UM-SJTU Joint Institute
Physics Laboratory
(Vp241)

Laboratory Report

Exercise 5
RC, RL, and RLC Circuits

Name: Han Yibei ID:519370910123 Group:11

Date: December 2, 2020

1 Abstract

This experiment is study the time constant, half period, the phase difference and current amplitude in circuits containing source, resistors, capacitors and resistors. Through the experiment, we could found that the time constant for RC, RL, and RLC are 11.2530±0.0014, 95.218±0.014, 30.952±0.06 [$\times 10^{-6} s$]. And their theoretical values are 10.0686, 98.921, 31.560 [$\times 10^{-6} s$] with relative error 11.76%, -3.74% and -1.93%. In RLC resonant circuits, the tested resonant frequency is 5040.000±0.001Hz, with relative error -0.06% to theoretical 5043±3Hz. The figure of experimental φ vs $\frac{f}{f_0}$ verifies the theoretical one. Calculations, figures and plots are used to analyze the experiment.

2 Introduction

2.1 Motivation

This experiment help the understading of RC, RL and RLC circuits and their characteristics, especially the damping situations. The RLC circuit is the foundation of engineering. Through the learning we can do better in circuit design.

2.2 Theoretical Background

2.2.1 Transient Processes in RC, RL, RLC Series Circuits

RC Series Circuits

In an RC circuit, the transient process include the capicitor's charge and dischargr. Fig.1 is an RC series circuit.

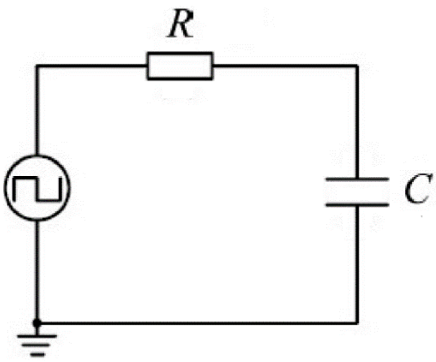

Figure 1: RC series circuit [1]

At the first half period of voltage $U(t) = \mathcal{E}$ and the capacitor charges. In the second half period, voltage is zero, and the capacitor discharges. The loop equation (KVL) for the charging process is

$$Rc\frac{dU_c}{dt} + U_c = \mathcal{E} \quad [1] \tag{1}$$

with $U_C(t=0)=0$, the solution of Eq.1 is

$$U_c = \mathcal{E}(1-e^{-\frac{t}{RC}}) \text{ and } U_R = iR = \mathcal{E}e^{-\frac{t}{RC}} \quad [1]$$

The capacitor U_C increases with time t, also the voltage U_R decreases. The curves of U(t), U_c(t), and U_R(t) are in Fig.2.

For the second half period:

$$Rc\frac{dU_c}{dt} + U_c = 0 \quad [1] \tag{2}$$

With $U_C(t=0)=\mathcal{E}$, the solution of Eq.2 is

$$U_c = \mathcal{E}e^{-\frac{t}{RC}} \text{ and } U_R = iR = \mathcal{E}e^{-\frac{t}{RC}} \quad [1]$$

where U_C and U_R decrease with time. $RC=\tau$ is the time constant.The half-life period $T_{1/2}$ is the time used for circuit to decrease half of its volatge. $T_{1/2} = \tau ln2 \approx 0.693\tau$.

