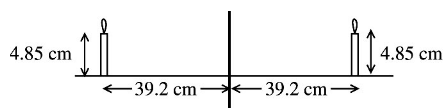
34

GEOMETRIC OPTICS

34.1. IDENTIFY and SET UP: Plane mirror: $s = -s'$ and $m = y'/y = -s'/s = +1$. We are given s and y and are asked to find s' and y'.

EXECUTE: The object and image are shown in Figure 34.1.



$s' = -s = -39.2$ cm.
$|y'| = |m||y| = (+1)(4.85 \text{ cm})$.
$|y'| = 4.85$ cm.

Figure 34.1

The image is 39.2 cm to the right of the mirror and is 4.85 cm tall.

EVALUATE: For a plane mirror the image is always the same distance behind the mirror as the object is in front of the mirror. The image always has the same height as the object.

34.2. IDENTIFY: Similar triangles say $\dfrac{h_{\text{tree}}}{h_{\text{mirror}}} = \dfrac{d_{\text{tree}}}{d_{\text{mirror}}}$.

SET UP: $d_{\text{mirror}} = 0.350$ m, $h_{\text{mirror}} = 0.0400$ m, and $d_{\text{tree}} = 28.0 \text{ m} + 0.350$ m.

EXECUTE: $h_{\text{tree}} = h_{\text{mirror}} \dfrac{d_{\text{tree}}}{d_{\text{mirror}}} = 0.040 \text{ m} \dfrac{28.0 \text{ m} + 0.350 \text{ m}}{0.350 \text{ m}} = 3.24$ m.

EVALUATE: The image of the tree formed by the mirror is 28.0 m behind the mirror and is 3.24 m tall.

34.3. IDENTIFY and SET UP: The virtual image formed by a plane mirror is the same size as the object and the same distance from the mirror as the object.

EXECUTE: $s' = -s$. The image of the tip is 12.0 cm behind the mirror surface and the image of the end of the eraser is 21.0 cm behind the mirror surface. The length of the image is 9.0 cm, the same as the length of the object. The image of the tip of the lead is the closest to the mirror surface.

EVALUATE: The same result would hold no matter how far the pencil was from the mirror.

34.4. IDENTIFY: $f = R/2$.

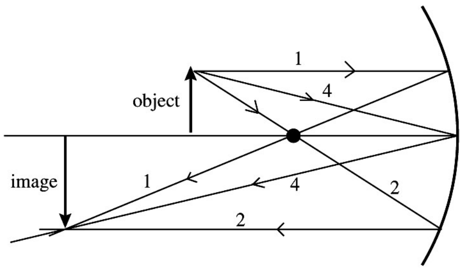
SET UP: For a concave mirror $R > 0$.

EXECUTE: **(a)** $f = \dfrac{R}{2} = \dfrac{34.0 \text{ cm}}{2} = 17.0$ cm.

EVALUATE: **(b)** The image formation by the mirror is determined by the law of reflection and that is unaffected by the medium in which the light is traveling. The focal length remains 17.0 cm.

34.5. IDENTIFY and SET UP: Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate s' and use $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$ to calculate y'. The image is real if s' is positive and is erect if $m > 0$. Concave means R and f are positive, $R = +22.0$ cm; $f = R/2 = +11.0$ cm.

EXECUTE: **(a)**



Three principal rays, numbered as in Section 34.2, are shown in Figure 34.5. The principal-ray diagram shows that the image is real, inverted, and enlarged.

Figure 34.5

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}.$

$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$ so $s'=\frac{sf}{s-f}=\frac{(16.5\text{ cm})(11.0\text{ cm})}{16.5\text{ cm}-11.0\text{ cm}}=+33.0\text{ cm}.$

$s'>0$ so real image, 33.0 cm to left of mirror vertex.

$m=-\frac{s'}{s}=-\frac{33.0\text{ cm}}{16.5\text{ cm}}=-2.00$ ($m<0$ means inverted image) $|y'|=|m||y|=2.00(0.600\text{ cm})=1.20\text{ cm}.$

EVALUATE: The image is 33.0 cm to the left of the mirror vertex. It is real, inverted, and is 1.20 cm tall (enlarged). The calculation agrees with the image characterization from the principal-ray diagram. A concave mirror used alone always forms a real, inverted image if $s>f$ and the image is enlarged if $f<s<2f$.

34.6. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}.$

SET UP: For a convex mirror, $R<0$. $R=-22.0\text{ cm}$ and $f=\frac{R}{2}=-11.0\text{ cm}.$

EXECUTE: **(a)** The principal-ray diagram is sketched in Figure 34.6.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}.$ $s'=\frac{sf}{s-f}=\frac{(16.5\text{ cm})(-11.0\text{ cm})}{16.5\text{ cm}-(-11.0\text{ cm})}=-6.6\text{ cm}.$ $m=-\frac{s'}{s}=-\frac{-6.6\text{ cm}}{16.5\text{ cm}}=+0.400.$

$|y'|=|m||y|=(0.400)(0.600\text{ cm})=0.240\text{ cm}.$ The image is 6.6 cm to the right of the mirror. It is 0.240 cm tall. $s'<0$, so the image is virtual. $m>0$, so the image is erect.

EVALUATE: The calculated image properties agree with the image characterization from the principal-ray diagram.

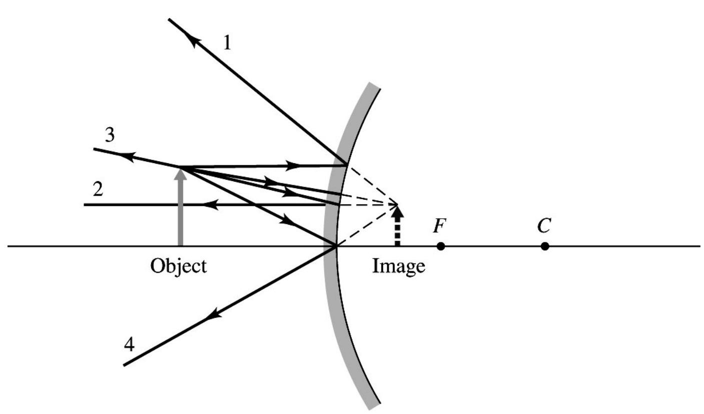


Figure 34.6

34.7. IDENTIFY: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $m=-\frac{s'}{s}$. $|m|=\frac{|y'|}{y}$. Find m and calculate y'.

