UM-SJTU Joint Institute
Physics Laboratory
(Vp241)

Laboratory Report

Exercise 5
RC, RL, and RLC Circuits

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Date: November 6, 2020

1 Abstract

In this experiment, different characteristics of RC,RL and RLC circuits are studied and proved. We basically studied the time constant, different regimes and their waveforms, and the reasonance circuit. Calculations, figures and plots are all used to better illustrate the result. Also, we discuss the uncertainty and possible error in this experiment.

2 Introduction

2.1 Motivation

This experiment helps us better understand the RC, RL, and RLC circuits and also the characteristics of them, including the phase-frequency and their phenomenon. Also, this experiment helps us know better about the real circuit and the use of signal generator.

2.2 Theoretical Background

2.2.1 Transient Processes in RC, RL, RLC Series Circuits

RC Series Circuits

In a RC circuit, the process of charging or discharging of the capacitor is an one of the transient process. For first half of the cycle, the square-wave voltage is $U(t) = \mathcal{E}$.

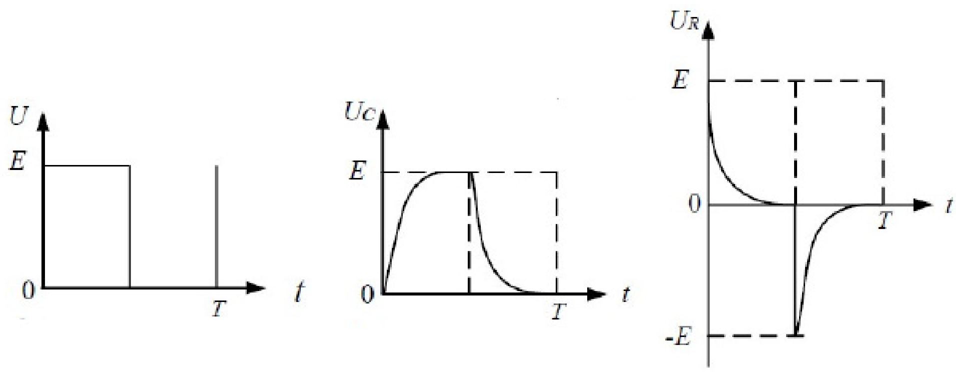

Figure 1: Charging/discharging curves for a RC series circuit.

For the second half-cycle, the square-wave voltage is zero,and the capacitor discharges through the resistor. The loop equation for the charging process is

$$RC\frac{dU_c}{dt} + U_C = \mathcal{E}$$

And we can get

$$U_R = iR = -\mathcal{E}e^{-\frac{t}{RC}}$$

where the magnitudes of both U_C and U_R decrease exponentially with time. And $RC = \tau$ is called the time constant of the circuit, and characterizes the dynamics of the transient process.

Another characteristics related to the time constant is the half-life period $T_{1/2}$, which indicates the time needed for U_C to decrease to a half of the initial value (or increase to a half of the terminal value), and may be also used to characterize the dynamics of the transient process. And the quantities are related by the equation

$$T_{1/2} = \tau \ln 2 \approx 0.693\tau.$$

RL Series Circuits
Similarly, we obtain

$$\tau = L/R \ \ and \ \ T_{1/2} = L \ln 2/R$$

RLC Series Circuit
Similarly we obtain

$$\frac{d^2U_c}{dt^2} + 2\beta\frac{dU_C}{dt} + \omega_0^2 U_C = \omega_0^2 \mathcal{E}$$

where $\beta = R/2L$ and $\omega_0 = 1/\sqrt{LC}$.

- If $\beta^2 - \omega_0^2 < 0$ (weak damping), the system is in the underdamped regime and the solution to the initial value problem is of the form

$$U_C = \mathcal{E} - \mathcal{E}e^{-\beta t}\left(\cos\omega t + \frac{\beta}{\omega}\sin\omega t\right)$$

  where $\omega = \sqrt{\omega_0^2 - \beta^2}$

- If $\beta^2 - \omega_0^2 > 0$, the system is in the overdamped regime with the solution of the form

$$U_C = \mathcal{E} - \frac{\mathcal{E}}{2\gamma}e^{-\beta t}\left[(\beta+\gamma)e^{\gamma t} - (\beta-\gamma)e^{-\gamma t}\right]$$

  where $\gamma = \sqrt{\beta^2 - \omega_0^2}$

- Finally, if $\beta^2 - \omega_0^2 = 0$, the system is said to be critically damped, and