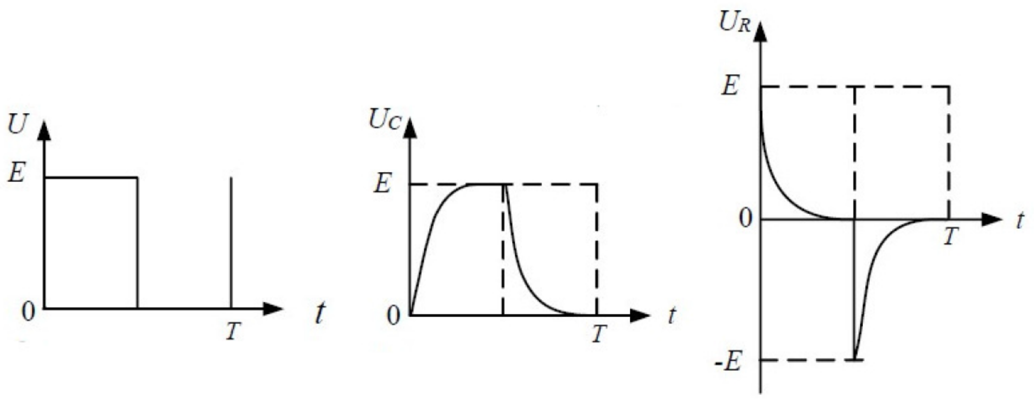

Figure 2: Charging/discharging curves for a RC series circuit [1]

RL Series Circuit As we did in RC series circuit

$$\tau = \frac{L}{R} \text{ and } T_{1/2} = \frac{L}{R}ln2 \quad [1]$$

RLC Series Circuit When applied voltage, the RLC circuit can be analyzed as:

$$LC\frac{d^2U_C}{dt} + U_c = \mathcal{E} \quad [1] \tag{3}$$

We can find that:

$$\beta = \frac{R}{2L} \text{ and } \omega_0 = \frac{1}{\sqrt{LC}} \quad [1] \tag{4}$$

Expressing Eq.3 by Eq.4, we can get:

$$\frac{d^2U_C}{dt^2} + 2\beta\frac{dU_C}{dt} + {\omega_0}^2 U_C = {\omega_0}^2\mathcal{E} \quad [1] \tag{5}$$

We can find the initial value for the equation is:

$$U_C(t=0) = 0 \text{ and } \frac{dU_C}{dt}|_{t=0} = 0 \quad [1] \tag{6}$$

The relation between β and ω_0 cause three regimes,

- If $|\beta| < |\omega_0|$, it's underdamped. The solution is:

$$U_C = \mathcal{E} - \mathcal{E}e^{-\beta t}(cos\omega t + \frac{\beta}{\omega}sin\omega t) \text{ and } \omega = \sqrt{{\omega_0}^2 - \beta^2} \quad [1]$$

- If $|\beta| > |\omega_0|$, it's overdamped. The solution is:

$$U_C = \mathcal{E} - \frac{\mathcal{E}}{2\gamma}e^{-\beta t}\left[(\beta+\gamma)\,e^{\gamma t} - (\beta-\gamma)\,e^{-\gamma t}\right] \text{ and } \gamma = \sqrt{\beta^2 - {\omega_0}^2} \quad [1]$$

- If $|\beta| = |\omega_0|$, it's critically damped. The solution is:

$$U_C = \mathcal{E} - \mathcal{E}(1+\beta t)e^{-\beta t} \quad [1]$$

When the power source is removed ($\mathcal{E}$= 0). The discharging process is similar to charging process. The voltage across the capacitor finally reach steady state. (Figure 3)

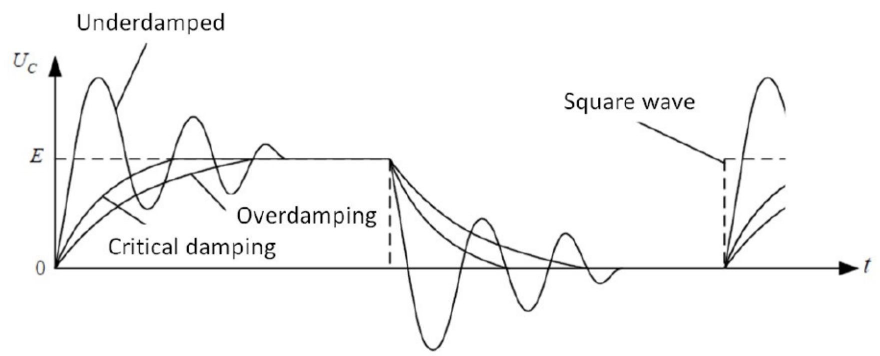

Figure 3: Three different regimes of transient processes in an RLC series circuit [1]