SET UP: $f=+1.75$ m.

EXECUTE: $s \gg f$ so $s'=f=1.75$ m.

$$m=-\frac{s'}{s}=-\frac{1.75\text{ m}}{5.58\times10^{10}\text{ m}}=-3.14\times10^{-11}.$$

$|y'|=|m||y|=(3.14\times10^{-11})(6.794\times10^{6}\text{ m})=2.13\times10^{-4}\text{ m}=0.213\text{ mm}.$

EVALUATE: The image is real and is 1.75 m in front of the mirror.

34.8. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: The mirror surface is convex so $R=-3.00$ cm. $s=18.0\text{ cm}-3.00\text{ cm}=15.0\text{ cm}$.

EXECUTE: $f=\frac{R}{2}=-1.50$ cm. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(15.0\text{ cm})(-1.50\text{ cm})}{15.0\text{ cm}-(-1.50\text{ cm})}=-1.3636$ cm, which rounds to –1.36 cm. The image is 1.36 cm behind the surface so it is $3.00\text{ cm}-1.36\text{ cm}=1.64\text{ cm}$ from the center of the ornament, on the same side of the center as the object.

$$m=-\frac{s'}{s}=-\frac{-1.3636\text{ cm}}{15.0\text{ cm}}=+0.0909.$$

EVALUATE: The image is virtual, upright and much smaller than the object.

34.9. IDENTIFY: The shell behaves as a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s'}{s}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{2}{-18.0\text{ cm}}-\frac{1}{-6.00\text{ cm}}\Rightarrow s=18.0\text{ cm}$ from the vertex.

$m=-\frac{s'}{s}=-\frac{-6.00\text{ cm}}{18.0\text{ cm}}=\frac{1}{3}\Rightarrow y'=\frac{1}{3}(1.5\text{ cm})=0.50\text{ cm}.$ The image is 0.50 cm tall, erect and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.10. IDENTIFY: The bottom surface of the bowl behaves as a spherical convex mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s'}{s}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{-2}{35\text{ cm}}-\frac{1}{60\text{ cm}}\Rightarrow s'=-13.5\text{ cm},$ which rounds to 14 cm behind the bowl.

$m=-\frac{s'}{s}=\frac{13.5\text{ cm}}{60\text{ cm}}=0.225\Rightarrow y'=(0.225)(5.0\text{ cm})=1.1\text{ cm}.$ The image is 1.1 cm tall, erect and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.11. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: For a concave mirror, $R>0$. $R=32.0$ cm and $f=\frac{R}{2}=16.0$ cm.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(12.0\text{ cm})(16.0\text{ cm})}{12.0\text{ cm}-16.0\text{ cm}}=-48.0$ cm. $m=-\frac{s'}{s}=-\frac{-48.0\text{ cm}}{12.0\text{ cm}}=+4.00.$

(b) $s'=-48.0$ cm, so the image is 48.0 cm to the right of the mirror. $s'<0$ so the image is virtual.

(c) The principal-ray diagram is sketched in Figure 34.11 (next page). The rules for principal rays apply only to paraxial rays. Principal ray 2, which travels to the mirror along a line that passes through the focus, makes a large angle with the optic axis and is not described well by the paraxial approximation. Therefore, principal ray 2 is not included in the sketch.

EVALUATE: A concave mirror forms a virtual image whenever $s<f$.

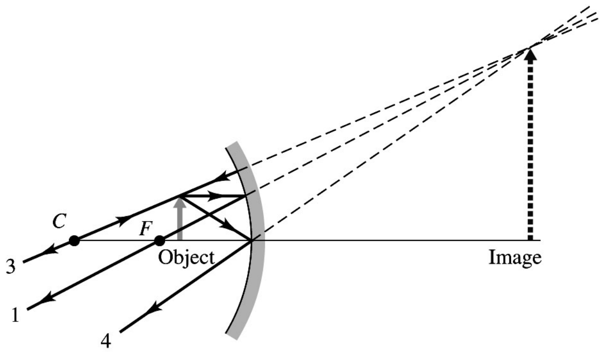


Figure 34.11

34.12. **IDENTIFY** and **SET UP:** For a spherical mirror, we have $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification is $m=-\frac{s'}{s}$. For a real image, $s' > 0$, so m is negative. The image height is the same as the object height, so $s' = s$.

EXECUTE: Using $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, with $s' = s$, we have $\frac{1}{f}=\frac{1}{s}+\frac{1}{s}=\frac{2}{s}=\frac{1}{18.0\text{ cm}}$, so $s = 36.0$ cm.

EVALUATE: The radius of curvature of the mirror is $R = 2f = 2(18.0\text{ cm}) = 36.0$ cm, which is the same as s. Therefore the object is at the center of curvature of the concave mirror.

34.13. **IDENTIFY:** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$.

SET UP: $m=+2.00$ and $s=1.25$ cm. An erect image must be virtual.

EXECUTE: **(a)** $s'=\frac{sf}{s-f}$ and $m=-\frac{f}{s-f}$. For a concave mirror, m can be larger than 1.00. For a convex mirror, $|f|=-f$ so $m=+\frac{|f|}{s+|f|}$ and m is always less than 1.00. The mirror must be concave $(f>0)$.

(b) $\frac{1}{f}=\frac{s'+s}{ss'}$. $f=\frac{ss'}{s+s'}$. $m=-\frac{s'}{s}=+2.00$ and $s'=-2.00s$. $f=\frac{s(-2.00s)}{s-2.00s}=+2.00s=+2.50$ cm. $R=2f=+5.00$ cm.

(c) The principal-ray diagram is drawn in Figure 34.13.

EVALUATE: The principal-ray diagram agrees with the description from the equations.