$$U_C = \mathcal{E} - \mathcal{E}(1+\beta t)e^{-\beta t}$$

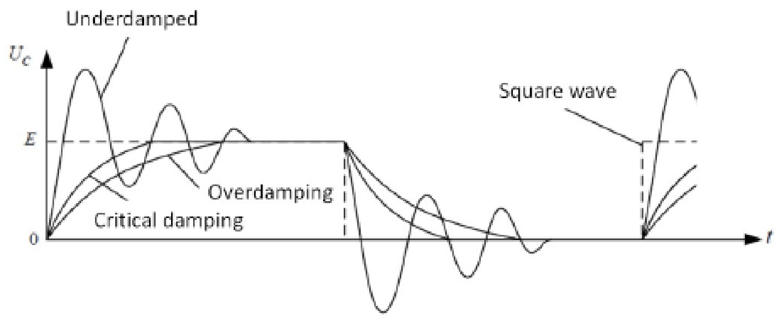

Figure 2: Three different regimes of transient processes in a RLC series circuit.

2.2.2 RC, RL Steady–State Circuits

When a sinusoidal alternating input voltage is provided to a RC (or RL) series circuit, the amplitude and the phase of the voltage will change with the frequency.

$$\varphi = \tan^{-1}\left(\frac{U_L}{U_R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right), \quad \varphi = \tan^{-1}\left(-\frac{U_C}{U_R}\right) = \tan^{-1}\left(-\frac{1}{\omega RC}\right)$$

2.2.3 RLC Resonant Circuit

RLC Series Circuit

A generic RLC series circuit is shown in Figure 3.

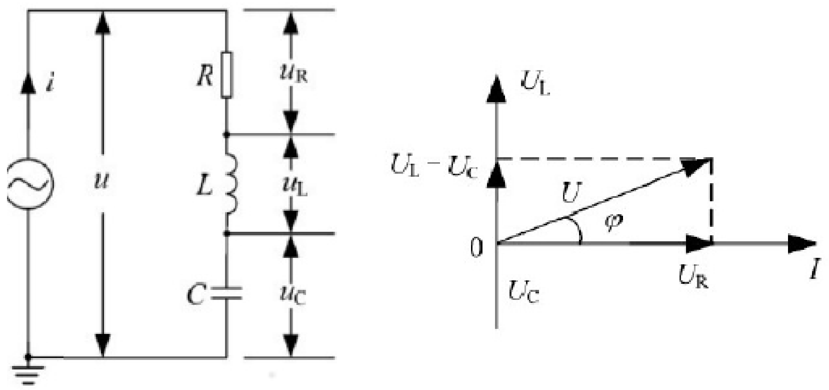

Figure 3: RLC series circuit.

The phase difference between the current and the voltage in the circuit

$$\varphi = \tan^{-1}\left(\frac{U_L - U_C}{U_R}\right) = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Resonance

If the frequency of the input signal provided by the source satisfies the condition

$$\omega_0 L = \frac{1}{\omega_0 C}, \quad \text{or, equivalently} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

the total impedance will reach a minimum, $Z_0 = R$. Note that the resistance R in a real circuit includes the internal resistance and all kinds of alternating-current power losses, so its actual value will be greater than the theoretical one.

When the current reaches its maximum, $I_\mathrm{m} = U/R$, the circuit is said to be at resonance. The frequency at which the resonance phenomenon occurs, is called the resonance frequency.

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

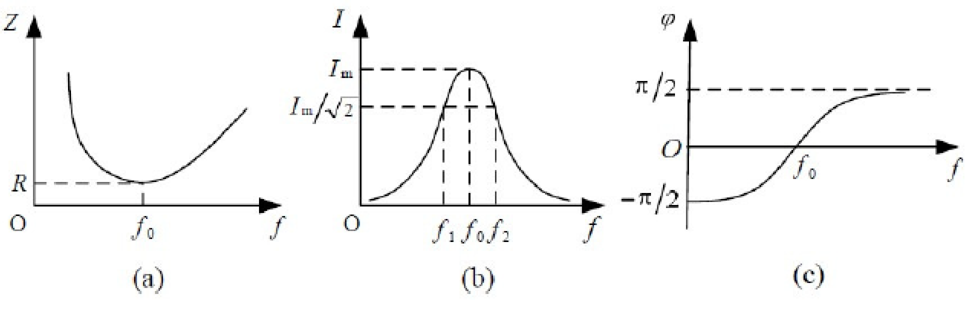

Figure 4: The impedance, the current and the phase difference as functions of the frequency for a RLC series circuit (generic sketches).

