VE216 Lecture 9

> Frequency Response

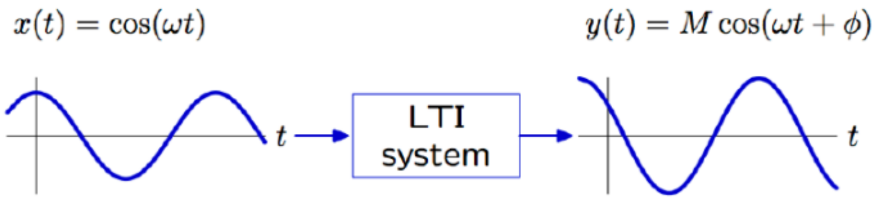
Frequency Response

A different way to characterize a system.

Previously we learnt to use **unit-sample/impulse response**.

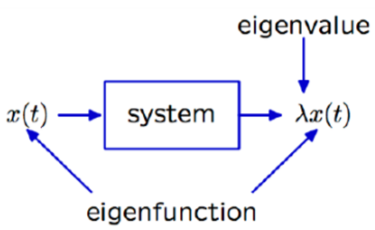
If the input to **LTI system** is a **eternal sinusoid**, the output is also a **eternal sinusoid**

- same **frequency**
- possibly different **amplitude** and **phase angle**



Calculation Method

Eigenfunction and eigenvalues.



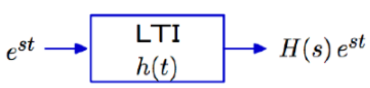
If **output signal** is a **scalar multiple** of **input signal**, then signal is **eigenfunction** with multiplier as **eigenvalue**.

Complex Exponentials

Complex exponentials are eigenfunctions of LTI systems.

$x(t) = e^{st}$ and $h(t)$ is impulse response, then:

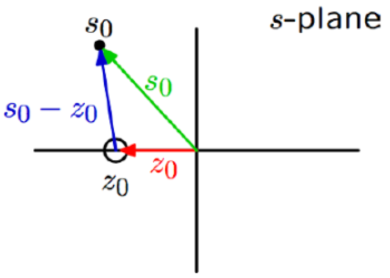
$(h * x)(t) = \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}d\tau = e^{st}\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau = H(s)e^{st}$



$cos\omega_0 t = \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})$

Vector Diagrams

$H(s_0) = K\frac{(s_0-z_0)(s_0-z_1)(s_0-z_2)\cdots}{(s_0-p_0)(s_0-p_1)(s_0-p_2)\cdots}$

So it can be represented into vector form 

$|H(s_0)| = |k|\frac{|(s_0-z_0)||(s_0-z_1)||(s_0-z_2)|\cdots}{|(s_0-p_0)||(s_0-p_1)||(s_0-p_2)|\cdots}$

$\angle H(s_0) = \angle K + \sum \angle(s_0 - z_k) - \sum \angle(s_0 - p_k)$

Frequency Response with Vector Diagram

$$\begin{aligned} y(t) &= \frac{1}{2}(H(j\omega_0)e^{j\omega_0 t} + H(-j\omega_0)e^{-j\omega_0 t}) \\ &= Re\{H(j\omega_0)e^{j\omega_0 t}\} \\ &= Re\{|H(j\omega_0)|e^{j\omega_0 t + j\angle H(j\omega_0)}\} \\ &= |H(j\omega_0)|cos(\omega_0 t + \angle H(j\omega_0)) \end{aligned}$$

Remark

Frequency response lives on the $j\omega$ axis of the Laplace transform.