UM-SJTU Joint Institute
Physics Laboratory
(Vp241)

Laboratory Report

Exercise 4
Polarization of Light

Name: Han Yibei ID:519370910123 Group:11

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1 Abstract

In this experiment, the polarization phenomenon of light is studied and Malus' law is verified with the relative error -0.5%; the way half-waveplate work, with $\Delta\theta$=2α and relative uncertainty 1.05%; the way quarter-wave plates work in optical systems about the generation and detection of linearly, elliptically and circularly polarized light, when $\alpha = 0°$, it's linearly polarized; when $\alpha = 45°$, it's circularly polarized; when $\alpha = 20°$, it's elliptically polarized; when $\alpha = 70°$, we can verified what we observed at $\alpha = 20°$. Calculations, figures and plots are used to illustrate the result. Also, we discuss the uncertainty and possible error in this experiment.

2 Introduction

2.1 Motivation

The polarization of light played an important role in the development of wave optics. And there existed a wide range of applications in numerous areas, such as optical measurement techniques, crystal structure research, and experimental stress analysis.(Ref[1])

2.2 Theoratical Background

Light can be described as transverse electromagnetic waves, with the plane of oscillations perpendicular to the direction of light propagation. The natural light is a mixture of waves with random transverse directions, and it can also be called unpolarized light. The unpolarized light have the uniform distribution of the transverse directions.

2.2.1 Polarization of Light

Light vector describes a time-dependent, propagating electric field. The light vector maintains a certain oscillation direction, is called linearly polarized light and the line of the direction is called the polarization axis (Fig.5).

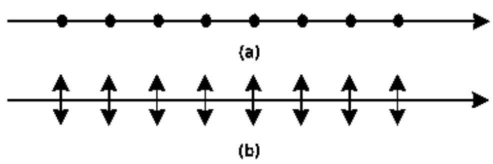

Figure 1: (a) Linearly polarized light with the polarization axis perpendicular to the page plane. (b) Linearly polarized light with the polarization axis parallel to the page plane (Ref[1])

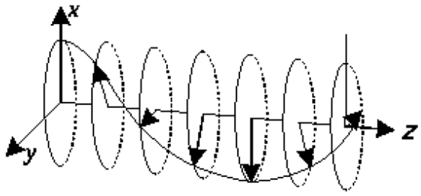

Figure 2: Elliptically xy plane polarized light propagating in the z direction (Ref[1])

The light with the light vector direction rotating and forms a circle with its endpoint, is called circularly polarized light. If the traces is an ellipse, the light is regarded as elliptically polarized (see Fig.2).

Although natural light is unpolarized, it can be regarded as a mixture of linearly polarized waves with equal amplitudes and different transverse directions. The direction of partially polarized light is the same as the direction of the the light vector with maximum amplitude.

2.2.2 Polarizer

Polarizer is used to produce polarized light using the principle of dichroism, which can pass the light polarized in a certain direction while absorbed other light polarized in different directions. It helps generate linearly polarized light from natural light. (Ref[1])

A polarization device can change incident natural light to polarized light (a polarizer), as well as detect and analyze dofferent type of polarized light (analyzer).

2.2.3 Malus' Law

The light coming out of a polarization device has change of the light brightness.

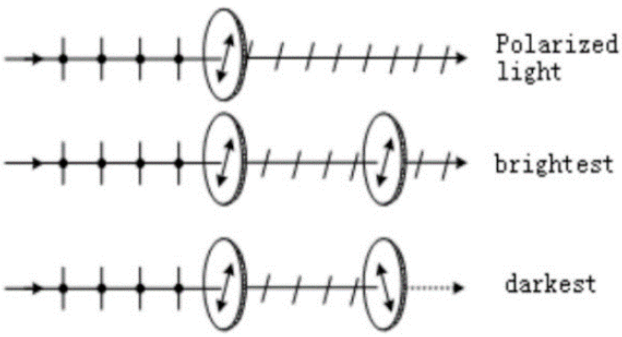

Figure 3: Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.(Ref[1])

With two polarizers arranged parallel as the Fig.3, left one plays the role of a polarizer while other is an analyzer. Let the angle between their polarization axes be θ. The light is incident normally on the polarizer and then continues to the analyzer. The intensity of the linearly polarized light leaving the analyzer is

$$I_{light} = I_{light,0} cos^2\theta(Ref[1]) \tag{1}$$

where I_{light} is the intensity of the linearly polarized light incident on the analyzer.

