VE 216 Summer 2020
Lab 2: AM Radio

(Note: Borrowed from UMich EECS 216)

1 Introduction

1.1 A Bit of History

The transmission and storage of information are vital functions of a modern society. The algorithmic and device technologies supporting these functions continue to advance at a rapid pace. Use of the internet and wireless services (e.g., cellphones) is now pervasive, and other forms of communications, such as digital radio and high definition television, are growing in popularity.

This laboratory is aimed at the following question - how can we transmit information wirelessly? To address this question, we need to understand something about the propagation of electromagnetic waves[1] and how to build a transmitter and receiver that will have sufficient power and sensitivity, respectively. Given the prevalence of wireless devices today, you may find it very surprising that the invention of the radio was a nontrivial matter. Well, it was not easy!

In the beginning, one used wires. The age of electronic communication arguably began in 1830 with the invention of the telegraph[2]. In 1844, news was conveyed for the first time using the telegraph, and the telegraph remained a popular means of sending information until the middle of the twentieth century. In the 1870s, voice transmission began with the introduction of the telephone, which was independently invented by Elisha Gray and Alexander Graham Bell. Both the telegraph and telephone were revolutionary inventions, but both require that the transmitter and receiver be physically connected via a wire.

It was not until the late 1800s that the theory of electromagnetics was developed through the efforts of Michael Faraday, James Clerk Maxwell and others. In 1887, Heinrich Hertz confirmed the predictions of Maxwell's theory by transmitting and detecting electromagnetic waves (i.e., radio waves). Although the apparatus used by Hertz was quite crude by today's standards, his work launched the modern era of wireless communications. It is a quirky bit of history that Hertz failed to fully appreciate the practical significance of his own scientific work! He stated that "It's of no use whatsoever. . . this is just an experiment that proves Maestro Maxwell was right – we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there." He went on to say "I do not think that the wireless waves I have discovered will have any practical applications." Obviously Hertz was a far better scientist than businessman!

In Hertz's experiment, batteries were used to charge up a large capacitor. After charging, the capacitor was connected to pair of wires whose opposite ends were placed in very close proximity, with only a small air gap between them. The voltage on the capacitor produced a strong electric field in the small air gap region, which in turn caused the air molecules inside the gap to ionize. In their ionized state, these molecules conduct electricity, and a spark of short duration was produced inside the gap when the capacitor discharged. The spark current was significant, since the large capacitor stored a substantial amount of charge. This discharge phenomenon is similar to that of a lightning bolt. The spark produces an electromagnetic (EM) wave that propagates away from the transmitter. As observed by Hertz, the presence of this wave can be detected at a distant point, since this wave will induce a small burst of current in a distant loop of wire. By varying

[1]We take as a given from everyday experience that sound waves simply don't travel far enough without dissipating, and that multiple people talking in a room leads to interference!

[2]The original digital communication system? Dots and dashes!

the time delay between successive transmitted bursts, Hertz's system would be used by others to transmit information wirelessly from one point to another.

In 1894 an improved detection device, known as a cohere, was developed by Oliver Lodge, Alexander Popov and Edouard Branly. This device consisted of a glass tube filled with metal filings. With a wire inserted into each end of the tube, the device functioned as a resistor. The value of the resistance, however, was found to change (in particular, decrease) in the presence of an electromagnetic wave. A circuit was built using this device that produced an increase in current in the presence of a received EM burst. The cohere never worked particularly well, but Guglielmo Marconi used a spark gap transmitter, a telegraph key and a cohere to demonstrate transatlantic telegraphy in 1901. The short bursts of EM waves produced by a spark gap transmitter were never suitable for voice transmission.

The spark gap transmission scheme turns out to suffer from a number of serious drawbacks. First, it is not suitable for voice transmission, only telegraphy. Second, it is difficult to efficiently couple the spark energy into an antenna for transmission. Third, the pulsed EM energy does not propagate well over long distances. Fourth, the pulse energy is distributed over a very wide spectral band, and as a consequence it impossible for the receiver to do station selection when multiple transmitters are operating (since all of the signals will occupy the same spectral band). Near the very end of the 19^{th} century, Nikola Tesla (his bust is displayed in the EECS atrium by the elevators) developed a device called the RF alternator that could be used together with tuned LC circuits to generate a quasi-sinusoidal carrier, thereby reducing the spectral bandwidth of the signal and permitting more efficient coupling to an antenna. The availability of high quality sinusoidal sources (e.g., oscillators), however, would have to await the development of vacuum tubes.

The early receivers also had their problems. They lacked sensitivity and consequently suffered from noise. A high quality nonlinear component was not yet available to permit efficient envelope detection, and the receivers had no mechanism for station selection even if multiple transmitters had been able to operate on different frequency bands. The first experimental wireless broadcast of voice (and music too) was demonstrated by Reginald Fessenden on December 24, 1906. His transmitter used a spark gap unit driven by an RF alternator that produced a quasi-sinusoidal carrier. The carrier was then amplitude modulated by placing a microphone in series with the antenna. The receiver contained a resonant circuit with a peak response tuned to the carrier frequency followed by a crude nonlinear element (a barretter detector), which was later replaced by an improved nonlinear element (an electrolytic detector) to perform envelope detection.

In 1904, J. A. Fleming used the Edison effect (i.e., the emission of electrons by a heated element as in the light bulb) to demonstrate a vacuum tube rectifier, that is, a diode. In 1906, Lee DeForest extended the vacuum tube design by adding a grid (in addition to the anode and cathode), to produce a vacuum tube triode (the forerunner of the solid state transistor). Oscillators for the transmitter could now be built using vacuum tube triodes. In 1906, G. W. Pickard patented a solid-state diode device based on a crystal. By 1912, reliable vacuum tubes started to become available and by 1915, vacuum tube transmitters started to become available. With the advent of vacuum tubes, the front-ends of the receivers were initially based on regenerative amplifier principles, leading to bandpass amplifiers that had very high gains and tunable center frequencies, and thus they could be used to select a station at a specific carrier frequency for reception. The operation of the regenerative amplifiers is based on feedback. The regenerative amplifier principle was apparently invented by Lee de Forest (and perhaps earlier by Robert Goddard and John Boltho), but demonstrated by E. H. Armstrong in 1914 while he was an undergraduate student at Columbia University. Between 1920 and 1923, regenerative receivers dominated the radio market.

The regenerative receivers, however, were expensive and difficult to build. Another approach was needed. In a direct conversion receiver, also known as a *heterodyne receiver*, the received signal is mixed, i.e., multiplied, with a local oscillator (LO) to shift the signal spectrum down to a lower frequency. Mathematically, this operation can be understood in terms of the following Fourier transform property

$$s(t)\cos(\omega_{LO}t) \leftrightarrow \frac{1}{2}S(j(\omega-\omega_{LO})) + \frac{1}{2}S(j(\omega+\omega_{LO})). \tag{1.1.1}$$

Highly frequency selective (i.e., narrow bandwidth) bandpass filters become easier to build as the center frequency of the passband decreases. Thus the heterodyne concept allows receivers to be built that have non-regenerative front-ends and that are selective enough to filter out unwanted signals. Both Fessenden and Lucien Levy developed the heterodyne concept. The idea of using a bandpass filter at a *fixed* center frequency, known as the intermediate frequency or IF for short, and choosing the LO frequency to shift the received signal into the IF band was invented and developed by Edwin Armstrong in 1920. Most radios today are of this type, which is known as a *superheterodyne* receiver. In 1960, the Sony corporation produced the first commercially available transistor-based superheterodyne AM radios, and they quickly became a popular household item. Although radio and communication systems are rapidly moving into the digital era, the heterodyning and filtering principles used in analog AM radios are also widely used in the front-ends of modern digital receivers.

1.2 Goals

In this laboratory you will construct and test your own superherodyne AM receiver, which operates on the basis of the same principles used in any radio in your car or home. You will use your radio to listen to both a local commercial AM station broadcast at a carrier frequency of 1600 KHz and to an AM signal transmitted in the laboratory. You will also measure the response of your circuit, at a variety of test points, to a simple AM signal produced by a function generator. The goals of this lab are:

- Learn about resonance phenomena and simple RLC bandpass filters.
- Learn a bit about antennas.
- Learn basic superheterodyne receiver operating principles, since theses principles play a critical role in many radio and signal processing systems.
- Use the frequency domain concepts learned in your VE 216 lectures to analyze the operation of a superheterodyne AM radio receiver. Learn about mixing and its effect on the signal spectrum. Observe the demodulation of an AM signal using an envelope detector.
- Construct a fully operational superheterodyne AM radio and demonstrate that it operates as predicted by theory.
- Gain an appreciation of the fact that the mathematical tools you are learning in VE 216 can be used to design and build interesting and useful systems.

