UM-SJTU Joint Institute

Physics Laboratory

(Vp241)

Laboratory Report

Exercise 2

The Hall Probe:

Characteristics and Applications

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1. Introduction

In 1879 E.H. Hall observed that when an electric current passes through a sample placed in a magnetic field, electric potential difference proportional to the current and to the magnetic field appears across the material in the direction perpendicular to both the current and the magnetic field. This effect is known as the Hall effect, and since its discovery it has found many practical applications. The principle of the Hall effect is used in devices for magnetic field measurements as well as in position and motion detectors.

In this lab, our objectives are:

- Study the principle of the Hall effect and its applications by using a Hall probe.
- Verify that the Hall voltage is proportional to the magnetic field.
- Study the sensitivity of an integrated Hall probe by calculating the magnetic field at the center of a solenoid. Measure the magnetic field distribution along the axis of the solenoid and compare it with the corresponding theoretical curve.

2. Theoretical Background

2.1 Hall Effect

When a conducting sheet is placed in a magnetic field so that the direction of the magnetic field B is perpendicular to the plane of the sheet and an electric current I passes through the sheet in the direction parallel with the plane shown in Figure 1, an electric potential difference will appear between the sides a and b of the sheet. This effect is called the Hall effect and the potential difference is called the Hall voltage U_H.

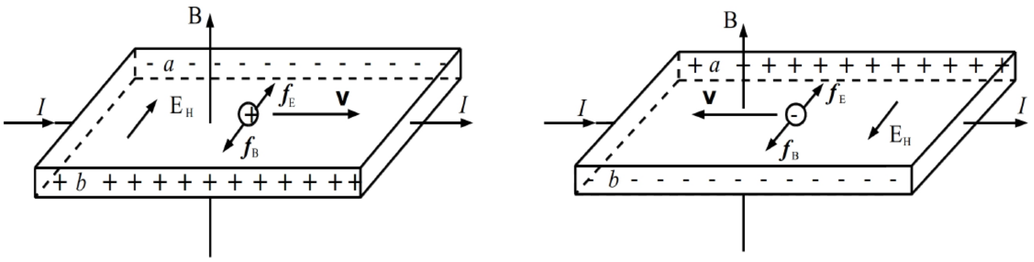


Figure 1 the illustration of the Hall effect

The Hall effect is called by the Lorentz force which acts on charges moving in a magnetic field. The Lorentz force F_B causes the moving charges to deflect and accumulate on one side of the sheet, which increases the magnitude of the transverse electric filed E_H(the Hall field). This field will produce an electric force F_E acting on the moving charges whose direction is opposite to F_B. When balance between F_E and F_B is reached, the charges no longer deflect and U_H stabilizes. When B is upward and I is to the right. If the sheet carries positive charge, then the voltage of a is lower than b. We can analyze the sign of U_H and determine the type of the charge carriers in semiconductors.

When the external magnetic field B is not too strong, the Hall voltage is proportional to both the current and the B, and inversely proportional to the thickness of the sheet d:

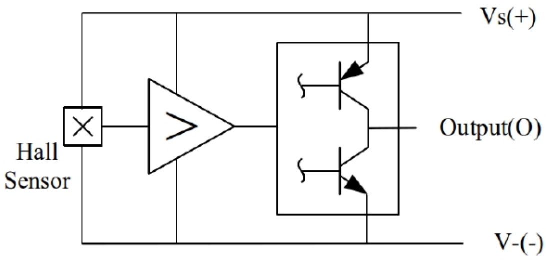
$$U_H = R_H \frac{IB}{d} = KIB \qquad (1)$$

Where R_H is called the Hall coefficient and K=$\frac{R_H}{d} = \frac{K_H}{I}$, where K_H is called sensitivity of the Hall element.

2.2 Integrated Hall Probe

When K_H and I are fixed, B can be found by measuring the Hall voltage with a Hall probe. Since the Hall voltage is very small, it should be amplified before the measurement.
The Hall probe and the circuit are designed using silicon. A device called integrated Hall probe is a single device combining both the Hall probe and the electric circuit. The integrated Hall probe SS495A consists of a Hall sensor, an amplifier, and a voltage compensator(Figure 2). The output voltage U can be read ignoring the residual voltage. The working voltage is $U_S = 5V$, and the output voltage U_0 is approximately 2.5V when B=0. The relation between the output voltage U and the magnitude of the magnetic field is:

$$B = \frac{U - U_0}{K_H} \qquad (2)$$



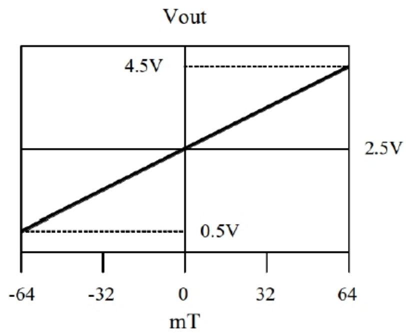


Figure 2 **Figure 3**

Figure 2 The integrated Hall probe SS495A

Figure 3 The relation between the output voltage U and B

2.3 Magnetic Field Distribution Inside a Solenoid

On the axis of a single layer solenoid, the magnetic field distribution can be represented as

$$B(x) = \mu_0 \frac{N}{L} I_M \left\{ \frac{L + 2x}{2\left[D^2 + (L + 2x)^2\right]^{\frac{1}{2}}} + \frac{L - 2x}{2\left[D^2 + (L - 2x)^2\right]^{\frac{1}{2}}} \right\} = C(x) I_M, \qquad (3)$$

where N is the number of turns of the solenoid,
L is its length,

I_M is the current through the solenoid wire,
D is the solenoid's diameter.
The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}$H/m.
In this lab, the solenoid has ten layers, and the magnetic field B(x) for each layer can be calculated using the equation above. Then the net magnetic on the axis of the solenoid can be found by adding contributions due to all layers. The theoretical value of the magnetic field inside the solenoid with $I_M = 0.1A$ is given in Table 1.

x [cm]	B [mT]	x [cm]	B [mT]
± 0.0	1.4366	± 8.0	1.4057
± 1.0	1.4363	± 9.0	1.3856
± 2.0	1.4356	± 10.0	1.3478
± 3.0	1.4343	± 11.0	1.2685
± 4.0	1.4323	± 11.5	1.1963
± 5.0	1.4292	± 12.0	1.0863
± 6.0	1.4245	± 12.5	0.9261
± 7.0	1.4173	± 13.0	0.7233

Table 1 Theoretical value of the magnetic field inside the solenoid.

2.4 Study of the Geomagnetic Field with a Hall Probe

The geomagnetic field of the Earth is similar to that of a bar magnet tilted about 11.5° from the spin axis of the Earth.
Figure 4 shows the magnetic filed lines of the Geomagnetic field.

