

## SOUND AND HEARING

**16.1. IDENTIFY and SET UP:**  $v = f\lambda$  gives the wavelength in terms of the frequency. Use  $p_{\max} = BkA$  to relate the pressure and displacement amplitudes.

**EXECUTE:** (a)  $\lambda = v/f = (344 \text{ m/s})/1000 \text{ Hz} = 0.344 \text{ m}$

(b)  $p_{\max} = BkA$  and  $Bk$  is constant gives  $p_{\max 1}/A_1 = p_{\max 2}/A_2$

$$A_2 = A_1 \left( \frac{p_{\max 2}}{p_{\max 1}} \right) = 1.2 \times 10^{-8} \text{ m} \left( \frac{30 \text{ Pa}}{3.0 \times 10^{-2} \text{ Pa}} \right) = 1.2 \times 10^{-5} \text{ m}$$

(c)  $p_{\max} = BkA = 2\pi BA/\lambda$

$$p_{\max} \lambda = 2\pi BA = \text{constant} \text{ so } p_{\max 1} \lambda_1 = p_{\max 2} \lambda_2 \text{ and } \lambda_2 = \lambda_1 \left( \frac{p_{\max 1}}{p_{\max 2}} \right) = (0.344 \text{ m}) \left( \frac{3.0 \times 10^{-2} \text{ Pa}}{1.5 \times 10^{-3} \text{ Pa}} \right) = 6.9 \text{ m}$$

$$f = v/\lambda = (344 \text{ m/s})/6.9 \text{ m} = 50 \text{ Hz}$$

**EVALUATE:** The pressure amplitude and displacement amplitude are directly proportional. For the same displacement amplitude, the pressure amplitude decreases when the frequency decreases and the wavelength increases.

**16.2. IDENTIFY:** Apply  $p_{\max} = BkA$  and solve for  $A$ .

$$\text{SET UP: } k = \frac{2\pi}{\lambda} \text{ and } v = f\lambda, \text{ so } k = \frac{2\pi f}{v} \text{ and } p = \frac{2\pi f B A}{v}.$$

$$\text{EXECUTE: } A = \frac{p_{\max} v}{2\pi B f} = \frac{(3.0 \times 10^{-2} \text{ Pa})(1480 \text{ m/s})}{2\pi(2.2 \times 10^9 \text{ Pa})(1000 \text{ Hz})} = 3.21 \times 10^{-12} \text{ m}.$$

**EVALUATE:** Both  $v$  and  $B$  are larger, but  $B$  is larger by a much greater factor, so  $v/B$  is a lot smaller and therefore  $A$  is a lot smaller.

**16.3. IDENTIFY:** Use  $p_{\max} = BkA$  to relate the pressure and displacement amplitudes.

**SET UP:** As stated in Example 16.1 the adiabatic bulk modulus for air is  $B = 1.42 \times 10^5 \text{ Pa}$ . Use  $v = f\lambda$  to calculate  $\lambda$  from  $f$ , and then  $k = 2\pi/\lambda$ .

**EXECUTE:** (a)  $f = 150 \text{ Hz}$

Need to calculate  $k$ :  $\lambda = v/f$  and  $k = 2\pi/\lambda$  so  $k = 2\pi f/v = (2\pi \text{ rad})(150 \text{ Hz})/344 \text{ m/s} = 2.74 \text{ rad/m}$ . Then

$p_{\max} = BkA = (1.42 \times 10^5 \text{ Pa})(2.74 \text{ rad/m})(0.0200 \times 10^{-3} \text{ m}) = 7.78 \text{ Pa}$ . This is below the pain threshold of 30 Pa.

(b)  $f$  is larger by a factor of 10 so  $k = 2\pi f/v$  is larger by a factor of 10, and  $p_{\max} = BkA$  is larger by a factor of 10.  $p_{\max} = 77.8 \text{ Pa}$ , above the pain threshold.

(c) There is again an increase in  $f$ ,  $k$ , and  $p_{\max}$  of a factor of 10, so  $p_{\max} = 778 \text{ Pa}$ , far above the pain threshold.

**EVALUATE:** When  $f$  increases,  $\lambda$  decreases so  $k$  increases and the pressure amplitude increases.

- 16.4. IDENTIFY:** Apply  $p_{\max} = BkA$ .  $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v}$ , so  $p_{\max} = \frac{2\pi fBA}{v}$ .

**SET UP:**  $v = 344$  m/s

**EXECUTE:**  $f = \frac{vp_{\max}}{2\pi BA} = \frac{(344 \text{ m/s})(10.0 \text{ Pa})}{2\pi(1.42 \times 10^5 \text{ Pa})(1.00 \times 10^{-6} \text{ m})} = 3.86 \times 10^3 \text{ Hz}$

**EVALUATE:** Audible frequencies range from about 20 Hz to about 20,000 Hz, so this frequency is audible.

- 16.5. IDENTIFY and SET UP:** Use the relation  $v = f\lambda$  to find the wavelength or frequency of various sounds.

**EXECUTE: (a)**  $\lambda = \frac{v}{f} = \frac{1531 \text{ m/s}}{17 \text{ Hz}} = 90 \text{ m}$ .

**(b)**  $f = \frac{v}{\lambda} = \frac{1531 \text{ m/s}}{0.015 \text{ m}} = 102 \text{ kHz}$ .

**(c)**  $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{25 \times 10^3 \text{ Hz}} = 1.4 \text{ cm}$ .

**(d)** For  $f = 78 \text{ kHz}$ ,  $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{78 \times 10^3 \text{ Hz}} = 4.4 \text{ mm}$ . For  $f = 39 \text{ kHz}$ ,  $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{39 \times 10^3 \text{ Hz}} = 8.8 \text{ mm}$ .

The range of wavelengths is 4.4 mm to 8.8 mm.

**(e)**  $\lambda = 0.25 \text{ mm}$  so  $f = \frac{v}{\lambda} = \frac{1550 \text{ m/s}}{0.25 \times 10^{-3} \text{ m}} = 6.2 \text{ MHz}$ .

**EVALUATE:** Nonaudible (to human) sounds cover a wide range of frequencies and wavelengths.

- 16.6. IDENTIFY:**  $v = f\lambda$ . Apply  $v = \sqrt{\frac{B}{\rho}}$  for the waves in the liquid and  $v = \sqrt{\frac{Y}{\rho}}$  for the waves in the metal bar.

**SET UP:** In part (b) the wave speed is  $v = \frac{d}{t} = \frac{1.50 \text{ m}}{3.90 \times 10^{-4} \text{ s}}$ .

**EXECUTE: (a)** Using  $v = \sqrt{\frac{B}{\rho}}$ , we have  $B = v^2 \rho = (\lambda f)^2 \rho$ , so

$B = [(8 \text{ m})(400 \text{ Hz})]^2 (1300 \text{ kg/m}^3) = 1.33 \times 10^{10} \text{ Pa}$ .

**(b)** Using  $v = \sqrt{\frac{Y}{\rho}}$ , we have  $Y = v^2 \rho = (L/t)^2 \rho = [(1.50 \text{ m})/(3.90 \times 10^{-4} \text{ s})]^2 (6400 \text{ kg/m}^3) = 9.47 \times 10^{10} \text{ Pa}$ .

**EVALUATE:** In the liquid,  $v = 3200$  m/s and in the metal,  $v = 3850$  m/s. Both these speeds are much greater than the speed of sound in air.

- 16.7. IDENTIFY:**  $d = vt$  for the sound waves in air and in water.

**SET UP:** Use  $v_{\text{water}} = 1482$  m/s at  $20^\circ\text{C}$ , as given in Table 16.1. In air,  $v = 344$  m/s.

**EXECUTE:** Since along the path to the diver the sound travels 1.2 m in air, the sound wave travels in water for the same time as the wave travels a distance  $22.0 \text{ m} - 1.20 \text{ m} = 20.8 \text{ m}$  in air. The depth of the diver is

$(20.8 \text{ m}) \frac{v_{\text{water}}}{v_{\text{air}}} = (20.8 \text{ m}) \frac{1482 \text{ m/s}}{344 \text{ m/s}} = 89.6 \text{ m}$ . This is the depth of the diver; the distance from the horn is 90.8 m.

**EVALUATE:** The time it takes the sound to travel from the horn to the person on shore is

$t_1 = \frac{22.0 \text{ m}}{344 \text{ m/s}} = 0.0640 \text{ s}$ . The time it takes the sound to travel from the horn to the diver is

$t_2 = \frac{1.2 \text{ m}}{344 \text{ m/s}} + \frac{89.6 \text{ m}}{1482 \text{ m/s}} = 0.0035 \text{ s} + 0.0605 \text{ s} = 0.0640 \text{ s}$ . These times are indeed the same. For three

figure accuracy the distance of the horn above the water can't be neglected.

**16.8. IDENTIFY:** Apply  $v = \sqrt{\frac{\gamma RT}{M}}$  to each gas.

**SET UP:** In each case, express  $M$  in units of kg/mol. For  $H_2$ ,  $\gamma = 1.41$ . For He and Ar,  $\gamma = 1.67$ .

**EXECUTE: (a)**  $v_{H_2} = \sqrt{\frac{(1.41)(8.3145 \text{ J/mol} \cdot \text{K})(300.15 \text{ K})}{(2.02 \times 10^{-3} \text{ kg/mol})}} = 1.32 \times 10^3 \text{ m/s}$

**(b)**  $v_{He} = \sqrt{\frac{(1.67)(8.3145 \text{ J/mol} \cdot \text{K})(300.15 \text{ K})}{(4.00 \times 10^{-3} \text{ kg/mol})}} = 1.02 \times 10^3 \text{ m/s}$

**(c)**  $v_{Ar} = \sqrt{\frac{(1.67)(8.3145 \text{ J/mol} \cdot \text{K})(300.15 \text{ K})}{(39.9 \times 10^{-3} \text{ kg/mol})}} = 323 \text{ m/s}$ .

**(d)** Repeating the calculation of Example 16.4 at  $T = 300.15 \text{ K}$  gives  $v_{air} = 348 \text{ m/s}$ , and so

$v_{H_2} = 3.80v_{air}$ ,  $v_{He} = 2.94v_{air}$  and  $v_{Ar} = 0.928v_{air}$ .

**EVALUATE:**  $v$  is larger for gases with smaller  $M$ .

**16.9. IDENTIFY:**  $v = f\lambda$ . The relation of  $v$  to gas temperature is given by  $v = \sqrt{\frac{\gamma RT}{M}}$ .

**SET UP:** Let  $T = 22.0^\circ\text{C} = 295.15 \text{ K}$ .

**EXECUTE:** At  $22.0^\circ\text{C}$ ,  $\lambda = \frac{v}{f} = \frac{325 \text{ m/s}}{1250 \text{ Hz}} = 0.260 \text{ m} = 26.0 \text{ cm}$ .  $\lambda = \frac{v}{f} = \frac{1}{f} \sqrt{\frac{\gamma RT}{M}}$ .  $\frac{\lambda}{\sqrt{T}} = \frac{1}{f} \sqrt{\frac{\gamma R}{M}}$ ,

which is constant, so  $\frac{\lambda_1}{\sqrt{T_1}} = \frac{\lambda_2}{\sqrt{T_2}}$ .  $T_2 = T_1 \left( \frac{\lambda_2}{\lambda_1} \right)^2 = (295.15 \text{ K}) \left( \frac{28.5 \text{ cm}}{26.0 \text{ cm}} \right)^2 = 354.6 \text{ K} = 81.4^\circ\text{C}$ .

**EVALUATE:** When  $T$  increases  $v$  increases and for fixed  $f$ ,  $\lambda$  increases. Note that we did not need to know either  $\gamma$  or  $M$  for the gas.

**16.10. IDENTIFY:**  $v = \sqrt{\frac{\gamma RT}{M}}$ . Take the derivative of  $v$  with respect to  $T$ . In part (b) replace  $dv$  by  $\Delta v$  and  $dT$  by  $\Delta T$  in the expression derived in part (a).

**SET UP:**  $\frac{d(x^{1/2})}{dx} = \frac{1}{2}x^{-1/2}$ . In  $v = \sqrt{\frac{\gamma RT}{M}}$ ,  $T$  must be in kelvins.  $20^\circ\text{C} = 293 \text{ K}$ .  $\Delta T = 1^\circ\text{C} = 1 \text{ K}$ .

**EXECUTE: (a)**  $\frac{dv}{dT} = \sqrt{\frac{\gamma R}{M}} \frac{dT^{1/2}}{dT} = \sqrt{\frac{\gamma R}{M}} \frac{1}{2}T^{-1/2} = \frac{1}{2T} \sqrt{\frac{\gamma RT}{M}} = \frac{v}{2T}$ . Rearranging gives  $\frac{dv}{v} = \frac{1}{2} \frac{dT}{T}$ , the desired result.

**(b)**  $\frac{\Delta v}{v} = \frac{1}{2} \frac{\Delta T}{T}$ .  $\Delta v = \frac{v}{2} \frac{\Delta T}{T} = \left( \frac{344 \text{ m/s}}{2} \right) \left( \frac{1 \text{ K}}{293 \text{ K}} \right) = 0.59 \text{ m/s}$ .

**EVALUATE:** Since  $\frac{\Delta T}{T} = 3.4 \times 10^{-3}$  and  $\frac{\Delta v}{v}$  is one-half this, replacing  $dT$  by  $\Delta T$  and  $dv$  by  $\Delta v$  is accurate. Using the result from part (a) is much simpler than calculating  $v$  for  $20^\circ\text{C}$  and for  $21^\circ\text{C}$  and subtracting, and is not subject to round-off errors.

**16.11. IDENTIFY and SET UP:** Use  $t = \text{distance/speed}$ . Calculate the time it takes each sound wave to travel the

$L = 60.0 \text{ m}$  length of the pipe. Use  $v = \sqrt{\frac{Y}{\rho}}$  to calculate the speed of sound in the brass rod.

**EXECUTE:** Wave in air:  $t = (60.0 \text{ m})/(344 \text{ m/s}) = 0.1744 \text{ s}$ .

Wave in the metal:  $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.0 \times 10^{10} \text{ Pa}}{8600 \text{ kg/m}^3}} = 3235 \text{ m/s}$ , so  $t = \frac{60.0 \text{ m}}{3235 \text{ m/s}} = 0.01855 \text{ s}$ .