2.2.2 RC, RL Steady-State Circuits

The phase change can be determined as:

$$\varphi = tan^{-1}(\frac{U_L}{U_R}) = tan^{-1}(\frac{\omega L}{R}) \quad [1]$$

$$\varphi = tan^{-1}(-\frac{U_C}{U_R}) = tan^{-1}(-\frac{1}{\omega RC}) \quad [1]$$

2.2.3 RLC Resonant Circuit

RLC Series Circuit Take I as a vector along the horizontal axis, the phase differences across the resistor, coil, and capacitor are

$$\varphi_R = 0 \qquad \varphi_L = \frac{\pi}{2} \qquad \varphi_C = -\frac{\pi}{2} \quad [1]$$

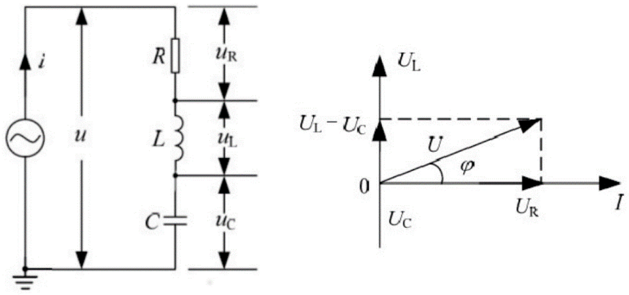

Figure 4: RLC series circuit [1]

The corresponding voltages are

$$U_R = IZ = IR \qquad U_L = IZ_L = I\omega L \qquad U_C = IZ_C = \frac{I}{\omega C} \quad [1]$$

Their amplitudes are:

$$U = \sqrt{U_R^2 + (U_L - U_C)^2} \qquad \text{or} \qquad U = I\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \quad [1]$$

The total impedance is:

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \quad [1]$$

with the total phase difference:

$$\varphi = tan^{-1}(\frac{U_L - U_C}{U_R}) = tan^{-1}(\frac{\omega L - \frac{1}{\omega C}}{R}) \quad [1]$$

Resonance The frequency that will cause resonant is:

$$\omega_0 L = \frac{1}{\omega_0 C} \longrightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad [1]$$

And the impedence now is smallest with amplitude $Z_0 = R$. When the current is $I_m = U = R$, its maximum. The resonance frequency is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \quad [1]$$

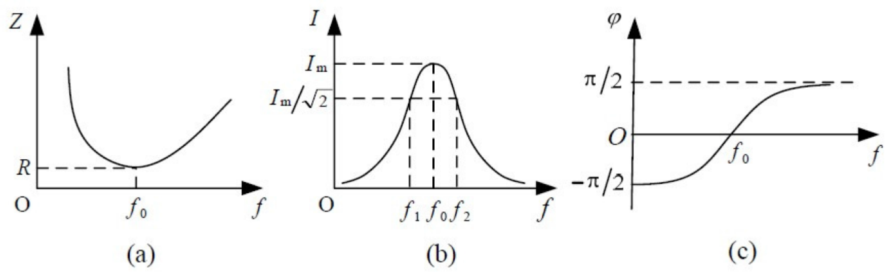

Figure 5: The impedance, the current and the phase difference as functions of the frequency for an RLC series circuit (generic sketches) [1]

Also, $\varphi = \varphi_u - \varphi_i all$. According to Equation, when $f < f_0$, $\varphi < 0$. The total voltage lag. The circuit is apacitive.

When $f = f_0$, $\varphi = 0$. The voltages is equal. The circuit is resistive.

When $f > f_0$, $\varphi > 0$. The total voltage lead. The circuit is inductive.