2 Background Information on Superheterodyne AM Radio Operation

Fundamentally, AM modulation and demodulation are based on the Fourier transform modulation property, Eq. (1.1.1). The actual translation of this mathematical fact into a practical electrical system of a functioning radio is discussed here. The development is rather long and covers the design of a radio from the antenna all the way to the last stage of demodulation. Don't despair, however, because the Pre-Lab assignment is quite short!

In Figure 2.1.1, you will find a block diagram of the superheterodyne AM radio that you will be building and testing in this lab.

2.1 Transmitted Signal

The transmitted AM (amplitude modulated) signal is of the form $x(t) = (A + bs(t))\cos(\omega_c t + \phi)$. In Lab 2, you observed signals $x(t)$ of this form with $s(t)$ a sinusoid or a triangular wave. The carrier is specified by $\cos(\omega_c t + \phi)$, $f_c = \frac{\omega_c}{2\pi}$ is the carrier frequency in Hz, $s(t)$ is the "information or message" that is being sent, e.g., voice, data, or music, and A and b are constants. The constants A and b are chosen to ensure the condition $A + bs(t) \geq 0$, which, from Lab 2, you know to be important for envelope detection. The modulation depth, specified as a percentage, is defined as

$$\frac{\max(bs(t)) - \min(bs(t))}{2A} \times 100\%, \tag{2.1.1}$$

as in Lab 2.

In commercial broadcast AM, the US Federal Communications Commission (FCC) specifies that the carrier frequency must be one of the following 117 values

$$f_c = 540\,KHz \;+\; (i-1)\times 10\text{ KHz},\quad \text{i} = 1,2,...,117 \tag{2.1.2}$$

and thus spans a range from 540 KHz to 1700 KHz in increments of 10 KHz. The carrier frequencies given by this formula correspond to the frequencies on your radio dial (AM).

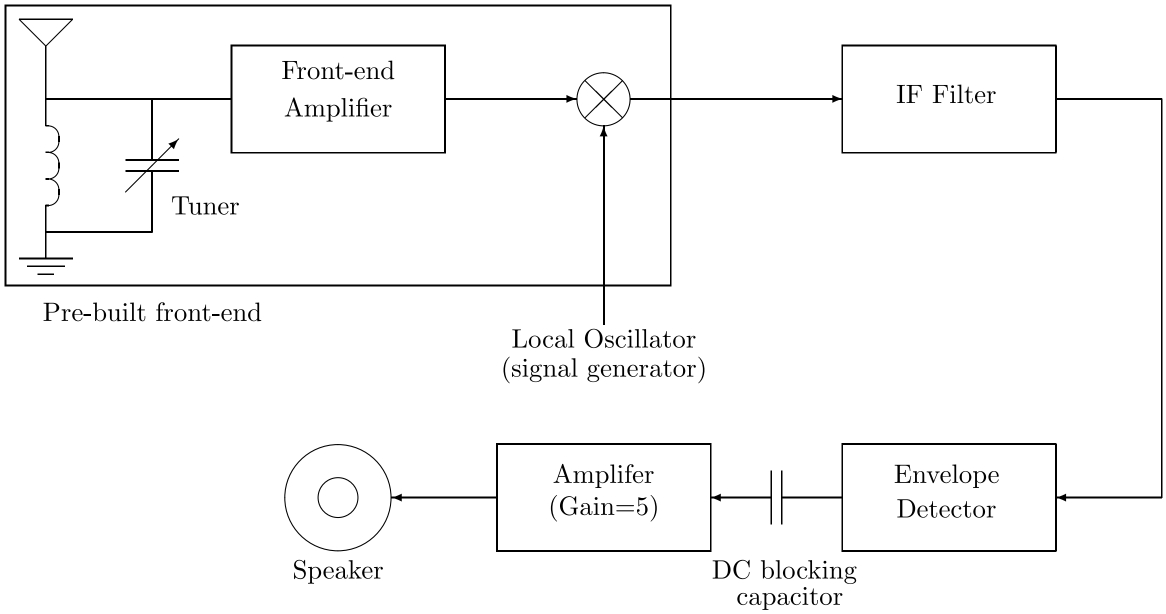


Figure 2.1.1: AM Superheterodyne Radio Block Diagram

The information signal itself, $s(t)$, is a baseband signal with a bandwidth of 5 KHz, i.e., $|S(j\omega)| \approx 0$ for $\omega > 2\pi \cdot 5000$ rad/s, where $S(j\omega)$ is the Fourier transform of a long segment of the signal (music, data or voice), $s(t)$. Thus the information signal contains little power at frequencies beyond 5 KHz. Younger adults can hear frequencies up to nearly 20 KHz, and thus commercial AM broadcasts do not transmit music with high fidelity, even in the absence of noise. This is not a limitation imposed by the method of amplitude modulation per se, but rather a consequence of the narrow bandwidth assigned by the FCC when broadcast AM radio was created. We could change this at any time if we were willing to absorb the huge cost of replacing the thousands of transmitters and millions of receivers in existence today.

2.2 Pre-built Front-end: Antenna & Tuned RLC Circuit

The "front-end" of the radio you will use in the lab has been pre-built and packaged for you. It consists of a tuned RLC circuit, a field effect transistor (FET) pre-amplifier and a mixer. The tuned RLC circuit, in addition to providing some filtering, also serves as an antenna. The antenna/tuned RLC circuit can be modeled by the circuit shown in Fig. 2.2.1 below.

As will be described later, the inductor value is fixed at approximately 960 μH, the antenna resistance R_{ant} is frequency dependent, increasing with frequency, and C is a user-controllable variable capacitor. It is a straightforward exercise to compute $\frac{V_{out}(j\omega)}{V_{ant}(j\omega)}$ as a function of frequency ω. It will be assumed that the output of the circuit is connected to a very high impedance input (e.g., the input of a field effect transistor (FET)), and thus is essentially open-circuited. The quantity $20\log\left|\frac{V_{out}(j\omega)}{V_{ant}(j\omega)}\right|$ dB has been computed and is plotted in Fig. 2.2.2 for two different RLC combinations, i.e., $R_{ant} = 200\ \Omega$, $L = 960\ \mu$H, $C = 15$ pF and $R_{ant} = 50\ \Omega$, $L = 960\ \mu$H, $C = 75$ pF. Note that this circuit acts as a bandpass filter whose center frequency

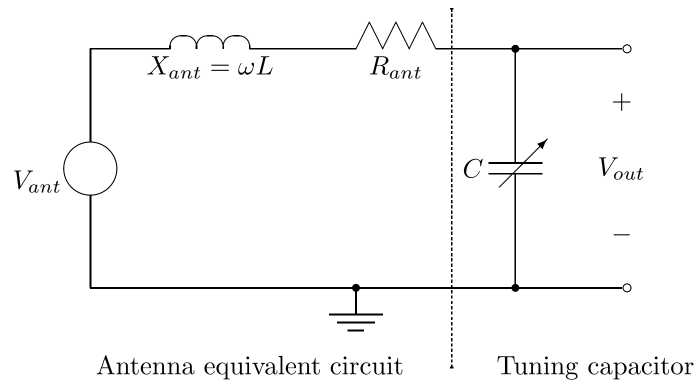


Figure 2.2.1: Antenna/Front-end Tuned RLC Circuit (preamplifier and mixer not shown)

is determined by the value of the capacitor, which is tunable by the radio listener. The capacitor value is chosen by the listener so that the center frequency of the bandpass filter corresponds to the carrier frequency of the desired AM station.

2.2.1 Antenna (or How to Efficiently Receive a Signal)

An antenna is a device that provides a means of transmitting or receiving radio waves, i.e., electromagnetic signals that propogate through free-space. Antennas are common components of almost every communication device, from radios to cellphones to laptops. To completely understand the operation of antennas requires a knowledge of electromagnetic theory that most of you have not yet acquired. You will learn this material in VE 230 and VE 330. Fortunately, however, a knowledge of some elementary physics VP 240) and circuit theory VE 215) suffices to understand the basic ideas. For those of you who already have taken VE 230/330 and wish to learn more about antennas, a good introductory book is [1].

