VE216 Lecture 17

> Discrete Time Frequency Representations

Complex Geometric Sequences

For the DT LTI $h[n]$ with input $x[n] = z^n$, we get
$$y[n] = (h * x)[n] = \sum_{k=-\infty}^{\infty} h[k] z^{n-k} = z^n \sum_{k=-\infty}^{\infty} h[k] z^{-k} = z^n H(z)$$

So $H(z) = \sum_{k=-\infty}^{\infty} h[k] z^{-k}$, this is for DT Transform.

Remember CT Transform $H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$

Rational System Functions

We can derive $\sum z^{-k} Y = \sum z^{-p} Z$, then we can derive the $H(z)X = Y$ here.

DT Vector Diagram

$H(z_0) = K \frac{\sum_{k=0}^{k_q} (z_0 - q_k)}{\sum_{r=0}^{r_p} (z_0 - p_r)}$, very similar to CT Vector diagrams.

$|H(z_0)| = |K| \frac{\sum_{k=0}^{k_q} |z_0 - q_k|}{\sum_{r=0}^{r_p} |z_0 - p_r|}$ and $\angle H(z_0) = \angle K + \sum_{k=0}^{k_q} \angle(z_0 - q_k) - \sum_{r=0}^{r_p} \angle(z_0 - p_r)$.

DT Frequency Response

$$\begin{aligned}
x[n] = \frac{1}{2}(e^{j\Omega_0 n} + e^{-j\Omega_0 n}) \leftrightarrow y[n] &= \frac{1}{2}(H(e^{j\Omega_0}) e^{j\Omega_0 n} + H(e^{-j\Omega_0}) e^{-j\Omega_0 n}) \\
&= \text{Re}\{H(e^{j\Omega_0}) e^{j\Omega_0 n}\} \\
&= \text{Re}\{|H(e^{j\Omega_0})| e^{j\angle H(e^{j\Omega_0})} e^{j\Omega_0 n}\} \\
&= |H(e^{j\Omega_0})| \cos(\Omega_0 n + \angle H(e^{j\Omega_0}))
\end{aligned}$$

CT DT Frequency Responses Difference

- CT Frequency Response: $H(s)$ on imaginary axis, $s = j\omega$
- DT Frequency Response: $H(z)$ on unit circle, $z = e^{j\Omega}$

DT Periodicity

Since $e^{j\Omega_2} = e^{j(\Omega_1 + 2\pi k)} = e^{j\Omega_1}$, then the "highest" DT frequency $\Omega = \pi$

DT Fourier Series

$x[n] = x[n+N] = \sum_{k=0}^{N-1} a_k e^{kj\Omega_0 n}$ with $\Omega_0 = \frac{2\pi}{N}$

Orthogonality

$\sum_{n=0}^{N-1} e^{j\Omega_0 kn} e^{-j\Omega_0 ln} = \sum_{n=0}^{N-1} e^{j\Omega_0 (k-l)n} = \begin{cases} N & k=l \\ 0 & k \neq l \end{cases} = N\delta[k-l]$

So we get $a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n}$

- Analysis equation: $a_k = a_{k+N} = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$ $\Omega_0 = \frac{2\pi}{N}$
- Synthesis equation: $x[n] = x[n+N] = \sum_{n=<N>} a_k e^{jk\Omega_0 n}$