3 Discription of Experiment

3.1 Apparatus

The measurement setup consists of the following main elements: "a signal generator, an oscilloscope, a digital multimeter, a wiring board, a resistor, a variable resistor, a capacitor, as well as an inductor."[1]

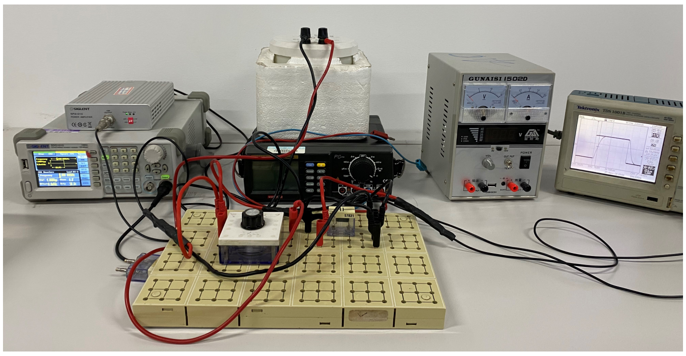

Figure 6: Apparatus

3.2 Device Information

The information of each measurement device is shown in Table 1.

apparatus	uncertainty
Signal generator	$\pm 0.001Hz/\pm 0.001Vpp$
Cursor(time)	$\pm 0.01/0.001\mu s$
Cursor(amplitude)	$\pm 0.02/0.002Vpp$
Multimeter(resistance)	$\pm 0.01\Omega$
Multimeter(capacitance)	$\pm 0.1nF$

Table 1: Information of Each Measurement Device

3.3 Measurement Procedure

3.3.1 RC, RL Series Circuit

First, build the RC or RL circuit. Change the amplitude and frenquency of signal generator to an appropriate value. Use cursor function to measure $T_1/2$. Calculate the time constant and compare it with the theoretical value.

3.3.2 RLC Series Circuit

First, build the RLC circuit with the variable resistor. Observe the waveform when it is underdamping, critically damping, and overdamping. Adjust the variable resistor to make the circuit critically damping. We will have βT1/2 = 1.68. Then the time constant is $\tau = 1/\beta = T1/2/1.68$.

3.3.3 RLC Resonant Circuit

Change the RLC circuit above and use a resistor with fixed value. Change the frequency, then observe U_R. Then calculate the corresponding phase difference. Compare the experimental value to the theoretical one. Plot the graphs $I = I_m$ vs. $f = f_0$ and $\varphi vs.f = f0$. Also, estimate the resonance frequency and calculate the quality factor Q.

3.3.4 Caution

We do these to make uncertainty smaller:

- Ground all the capacitors, inductors and resistors to the same point.
- Turn on power supply only after conpleting circuit
- Turn off power supply before changing the circuit

4 Result

4.1 RC series circuit

Quantities	Value
R[Ω]	101.09$\pm$0.01
F[Hz]	1000.000$\pm$0.001
$\mathcal{E}$[Vpp]	4.000$\pm$0.001
C[nf]	99.6$\pm$0.1
T[μs]	7.800$\pm$0.001

Table 2: Measurement Data for RC circuit

We than calculate the time constant τ_{exp}:

$$\tau_{exp} = \frac{T_{1/2}}{ln2} = \frac{7.800}{ln2} = 11.2530 \pm 0.0014\mu s = 11.2530 \pm 0.0014[\times 10^{-6}]s$$

Theoretically, the time constant τ_{theo} should be:

$$\tau_{theo} = RC = 101.09 \times 99.6 \times 10^{-9} = 10.0686[\times 10^{-6}]s$$

The relative error is:

$$u_r == \frac{\tau_{exp} - \tau_{theo}}{\tau_{theo}} = \frac{11.2530 - 10.0686}{10.0686} = 11.76\%$$

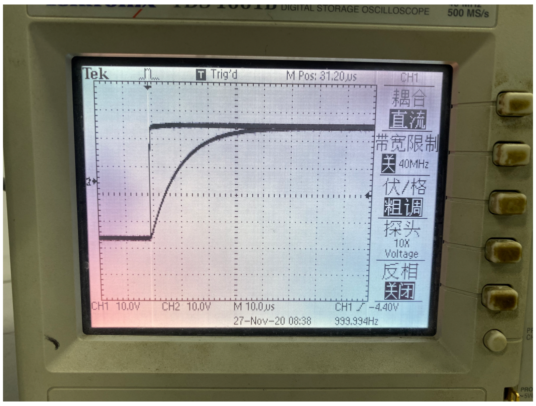

Figure 7: Waveform of RC circuit

4.2 RL series circuit

Quantities	Value
R[Ω]	101.09±0.01
F[Hz]	500.000±0.001
$\mathcal{E}$[Vpp]	4.000±0.001
L[H]	0.01
T[μs]	66.00 ±0.01