Antennas can take many different forms depending on the application. In your radio, the receiving antenna is a coil of wire wound around a ferrite core, i.e., an electrically non-conducting material with special magnetic properties. Such a coil depicted in Fig. 2.2.3(a) is known as a “loopstick” and is commonly used for the antenna in AM radios.

The carrier wavelength, λ_c, is given by $\lambda_c = c/f_c$, where c is the vacuum speed of light, 3x10^8 m/s, and f_c is the carrier frequency in Hz, which for commercial AM lies between 540 KHz and 1700 KHz. Thus the carrier wavelength is on the order of a few hundred meters. The electrical properties of an antenna can be modeled by a Thevenin equivalent circuit consisting of a voltage source, V_{ant}, in series with an impedance, $Z_{ant} = R_{ant} + jX_{ant}$ as shown in Fig. 2.2.4.

Z$_{ant}$ is the same whether the antenna is transmitting or receiving a signal. V_{ant}, on the other hand, is only non-zero when the antenna is receiving a signal. Time-varying electric and magnetic fields are associated with the transmission of radio waves. According to Faraday’s Law of electromagnetics, a time-varying magnetic field radiated from a distant transmitter will induce an (open-loop) voltage drop, V_{ant}, across the ends of the coil in a loopstick receiving antenna (in particular, the voltage induced across the ends of a loop of wire equals the rate of change of flux through that loop). The voltage drop will be proportional to the product of the square-root of the power of the transmitted radio wave measured at the receiving antenna, the cross-sectional area of the coil, the number of turns in the coil and the relative permeability of the ferrite core.

R_{ant} is the sum of three resistive terms (1) the radiation resistance R_r of the antenna, (2) the resistance R_{coil} of the coil wire and (3) the magnetic losses (due to hysteresis) in the core R_{core},

$$R_{ant} = R_r + R_{coil} + R_{core}. \tag{2.2.1}$$

The coil wire and the core magnetic losses are parasitic effects, and with ideal materials (i.e., perfectly conducting coil wires and ferrites without hysteresis), these resistive terms would be zero. The radiation

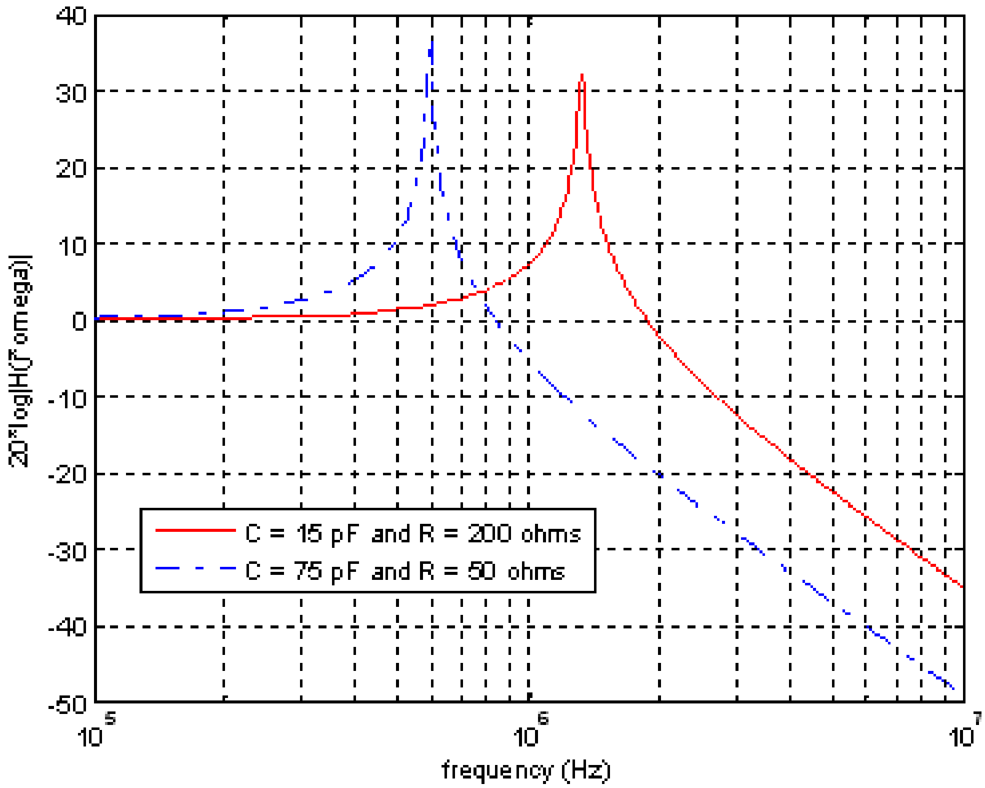


Figure 2.2.2: Frequency Response of Tuned RLC Front-End

resistance, as the name implies, is associated with the radiation and reception of electromagnetic radiation (i.e., radio waves) and is not actually a loss term. If a sinusoidal voltage source is applied across the ends of the antenna coil when used as a transmitting antenna, a current will flow in the coil. The ratio of the applied voltage to current, assuming that R_{coil} and R_{core} are zero, is given by the radiation resistance. Furthermore the time-averaged power radiated away from the antenna in the form of an electromagnetic wave is given by the product of 1/2 the square of the current and the radiation resistance (i.e., the power that would be dissipated by a real resistor of R_{ant} ohms). Similarly when operated as a receiving antenna, the current flowing through an attached load of impedance Z_L will be given by $I_L = \frac{V_{ant}}{(Z_{ant}+Z_L)}$. This current not only delivers signal power to the load but also drives the antenna to re-radiate some of the received electromagnetic signal. The time-averaged power that is re-radiated is given by the product of 1/2 the square of the current and the radiation resistance.

The radiation resistance of a loopstick receiving antenna is negligible ($\ll 1$ ohm) when the size of the antenna is small relative to a wavelength (as it is in our case) and can be ignored. The coil and ferrite core resistance values increase with frequency, but together remain below a few hundred ohms at commercial AM carrier frequencies. The reactance of the antenna is inductive and corresponds to an inductor value of approximately 960 μH for our loopstick. The use of a ferrite core, as opposed to a hollow air core, greatly increases (by a factor of several hundred to a thousand) the strength of the magnetic field inside the coil, and hence the Thevenin equivalent voltage, V_{ant}. It also increases the inductance by an identical multiplicative factor.

In general, why should one care about the impedance of an antenna? Well suppose that the antenna is being used for transmission. The output of the transmitter, which generates some voltage, is applied across the antenna terminals. The transmitter output can be modeled as Thevein equivalent voltage, V_{trans}, in

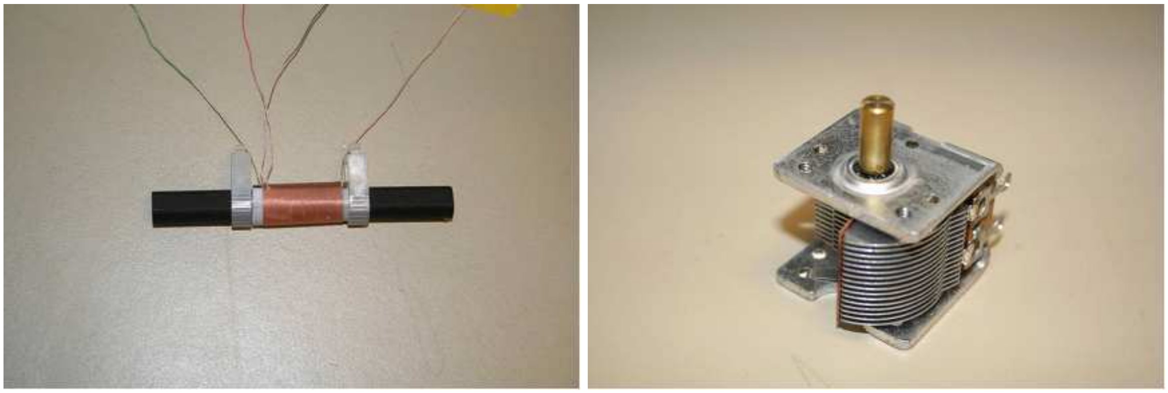


Figure 2.2.3: (a) A Loopstick Antenna (b) Variable Capacitor

series with a Thevenin equivalent impedance, Z_{trans}. The power radiated by the antenna will then be given by

$$P_{rad} = \frac{1}{2}\left|\frac{V_{trans}}{Z_{ant} + Z_{trans}}\right|^2 R_{ant}. \tag{2.2.2}$$

