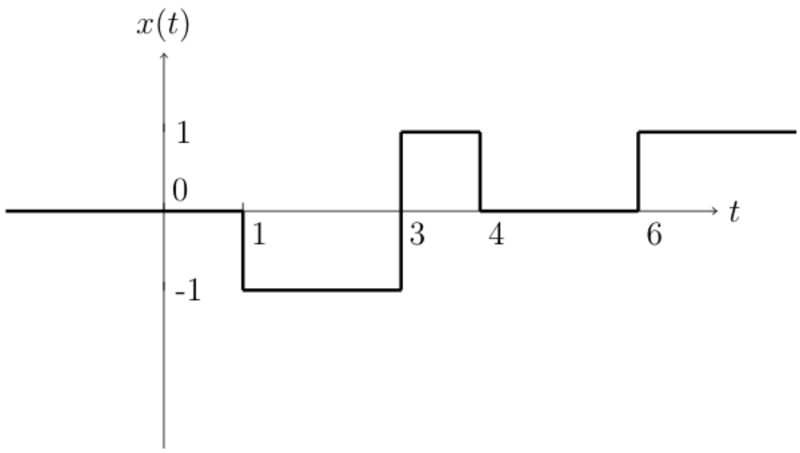
$$\begin{aligned} y_1(t) &= \int_{-\infty}^{t/3} x_1(\tau) d\tau \\ y_2(t) &= x \int_{-\infty}^{t/3} x_2(\tau) d\tau \text{ and} \\ x(t) &= \alpha x_1(t) + \beta x_2(t) \end{aligned}$$

then the output y(t) corresponding to the input x(t) is

$$\begin{aligned} y(t) &= \int_{-\infty}^{t/3} (\alpha x_1 + \beta x_2)(\tau) d\tau \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

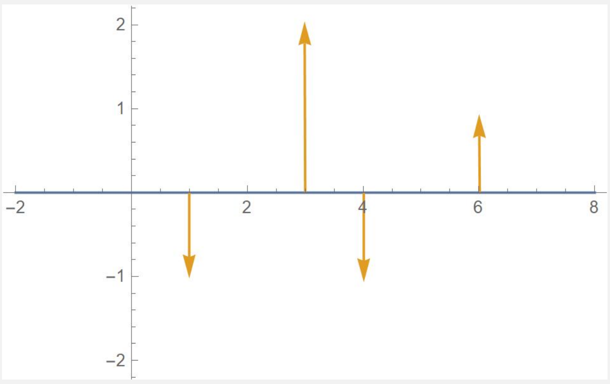
Causality: The system depends on t in past, future or current depends on $x(\tau)$. So we cannot tell whether it is causal or not.
Stability: $y(t) = x(t-3) + x(3-t) < x_{Max} + x_{Max} = 2x_{Max}$, let $y_{Max} = 2x_{Max} + 1$, $|y(t)| < y_{Max}$

5. Consider the signal illustrated below.

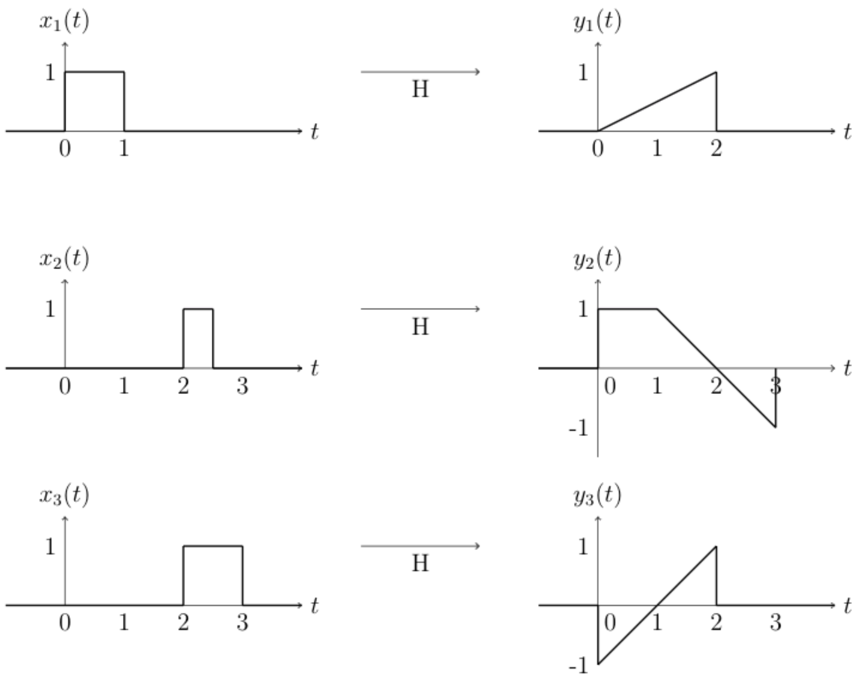


(a) [5!] Express the signal $x(t)$ using a sum of step functions.
(b) [5!] Find the derivative of the signal and carefully sketch it.