Table 3: Measurement Data for RC circuit

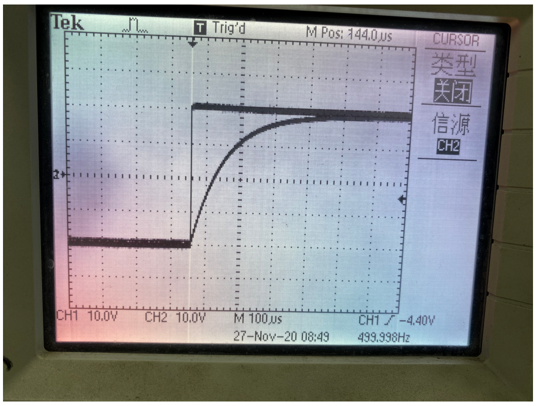

Figure 8: Waveform of RL circuit

We than calculate the time constant τ_{exp}:

$$\tau_{exp} = \frac{T_{1/2}}{ln2} = \frac{7.800}{ln2} = 95.218 \pm 0.014\mu s = 95.218 \pm 0.014[\times 10^{-6}]s$$

Theoretically, the time constant τ_{theo} should be:

$$\tau_{theo} = \frac{L}{R} = \frac{0.01}{101.09} = 98.921[\times 10^{-6}]s$$

The relative error is:

$$u_r == \frac{\tau_{exp} - \tau_{theo}}{\tau_{theo}} = \frac{95.218 - 98.921}{98.921} = -3.74\%$$

4.3 RLC series circuit

Quantities	Value
R[Ω]	101.09±0.01
F[Hz]	1000.000±0.001
$\mathcal{E}$[Vpp]	4.000±0.001
C[nf]	99.6±0.1
L[H]	0.01
T[μs]	52.00 ±0.01

Table 4: Measurement Data for RLC circuit

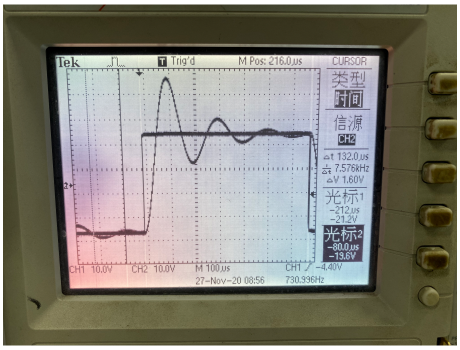

Figure 9: Under-damped regime of RLC circuit

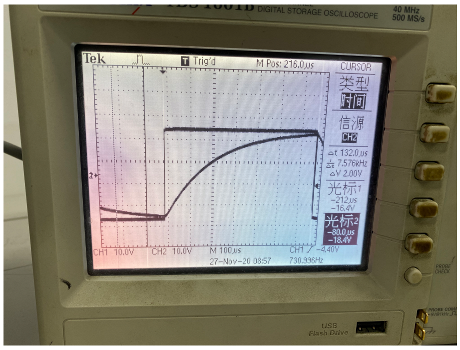

Figure 10: Over-damped regime of RLC circuit

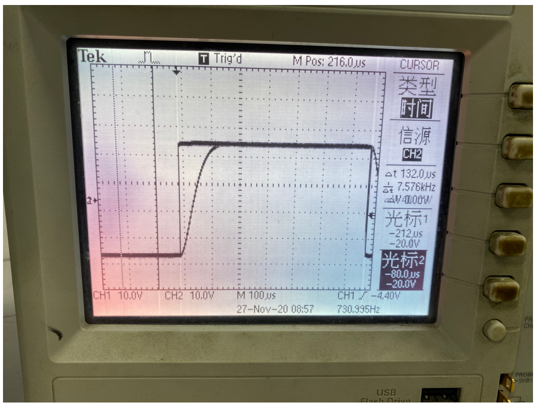

Figure 11: Critically-damped rigime of RLC circuit

We than calculate the time constant τ_{exp}:

$$\tau_{exp} = \frac{T_{1/2}}{\beta t} = \frac{52.00}{1.68} = 30.952 \pm 0.006\mu s = 30.952 \pm 0.006[\times 10^{-6}]s$$