This power will be a maximum when the transmitter is designed to yield $R_{trans} \approx 0$ and X_{trans}= - X_{ant}, yielding[3]

$$\max(P_{rad}) = \frac{1}{2}\left|\frac{V_{trans}}{R_r + R_{coil} + R_{core}}\right|^2 R_r^2. \tag{2.2.3}$$

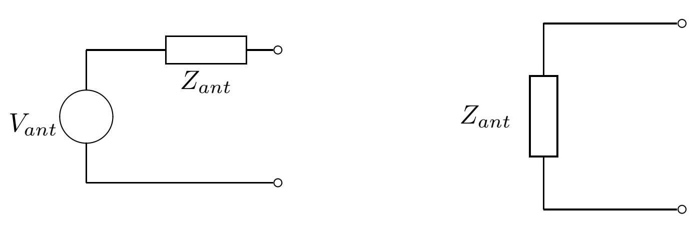
Thus the fraction of the power radiated by the antenna (as opposed to being dissipated "needlessly" in the coil and core) is given by

$$\left(\frac{R_r}{R_r + R_{coil} + R_{core}}\right)^2.$$

Clearly the efficiency increases as the size of $R_{coil} + R_{core}$ can be reduced relative to the radiation resistance. Unfortunately, antennas that are small relative to the wavelength of the signal being transmitted generally have very small radiation resistances and thus are inefficient radiators.

A similar analysis can be performed when the antenna is used for reception rather than transmission. The received power dissipated in $R_{coil} + R_{core}$ is wasted. Furthermore, the issue of noise becomes important. The ability to communicate with high fidelity is ultimately limited by the presence of noise/interference. Some noise is present in the atmosphere, for example that due to lightning strikes around the world, while some is produced by the electronics themselves inside the radio. It is a fundamental fact of physics that it is

[3]Refer back to Eq. (2.2.1) for the definition of each of the terms in the resistance.



(a) (b)

Figure 2.2.4: Thevenin Equivalent Circuit Model for an Antenna (a) Receiving, (b) Transmitting; see also Fig. 2.2.1.

impossible to completely remove the electronics noise because it is due to the thermal agitation of electrons in the circuit elements. In situations where the external atmospheric noise is negligible in comparison to the signal level and the signal level is very weak, it is important to deliver as much of the received signal as possible to the load in order to overcome the noise due to the radio electronics. We know that maximum power delivery from the antenna to the load occurs when the impedance of the load is matched to that of the antenna. Let us assume for the moment that we have an antenna with very low parasitic resistance, $R_{coil} + R_{core}$, which is good from an efficiency point of view. In order to deliver maximum power from the antenna to the load under such conditions, the front-end receiver electronics must be designed to be impedance matched, i.e., $Z_{load} = Z^*_{ant}$. For small (relative to the wavelength) antennas, however, this is very difficult to achieve because R_r is very small.

In the AM radio band, atmospheric noise and interference are generally much larger than the electronics noise. Thus the fidelity is not limited by the electronics noise and therefore *impedance matching of the load to the antenna is not critical as long as a reasonable signal level is provided to the load.*

2.2.2 RLC Resonant Circuit (or the First Step in Selecting a Station)

The variable capacitor indicated in Fig. 2.2.1 consists of two sets of interleaved parallel metallic plates as shown in Fig. 2.2.3 (b). One set of the plates can be rotated relative to the other, thus varying the overlap between the two plate sets, which in turn causes the capacitance to change. The capacitance ranges from about 10 pF when there is no overlap between the plates to 400 pF when the plates are fully overlapped.

The loopstick antenna together with the capacitor shown in Fig. 2.2.1 is a series RLC resonance circuit. Since this type of circuit and *resonance phenomena* in general play such an important role in electrical engineering, we will digress briefly to discuss these topics in some further detail below. You should be able to derive for yourself Eqs. (2.2.4)-(2.2.7) below using the material that you learned in VE 215.

The current flowing in the RLC circuit will be a maximum when the series RLC impedance is minimum. This impedance minimum occurs at a frequency for which the reactance of the inductor (i.e., $j\omega L$) exactly cancels the reactance of the capacitor (i.e., $-j/\omega C$). When this condition occurs, the circuit is said to be at *resonance*. The resonance frequency is given by

$$f_{res} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}\text{Hz} \quad \text{or} \quad \omega_{\text{res}} = \frac{1}{\sqrt{\text{LC}}}\text{rad/s} \tag{2.2.4}$$

and at the resonance frequency, the current achieves its peak value of V_{ant}/R_{ant}. The magnitude of the current decreases monotonically as one moves away from resonance. The current drops to $1/\sqrt{2}$ of its peak value when the source frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2} \;\pm\; \frac{1}{2\pi}\frac{R}{2L}\text{Hz} \tag{2.2.5}$$

Thus the 3 dB bandwidth, BW_{3dB}, of the resonant response is equal to

$$BW_{3dB} = \frac{1}{2\pi}\frac{R}{L}\text{Hz} \tag{2.2.6}$$

While the resonance frequency does not lie exactly at the midpoint between the two 3 dB points, it is very nearly centered when $BW_{3dB} \ll f_{res}$.

The ratio f_{res}/BW_{3dB} measures the "sharpness" of the resonance and is known as the quality factor, or Q of the circuit. Using Eqs. (2.2.4) - (2.2.6), it is easily shown that

$$Q = 2\pi f_{res}\frac{L}{R} \tag{2.2.7}$$

A high Q corresponds to a very sharp resonance, i.e, $BW_{3dB} \ll f_{res}$. Note that the Q is inversely proportional to R, which is the source of power dissipation in the resonator. Consequently Q increases as the resonator losses decrease. The combination of the loopstick antenna and variable capacitor act as a tunable bandpass filter as we saw in Fig. 2.2.2. *Placing the resonance peak at the carrier frequency of the desired radio station is the first step in receiving a signal and rejecting unwanted signals.*

Continuing with our discussion of the radio front-end, the inductance of the loopstick antenna has been measured to be approximately 960 μH while the variable capacitor has a range of about 10 pF to 400 pF. The bandwidth of the front-end resonant circuit is given by Eq. (2.2.6). Thus in order to estimate the bandwidth, we need to know the value of R_{ant}, which as we indicated earlier is determined primarily by the loopstick coil resistance and the losses in the ferrite core. By replacing the voltage source, V_{ant}, shown in Fig. 2.2.1 by a function generator and measuring the resonant 3 dB bandwidth for different capacitance values, one can determine R_{ant} as a function of the resonance frequency. As noted earlier, the value of R_{ant} will increase with frequency, and thus the Q of the filter will decrease, i.e., the filter will become less frequency selective, as the resonance frequency increases.

You have also seen in lecture that the time-domain and frequency-domain descriptions of LTI systems are related via the Fourier transform. Namely the time response of an LTI system is given by the convolution of the input with the system's impulse response, and the frequency response is the product of the Fourier transform of the input and the frequency response function. In addition, the frequency response function is equal to the Fourier transform of the impulse response. Finally, the impulse response is equal to the derivative of the step response.

If we consider the series RLC circuit shown in Fig. 2.2.1, it is easy to verify that V_{ant} and V_{out} are related by the following differential equation by noting that the sum of the voltage drops around the loop must be zero

$$LC\frac{d^2V_{out}}{dt^2} + RC\frac{dV_{out}}{dt} + V_{out} = V_{ant} \tag{2.2.8}$$

(In order to derive Eq. (2.2.8), you also need to recall that the instantaneous current flowing through a capacitor is given by $C\frac{dv}{dt}$ and the instantaneous voltage drop across an inductor by $L\frac{di}{dt}$.) If we set the V_{ant} to be a unit step function, then the solution of Eq. (2.2.8) is given by (you will learn how to find this solution later in the course using Laplace transform techniques)

$$V_{out}(t) = [1 - (\sin\phi)^{-1}e^{-(R/2L)t}\sin(\omega\, t + \phi)]u(t) \tag{2.2.9}$$

where

$$\omega = \sqrt{\omega_{res}^2 - (R/2L)^2}\ \text{rad/s} \tag{2.2.10}$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{\omega_{res}^2 - (R/2L)^2}}{(-R/2L)}\right) \tag{2.2.11}$$

$$\omega_{res} = \frac{1}{\sqrt{LC}}\ \text{rad/s} \tag{2.2.12}$$

Observe that the step response has an exponentially decaying sinusoidal component. When the Q is high, the oscillating frequency will be approximately equal to the resonance frequency, ω_{res}, while the resonant bandwidth (see Eq. (2.2.6)) is related to the exponential decay rate, $R/2L$. These observations can be used to determine the resonance frequency and the bandwidth experimentally.