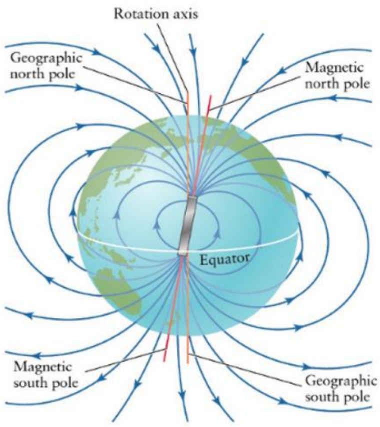


Figure 4 Magnetic field of the Earth

Figure 5 shows the geomagnetic field distribution of China in 1970. The magnetic inclination is about 44.5° and the magnitude of the magnetic field in Shanghai is about 48000nT.

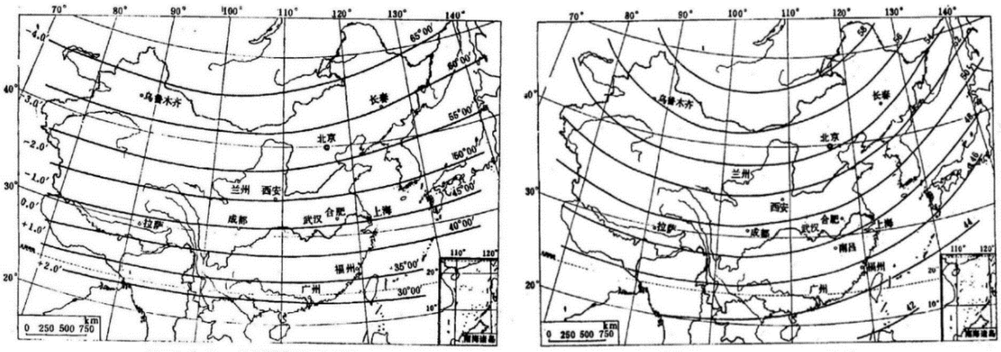


Figure 5 Geomagnetic inclination in China,1970(left). The magnitude of the geomagnetic field in China,1970(right).

3. Experimental Setup

3.1 Apparatus

Figure 6 shows the integrated Hall probe SS495A, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

$$K_H = 31.25 \pm 1.25 V/T or 3.125 \pm 0.125 mV/G$$

Figure 6 Experimental setup

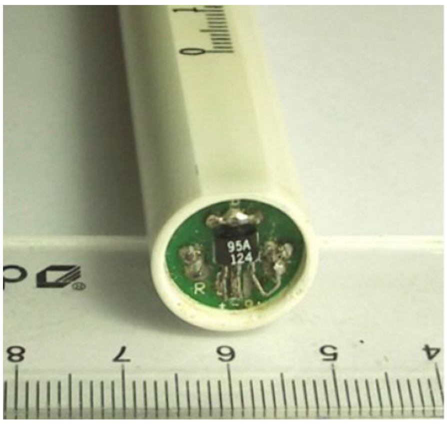


Figure 7 Integrated Hall probe SS495A

3.2 Device Information

The information of each measurement device is shown in Table 2.

apparatus	range	Minimum scale of value	Maximum uncertainty
Voltage source	/	0.01V	0.5%
Voltimeter	/	0.001Vor0.0001V	$0.05\% + 6 \times 10^{-3} or 6 \times 10^{-4}$V
Current Source	/	0.01A	2%
Graduated Ruler	0~30cm	0.1cm	$\pm$0.05cm

Table 2 Information of Each Measurement Device

4. Measurement Procedure

4.1 Relation Between Sensitivity K_H and Working Voltage U_S

1. Place the integrated Hall probe at the center of the solenoid. Set the working voltage at 5 V and measure the output voltage $U_0(I_M = 0)$ and U (I_M= 250 mA). Take the theoretical value of B(x = 0) from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).
2. Measure K_H for different values of U_S (from 2.8 V to 10 V). Calculate K_H/U_S and plot the curve K_H/U_S vs. U_S.

4.2 Relation between Output Voltage U and Magnetic Field B

1. With B=0, U_S=5 V, connect the 2.4~2.6 V output terminal of the DC voltage divider and the negative port of the voltmeter. Adjust the voltage until U_0= 0.
2. Place the integrated Hall probe at the center of the solenoid and measure the output voltage U for different values of I_M ranging from 0 to 500 mA, with intervals of 50mA.
3. Explain the relation between B(x = 0) and the Hall voltage U_H. Pay attention to the fact that the output voltage U is the amplified signal from U_H. The theoretical value of B(x = 0) can be found from Table 1.
4. Plot the curve U vs. B and find the sensitivity K_H by a linear fit (use a computer). Compare the value you obtained with the theoretical value in given in the Apparatus section and the value you have found in the first part.

4.3 Magnetic Field Distribution Inside the Solenoid

1. Measure the magnetic field distribution along the axis of the solenoid for I_M = 250mA, record the output voltage U and the corresponding position x. Then find B = B(x). (Use the value of K_H found in the previous part of the experiment).
2. Use a computer to plot the theoretical and the experimental curve showing the magnetic field distribution inside the solenoid. Use dots for the data measured and a solid line for the theoretical curve. The origin of the plot should be at the center of the solenoid.

5. Results

5.1 Relation Between Sensitivity K_H and Working Voltage U_S

5.1.1 Calculation of K_H

U_S[V]$\pm$0.5%[V]	5.00
$U_0(I_M$=0)[V]$\pm 0.05\% + 6 \times 10^{-3}$[V]	2.517
$U(I_M$=250mA)[V]$\pm 0.05\% + 6 \times 10^{-3}$[V]	2.636

Table 3 Data for U0 and U with Us=5V

From Table 1 we get when x=0cm, I_M=0.1A, B_0=1.4366mT.
Now I=250mA. Since B is proportional to I, we calculate B:

$$B = B_0 \times \frac{I}{I_M} = 1.4366 \times 10^{-3} \times \frac{250 \times 10^{-3}}{0.1} = 3.5915 \times 10^{-3} T$$

Using equation(2), we calculate $K_H *$:

$$K_H * = \frac{U - U_0}{B} = \frac{2.636 - 2.517}{3.5915 \times 10^{-3}} = 33.13 \pm 3 V/T$$

5.1.2 Measurement of U_0 and U under different U_s

We change different value of U_S.
U_0 is measured when $\boldsymbol{I_M}$=0
U is measured when $\boldsymbol{I_M}$=250mA.
The experimental data are recorded in Table 4.