Quality Factor

For a circuit driven at the resonance frequency, the ratio of UL (or UC) to U is called the quality factor Q of a resonant circuit

$$Q = \frac{U_L}{U} = \omega_0 LR$$

The quality factor can also be found as

$$Q = \frac{f_0}{f_2 - f_1}$$

where f1 and f2 are two frequencies such that $I(f_1) = I(f_2) = I_m/\sqrt{2}$.

The equations and theories are all collected from books and manual, see reference 123

3 Description of experiment

3.1 Apparatus

The measurement setup consists of the following main elements: a signal generator, an oscilloscope, a digital multimeter, a wiring board, a fixed resistor , a variable resistor, a capacitor, as well as one inductor. And a collection of the uncertainty is shown in Table 1

Quantity	Apparatus	Precision
R	digital multimeter	0.01[Ω]
f	signal generator	0.001[Hz]
ϵ	signal generator	0.001[V_{pp}]
C	digital multimeter	0.01[nF]
$T_{\frac{1}{2}}$	oscilloscope	0.01[μs]

Table 1: Apparatus Uncertainty

3.2 Measurement Procedure

3.2.1 RC, RL Series Circuit

1. First we assemble a circuit with resistor and capacitor, connect the wave source and signal generator. Observe the change of the waveform when the time constant is smaller or greater than the period of the square–wave. Choose the frequency that allows the capacitor to fully charge/discharge.

2. Measure the $T_{1/2}$, calculate the time constant and compare with the theoretical value.

3. Change capacitor into inductor, repeat the above steps.

3.2.2 RLC Series Circuit

1. Use a capacitor, inductor and variable resistor to assemble a RLC series circuit. Observe the waveform when the capacitor voltage in the underdamped, critically damped, and over-damped regimes.

2. Adjust the variable resistor to the critically damped regime. Find $T_{1/2}$ and get τ, compare the theoretical value.

3.2.3 RLC Resonant Circuit

Apply a sinusoidal input voltage U_i to the RLC series circuit, change the frequency, then observe the change of the voltage U_R for a fixed resistor R, as well as the phase difference between U_R and U_i. Measure how U_R changes with U_i and calculate the phase difference according to Figure 4. Plot the graphs I/I_m vs. f/f_0 and φ vs. f/f_0. Estimate the resonance frequency and calculate the quality factor Q.

4 Results

4.1 Measurement of RC series circuit

From the lab we captured a screen shot of the RC circuit. We see the charging/discharging ofthe capacitor from the curve. The parameters of the RC series circuit are shown in Table 2.

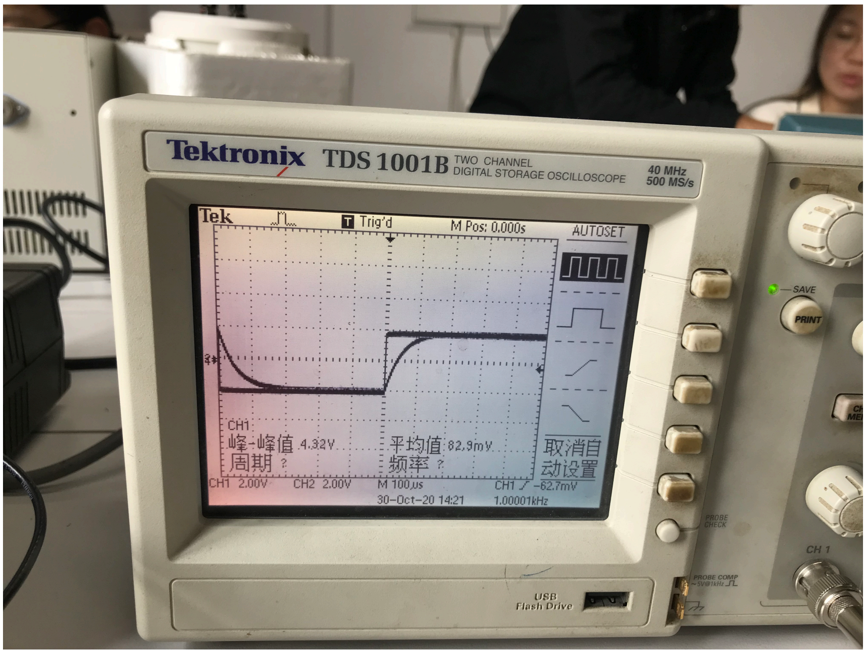

Figure 5: Screen capture of RC circuit.