2.3 First Stage of Demodulation: The Amplifier & Mixer in the Front-end

The output of the circuit shown in Fig. 2.2.1 is fed into a single field-effect transistor (FET) amplifier to boost the signal strength to a level suitable for the mixer to operate. The output of this amplifier feeds one of the two mixer inputs. The mixer is a nonlinear device that produces at its output the product of the voltages appearing at its two input ports (designated signal port and LO port). The local oscillator (LO) input is a sinusoid whose frequency is varied to select the channel (i.e., 540 KHz through 1700 KHz) to which one wishes to listen. For our radio, the LO is the signal generator on your lab bench. Thus for a transmitted AM signal

$$x(t) = (A + bs(t))\cos(\omega_c t + \phi) \tag{2.3.1}$$

the output of the mixer will be given by

$$(A + bs(t))\cos(\omega_c t + \phi)\cos(\omega_{LO} t + \theta) \tag{2.3.2}$$

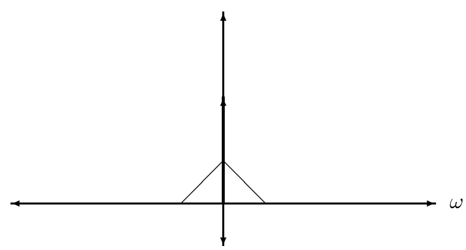
where $\theta - \phi$ is the relative phase difference between the carrier and the LO. The receiver has no way of knowing the value of this phase difference. A little bit of trigonometry (i.e., $\cos(x)\cos(y) = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$) indicates that

$$(A + bs(t))\cos(\omega_c t + \phi)\cos(\omega_{LO} t + \theta) = \frac{1}{2}(A + bs(t))\cos((\omega_c + \omega_{LO})t + \phi + \theta) + \frac{1}{2}(A + bs(t))\cos((\omega_c - \omega_{LO})t + \phi - \theta) \quad (2.3.3)$$

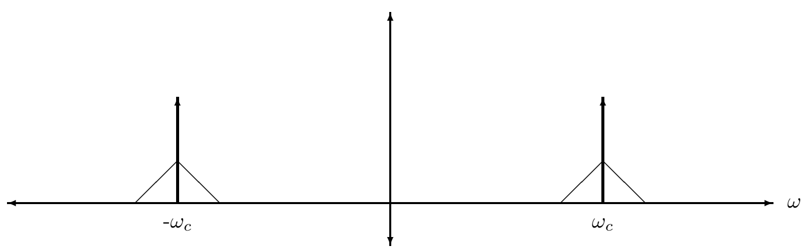
Note that mixing has both translated the carrier frequency of the original signal up to a frequency of $\omega_c + \omega_{LO}$ and down to a frequency $\omega_c - \omega_{LO}$, as guaranteed by the following Fourier transform property: $x(t) \leftrightarrow X(j\omega)$ implies that

$$x(t)\cos(\omega_o t) \leftrightarrow \frac{1}{2}X(j(\omega + \omega_o)) + \frac{1}{2}X(j(\omega - \omega_o)). \quad (2.3.4)$$

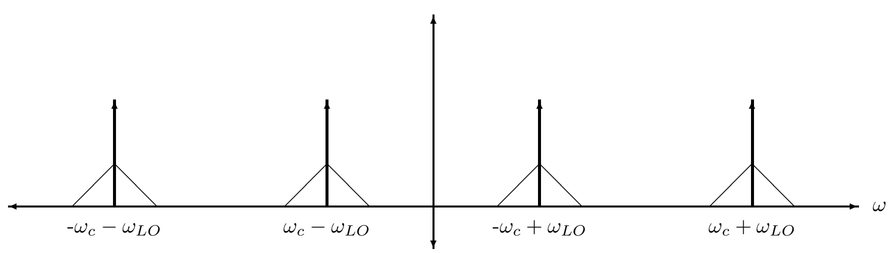
A frequency domain representation of the operation of the modulator and mixer is illustrated in Fig. 2.3.1.



(a) Original Baseband Signal Spectrum
$A + bs(t) \leftrightarrow 2\pi A\delta(\omega) + bS(j\omega)$



(b) AM Signal Spectrum
$x(t) = (A + bs(t))\cos(\omega_c t) \leftrightarrow X(j\omega)$
$X(j\omega) = A\pi\delta(\omega - \omega_c) + \frac{1}{2}bS(j(\omega - \omega_c)) + A\pi\delta(\omega + \omega_c) + \frac{1}{2}bS(j(\omega + \omega_c))$



(c) Signal Spectrum After Mixing with LO (drawn for the case where $\omega_{LO} > \omega_c$)

Figure 2.3.1: Signal Spectrum (a) original signal, (b) signal at transmitter after AM modulation, (c) signal at the output of the radio front-end mixer.

For simplicity of illustration,we have assumed that the spectrum of $s(t)$ is purely real, has a triangular shape[4], and that $\theta = \phi = 0$.

[4]You should know a time-domain signal whose Fourier transform has these properties.

Notes:

1. The figures are not drawn to scale.
2. The spectrum, $S(j\omega)$, of the original baseband signal (e.g., music or voice), $s(t)$, is generally complex-valued, having a magnitude and phase. For simplicity of illustration, we have chosen a spectrum that is real-valued. An actual spectrum would look quite different. Note that for commercial AM $|S(j\omega)| \approx 0 \quad \text{for} \quad |\omega| > 2\pi(5000)$ rad/s.
3. In part (b) of Fig. 2.3.1, the phase, ϕ, of the carrier has been assumed to be zero. In general, this phase will be non-zero. The introduction of a non-zero phase will not affect the position or the shape of the spectrum shown in Fig. 2.3.1 (b). Note that it will simply cause the spectrum to be multiplied by a scale factor of either $e^{\pm j\phi}$ by recalling that

$$\cos(\omega_{LO}t + \phi) = \frac{1}{2}e^{j\phi}e^{j\omega_{LO}t} + \frac{1}{2}e^{-j\phi}e^{-j\omega_{LO}t} \tag{2.3.5}$$

A similar comment is applicable to Fig. 2.3.1 (c) and the phase, θ, of the LO.

2.4 IF Filter or How to Practically Select a Station and Demodulate a Signal

The intermediate frequency (IF) filter shown in Fig. 2.1.1 is a bandpass filter centered at f_{IF} (the IF frequency). In the ideal case, the frequency response function of this filter would be that shown in Fig. 2.4.1 below. Observe (see Fig. 2.3.1 (c)) that by choosing the LO frequency appropriately we can use the mixer

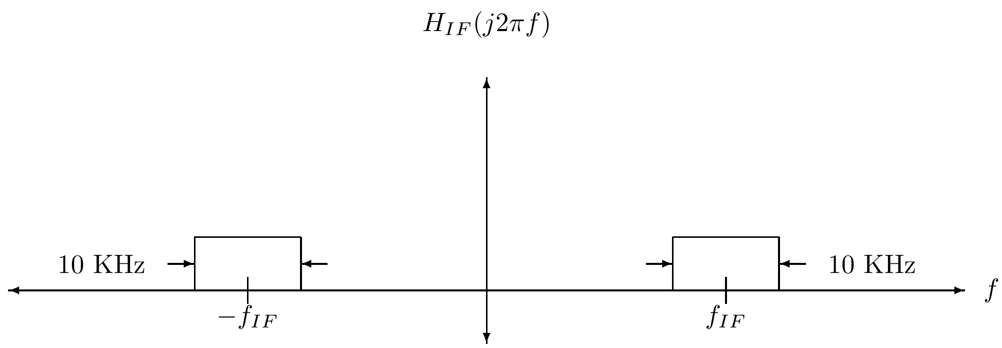


Figure 2.4.1: Ideal IF Filter

to shift the frequency of the modulated signal so that the modulated signal passes through the IF filter undistorted (assuming, of course, that the bandwidth of the IF filter exceeds the bandwidth of the message signal). Either of two frequency choices are possible for the LO, namely (assuming $f_c > f_{IF}$)

$$\omega_{IF} = \omega_c - \omega_{LO} \rightarrow f_{LO} = f_c - f_{IF} \tag{2.4.1}$$

or

$$\omega_{IF} = -\omega_c + \omega_{LO} \rightarrow f_{LO} = f_c + f_{IF} \tag{2.4.2}$$

as indicated below in Fig. 2.4.2 and 2.4.3, respectively.