Answer:
(a) $\boxed{\text{x(t)=-u(t-1)+2u(t-3)-u(t-4)+u(t-6)}}$
(b) $\boxed{x'(t) = -\delta(t-1) + 2\delta(t-3) - \delta(t-4) + \delta(t-6)}$



6. A system H has following input-output pairs. Answer the following question, and justify your answers

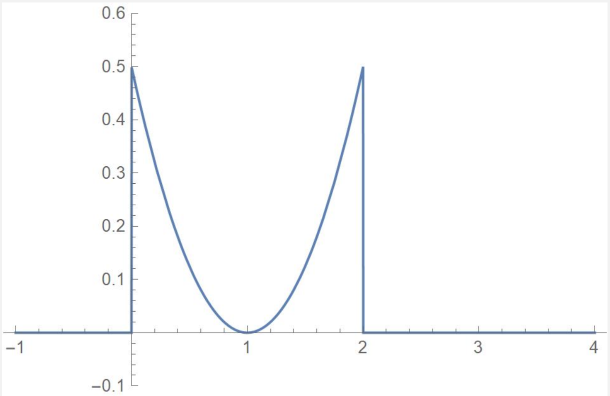


(a) [5!] Could this system be causal?
(b) [5!] Could this system be time invariant?

Answer:
(a) No
In figure 2 and 3, the 0-2 both zero. But the output is not the same.
(b) No
In figure 1 and 3, the input change 2 unit, but the output did not only change 2 unit.

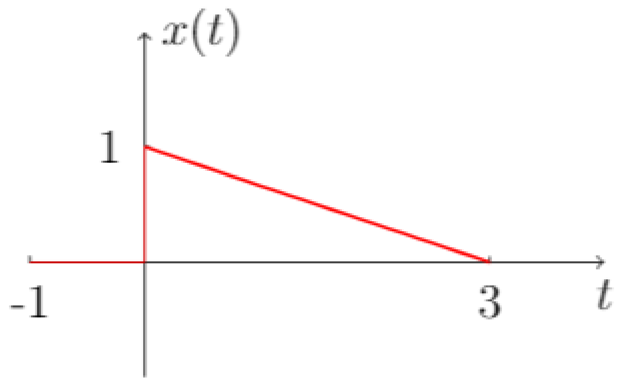
7. Consider the signal $s(t) = (\frac{t-1}{2})^2 rect(\frac{t-1}{2})$
(a) [3!] Make a sketch of $s(t)$
(b) [7!] Evaluate $\int_{-\infty}^{\infty} s(t)x(t)dt$, where $x(t) = \delta(2t-1) + \delta(t-2) - \delta(3t-5)$
(You do not need to give the numeric value. It is OK to leave the expression with $s(t)$)

Answer:
(a)

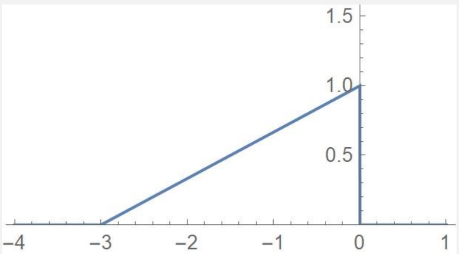


(b) $\boxed{\frac{1}{2}s(\frac{1}{2})+s(2)-\frac{1}{3}s(\frac{5}{3})}$

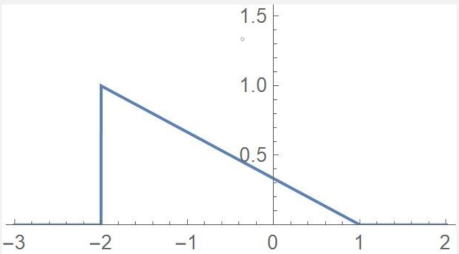
8. For $x(t)$ indicated in the following figure, sketch the following:
(a) [2!] $x(-t)$
(b) [3!] $x(t+2)$
(c) [5!] $x(2t+2)$
(d) [5!] $x(1-3t)$



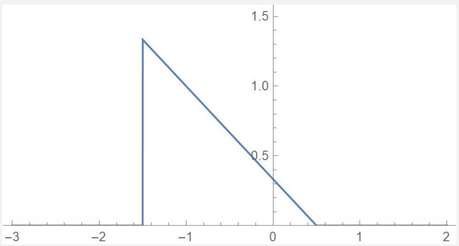
Answer:
(a) x(-t)



(b) x(t+2)



(c) x(2t+2)



(d) x(1-3t)