R[Ω]±0.01[Ω]	f[kHz]±1 × 10^{-6}[kHz]	$\epsilon[V] \pm 0.001[V]$	C[nF] ± 0.01[nF]	T[μs] ± 0.01[μs]
99.94	1.000000	4.000	232.02	32.00

Table 2: $T_{1/2}$ measurement data for RC series circuit.

We then have experimental and theoretical value to be:

$$\tau_{ex} = \frac{T_{1/2}}{ln2} = 46.167 \pm 0.014[\mu s]$$

$$\tau_{th} = RC = 23.188 \pm 0.003[\mu s]$$

The relative error is 99.1%.

4.2 Measurement of RL Series Circuit.

The parameters of the RC series circuit are shown in Table 3. From which we can calculate

R[Ω]±0.01[Ω]	f[kHz]$\pm 1 \times 10^{-6}[kHz]$	$\epsilon[V] \pm 0.001[V]$	L$[H]$	T$[\mu s] \pm 0.01[\mu s]$
99.94	1.000000	4.000	0.01	72.00

Table 3: $T_{1/2}$ measurement data for RL series circuit.

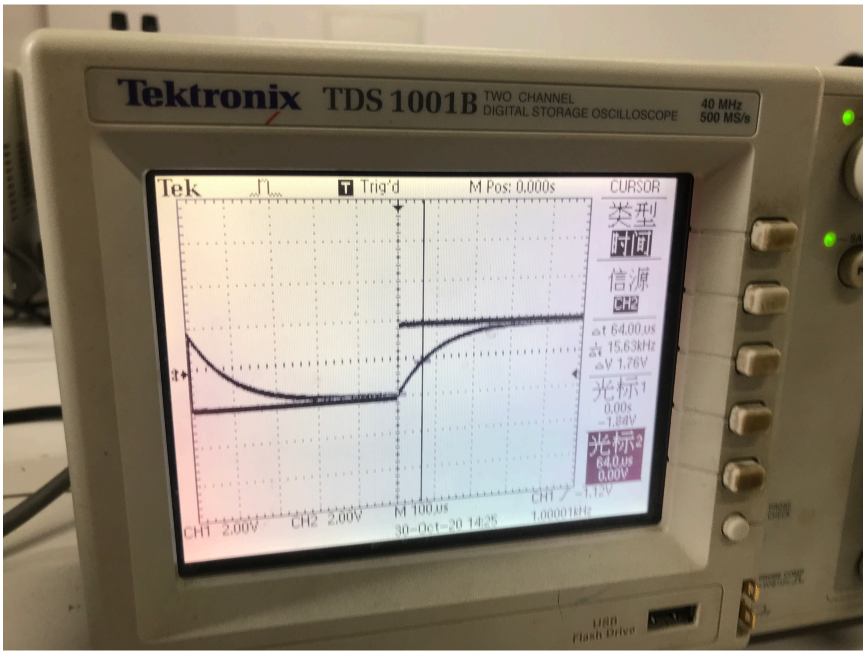

Figure 6: Screen capture of RL circuit.

We then have experimental and theoretical value to be:

$$\tau_{ex} = \frac{T_{1/2}}{ln2} = 103.874 \pm 0.014[\mu s]$$

$$\tau_{th} = \frac{L}{R} = 100.1 \pm 1.0[\mu s]$$

The relative error is 3.8%.

4.3 Measurement of RLC Series Circuit.

In the lab we captured the wave shown on the oscillator under three damping situations. Adjusting the resistance we also found the point where the critical damping occurs. The data under critically-damped situation are recorded in Table 4.

R[Ω]±0.01[Ω]	C[nF] ± 0.01[nF]	f[kHz]±1 × 10^{-6}[kHz]	ϵ[V] ± 0.001[V]	L[H]	T[μs] ± 0.01[μs]
99.94	233.02	1.000000	4.000	0.01	112.00

Table 4: $T_{1/2}$ measurement data for critically damped RLC series circuit.