In either case the output of the IF filter (up to a multiplicative constant) becomes

$$(A + bs(t))\cos(\omega_{IF}t + \phi - \theta).$$

Thus, if the output of the IF filter is fed into a properly designed envelope detector (see Lab 2), the output of the envelope detector will be $A + bs(t)$ (up to a multiplicative constant). Finally the constant term, A, can be removed by placing a capacitor in series with the output of the envelope detector (see Fig. 2.1.1) to block the DC component, leaving just the information signal $s(t)$ (up to a multiplicative constant). Thus our radio will have recovered the transmitted information signal, $s(t)$! A similar result is obtained when $f_c < f_{IF}$.

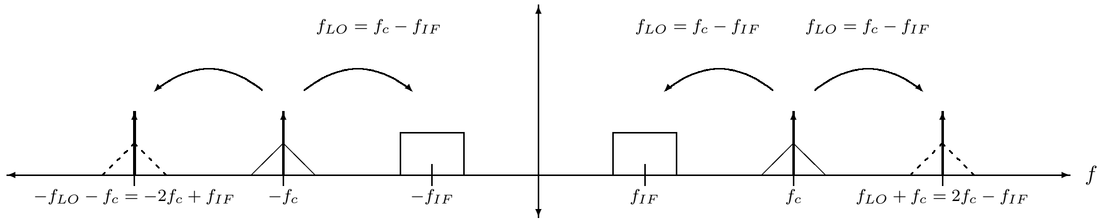


Figure 2.4.2: Using LO to Mix into IF Band when $f_{LO} = f_c - f_{IF}$

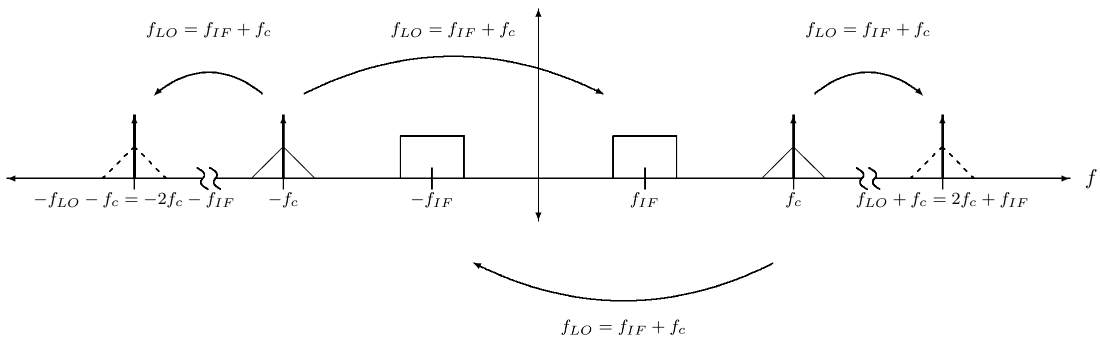


Figure 2.4.3: Using LO to Mix into IF Band when $f_{LO} = f_{IF} + f_c$

2.5 A Simple Butterworth Filter Realization of the IF Filter

We have seen that the IF filter should be a bandpass filter. Bandpass filters can take many different forms. For example, a bandpass Butterworth filter of order N is characterized by the following (magnitude) frequency response function

$$|H(j\omega)|^2 = \frac{H_o}{1 + [2(\omega - \omega_o)/\beta]^{2N}}, \tag{2.5.1}$$

where the center frequency of the filter is ω_o rad/s, the peak gain is H_o and the 3-dB bandwidth is β rad/s. This filter approaches an ideal bandpass filter as N gets large, since for N large, the frequency response function is nearly constant and equal to H$_o$ over the passband, $|\omega - \omega_o| < \beta$, and decreases rapidly towards zero as one moves outside of the passband. Note that the 3-dB bandwidth alone does not indicate the sharpness of the transition from passband to stop band. For the Butterworth filter, both the 3-dB bandwidth and the filter order N determine the filter performance.

In this lab, we will construct a very simple, op amp-based, IF filter. This filter does not have a particularly sharp passband-to-stopband transition, but it is relatively simple to build and will be sufficient for our application.

The frequency response function of our bandpass filter is given by

$$H(s) = H_o \frac{\beta s}{s^2 + \beta s + \omega_o^2} \tag{2.5.2}$$

where $s = j\omega$. Note that Eq. (2.5.2) can be rewritten as

$$H(j\omega) = H_o \frac{\beta j \omega_o}{(\omega_o + \omega)(\omega_o - \omega) + j\beta\omega} = H_o \frac{1}{\frac{\omega}{\omega_o} + j\frac{(\omega - \omega_o)}{\beta\omega_o/(\omega_o + \omega)}} \tag{2.5.3}$$

For ω in the vicinity of ω_o, Eq. (2.5.3) reduces to

$$\begin{aligned} H(j\omega) &\approx H_o \frac{1}{\frac{\omega_o}{\omega_o} + j\frac{(\omega-\omega_o)}{\beta\omega_o/(\omega_o+\omega_o)}} \\ &= H_o \frac{1}{1 + j\frac{(\omega-\omega_o)}{\beta/2}}. \end{aligned} \tag{2.5.4}$$

It follows that the frequency response function in Eq. (2.5.4) achieves a maximum value of H_o at a frequency of ω_o rad/s with a 3-dB bandwidth equal to β rad/s. Moreover, this result applies to our original frequency response function as long as $\beta + \omega_o \approx \omega_o$, that is, when β is a small fraction of ω_o, which one typically writes as $\beta \ll \omega_o$.

The bottom line is that the bandpass filter in Eq. (2.5.3) achieves a maximum value of H_o at a frequency of ω_o rad/s with an *approximate* 3-dB bandwidth of

$$BW_{3dB} \approx \beta \quad \text{vaild when} \quad \beta \ll \omega_o. \tag{2.5.5}$$

Image Frequencies: This discussion is carried out for the case that the carrier frequency is *greater* than the center frequency of the IF filter. Analyzing the other case is left to the reader.

By examining Fig. 2.4.3, one can see that when the LO frequency f_{LO} is set equal to $f_c + f_{IF}$, not only does the signal centered at f_c get mixed into the IF band, but so does any signal centered at

$$f_{imag} = f_{IF} + f_{LO} = f_c + 2f_{IF}. \tag{2.5.6}$$

The frequency band centered at f_{imag} is known as the *image* band. Using Eq. (2.5.6), we conclude that

$$f_{imag} - f_c = 2f_{IF} \quad (\text{valid when } \ f_{LO} = f_c + f_{IF} \ \ \& \ \ f_c > f_{IF}). \tag{2.5.7}$$

Thus the separation between the carrier frequency of the desired station (i.e., the one you wish to receive) and the center frequency of the image band (which also ends up in the passband of the IF filter after the mixer) is equal to twice the IF frequency.

A similar result is obtained when the situation illustrated in Fig. 2.4.2 is considered. Specifically, when $f_{LO} = f_c - f_{IF}$,

$$f_{imag} = |f_{If} - f_{LO}| \ = \ |2f_{IF} - f_c|. \tag{2.5.8}$$

Then the separation between the carrier frequency of the desired station (i.e., the one you wish to receive) and the center frequency of the image band is equal to

$$f_c - f_{imag} = \begin{cases} 2f_{IF}, & f_c > 2f_{IF} \\ 2(f_c - f_{IF}), & f_c < 2f_{IF} \end{cases} \quad (\text{valid when } \ f_{LO} = f_c - f_{IF} \ \ \& \ \ f_c > f_{IF}). \tag{2.5.9}$$

We conclude that when the LO frequency is chosen to be the higher of the two possible choices, namely $f_c + f_{IF}$, then the IF filter cannot separate two AM stations whose carrier frequencies differ by twice the IF frequency, and according to Eq. (2.5.9)*, the separation between the desired and image stations may be even less when the lower LO frequency is used*[5]. In either case, the RLC front-end (recall the antenna and resonance tuning circuit) must sufficiently attenuate signals in the image band when tuned to the carrier frequency of the desired station; otherwise, the image band will corrupt the demodulation of the desired station. Consequently, the higher LO frequency is often chosen in order to simplify[6] the design of the RLC filter in the front-end. AM radios are designed so that both the LO frequency and the resonance frequency of the RLC front-end are set together when a station is selected. The capacitor in the front-end RLC circuit is adjusted so that the resonance frequency of this circuit is equal to the carrier frequency, f_c, of the station that is to be received, while the LO frequency is set to one of the two frequencies $|f_c \pm f_{IF}|$.

