Vp250 Problem Set 4

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Problem 1

(a) The electric field between two parallel plate is $E = \frac{V_{AB}}{d}$. And as the drop is at rest:

$$mg = qE$$

The mass $m = \frac{4}{3}\pi r^3 \rho$. So we have $\frac{4}{3}\pi r^3 \rho g = q\frac{V_{AB}}{d} \Rightarrow q = \frac{4\pi}{3}\frac{\rho r^3 g d}{V_{AB}}$.

(b) When the drop reaches terminal speed

$$mg = 6\pi\eta r v_\infty \Rightarrow \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_\infty \Rightarrow r = 3\sqrt{\frac{\eta v_\infty}{2\rho g}}$$

Replace r with $3\sqrt{\frac{\eta v_\infty}{2\rho g}}$ in the answer of (a):

$$q = \frac{4\pi}{3}\frac{\rho(3\sqrt{\frac{\eta v_\infty}{2\rho g}})^3 g d}{V_{AB}} = \frac{18\pi d}{V_{AB}}\sqrt{\frac{\eta^3 v_\infty^3}{2\rho g}}$$

(c) The terminal speed:

$$v_\infty = \frac{1 \cdot 10^{-3}}{39.3} = 2.54 \cdot 10^{-5} m/s$$

So the charge:

$$q = \frac{18\pi \cdot 0.001}{9.16}\sqrt{\frac{(1.81 \cdot 10^{-5})^3(2.54 \cdot 10^{-5})^3}{2 \cdot 824 \cdot 9.8}} = 4.78 \cdot 10^{-19} C$$

The number of the electrons:

$$\frac{q}{e} = \frac{4.78 \cdot 10^{-19}}{1.602 \cdot 10^{-19}} = 2.98 \approx 3$$

The radius:

$$r = 3\sqrt{\frac{\eta v_\infty}{2\rho g}} = 3\sqrt{\frac{1.81 \cdot 10^{-5} \cdot 2.54 \cdot 10^{-5}}{2 \cdot 824 \cdot 9.8}} = 5.06 \cdot 10^{-7} m$$

Problem 2

(a) The capacitance $C = \varepsilon_0 \frac{A}{x}$. So the energy stored in this capacitor is $U = \frac{Q^2}{2C} = \frac{xQ^2}{2\varepsilon_0 A}$.

(b) The capacitance after the change: $C' = \varepsilon_0 \frac{A}{x + \mathrm{d}x}$.

The energy stored after the change is $U' = \frac{Q^2}{2C'} = \frac{(x + \mathrm{d}x)Q^2}{2\varepsilon_0 A}$ So the change of energy is $\Delta U = U' - U = \frac{\mathrm{d}xQ^2}{2\varepsilon_0 A}$

(c) $\delta W = \frac{Q^2}{2\varepsilon_0 A}\mathrm{d}x = F\mathrm{d}x$. So $F = \frac{Q^2}{2\varepsilon_0 A}$

(d) The electric field $E = \frac{V}{x}$. And $C = \frac{Q}{V} = \varepsilon_0 \frac{A}{x}$. So $E = \frac{Q}{\varepsilon_0 A}$ and then $QE = \frac{Q^2}{\varepsilon_0 A} \neq F$.
The formula QE is for the total electric field induced by the two plates. But the force we calculate is the force acted on one plate by the other. So it should be divided by two.

Problem 3

The equivalent capacitance of n modules is $C_n = \frac{\frac{1}{2}(1+C_{n-1})}{\frac{1}{2}+1+C_{n-1}} = \frac{1+C_{n-1}}{3+2C_{n-1}}$. So the formula for equivalent capacitance of n modules is

$$C_n = \frac{1-(2-\sqrt{3})^{2n}}{(\sqrt{3}-1)(2-\sqrt{3})^{2n}+\sqrt{3}+1}$$

When n goes to infnity, $C_n \to \frac{1}{\sqrt{3}+1}F$. So even you apply a new module to the circuit, the value will not change.

Problem 4

(a) The capacitance before the dielectric inserted into: $C_0 = \varepsilon_0 \frac{L^2}{D} = \varepsilon_0 \left(\frac{L(L-x)}{D} + \frac{Lx}{D}\right)$.

Then the capacitance after the dielectric inserted into is: $C = \varepsilon_r \varepsilon_0 \frac{Lx}{D} + \varepsilon_0 \frac{L(L-x)}{D}$

(b) The energy stored in the capacitor originially is $U = \frac{CV^2}{2} = \frac{1}{2}\left(\varepsilon_r \varepsilon_0 \frac{Lx}{D} + \varepsilon_0 \frac{L(L-x)}{D}\right)V^2$. After the extra $\mathrm{d}x$, the energy: $U' = \frac{1}{2}\left(\varepsilon_r \varepsilon_0 \frac{L(x+\mathrm{d}x)}{D} + \varepsilon_0 \frac{L(L-x-\mathrm{d}x)}{D}\right)V^2$.

So the energy change $\delta U = U' - U = \frac{1}{2}\varepsilon_0 \left(\varepsilon_r \frac{L\mathrm{d}x}{D} - \frac{L\mathrm{d}x}{D}\right)V^2 = \frac{\varepsilon_0 V^2 L(\varepsilon_r - 1)}{2D}\mathrm{d}x$.

(c) The energy stored in the capacitor originially is $U = \frac{Q^2}{2C}$.
Differentiate U w.r.t x and replace Q by CV:

$$\frac{\mathrm{d}U}{\mathrm{d}x} = -\frac{Q^2}{2}\frac{\varepsilon_r\varepsilon_0 \frac{L}{D} - \varepsilon_0 \frac{L}{D}}{\left(\varepsilon_r \varepsilon_0 \frac{Lx}{D} + \varepsilon_0 \frac{L(L-x)}{D}\right)^2} = \frac{\varepsilon_0 L V^2}{2D}(1-\varepsilon_r)$$

So $\delta U = \frac{\varepsilon_0 L V^2}{2D}(1-\varepsilon_r)\mathrm{d}x$, which is negative to the δU in part (b).

(d) For part (b):

$$\mathrm{d}U = -F\mathrm{d}x \Rightarrow \frac{\varepsilon_0 V^2 L(\varepsilon_r - 1)}{2D}\mathrm{d}x = -F\mathrm{d}x \Rightarrow F = \frac{\varepsilon_0 V^2 L(1-\varepsilon_r)}{2D}$$

Since $\varepsilon_r > 1$, the value of F is less than 0, which means that the force has oppposite direction to its movement pushing it out of the capacitor.
For part (c):

$$\mathrm{d}U = -F\mathrm{d}x \Rightarrow \frac{\varepsilon_0 V^2 L(1-\varepsilon_r)}{2D}\mathrm{d}x = -F\mathrm{d}x \Rightarrow F = \frac{\varepsilon_0 V^2 L(\varepsilon_r - 1)}{2D}$$

Since $\varepsilon_r > 1$, the value of F is greater than 0, which means that the force is in the direction of the movement pulling the slab into the capacitor.

(e) In condition (b), the capacitor is connected to a battery, which also does work to the slab. So the change of potential energy δU is not all by the influence of the force F. And then the force we calculate by that is the net force acted on the slab by F and some force component from the battery.