For a linearly polarized light, the transmitted light will change periodically when rotating the polarizer and will exist certain point when intensity is 0. For a elliptically or partially polarized light, the transmitted light will also change periodically. For the natural light and circularly polarized light, the intensity won't change at all. So er can distinguish the polarized light type from using the polarizer.

2.2.4 Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates

When linearly polarized light is incident on a crystal plate parallel to the optical axis, and the angle between the polarizing axis and the optical axis is α. The linearly polarized light can be divide into two parts: an extraordinary wave with parallel oscillation direction pwith the optical axis and an ordinary wave with perpendicular direction with the optical axis. The final optical path difference over the thickness d is $\Delta = (n_e - n_o)d$ and the phase difference is

$$\delta = \frac{2\pi}{\lambda}(n_e - n_o)d(Ref[1])$$

where λ is the wavelength, n_e is the refractive index for the extraordinary axis, and n_o is the refractive index for the ordinary axis.

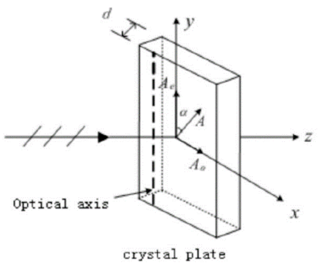

Figure 4: Linearly polarized light passing through a waveplate.(Ref[1])

$$\begin{cases} E_x = A_o cos\omega t \\ E_y = A_o cos(\omega t + \delta) \end{cases} \longrightarrow \frac{E_x^2}{A_o^2} + \frac{E_x^2}{A_o^2} - 2\frac{E_x E_y}{A_o A_e} cos\delta = sin^2\delta(Ref[1]) \tag{2}$$

Some special cases can be discussed.

When $\Delta = k\lambda$ (k=0,1,2,...), $\delta = 0$. The equation is $E_y = \frac{A_e}{A_o}E_x$. The light through this kind of waveplate remain unchange and this kind of waveplate called full-wave plate.(Ref[1])

When $\Delta = (2k+1)\lambda/2$ (k=0,1,2...), and $\delta = \pi$. The equation is $E_y = -\frac{A_e}{A_o}E_x$. The light through it remain the same type but the axis rotate 2α. This kind of waveplant is called 1/2-wave plate.(Ref[1])

When $\Delta = (2k+1)\lambda/4$ (k=0,1,2...), and $\delta = \pm\pi/2$. The equation is $\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} = 1$. This kind of waveplate is called 1/4-wave plate. When $\alpha = 0$ or $\pi/2$, the light is still linearly polarized. When $\alpha = \pi/4$, the transmitted life is circularly polarized. Otherwise, the transmitted light is elliptically polarized.(Ref[1])

3 Discription of Experiment

3.1 Apparatus

The measurement setup consists of: a semiconductor laser, a tungsten iodine lamp, a silicon photo-cell, a UT51 digital universal meter, as well as two polarizers, 1/2-wave and 1/4-wave plates (the uncertainty of the angle is 2°) and a lens with a glass sheet.(Ref[1]) The detailed information is shown in Table 6.

Apparatus	Uncertainty
Universal meter	$\pm 0.001\mu$A
Polarizer	$\pm 2°$
Analyzer	$\pm 2°$

Table 1: Apparatus Information

3.2 Measurement Procedure

3.2.1 Apparatus Adjustment

At the beginning, we should choose the appropriate photo-cell aperture(see Fig) used in different experiments. And close the light to reduce uncertainty.

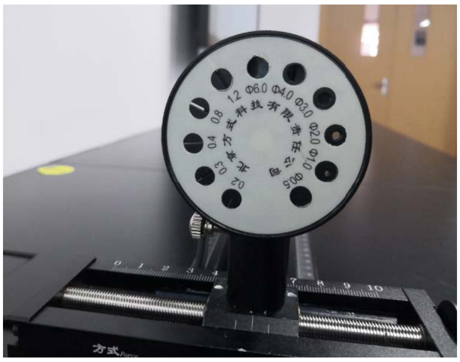

Figure 5: Photo-cell(Ref[1])

In this experiment, only the $\phi 6.0$ aperture, which preserves the incident light intensity, is needed. Then adjust the laser and the photo-cell to enable the light pass through certain aperture. Also make sure that the light pass through the center of the lens and glass sheets between photo-cell and laser. Adjust the distance between the lens and the laser to make the light intensity in an appropriate range. Final adjustment step is to set the digital universal meter in the appropriate mode and range.