The time interval between the two sounds is  $\Delta t = 0.1744 \text{ s} - 0.01855 \text{ s} = 0.156 \text{ s}$ .

**EVALUATE:** The restoring forces that propagate the sound waves are much greater in solid brass than in air, so  $v$  is much larger in brass.

- 16.12. IDENTIFY:** For transverse waves,  $v_{\text{trans}} = \sqrt{\frac{F}{\mu}}$ . For longitudinal waves,  $v_{\text{long}} = \sqrt{\frac{Y}{\rho}}$ .

**SET UP:** The mass per unit length  $\mu$  is related to the density (assumed uniform) and the cross-section area  $A$  by  $\mu = A\rho$ .

**EXECUTE:**  $v_{\text{long}} = 30v_{\text{trans}}$  gives  $\sqrt{\frac{Y}{\rho}} = 30\sqrt{\frac{F}{\mu}}$  and  $\frac{Y}{\rho} = 900\frac{F}{A\rho}$ . Therefore,  $F/A = \frac{Y}{900}$ .

**EVALUATE:** Typical values of  $Y$  are on the order of  $10^{11}$  Pa, so the stress must be about  $10^8$  Pa. If  $A$  is on the order of  $1 \text{ mm}^2 = 10^{-6} \text{ m}^2$ , this requires a force of about 100 N.

- 16.13. IDENTIFY and SET UP:** Sound delivers energy (and hence power) to the ear. For a whisper,  $I = 1 \times 10^{-10} \text{ W/m}^2$ . The area of the tympanic membrane is  $A = \pi r^2$ , with  $r = 4.2 \times 10^{-3} \text{ m}$ . Intensity is energy per unit time per unit area.

**EXECUTE:** (a)  $E = IAt = (1 \times 10^{-10} \text{ W/m}^2)\pi(4.2 \times 10^{-3} \text{ m})^2(1 \text{ s}) = 5.5 \times 10^{-15} \text{ J}$ .

(b)  $K = \frac{1}{2}mv^2$  so  $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.5 \times 10^{-15} \text{ J})}{2.0 \times 10^{-6} \text{ kg}}} = 7.4 \times 10^{-5} \text{ m/s} = 0.074 \text{ mm/s}$ .

**EVALUATE:** Compared to the energy of ordinary objects, it takes only a very small amount of energy for hearing. As part (b) shows, a mosquito carries a lot more energy than is needed for hearing.

- 16.14. IDENTIFY:** The sound intensity level decreases by 13.0 dB, and from this we can find the change in the intensity.

**SET UP:**  $\beta = 10 \log(I/I_0)$ .  $\Delta\beta = 13.0 \text{ dB}$ .

**EXECUTE:** (a)  $\Delta\beta = \beta_2 - \beta_1 = 10 \text{ dB} \log(I_2/I_0) - 10 \text{ dB} \log(I_1/I_0) = 10 \text{ dB} \log(I_2/I_1) = 13.0 \text{ dB}$ , so  $1.3 = \log(I_2/I_1)$  which gives  $I_2/I_1 = 20.0$ .

(b) **EVALUATE:** According to the equation in part (a) the difference in two sound intensity levels is determined by the ratio of the sound intensities. So you don't need to know  $I_1$ , just the ratio  $I_2/I_1$ .

- 16.15. IDENTIFY and SET UP:** We want the sound intensity level to increase from 20.0 dB to 60.0 dB. The previous problem showed that  $\beta_2 - \beta_1 = (10 \text{ dB})\log\left(\frac{I_2}{I_1}\right)$ . We also know that  $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$ .

**EXECUTE:** Using  $\beta_2 - \beta_1 = (10 \text{ dB})\log\left(\frac{I_2}{I_1}\right)$ , we have  $\Delta\beta = +40.0 \text{ dB}$ . Therefore  $\log\left(\frac{I_2}{I_1}\right) = 4.00$ , so

$\frac{I_2}{I_1} = 1.00 \times 10^4$ . Using  $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$  and solving for  $r_2$ , we get  $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (15.0 \text{ m})\sqrt{\frac{1}{1.00 \times 10^4}} = 15.0 \text{ cm}$ .

**EVALUATE:** A change of  $10^2$  in distance gives a change of  $10^4$  in intensity. Our analysis assumes that the sound spreads from the source uniformly in all directions.

- 16.16. IDENTIFY:** Knowing the sound level in decibels, we can determine the rate at which energy is delivered to the eardrum.

**SET UP:** Intensity is energy per unit time per unit area.  $\beta = (10 \text{ dB})\log\left(\frac{I}{I_0}\right)$ , with  $I_0 = 1 \times 10^{-12} \text{ W/m}^2$ .

The area of the eardrum is  $A = \pi r^2$ , with  $r = 4.2 \times 10^{-3} \text{ m}$ . Part (b) of Problem 16.13 gave  $v = 0.074 \text{ mm/s}$ .

**EXECUTE:** (a)  $\beta = 110 \text{ dB}$  gives  $11.0 = \log\left(\frac{I}{I_0}\right)$  and  $I = (10^{11})I_0 = 0.100 \text{ W/m}^2$ .

$E = IAt = (0.100 \text{ W/m}^2)\pi(4.2 \times 10^{-3} \text{ m})^2(1 \text{ s}) = 5.5 \mu\text{J}$ .

(b)  $K = \frac{1}{2}mv^2$  so  $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.5 \times 10^{-6} \text{ J})}{2.0 \times 10^{-6} \text{ kg}}} = 2.3 \text{ m/s}$ . This is about 31,000 times faster than the speed

in Problem 16.13b.

**EVALUATE:** Even though the sound wave intensity level is very high, the rate at which energy is delivered to the eardrum is very small, because the area of the eardrum is very small.

**16.17. IDENTIFY and SET UP:** Apply  $p_{\max} = BkA$ ,  $I = \frac{1}{2}B\omega kA^2$ , and  $\beta = (10 \text{ dB})\log\left(\frac{I}{I_0}\right)$ .

**EXECUTE: (a)**  $\omega = 2\pi f = (2\pi \text{ rad})(320 \text{ Hz}) = 2011 \text{ rad/s}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v} = \frac{2011 \text{ rad/s}}{344 \text{ m/s}} = 5.84 \text{ rad/m}$$

$B = 1.42 \times 10^5 \text{ Pa}$  (Example 16.1)

Then  $p_{\max} = BkA = (1.42 \times 10^5 \text{ Pa})(5.84 \text{ rad/m})(5.00 \times 10^{-6} \text{ m}) = 4.14 \text{ Pa}$ .

(b) Using  $I = \frac{1}{2}\omega BkA^2$  gives

$$I = \frac{1}{2}(2011 \text{ rad/s})(1.42 \times 10^5 \text{ Pa})(5.84 \text{ rad/m})(5.00 \times 10^{-6} \text{ m})^2 = 2.08 \times 10^{-2} \text{ W/m}^2.$$

(c)  $\beta = (10 \text{ dB})\log\left(\frac{I}{I_0}\right)$ :  $\beta = (10 \text{ dB})\log(I/I_0)$ , with  $I_0 = 1 \times 10^{-12} \text{ W/m}^2$ .

$$\beta = (10 \text{ dB})\log[(2.08 \times 10^{-2} \text{ W/m}^2)/(1 \times 10^{-12} \text{ W/m}^2)] = 103 \text{ dB}.$$

**EVALUATE:** Even though the displacement amplitude is very small, this is a very intense sound. Compare the sound intensity level to the values in Table 16.2.

**16.18. IDENTIFY:** Changing the sound intensity level will decrease the rate at which energy reaches the ear.

**SET UP:** Example 16.9 shows that  $\beta_2 - \beta_1 = (10 \text{ dB})\log\left(\frac{I_2}{I_1}\right)$ .

**EXECUTE: (a)**  $\Delta\beta = -30 \text{ dB}$  so  $\log\left(\frac{I_2}{I_1}\right) = -3$  and  $\frac{I_2}{I_1} = 10^{-3} = 1/1000$ .

(b)  $I_2/I_1 = \frac{1}{2}$  so  $\Delta\beta = 10\log\left(\frac{1}{2}\right) = -3.0 \text{ dB}$

**EVALUATE:** Because of the logarithmic relationship between the intensity and intensity level of sound, a small change in the intensity level produces a large change in the intensity.

**16.19. IDENTIFY:** Use  $I = \frac{vp_{\max}^2}{2B}$  to relate  $I$  and  $p_{\max}$ .  $\beta = (10 \text{ dB})\log(I/I_0)$ . The equation  $p_{\max} = BkA$  says the

pressure amplitude and displacement amplitude are related by  $p_{\max} = BkA = B\left(\frac{2\pi f}{v}\right)A$ .

**SET UP:** At  $20^\circ\text{C}$  the bulk modulus for air is  $1.42 \times 10^5 \text{ Pa}$  and  $v = 344 \text{ m/s}$ .  $I_0 = 1 \times 10^{-12} \text{ W/m}^2$ .

**EXECUTE: (a)**  $I = \frac{vp_{\max}^2}{2B} = \frac{(344 \text{ m/s})(6.0 \times 10^{-5} \text{ Pa})^2}{2(1.42 \times 10^5 \text{ Pa})} = 4.4 \times 10^{-12} \text{ W/m}^2$

(b)  $\beta = (10 \text{ dB})\log\left(\frac{4.4 \times 10^{-12} \text{ W/m}^2}{1 \times 10^{-12} \text{ W/m}^2}\right) = 6.4 \text{ dB}$

(c)  $A = \frac{vp_{\max}}{2\pi fB} = \frac{(344 \text{ m/s})(6.0 \times 10^{-5} \text{ Pa})}{2\pi(400 \text{ Hz})(1.42 \times 10^5 \text{ Pa})} = 5.8 \times 10^{-11} \text{ m}$

**EVALUATE:** This is a very faint sound and the displacement and pressure amplitudes are very small. Note that the displacement amplitude depends on the frequency but the pressure amplitude does not.

**16.20. IDENTIFY and SET UP:** Apply the relation  $\beta_2 - \beta_1 = (10 \text{ dB})\log(I_2/I_1)$  that is derived in Example 16.9.

**EXECUTE:** (a)  $\Delta\beta = (10 \text{ dB})\log\left(\frac{4I}{I}\right) = 6.0 \text{ dB}$

(b) The total number of crying babies must be multiplied by four, for an increase of 12 kids.

**EVALUATE:** For  $I_2 = \alpha I_1$ , where  $\alpha$  is some factor, the increase in sound intensity level is  $\Delta\beta = (10 \text{ dB})\log\alpha$ . For  $\alpha = 4$ ,  $\Delta\beta = 6.0 \text{ dB}$ .

**16.21. IDENTIFY and SET UP:** Let 1 refer to the mother and 2 to the father. Use the result derived in Example 16.9 for the difference in sound intensity level for the two sounds. Relate intensity to distance from the source using  $I_1/I_2 = r_2^2/r_1^2$ .

**EXECUTE:** From Example 16.9,  $\beta_2 - \beta_1 = (10 \text{ dB})\log(I_2/I_1)$

Using  $I_1/I_2 = r_2^2/r_1^2$  gives us

$$\Delta\beta = \beta_2 - \beta_1 = (10 \text{ dB})\log(I_2/I_1) = (10 \text{ dB})\log(r_1/r_2)^2 = (20 \text{ dB})\log(r_1/r_2)$$

$$\Delta\beta = (20 \text{ dB})\log(1.50 \text{ m}/0.30 \text{ m}) = 14.0 \text{ dB}.$$

**EVALUATE:** The father is 5 times closer so the intensity at his location is 25 times greater.

**16.22. IDENTIFY:**  $\beta = (10 \text{ dB})\log\frac{I}{I_0}$ .  $\beta_2 - \beta_1 = (10 \text{ dB})\log\frac{I_2}{I_1}$ . Solve for  $\frac{I_2}{I_1}$ .

**SET UP:** If  $\log y = x$  then  $y = 10^x$ . Let  $\beta_2 = 70 \text{ dB}$  and  $\beta_1 = 95 \text{ dB}$ .

**EXECUTE:**  $70.0 \text{ dB} - 95.0 \text{ dB} = -25.0 \text{ dB} = (10 \text{ dB})\log\frac{I_2}{I_1}$ .  $\log\frac{I_2}{I_1} = -2.5$  and  $\frac{I_2}{I_1} = 10^{-2.5} = 3.2 \times 10^{-3}$ .

**EVALUATE:**  $I_2 < I_1$  when  $\beta_2 < \beta_1$ .

**16.23. IDENTIFY:** The intensity of sound obeys an inverse square law.

**SET UP:**  $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$ .  $\beta = (10 \text{ dB})\log\left(\frac{I}{I_0}\right)$ , with  $I_0 = 1 \times 10^{-12} \text{ W/m}^2$ .

**EXECUTE:** (a)  $\beta = 53 \text{ dB}$  gives  $5.3 = \log\left(\frac{I}{I_0}\right)$  and  $I = (10^{5.3})I_0 = 2.0 \times 10^{-7} \text{ W/m}^2$ .

(b)  $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (3.0 \text{ m})\sqrt{\frac{4}{1}} = 6.0 \text{ m}$ .

(c)  $\beta = \frac{53 \text{ dB}}{4} = 13.25 \text{ dB}$  gives  $1.325 = \log\left(\frac{I}{I_0}\right)$  and  $I = 2.1 \times 10^{-11} \text{ W/m}^2$ .

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (3.0 \text{ m})\sqrt{\frac{2.0 \times 10^{-7} \text{ W/m}^2}{2.1 \times 10^{-11} \text{ W/m}^2}} = 290 \text{ m}.$$

**EVALUATE:** (d) Intensity obeys the inverse square law but noise level does not.

**16.24. IDENTIFY:** We must use the relationship between intensity and sound level.

**SET UP:** Example 16.9 shows that  $\beta_2 - \beta_1 = (10 \text{ dB})\log\left(\frac{I_2}{I_1}\right)$ .

**EXECUTE:** (a)  $\Delta\beta = 5.00 \text{ dB}$  gives  $\log\left(\frac{I_2}{I_1}\right) = 0.5$  and  $\frac{I_2}{I_1} = 10^{0.5} = 3.16$ .