Theoretically, the time constant τ_{theo} should be:

$$\tau_{theo} = \sqrt{LC} = \sqrt{0.01 \times 99.6 \times 10^{-9}} = 31.560[\times 10^{-6}]s$$

The relative error is:

$$u_r == \frac{\tau_{exp} - \tau_{theo}}{\tau_{theo}} = \frac{30.952 - 31.560}{31.560} = -1.93\%$$

4.4 RLC resonant circuit

Quantities	Value
R[Ω]	101.09±0.01
C[nf]	99.6±0.1
L[H]	0.01
$\mathcal{E}$[Vpp]	4.000±0.001
f_0[Hz]	5040.000 ±0.001

Table 5: Measurement Data for RLC Resonant circuit

4.4.1 Relationship between $\frac{I}{I_m}$ and $\frac{f}{f_0}$

U[V]	u_U[v]	f[Hz]±0.001[Hz]	f/f_0	u_{f/f_0}	I/I_m	u_{I/I_m}
0.400	0.002	540.000	0.1071429	0.0000002	0.1000	0.0005
0.800	0.002	1740.000	0.3452381	0.0000002	0.2000	0.0005
1.24	0.02	2740.000	0.5436508	0.0000002	0.310	0.005
1.64	0.02	3340.000	0.6626984	0.0000002	0.410	0.005
1.96	0.02	3640.000	0.7222222	0.0000002	0.490	0.006
2.40	0.02	3940.000	0.781746	0.0000003	0.600	0.0060
2.72	0.02	4140.000	0.8214286	0.0000003	0.680	0.006
3.04	0.02	4340.000	0.8611111	0.0000003	0.760	0.006
3.44	0.02	4540.000	0.9007937	0.0000003	0.860	0.007
3.76	0.02	4740.000	0.9404762	0.0000003	0.940	0.007
4.00	0.02	5040.000	1.0000000	0.0000003	1.000	0.007
3.76	0.02	5340.000	1.0595238	0.0000003	0.940	0.007
3.56	0.02	5440.000	1.0793651	0.0000003	0.890	0.007
3.16	0.02	5740.000	1.1388889	0.0000003	0.790	0.006
2.72	0.02	6040.000	1.1984127	0.0000003	0.680	0.006
2.40	0.02	6340.000	1.2579365	0.0000003	0.600	0.006
1.96	0.02	6840.000	1.3571429	0.0000003	0.490	0.006
1.64	0.02	7640.000	1.5158730	0.0000004	0.410	0.005
1.24	0.02	9040.000	1.7936508	0.0000004	0.310	0.005
0.800	0.002	13340.000	2.6468254	0.0000006	0.2000	0.0005
0.560	0.002	20940.000	4.1547619	0.0000008	0.1400	0.0005

Table 6: Relation between I/I_m and f/f_0

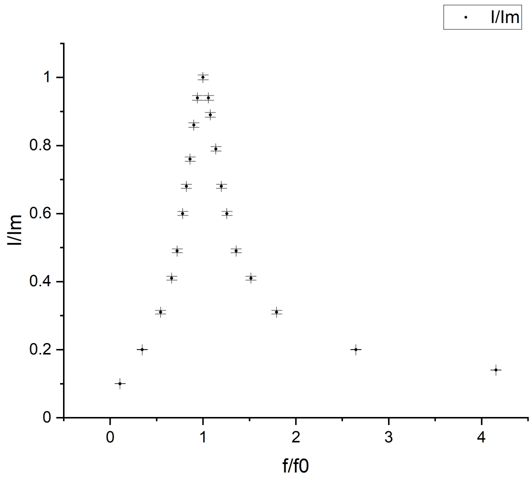

Figure 12: Relation between I/I_m and f/f_0

4.4.2 Phase Shift

$$\varphi_{theo} = tan^{-1}(\frac{2\pi fL - \frac{1}{2\pi fC}}{R}) \qquad \varphi_{exp} = cos^{-1}(\frac{U_R}{U_m})$$