	$U_S(V)$	$U_0(V)$	$U(V)$	$u_{U_S}(V)$	$u_{U_0}(V)$	$u_U(V)$
1	2.80	1.4336	1.5040	0.014	0.001	0.001
2	3.29	1.6768	1.7576	0.016	0.001	0.001
3	3.67	1.8665	1.9562	0.018	0.002	0.002
4	4.04	2.0523	2.146	0.020	0.002	0.002
5	4.65	2.347	2.460	0.023	0.007	0.007
6	4.98	2.508	2.619	0.025	0.007	0.007
7	5.25	2.640	2.763	0.026	0.007	0.007
8	5.83	2.919	3.054	0.029	0.007	0.008
9	6.32	3.157	3.293	0.032	0.008	0.008
10	6.75	3.364	3.512	0.034	0.008	0.008
11	7.01	3.489	3.639	0.035	0.008	0.008
12	7.53	3.746	3.901	0.038	0.008	0.008
13	7.92	3.935	4.091	0.040	0.008	0.008
14	8.30	4.117	4.275	0.042	0.008	0.008
15	8.86	4.395	4.566	0.044	0.008	0.008
16	9.27	4.590	4.761	0.046	0.008	0.008
17	9.74	4.816	4.992	0.049	0.008	0.008
18	10.00	4.951	5.121	0.050	0.008	0.009

Table 4 Data for U0 and U with different Us

5.1.3 Relation between K_H and U_s

Then we calculate $\frac{K_H}{U_S} = \frac{U-U_0}{BU_S}$

Using the first row of data as an example,

$$\frac{K_H}{U_S} = \frac{U - U_0}{BU_S} = \frac{1.5040 - 1.4336}{3.5915 \times 10^{-3} \times 2.80} = 7.00 \pm 0.14T^{-1}$$

The values of $\frac{K_H}{U_S}$ with different U_S and their uncertainties are shown in Table5.

	$U_S(V)$	$\frac{K_H}{U_S}(T^{-1})$	u_{U_S}(V)	$u_{\frac{K_H}{U_S}}(T^{-1})$
1	2.80	7.00	0.014	0.14
2	3.29	6.84	0.016	0.12
3	3.67	6.81	0.018	0.22
4	4.04	6.46	0.020	0.20
5	4.65	6.77	0.023	0.61
6	4.98	6.21	0.025	0.58
7	5.25	6.52	0.026	0.55
8	5.83	6.45	0.029	0.51
9	6.32	5.99	0.032	0.48
10	6.75	6.10	0.034	0.45
11	7.01	5.96	0.035	0.44
12	7.53	5.73	0.038	0.41
13	7.92	5.48	0.040	0.40
14	8.30	5.30	0.042	0.39
15	8.86	5.37	0.044	0.37
16	9.27	5.14	0.046	0.36
17	9.74	5.03	0.049	0.34
18	10.00	4.73	0.050	0.34

Table 5 Us and KH/Us

Then we plot the dots, shown in Figure8:

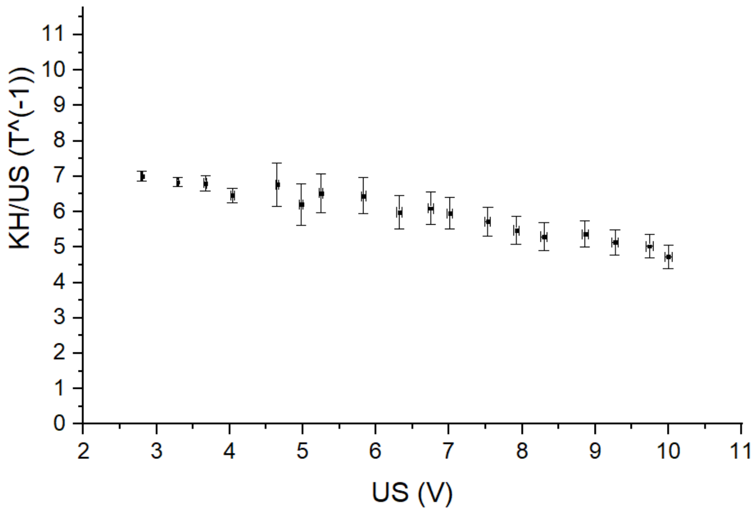


Figure 8 Plot of KH/Us and Us

5.2 Relation between Output Voltage U and Magnetic Field B

5.2.1 Measurement of I_M and U

We measure output voltage U under different I_M, and record the results in Table6:

	I_M[A]	u_{I_M}[A]	U[V]	u_U[V]
1	0.00	0	0.0000	0.0006
2	0.05	0.001	0.0228	0.0006
3	0.10	0.002	0.0449	0.0006
4	0.15	0.003	0.0699	0.0006
5	0.20	0.004	0.0894	0.0006
6	0.25	0.005	0.1165	0.0007
7	0.30	0.006	0.1405	0.0007
8	0.35	0.007	0.1612	0.0007
9	0.40	0.008	0.1844	0.0007
10	0.45	0.009	0.2090	0.0007
11	0.50	0.01	0.2310	0.0007

Table 6 Measurement data for the IM vs. U relation

From Table 1 we get when x=0cm, I_M=0.1A, B_0=1.4366mT.

Now I=I_M. Since B is proportional to I, we calculate B. Take I_M=0.05A for example:

$$B = B_0 \times \frac{I_M}{I_0} = 1.4366 \times 10^{-3} \times \frac{0.05}{0.1} = 0.00072 \pm 0.00001T$$

We arrange the data and the uncertainty in Table7:

	U[V]	u_U[V]	B[T]	u_B[T]
1	0.0000	0.0006	0	0
2	0.0228	0.0006	0.00072	0.00001
3	0.0449	0.0006	0.0014	0.00003
4	0.0699	0.0006	0.0022	0.00004
5	0.0894	0.0006	0.0029	0.00006
6	0.1165	0.0007	0.0036	0.00007
7	0.1405	0.0007	0.0043	0.00009
8	0.1612	0.0007	0.0050	0.00010
9	0.1844	0.0007	0.0058	0.00011
10	0.2090	0.0007	0.0065	0.00013
11	0.2310	0.0007	0.0072	0.00014

Table 7 Relation of U and B

Then we apply linear fit to U and B, shown in Figure 9:

