VE216 Lecture 19

Fourier Representations Relations

DT Fourier Series

DT Fourier transform

$$a_k = a_{k+N} = \frac{1}{N} \sum_{n = < N >} x[n] e^{-j\frac{2\pi}{N}kn} \qquad X(e^{j\Omega}) = \sum_{n = -\infty}^{\infty} x[n] e^{-j\Omega n}$$

$$x[n] = x[n+N] = \sum_{k=< N>} a_k e^{j\frac{2\pi}{N}kn} \qquad x[n] = \frac{1}{2\pi} \int_{<2\pi>} X(e^{j\Omega})e^{j\Omega n} d\Omega$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

CT Fourier Series

CT Fourier transform

$$a_k = \frac{1}{T} \int_T x(t) e^{-j\frac{2\pi}{T}kt} dt$$

$$\begin{split} a_k &= \frac{1}{T} \int_T x(t) e^{-j\frac{2\pi}{T}kt} dt & X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ x(t) &= x(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt} & x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \end{split}$$

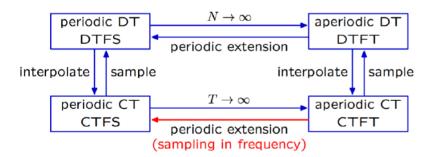
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega$$

Relation between Fourier Transform and Series

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

Relation among Fourier Representations



Relation between CT and DT Transforms

$$x_p(t) = x(t)p(t)$$
 where $p(t) = \sum_{k=-\infty}^{\infty} \delta(t+kT)$

$$X(e^{j\Omega})=\sum_n x[n]e^{-j\Omega n}$$
 and $X_p(j\omega)=\int x_p(t)e^{-j\omega t}dt=\sum_n x[n]e^{-j\omega nT}$

$$\Omega = \omega T$$