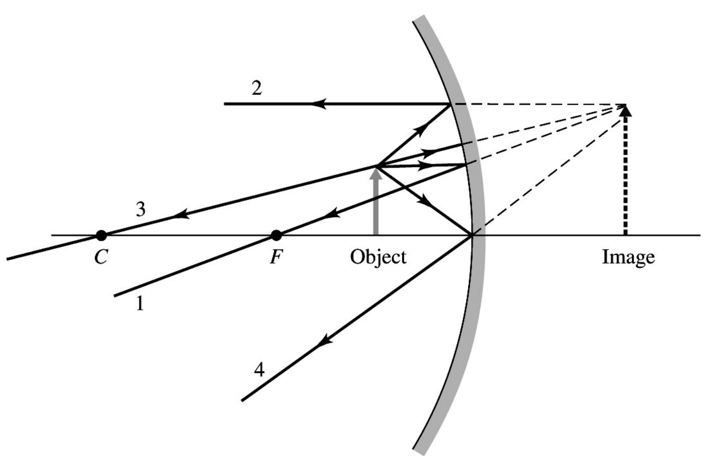


Figure 34.13

34.14. **IDENTIFY** and **SET UP:** For a spherical mirror, we have $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification is $m=-\frac{s'}{s}$. For a convex mirror, the image is virtual, so $s' < 0$, so m is positive. The image height is $\frac{1}{2}$ the same as the object height, so $m = +\frac{1}{2}$. Therefore $+\frac{1}{2}=-\frac{s'}{s}$, which gives $s' = -s/2$.

EXECUTE: Using $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, we have $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}-\frac{2}{s}=-\frac{1}{s}=\frac{1}{-12.0\text{ cm}}$, so $s = +12.0$ cm.

EVALUATE: $s' = -s/2 = -6.00$ cm, so the image is virtual, erect, and 6.0 cm from the vertex of the mirror on the side opposite the object.

34.15. **IDENTIFY:** In part (a), the shell is a concave mirror, but in (b) it is a convex mirror. The magnitude of its focal length is the same in both cases, but the sign reverses.

SET UP: For the orientation of the shell shown in the figure in the problem, $R=+12.0$ cm. When the glass is reversed, so the seed faces a convex surface, $R=-12.0$ cm. $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$.

EXECUTE: **(a)** $R=+12.0$ cm. $\frac{1}{s'}=\frac{2}{R}-\frac{1}{s}=\frac{2s-R}{Rs}$ and $s'=\frac{Rs}{2s-R}=\frac{(12.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm}-12.0\text{ cm}}=+10.0$ cm.

$m=-\frac{s'}{s}=-\frac{10.0\text{ cm}}{15.0\text{ cm}}=-0.667.$ $y'=my=-2.20$ mm. The image is 10.0 cm to the left of the shell vertex and is 2.20 mm tall.

(b) $R=-12.0$ cm. $s'=\frac{(-12.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm}+12.0\text{ cm}}=-4.29$ cm. $m=-\frac{-4.29\text{ cm}}{15.0\text{ cm}}=+0.286.$

$y'=my=0.944$ mm. The image is 4.29 cm to the right of the shell vertex and is 0.944 mm tall.

EVALUATE: In (a), $s>R/2$ and the mirror is concave, so the image is real. In (b) the image is virtual because a convex mirror always forms a virtual image.

34.16. **IDENTIFY:** The surface is flat so $R\to\infty$ and $\frac{n_a}{s}+\frac{n_b}{s'}=0.$

SET UP: The light travels from the fish to the eye, so $n_a=1.333$ and $n_b=1.00$. When the fish is viewed, $s=7.0$ cm. The fish is $20.0\text{ cm}-7.0\text{ cm}=13.0\text{ cm}$ above the mirror, so the image of the fish is 13.0 cm below the mirror and $20.0\text{ cm}+13.0\text{ cm}=33.0\text{ cm}$ below the surface of the water. When the image is viewed, $s=33.0$ cm.

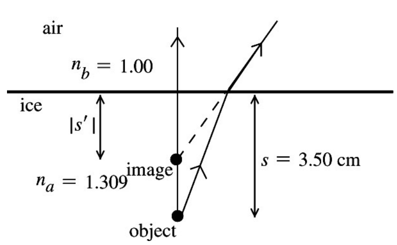
EXECUTE: **(a)** $s'=-\left(\frac{n_b}{n_a}\right)s=-\left(\frac{1.00}{1.333}\right)(7.0\text{ cm})=-5.25$ cm. The apparent depth is 5.25 cm.

(b) $s'=-\left(\frac{n_b}{n_a}\right)s=-\left(\frac{1.00}{1.333}\right)(33.0\text{ cm})=-24.8$ cm. The apparent depth of the image of the fish in the mirror is 24.8 cm.

EVALUATE: In each case the apparent depth is less than the actual depth of what is being viewed.

34.17. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$, with $R\to\infty$. $|s'|$ is the apparent depth.

SET UP: The image and object are shown in Figure 34.17.



$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$;
$R\to\infty$ (flat surface), so
$\frac{n_a}{s}+\frac{n_b}{s'}=0.$

Figure 34.17

EXECUTE: $s' = -\dfrac{n_b s}{n_a} = -\dfrac{(1.00)(3.50\text{ cm})}{1.309} = -2.67\text{ cm}.$

The apparent depth is 2.67 cm.

EVALUATE: When the light goes from ice to air (larger to smaller n), it is bent away from the normal and the virtual image is closer to the surface than the object is.

34.18. IDENTIFY and SET UP: For a plane refracting surface, we have $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0.$ Light is coming from the object, so $s = 3.60$ m and n_a is the target variable. The image is formed by light in the air, where you are, so $n_b = 1.00$ and $s' = -2.45$ m.

EXECUTE: Using $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0$ we have $\dfrac{n_a}{3.60\text{ m}} + \dfrac{1.00}{-2.45\text{ m}} = 0,$ which gives $n_a = 1.47$.

EVALUATE: We get $n_a > 1$, which it must be.

34.19. IDENTIFY: Think of the surface of the water as a section of a sphere having an infinite radius of curvature.

SET UP: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0.$ $n_a = 1.00.$ $n_b = 1.333.$

EXECUTE: The image is $5.20\text{ m} - 0.80\text{ m} = 4.40\text{ m}$ above the surface of the water, so $s' = -4.40$ m.

$s = -\dfrac{n_a}{n_b} s' = -\left(\dfrac{1.00}{1.333}\right)(-4.40\text{ m}) = +3.30\text{ m}.$

EVALUATE: The diving board is closer to the water than it looks to the swimmer.

34.20. IDENTIFY: Think of the surface of the water as a section of a sphere having an infinite radius of curvature.

SET UP: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0.$ $n_a = 1.333.$ $n_b = 1.00.$

EXECUTE: The image is 4.00 m below surface of the water, so $s' = -4.00$ m.

$s = -\dfrac{n_a}{n_b} s' = -\left(\dfrac{1.333}{1.00}\right)(-4.00\text{ m}) = 5.33\text{ m}.$

EVALUATE: The water is 1.33 m deeper than it appears to the person.