(b)  $\frac{I_2}{I_1} = 100$  gives  $\Delta\beta = 10\log(100) = 20 \text{ dB}$ .

(c)  $\frac{I_2}{I_1} = 2$  gives  $\Delta\beta = 10\log 2 = 3.0 \text{ dB}$ .

**EVALUATE:** Every doubling of the intensity increases the decibel level by 3.0 dB.

- 16.25. IDENTIFY and SET UP:** An open end is a displacement antinode and a closed end is a displacement node. Sketch the standing wave pattern and use the sketch to relate the node-to-antinode distance to the length of the pipe. A displacement node is a pressure antinode and a displacement antinode is a pressure node.  
**EXECUTE: (a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.25a. The open ends are displacement antinodes.

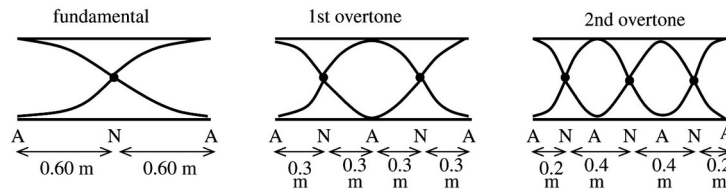


Figure 16.25a

Location of the displacement nodes (N) measured from the left end:

fundamental 0.60 m

1st overtone 0.30 m, 0.90 m

2nd overtone 0.20 m, 0.60 m, 1.00 m

Location of the pressure nodes (displacement antinodes (A)) measured from the left end:

fundamental 0, 1.20 m

1st overtone 0, 0.60 m, 1.20 m

2nd overtone 0, 0.40 m, 0.80 m, 1.20 m

**(b)** The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.25b.

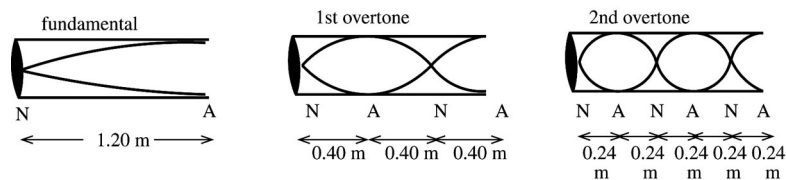


Figure 16.25b

Location of the displacement nodes (N) measured from the closed end:

fundamental 0

1st overtone 0, 0.80 m

2nd overtone 0, 0.48 m, 0.96 m

Location of the pressure nodes (displacement antinodes (A)) measured from the closed end:

fundamental 1.20 m

1st overtone 0.40 m, 1.20 m

2nd overtone 0.24 m, 0.72 m, 1.20 m

**EVALUATE:** The node-to-node or antinode-to-antinode distance is  $\lambda/2$ . For the higher overtones the frequency is higher and the wavelength is smaller.

- 16.26. IDENTIFY:** For an open pipe,  $f_1 = \frac{v}{2L}$ . For a stopped pipe,  $f_1 = \frac{v}{4L}$ .  $v = f\lambda$ .

**SET UP:**  $v = 344$  m/s. For a pipe, there must be a displacement node at a closed end and an antinode at the open end.

**EXECUTE: (a)**  $L = \frac{v}{2f_1} = \frac{344 \text{ m/s}}{2(524 \text{ Hz})} = 0.328 \text{ m}$ .

(b) There is a node at one end, an antinode at the other end and no other nodes or antinodes in between, so  $\frac{\lambda_1}{4} = L$  and  $\lambda_1 = 4L = 4(0.328 \text{ m}) = 1.31 \text{ m}$ .

(c)  $f_1 = \frac{v}{4L} = \frac{1}{2} \left( \frac{v}{2L} \right) = \frac{1}{2} (524 \text{ Hz}) = 262 \text{ Hz}$ .

**EVALUATE:** We could also calculate  $f_1$  for the stopped pipe as  $f_1 = \frac{v}{\lambda_1} = \frac{344 \text{ m/s}}{1.31 \text{ m}} = 262 \text{ Hz}$ , which agrees with our result in part (c).

**16.27. IDENTIFY:** For a stopped pipe, the standing wave frequencies are given by  $f_n = nv/4L$ .

**SET UP:** The first three standing wave frequencies correspond to  $n = 1, 3$ , and  $5$ .

**EXECUTE:**  $f_1 = \frac{(344 \text{ m/s})}{4(0.17 \text{ m})} = 506 \text{ Hz}$ ,  $f_3 = 3f_1 = 1517 \text{ Hz}$ ,  $f_5 = 5f_1 = 2529 \text{ Hz}$ .

**EVALUATE:** All three of these frequencies are in the audible range, which is about 20 Hz to 20,000 Hz.

**16.28. IDENTIFY:** The vocal tract is modeled as a stopped pipe, open at one end and closed at the other end, so we know the wavelength of standing waves in the tract.

**SET UP:** For a stopped pipe,  $\lambda_n = 4L/n$  ( $n = 1, 3, 5, \dots$ ) and  $v = f\lambda$ , so  $f_1 = \frac{v}{4L}$  with  $f_1 = 220 \text{ Hz}$ .

**EXECUTE:**  $L = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(220 \text{ Hz})} = 39.1 \text{ cm}$ . This result is a reasonable value for the mouth to diaphragm

distance for a typical adult.

**EVALUATE:** 1244 Hz is not an integer multiple of the fundamental frequency of 220 Hz; it is 5.65 times the fundamental. The production of sung notes is more complicated than harmonics of an air column of fixed length.

**16.29. IDENTIFY:** A pipe open at one end and closed at the other is a stopped pipe.

**SET UP:** For an open pipe, the fundamental is  $f_1 = v/2L$ , and for a stopped pipe, it is  $f_1 = v/4L$ .

**EXECUTE:** (a) For an open pipe,  $f_1 = \frac{v}{2L} = \frac{344 \text{ m/s}}{2(4.88 \text{ m})} = 35.2 \text{ Hz}$ .

(b) For a stopped pipe,  $f_1 = \frac{v}{4L} = \frac{35.2 \text{ Hz}}{2} = 17.6 \text{ Hz}$ .

**EVALUATE:** Even though the pipes both have the same length, their fundamental frequencies are very different, depending on whether they are open or closed at their ends.

**16.30. IDENTIFY:** There must be a node at each end of the pipe. For the fundamental there are no additional nodes and each successive overtone has one additional node.  $v = f\lambda$ .

**SET UP:**  $v = 344 \text{ m/s}$ . The node to node distance is  $\lambda/2$ .

**EXECUTE:** (a)  $\frac{\lambda_1}{2} = L$  so  $\lambda_1 = 2L$ . Each successive overtone adds an additional  $\lambda/2$  along the pipe, so

$n \left( \frac{\lambda_n}{2} \right) = L$  and  $\lambda_n = \frac{2L}{n}$ , where  $n = 1, 2, 3, \dots$ .  $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$ .

(b)  $f_1 = \frac{v}{2L} = \frac{344 \text{ m/s}}{2(2.50 \text{ m})} = 68.8 \text{ Hz}$ .  $f_2 = 2f_1 = 138 \text{ Hz}$ .  $f_3 = 3f_1 = 206 \text{ Hz}$ . All three of these frequencies

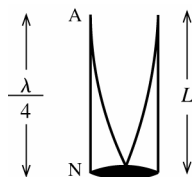
are audible.

**EVALUATE:** A pipe of length  $L$  closed at both ends has the same standing wave wavelengths, frequencies and nodal patterns as for a string of length  $L$  that is fixed at both ends.

**16.31. IDENTIFY and SET UP:** Use the standing wave pattern to relate the wavelength of the standing wave to the length of the air column and then use  $v = f\lambda$  to calculate  $f$ . There is a displacement antinode at the top (open) end of the air column and a node at the bottom (closed) end, as shown in Figure 16.31.



EXECUTE: (a)



$$\lambda/4 = L$$

$$\lambda = 4L = 4(0.140 \text{ m}) = 0.560 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{344 \text{ m/s}}{0.560 \text{ m}} = 614 \text{ Hz}$$

Figure 16.31

(b) Now the length  $L$  of the air column becomes  $\frac{1}{2}(0.140 \text{ m}) = 0.070 \text{ m}$  and  $\lambda = 4L = 0.280 \text{ m}$ .

$$f = \frac{v}{\lambda} = \frac{344 \text{ m/s}}{0.280 \text{ m}} = 1230 \text{ Hz}$$

EVALUATE: Smaller  $L$  means smaller  $\lambda$  which in turn corresponds to larger  $f$ .

16.32. IDENTIFY: The wire will vibrate in its second overtone with frequency  $f_3^{\text{wire}}$  when  $f_3^{\text{wire}} = f_1^{\text{pipe}}$ . For a stopped pipe,  $f_1^{\text{pipe}} = \frac{v}{4L_{\text{pipe}}}$ . The second overtone standing wave frequency for a wire fixed at both ends

$$\text{is } f_3^{\text{wire}} = 3 \left( \frac{v_{\text{wire}}}{2L_{\text{wire}}} \right). \quad v_{\text{wire}} = \sqrt{F/\mu}.$$

SET UP: The wire has  $\mu = \frac{m}{L_{\text{wire}}} = \frac{7.25 \times 10^{-3} \text{ kg}}{0.620 \text{ m}} = 1.169 \times 10^{-2} \text{ kg/m}$ . The speed of sound in air is  $v = 344 \text{ m/s}$ .

$$\text{EXECUTE: } v_{\text{wire}} = \sqrt{\frac{4110 \text{ N}}{1.169 \times 10^{-2} \text{ kg/m}}} = 592.85 \text{ m/s. } f_3^{\text{wire}} = f_1^{\text{pipe}} \text{ gives } 3 \frac{v_{\text{wire}}}{2L_{\text{wire}}} = \frac{v}{4L_{\text{pipe}}}.$$

$$L_{\text{pipe}} = \frac{2L_{\text{wire}}v}{12v_{\text{wire}}} = \frac{2(0.620 \text{ m})(344 \text{ m/s})}{12(592.85 \text{ m/s})} = 0.0600 \text{ m} = 6.00 \text{ cm}.$$

EVALUATE: The fundamental for the pipe has the same frequency as the third harmonic of the wire. But the wave speeds for the two objects are different and the two standing waves have different wavelengths.

16.33. IDENTIFY: The second overtone is the third harmonic, with  $f = 3f_1$ .

$$\text{SET UP: } v = \sqrt{\frac{F}{\mu}}. \quad f_1 = v/2L. \quad v = f\lambda. \quad \lambda_n = 2L/n, \text{ so } \frac{3\lambda}{2} = L \text{ for the third harmonic.}$$

$$\text{EXECUTE: (a) } v = \sqrt{\frac{35.0 \text{ N}}{(5.625 \times 10^{-3} \text{ kg})/(0.750 \text{ m})}} = 68.3 \text{ m/s}.$$

$$f = \frac{3v}{2L} = \frac{3(68.3 \text{ m/s})}{2(0.750 \text{ m})} = 137 \text{ Hz}$$

$$\lambda = \frac{v}{f} = \frac{68.3 \text{ m/s}}{137 \text{ Hz}} = 0.50 \text{ m}$$

$$\text{(b) } f = 137 \text{ Hz, the same as for the wire, so } \lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{137 \text{ Hz}} = 2.51 \text{ m}.$$

EVALUATE:  $\lambda$  is larger in air because  $v$  is larger there.

16.34. IDENTIFY and SET UP: The path difference for the two sources is  $d$ . For destructive interference, the path difference is a half-integer number of wavelengths. For constructive interference, the path difference is an integer number of wavelengths.  $v = f\lambda$ .

**EXECUTE:** (a)  $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{725 \text{ Hz}} = 0.474 \text{ m}$ . Destructive interference will first occur when

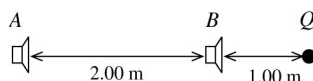
$$d = \lambda/2 = 0.237 \text{ m}.$$

(b) Destructive interference will next occur when  $d = 3\lambda/2 = 0.711 \text{ m}$ .

(c) Constructive interference will first occur when  $d = \lambda = 0.474 \text{ m}$ .

**EVALUATE:** Constructive interference should first occur midway between the first two points where destructive interference occurs. This midpoint is  $(0.237 \text{ m} + 0.711 \text{ m})/2 = 0.474 \text{ m}$ , which is just what we found in part (c).

- 16.35. (a) IDENTIFY and SET UP:** Path difference from points  $A$  and  $B$  to point  $Q$  is  $3.00 \text{ m} - 1.00 \text{ m} = 2.00 \text{ m}$ , as shown in Figure 16.35. Constructive interference implies path difference  $= n\lambda$ ,  $n = 1, 2, 3, \dots$



**Figure 16.35**

**EXECUTE:**  $2.00 \text{ m} = n\lambda$  so  $\lambda = 2.00 \text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{2.00 \text{ m}} = \frac{n(344 \text{ m/s})}{2.00 \text{ m}} = n(172 \text{ Hz}), \quad n = 1, 2, 3, \dots$$

The lowest frequency for which constructive interference occurs is 172 Hz.

(b) **IDENTIFY and SET UP:** Destructive interference implies path difference  $= (n/2)\lambda$ ,  $n = 1, 3, 5, \dots$

**EXECUTE:**  $2.00 \text{ m} = (n/2)\lambda$  so  $\lambda = 4.00 \text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{4.00 \text{ m}} = \frac{n(344 \text{ m/s})}{(4.00 \text{ m})} = n(86 \text{ Hz}), \quad n = 1, 3, 5, \dots$$

The lowest frequency for which destructive interference occurs is 86 Hz.

**EVALUATE:** As the frequency is slowly increased, the intensity at  $Q$  will fluctuate, as the interference changes between destructive and constructive.

- 16.36. IDENTIFY:** Constructive interference occurs when the difference of the distances of each source from point  $P$  is an integer number of wavelengths. The interference is destructive when this difference of path lengths is a half integer number of wavelengths.

**SET UP:** The wavelength is  $\lambda = v/f = (344 \text{ m/s})/(206 \text{ Hz}) = 1.67 \text{ m}$ . Since  $P$  is between the speakers,  $x$  must be in the range 0 to  $L$ , where  $L = 2.00 \text{ m}$  is the distance between the speakers.