And then we can calculate the theoretical and experimental value for τ.

$$\tau_{ex} = \frac{T_{1/2}}{1.68} = 66.667 \pm 0.006[\mu s]$$

$$\tau_{th} = \sqrt{LC} = 48.2721 \pm 0.0010[\mu s]$$

The relative error is 38.1%.

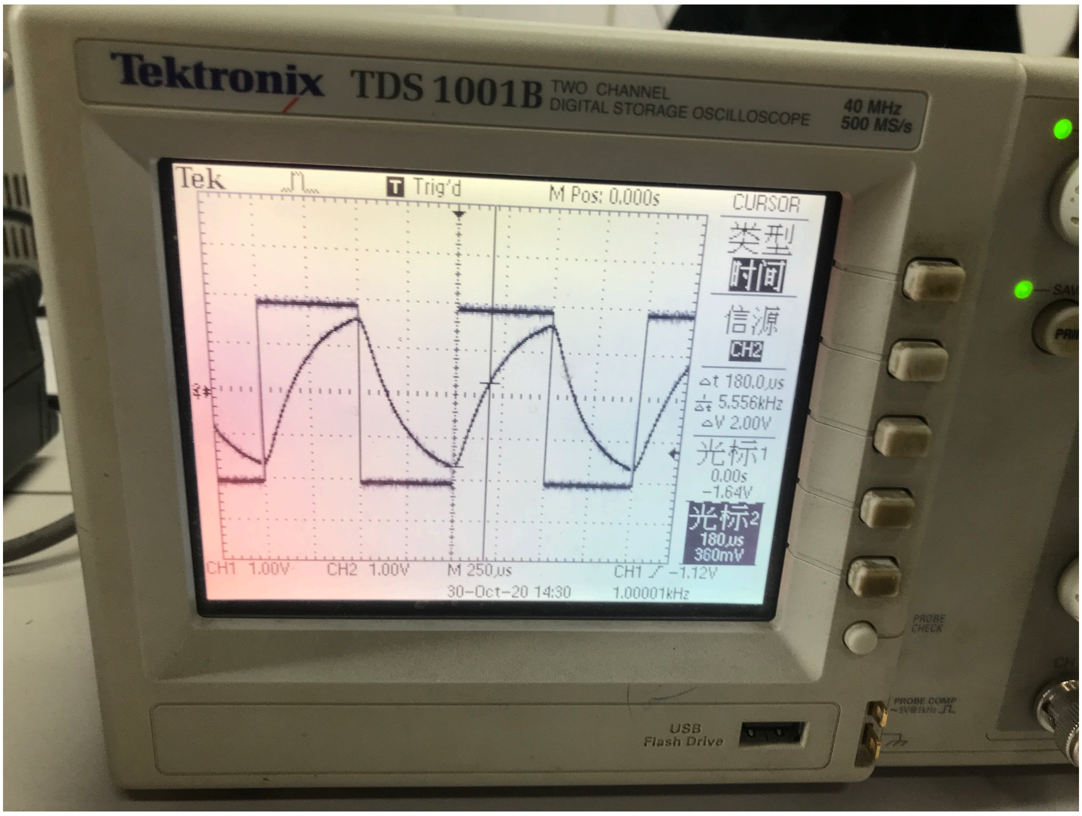

Figure 7: Screen capture of over-damped situation.