34.21. IDENTIFY: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}.$ $m = -\dfrac{n_a s'}{n_b s}.$ Light comes from the fish to the person's eye.

SET UP: $R = -14.0$ cm. $s = +14.0$ cm. $n_a = 1.333$ (water). $n_b = 1.00$ (air). Figure 34.21 shows the object and the refracting surface.

EXECUTE: **(a)** $\dfrac{1.333}{14.0\text{ cm}} + \dfrac{1.00}{s'} = \dfrac{1.00 - 1.333}{-14.0\text{ cm}}.$ $s' = -14.0$ cm. $m = -\dfrac{(1.333)(-14.0\text{ cm})}{(1.00)(14.0\text{ cm})} = +1.33.$

The fish's image is 14.0 cm to the left of the bowl surface so is at the center of the bowl and the magnification is 1.33.

(b) The focal point is at the image location when $s \to \infty.$ $\dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}.$ $n_a = 1.00.$ $n_b = 1.333.$

$R = +14.0$ cm. $\dfrac{1.333}{s'} = \dfrac{1.333 - 1.00}{14.0\text{ cm}}.$ $s' = +56.0$ cm. s' is greater than the diameter of the bowl, so the surface facing the sunlight does not focus the sunlight to a point inside the bowl. The focal point is outside the bowl and there is no danger to the fish.

EVALUATE: In part (b) the rays refract when they exit the bowl back into the air so the image we calculated is not the final image.

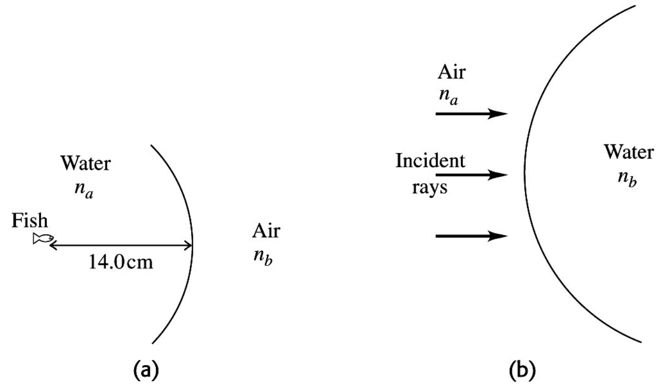


Figure 34.21

34.22. IDENTIFY: Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

SET UP: For a convex surface, $R>0$. $R=+3.00$ cm. $n_a=1.00$, $n_b=1.60$.

EXECUTE: **(a)** $s\to\infty$. $\frac{n_b}{s'}=\frac{n_b-n_a}{R}$. $s'=\left(\frac{n_b}{n_b-n_a}\right)R=\left(\frac{1.60}{1.60-1.00}\right)(+3.00\text{ cm})=+8.00\text{ cm}$. The image is 8.00 cm to the right of the vertex.

(b) $s=12.0$ cm. $\frac{1.00}{12.0\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{3.00\text{ cm}}$. $s'=+13.7$ cm. The image is 13.7 cm to the right of the vertex.

(c) $s=2.00$ cm. $\frac{1.00}{2.00\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{3.00\text{ cm}}$. $s'=-5.33$ cm. The image is 5.33 cm to the left of the vertex.

EVALUATE: The image can be either real $(s'>0)$ or virtual $(s'<0)$, depending on the distance of the object from the refracting surface.

34.23. IDENTIFY: The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and oil.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.45}{s}+\frac{1.60}{1.20\text{ m}}=\frac{0.15}{0.0300\text{ m}}\Rightarrow s=39.5\text{ cm}$.

EVALUATE: The presence of the oil changes the location of the image.

34.24. IDENTIFY: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$. $m=-\frac{n_a s'}{n_b s}$.

SET UP: $R=+4.00$ cm. $n_a=1.00$. $n_b=1.60$. $s=24.0$ cm.

EXECUTE: $\frac{1}{24.0\text{ cm}}+\frac{1.60}{s'}=\frac{1.60-1.00}{4.00\text{ cm}}$. $s'=+14.8$ cm. $m=-\frac{(1.00)(14.8\text{ cm})}{(1.60)(24.0\text{ cm})}=-0.385$.

$|y'|=|m||y|=(0.385)(1.50\text{ mm})=0.578\text{ mm}$. The image is 14.8 cm to the right of the vertex and is 0.578 mm tall. $m<0$, so the image is inverted.

EVALUATE: The image is real.

34.25. IDENTIFY: Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ and $m=-\frac{n_a s'}{n_b s}$. Calculate s' and y'. The image is erect if $m>0$.

SET UP: The object and refracting surface are shown in Figure 34.25 (next page).

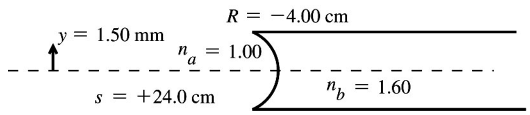


Figure 34.25

EXECUTE: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}.$

$\dfrac{1.00}{24.0\text{ cm}}+\dfrac{1.60}{s'}=\dfrac{1.60-1.00}{-4.00\text{ cm}}.$

Multiplying by 24.0 cm gives $1.00+\dfrac{38.4}{s'}=-3.60.$

$\dfrac{38.4\text{ cm}}{s'}=-4.60$ and $s'=-\dfrac{38.4\text{ cm}}{4.60}=-8.35$ cm.

Eq. (34.12): $m=-\dfrac{n_a s'}{n_b s}=-\dfrac{(1.00)(-8.35\text{ cm})}{(1.60)(+24.0\text{ cm})}=+0.217.$

$|y'|=|m||y|=(0.217)(1.50\text{ mm})=0.326$ mm.

EVALUATE: The image is virtual $(s'<0)$ and is 8.35 cm to the left of the vertex. The image is erect $(m>0)$ and is 0.326 mm tall. R is negative since the center of curvature of the surface is on the incoming side.

34.26. IDENTIFY: The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and liquid.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}.$

EXECUTE: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{n_a}{14.0\text{ cm}}+\dfrac{1.60}{-9.00\text{ cm}}=\dfrac{1.60-n_a}{-4.00\text{ cm}}\Rightarrow n_a=1.24.$

EVALUATE: The result is a reasonable refractive index for liquids.