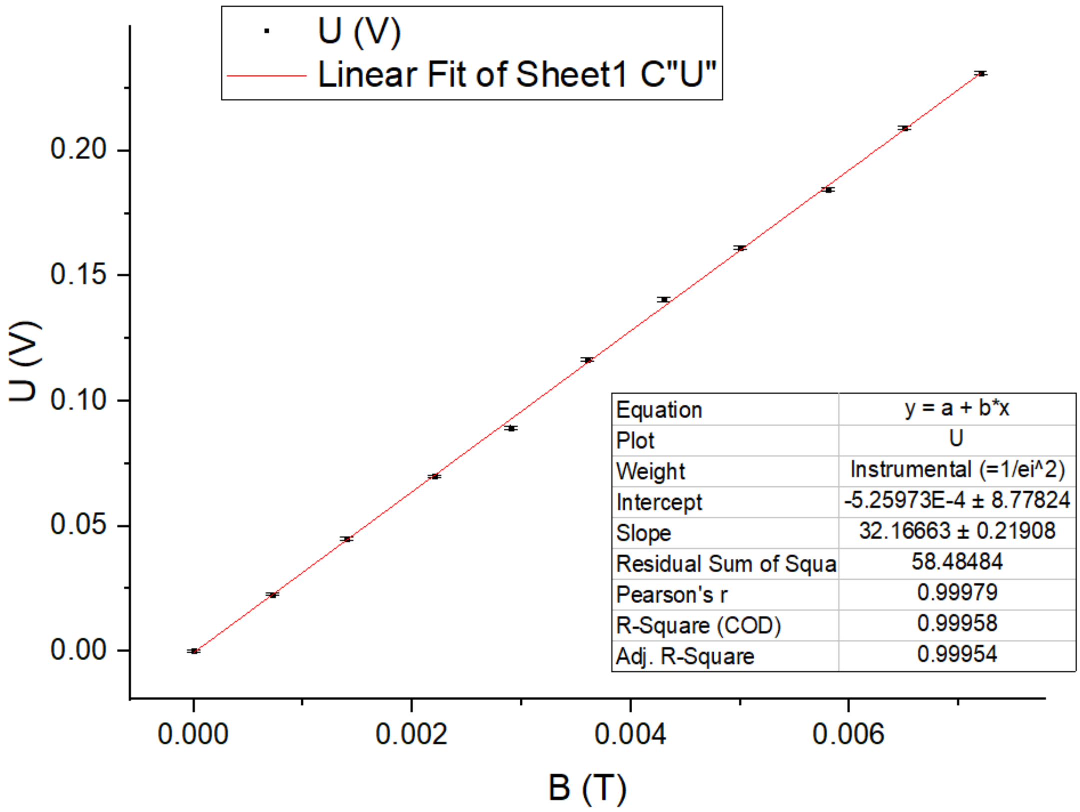


Figure 9 Linear Fit of U and B

The slope of the line is 32.17V/T, and the standard error is 0.21908V/T. The CI half-width is 0.21908$\times$ $t_{9,0.95} = 0.21908 \times 2.36 = 0.51703$, rounded to be 0.5V/T. Therefore, the experimental $K_H = 32.17 \pm 0.5V/T$.

In 5.1, we calculate $K_H *$ is 33.13$\pm 3V/T$.

The relative error compared with $K_H *$ is

$$u_{r1} = \frac{32.17 - 33.13}{33.13} \times 100\% = -2.9\%$$

We take $K_H = 31.25 \pm 1.25V$ marked on the apparatus as the theoretical value, the relative error compared with the theoretical value is

$$u_{r2} = \frac{32.17 - 31.25}{31.25} \times 100\% = 2.9\%$$

5.3 Magnetic Field Distribution Inside the Solenoid

We measured the magnetic field inside the Solenoid along the axis at different distance. The experimental data is shown in Table8:

	x[cm]$\pm 0.05[cm]$	U[V]	$u_U[V]$		x[cm]$\pm 0.05[cm]$	U[V]	$u_U[V]$
1	0.00	0.0057	0.0006	27	12.50	0.1152	0.0007
2	0.30	0.0081	0.0006	28	14.00	0.1152	0.0007
3	0.60	0.0105	0.0006	29	15.00	0.1154	0.0007
4	0.80	0.0131	0.0006	30	16.50	0.1156	0.0007
5	1.00	0.0151	0.0006	31	18.00	0.1155	0.0007
6	1.20	0.0166	0.0006	32	20.00	0.1152	0.0007
7	1.40	0.0196	0.0006	33	21.00	0.1148	0.0007
8	1.80	0.0268	0.0006	34	22.00	0.1145	0.0007
9	2.00	0.0317	0.0006	35	23.00	0.1138	0.0007
10	2.30	0.0388	0.0006	36	24.50	0.1120	0.0007
11	2.60	0.0478	0.0006	37	25.50	0.1097	0.0007
12	2.80	0.0554	0.0006	38	26.50	0.1040	0.0007
13	3.20	0.0686	0.0006	39	27.00	0.0999	0.0007
14	3.50	0.0772	0.0006	40	27.50	0.0920	0.0006
15	3.80	0.0854	0.0006	41	27.90	0.0852	0.0006
16	4.50	0.0903	0.0006	42	28.20	0.0771	0.0006
17	5.00	0.0979	0.0006	43	28.50	0.0679	0.0006
18	5.50	0.1038	0.0007	44	28.80	0.0581	0.0006
19	6.00	0.1063	0.0007	45	29.00	0.0503	0.0006
20	6.50	0.1079	0.0007	46	29.30	0.0411	0.0006
21	7.20	0.1094	0.0007	47	29.50	0.0346	0.0006
22	7.80	0.1103	0.0007	48	29.60	0.0325	0.0006
23	8.50	0.1137	0.0007	49	29.70	0.0303	0.0006
24	9.50	0.1142	0.0007	50	29.80	0.0276	0.0006
25	10.50	0.1147	0.0007	51	29.90	0.0256	0.0006
26	11.50	0.1150	0.0007	52	30.00	0.0234	0.0006

Table 8 U with different x

Based on the value of U, we calculate the corresponding B by the equation:

$$B = \frac{U}{K_H}$$

Take U=0.0057V for example, $B = \frac{U}{K_H} = \frac{0.0057}{32.17} = 0.00018 \pm 0.00002T$