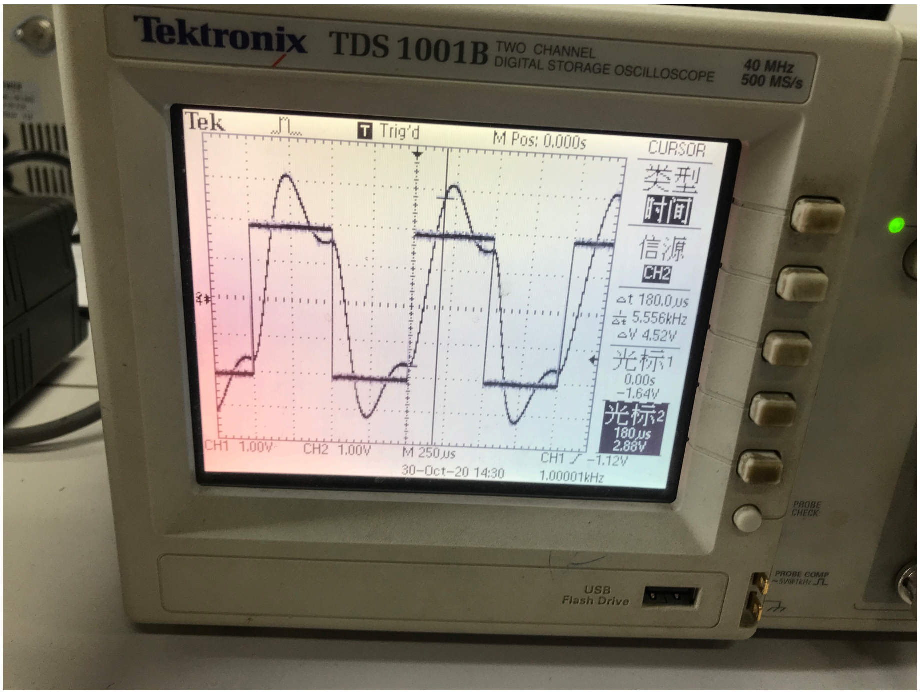

Figure 8: Screen capture of under-damped situation.

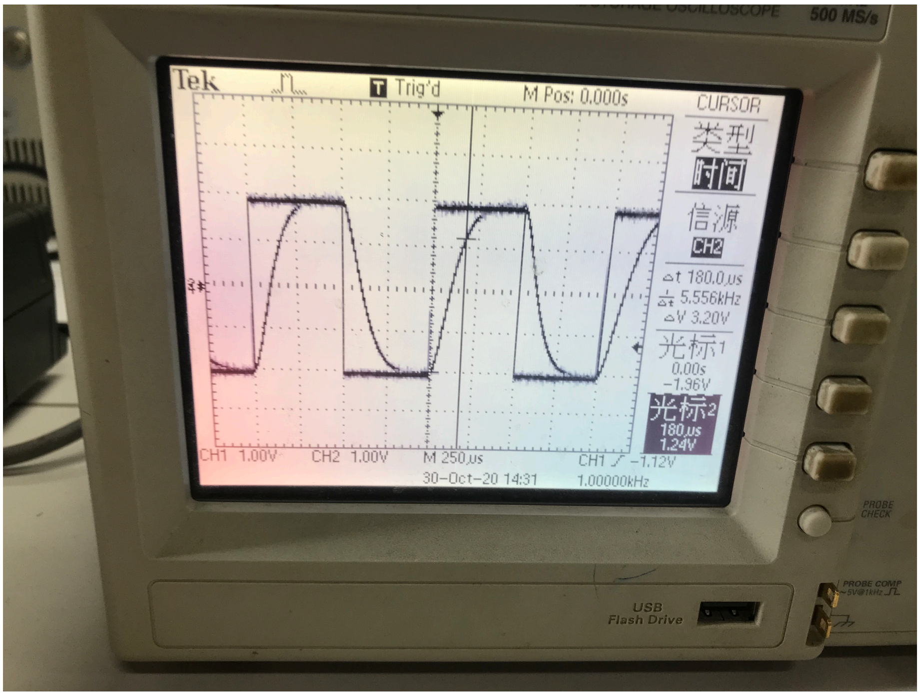

Figure 9: Screen capture of critical-damped situation.

4.4 Measurement of RLC Resonant Circuit.

All related data is shown in Table . It's worth mentioning that the units for f is Hz, for v is V, for ϕ is rad, and for all relative parameters there are no unit. Changing the input frequency, we observe the change of U_R for a fixed resistor R. The parameters are unchanged and the changing of U_R is recorded in Table 5.