3.2.2 Demonstration of Malus' Law

Assemble the measurement setup as shown in Fig.6.First assemble the polarizer and check whether the present light is linearly polarized. Then put the analyzer on. Rotate the analyzer to find the maximum electric current I_0. Then, rotate the analyzer from 0° to 90°, at the interval of 5°, record the magnitude of the current. Record the values plot the graph I/I_0 vs. $cos^2\theta$. Perform linear fitting and compare the data with the theoretical result.

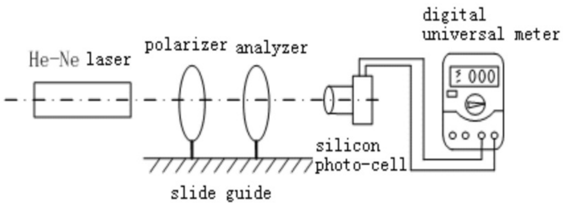

Figure 6: Experimental setup for a demonstration of Malus' law.(Ref[1])

3.2.3 Linearly Polarized Light and the Half-wave Plate

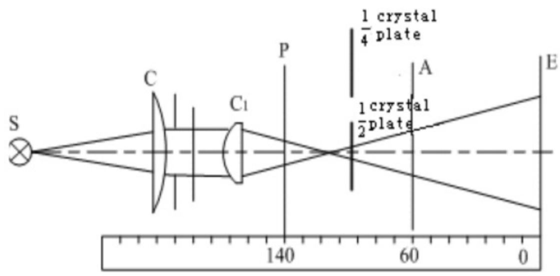

Figure 7: Experimental setup for the 1/2-wave plate.(Ref[1])

Set up the equipment on the optical bench as shown in Fig.7. Make A and P perpendicular to each other before placing the half-wave plate. After inserting the half-wave plate, make the new extinction position as the reference position. Rotate the half-wave plate at the interval of 10°from the initial position, from 90°to 10°, then record $\Delta\theta$ in a table. Plot the graph $\Delta\theta$ vs. α.

3.2.4 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

Set up the equipment on the optical bench as shown in Fig.77. Make A and P perpendicular to each other before placing the half-wave plate. After inserting the 1/4-wave plate, make the new extinction position as the reference position. Rotate the analyzer for 360° and record the light intensity for every 10°. Record the data in a table and also record the maximum intensity I0. Repeat for α=20°, 45°. For α=70°, just recording the maxium intensity and its corresponding changed angle $\Delta\theta$. Use a computer to plot the relation between the rotation angle of the analyzer and the light amplitude in polar coordinates. Mark the position recorded in $\alpha = 70°$and compare it with the data recorded in $\alpha = 20°$.

3.2.5 Caution

Do not direct the laser beam into the eye. Do not touch the surface of the polarizers or the wave plates. Please leave the equipment in order before leaving.

4 Result

4.1 Malus' Law Demonstration

We want to find the relationship between $cos^2\theta$ and I/I_0. All the result and uncertainty is shown in Tabel 2. Then we apply linear fit to I/I_0 vs. $cos^2\theta$, shown in Fig.8.

