PROBLEM SET 3

Due: 29 September 2020, 12.30 p.m.

Problem 1. (*optional exercise) Consider the following force field $\mathbf{F}(\mathbf{r}) = \left(-\frac{Ay}{x^2+y^2}, \frac{Ax}{x^2+y^2}, 0\right)$, where A is a positive constant with appropriate units.

(a) Sketch this field on the xy–plane (you may use a computer).

(b) Calculate rot $\mathbf{F}$ to check that the field has zero curl at every point of space, except the z axis.

(c) Show that work done by force $\mathbf{F}$ in moving a particle so that it makes one full turn along the unit circle Γ in the xy–plane in the positive (anti-clockwise) direction is equal to $2\pi A$. You may find polar coordinates useful.

(d) Is there a contradiction between the results of (b) and (c)? Why or why not? Is this field conservative?

Problem 2. Four electrons are kept at the corners of a square 10 nm on a side, with an alpha particle in its center (*i.e.*, at the intersection point of the diagonals). How much work is needed (give the result in eV) to move that alpha particle to the midpoint of one of the sides of the square?

(*2 points*)

Problem 3. Three identical charges $+Q$ are placed at the corners of a square of side a.

(a) What is the electric field at the fourth corner (the one missing a charge) due to the first three charges?

(b) What is the electric potential at that corner?

(c) How much work does it take to bring another charge, $+Q$, from infinity and place it at that corner?

(d) How much energy did it take to assemble the configuration of three charges mentioned in the first sentence?

(*4 × 1 points*)

Problem 4. For the non-uniformly charged insulating hollow ball discussed in Problem 3 of Problem Set 2, find the electric potential (remember to indicate the reference point) at any point of space $\mathbf{r}$, and sketch the graph of $V(|\mathbf{r}|)$. You may use all results you have derived in your previous homework.

(*4 points*)

Problem 5. Find the electric potential at a distance s from an infinitely long straight wire charged with constant density λ. Comment on your choice of the reference point. Compute the gradient of the potential to check that it yields the correct electric field (find the electric field using Gauss's law).

(*4 points*)

Problem 6. A uniformly charged solid sphere has radius R and total charge q. Find the energy of this charge configuration, *i.e.* the work needed to be done to assemble it by bringing infinitesimal charges from far away. (As we mentioned in the lecture, this energy is also often called the "self-energy" of the charge distribution.)

Hint. Use symmetry.

(*4 points*)