**EXECUTE:** The difference in path length is  $\Delta l = (L - x) - x = L - 2x$ , or  $x = (L - \Delta l)/2$ . For destructive interference,  $\Delta l = (n + (1/2))\lambda$ , and for constructive interference,  $\Delta l = n\lambda$ .

(a) Destructive interference:  $n = 0$  gives  $\Delta l = 0.835 \text{ m}$  and  $x = 0.58 \text{ m}$ .  $n = -1$  gives  $\Delta l = -0.835 \text{ m}$  and  $x = 1.42 \text{ m}$ . No other values of  $n$  place  $P$  between the speakers.

(b) Constructive interference:  $n = 0$  gives  $\Delta l = 0$  and  $x = 1.00 \text{ m}$ .  $n = 1$  gives  $\Delta l = 1.67 \text{ m}$  and  $x = 0.17 \text{ m}$ .  $n = -1$  gives  $\Delta l = -1.67 \text{ m}$  and  $x = 1.83 \text{ m}$ . No other values of  $n$  place  $P$  between the speakers.

(c) Treating the speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting from the walls, ceiling and floor.

**EVALUATE:** Points of constructive interference are a distance  $\lambda/2$  apart, and the same is true for the points of destructive interference.

- 16.37. IDENTIFY:** For constructive interference the path difference is an integer number of wavelengths and for destructive interference the path difference is a half-integer number of wavelengths.

**SET UP:**  $\lambda = v/f = (344 \text{ m/s})/(688 \text{ Hz}) = 0.500 \text{ m}$

**EXECUTE:** To move from constructive interference to destructive interference, the path difference must change by  $\lambda/2$ . If you move a distance  $x$  toward speaker  $B$ , the distance to  $B$  gets shorter by  $x$  and the distance to  $A$  gets longer by  $x$  so the path difference changes by  $2x$ .  $2x = \lambda/2$  and  $x = \lambda/4 = 0.125 \text{ m}$ .

**EVALUATE:** If you walk an additional distance of 0.125 m farther, the interference again becomes constructive.

- 16.38. IDENTIFY:** Destructive interference occurs when the path difference is a half integer number of wavelengths.

**SET UP:**  $v = 344$  m/s, so  $\lambda = v/f = (344 \text{ m/s})/(172 \text{ Hz}) = 2.00$  m. If  $r_A = 8.00$  m and  $r_B$  are the distances of the person from each speaker, the condition for destructive interference is  $r_B - r_A = \left(n + \frac{1}{2}\right)\lambda$ , where  $n$  is any integer.

**EXECUTE:** Requiring  $r_B = r_A + \left(n + \frac{1}{2}\right)\lambda > 0$  gives  $n + \frac{1}{2} > -r_A/\lambda = 0 - (8.00 \text{ m})/(2.00 \text{ m}) = -4$ , so the smallest value of  $r_B$  occurs when  $n = -4$ , and the closest distance to  $B$  is

$$r_B = 8.00 \text{ m} + \left(-4 + \frac{1}{2}\right)(2.00 \text{ m}) = 1.00 \text{ m}.$$

**EVALUATE:** For  $r_B = 1.00$  m, the path difference is  $r_A - r_B = 7.00$  m. This is  $3.5\lambda$ .

- 16.39. IDENTIFY:** For constructive interference, the path difference is an integer number of wavelengths. For destructive interference, the path difference is a half-integer number of wavelengths.

**SET UP:** One speaker is 4.50 m from the microphone and the other is 4.92 m from the microphone, so the path difference is 0.42 m.  $f = v/\lambda$ .

**EXECUTE: (a)**  $\lambda = 0.42$  m gives  $f = \frac{v}{\lambda} = 820$  Hz;  $2\lambda = 0.42$  m gives  $\lambda = 0.21$  m and

$f = \frac{v}{\lambda} = 1640$  Hz;  $3\lambda = 0.42$  m gives  $\lambda = 0.14$  m and  $f = \frac{v}{\lambda} = 2460$  Hz, and so on. The frequencies for constructive interference are  $n(820 \text{ Hz})$ ,  $n = 1, 2, 3, \dots$

**(b)**  $\lambda/2 = 0.42$  m gives  $\lambda = 0.84$  m and  $f = \frac{v}{\lambda} = 410$  Hz;  $3\lambda/2 = 0.42$  m gives  $\lambda = 0.28$  m and

$f = \frac{v}{\lambda} = 1230$  Hz;  $5\lambda/2 = 0.42$  m gives  $\lambda = 0.168$  m and  $f = \frac{v}{\lambda} = 2050$  Hz, and so on. The frequencies for destructive interference are  $(2n + 1)(410 \text{ Hz})$ ,  $n = 0, 1, 2, \dots$

**EVALUATE:** The frequencies for constructive interference lie midway between the frequencies for destructive interference.

- 16.40. IDENTIFY:**  $f_{\text{beat}} = |f_1 - f_2|$ .  $v = f\lambda$ .

**SET UP:**  $v = 344$  m/s. Let  $\lambda_1 = 64.8$  cm and  $\lambda_2 = 65.2$  cm.  $\lambda_2 > \lambda_1$  so  $f_1 > f_2$ .

**EXECUTE:**  $f_1 - f_2 = v \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) = \frac{v(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} = \frac{(344 \text{ m/s})(0.04 \times 10^{-2} \text{ m})}{(0.648 \text{ m})(0.652 \text{ m})} = 0.33 \text{ beats/s}$ , which rounds to 0.3 beats/s.

**EVALUATE:** We could have calculated  $f_1$  and  $f_2$  and subtracted, but doing it this way we would have to be careful to retain enough figures in intermediate calculations to avoid round-off errors.

- 16.41. IDENTIFY:** The beat is due to a difference in the frequencies of the two sounds.

**SET UP:**  $f_{\text{beat}} = f_1 - f_2$ . Tightening the string increases the wave speed for transverse waves on the string and this in turn increases the frequency.

**EXECUTE: (a)** If the beat frequency increases when she raises her frequency by tightening the string, it must be that her frequency is 433 Hz, 3 Hz above concert  $A$ .

**(b)** She needs to lower her frequency by loosening her string.

**EVALUATE:** The beat would only be audible if the two sounds are quite close in frequency. A musician with a good sense of pitch can come very close to the correct frequency just from hearing the tone.

- 16.42. IDENTIFY:** The motors produce sound having the same frequency as the motor. If the motors are almost, but not quite, the same, a beat will result.

**SET UP:**  $f_{\text{beat}} = f_1 - f_2$ . 1 rpm = 60 Hz.

**EXECUTE: (a)** 575 rpm = 9.58 Hz. The frequency of the other propeller differs by 2.0 Hz, so the frequency of the other propeller is either 11.6 Hz or 7.6 Hz. These frequencies correspond to 696 rpm or 456 rpm.

(b) When the speed and rpm of the second propeller is increased the beat frequency increases, so the frequency of the second propeller moves farther from the frequency of the first and the second propeller is turning at 696 rpm.

**EVALUATE:** If the frequency of the second propeller was 7.6 Hz then it would have moved close to the frequency of the first when its frequency was increased and the beat frequency would have decreased.

**16.43. IDENTIFY:**  $f_{\text{beat}} = |f_a - f_b|$ . For a stopped pipe,  $f_1 = \frac{v}{4L}$ .

**SET UP:**  $v = 344$  m/s. Let  $L_a = 1.14$  m and  $L_b = 1.16$  m.  $L_b > L_a$  so  $f_{1a} > f_{1b}$ .

**EXECUTE:**  $f_{1a} - f_{1b} = \frac{v}{4} \left( \frac{1}{L_a} - \frac{1}{L_b} \right) = \frac{v(L_b - L_a)}{4L_a L_b} = \frac{(344 \text{ m/s})(2.00 \times 10^{-2} \text{ m})}{4(1.14 \text{ m})(1.16 \text{ m})} = 1.3 \text{ Hz}$ . There are 1.3 beats per second.

**EVALUATE:** Increasing the length of the pipe increases the wavelength of the fundamental and decreases the frequency.

**16.44. IDENTIFY:** Follow the steps of Example 16.18.

**SET UP:** In the first step,  $v_s = +20.0$  m/s instead of  $-30.0$  m/s. In the second step,  $v_L = -20.0$  m/s instead of  $+30.0$  m/s.

**EXECUTE:**  $f_W = \left( \frac{v}{v + v_s} \right) f_S = \left( \frac{340 \text{ m/s}}{340 \text{ m/s} + 20.0 \text{ m/s}} \right) (300 \text{ Hz}) = 283 \text{ Hz}$ . Then

$$f_L = \left( \frac{v + v_L}{v} \right) f_W = \left( \frac{340 \text{ m/s} - 20.0 \text{ m/s}}{340 \text{ m/s}} \right) (283 \text{ Hz}) = 266 \text{ Hz}.$$

**EVALUATE:** When the car is moving toward the reflecting surface, the received frequency back at the source is higher than the emitted frequency. When the car is moving away from the reflecting surface, as is the case here, the received frequency back at the source is lower than the emitted frequency.

**16.45. IDENTIFY:** Apply the Doppler shift equation  $f_L = \left( \frac{v + v_L}{v + v_s} \right) f_S$ .

**SET UP:** The positive direction is from listener to source.  $f_S = 1200$  Hz.  $f_L = 1240$  Hz.

**EXECUTE:**  $v_L = 0$ .  $v_s = -25.0$  m/s.  $f_L = \left( \frac{v}{v + v_s} \right) f_S$  gives

$$v = \frac{v_s f_L}{f_S - f_L} = \frac{(-25 \text{ m/s})(1240 \text{ Hz})}{1200 \text{ Hz} - 1240 \text{ Hz}} = 780 \text{ m/s}.$$

**EVALUATE:**  $f_L > f_S$  since the source is approaching the listener.

**16.46. IDENTIFY and SET UP:** Apply  $\lambda = \frac{v - v_s}{f_S}$  and  $\lambda = \frac{v + v_s}{f_S}$  for the wavelengths in front of and behind the

source. Then  $f = v/\lambda$ . When the source is at rest  $\lambda = \frac{v}{f_S} = \frac{344 \text{ m/s}}{400 \text{ Hz}} = 0.860$  m.

**EXECUTE: (a)**  $\lambda = \frac{v - v_s}{f_S} = \frac{344 \text{ m/s} - 25.0 \text{ m/s}}{400 \text{ Hz}} = 0.798$  m

**(b)**  $\lambda = \frac{v + v_s}{f_S} = \frac{344 \text{ m/s} + 25.0 \text{ m/s}}{400 \text{ Hz}} = 0.922$  m

**(c)**  $f_L = v/\lambda$  (since  $v_L = 0$ ), so  $f_L = (344 \text{ m/s})/0.798 \text{ m} = 431$  Hz

**(d)**  $f_L = v/\lambda = (344 \text{ m/s})/0.922 \text{ m} = 373$  Hz

**EVALUATE:** In front of the source (source moving toward listener) the wavelength is decreased and the frequency is increased. Behind the source (source moving away from listener) the wavelength is increased and the frequency is decreased.

**16.47. IDENTIFY:** Apply the Doppler shift equation  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ .

**SET UP:** The positive direction is from listener to source.  $f_S = 392$  Hz.

**EXECUTE: (a)**  $v_S = 0$ .  $v_L = -15.0$  m/s.  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} - 15.0 \text{ m/s}}{344 \text{ m/s}} \right) (392 \text{ Hz}) = 375 \text{ Hz}$

**(b)**  $v_S = +35.0$  m/s.  $v_L = +15.0$  m/s.  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} + 15.0 \text{ m/s}}{344 \text{ m/s} + 35.0 \text{ m/s}} \right) (392 \text{ Hz}) = 371 \text{ Hz}$

**(c)**  $f_{\text{beat}} = f_1 - f_2 = 4$  Hz

**EVALUATE:** The distance between whistle *A* and the listener is increasing, and for whistle *A*  $f_L < f_S$ .

The distance between whistle *B* and the listener is also increasing, and for whistle *B*  $f_L < f_S$ .

**16.48. IDENTIFY:** Apply  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ .

**SET UP:**  $f_S = 1000$  Hz. The positive direction is from the listener to the source.  $v = 344$  m/s.

**EXECUTE: (a)**  $v_S = -(344 \text{ m/s})/2 = -172$  m/s,  $v_L = 0$ .

$f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s}}{344 \text{ m/s} - 172 \text{ m/s}} \right) (1000 \text{ Hz}) = 2000 \text{ Hz}$

**(b)**  $v_S = 0$ ,  $v_L = +172$  m/s.  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} + 172 \text{ m/s}}{344 \text{ m/s}} \right) (1000 \text{ Hz}) = 1500 \text{ Hz}$

**EVALUATE:** The answer in (b) is much less than the answer in (a). It is the velocity of the source and listener relative to the air that determines the effect, not the relative velocity of the source and listener relative to each other.

**16.49. IDENTIFY:** The distance between crests is  $\lambda$ . In front of the source  $\lambda = \frac{v - v_S}{f_S}$  and behind the source

$$\lambda = \frac{v + v_S}{f_S}. \quad f_S = 1/T.$$

**SET UP:**  $T = 1.6$  s.  $v = 0.32$  m/s. The crest to crest distance is the wavelength, so  $\lambda = 0.12$  m.

**EXECUTE: (a)**  $f_S = 1/T = 0.625$  Hz.  $\lambda = \frac{v - v_S}{f_S}$  gives

$$v_S = v - \lambda f_S = 0.32 \text{ m/s} - (0.12 \text{ m})(0.625 \text{ Hz}) = 0.25 \text{ m/s}.$$

**(b)**  $\lambda = \frac{v + v_S}{f_S} = \frac{0.32 \text{ m/s} + 0.25 \text{ m/s}}{0.625 \text{ Hz}} = 0.91 \text{ m}$

**EVALUATE:** If the duck was held at rest but still paddled its feet, it would produce waves of wavelength

$$\lambda = \frac{0.32 \text{ m/s}}{0.625 \text{ Hz}} = 0.51 \text{ m}.$$

In front of the duck the wavelength is decreased and behind the duck the wavelength is increased. The speed of the duck is 78% of the wave speed, so the Doppler effects are large.