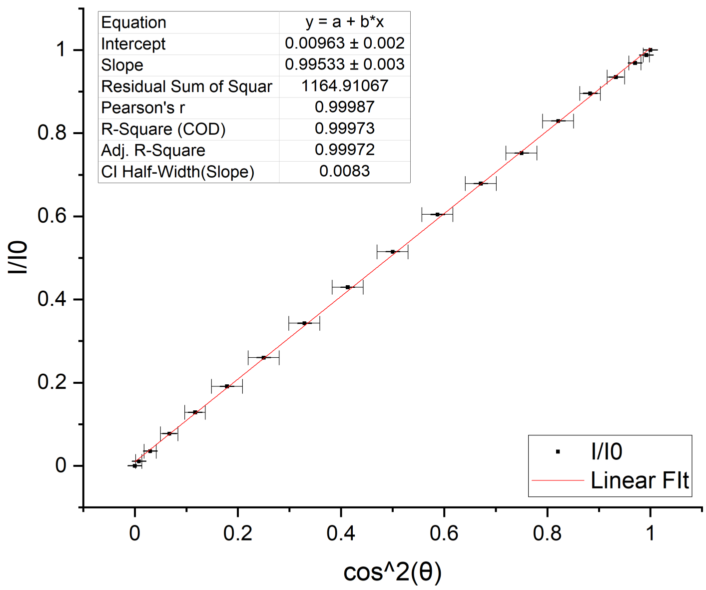

Figure 8: Linear fit of I/I_0 vs. $cos^2\theta$

From the figure we can see that the Pearson's r is 0.999 and the intercept is close to 0, which means I/I_0 is proportional to $cos^2\theta$. The slope is 0.995 ± 0.008, and the theoretical value of the slope should be 1. The relative error is $u_r = \frac{0.995-1}{1} \times 100\% = -0.5\%$, which is very small.

4.2 Linearly Polarized Light and the Half-wave Plate

For the rotation angle of the analyzer, we need to perform the calculation, and the results and uncertainties is in Table 3.

From the we can see that the Pearson's r is 0.999 which is very close to 1, and the intercept isvery close to 0. These mean that $\Delta\theta$ is proportional to θ. The slope is 2.021 ± 0.016. The theoretical value of the slope should be 2(Ref[1]). The relative error is $u_r = (2.021 - 2)/2 \times 100\% = 1.05\%$, which is very small.

We can know from the procedure and the data that as the 1/2-wave plate rotates for 360°light extinction can be observed for 2 times. And after the linearly polarized light passes through the 1/2-wave plate with α, the polarization axis will be rotated for 2α.

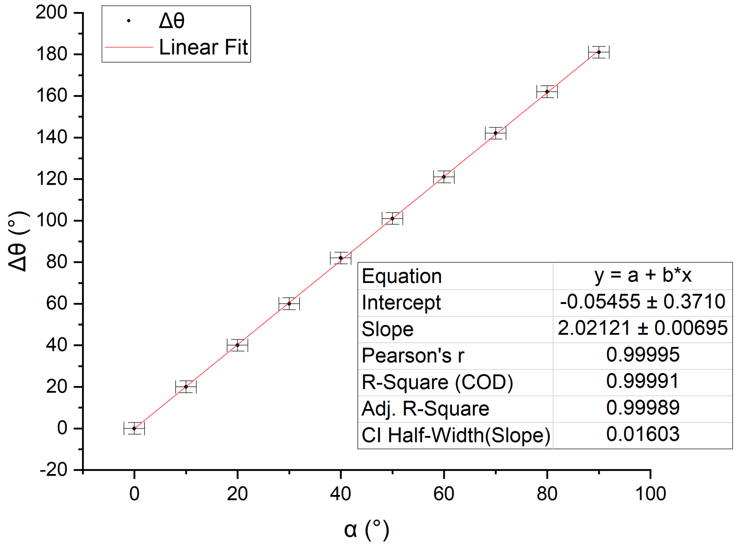

Figure 9: Linear fit of $\Delta\theta$ vs. α

4.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

4.3.1 Rotation angle 0°

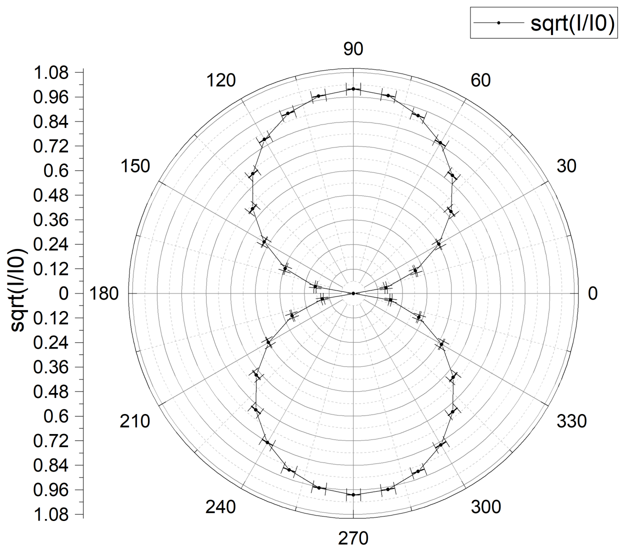

Figure 10: Plot of $\sqrt{I/I_0}$ vs. θ

We want the relationship between $\sqrt{I/I_0}$ and θ, we calculate all the result, shown in Table 4. Then we plot $\sqrt{I/I_0}$ vs. θ in the polar coordinate, shown in Fig.10.