34.27. IDENTIFY: Use the lensmaker's equation $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$ to calculate f. Then apply the thin-lens equation $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ and $m=\dfrac{y'}{y}=-\dfrac{s'}{s}.$

SET UP: $R_1\to\infty$. $R_2=-13.0$ cm. If the lens is reversed, $R_1=+13.0$ cm and $R_2\to\infty$.

EXECUTE: **(a)** $\dfrac{1}{f}=(0.70)\left(\dfrac{1}{\infty}-\dfrac{1}{-13.0\text{ cm}}\right)=\dfrac{0.70}{13.0\text{ cm}}$ and $f=18.6$ cm. $\dfrac{1}{s'}=\dfrac{1}{f}-\dfrac{1}{s}=\dfrac{s-f}{sf}.$

$s'=\dfrac{sf}{s-f}=\dfrac{(22.5\text{ cm})(18.6\text{ cm})}{22.5\text{ cm}-18.6\text{ cm}}=107$ cm. $m=-\dfrac{s'}{s}=-\dfrac{107\text{ cm}}{22.5\text{ cm}}=-4.76.$

$y'=my=(-4.76)(3.75\text{ mm})=-17.8$ mm. The image is 107 cm to the right of the lens and is 17.8 mm tall. The image is real and inverted.

(b) $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{13.0\text{ cm}}-\dfrac{1}{\infty}\right)$ and $f=18.6$ cm. The image is the same as in part (a).

EVALUATE: Reversing a lens does not change the focal length of the lens.

34.28. IDENTIFY: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$. The sign of f determines whether the lens is converging or diverging.

SET UP: $s=16.0$ cm. $s'=-12.0$ cm.

EXECUTE: **(a)** $f=\dfrac{ss'}{s+s'}=\dfrac{(16.0\text{ cm})(-12.0\text{ cm})}{16.0\text{ cm}+(-12.0\text{ cm})}=-48.0$ cm. $f<0$ and the lens is diverging.

(b) $m = -\dfrac{s'}{s} = -\dfrac{-12.0\text{ cm}}{16.0\text{ cm}} = +0.750.\ |y'| = |m|\,y = (0.750)(8.50\text{ mm}) = 6.38\text{ mm}.$ $m > 0$ and the image is erect.

(c) The principal-ray diagram is sketched in Figure 34.28.

EVALUATE: A diverging lens always forms an image that is virtual, erect, and reduced in size.

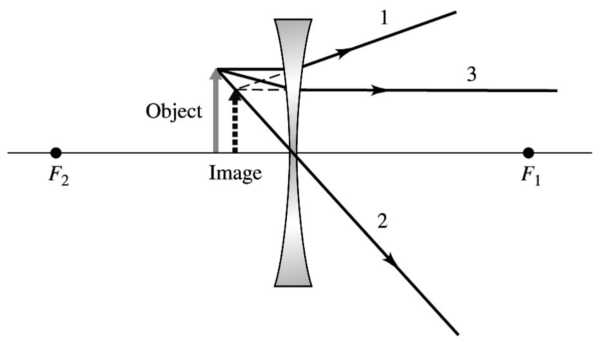


Figure 34.28

34.29. **IDENTIFY:** Use the lensmaker's equation and the thin-lens equation.

SET UP: Combine the lensmaker's equation and the thin-lens equation to get $\dfrac{1}{s} + \dfrac{1}{s'} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, and use the fact that the magnification of the lens is $m = -\dfrac{s'}{s}$.

EXECUTE: **(a)** $\dfrac{1}{s} + \dfrac{1}{s'} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) \Rightarrow \dfrac{1}{24.0\text{ cm}} + \dfrac{1}{s'} = (1.52-1)\left(\dfrac{1}{-7.00\text{ cm}} - \dfrac{1}{-4.00\text{ cm}}\right)$

$$\Rightarrow s' = 71.2\text{ cm, to the right of the lens.}$$

(b) $m = -\dfrac{s'}{s} = -\dfrac{71.2\text{ cm}}{24.0\text{ cm}} = -2.97.$

EVALUATE: Since the magnification is negative, the image is inverted.

34.30. **IDENTIFY:** Apply $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$ to relate s' and s and then use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$.

SET UP: Since the image is inverted, $y' < 0$ and $m < 0$.

EXECUTE: $m = \dfrac{y'}{y} = \dfrac{-4.50\text{ cm}}{3.20\text{ cm}} = -1.406.$ $m = -\dfrac{s'}{s}$ gives $s' = +1.406s$. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ gives $\dfrac{1}{s} + \dfrac{1}{1.406s} = \dfrac{1}{70.0\text{ cm}}$ so $s = 119.8\text{ cm}$, which rounds to 120 cm. $s' = (1.406)(119.8\text{ cm}) = 168\text{ cm}$. The object is 120 cm to the left of the lens. The image is 168 cm to the right of the lens and is real.

EVALUATE: For a single lens an inverted image is always real.

34.31. **IDENTIFY:** The thin-lens equation applies in this case.

SET UP: The thin-lens equation is $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, and the magnification is $m = -\dfrac{s'}{s} = \dfrac{y'}{y}$.

EXECUTE: $m = \dfrac{y'}{y} = \dfrac{34.0\text{ mm}}{8.00\text{ mm}} = 4.25 = -\dfrac{s'}{s} = -\dfrac{-12.0\text{ cm}}{s} \Rightarrow s = 2.82\text{ cm}$. The thin-lens equation gives $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow f = 3.69\text{ cm}.$

EVALUATE: Since the focal length is positive, this is a converging lens. The image distance is negative because the object is inside the focal point of the lens.

34.32. **IDENTIFY:** Apply $m = -\frac{s'}{s}$ to relate s and s'. Then use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: Since the image is to the right of the lens, $s' > 0$. $s' + s = 6.00$ m.

EXECUTE: **(a)** $s' = 80.0s$ and $s + s' = 6.00$ m gives $81.00s = 6.00$ m and $s = 0.0741$ m. $s' = 5.93$ m.

(b) The image is inverted since both the image and object are real $(s' > 0, s > 0)$.

(c) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.0741\text{ m}} + \frac{1}{5.93\text{ m}} \Rightarrow f = 0.0732$ m, and the lens is converging.

EVALUATE: The object is close to the lens and the image is much farther from the lens. This is typical for slide projectors.

34.33. **IDENTIFY:** Apply $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

SET UP: For a distant object the image is at the focal point of the lens. Therefore, $f = 1.87$ cm. For the double-convex lens, $R_1 = +R$ and $R_2 = -R$, where $R = 2.50$ cm.