[5]When the lower LO frequency $f_{LO} = f_c - f_{IF}$ is used and $f_c < 2f_{IF}$, then the separation of the carrier and image frequencies is $2(f_c - f_{IF}) < 2f_{IF}$, which is smaller than the separation of the carrier and image frequencies achieved with the higher LO frequency $f_{LO} = f_c + f_{IF}$. When $f_c > 2f_{IF}$, both LO frequencies give the same separation.

[6]The smaller the distance between the carrier and image frequencies becomes, the narrower the bandwidth must be in the RLC filter in order to attenuate the image band.

The frequency selectivity (i.e., its Q) of a bandpass filter is a measure of its ability to attenuate signals that do not lie near the center frequency of its passband, i.e., within a small fraction of the center frequency of the passband. Consequently if the IF frequency is properly chosen, then the front-end RLC circuit does not need to have much frequency selectivity in order to reject the image frequency because $\frac{f_c - f_{imag}}{f_c} = 2\frac{f_{IF}}{f_c}$ when $f_{LO} = f_c + f_{IF}$. For example, if f_{IF}= 455 KHz, which is a common choice in commercial radios, then the desired and image frequencies are separated by 910 KHz and the front-end series RLC filter does not require much frequency selectivity. The IF filter for the radio that you will build in this lab will be centered at approximately 100 KHz.

2.6 Why a Superheterodyne Receiver?

A radio, like the one built in this lab experiment, which has an IF filter with a *fixed* center frequency and a variable frequency LO (that can be used to shift the desired station into the IF band by mixing), is known as a *superheterodyne* receiver. As mentioned earlier, the superheterodyne receiver was invented by Edwin Armstrong in 1920 and is still widely used today.

It is natural to ask why we shouldn't eliminate the LO, the mixer, and the fixed IF filter and replace these elements by a single bandpass filter with a tunable center frequency. While such an approach is possible in principle, it is difficult in practice to build *tunable* bandpass filters that have adequate frequency selectivity and gain.

Perhaps an even better approach, then, would be to mix the frequency of the desired station down to baseband (i.e., choose $f_{LO} = f_c$) and then simply low pass filter the output of the mixer. This approach, however, may lead to a very weak signal, and one that will experience power fluctuations as the following analysis indicates.

Consider the signal at the output of the mixer:

$$
\begin{aligned}
&(A + bs(t))\cos(\omega_c t + \phi)\cos(\omega_{LO} t + \theta) \\
&\qquad = \frac{1}{2}(A + bs(t))\cos((\omega_c + \omega_{LO})t + \theta + \phi) + \frac{1}{2}(A + bs(t))\cos((\omega_c - \omega_{LO})t + \phi - \theta) \quad (2.6.1)
\end{aligned}
$$

When $f_{LO} = f_c$, this signal becomes

$$
(A + bs(t))\cos(\omega_c t + \phi)\cos(\omega_c t + \theta) = \frac{1}{2}(A + bs(t))\cos(2\omega_c t + \phi + \theta) + \frac{1}{2}(A + bs(t))\cos(\phi - \theta) \quad (2.6.2)
$$

The term at twice the carrier frequency will be strongly attenuated by the low-pass filter, leaving $\frac{1}{2}(A + bs(t))\cos(\phi - \theta)$ at the output[7]. Note that for many values of $\phi - \theta$, for example, $\phi - \theta \approx \pi/2$, the demodulated signal will be greatly diminished in strength. Indeed, for the case $\phi - \theta = \pi/2$, the signal strength is identically zero! It follows that when $|\cos(\phi - \theta)| \approx 0$, the presence of noise will significantly degrade the quality of reception. Furthermore, the relative phase difference between the carrier and the LO would vary when the distance between the transmitter and the receiver changes, as in a moving car. Thus, if the radio is moving, the demodulated signal strength will be time-varying (the radio would fade in and out). Although radios can be built to track the relative phase and make LO adjustments to keep the relative phase small (a method known as coherent reception), such radios are more expensive.

The use of envelope detection obviates the need to track the relative phase. There is, however, a price to be paid for this simplicity. Envelope detection requires that a DC bias, A, be added to the signal before modulation. This DC bias is chosen to ensure that the quantity $A + bs(t)$ always remains positive, otherwise the message signal $s(t)$ cannot be uniquely recovered from the envelope, $|A + bs(t)|$. The presence of the DC bias term means that the transmitter must use additional power, since it needs to transmit both the signal and the DC bias term. Because there are many fewer transmitters than receivers, it was decided in the early days that it was better to burden the transmitter rather than the receiver with the extra cost. Given the low cost and advanced state of electronics today, such a trade-off may no longer be favorable.

[7]Note that the constant term A can be removed by a DC blocking capacitor; see Fig. 2.1.1.

3 Pre-lab Assignment

3.1

(a) Consider the RLC circuit shown in Fig. 2.2.1. Compute the frequency response function $\frac{V_{out}(j\omega)}{V_{ant}(j\omega)}$. Your answer should be of the following form

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{ant}(j\omega)} = \frac{1}{a_1 s^2 + a_2 s + 1} \tag{3.1.1}$$

where $s \equiv j\omega$.

(b) Assuming $R = 20\,\Omega$, $L = 960\mu$H and $C = 100$ pF, compute and plot (use Matlab) the quantity $20\log_{10}|H(j\omega)|$, where $H(j\omega)$ is given by Eq. (3.1.1), over the frequency range 100 KHz to 10 MHz. Repeat for $R = 90\Omega$, $L = 960\mu$H and $C = 30$ pF.

For your plots use the `logspace` command, `freq = logspace(5, 7, 10000)`, to specify the frequency points at which the frequency response function is to be computed. Plotting should be done using the `semilogx` and `grid` Matlab commands as was done to generate Fig. 2.2.2. Both plots should appear on a single graph with appropriate legends and axis titles. On your plot, specify the frequency at the peak, the 3-dB bandwidth BW_{3dB} and quality factor Q for each of the two series RLC circuits. Compare your results with the values computed using Eqs. (2.2.4) and (2.2.6). When you're done with this problem, you should present all of your information in two tables as in Table 3.1 below, with one table containing values read off from the plot and the other containing values computed from the equations.

	Peak freq. (kHz)	3dB BW (kHz)	Quality Factor
C = 100 pF			
C = 30 pF			

Table 3.1: Values from Plot or Equations

3.2

Your radio's IF filter will be constructed using the op-amp circuit of Figure 3.2.1.

$$R_1 = 1\ \text{K}\Omega \quad \text{R}_3 = 10\ \text{K}\Omega$$
$$R_2 = 120\ \Omega \quad C = 1.5\ \text{nF}$$

(a) Derive the frequency response function, $H_{IF}(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$, of this IF filter. If your derivation has been done correctly, you can put it into the following form

$$H_{IF}(j\omega) = \frac{a_1 s}{a_2 s^2 + a_3 s + a_4} \tag{3.2.1}$$

where

$$s = j\omega, \qquad a_1 = -\ R_2 R_3 C/(R_1 + R_2), \qquad a_4 = 1.$$

Find the values of a_2 and a_3 in terms of R_1, R_2, R_3 and C. Also compute the *numerical* values of a_1 through a_3 using the specified resistors and capacitor values.

Hint: Use the Golden Rules of op-amps (The output attempts to do whatever is necessary to make the voltage difference between the inputs zero. The inputs draw no current.) to write node equations at points A and B. Solve these two simultaneous equations to compute the desired quantities.