4.3.2 Rotation angle 20°

We want the relationship between $\sqrt{I/I_0}$ and θ, we calculate all the result, shown in Table 5. Then we plot $\sqrt{I/I_0}$ vs. θ in the polar coordinate, shown in Fig.11.

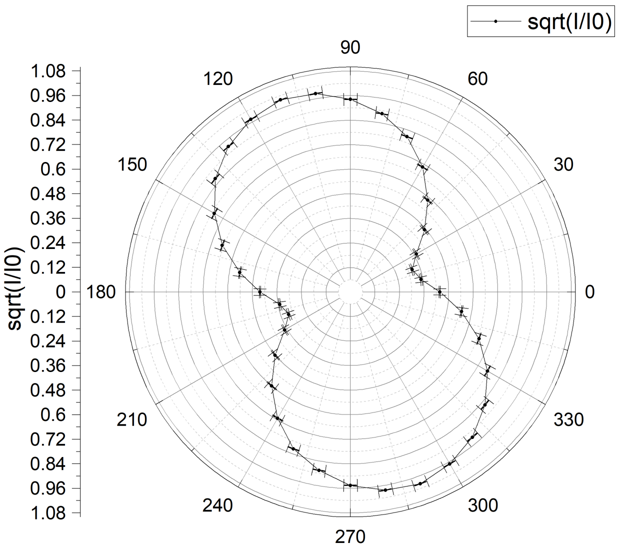

Figure 11: Plot of $\sqrt{I/I_0}$ vs. θ

4.3.3 Rotation angle 45°

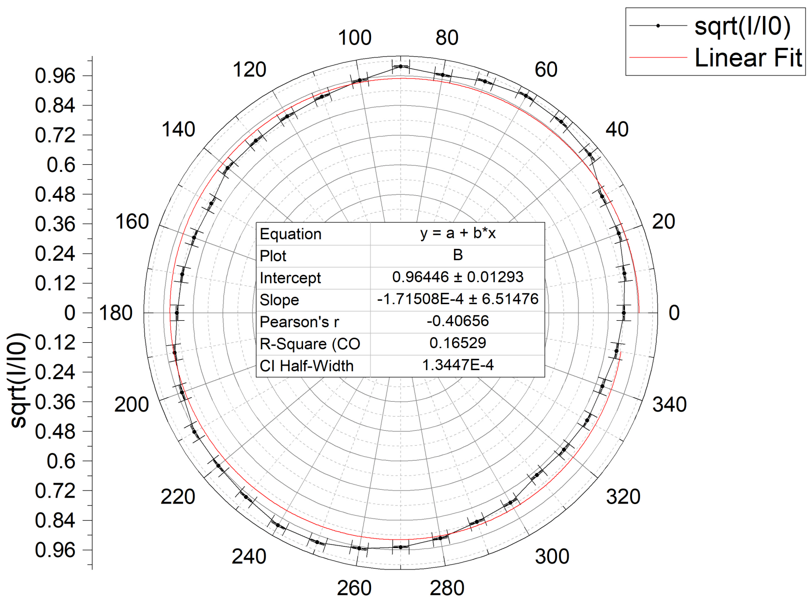

Figure 12: Plot of $\sqrt{I/I_0}$ vs. θ

We want the relationship between $\sqrt{I/I_0}$ and θ, we calculate all the result, shown in Table 12. Then we plot $\sqrt{I/I_0}$ vs. θ in the polar coordinate, shown in Fig.12. The slope is 1.3447E-4, is very closed to 0. But the Pearson's r is relatively small, the reason will be discussed in conlusion part.

4.3.4 Rotation angle 70°

When the current reaches its maximum, we recorded the angle of the analyzer and the current in Table 7. We also mark the maximum current point on the previous plot of 1/4-wave plate of the rotation angle 20°, shown in Fig.13.

