# Vp250 Problem Set 5

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## Problem 1

(a) For two resistors in series, the equivalent resistance is $R = R_1 + R_2$.
Since $R_1 = \rho_1 \frac{L}{A} = \rho_{01}(1+\alpha_1(T-T_0))\frac{L}{A}$,
and $R_2 = \rho_2 \frac{L}{A} = \rho_{02}(1+\alpha_2(T-T_0))\frac{L}{A}$,
we can get

$$\begin{aligned} R &= \frac{L}{A}[\rho_{01} + \rho_{02} + (\rho_{01}\alpha_1 + \rho_{02}\alpha_2)(T-T_0)] \\ &= \frac{L}{A}(\rho_{01}+\rho_{02})\left[1 + \frac{\rho_{01}\alpha_1 + \rho_{02}\alpha_2}{\rho_{01}+\rho_{02}}(T-T_0)\right] \end{aligned}$$

So the effective temperature coefficient is $\frac{\rho_{01}\alpha_1 + \rho_{02}\alpha_2}{\rho_{01}+\rho_{02}}$.

(b) For two resistors in parallel, the equivalent resistance is $R = \frac{R_1 R_2}{R_1 + R_2}$.
Since $R_1 = \rho_1 \frac{L}{A} = \rho_{01}(1+\alpha_1(T-T_0))\frac{L}{A}$,
and $R_2 = \rho_2 \frac{L}{A} = \rho_{02}(1+\alpha_2(T-T_0))\frac{L}{A}$,
we can get

$$\begin{aligned} R &= \frac{L}{A} \cdot \frac{\rho_{01}(1+\alpha_1(T-T_0)) \cdot \rho_{02}(1+\alpha_2(T-T_0))}{\rho_{01}(1+\alpha_1(T-T_0)) + \rho_{02}(1+\alpha_2(T-T_0))} \\ &= \frac{L}{A} \cdot \frac{\rho_{01}\rho_{02}(1+\alpha_1\alpha_2(T-T_0)^2 + (\alpha_2+\alpha_1)(T-T_0))}{\rho_{01}+\rho_{02}+(\rho_{01}\alpha_1+\rho_{02}\alpha_2)(T-T_0)} \\ &(\text{Take } T_0 = 0) \\ &= \frac{L}{A} \cdot \frac{\rho_{01}\rho_{02}(1+\alpha_1\alpha_2 T^2 + (\alpha_2+\alpha_1)T)}{\rho_{01}+\rho_{02}+(\rho_{01}\alpha_1+\rho_{02}\alpha_2)T} \end{aligned}$$

Differenciate R w.r.t T and insert T=0, we get:

$$R' = \frac{L}{A} \cdot \frac{\rho_{01}\rho_{02}(\alpha_1\rho_{02} + \alpha_2\rho_{01})}{(\rho_{01}+\rho_{02})^2}$$

Since $R = \frac{L}{2A}\rho_0(1+\alpha(T-T_0))$, $R' = \frac{L}{2A}\rho\alpha$. The effective temperature coefficient is

$$\alpha = \frac{R' 2A}{\rho_0 L} = \frac{\alpha_1\rho_{02} + \alpha_2\rho_{01}}{\rho_{01}+\rho_{02}}$$

# Problem 2

$$R_{eq} = 20 || \, (3 + (4+6) \, || \, (8||9)) = \frac{14460}{3143} \approx 4.6\Omega$$

# Problem 3

$$R_{eq} = R||R||R + R||R||R||R||R||R + R||R||R = \frac{5}{6}R$$

# Problem 4

(a)

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R + 2r} = \frac{12-8}{8+2\cdot 1} = 0.4A$$

direction: downwards

(b)

$$P_R = I^2 R = 0.4^2 \cdot 8 = 1.28W$$
$$P_r = 2I^2 r = 2 \cdot 0.4^2 \cdot 1 = 0.32W$$

Then the total power is:

$$P = P_R + P_r = 1.6W$$

(c) In $\varepsilon_1$, rate:

$$P_1 = \varepsilon_1 I = 12 \cdot 0.4 = 4.8W$$

(d) In $\varepsilon_2$, rate:

$$P_2 = \varepsilon_2 I = 8 \cdot 0.4 = 3.2W$$

(e) The overall rate of production of electrical energy is 4.8W.
The overall rate of assumption of energy is the combination of the power consumed in $R$, two internal resistance and the charged battery. And their sum is $1.28 + 0.32 + 3.2 = 4.8W$, which is equal to the production.

