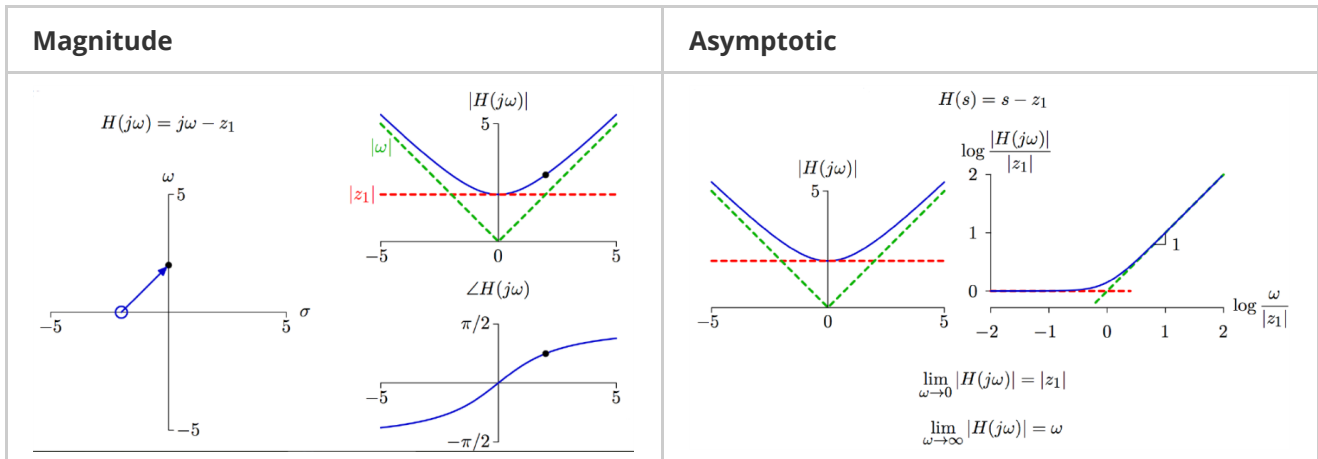


# VE216 Lecture 11

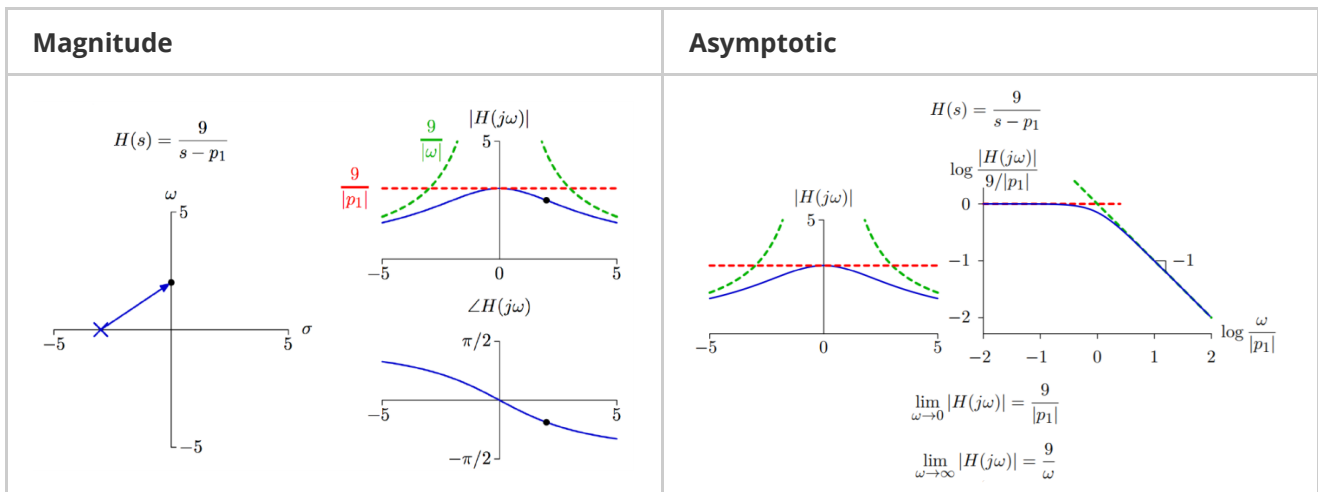
CT Frequency Response and Bode Plots

## Asymptotic Magnitude Behavior

### Isolated Zero



### Isolated Pole



## Complicated Systems Asymptotic Behavior

$$H(s_0) = K \frac{\prod_{q=1}^Q (s_0 - z_q)}{\prod_{p=1}^P (s_0 - p_p)}, \text{ then } |H(s_0)| = \left| K \frac{\prod_{q=1}^Q (s_0 - z_q)}{\prod_{p=1}^P (s_0 - p_p)} \right| = |K| \frac{\prod_{q=1}^Q |s_0 - z_q|}{\prod_{p=1}^P |s_0 - p_p|}$$