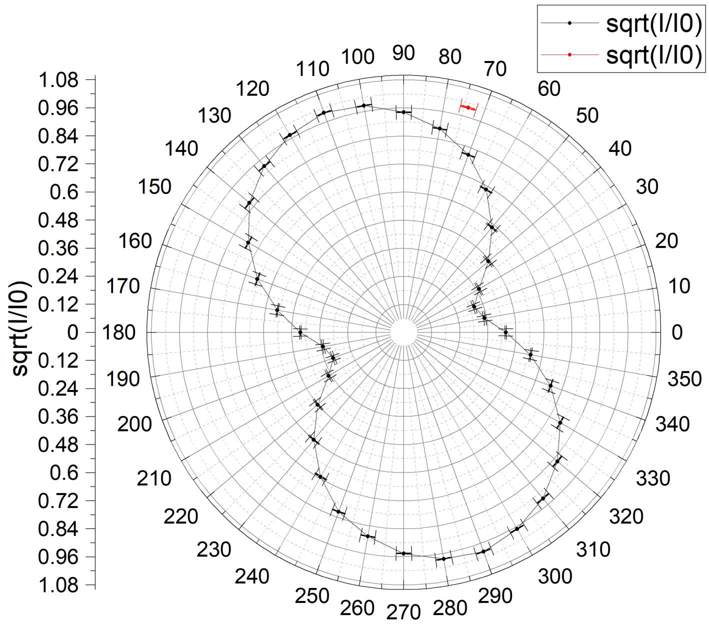

Figure 13: Marked position

5 Conclusions

5.1 Conclusions

In this lab, we understand some properties of light. Particularly, we study the polarization phenomenon and verify Malus' law, as well as understand the way half- and quarter-wave plates work in optical systems. Generation and detection of elliptically and circularly polarized light are also be investigated.

5.1.1 Malus' Law

We apply linear fit to I/I_0 vs. $cos^2\theta$. The result shows that the Pearson's r is very close to 0 and the intercept is close to 1, which means I/I_0 is proportional to $cos^2\theta$. The slope is is very close to the theoretical value 1, with the relative error -0.5%.

From the result we verify Malus' Law: $I_{light} = I_{light,0} \cdot cos^2\theta$.

5.1.2 Linearly Polarized Light

We apply linear fit to $\Delta\theta vs.\theta$.The result shows that the Pearson's r is very close to 1, and the intercept is very close to 0. These mean that $\Delta\theta$ is proportional to θ. The slope is very close to the theoretical value 2(Ref[1]) with the relative error 1.05%.

This means that the polarization axis of a polarized light passing through the half-wave plate is rotated by an angle 2α .

5.1.3 Circularly and Elliptically Polarized Light

5.1.3.1 Wave-plate Rotation angle=0°

Observing the polar plot of $\sqrt{I/I_0}$ vs. θ, we can see that there are two maximum light intensity near θ=0°and θ=180°and two light extinction near θ=90°and θ=270°. Since the minimum intensity is zero, so in this case, the transmitted light is linearly polarized. Its polarization axis is parallel to the optical axis of the 1/4-wave plate.

5.1.3.2 Wave-plate Rotation angle=20°

Observing the polar plot of $\sqrt{I/I_0}$ vs. θ, we can see that there are two maximum light intensity near θ=110°and θ=290°and two light intensity minimums near θ=20°and θ=200°, In this case, the transmitted light is elliptically polarized.

5.1.3.3 Wave-plate Rotation angle=45°

Observing the polar plot of $\sqrt{I/I_0}$ vs. θ, there is no light extinction, and the maximum and minimum light intensity are about the same. When θ is changing, the light intensity changes slightly. In this case, the transmitted light is circularly polarized.

5.1.3.4 Wave-plate Rotation angle=70°

$$For\ \alpha = \theta\ \frac{E_x}{A_o^2} + \frac{E_y^2}{A_e^2} = 1\ (|cos\theta| > |sin\theta|)\ \ max\ at\ 90°$$

$$For\ \alpha = 90° - \theta\ \frac{E_x}{A_o^2} + \frac{E_y^2}{A_e^2} = 1\ (|cos\theta| > |sin\theta|)\ \ max\ at\ 0°$$

$$Angle\ between\ max = |\theta + \theta_{max\theta} + (90° - \theta) + \theta_{max(90°-\theta)}| = 180°$$

Observing the marked position on Fig.13, we find that the θ of red spot and the maximum spot of 20°and 70°has the sum of about 184°. $u_r = \frac{184-180}{180} \times 100\% = 2.2\%$. From the equation above we can see that the two maximum current should be the same since the two elliptical has the same long axis. But there exit certain error in the experimrnt, the reason will be discussed in next error analysis part.

We conclude that when rotation angles are complementary, the position where the light intensity reaches its maximum have the sum about 180°. The maximum light intensity is almost the same.