f	U_R	I/Im	μI/Im	f/f0	μf/f0	ϕ_{ex}	$\mu\phi_{ex}$	ϕ_{theo}	$\mu\phi_{theo}$
316.970	0.40	0.1042	0.0003	0.1368000	0.0000004	1.4664	0.0003	-1.524000	0.000005
476.060	0.64	0.1667	0.0003	0.2054600	0.0000004	1.4033	0.0003	-1.499800	0.000007
696.940	0.96	0.2500	0.0003	0.3007900	0.0000005	1.3181	0.0003	-1.464500	0.000011
757.990	1.12	0.2917	0.0003	0.3271400	0.0000005	1.2748	0.0003	-1.454200	0.000012
1170.100	1.76	0.4583	0.0003	0.5049800	0.0000005	1.0947	0.0003	-1.377400	0.000019
1337.000	2.08	0.5417	0.0003	0.5770300	0.0000005	0.9984	0.0004	-1.34080	0.00002
1487.100	2.40	0.6250	0.0003	0.6418000	0.0000005	0.8957	0.0004	-1.30410	0.00003
1757.000	3.04	0.7917	0.0003	0.7583000	0.0000005	0.6573	0.0005	-1.22610	0.00003
1947.000	3.36	0.8750	0.0003	0.8403100	0.0000006	0.5054	0.0007	-1.15840	0.00004
2017.000	3.52	0.9167	0.0004	0.8705200	0.0000006	0.4111	0.0009	-1.13010	0.00004
2317.000	3.84	1.0000	0.0004	1.0000000	0.0000006	0.0000	Inf	-0.98059	0.00005
2377.000	3.76	0.9792	0.0004	1.0259000	0.0000006	0.2045	0.0018	-0.94396	0.00005
2467.000	3.68	0.9583	0.0004	1.0647000	0.0000006	0.2897	0.0013	-0.88386	0.00005
2607.000	3.52	0.9167	0.0004	1.1252000	0.0000006	0.4111	0.0009	-0.77655	0.00005
2847.000	3.20	0.8333	0.0003	1.2287000	0.0000007	0.5857	0.0006	-0.54817	0.00004
3097.000	2.88	0.7500	0.0003	1.3366000	0.0000007	0.7227	0.0005	-0.25401	0.00002
3307.000	2.72	0.7083	0.0003	1.4273000	0.0000008	0.7837	0.0005	0.012573	0.000002
3587.000	2.40	0.6250	0.0003	1.5481000	0.0000008	0.8957	0.0004	0.33657	0.00003
3967.100	2.08	0.5417	0.0003	1.7121000	0.0000009	0.9984	0.0004	0.65703	0.00005
7027.000	1.12	0.2917	0.0003	3.0328000	0.0000014	1.2748	0.0003	1.28830	0.00003
10027.000	0.80	0.2083	0.0003	4.3275000	0.0000019	1.3609	0.0003	1.394800	0.000017

Table 5: Measurement and Calculated Data for RLC Resonant Circuit

From the table above we can find $U_m = 3.84 \pm 0.01[V], f_0 = 2317.020[Hz]$, while we can calculate the theoretical value of f0 is:

$$f_{0_{theo}} = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 3297.03[Hz]$$

Thus we have the reltaive error to be -29.7%

Also using the data, we can get $f_1 = 1757.000[Hz]$ and $f_2 = 3307.050[Hz]$ To calculate the quality factor Q, we have

$$Q_{ex} = \frac{f_0}{f_2 - f_1} = 1.4948033 \pm 1.5 \times 10^{-6}$$

$$Q_{th} = \frac{\sqrt{LC}}{RC} = 2.07282 \pm 2 \times 10^{-4}$$

And we can get the reltaive error to be -28.0%.

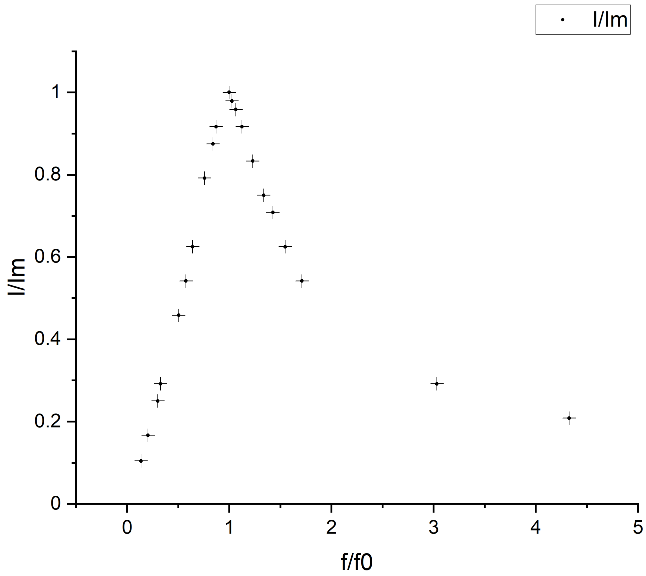

Figure 10: I/Im vs. f/f0 plot with uncertainty.