We arrange all the data and uncertainties in Table 9:

	x[cm]$\pm 0.05[cm]$	B[T]	$u_B[T]$		x[cm]$\pm 0.05[cm]$	B[T]	$u_B[T]$
1	0.00	0.00018	0.00002	27	12.50	0.00358	0.00006
2	0.30	0.00025	0.00002	28	14.00	0.00358	0.00006
3	0.60	0.00033	0.00002	29	15.00	0.00359	0.00006
4	0.80	0.00041	0.00002	30	16.50	0.00359	0.00006
5	1.00	0.00047	0.00002	31	18.00	0.00359	0.00006
6	1.20	0.00052	0.00002	32	20.00	0.00358	0.00006
7	1.40	0.00061	0.00002	33	21.00	0.00357	0.00006
8	1.80	0.00083	0.00002	34	22.00	0.00356	0.00006
9	2.00	0.00099	0.00002	35	23.00	0.00354	0.00006
10	2.30	0.00121	0.00003	36	24.50	0.00348	0.00006
11	2.60	0.00149	0.00003	37	25.50	0.00341	0.00006
12	2.80	0.00172	0.00003	38	26.50	0.00323	0.00005
13	3.20	0.00213	0.00004	39	27.00	0.00311	0.00005
14	3.50	0.00240	0.00004	40	27.50	0.00286	0.00005
15	3.80	0.00265	0.00005	41	27.90	0.00265	0.00005
16	4.50	0.00281	0.00005	42	28.20	0.00240	0.00004
17	5.00	0.00304	0.00005	43	28.50	0.00211	0.00004
18	5.50	0.00323	0.00005	44	28.80	0.00181	0.00003
19	6.00	0.00330	0.00005	45	29.00	0.00156	0.00003
20	6.50	0.00335	0.00006	46	29.30	0.00128	0.00003
21	7.20	0.00340	0.00006	47	29.50	0.00108	0.00003
22	7.80	0.00343	0.00006	48	29.60	0.00101	0.00002
23	8.50	0.00353	0.00006	49	29.70	0.00094	0.00002
24	9.50	0.00355	0.00006	50	29.80	0.00086	0.00002
25	10.50	0.00357	0.00006	51	29.90	0.00080	0.00002
26	11.50	0.00357	0.00006	52	30.00	0.00073	0.00002

Table 9 B with different x

We then calculate the theoretical value of B using the data from Table1:

$$B(theo) = B_0 \times \frac{I}{I_M}$$

Take $B_0 = 1.4363 \times 10^{-3}$ for example:

$$B(theo) = B_0 \times \frac{I}{I_M} = 1.4363 \times 10^{-3} \times \frac{250 \times 10^{-3}}{0.1} = 0.003591T$$

The x we take is the scale on the ruler, but the true x should be the distance from the center of the solenoid. Therefore, when we plot the figure, the horizontal coordinate should be x-13(cm).

For all the data, we calculate B(theo), shown in Table 10:

x[cm]	$B_0[mT]$	B(theo)[T]	x[cm]	$B_0[mT]$	B(theo)[T]
-13.0	0.7233	0.0018	1.0	1.4363	0.0036
-12.5	0.9261	0.0023	2.0	1.4356	0.0036
-12.0	1.0863	0.0027	3.0	1.4343	0.0036
-11.5	1.1963	0.0030	4.0	1.4323	0.0036
-11.0	1.2685	0.0032	5.0	1.4292	0.0036
-10.0	1.3478	0.0034	6.0	1.4245	0.0036
-9.0	1.3856	0.0035	7.0	1.4173	0.0035
-8.0	1.4057	0.0035	8.0	1.4057	0.0035
-7.0	1.4173	0.0035	9.0	1.3856	0.0035
-6.0	1.4245	0.0036	10.0	1.3478	0.0034
-5.0	1.4292	0.0036	11.0	1.2685	0.0032
-4.0	1.4323	0.0036	11.5	1.1963	0.0030
-3.0	1.4343	0.0036	12.0	1.0863	0.0027
-2.0	1.4356	0.0036	12.5	0.9261	0.0023
-1.0	1.4363	0.0036	13.0	0.7233	0.0018
0.0	1.4366	0.0036			

Table 10 Theoretical value of B at different x

We plot the experimental B and theoretical B in one figure:

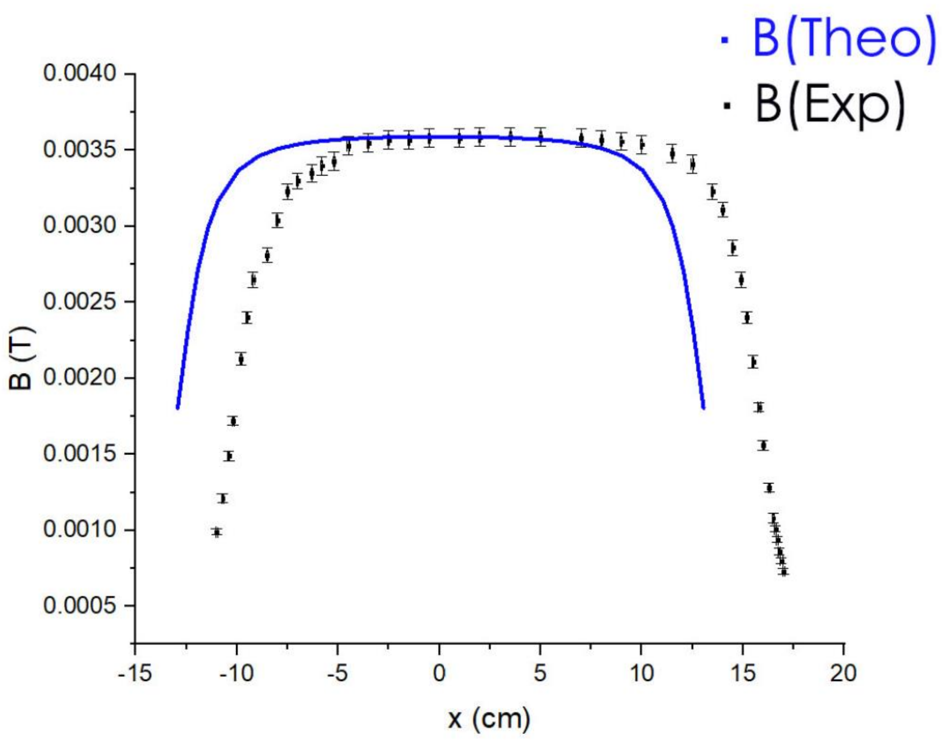


Figure 10 B(exp) and B(Theo) vs. x

6. Conclusions and discussion

6.1 Conclusions

In this lab, we

- Study the principle of the Hall effect and its applications by using a Hall probe.
- Calculate the sensitivity K_H based on our experimental data and compare it with the theoretical value.
- Verify that the Hall voltage $U_H$is proportional to the magnetic field B.
- Measure the magnetic field distribution along the axis of the solenoid and compare it with the corresponding theoretical curve.

In 5.1.1, when Us=5V, we measured $K_H *$=$33.13 \pm 3V/T$.

In 5.1.3, we plot $\frac{K_H}{U_S}$ vs. U_S and found that $\frac{K_H}{U_S}$ almost remains the same although U_S is changing. This shows that K_H is proportional to U_S. However, from the plot we can see $\frac{K_H}{U_S}$ has an incline to decrease when U_S is increasing, and this error will be analyzed in 6.2.