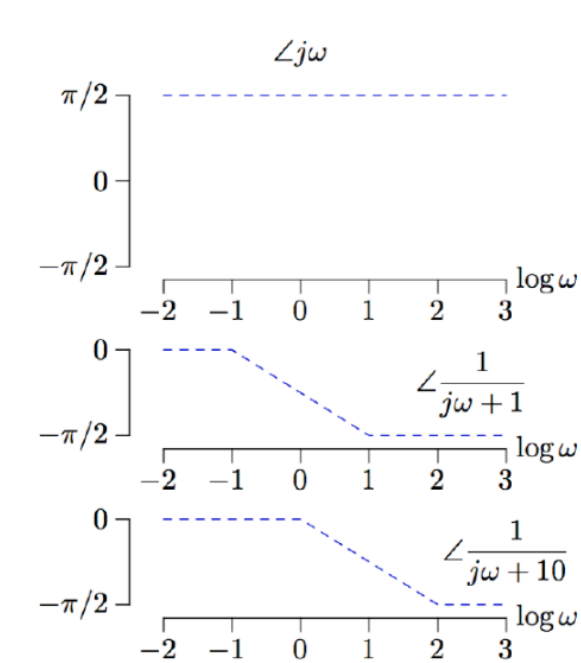
$$\text{Thus } \log |H(j\omega)| = \log |K| + \sum_{q=1}^Q \log |j\omega - z_q| - \sum_{p=1}^P \log |j\omega - p_p|$$

With proportion to the  $\log(\omega)$ , we get the bode plot.

## Bode Plot Angle

According to the previous lectures:

$$\angle H(s_0) = \angle \left( K \frac{\prod_{q=1}^Q (s_0 - z_q)}{\prod_{p=1}^P (s_0 - p_p)} \right) = \angle K + \sum_{q=1}^Q \angle(s_0 - z_q) - \sum_{p=1}^P \angle(s_0 - p_p)$$



If we need more calculation, then we can add them together as a graph.

## Bode Plot: dB

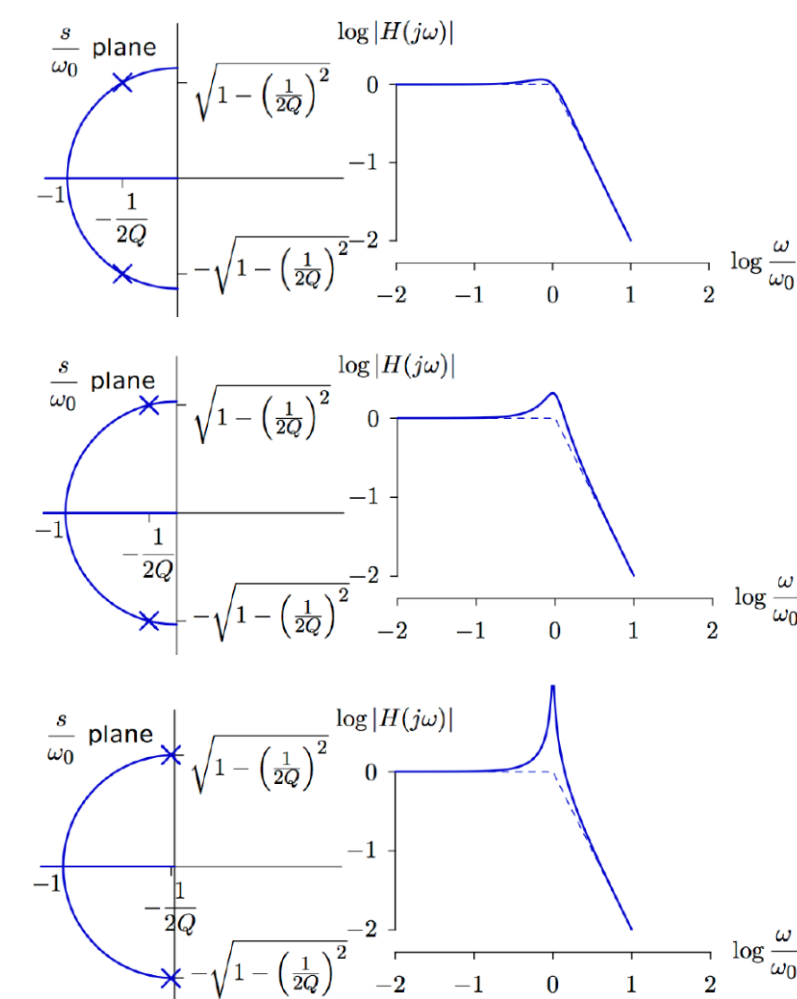
$$|H(j\omega)|[\text{dB}] = 20 \log_{10} |H(j\omega)|$$