**16.50. IDENTIFY:** Apply the Doppler effect formula  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ .

**(a) SET UP:** The positive direction is from the listener toward the source, as shown in Figure 16.50a.

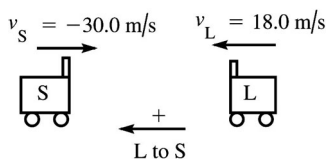


Figure 16.50a

**EXECUTE:**  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} + 18.0 \text{ m/s}}{344 \text{ m/s} - 30.0 \text{ m/s}} \right) (352 \text{ Hz}) = 406 \text{ Hz}.$

**EVALUATE:** Listener and source are approaching and  $f_L > f_S$ .

**(b) SET UP:** See Figure 16.50b.

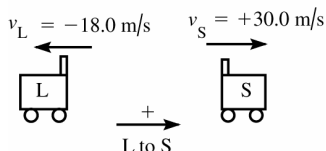


Figure 16.50b

**EXECUTE:**  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} - 18.0 \text{ m/s}}{344 \text{ m/s} + 30.3 \text{ m/s}} \right) (352 \text{ Hz}) = 307 \text{ Hz}.$

**EVALUATE:** Listener and source are moving away from each other and  $f_L < f_S$ .

**16.51. IDENTIFY:** Apply  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ .

**SET UP:** The positive direction is from the motorcycle toward the car. The car is stationary, so  $v_S = 0$ .

**EXECUTE:**  $f_L = \frac{v + v_L}{v + v_S} f_S = (1 + v_L/v) f_S$ , which gives

$$v_L = v \left( \frac{f_L}{f_S} - 1 \right) = (344 \text{ m/s}) \left( \frac{490 \text{ Hz}}{520 \text{ Hz}} - 1 \right) = -19.8 \text{ m/s. You must be traveling at 19.8 m/s.}$$

**EVALUATE:**  $v_L < 0$  means that the listener is moving away from the source.

**16.52. IDENTIFY:** We have a moving source and a stationary observer. The beat frequency is due to interference between the Doppler-shifted sound from the horn in the moving car and the horn in the stationary car. The beat frequency is equal to the difference between these two frequencies.

**SET UP:** Apply  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{v}{v + v_S} \right) f_S$ , where  $f_S = 260 \text{ Hz}$ . Since the source is moving toward you, you will hear the moving car horn at a higher pitch than your horn, and the beat frequency will be given by  $f_{\text{beat}} = f_L - f_S$ .

**EXECUTE:** We can determine  $f_L$  from the beat frequency:  $f_{\text{beat}} = 6.0 \text{ Hz} = f_L - f_S = f_L - 260 \text{ Hz}$ ; thus,

$$f_L = 266 \text{ Hz. Assuming that } v = 344 \text{ m/s, we obtain } 266 \text{ Hz} = \left( \frac{344 \text{ m/s}}{344 \text{ m/s} + v_S} \right) (260 \text{ Hz}). \text{ Solving for } v_S$$

$$\text{we obtain } v_S = \left( \frac{260 \text{ Hz}}{266 \text{ Hz}} - 1 \right) (344 \text{ m/s}) = -7.8 \text{ m/s. Thus, your friend is moving at 7.8 m/s toward you.}$$

**EVALUATE:** What frequency will your friend hear? In this case, you have a stationary source (your horn) and a moving observer (your friend). The positive direction points from listener to source. Thus, we have

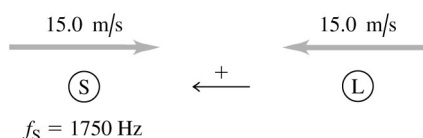
$$f_L = \left( \frac{344 \text{ m/s} + 7.8 \text{ m/s}}{344 \text{ m/s}} \right) (260 \text{ Hz}) = 265.8 \text{ Hz} \approx 266 \text{ Hz. At low speeds, there is little difference in the}$$

Doppler shift of a moving source or that of a moving observer.

**16.53. IDENTIFY:** Each bird is a moving source of sound and a moving observer, so each will experience a Doppler shift.

**SET UP:** Let one bird be the listener and the other be the source. Use coordinates as shown in Figure 16.53,

with the positive direction from listener to source.  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ .



**Figure 16.53**

**EXECUTE: (a)**  $f_S = 1750$  Hz,  $v_S = -15.0$  m/s, and  $v_L = +15.0$  m/s.

$$f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} + 15.0 \text{ m/s}}{344 \text{ m/s} - 15.0 \text{ m/s}} \right) (1750 \text{ Hz}) = 1910 \text{ Hz}.$$

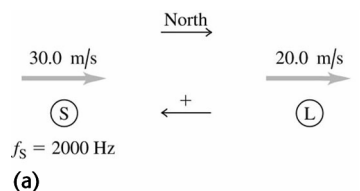
**(b)** One canary hears a frequency of 1910 Hz and the waves move past it at  $344 \text{ m/s} + 15 \text{ m/s}$ , so the wavelength it detects is  $\lambda = \frac{344 \text{ m/s} + 15 \text{ m/s}}{1910 \text{ Hz}} = 0.188 \text{ m}$ . For a stationary bird,  $\lambda = \frac{344 \text{ m/s}}{1750 \text{ Hz}} = 0.197 \text{ m}$ .

**EVALUATE:** The approach of the two birds raises the frequency, and the motion of the source toward the listener decreases the wavelength.

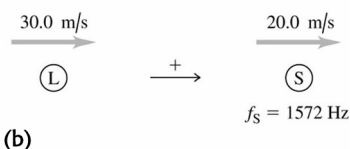
**16.54. IDENTIFY:** There is a Doppler shift due to the motion of the fire engine as well as due to the motion of the truck, which reflects the sound waves.

**SET UP:** We use the Doppler shift equation  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ .

**EXECUTE: (a)** First consider the truck as the listener, as shown in Figure 16.54a.



**(a)**



**(b)**

**Figure 16.54**

$f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} - 20.0 \text{ m/s}}{344 \text{ m/s} - 30.0 \text{ m/s}} \right) (2000 \text{ Hz}) = 2064 \text{ Hz}$ . Now consider the truck as a source, with  $f_S = 2064$  Hz, and the fire engine driver as the listener (Figure 16.54b).

$f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{344 \text{ m/s} + 30.0 \text{ m/s}}{344 \text{ m/s} + 20.0 \text{ m/s}} \right) (2064 \text{ Hz}) = 2120 \text{ Hz}$ . The objects are getting closer together so the frequency is increased.

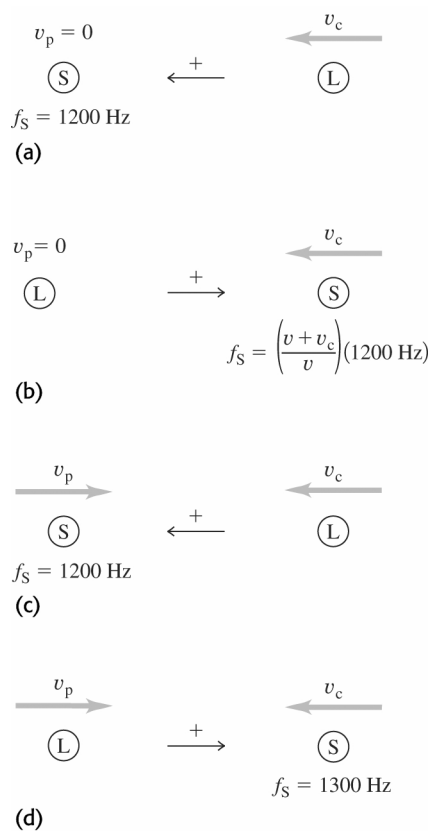
(b) The driver detects a frequency of 2120 Hz and the waves returning from the truck move past him at  $344 \text{ m/s} + 30.0 \text{ m/s}$ , so the wavelength he measures is  $\lambda = \frac{344 \text{ m/s} + 30 \text{ m/s}}{2120 \text{ Hz}} = 0.176 \text{ m}$ . The wavelength of waves emitted by the fire engine when it is stationary is  $\lambda = \frac{344 \text{ m/s}}{2000 \text{ Hz}} = 0.172 \text{ m}$ .

**EVALUATE:** In (a) the objects are getting closer together so the frequency is increased. In (b), the quantities to use in the equation  $v = f\lambda$  are measured *relative to the observer*.

**16.55. IDENTIFY:** Apply the Doppler shift formulas. We first treat the stationary police car as the source and then as the observer as he receives his own sound reflected from the on-coming car.

**SET UP:**  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ .

**EXECUTE: (a)** Since the frequency is increased the moving car must be approaching the police car. Let  $v_c$  be the speed of the moving car. The speed  $v_p$  of the police car is zero. First consider the moving car as the listener, as shown in Figure 16.55a.



**Figure 16.55**

$$f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S = \left( \frac{v + v_c}{v} \right) (1200 \text{ Hz})$$

Then consider the moving car as the source and the police car as the listener (Figure 16.55b):

$$f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S \text{ gives } 1250 \text{ Hz} = \left( \frac{v}{v - v_c} \right) \left( \frac{v + v_c}{v} \right) (1200 \text{ Hz}).$$



Solving for  $v_c$  gives

$$v_c = \left( \frac{50}{2450} \right) v = \left( \frac{50}{2450} \right) (344 \text{ m/s}) = 7.02 \text{ m/s}$$

(b) Repeat the calculation of part (a), but now  $v_p = 20.0 \text{ m/s}$ , toward the other car.

Waves received by the car (Figure 16.55c):

$$f_L = \left( \frac{v + v_c}{v - v_p} \right) f_S = \left( \frac{344 \text{ m/s} + 7 \text{ m/s}}{344 \text{ m/s} - 20 \text{ m/s}} \right) (1200 \text{ Hz}) = 1300 \text{ Hz}$$

Waves reflected by the car and received by the police car (Figure 16.55d):

$$f_L = \left( \frac{v + v_p}{v - v_c} \right) f_S = \left( \frac{344 \text{ m/s} + 20 \text{ m/s}}{344 \text{ m/s} - 7 \text{ m/s}} \right) (1300 \text{ Hz}) = 1404 \text{ Hz}$$

**EVALUATE:** The cars move toward each other with a greater relative speed in (b) and the increase in frequency is much larger there.

**16.56. IDENTIFY:** Apply  $f_R = \sqrt{\frac{c - v}{c + v}} f_S$ .

**SET UP:** Require  $f_R = 1.100 f_S$ . Since  $f_R > f_S$  the star would be moving toward us and  $v < 0$ , so  $v = -|v|$ .  $c = 3.00 \times 10^8 \text{ m/s}$ .

**EXECUTE:**  $f_R = \sqrt{\frac{c + |v|}{c - |v|}} f_S$ .  $f_R = 1.100 f_S$  gives  $\frac{c + |v|}{c - |v|} = (1.100)^2$ . Solving for  $|v|$  gives

$$|v| = \frac{[(1.100)^2 - 1]c}{1 + (1.100)^2} = 0.0950c = 2.85 \times 10^7 \text{ m/s}.$$

**EVALUATE:**  $\frac{v}{c} = 9.5\%$ .  $\frac{\Delta f}{f_S} = \frac{f_R - f_S}{f_S} = 10.0\%$ .  $\frac{v}{c}$  and  $\frac{\Delta f}{f_S}$  are approximately equal.

**16.57. IDENTIFY:** Apply  $\sin \alpha = v/v_s$  to calculate  $\alpha$ . Use the method of Example 16.19 to calculate  $t$ .

**SET UP:** Mach 1.70 means  $v_s/v = 1.70$ .

**EXECUTE: (a)** In  $\sin \alpha = v/v_s$ ,  $v/v_s = 1/1.70 = 0.588$  and  $\alpha = \arcsin(0.588) = 36.0^\circ$ .

**(b)** As in Example 16.19,  $t = \frac{1250 \text{ m}}{(1.70)(344 \text{ m/s})(\tan 36.0^\circ)} = 2.94 \text{ s}$ .

**EVALUATE:** The angle  $\alpha$  decreases when the speed  $v_s$  of the plane increases.

**16.58. IDENTIFY:** Apply  $\sin \alpha = v/v_s$ .

**SET UP:** The Mach number is the value of  $v_s/v$ , where  $v_s$  is the speed of the shuttle and  $v$  is the speed of sound at the altitude of the shuttle.

**EXECUTE: (a)**  $\frac{v}{v_s} = \sin \alpha = \sin 58.0^\circ = 0.848$ . The Mach number is  $\frac{v_s}{v} = \frac{1}{0.848} = 1.18$ .

**(b)**  $v_s = \frac{v}{\sin \alpha} = \frac{331 \text{ m/s}}{\sin 58.0^\circ} = 390 \text{ m/s}$

**(c)**  $\frac{v_s}{v} = \frac{390 \text{ m/s}}{344 \text{ m/s}} = 1.13$ . The Mach number would be 1.13.  $\sin \alpha = \frac{v}{v_s} = \frac{344 \text{ m/s}}{390 \text{ m/s}}$  and  $\alpha = 61.9^\circ$ .

**EVALUATE:** The smaller the Mach number, the larger the angle of the shock-wave cone.

**16.59. IDENTIFY:** The sound intensity level is  $\beta = (10 \text{ dB})\log(I/I_0)$ , so the same sound intensity level  $\beta$  means

the same intensity  $I$ . The intensity is related to pressure amplitude by  $I = \frac{vp_{\max}^2}{2B}$  and to the displacement

amplitude by  $I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2$ .

**SET UP:**  $v = 344 \text{ m/s}$ .  $\omega = 2\pi f$ . Each octave higher corresponds to a doubling of frequency, so the note sung by the bass has frequency  $(932 \text{ Hz})/8 = 116.5 \text{ Hz}$ . Let 1 refer to the note sung by the soprano and 2 refer to the note sung by the bass.  $I_0 = 1 \times 10^{-12} \text{ W/m}^2$ .

**EXECUTE:** (a)  $I = \frac{vp_{\max}^2}{2B}$  and  $I_1 = I_2$  gives  $p_{\max,1} = p_{\max,2}$ ; the ratio is 1.00.

(b)  $I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2 = \frac{1}{2}\sqrt{\rho B}4\pi^2 f^2 A^2$ .  $I_1 = I_2$  gives  $f_1 A_1 = f_2 A_2$ .  $\frac{A_2}{A_1} = \frac{f_1}{f_2} = 8.00$ .