5.2 Error Analysis

- Environment light will largely affect the result of this experiment.
- It is hard to make sure that the light passes through the center of the lens.
- We cannot ensure that the light is incident normally on the polarizer and the analyzer.
- The readings of the universal meter are unstable. The same angle may not result in the same light intensity at a second time.
- The uncertainty of the angle is relatively large.
- There are fingerprints and other stains on the surface of the plate and analyzer, which may cause error when the light is polarized.
- Since the light intensity is indirectly found by measuring the electric current, there might be errors in the process of transforming the two factors.
- The class instructor came in and touched the laser beam, so the light intensity will be slightly changed, thus causing the difference between the maximum intensity in 70°and 20°.

5.3 Improvements

- Do this experiment in a completely dark environment.
- Use digital device to record the angle of the analyzer, so that the uncertainty of the angle can be reduced.

6 Reference

1. Jiang Shaopeng, Qin Tian, Cao Jianjun, Lin Xinyu, Zhong Xiaoxue, Mateusz Krzyzosiak, M. Krzyzosiak (2019). Exercise 4 - lab manual [rev. 5].pdf Shanghai: UMJI-SJTU.

APPENDIX

A Data Table

$\theta[°]\pm2[°]$	I$[\mu A]\pm0.001\mu A$	I/I_0	u_{I/I_0}	$cos^2\theta$	$u_{cos^2\theta}$
0	0.738	1.0000	0.0008	1.000	0.000
5	0.729	0.9878	0.0008	0.992	0.006
10	0.715	0.9688	0.0008	0.970	0.012
15	0.690	0.9350	0.0008	0.933	0.017
20	0.661	0.8957	0.0008	0.88	0.02
25	0.612	0.8293	0.0008	0.82	0.03
30	0.555	0.7520	0.0008	0.75	0.03
35	0.501	0.6789	0.0008	0.67	0.03
40	0.446	0.6043	0.0008	0.59	0.03
45	0.380	0.5149	0.0008	0.50	0.03
50	0.317	0.4295	0.0007	0.41	0.03
55	0.253	0.3428	0.0007	0.33	0.03
60	0.192	0.2602	0.0007	0.25	0.03
65	0.141	0.1911	0.0007	0.18	0.03
70	0.095	0.1287	0.0007	0.12	0.02
75	0.057	0.0772	0.0007	0.067	0.017
80	0.026	0.0352	0.0007	0.030	0.012
85	0.008	0.0108	0.0007	0.008	0.006
90	0.000	0.0000	0.0007	0.000	0.000

Table 2: Calculation of $cos^2\theta$ and I/I0

α	u_α	$\Delta\theta$	$u_{\Delta\theta}$
0	2.00	0	2.83
10	2.00	20	2.83
20	2.00	40	2.83
30	2.00	60	2.83
40	2.00	82	2.83
50	2.00	101	2.83
60	2.00	121	2.83
70	2.00	142	2.83
80	2.00	162	2.83
90	2.00	181	2.83

Table 3: Measurement data for the 1/2-wave plate

Maximum Electric Current I_0=0.500±0.001μA									
θ	u_θ	I	$\sqrt{I/I_0}$	$u_{\sqrt{I/I_0}}$	θ	u_θ	I	$\sqrt{I/I_0}$	$u_{\sqrt{I/I_0}}$
0	2	0	0	/	180	2	0	0	/
10	2	0.013	0.161	0.006	190	2	0.012	0.155	0.007
20	2	0.052	0.322	0.003	200	2	0.051	0.319	0.003
30	2	0.116	0.482	0.002	210	2	0.115	0.480	0.002
40	2	0.194	0.6229	0.0019	220	2	0.192	0.6197	0.0019
50	2	0.283	0.7523	0.0017	230	2	0.276	0.7430	0.0017
60	2	0.361	0.8497	0.0015	240	2	0.354	0.8414	0.0016
70	2	0.428	0.9252	0.0015	250	2	0.421	0.9176	0.0015
80	2	0.482	0.9818	0.0014	260	2	0.466	0.9654	0.0014
90	2	0.499	0.9990	0.0014	270	2	0.485	0.9849	0.0014
100	2	0.48	0.9798	0.0014	280	2	0.473	0.9726	0.0014
110	2	0.438	0.9359	0.0015	290	2	0.428	0.9252	0.0015
120	2	0.378	0.8695	0.0015	300	2	0.365	0.8544	0.0015
130	2	0.292	0.7642	0.0016	310	2	0.285	0.7550	0.0017
140	2	0.207	0.6434	0.0018	320	2	0.203	0.6372	0.0019
150	2	0.127	0.504	0.002	330	2	0.124	0.498	0.002
160	2	0.063	0.355	0.003	340	2	0.058	0.341	0.003
170	2	0.018	0.190	0.005	350	2	0.017	0.184	0.006