# Problem 5

Set the current through $R_1$ to be $I_1$ (downwards positive), the current through $R_2$ to be $I_2$ (upwards positive), the current through $R_3$ to be $I_3$ (leftwards positive).
For the mesh in the left:

$$\varepsilon_1 - I_1 R_1 - \varepsilon_2 - I_2 R_2 = 0$$

For the mesh in the right:

$$\varepsilon_3 - I_3 R_3 + I_2 R_2 = 0$$

By Junction Rule:

$$I_1 = I_2 + I_3$$

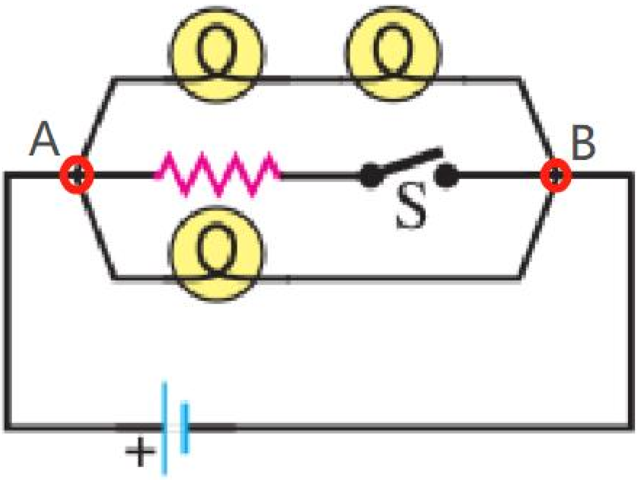
Then we get:

$$I_1 = \frac{99}{19} A$$
$$I_2 = -\frac{21}{19} A$$
$$I_3 = \frac{120}{19} A$$

So the current through $R_1$ is $\frac{99}{19} = 5.21A$ (direction: downwards)
The current through $R_2$ is $\frac{21}{19} = 1.11A$ (downwards).
The current through $R_3$ is $\frac{120}{19} = 6.32A$ (leftwards).

# Problem 6

(a) No change. Because the voltage between the two node A and B will not change after turning on the switch S. Then the power consumed by the bulb will not change too.




(b) The brightness of the bulb will be lower.

Because when the switch is turned on, the equivalent resistance between node A and B has $\frac{1}{R} = \frac{1}{R_{additional}} + \frac{1}{R_{original}}$. But the resistance before turning on the switch has $\frac{1}{R} = \frac{1}{R_{original}}$. We can see that $\frac{1}{R}$ becomes larger and so $R$ becomes smaller, which means the voltage between A and B drops. So the power consumed by the bulb is lower.

## Problem 7

Set the current through $\varepsilon$ as I. When the galvanometer reads 0, the current through P and X is:

$$I_{PX} = I\frac{M+N}{M+N+P+X}$$

The current through M and N is:

$$I_{MN} = I\frac{P+X}{M+N+P+X}$$

By loop rule:

$$I_{PX}P = I_{MN}N$$

So we get

$$IP\frac{M+N}{M+N+P+X} = IN\frac{P+X}{M+N+P+X}$$
$$\Rightarrow P(M+N) = N(P+X) \Rightarrow X = \frac{PM}{N}$$

## Problem 8

(a)

$$q(t) = 7 \cdot 10^{-6} e^{-\frac{t_d}{670 \cdot 10^3 \cdot 0.92 \cdot 10^{-6}}} = 1.6 \cdot 10^{-19} \Rightarrow t_d = 19.36s$$

(b) Yes.

$$q(t) = Q_{max}e^{-\frac{t_d}{RC}} = e \Rightarrow -\frac{t_d}{RC} = \ln\frac{e}{Q_{max}} \Rightarrow t_d = -RC\ln\frac{e}{Q_{max}}$$

The time constant is $RC$. $\ln\frac{e}{Q_{max}}$ is a constant for given $Q_{max}$. So the time required to reach this state always the same number of time constants, independent of R and C.