(c)  $\beta = 72.0 \text{ dB}$  gives  $\log(I/I_0) = 7.2$ .  $\frac{I}{I_0} = 10^{7.2}$  and  $I = 1.585 \times 10^{-5} \text{ W/m}^2$ .  $I = \frac{1}{2}\sqrt{\rho B}4\pi^2 f^2 A^2$ .

$$A = \frac{1}{2\pi f} \sqrt{\frac{2I}{\sqrt{\rho B}}} = \frac{1}{2\pi(932 \text{ Hz})} \sqrt{\frac{2(1.585 \times 10^{-5} \text{ W/m}^2)}{\sqrt{(1.20 \text{ kg/m}^3)(1.42 \times 10^5 \text{ Pa})}}} = 4.73 \times 10^{-8} \text{ m} = 47.3 \text{ nm}.$$

**EVALUATE:** Even for this loud note the displacement amplitude is very small. For a given intensity, the displacement amplitude depends on the frequency of the sound wave but the pressure amplitude does not.

**16.60. IDENTIFY:** Use the equations that relate intensity level and intensity, intensity and pressure amplitude, pressure amplitude and displacement amplitude, and intensity and distance.

(a) **SET UP:** Use the intensity level  $\beta$  to calculate  $I$  at this distance.  $\beta = (10 \text{ dB})\log(I/I_0)$

**EXECUTE:**  $52.0 \text{ dB} = (10 \text{ dB})\log[I/(10^{-12} \text{ W/m}^2)]$

$\log[I/(10^{-12} \text{ W/m}^2)] = 5.20$  implies  $I = 1.585 \times 10^{-7} \text{ W/m}^2$

**SET UP:** Then use  $I = \frac{p_{\max}^2}{2\rho v}$  to calculate  $p_{\max}$ :

$$I = \frac{p_{\max}^2}{2\rho v} \text{ so } p_{\max} = \sqrt{2\rho v I}$$

From Example 16.5,  $\rho = 1.20 \text{ kg/m}^3$  for air at  $20^\circ\text{C}$ .

**EXECUTE:**  $p_{\max} = \sqrt{2\rho v I} = \sqrt{2(1.20 \text{ kg/m}^3)(344 \text{ m/s})(1.585 \times 10^{-7} \text{ W/m}^2)} = 0.0114 \text{ Pa}$

(b) **SET UP:** Use  $p_{\max} = BkA$  so  $A = \frac{p_{\max}}{Bk}$

For air  $B = 1.42 \times 10^5 \text{ Pa}$  (Example 16.1).

**EXECUTE:**  $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{(2\pi \text{ rad})(587 \text{ Hz})}{344 \text{ m/s}} = 10.72 \text{ rad/m}$

$$A = \frac{p_{\max}}{Bk} = \frac{0.0114 \text{ Pa}}{(1.42 \times 10^5 \text{ Pa})(10.72 \text{ rad/m})} = 7.49 \times 10^{-9} \text{ m}$$

(c) **SET UP:**  $\beta_2 - \beta_1 = (10 \text{ dB})\log(I_2/I_1)$  (Example 16.9).

Inverse-square law:  $I_1/I_2 = r_2^2/r_1^2$  so  $I_2/I_1 = r_1^2/r_2^2$

**EXECUTE:**  $\beta_2 - \beta_1 = (10 \text{ dB})\log(r_1/r_2)^2 = (20 \text{ dB})\log(r_1/r_2)$ .

$\beta_2 = 52.0 \text{ dB}$  and  $r_2 = 5.00 \text{ m}$ . Then  $\beta_1 = 30.0 \text{ dB}$  and we need to calculate  $r_1$ .

$$52.0 \text{ dB} - 30.0 \text{ dB} = (20 \text{ dB})\log(r_1/r_2)$$

$$22.0 \text{ dB} = (20 \text{ dB})\log(r_1/r_2)$$

$$\log(r_1/r_2) = 1.10 \text{ so } r_1 = 12.6r_2 = 63.0 \text{ m}.$$

**EVALUATE:** The decrease in intensity level corresponds to a decrease in intensity, and this means an increase in distance. The intensity level uses a logarithmic scale, so simple proportionality between  $r$  and  $\beta$  doesn't apply.

- 16.61. IDENTIFY:** The flute acts as a stopped pipe and its harmonic frequencies are given by  $f_n = nf_1$ ,  $n = 1, 3, 5, \dots$ . The resonant frequencies of the string are  $f_n = nf_1$ ,  $n = 1, 2, 3, \dots$ . The string resonates when the string frequency equals the flute frequency.

**SET UP:** For the string  $f_{1s} = 600.0$  Hz. For the flute, the fundamental frequency is

$$f_{1f} = \frac{v}{4L} = \frac{344.0 \text{ m/s}}{4(0.1075 \text{ m})} = 800.0 \text{ Hz.}$$

Let  $n_f$  label the harmonics of the flute and let  $n_s$  label the

harmonics of the string. **EXECUTE:** For the flute and string to be in resonance,  $n_f f_{1f} = n_s f_{1s}$ , where  $f_{1s} = 600.0$  Hz is the

fundamental frequency for the string.  $n_s = n_f (f_{1f}/f_{1s}) = \frac{4}{3} n_f$ .  $n_s$  is an integer when  $n_f = 3N$ ,  $N = 1, 3, 5, \dots$

(the flute has only odd harmonics).  $n_f = 3N$  gives  $n_s = 4N$ .

Flute harmonic  $3N$  resonates with string harmonic  $4N$ ,  $N = 1, 3, 5, \dots$

**EVALUATE:** We can check our results for some specific values of  $N$ . For  $N = 1$ ,  $n_f = 3$  and

$f_{3f} = 2400$  Hz. For this  $N$ ,  $n_s = 4$  and  $f_{4s} = 2400$  Hz. For  $N = 3$ ,  $n_f = 9$  and  $f_{9f} = 7200$  Hz, and

$n_s = 12$ ,  $f_{12s} = 7200$  Hz. Our general results do give equal frequencies for the two objects.

- 16.62. IDENTIFY:**  $f_{\text{beat}} = |f_A - f_B|$ .  $f_1 = \frac{v}{2L}$  and  $v = \sqrt{\frac{FL}{m}}$  gives  $f_1 = \frac{1}{2} \sqrt{\frac{F}{mL}}$ . Apply  $\Sigma \tau_z = 0$  to the bar to find the tension in each wire.

**SET UP:** For  $\Sigma \tau_z = 0$  take the pivot at wire  $A$  and let counterclockwise torques be positive. The free-body diagram for the bar is given in Figure 16.62. Let  $L$  be the length of the bar.

**EXECUTE:**  $\Sigma \tau_z = 0$  gives  $F_B L - w_{\text{lead}}(3L/4) - w_{\text{bar}}(L/2) = 0$ .

$F_B = 3w_{\text{lead}}/4 + w_{\text{bar}}/2 = 3(185 \text{ N})/4 + (165 \text{ N})/2 = 221 \text{ N}$ .  $F_A + F_B = w_{\text{bar}} + w_{\text{lead}}$  so

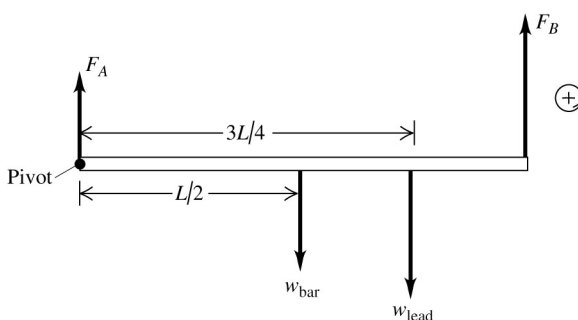
$$F_A = w_{\text{bar}} + w_{\text{lead}} - F_B = 165 \text{ N} + 185 \text{ N} - 221 \text{ N} = 129 \text{ N.}$$

$$f_{1A} = \frac{1}{2} \sqrt{\frac{129 \text{ N}}{(5.50 \times 10^{-3} \text{ kg})(0.750 \text{ m})}} = 88.4 \text{ Hz.}$$

$$f_{1B} = f_{1A} \sqrt{\frac{221 \text{ N}}{129 \text{ N}}} = 115.7 \text{ Hz.}$$

$$f_{\text{beat}} = f_{1B} - f_{1A} = 27.3 \text{ Hz.}$$

**EVALUATE:** The frequency increases when the tension in the wire increases.



**Figure 16.62**

- 16.63. IDENTIFY and SET UP:** The frequency of any harmonic is an integer multiple of the fundamental. For a stopped pipe only odd harmonics are present. For an open pipe, all harmonics are present. See which pattern of harmonics fits to the observed values in order to determine which type of pipe it is. Then solve for the fundamental frequency and relate that to the length of the pipe.

**EXECUTE:** (a) For an open pipe the successive harmonics are  $f_n = nf_1$ ,  $n = 1, 2, 3, \dots$ . For a stopped pipe the successive harmonics are  $f_n = nf_1$ ,  $n = 1, 3, 5, \dots$ . If the pipe is open and these harmonics are successive, then  $f_n = nf_1 = 1372 \text{ Hz}$  and  $f_{n+1} = (n+1)f_1 = 1764 \text{ Hz}$ . Subtract the first equation from the second:  $(n+1)f_1 - nf_1 = 1764 \text{ Hz} - 1372 \text{ Hz}$ . This gives  $f_1 = 392 \text{ Hz}$ . Then  $n = \frac{1372 \text{ Hz}}{392 \text{ Hz}} = 3.5$ . But  $n$  must be an integer, so the pipe can't be open. If the pipe is stopped and these harmonics are successive, then  $f_n = nf_1 = 1372 \text{ Hz}$  and  $f_{n+2} = (n+2)f_1 = 1764 \text{ Hz}$  (in this case successive harmonics differ in  $n$  by 2). Subtracting one equation from the other gives  $2f_1 = 392 \text{ Hz}$  and  $f_1 = 196 \text{ Hz}$ . Then  $n = 1372 \text{ Hz}/f_1 = 7$  so  $1372 \text{ Hz} = 7f_1$  and  $1764 \text{ Hz} = 9f_1$ . The solution gives integer  $n$  as it should; the pipe is stopped.

(b) From part (a) these are the seventh and ninth harmonics.

(c) From part (a)  $f_1 = 196 \text{ Hz}$ .

For a stopped pipe  $f_1 = \frac{v}{4L}$  and  $L = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(196 \text{ Hz})} = 0.439 \text{ m}$ .

**EVALUATE:** It is essential to know that these are successive harmonics and to realize that 1372 Hz is not the fundamental. There are other lower frequency standing waves; these are just two successive ones.

**16.64. IDENTIFY:** For a stopped pipe the frequency of the fundamental is  $f_1 = \frac{v}{4L}$ . The speed of sound in air

depends on temperature, as shown by  $v = \sqrt{\frac{\gamma RT}{M}}$ .

**SET UP:** Example 16.4 shows that the speed of sound in air at  $20^\circ\text{C}$  is 344 m/s.

**EXECUTE:** (a)  $L = \frac{v}{4f} = \frac{344 \text{ m/s}}{4(349 \text{ Hz})} = 0.246 \text{ m}$

(b) The frequency will be proportional to the speed, and hence to the square root of the Kelvin temperature. The temperature necessary to have the frequency be higher is  $(293.15 \text{ K})[(370 \text{ Hz})/(349 \text{ Hz})]^2 = 329.5 \text{ K}$ , which is  $56.3^\circ\text{C}$ .

**EVALUATE:**  $56.3^\circ\text{C} = 133^\circ\text{F}$ , so this extreme rise in pitch won't occur in practical situations. But changes in temperature can have noticeable effects on the pitch of the organ notes.

**16.65. IDENTIFY:** Destructive interference occurs when the path difference is a half-integer number of wavelengths. Constructive interference occurs when the path difference is an integer number of wavelengths.

**SET UP:**  $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{784 \text{ Hz}} = 0.439 \text{ m}$

**EXECUTE:** (a) If the separation of the speakers is denoted  $h$ , the condition for destructive interference is  $\sqrt{x^2 + h^2} - x = \beta\lambda$ , where  $\beta$  is an odd multiple of one-half. Adding  $x$  to both sides, squaring, canceling

the  $x^2$  term from both sides, and solving for  $x$  gives  $x = \frac{h^2}{2\beta\lambda} - \frac{\beta}{2}\lambda$ . Using  $\lambda = 0.439 \text{ m}$  and

$h = 2.00 \text{ m}$  yields 9.01 m for  $\beta = \frac{1}{2}$ , 2.71 m for  $\beta = \frac{3}{2}$ , 1.27 m for  $\beta = \frac{5}{2}$ , 0.53 m for  $\beta = \frac{7}{2}$ , and 0.026 m for  $\beta = \frac{9}{2}$ . These are the only allowable values of  $\beta$  that give positive solutions for  $x$ .

(b) Repeating the above for integral values of  $\beta$ , constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m. Note that these are between, but not midway between, the answers to part (a).

(c) If  $h = \lambda/2$ , there will be destructive interference at speaker B. If  $\lambda/2 > h$ , the path difference can never be as large as  $\lambda/2$ . (This is also obtained from the above expression for  $x$ , with  $x = 0$  and  $\beta = \frac{1}{2}$ .) The minimum frequency is then  $v/2h = (344 \text{ m/s})/(4.0 \text{ m}) = 86 \text{ Hz}$ .

**EVALUATE:** When  $f$  increases,  $\lambda$  is smaller and there are more occurrences of points of constructive and destructive interference.

**16.66. IDENTIFY:** Apply  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ . The wall first acts as a listener and then as a source.

**SET UP:** The positive direction is from listener to source. The bat is moving toward the wall so the Doppler effect increases the frequency and the final frequency received,  $f_{L2}$ , is greater than the original source frequency,  $f_{S1}$ .  $f_{S1} = 1700$  Hz.  $f_{L2} - f_{S1} = 8.00$  Hz.

