Homework 6

VE216 - Introduction to Signal and Systems, Qiao Heng, Spring 2021

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HW Notes:

- Problems where the number of points are followed by an exclamation are basic skill problems and will be graded without partial credit.
- $\boxed{\text{Box}}$ your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit. For full credit, ~~cross out~~ any incorrect intermediate step.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

Problems:

1. [5] Is this system stable? Explain. (Note: the system is causal.)

$$2 \cdot 10^6 y(t) + 10^5 \frac{d}{dt} y(t) + 60 \frac{d^2}{dt^2} y(t) + \frac{d^3}{dt^3} y(t) = 8 \cdot 10^6 x(t) - 10^4 \frac{d}{dt} x(t)$$

Answer:

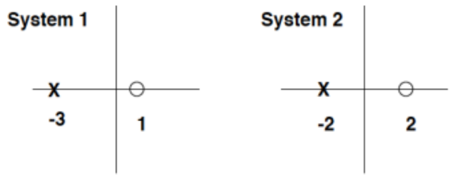
$$y(t)(2 \times 10^6 + 10^5 s + 60s^2 + s^3) = x(t)(8 \times 10^6 - 10^4 s)$$

$$\frac{y(t)}{x(t)} = \frac{8 \times 10^6 - 10^4 s}{2 \times 10^6 + 10^5 s + 60s^2 + s^3}$$

Let $2 \times 10^6 + 10^5 s + 60s^2 + s^3 = 0$ we can get $s_1 = 20.16$ $s_{2,3} = -19.9 \pm 314.3j$

The poles are all in the LHS and the ROC contains the $j\omega$ axis, so it is $\boxed{\text{stable}}$

2. [10] A unit step signal is applied to a system consisting of two LTI system connected in parallel. The pole-zero plots of each of the system are shown below. Determine the output signal. Assume that each of the system has unit gain at DC.



Hint:first find the Laplace transform $Y(s)$ of the output signal using the convolution and linearity properties of the Laplace transform, Then take the inverse Laplace transform to get $y(t)$ using PFE. The "unit gain at DC" specifies $H_1(0)$ and $H_2(0)$, which you can use to determine the scaling factor.

Answer:

$$H_1(s) = G_1 \times \frac{s-1}{s+3} \qquad Y_2(s) = G_2 \times \frac{s-2}{s+2}$$

$$G = \frac{b_m}{a_0} \quad 1 = G_1 \times -\frac{1}{3} \quad 1 = G_2 \times -1$$

$$Y(s) = X(s)(H_1(s) + H_2(s)) = X(s)(\frac{3-3s}{3+s} + \frac{2-s}{2+s})$$

$$= -\frac{1}{s}\frac{4(s^2+s-3)}{(s+2)(s+3)} = \frac{2}{s} - \frac{4}{s+3} - \frac{2}{s+2}$$

$Real\{s\} > 0$ and $y(t) = 2u(t) - 4e^{-3t}u(t) - 2e^{-2t}u(t)$

3. [20] Consider an LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.
(a) Determinse the Laplace transform of $x(t)$ and $h(t)$.
(b) Using the convolution property, determine the Laplace transform $Y(s)$ of the output $y(t)$.
(c) From the Laplace transform of $y(t)$ as obtained in part(b), determine $y(t)$.
(d) Verify your result in part (c) by explicitly convolving $x(t)$ and $h(t)$.

Answer:
(a) $x(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1} \quad h(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}$
(b) $Y(s) = X(s) \cdot H(s) = \frac{1}{(s+2)(s+1)}$
(c) $\frac{1}{(s+2)(s+3)} = \frac{1}{s+1} - \frac{1}{s+2}$
$\Rightarrow y(t) = e^{-t}u(t) + e^{-2t}u(t)$
(d) $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau = e^{-t}u(t) + e^{-2t}u(t)$

4. [20] The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}$$

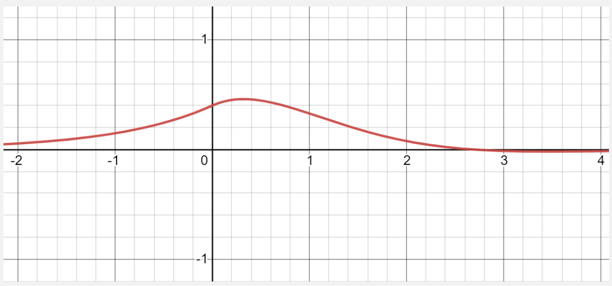
Determine and sketch the response $y(t)$ when the input is

$$e^{-|t|}, \quad -\infty < t < \infty$$

Answer:
$x(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{2}{1-s^2}$, then $Y(s) = X(s) \cdot H(s) = \frac{1}{(s^2+2s+2)(1-s)}$

$$Y(s) = \frac{2}{5}(\frac{s+1}{(s+1)^2+1}) + \frac{2}{(s+1)^2+1} - \frac{1}{s-1}$$

$$y(t) = \frac{2}{5}coste^{-t}u(t) + \frac{4}{5}sinte^{-t}u(t) + \frac{2}{5}e^{t}u(-t)$$



5. [45] In this problem, we consider the construction of various type of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$, and system function

$$H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}$$

To derive the direct-diagram representation of S, we first consider a causal LTI system S1 that has the same input $x(t)$ as S, but whose system function is

$$H(s) = \frac{1}{s^2 + 3s + 2}$$

With the output of S_1 denoted by $y_1(t)$, the direct-form diagram representation of S_1 is shown in Figure below. The signals $e(t)$ and $f(t)$ indicates in the figure represent respective inputs into the two integrators.

(a) Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $\frac{dy_1 t}{dt}$, and $\frac{d^2 y_1(t)}{dt^2}$.

(b) How is $\frac{dy_1 t}{dt}$ related to $f(t)$.

(c) How is $\frac{d^2 y_1(t)}{dt^2}$ related to $e(t)$.

(d) Express $y(t)$ as a linear combination of $e(t)$, $f(t)$, $y_1(t)$.

(e) Use the result from the previous part to extend the direct-form block diagram representation of S_1 and create a block diagram representation of S.

(f) Observing that

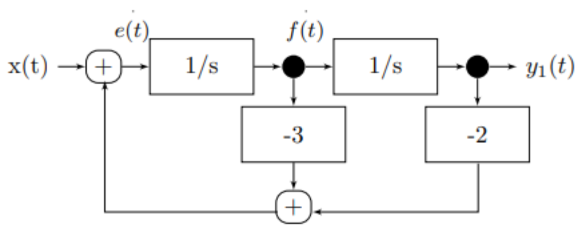
$$H(s) = \left(\frac{2(s-1)}{s+2}\right)\left(\frac{s+3}{s+1}\right)$$

draw a block diagram representation for S as a cascade combination of two subsystems.

(g) Observing that

$$H(S) = 2 + \frac{6}{s+2} - \frac{8}{s+1}$$

draw a block-diagram representation for S as parallel combination of three subsystems.



Answer:

(a)

$$y(t) = 2\frac{d^2y_1t}{dt^2} + 4\frac{dy_1(t)}{dt} - 6y_1(t)$$

(b) $\frac{dy_1t}{dt} = f(t)$

(c) $\frac{d^2y_1t}{dt^2} = e(t)$

(d) $y(t) = 2e(t) + 4f(t) - 6y_1(t)$