For  $H(j\omega) = \frac{1}{j\omega + 1}$ , the approximation is here

$X$	$20 \log_{10} X$
1	0 dB
$\sqrt{2}$	3 dB
2	6 dB
10	20 dB
100	40 dB

# Frequency Response of High-Q System

$$H(s) = \frac{1}{1 + \frac{1}{Q} \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

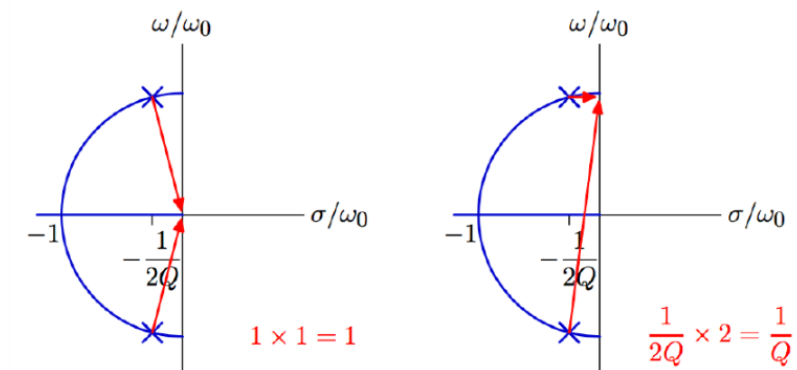


## Peak Magnitude Dependence

Assume  $Q > 3$ . Then we see on the  $\frac{s}{\omega_0}$  plane:

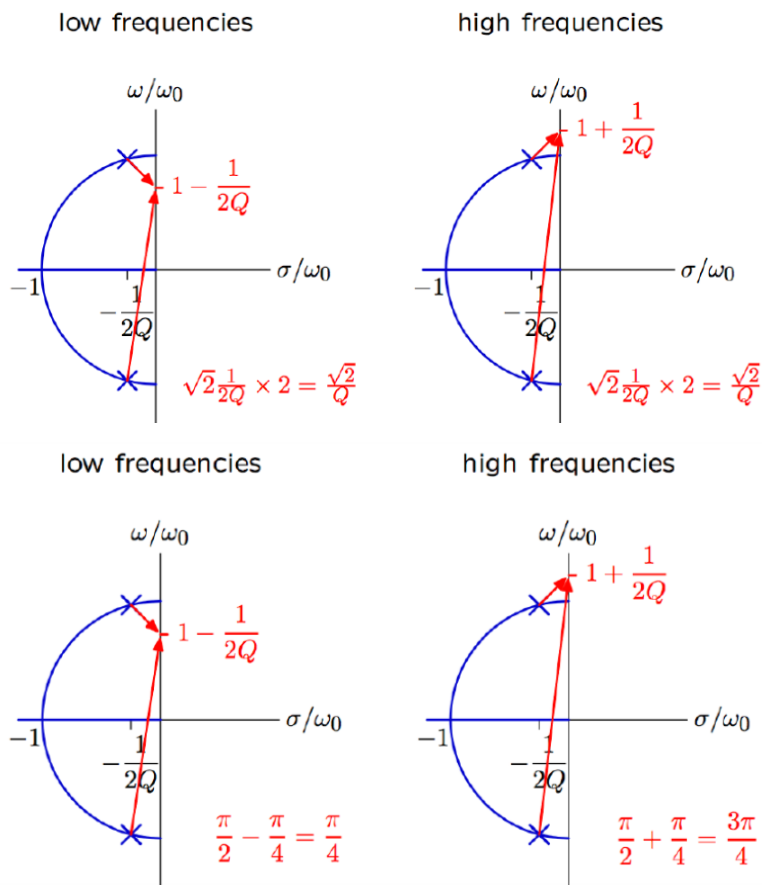
low frequencies

high frequencies



So the peak magnitude increase with  $Q$ .

## 3dB Bandwidth and Phase Change



Change in phase approximately  $\pi/2$ .

## Frequency Response of a High-Q System

As  $Q$  increases, the phase changes more abruptly with  $\omega$ .