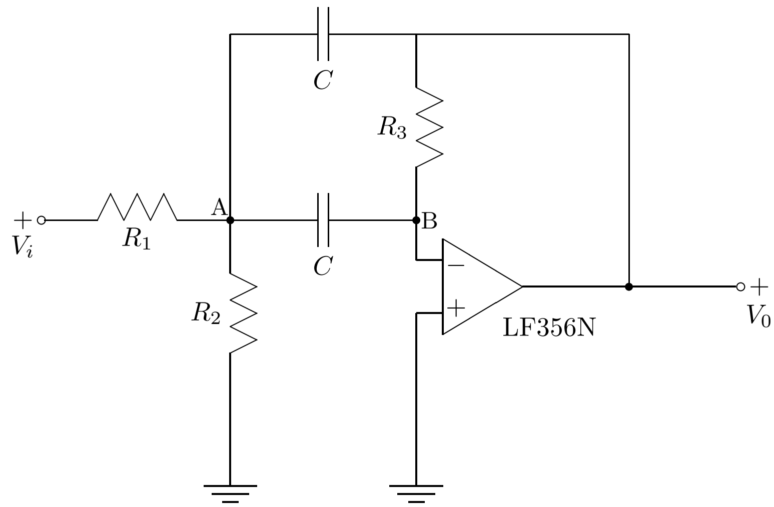


Figure 3.2.1: IF Filter

(b) The IF filter shown in Fig. 3.2.1 is a bandpass filter as indicated by Eqs. (2.5.2)-(2.5.5). Derive expressions for the frequency (in Hz) at the center of the passband, the peak value of the frequency response function and the filter's 3dB bandwidth (in Hz) in terms of the variables R_1, R_2, R_3 and C, using the approximation indicated in Eq. (2.5.5). Also compute the *numerical* values of these three quantities using the specified values for the resistors and capacitors. Just to be extra clear, your answer will have two parts: a set of formulas *and* a set of numerical values.

(c) Use the Matlab `bode` command to plot the frequency response function (magnitude and phase) of this IF filter. The peak response should be at approximately 102 KHz. Write on the face of your plot the numerical value of a_3, the peak value of the frequency response function, the frequency at the peak, and the 3 dB filter bandwidth. Also on your plot write the corresponding values computed on the basis of Eqs. (2.5.2) - (2.5.5) and comment on the quality of the approximation used in Eq. (2.5.5) (that is, how close is the analytical value of BW_{3dB} to the "exact" value read off your plot).

3.3

Assume that you wish to receive a radio station operating at a carrier frequency of 1600 KHz and the IF filter is centered at 100 KHz.

a) Find the two possible choices for the LO frequency. For each of these cases indicate the center frequency location of the image band.

b) Repeat for the case when the carrier frequency is at 530 KHz.

Once you have calculated the values in a) and b), write them in a table like Table 3.2.

f_c (KHz)	f_{LO1} (KHz)	f_{LO2} (KHz)	f_{image1} (KHz)	f_{image2} (KHz)
1600				
530				

Table 3.2: Required LO frequencies and Corresponding Image Frequencies

3.4

Sketch the equivalents of Figs. 2.4.2 and 2.4.3 for the case that $\frac{1}{2}f_{IF} < f_c < f_{IF}$; recall that Figs. 2.4.2 and 2.4.3 assumed that $f_c > f_{IF}$. You will have two sketches, one for the case $f_{LO} = f_{IF} - f_c$ and another for the case $f_{LO} = f_{IF} + f_c$.

3.5 This problem is optional (i.e., not required, and worth zero points, even if completed). No hints will be given in office hours. The result here does NOT apply to the mixer we will use in the lab. It is a very hard test of your knowledge of the Fourier transform applied to modulation.

Consider the AM radio block diagram shown below in Fig. 3.5.1. The "special" mixer performs the operation

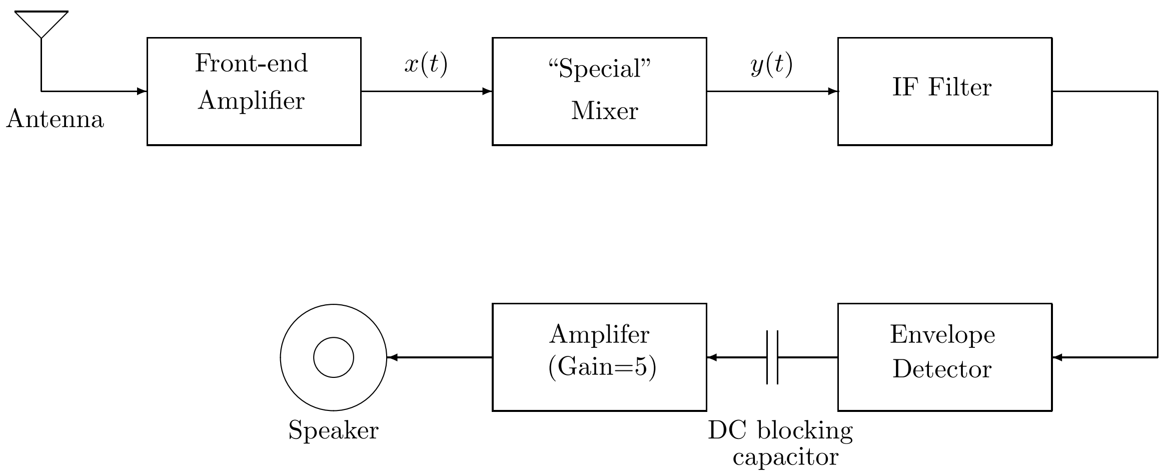


Figure 3.5.1: AM Radio With "Special" Mixer

indicated in Fig. 3.5.2.

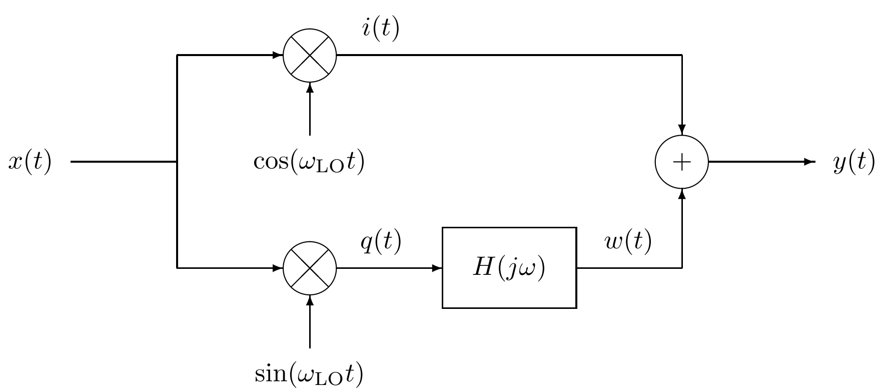


Figure 3.5.2: "Special" Mixer

The filter shown has $|H(j\omega)| = 1$, and therefore it passes all frequency components without altering their magnitudes. Any filter whose frequency response function satisfies the condition $|H(j\omega)| = c$, where c is a constant independent of frequency is known as an *all-pass* filter. The phase response chosen for the filter shown in Fig. 3.5.2 is

$$\angle H(j\omega) = \begin{cases} \frac{\pi}{2}, & \omega > 0 \\ -\frac{\pi}{2}, & \omega < 0 \end{cases} \tag{3.5.1}$$

(a) Suppose that the input, $x(t)$, shown in Fig. 3.5.2 consists of the desired radio station at a carrier frequency of ω_c plus a second station centered at the image frequency, $\omega_{imag} = \omega_c + 2\omega_{IF}$ where the LO frequency is given by $\omega_{LO} = \omega_c + \omega_{IF}$. Thus we can write

$$x(t) = [A_1 + b_1 s_1(t)]\cos(\omega_c t) + [A_2 + b_2 s_2(t)]\cos(\omega_{imag} t). \tag{3.5.2}$$

Derive expressions for $I(j\omega)$, $Q(j\omega)$, $W(j\omega)$ and $Y(j\omega)$ in terms of A_1, b_1, A_2, b_2, $S_1(j\omega)$ and $S_2(j\omega)$ for frequencies that lie inside the IF filter band. Also find an expression for the output of the IF filter as a function of time when the IF input is $y(t)$.

(b) Consider the AM Radio block diagram shown in Fig. 3.5.1 and 3.5.2. The antenna that appears in Fig. 3.5.1 consists of a loopstick. The front-end does *not* contain a resonant series RLC circuit for image rejection. Does this radio have an image problem? Explain you answer.

Suggest an appropriate name for the "special" mixer shown in Fig. 3.5.2.

References

[1] Warren L. Stutzman and Gary A. Thiele. *Antenna Theory and Design*. John Wiley and Sons, second edition, 1998.