In 5.2, we measured the output voltage U with different I_M. Since I_M is proportional to B, we thus get the relation between U and B. We find that U is proportional to B.

The slope indicates that $K_H = 32.17 \pm 0.5V/T$.

We take $K_H = 31.25 \pm 1.25V$ marked on the apparatus as the theoretical value. Also, in 5.1.1 we measured $K_H *$=$33.13 \pm 3V/T$. Therefore, K_H from linear fit has a relative error of -2.9% compared with $K_H *$ and has a relative error of 2.9% compared with the theoretical value.

In 5.3, we measured the magnetic field B at different x along the axis of the solenoid. From Table 1 we obtain the theoretical distribution of the magnetic field, and we plot them in Figure 10 and compare them. Their shapes coincide but there is a slight position deviation of the two shape, and the error will be discussed in 6.2.

6.2 Error Analysis

- In 5.1, the reason why $\frac{K_H}{U_S}$ has an incline to decrease when U_S is increasing might be the wrong experimental procedure. I first set I_M=0A, change the Us from 2.8 to 10V, finish this round of measurement and then set I_M=250mA, set the Us equal to the round1 values and then did the second round of measurement. Therefore, with the same Us, U0 and U are measured under a large time interval, which results in deviations in these two factors.
- In 5.3, the reason why the two shapes has slight deviation might be: if the theoretical shape is moved 2cm to the right then the two shapes can cover each other. Hence, maybe the center of the solenoid is actually at 15cm on the scale.

- Other errors might exist because:
  Instability of displaying of U0 and U;
  In procedure 4.2.1, U0 is very hard to be adjusted to completely 0.
  The Hall probe might move its position when measuring;
  Rise of temperature causing R to change.

6.3 Improvements

- Since we have to change Us and Im by switching the channel, it's hard to obtain the value we want. I suggest the source value can be set digitally.
- Add a procedure to let students determine the center of the solenoid.

7. Reference

[1] M. Krzyzosiak (2019). Exercise 2 - lab manual [rev. 3.8].pdf Shanghai: UMJI-SJTU.

A. Measurement uncertainty analysis

A.1 Uncertainty in Analysis of the Relation Between Sensitivity K_H and Working Voltage U_S

A.1.1 Uncertainty in Calculation of K_H

$$u_{Us} = Us \times 0.5\% = 5.000.5\% = 0.03V$$

$$u_{U_0} = U_0 \times 0.05\% + 6 \times 10^{-3} = 2.517 \times 0.05\% + 6 \times 10^{-3} = 0.007V$$

$$u_U = U \times 0.05\% + 6 \times 10^{-3} = 2.636 \times 0.05\% + 6 \times 10^{-3} = 0.007V$$

$$K_H = \frac{U - U_0}{B}$$

$$\frac{\partial K_H}{\partial U} = \frac{1}{B} = \frac{1}{3.5915 \times 10^{-3}} = 278.44$$

$$\frac{\partial K_H}{\partial U_0} = -\frac{1}{B} = -\frac{1}{3.5915 \times 10^{-3}} = -278.44$$

$$u_{K_H} = \sqrt{(\frac{\partial K_H}{\partial U})^2 {u_U}^2 + (\frac{\partial K_H}{\partial U_0})^2 {u_{U_0}}^2} = \sqrt{(278.44)^2 \times 0.007^2 + (-278.44)^2 \times 0.007^2} = 3V/T$$

A.1.2 Uncertainty in Measurement of U_0 and U under different U_s

The uncertainty of Us is:

$$u_{Us} = Us \times 0.5\%$$

The uncertainty of U_0 is:

$$u_{U_0} = U_0 \times 0.05\% + 6 \times 10^{-4}$$

When U_0 has four decimal numbers.

$$u_{U_0} = U_0 \times 0.05\% + 6 \times 10^{-3}$$

When U_0 has three decimal numbers.

The uncertainty of U is:

$$u_U = U \times 0.05\% + 6 \times 10^{-4}$$

When U has four decimal numbers.

$$u_U = U \times 0.05\% + 6 \times 10^{-3}$$

When U has three decimal numbers.

Take the 1st row of data for example:

$$u_{Us} = Us \times 0.5\% = 2.80 \times 0.5\% = 0.014V$$

$$u_{U_0} = U_0 \times 0.05\% + 6 \times 10^{-4} = 1.4336 \times 0.05\% + 6 \times 10^{-4} = 0.0013V$$

$$u_U = U \times 0.05\% + 6 \times 10^{-4} = 1.5040 \times 0.05\% + 6 \times 10^{-4} = 0.0014V$$

We arrange all the data and uncertainties in Table 11:

	$U_S(V)$	$U_0(V)$	$U(V)$	$u_{U_S}(V)$	$u_{U_0}(V)$	$u_U(V)$
1	2.80	1.4336	1.5040	0.014	0.001	0.001
2	3.29	1.6768	1.7576	0.016	0.001	0.001
3	3.67	1.8665	1.9562	0.018	0.002	0.002
4	4.04	2.0523	2.146	0.020	0.002	0.002
5	4.65	2.347	2.460	0.023	0.007	0.007
6	4.98	2.508	2.619	0.025	0.007	0.007
7	5.25	2,640	2.763	0.026	0.007	0.007
8	5.83	2.919	3.054	0.029	0.007	0.008
9	6.32	3.157	3.293	0.032	0.008	0.008
10	6.75	3.364	3.512	0.034	0.008	0.008
11	7.01	3.489	3.639	0.035	0.008	0.008
12	7.53	3.746	3.901	0.038	0.008	0.008
13	7.92	3.935	4.091	0.040	0.008	0.008
14	8.30	4.117	4.275	0.042	0.008	0.008
15	8.86	4.395	4.566	0.044	0.008	0.008
16	9.27	4.590	4.761	0.046	0.008	0.008
17	9.74	4.816	4.992	0.049	0.008	0.008
18	10.00	4.951	5.121	0.050	0.008	0.009