Table 4: Calculation and Data of 0° 1/4-waveplate

Maximum Electric Current $I_0 = 0.3980.001\mu A$									
θ	u_θ	I	$\sqrt{I/I_0}$	$u_{\sqrt{I/I_0}}$	θ	u_θ	I	$\sqrt{I/I_0}$	$u_{\sqrt{I/I_0}}$
0	2	0.076	0.437	0.003	180	2	0.078	0.443	0.003
10	2	0.049	0.351	0.004	190	2	0.049	0.351	0.004
20	2	0.041	0.321	0.004	200	2	0.041	0.321	0.004
30	2	0.055	0.372	0.004	210	2	0.055	0.372	0.004
40	2	0.089	0.473	0.003	220	2	0.092	0.481	0.003
50	2	0.137	0.587	0.002	230	2	0[1]43	0.599	0.002
60	2	0.198	0.705	0.002	240	2	0.202	0.712	0.002
70	2	0.260	0.808	0.002	250	2	0.265	0.816	0.002
80	2	0.312	0.8854	0.0019	260	2	0.312	0.8854	0.0019
90	2	0.353	0.9418	0.0018	270	2	0.356	0.9458	0.0018
100	2	0.386	0.9848	0.0018	280	2	0.385	0.9835	0.0018
110	2	0.397	0.9987	0.0018	290	2	0.396	0.9975	0.0018
120	2	0.378	0.9746	0.0018	300	2	0.375	0.9707	0.0018
130	2	0.343	0.9283	0.0018	310	2	0.342	0.9270	0.0018
140	2	0.296	0.8624	0.0019	320	2	0.293	0.8580	0.0019
150	2	0.235	0.768	0.002	330	2	0.238	0.773	0.002
160	2	0.177	0.667	0.002	340	2	0.178	0.669	0.002
170	2	0.120	0.549	0.003	350	2	0.121	0.551	0.003

Table 5: Calculation and Data of 20° 1/4-waveplate

Maximum Electric Current $I_0 = 0.211 \pm 0.001\mu A$									
θ	u_θ	I	$\sqrt{I/I_0}$	$u_{\sqrt{I/I_0}}$	θ	u_θ	I	$\sqrt{I/I_0}$	$u_{\sqrt{I/I_0}}$
0	2	0.172	0.903	0.004	180	2	0.173	0.905	0.004
10	2	0.178	0.918	0.004	190	2	0.182	0.929	0.003
20	2	0.186	0.939	0.003	200	2	0.187	0.941	0.003
30	2	0.187	0.941	0.003	210	2	0.196	0.964	0.003
40	2	0.210	0.998	0.003	220	2	0.196	0.964	0.003
50	2	0.215	1.009	0.003	230	2	0.200	0.974	0.003
60	2	0.217	1.014	0.003	240	2	0.208	0.993	0.003
70	2	0.210	0.998	0.003	250	2	0.206	0.988	0.003
80	2	0.202	0.978	0.003	260	2	0.198	0.969	0.003
90	2	0.210	0.998	0.003	270	2	0.190	0.949	0.003
100	2	0.193	0.956	0.003	280	2	0.181	0.926	0.003
110	2	0.183	0.931	0.003	290	2	0.171	0.900	0.004
120	2	0.178	0.918	0.004	300	2	0.166	0.887	0.004
130	2	0.175	0.911	0.004	310	2	0.155	0.857	0.004
140	2	0.176	0.913	0.004	320	2	0.157	0.863	0.004
150	2	0.165	0.884	0.004	330	2	0.160	0.871	0.004
160	2	0.167	0.890	0.004	340	2	0.159	0.868	0.004
170	2	0.170	0.898	0.004	350	2	0.166	0.887	0.004

Table 6: Calculation and Data of 45° 1/4-waveplate

I[μA] $\pm 0.001\mu A$	0.317
θ[°]±2°	74
$\sqrt{I/I_0}$	1.000
$u_{\sqrt{I/I_0}}$	0.002

Table 7: Calculation and Data of 70°1/4-waveplate