**EXECUTE:** The wall receives the sound:  $f_S = f_{S1}$ .  $f_L = f_{L1}$ .  $v_S = -v_{\text{bat}}$  and  $v_L = 0$ .  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$

gives  $f_{L1} = \left( \frac{v}{v - v_{\text{bat}}} \right) f_{S1}$ . The wall receives the sound:  $f_{S2} = f_{L1}$ .  $v_S = 0$  and  $v_L = +v_{\text{bat}}$ .

$$f_{L2} = \left( \frac{v + v_{\text{bat}}}{v} \right) f_{S2} = \left( \frac{v + v_{\text{bat}}}{v} \right) \left( \frac{v}{v - v_{\text{bat}}} \right) f_{S1} = \left( \frac{v + v_{\text{bat}}}{v - v_{\text{bat}}} \right) f_{S1}.$$

$$f_{L2} - f_{S1} = \Delta f = \left( \frac{v + v_{\text{bat}}}{v - v_{\text{bat}}} - 1 \right) f_{S1} = \left( \frac{2v_{\text{bat}}}{v - v_{\text{bat}}} \right) f_{S1}. \quad v_{\text{bat}} = \frac{v \Delta f}{2f_{S1} + \Delta f} = \frac{(344 \text{ m/s})(8.00 \text{ Hz})}{2(1700 \text{ Hz}) + 8.00 \text{ Hz}} = 0.808 \text{ m/s}.$$

**EVALUATE:**  $f_{S1} < \Delta f$ , so we can write our result as the approximate but accurate expression

$$\Delta f = \left( \frac{2v_{\text{bat}}}{v} \right) f_{S1}.$$

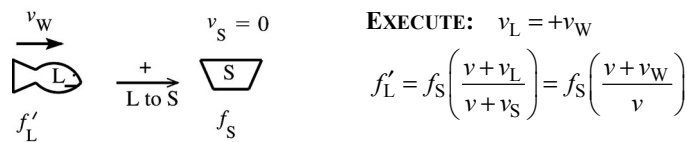
**16.67. (a) IDENTIFY and SET UP:** Use  $v = f\lambda$  to calculate  $\lambda$ .

$$\text{EXECUTE: } \lambda = \frac{v}{f} = \frac{1482 \text{ m/s}}{18.0 \times 10^3 \text{ Hz}} = 0.0823 \text{ m}.$$

**(b) IDENTIFY:** Apply the Doppler effect equation,  $f_L = \left( \frac{v + v_L}{v} \right) f_S = \left( 1 + \frac{v_L}{v} \right) f_S$ . The Problem-Solving

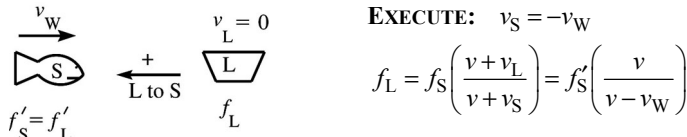
Strategy in the text (Section 16.8) describes how to do this problem. The frequency of the directly radiated waves is  $f_S = 18,000$  Hz. The moving whale first plays the role of a moving listener, receiving waves with frequency  $f'_L$ . The whale then acts as a moving source, emitting waves with the same frequency,  $f'_S = f'_L$  with which they are received. Let the speed of the whale be  $v_W$ .

**SET UP:** whale receives waves (Figure 16.67a)



**Figure 16.67a**

**SET UP:** whale re-emits the waves (Figure 16.67b)



**Figure 16.67b**

$$\text{But } f'_S = f'_L \text{ so } f_L = f_S \left( \frac{v + v_W}{v} \right) \left( \frac{v}{v - v_W} \right) = f_S \left( \frac{v + v_W}{v - v_W} \right).$$

$$\text{Then } \Delta f = f_S - f_L = f_S \left( 1 - \frac{v + v_W}{v - v_W} \right) = f_S \left( \frac{v - v_W - v - v_W}{v - v_W} \right) = \frac{-2f_S v_W}{v - v_W}.$$

$$\Delta f = \frac{-2(1.80 \times 10^4 \text{ Hz})(4.95 \text{ m/s})}{1482 \text{ m/s} - 4.95 \text{ m/s}} = -120 \text{ Hz}.$$

**EVALUATE:**  $\Delta f$  is negative, which means that  $f_L > f_S$ . This is reasonable because the listener and source are moving toward each other so the frequency is raised.

- 16.68. IDENTIFY:** Apply  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ . The heart wall first acts as the listener and then as the source.

**SET UP:** The positive direction is from listener to source. The heart wall is moving toward the receiver so the Doppler effect increases the frequency and the final frequency received,  $f_{L2}$ , is greater than the source frequency,  $f_{S1}$ .  $f_{L2} - f_{S1} = 72 \text{ Hz}$ .

$$\text{EXECUTE: Heart wall receives the sound: } f_S = f_{S1}, f_L = f_{L1}, v_S = 0, v_L = -v_{\text{wall}}, f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$$

$$\text{gives } f_{L1} = \left( \frac{v - v_{\text{wall}}}{v} \right) f_{S1}.$$

Heart wall emits the sound:  $f_{S2} = f_{L1}$ ,  $v_S = +v_{\text{wall}}$ ,  $v_L = 0$ .

$$f_{L2} = \left( \frac{v}{v + v_{\text{wall}}} \right) f_{S2} = \left( \frac{v}{v + v_{\text{wall}}} \right) \left( \frac{v - v_{\text{wall}}}{v} \right) f_{S1} = \left( \frac{v - v_{\text{wall}}}{v + v_{\text{wall}}} \right) f_{S1}.$$

$$f_{L2} - f_{S1} = \left( 1 - \frac{v - v_{\text{wall}}}{v + v_{\text{wall}}} \right) f_{S1} = \left( \frac{2v_{\text{wall}}}{v + v_{\text{wall}}} \right) f_{S1}, v_{\text{wall}} = \frac{(f_{L2} - f_{S1})v}{2f_{S1} - (f_{L2} - f_{S1})}, f_{S1} \gg f_{L2} - f_{S1} \text{ and}$$

$$v_{\text{wall}} = \frac{(f_{L2} - f_{S1})v}{2f_{S1}} = \frac{(72 \text{ Hz})(1500 \text{ m/s})}{2(2.00 \times 10^6 \text{ Hz})} = 0.0270 \text{ m/s} = 2.70 \text{ cm/s}.$$

**EVALUATE:**  $f_{S1} = 2.00 \times 10^6 \text{ Hz}$  and  $f_{L2} - f_{S1} = 72 \text{ Hz}$ , so the approximation we made is very accurate. Within this approximation, the frequency difference between the reflected and transmitted waves is directly proportional to the speed of the heart wall.

- 16.69. IDENTIFY:** Follow the method of Example 16.18 and apply the Doppler shift formula twice, once with the insect as the listener and again with the insect as the source.

**SET UP:** Let  $v_{\text{bat}}$  be the speed of the bat,  $v_{\text{insect}}$  be the speed of the insect, and  $f_i$  be the frequency with which the sound waves both strike and are reflected from the insect. The positive direction in each application of the Doppler shift formula is from the listener to the source.

**EXECUTE:** The frequencies at which the bat sends and receives the signals are related by

$$f_L = f_i \left( \frac{v + v_{\text{bat}}}{v - v_{\text{insect}}} \right) = f_S \left( \frac{v + v_{\text{insect}}}{v - v_{\text{bat}}} \right) \left( \frac{v + v_{\text{bat}}}{v - v_{\text{insect}}} \right). \text{ Solving for } v_{\text{insect}},$$

$$v_{\text{insect}} = v \left[ \frac{1 - \frac{f_S}{f_L} \left( \frac{v + v_{\text{bat}}}{v - v_{\text{bat}}} \right)}{1 + \frac{f_S}{f_L} \left( \frac{v + v_{\text{bat}}}{v - v_{\text{bat}}} \right)} \right] = v \left[ \frac{f_L(v - v_{\text{bat}}) - f_S(v + v_{\text{bat}})}{f_L(v - v_{\text{bat}}) + f_S(v + v_{\text{bat}})} \right].$$

Letting  $f_L = f_{\text{refl}}$  and  $f_S = f_{\text{bat}}$  gives the result.

**(b)** If  $f_{\text{bat}} = 80.7 \text{ kHz}$ ,  $f_{\text{refl}} = 83.5 \text{ kHz}$ , and  $v_{\text{bat}} = 3.9 \text{ m/s}$ , then  $v_{\text{insect}} = 2.0 \text{ m/s}$ .

**EVALUATE:**  $f_{\text{refl}} > f_{\text{bat}}$  because the bat and insect are approaching each other.

- 16.70. IDENTIFY:** Apply the Doppler effect formula  $f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S$ . In the SHM the source moves toward and

away from the listener, with maximum speed  $\omega_p A_p$ .

**SET UP:** The direction from listener to source is positive.

**EXECUTE: (a)** The maximum velocity of the siren is  $\omega_p A_p = 2\pi f_p A_p$ . You hear a sound with frequency  $f_L = f_{\text{siren}} v / (v + v_S)$ , where  $v_S$  varies between  $+2\pi f_p A_p$  and  $-2\pi f_p A_p$ .  $f_{L-\text{max}} = f_{\text{siren}} v / (v - 2\pi f_p A_p)$  and  $f_{L-\text{min}} = f_{\text{siren}} v / (v + 2\pi f_p A_p)$ .

**(b)** The maximum (minimum) frequency is heard when the platform is passing through equilibrium and moving up (down).

**EVALUATE:** When the platform is moving upward the frequency you hear is greater than  $f_{\text{siren}}$  and when it is moving downward the frequency you hear is less than  $f_{\text{siren}}$ . When the platform is at its maximum displacement from equilibrium its speed is zero and the frequency you hear is  $f_{\text{siren}}$ .

- 16.71. IDENTIFY:** The sound from the speaker moving toward the listener will have an increased frequency, while the sound from the speaker moving away from the listener will have a decreased frequency. The difference in these frequencies will produce a beat.

**SET UP:** The greatest frequency shift from the Doppler effect occurs when one speaker is moving away and one is moving toward the person. The speakers have speed  $v_0 = r\omega$ , where  $r = 0.75$  m.

$$f_L = \left( \frac{v + v_L}{v + v_S} \right) f_S, \text{ with the positive direction from the listener to the source. } v = 344 \text{ m/s.}$$

**EXECUTE: (a)**  $f = \frac{v}{\lambda} = \frac{344 \text{ m/s}}{0.313 \text{ m}} = 1100 \text{ Hz}$ .  $\omega = (75 \text{ rpm}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 7.85 \text{ rad/s}$  and  $v_0 = (0.75 \text{ m})(7.85 \text{ rad/s}) = 5.89 \text{ m/s}$ .

For speaker *A*, moving toward the listener:  $f_{LA} = \left( \frac{v}{v - 5.89 \text{ m/s}} \right) (1100 \text{ Hz}) = 1119 \text{ Hz}$ .

For speaker *B*, moving away from the listener:  $f_{LB} = \left( \frac{v}{v + 5.89 \text{ m/s}} \right) (1100 \text{ Hz}) = 1081 \text{ Hz}$ .

$$f_{\text{beat}} = f_1 - f_2 = 1119 \text{ Hz} - 1081 \text{ Hz} = 38 \text{ Hz}.$$

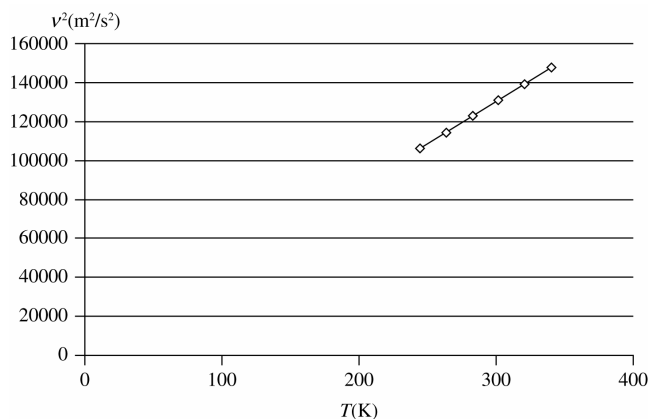
**(b)** A person can hear individual beats only up to about 7 Hz and this beat frequency is much larger than that.

**EVALUATE:** As the turntable rotates faster the beat frequency at this position of the speakers increases.

- 16.72. IDENTIFY and SET UP:** Assuming that the gas is nearly ideal, the speed of sound in it is given by

$$v = \sqrt{\frac{\gamma RT}{M}}, \text{ where } T \text{ is in absolute (Kelvin) units and } M \text{ is the molar mass of the gas.}$$

**EXECUTE: (a)** Squaring  $v = \sqrt{\frac{\gamma RT}{M}}$ , gives  $v^2 = \left( \frac{\gamma R}{M} \right) T$ . On a graph of  $v^2$  versus  $T$ , we would expect a straight line with slope equal to  $\frac{\gamma R}{M}$ . Figure 16.72 shows the graph of the data given in the problem.



**Figure 16.72**

(b) The best-fit equation for the graph in Figure 16.72 is  $v^2 = (416.47 \text{ m}^2/\text{K} \cdot \text{s}^2) T + 296.65 \text{ m}^2/\text{s}^2$ . Solving our expression for the slope for  $M$  gives  $M = \gamma R / \text{slope}$ . Putting in the numbers gives

$$M = (1.40)(8.3145 \text{ J/mol} \cdot \text{K}) / (416.47 \text{ m}^2/\text{K} \cdot \text{s}^2) = 0.0279 \text{ kg/mol} = 27.9 \text{ g/mol}.$$

**EVALUATE:** Nitrogen  $\text{N}_2$  is a diatomic gas with a molecular mass of 28.0 g/mol, so the gas is probably nitrogen.

- 16.73. IDENTIFY and SET UP:** There is a node at the piston, so the distance the piston moves is the node to node distance,  $\lambda/2$ . Use  $v = f\lambda$  to calculate  $v$  and  $v = \sqrt{\frac{\gamma RT}{M}}$  to calculate  $\gamma$  from  $v$ .