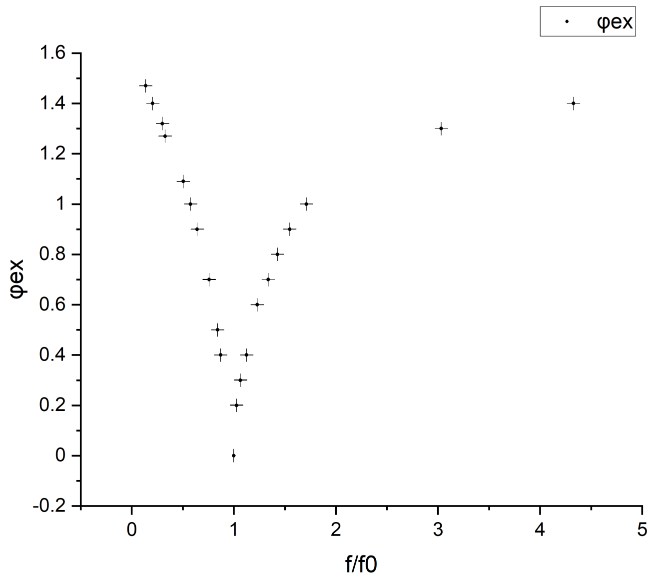

Figure 11: ϕth vs. f/f0 plot with uncertainty.

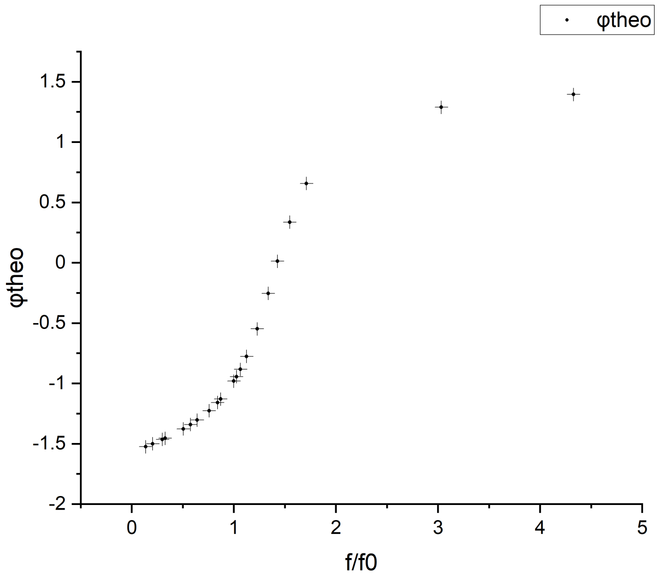

Figure 12: ϕex vs. f/f0 plot with uncertainty.

5 Conclusions

As a whole, we get three sets of experimental and theoretical value of time constant τ. For three sets of data, we each get the relative error to be 99.1%, 3.8%, 38.1%. And get three graphs of different damping regimes which basically conform to the theoretical graph. In the reasonance part, the experimental and theoretical value of resonance frequency f0 is found and they're calculated to have -29.7% reltiave error. Also, the theoretical and experimental value of quality factor Q is discussed and get the relative error -28.0%. The relationship of f/f0 with I/Im and ϕ_{ex} and ϕ_{th} are studied by graph.

However, we can find that the outcome is not that ideal, we can see most of the relative error is very large, so that the outcome is not that convincing and it seems that the experiment isn't that successful to prove the theroy. However, I first notice that the relative error for the second set of time constant is very small while the first one is the largest. And I consider it to be somehow related to C and L since the first theoretical value is determined by C and the second one is determined by L.

And in the quality factor part, I then try another approach, which is $Q = \omega_0 L/R$ and get Q to be roughly 1.46. And the relative error than becomes 2.1%, which justifies the idea that the value of C somehow get wrong. Adjusting C according to the first experimental value to be around $0.467\mu f$ and we get the other uncertainty to be 3.8%, $2.3\%, -4.6\%, 2.7\%$, which are rather small. According to the experiment procedure, the problem is mainly caused by the measurement device.

From this point of view, we can basically say that although there're some problems, the experiment illustrates the theories well. And for the suggestion, I think that many of the equipments should be renewed since the uncertainty is too large.

6 References

1.Bell, David A. Fundamentals of Electric Circuits. Oxford Univ. Press, 2009.
2.Qin Tian, Feng Yaming, Gu Yichen, Mateusz Krzyzosiak, Physics Laboratory VP241 Exercise 5 RC, RL, and RLC Circuits.
3.Young, Hugh D., et al. University Physics. Pearson, 2014.