Table 11 uncertainties of Us U0 and U

A.1.3 Uncertainty in the analysis of the relation between K_H and U_S

$$\frac{K_H}{U_S} = \frac{U - U_0}{BU_S}$$

$$\frac{\partial \frac{K_H}{U_S}}{\partial \mathrm{U}} = \frac{1}{BU_S} \qquad \frac{\partial \frac{K_H}{U_S}}{\partial U_0} = \frac{-1}{BU_S} \qquad \frac{\partial \frac{K_H}{U_S}}{\partial U_S} = \frac{U_0 - U}{BU_S{}^2}$$

$$u_{\frac{K_H}{U_S}} = \sqrt{\left(\frac{\partial \frac{K_H}{U_S}}{\partial \mathrm{U}}\right)^2 u_U{}^2 + \left(\frac{\partial \frac{K_H}{U_S}}{\partial U_0}\right)^2 u_{U_0}{}^2 + \left(\frac{\partial \frac{K_H}{U_S}}{\partial U_S}\right)^2 u_{U_S}{}^2}$$

$$= \sqrt{\left(\frac{1}{BU_S}\right)^2 u_U{}^2 + \left(\frac{-1}{BU_S}\right)^2 u_{U_0}{}^2 + \left(\frac{U_0 - U}{BU_S{}^2}\right)^2 u_{U_S}{}^2}$$

Take the 1st row of data for example:

$$u_{\frac{K_H}{U_S}} = \sqrt{\left(\frac{0.001}{3.5915 \times 10^{-3} \times 2.80}\right)^2 + \left(\frac{-0.001}{3.5915 \times 10^{-3} \times 2.80}\right)^2 + \left(\frac{1.4336 - 1.5040}{3.5915 \times 10^{-3} \times 2.80^2}\right)^2 \times 0.014^2}$$

$$= 0.14T^{-1}$$

We arrange all the data and uncertainties in Table 12

:

	$U_S(V)$	$\frac{K_H}{U_S}(T^{-1})$	u_{Us}(V)	$u_{\frac{K_H}{U_S}}(T^{-1})$
1	2.80	7.00	0.014	0.14
2	3.29	6.84	0.016	0.12
3	3.67	6.81	0.018	0.22
4	4.04	6.46	0.020	0.20
5	4.65	6.77	0.023	0.61
6	4.98	6.21	0.025	0.58
7	5.25	6.52	0.026	0.55
8	5.83	6.45	0.029	0.51
9	6.32	5.99	0.032	0.48
10	6.75	6.10	0.034	0.45
11	7.01	5.96	0.035	0.44
12	7.53	5.73	0.038	0.41
13	7.92	5.48	0.040	0.40
14	8.30	5.30	0.042	0.39
15	8.86	5.37	0.044	0.37
16	9.27	5.14	0.046	0.36
17	9.74	5.03	0.049	0.34
18	10.00	4.73	0.050	0.34

Table 12 Uncertainties of Us and KH/US

A.2 Uncertainty in the Analysis of the Relation between Output Voltage U and Magnetic Field B

A.2.1 Uncertainty in Measurement of I_M and U

$$u_{I_M} = I_M \times 2\%$$

$$u_U = U \times 0.05\% + 6 \times 10^{-4}$$

Take the 2nd row of data for example:

$$u_{I_M} = I_M \times 2\% = 0.05 \times 2\% = 0.001A$$

$$u_U = U \times 0.05\% + 6 \times 10^{-4} = 0.0228 \times 0.05\% + 6 \times 10^{-4} = 0.0006V$$

We arrange the data and the uncertainty in Table 13

	I_M[A]	u_{I_M}[A]	U[V]	u_U[V]
1	0.00	0	0.0000	0.0006
2	0.05	0.001	0.0228	0.0006
3	0.10	0.002	0.0449	0.0006
4	0.15	0.003	0.0699	0.0006
5	0.20	0.004	0.0894	0.0006
6	0.25	0.005	0.1165	0.0007
7	0.30	0.006	0.1405	0.0007
8	0.35	0.007	0.1612	0.0007
9	0.40	0.008	0.1844	0.0007
10	0.45	0.009	0.2090	0.0007
11	0.50	0.01	0.2310	0.0007

Table 13 Uncertainties of Im and U

A.2.2 Uncertainty in Calculation of B

$$B = B_0 \times \frac{I_M}{I_0} = 1.4366 \times 10^{-3} \times \frac{I_M}{0.1}$$

$$u_B = 1.4366 \times 10^{-2} \times u_{I_M}$$

Take the 2nd row of data for example:

$$u_B = 1.4366 \times 10^{-2} \times u_{I_M} = 1.4366 \times 10^{-2} \times 0.001 = 0.00001T$$

For all the data, the uncertainties are listed in table 14:

	I_M[A]	u_{I_M}[A]	B[T]	u_B[T]
1	0.00	0	0	0
2	0.05	0.001	0.00072	0.00001
3	0.10	0.002	0.0014	0.00003
4	0.15	0.003	0.0022	0.00004
5	0.20	0.004	0.0029	0.00006
6	0.25	0.005	0.0036	0.00007
7	0.30	0.006	0.0043	0.00009
8	0.35	0.007	0.0050	0.00010

9	0.40	0.008	0.0058	0.00011
10	0.45	0.009	0.0065	0.00013
11	0.50	0.01	0.0072	0.00014

Table 14 Uncertainties of Im and B

A.3 Uncertainty in the Analysis of Magnetic Field Distribution Inside the Solenoid

A.3.1 Uncertainty of the U

$$u_U = \mathrm{U} \times 0.05\% + 6 \times 10^{-4}$$

Take U=0.0081 for example,

$$u_U = \mathrm{U} \times 0.05\% + 6 \times 10^{-4} = 0.0081 \times 0.05\% + 6 \times 10^{-4} = 0.0006V$$

For all the data, the uncertainties are listed in table 15:

	U[V]	$u_U[V]$		U[V]	$u_U[V]$
1	0.0057	0.0006	27	0.1152	0.0007
2	0.0081	0.0006	28	0.1152	0.0007
3	0.0105	0.0006	29	0.1154	0.0007
4	0.0131	0.0006	30	0.1156	0.0007
5	0.0151	0.0006	31	0.1155	0.0007
6	0.0166	0.0006	32	0.1152	0.0007
7	0.0196	0.0006	33	0.1148	0.0007
8	0.0268	0.0006	34	0.1145	0.0007
9	0.0317	0.0006	35	0.1138	0.0007
10	0.0388	0.0006	36	0.1120	0.0007
11	0.0478	0.0006	37	0.1097	0.0007
12	0.0554	0.0006	38	0.1040	0.0007
13	0.0686	0.0006	39	0.0999	0.0007
14	0.0772	0.0006	40	0.0920	0.0006
15	0.0854	0.0006	41	0.0852	0.0006
16	0.0903	0.0006	42	0.0771	0.0006
17	0.0979	0.0006	43	0.0679	0.0006
18	0.1038	0.0007	44	0.0581	0.0006
19	0.1063	0.0007	45	0.0503	0.0006
20	0.1079	0.0007	46	0.0411	0.0006
21	0.1094	0.0007	47	0.0346	0.0006
22	0.1103	0.0007	48	0.0325	0.0006
23	0.1137	0.0007	49	0.0303	0.0006
24	0.1142	0.0007	50	0.0276	0.0006
25	0.1147	0.0007	51	0.0256	0.0006
26	0.1150	0.0007	52	0.0234	0.0006