f/f_0	u_{f/f_0}	φ_{ex}	$u_{\varphi_{ex}}$	φ_{theo}	$u_{\varphi_{theo}}$
0.1071429	0.0000002	-1.4106	0.0005	-1.536252	0.000003
0.3452381	0.0000002	-1.3694	0.0005	-1.446488	0.000012
0.5436508	0.0000002	-1.256	0.006	-1.32964	0.00002
0.6626984	0.0000002	-1.148	0.006	-1.21079	0.00003
0.7222222	0.0000002	-1.059	0.006	-1.22688	0.00004
0.7817460	0.0000003	-0.927	0.007	-1.00165	0.00004
0.8214286	0.0000003	-0.823	0.008	-0.89408	0.00005
0.8611111	0.0000003	-0.707	0.010	-0.75697	0.00005
0.9007937	0.0000003	-0.536	0.013	-0.58333	0.00005
0.9404762	0.0000003	-0.35	0.02	-0.37074	0.00003
1.0000000	0.0000003	0	Inf	-0.0037519	0.0000013
1.0595238	0.0000003	0.35	0.02	0.34459	0.00003
1.0793651	0.0000003	0.473	0.015	0.44384	0.00004
1.1388889	0.0000003	0.660	0.010	0.68310	0.00005
1.1984127	0.0000003	0.823	0.008	0.84944	0.00005
1.2579365	0.0000003	0.927	0.007	0.96619	0.00005
1.3571429	0.0000003	1.059	0.006	1.09493	0.00004
1.5158730	0.0000004	1.148	0.006	1.21361	0.00003
1.7936508	0.0000004	1.256	0.006	1.31794	0.00002
2.6468254	0.0000006	1.3694	0.0005	1.430998	0.000014
4.1547619	0.0000008	1.4303	0.0005	1.489412	0.000008

Table 7: Calculation result of phase shift

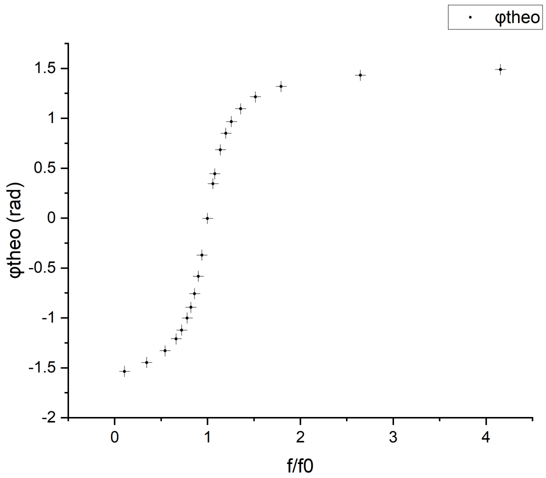

Figure 13: φ_{theo} vs $\frac{f}{f_0}$

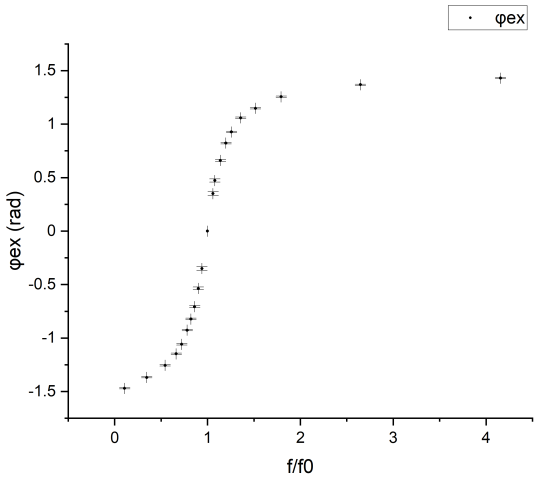

Figure 14: φ_{ex} vs $\frac{f}{f_0}$

The theoretical resonate frequency

$$f_{0,theo} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 99.6 \times 10^{-9}}} = 5043[Hz] \pm 3[Hz]$$

$$u_r == \frac{f_0 - f_{0,theo}}{f_{0,theo}} = \frac{5040.000 - 5043}{5043} = -0.06\%$$