EXECUTE: $\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(n-1)}{R}$. $n = \frac{R}{2f} + 1 = \frac{2.50\text{ cm}}{2(1.87\text{ cm})} + 1 = 1.67$.

EVALUATE: $f > 0$ and the lens is converging. A double-convex lens surrounded by air is always converging.

34.34. **IDENTIFY:** We know the focal length and magnification and are asked to find the locations of the object and image.

SET UP: $m = \frac{y'}{y} = -\frac{s'}{s}$. Since the image is erect, $y' > 0$ and $m > 0$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

EXECUTE: $m = \frac{y'}{y} = \frac{1.30\text{ cm}}{0.400\text{ cm}} = +3.25$. $m = -\frac{s'}{s} = +3.25$ gives $s' = -3.25s$. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $\frac{1}{s} + \frac{1}{-3.25s} = \frac{1}{9.00\text{ cm}}$ so $s = 6.23$ cm. $s' = -(3.25)(6.23\text{ cm}) = -20.2$ cm. The object is 6.23 cm to the left of the lens. The image is 20.2 cm to the left of the lens and is virtual.

EVALUATE: The image is virtual because the object distance is less than the focal length.

34.35. **IDENTIFY:** First use the lensmaker's formula to find the radius of curvature of the cornea.

SET UP: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. $R_1 = +5.0$ mm. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $m = \frac{y'}{y} = -\frac{s'}{s}$.

EXECUTE: **(a)** $\frac{1}{f(n-1)} = \frac{1}{R_1} - \frac{1}{R_2}$. $\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f(n-1)} = \frac{1}{+5.0\text{ mm}} - \frac{1}{(18.0\text{ mm})(0.38)}$ so $R_2 = 18.6$ mm.

(b) $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$. $s' = \frac{sf}{s-f} = \frac{(25\text{ cm})(1.8\text{ cm})}{25\text{ cm} - 1.8\text{ cm}} = 1.9\text{ cm} = 19$ mm.

(c) $m = -\frac{s'}{s} = -\frac{1.9\text{ cm}}{25\text{ cm}} = -0.076$. $y' = my = (-0.076)(8.0\text{ mm}) = -0.61$ mm. $s' > 0$ so the image is real. $m < 0$ so the image is inverted.

EVALUATE: The cornea alone would focus an object at a distance of 19 mm, which is not at the retina. We must consider the effects of the lens of the eye and the fact that the eye is filled with liquid having an index of refraction.

34.36. **IDENTIFY:** Apply the lensmaker's formula to calculate the radii of the surfaces.

SET UP: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, where $n = 1.55$ and $f = 20.0$ cm.

EXECUTE: Since $f > 0$ we choose $R_1 = R$ and $R_2 = -R$, where R is the magnitude of the radius of curvature. Thus we have $\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(n-1)}{R}$. Solving for R we obtain $R = 2(n-1)f = 2(1.55-1)(20.0\text{ cm}) = 22$ cm.

EVALUATE: For identical convex surfaces, the relation between f and R is $f = \dfrac{1}{n-1} \cdot \dfrac{R}{2}$. This is reminiscent of the relation for spherical mirrors, which is $f = \dfrac{R}{2}$.

34.37. IDENTIFY: First use the figure that accompanies the problem to decide if each radius of curvature is positive or negative. Then apply the lensmaker's formula to calculate the focal length of each lens.

SET UP: Use $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ to calculate f and then use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to locate the image. $s = 18.0$ cm.

EXECUTE: **(a)** $\dfrac{1}{f} = (0.5)\left(\dfrac{1}{10.0\text{ cm}} - \dfrac{1}{-15.0\text{ cm}}\right)$ and $f = +12.0$ cm. $\dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{s} = \dfrac{s-f}{sf}$.

$s' = \dfrac{f}{s-f} = \dfrac{(18.0\text{ cm})(12.0\text{ cm})}{18.0\text{ cm} - 12.0\text{ cm}} = +36.0$ cm. The image is 36.0 cm to the right of the lens.

(b) $\dfrac{1}{f} = (0.5)\left(\dfrac{1}{10.0\text{ cm}} - \dfrac{1}{\infty}\right)$ so $f = +20.0$ cm. $s' = \dfrac{sf}{s-f} = \dfrac{(18.0\text{ cm})(20.0\text{ cm})}{18.0\text{ cm} - 20.0\text{ cm}} = -180$ cm. The image is 180 cm to the left of the lens.

(c) $\dfrac{1}{f} = (0.5)\left(\dfrac{1}{-10.0\text{ cm}} - \dfrac{1}{15.0\text{ cm}}\right)$ so $f = -12.0$ cm. $s' = \dfrac{sf}{s-f} = \dfrac{(18.0\text{ cm})(-12.0\text{ cm})}{18.0\text{ cm} + 12.0\text{ cm}} = -7.20$ cm. The image is 7.20 cm to the left of the lens.

(d) $\dfrac{1}{f} = (0.5)\left(\dfrac{1}{-10.0\text{ cm}} - \dfrac{1}{-15.0\text{ cm}}\right)$ so $f = -60.0$ cm. $s' = \dfrac{sf}{s-f} = \dfrac{(18.0\text{ cm})(-60.0\text{ cm})}{18.0\text{ cm} + 60.0\text{ cm}} = -13.8$ cm. The image is 13.8 cm to the left of the lens.

EVALUATE: The focal length of a lens is determined by *both* of its radii of curvature.

34.38. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.

SET UP: $f = +12.0$ cm and $s' = -17.0$ cm.

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} = \dfrac{1}{12.0\text{ cm}} - \dfrac{1}{-17.0\text{ cm}} \Rightarrow s = 7.0$ cm.

$m = -\dfrac{s'}{s} = -\dfrac{(-17.0)}{7.0} = +2.4 \Rightarrow y = \dfrac{y'}{m} = \dfrac{0.800\text{ cm}}{+2.4} = +0.34$ cm, so the object is 0.34 cm tall, erect, same side as the image. The principal-ray diagram is sketched in Figure 34.38. The image is erect.

EVALUATE: When the object is inside the focal point, a converging lens forms a virtual, enlarged image.