Table 15 Uncertainties of U

A.3.2 Uncertainty of B

$$B = \frac{U}{K_H}$$

$$\frac{\partial B}{\partial U} = \frac{1}{K_H} = \frac{1}{32.17} = 0.0311$$

$$\frac{\partial B}{\partial K_H} = -\frac{U}{{K_H}^2}$$

$$u_B = \sqrt{(\frac{\partial B}{\partial U})^2 {u_U}^2 + (\frac{\partial B}{\partial K_H})^2 {u_{K_H}}^2} = \sqrt{(0.0311)^2 {u_U}^2 + (-\frac{U}{{K_H}^2})^2 {u_{K_H}}^2}$$

Take the 1st row of data for example:

$$u_B == \sqrt{(0.0311)^2 \times 0.0006^2 + \left(-\frac{0.0057}{32.17^2}\right)^2 \times 0.5^2} = 0.00002T$$

For all the data, the uncertainties are listed in Table 16:

	B[T]	$u_B[T]$		B[T]	$u_B[T]$
1	0.00018	0.00002	27	0.00358	0.00006
2	0.00025	0.00002	28	0.00358	0.00006
3	0.00033	0.00002	29	0.00359	0.00006
4	0.00041	0.00002	30	0.00359	0.00006
5	0.00047	0.00002	31	0.00359	0.00006
6	0.00052	0.00002	32	0.00358	0.00006
7	0.00061	0.00002	33	0.00357	0.00006
8	0.00083	0.00002	34	0.00356	0.00006
9	0.00099	0.00002	35	0.00354	0.00006
10	0.00121	0.00003	36	0.00348	0.00006
11	0.00149	0.00003	37	0.00341	0.00006
12	0.00172	0.00003	38	0.00323	0.00005
13	0.00213	0.00004	39	0.00311	0.00005
14	0.00240	0.00004	40	0.00286	0.00005
15	0.00265	0.00005	41	0.00265	0.00005
16	0.00281	0.00005	42	0.00240	0.00004
17	0.00304	0.00005	43	0.00211	0.00004
18	0.00323	0.00005	44	0.00181	0.00003
19	0.00330	0.00005	45	0.00156	0.00003
20	0.00335	0.00006	46	0.00128	0.00003
21	0.00340	0.00006	47	0.00108	0.00003
22	0.00343	0.00006	48	0.00101	0.00002
23	0.00353	0.00006	49	0.00094	0.00002
24	0.00355	0.00006	50	0.00086	0.00002
25	0.00357	0.00006	51	0.00080	0.00002
26	0.00357	0.00006	52	0.00073	0.00002

Table 16 Uncertainties of B

UM-SJTU Physics Laboratory Vp241
Data Sheet (Exercise 2)

Name: 何雯欣 Student ID: 518370910117

Name: ______ Student ID: ______

Group: 1 Date: 2019/10/25

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with a pencil or modified with a correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used. You are required to hand in the original data with your lab report, so please keep the data sheet properly.

U_S [V] ± 0.5% [V]	$U_0(I_M = 0)$ [V] ± 0.05% + 6×10⁻³ [V]	$U(I_M = 250\text{ mA})$ [V] ± 0.05% + 6×10⁻³ [V]
5.00	2.517	2.636

Table 1. Data for U_0 and U with $U_S = 5$ V.

	U_S [V] ± 0.5% [V]	U_0 [V] ± 0.05% + 6×10⁻⁴ [V]	U [V] ± 0.05% + 6×10⁻⁴ [V]
1	2.80	1.4336	1.5040
2	3.29	1.6768	1.7576
3	3.67	1.8665	1.9562
4	4.04	2.0523	2.146
5	4.65	2.347	2.460
6	4.98	2.508	2.619
7	5.25	2.640	2.763
8	5.83	2.919	3.054
9	6.32	3.157	3.293
10	6.75	3.364	3.512
11	7.01	3.489	3.639
12	7.53	3.746	3.901
13	7.92	3.935	4.091
14	8.30	4.117	4.275
15	8.86	4.395	4.566
16	9.27	4.590	4.761
17	9.74	4.816	4.992
18	10.00	4.951	5.121

Table 2. Data for U_0 and U with different U_S.

Instructor's signature: B

$+6\times10^{-4}$ V

	I_M [A] ± 2% [A]	U [V] ± 0.05% [V]
1	0.00	0.0000
2	0.05	0.0228
3	0.10	0.0449
4	0.15	0.0699
5	0.20	0.0894
6	0.25	0.1165
7	0.30	0.1405
8	0.35	0.1612
9	0.40	0.1844
10	0.45	0.2090
11	0.50	0.2310

Table 3. Measurement data for the I_M vs. U relation.

Instructor's signature: B.

	x [cm] ± 0.05 [cm]	U [V] ± 0.05% $+6\times10^{-4}$ [V]		x [cm] ± 0.05 [cm]	U [V] ± 0.05% $+6\times10^{-4}$ [V]
1	0.00	0.0057	27	12.50	0.1152
2	0.30	0.0081	28	14.00	0.1152
3	0.60	0.0105	29	15.00	0.1154
4	~~1.00~~ 0.80	~~[illegible]~~ 0.0131	30	16.50	0.1156
5	1.00	0.0151	31	18.00	0.1155
6	1.20	0.0166	32	20.00	0.1152
7	1.40	0.0196	33	21.00	0.1148
8	1.80	0.0268	34	22.00	0.1145
9	2.00	0.0317	35	23.00	0.1138
10	2.30	0.0388	36	24.50	0.1120
11	2.60	0.0478	37	25.50	0.1097
12	2.80	0.0554	38	26.50	0.1040
13	3.20	0.0686	39	27.00	0.0999
14	3.50	0.0772	40	27.50	0.0920
15	3.80	0.0854	41	27.90	0.0852
16	4.50	0.0903	42	28.20	0.0771
17	5.00	0.0979	43	28.50	0.0679
18	5.50	0.1038	44	28.80	0.0581
19	6.00	0.1063	45	29.00	0.0503
20	6.50	0.1079	46	29.30	0.0411
21	7.20	0.1094	47	29.50	0.0346
22	7.80	0.1103	48	29.60	0.0325
23	8.50	0.1137	49	29.70	0.0303
24	9.50	0.1142	50	29.80	0.0276
25	10.50	0.1147	51	29.90	0.0256
26	11.50	0.1150	52	30.00	0.0234

Table 4. Data for the U vs. x relation.

Instructor's signature: B.

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