4.5 Quality factor

$$Q_{theo} = \frac{\sqrt{LC}}{RC} = \frac{\sqrt{0.01 \times 99.6 \times 10^{-9}}}{101.09 \times 99.6 \times 10^{-9}} = 3.1344 \pm 0.0003$$

when $I(f_1) = I(f_2) = Im/\sqrt{2}$,$U_R(f_1) = U_R(f_2) = \frac{U_m}{\sqrt{2}} = \frac{4.00}{\sqrt{2}} = 2.83Vpp$ The closet frequency is f_1=4140Hz f_2=6040Hz.

$$Q_{exp} = \frac{f_0}{f_2 - f_1} = \frac{5040}{6040 - 4140} = 1.438 \pm 1 \times 10^{-6} = 2.652632 \pm 0.000006$$

$$r = \frac{Q_{exp} - Q_{theo}}{Q_{theo}} = \frac{2.6526 - 3.1344}{3.1344} = -15.37\%$$

5 Conclusions and Discussions

In this lab, we can achieve the following goals and significances:

- Understand the physics of alternating-current circuits, especially the dynamic process in RC, RL, and RLC series circuits.
- Study methods for the amplitude-frequency and the phase-frequency characteristics
- Find the resonance frequency and quality factor of RLC circuit.

In RC circuit, the time constant is found: $\tau_{exp} = 51.94 \pm 0.01[\mu s]$ and $\tau_{theo} = 46.00 \pm 0.01[\mu s]$ The relative error is 12.9%

In RL circuit, the time constant is found: $\tau_{exp} = 100.99 \pm 0.01[\mu s]$ and $\tau_{theo} = 100.00 \pm 0.01[\mu s]$ The relative error is 0.99%

In RLC circuit, the time constant is found: $\tau_{exp} = 83.33 \pm 0.06[\mu s]$ and $\tau_{theo} = 67.86 \pm 0.007[\mu s]$ The relative error is 22.8%

When $f = f_0$, the circuit reaches resonance and I reaches its maximum. Far from f_0, I becomes smaller and the experimental figure match the theoretical one.

We can find the phase shift increase in Fig.13 and Fig.14. Theoretically, when f=f_0, the phase shift is 0. When f<f_0, the phase shift is negative. When f>f_0, the phase shift is positive. The phase shift is close to $\pi/2$ if f/f_0 is really small or big.

The resonant frequency is found at $f_0 = 2300.000 \pm 0.001Hz$. Compared with the theoretical value: $f_{0,theo} = 2345.3 \pm 0.3Hz$, the relative error of f_0 is -1.93%.

The quality factor is found: $Q_{theo} = 1.472 \pm 0.0002$ and $Q_{exp} = 1.438 \pm 1 \times 10^{-6}$ The relative error is -2.31%.

The uncertainty may caused by the following reasons.

- The large relative error of time constant may because of the inaccuracy while reading $T_{1/2}$. Since U cannot reach precisely half of U_m.
- In RLC part, it's hard to determine the critically-damped simply by eyes.
- The oscilliscope may have resistance.
- The resistance may change because of rise of temperature.
- When calculating the experimental quality factor, we just take the closest values from $\frac{I_m}{\sqrt{2}}$ as f_1 and f_2 from our experimental data.

Change the resistor digitally to more precisely reach the critically-damped regime. Improve the accuracy of the equipment. Repeat the experiment of resonant series to find more precise f_1 and f_2. These may help us improve the accuracy.

6 Reference

[1] Qin Tian, Feng Yaming, Gu Yichen, Mateusz Krzyzosiak. Physics Laboratory VP241 Exercise 5: RC, RL, and RLC Circuits.

APPENDIX

A Data Sheet