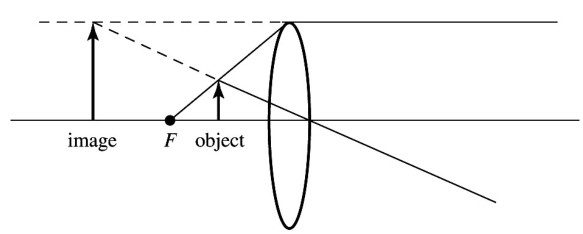


Figure 34.38

34.39. IDENTIFY: Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate the object distance s. m calculated from $m = -\dfrac{s'}{s}$ determines the size and orientation of the image.

SET UP: $f = -48.0$ cm. Virtual image 17.0 cm from lens so $s' = -17.0$ cm.

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, so $\dfrac{1}{s} = \dfrac{1}{f} - \dfrac{1}{s'} = \dfrac{s'-f}{s'f}$.

$$s=\frac{s'f}{s'-f}=\frac{(-17.0\text{ cm})(-48.0\text{ cm})}{-17.0\text{ cm}-(-48.0\text{ cm})}=+26.3\text{ cm}.$$

$$m=-\frac{s'}{s}=-\frac{-17.0\text{ cm}}{+26.3\text{ cm}}=+0.646.$$

$$m=\frac{y'}{y}\text{ so }|y|=\frac{|y'|}{|m|}=\frac{8.00\text{ mm}}{0.646}=12.4\text{ mm}.$$

The principal-ray diagram is sketched in Figure 34.39.

EVALUATE: Virtual image, real object $(s>0)$ so image and object are on same side of lens. $m>0$ so image is erect with respect to the object. The height of the object is 12.4 mm.

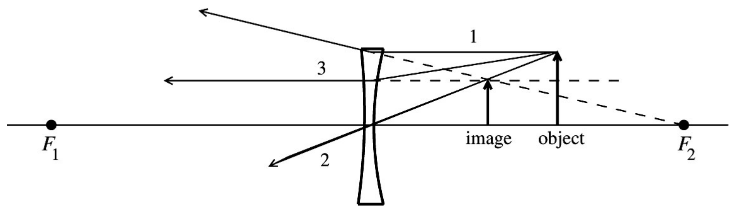


Figure 34.39

34.40. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: The sign of f determines whether the lens is converging or diverging. $s=16.0$ cm. $s'=+36.0$ cm. Use $m=-\frac{s'}{s}$ to find the size and orientation of the image.

EXECUTE: **(a)** $f=\frac{ss'}{s+s'}=\frac{(16.0\text{ cm})(36.0\text{ cm})}{16.0\text{ cm}+36.0\text{ cm}}=11.1$ cm. $f>0$ and the lens is converging.

(b) $m=-\frac{s'}{s}=-\frac{36.0\text{ cm}}{16.0\text{ cm}}=-2.25$. $|y'|=|m|\,y=(2.25)(8.00\text{ mm})=18.0$ mm. $m<0$ so the image is inverted.

(c) The principal-ray diagram is sketched in Figure 34.40.

EVALUATE: The image is real so the lens must be converging.

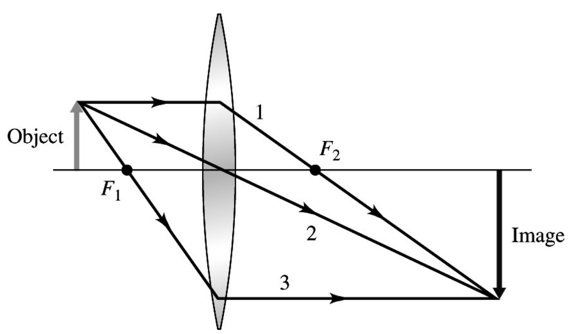


Figure 34.40

34.41. **IDENTIFY:** The first lens forms an image that is then the object for the second lens.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. $m_1=\frac{y_1'}{y_1}$ and $m_2=\frac{y_2'}{y_2}$.

EXECUTE: **(a)** Lens 1: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s_1'=\frac{s_1 f_1}{s_1-f_1}=\frac{(50.0\text{ cm})(40.0\text{ cm})}{50.0\text{ cm}-40.0\text{ cm}}=+200\text{ cm}.$

$m_1=-\frac{s_1'}{s_1}=-\frac{200\text{ cm}}{50\text{ cm}}=-4.00.$ $y_1'=m_1y_1=(-4.00)(1.20\text{ cm})=-4.80\text{ cm}.$ The image I_1 is 200 cm to the right of lens 1, is 4.80 cm tall and is inverted.

(b) Lens 2: $y_2=-4.80$ cm. The image I_1 is $300\text{ cm}-200\text{ cm}=100\text{ cm}$ to the left of lens 2, so $s_2=+100$ cm. $s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(100\text{ cm})(60.0\text{ cm})}{100\text{ cm}-60.0\text{ cm}}=+150\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{150\text{ cm}}{100\text{ cm}}=-1.50.$

$y_2'=m_2y_2=(-1.50)(-4.80\text{ cm})=+7.20\text{ cm}.$ The image is 150 cm to the right of the second lens, is 7.20 cm tall, and is erect with respect to the original object.

EVALUATE: The overall magnification of the lens combination is $m_{\text{tot}}=m_1m_2$.

34.42. **IDENTIFY:** The first lens forms an image that is then the object for the second lens. We follow the same general procedure as in Problem 34.41.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. $m_1=\frac{y_1'}{y_1}$ and $m_2=\frac{y_2'}{y_2}$. For a diverging lens, $f<0$.

EXECUTE: **(a)** $f_1=+40.0$ cm. I_1 is the same as in Problem 34.39. For lens 2,

$s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(100\text{ cm})(-60.0\text{ cm})}{100\text{ cm}-(-60.0\text{ cm})}=-37.5\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{-37.5\text{ cm}}{100\text{ cm}}=+0.375.$

$y_2'=m_2y_2=(+0.375)(-4.80\text{ cm})=-1.80\text{ cm}.$ The final image is 37.5 cm to the left of the second lens (262.5 cm to the right of the first lens). The final image is inverted and is 1.80 cm tall.

(b) $f_1=-40.0$ cm. $s_1'=\frac{s_1f_1}{s_1-f_1}=\frac{(50.0\text{ cm})(-40.0\text{ cm})}{50.0\text{ cm}-(-40.0\text{ cm})}=-22.2\text{ cm}.$ $m_1=-\frac{s_1'}{s_1}=-\frac{-22.2\text{ cm}}{50.0\text{ cm}}=+0.444.$

$y_1'=m_1y_1=(0.444)(1.20\text{ cm})=0.533\text{ cm}.$ The image I_1 is 22.2 cm to the left of lens 1 so is $22.2\text{ cm}+300\text{ cm}=322.2\text{ cm}$ to the left of lens 2 and $s_2=+322.2$ cm. $y_2=y_1'=0.533$ cm.