**EXECUTE: (a)**  $\lambda/2 = 37.5 \text{ cm}$ , so  $\lambda = 2(37.5 \text{ cm}) = 75.0 \text{ cm} = 0.750 \text{ m}$ .

$$v = f\lambda = (500 \text{ Hz})(0.750 \text{ m}) = 375 \text{ m/s}$$

$$\text{(b) Solve } v = \sqrt{\gamma RT/M} \text{ for } \gamma: \gamma = \frac{Mv^2}{RT} = \frac{(28.8 \times 10^{-3} \text{ kg/mol})(375 \text{ m/s})^2}{(8.3145 \text{ J/mol} \cdot \text{K})(350 \text{ K})} = 1.39.$$

(c) **EVALUATE:** There is a node at the piston so when the piston is 18.0 cm from the open end the node is inside the pipe, 18.0 cm from the open end. The node to antinode distance is  $\lambda/4 = 18.8 \text{ cm}$ , so the antinode is 0.8 cm beyond the open end of the pipe.

The value of  $\gamma$  we calculated agrees with the value given for air in Example 16.4.

- 16.74. IDENTIFY and SET UP:** We know from the problem that  $f_R = f_S \left(1 - \frac{v}{c}\right)^{1/2} \left(1 + \frac{v}{c}\right)^{1/2}$ . The radius of the nebula is  $R = vt$ , where  $t$  is the time since the supernova explosion. When the source and receiver are moving toward each other,  $v$  is negative and  $f_R > f_S$ . The light from the explosion reached earth 960 years ago, so that is the amount of time the nebula has expanded.  $1 \text{ ly} = 9.461 \times 10^{15} \text{ m}$ .

**EXECUTE: (a)** According to the binomial theorem,  $(1 \pm x)^n \approx 1 \pm nx$  if  $|x| \ll 1$ . Applying this to the two square roots, where  $n = \pm \frac{1}{2}$  and  $x = v/c$ , the equation  $f_R = f_S \left(1 - \frac{v}{c}\right)^{1/2} \left(1 + \frac{v}{c}\right)^{1/2}$  becomes

$$f_R \approx f_S \left(1 - \frac{1}{2} \frac{v}{c}\right) \left(1 + \frac{1}{2} \frac{v}{c}\right) \approx f_S \left(1 - \frac{1}{2} \frac{v}{c}\right)^2 \approx f_S \left[1 - 2 \left(\frac{1}{2} \frac{v}{c}\right)\right] \approx f_S \left(1 - \frac{v}{c}\right).$$

(b) Solving the equation we derived in part (a) for  $v$  gives

$$v = c \frac{f_S - f_R}{f_S} = (3.00 \times 10^8 \text{ m/s}) \frac{-0.018 \times 10^{14} \text{ Hz}}{4.568 \times 10^{14} \text{ Hz}} = -1.2 \times 10^6 \text{ m/s}, \text{ with the minus sign indicating that the}$$

gas is approaching the earth, as is expected since  $f_R > f_S$ .

(c) As of 2014, the supernova occurred 960 years ago. The diameter  $D$  is therefore

$$D = 2(960 \text{ y})(3.156 \times 10^7 \text{ s/y})(1.2 \times 10^6 \text{ m/s}) = 7.15 \times 10^{16} \text{ m} = 7.6 \text{ ly}.$$

(d) The ratio of the width of the nebula to  $2\pi$  times the distance from the earth is the ratio of the angular width (taken as 5 arc minutes) to an entire circle, which is  $60 \times 360$  arc minutes. The distance to the

nebula is then  $\left(\frac{2}{2\pi}\right)(3.75 \text{ ly}) \frac{(60)(360)}{5} = 5.2 \times 10^3 \text{ ly}$ . The time it takes light to travel this distance is

5200 yr, so the explosion actually took place 5200 yr before 1054 C.E., or about 4100 B.C.E.

**EVALUATE:**  $\left|\frac{v}{c}\right| = 4.0 \times 10^{-3}$ , so even though  $|v|$  is very large the approximation required for  $v = c \frac{\Delta f}{f}$  is accurate.

- 16.75. IDENTIFY:** The phase of the wave is determined by the value of  $x - vt$ , so  $t$  increasing is equivalent to  $x$  decreasing with  $t$  constant. The pressure fluctuation and displacement are related by the equation

$$p(x, t) = -B \frac{\partial y(x, t)}{\partial x}.$$



**SET UP:**  $y(x, t) = -\frac{1}{B} \int p(x, t) dx$ . If  $p(x, t)$  versus  $x$  is a straight line, then  $y(x, t)$  versus  $x$  is a parabola.

For air,  $B = 1.42 \times 10^5$  Pa.

**EXECUTE: (a)** The graph is sketched in Figure 16.75a.

**(b)** From  $p(x, t) = BkA \sin(kx - \omega t)$ , the function that has the given  $p(x, 0)$  at  $t = 0$  is given graphically in Figure 16.75b. Each section is a parabola, not a portion of a sine curve. The period is  $\lambda/v = (0.200 \text{ m})/(344 \text{ m/s}) = 5.81 \times 10^{-4} \text{ s}$  and the amplitude is equal to the area under the  $p$  versus  $x$  curve between  $x = 0$  and  $x = 0.0500 \text{ m}$  divided by  $B$ , or  $7.04 \times 10^{-6} \text{ m}$ .

**(c)** Assuming a wave moving in the  $+x$ -direction,  $y(0, t)$  is as shown in Figure 16.75c.

**(d)** The maximum velocity of a particle occurs when a particle is moving through the origin, and the particle speed is  $v_y = -\frac{\partial y}{\partial x} v = \frac{pv}{B}$ . The maximum velocity is found from the maximum pressure, and

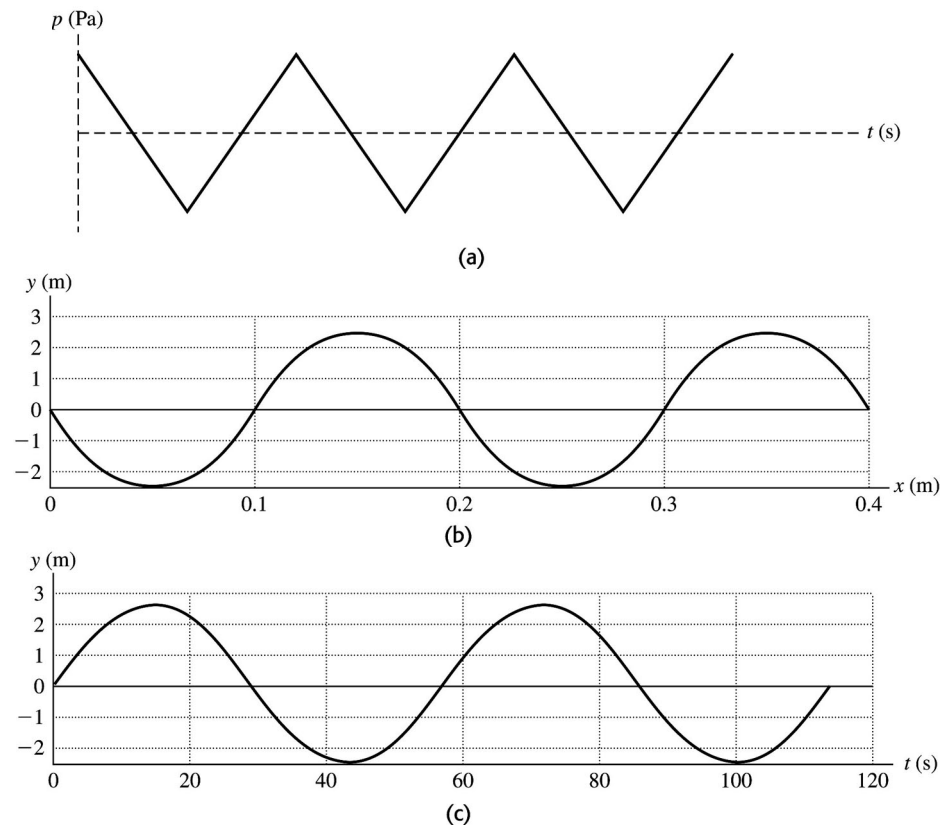
$v_{y\text{max}} = (40 \text{ Pa})(344 \text{ m/s})/(1.42 \times 10^5 \text{ Pa}) = 9.69 \text{ cm/s}$ . The maximum acceleration is the maximum pressure gradient divided by the density,

$$a_{\text{max}} = \frac{(80.0 \text{ Pa})/(0.100 \text{ m})}{(1.20 \text{ kg/m}^3)} = 6.67 \times 10^2 \text{ m/s}^2.$$

**(e)** The speaker cone moves with the displacement as found in part (c); the speaker cone alternates between moving forward and backward with constant magnitude of acceleration (but changing sign).

The acceleration as a function of time is a square wave with amplitude  $667 \text{ m/s}^2$  and frequency  $f = v/\lambda = (344 \text{ m/s})/(0.200 \text{ m}) = 1.72 \text{ kHz}$ .

**EVALUATE:** We can verify that  $p(x, t)$  versus  $x$  has a shape proportional to the slope of the graph of  $y(x, t)$  versus  $x$ . The same is also true of the graphs versus  $t$ .



**Figure 16.75**

- 16.76. IDENTIFY and SET UP:** Consider the derivation of the speed of a longitudinal wave in Section 16.2.  
**EXECUTE: (a)** The quantity of interest is the change in force per fractional length change. The force constant  $k'$  is the change in force per length change, so the force change per fractional length change is  $k'L$ , the applied force at one end is  $F = (k'L)(v_y/v)$  and the longitudinal impulse when this force is applied for a time  $t$  is  $k'Ltv_y/v$ . The change in longitudinal momentum is  $((vt)m/L)v_y$  and equating the expressions, canceling a factor of  $t$  and solving for  $v$  gives  $v^2 = L^2k'/m$ .  
**(b)**  $v = (2.00 \text{ m})\sqrt{(1.50 \text{ N/m})/(0.250 \text{ kg})} = 4.90 \text{ m/s}$   
**EVALUATE:** A larger  $k'$  corresponds to a stiffer spring and for a stiffer spring the wave speed is greater.
- 16.77. IDENTIFY and SET UP:** The time between pulses is limited by the time for the wave to travel from the transducer to the structure and then back again. Use  $x = v_x t$  and  $f = 1/T$ .  
**EXECUTE: (a)** The wave travels 10 cm in and 10 cm out, so  $t = x/v_x = (0.20 \text{ m})/(1540 \text{ m/s}) = 0.13 \times 10^{-3} \text{ s} = 0.13 \text{ ms}$ . The period can be no shorter than this, so the highest pulse frequency is  $f = 1/t = 1/(0.13 \text{ ms}) = 7700 \text{ Hz}$ , which is choice (b).  
**EVALUATE:** The pulse frequency is not the same thing as the frequency of the ultrasound waves, which is around 1.0 MHz.
- 16.78. IDENTIFY and SET UP:** Call  $S$  the intensity level of the beam. The beam attenuates by 100 dB per meter, so in 10 cm (0.10 m) it would attenuate by 1/10 of this amount. Therefore  $\Delta S = -10 \text{ dB}$ .  $S = 10 \text{ dB}$   $\log(I/I_0)$ .  
**EXECUTE: (a)**  $\Delta S = S_2 - S_1 = 10 \text{ dB} \log(I_2/I_0) - 10 \text{ dB} \log(I_1/I_0) = 10 \text{ dB} \log(I_2/I_0)$  since  $I_1 = I_0$ . Therefore  $-10 \text{ dB} = 10 \text{ dB} \log(I_2/I_0)$ , which gives  $I_2/I_0 = 10^{-1}$ , so  $I_2 = 1/10 I_0$ , which is choice (d).  
**EVALUATE:** In the next 10 cm, the beam would attenuate by another factor of 1/10, so it would be 1/100 of the initial intensity.
- 16.79. IDENTIFY and SET UP:** The beam goes through 5.0 cm of tissue and 2.0 cm of bone. Use  $d = vt$  to calculate the total time in this case and compare it with the time to travel 7.0 cm through only tissue.  
**EXECUTE:**  $d = vt$  gives  $t = x/v$ . Calculate the time to go through 2.0 cm of bone and 5.0 cm of tissue and then get the total time  $t_{\text{tot}} = t_T + t_B = x_T/v_T + x_B/v_B$ , so  $t_{\text{tot}} = x_T/v_T + x_B/v_B$ . Putting in the numbers gives  $t_{\text{tot}} = (0.050 \text{ m})/(1540 \text{ m/s}) + (0.020 \text{ m})/(3080 \text{ m/s}) = 3.896 \times 10^{-5} \text{ s}$ . If the wave went through only tissue during this time, it would have traveled  $x = v_T t_{\text{tot}} = (1540 \text{ m/s})(3.896 \times 10^{-5} \text{ s}) = 6.0 \times 10^{-2} \text{ m} = 6.0 \text{ cm}$ . So the beam traveled 7.0 cm, but you think it traveled 6.0 cm, so the structure is actually 1.0 cm deeper than you think, which makes choice (a) the correct one.  
**EVALUATE:** A difference of 1.0 cm when a structure is 7.0 cm below the surface can be very significant.
- 16.80. IDENTIFY and SET UP:** In a standing wave pattern, the distance between antinodes is one-half the wavelength of the waves. Use  $v = f\lambda$  to find the wavelength.  
**EXECUTE:**  $\lambda = v/f = (1540 \text{ m/s})/(1.00 \text{ MHz}) = 1.54 \times 10^{-3} \text{ m} = 1.54 \text{ mm}$ . The distance  $D$  between antinodes is  $D = \lambda/2 = (1.54 \text{ mm})/2 = 0.77 \text{ mm} \approx 0.75 \text{ mm}$ , which is choice (b).  
**EVALUATE:** Decreasing the frequency could reduce the distance between antinodes if this is desired.
- 16.81. IDENTIFY and SET UP:** The antinode spacing is  $\lambda/2$ . Use  $v = f\lambda$ .  
**EXECUTE: (a)** The antinode spacing  $d$  is  $\lambda/2$ . Using  $v = f\lambda$ , we have  $d = \lambda/2 = v/2f$ . For the numbers in this problem, we have  $d = (1540 \text{ m/s})/[2(1.0 \text{ kHz})] = 0.77 \text{ m} = 77 \text{ cm}$ . The cranium is much smaller than 77 cm, so there will be no standing waves within it at 1.0 kHz, which is choice (b).  
**EVALUATE:** Using 1.0 MHz waves, the distance between antinodes would be 1000 times smaller, or 0.077 cm, so there could certainly be standing waves within the cranium.