$s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(322.2\text{ cm})(60.0\text{ cm})}{322.2\text{ cm}-60.0\text{ cm}}=+73.7\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{73.7\text{ cm}}{322.2\text{ cm}}=-0.229.$

$y_2'=m_2y_2=(-0.229)(0.533\text{ cm})=-0.122\text{ cm}.$ The final image is 73.7 cm to the right of the second lens, is inverted and is 0.122 cm tall.

(c) $f_1=-40.0$ cm. $f_2=-60.0$ cm. I_1 is as calculated in part (b).

$s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(322.2\text{ cm})(-60.0\text{ cm})}{322.2\text{ cm}-(-60.0\text{ cm})}=-50.6\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{-50.6\text{ cm}}{322.2\text{ cm}}=+0.157.$

$y_2'=m_2y_2=(0.157)(0.533\text{ cm})=0.0837\text{ cm}.$ The final image is 50.6 cm to the left of the second lens (249.4 cm to the right of the first lens), is upright and is 0.0837 cm tall.

EVALUATE: The overall magnification of the lens combination is $m_{\text{tot}}=m_1m_2$.

34.43. **IDENTIFY:** The first lens forms an image that is then the object for the second lens. We follow the same general procedure as in Problem 34.41.

SET UP: $m_{\text{tot}}=m_1m_2$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}$.

EXECUTE: **(a)** Lens 1: $f_1=-12.0$ cm, $s_1=20.0$ cm. $s'=\frac{(20.0\text{ cm})(-12.0\text{ cm})}{20.0\text{ cm}+12.0\text{ cm}}=-7.5\text{ cm}.$

$m_1=-\frac{s_1'}{s_1}=-\frac{-7.5\text{ cm}}{20.0\text{ cm}}=+0.375.$

Lens 2: The image of lens 1 is 7.5 cm to the left of lens 1 so is $7.5\text{ cm}+9.00\text{ cm}=16.5\text{ cm}$ to the left of lens 2.

$s_2=+16.5$ cm. $f_2=+12.0$ cm. $s_2'=\frac{(16.5\text{ cm})(12.0\text{ cm})}{16.5\text{ cm}-12.0\text{ cm}}=44.0\text{ cm}.$ $m_2=-\frac{s_2'}{s_2}=-\frac{44.0\text{ cm}}{16.5\text{ cm}}=-2.67.$ The final image is 44.0 cm to the right of lens 2 so is 53.0 cm to the right of the first lens.

(b) $s_2' > 0$ so the final image is real.

(c) $m_{\text{tot}} = m_1 m_2 = (+0.375)(-2.67) = -1.00$. The image is 2.50 mm tall and is inverted.

EVALUATE: The light travels through the lenses in the direction from left to right. A real image for the second lens is to the right of that lens and a virtual image is to the left of the second lens.

34.44. IDENTIFY: Use the lensmaker's equation to find the radius of curvature of the lens of the eye.

SET UP: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$. If R is the radius of the lens, then $R_1 = R$ and $R_2 = -R$. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.

EXECUTE: **(a)** $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{-R}\right) = \dfrac{2(n-1)}{R}$.

$R = 2(n-1)f = 2(0.44)(8.0\text{ mm}) = 7.0\text{ mm}$.

(b) $\dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{s} = \dfrac{s-f}{sf}$. $s' = \dfrac{sf}{s-f} = \dfrac{(30.0\text{ cm})(0.80\text{ cm})}{30.0\text{ cm} - 0.80\text{ cm}} = 0.82\text{ cm} = 8.2\text{ mm}$. The image is 8.2 mm from the lens, on the side opposite the object. $m = -\dfrac{s'}{s} = -\dfrac{0.82\text{ cm}}{30.0\text{ cm}} = -0.0273$.

$y' = my = (-0.0273)(16\text{ cm}) = 0.44\text{ cm} = 4.4\text{ mm}$. $s' > 0$ so the image is real. $m < 0$ so the image is inverted.

EVALUATE: The lens is converging and has a very short focal length. As long as the object is farther than 8.0 mm from the eye, the lens forms a real image.

34.45. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$.

SET UP: The image is to be formed on the sensor, so $s' = +20.4$ cm.

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} + \dfrac{1}{20.4\text{ cm}} = \dfrac{1}{20.0\text{ cm}} \Rightarrow s = 1020\text{ cm} = 10.2\text{ m}$.

EVALUATE: The object distance is much greater than f, so the image is just outside the focal point of the lens.

34.46. IDENTIFY: The projector lens can be modeled as a thin lens.

SET UP: The thin-lens equation is $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, and the magnification of the lens is $m = -\dfrac{s'}{s}$.

EXECUTE: **(a)** $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{f} = \dfrac{1}{0.150\text{ m}} + \dfrac{1}{9.00\text{ m}} \Rightarrow f = 147.5\text{ mm}$, so use a $f = 148\text{ mm}$ lens.

(b) $m = -\dfrac{s'}{s} \Rightarrow |m| = 60 \Rightarrow$ Area $= 1.44\text{ m} \times 2.16\text{ m}$.

EVALUATE: The lens must produce a real image to be viewed on the screen. Since the magnification comes out negative, the slides to be viewed must be placed upside down in the tray.

34.47. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.

SET UP: $s = 3.90$ m. $f = 0.085$ m.

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{3.90\text{ m}} + \dfrac{1}{s'} = \dfrac{1}{0.085\text{ m}} \Rightarrow s' = 0.0869\text{ m}$.

$y' = -\dfrac{s'}{s}y = -\dfrac{0.0869}{3.90}1750\text{ mm} = -39.0\text{ mm}$, so it will not fit on the 24-mm$\times$36-mm sensor.

EVALUATE: The image is just outside the focal point and $s' \approx f$. To have $|y'| = 36$ mm, so that the image will fit on the sensor, $s = -\dfrac{s'y}{y'} \approx -\dfrac{(0.085\text{ m})(1.75\text{ m})}{-0.036\text{ m}} = 4.1\text{ m}$. The person would need to stand about 4.1 m from the lens.

34.48. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. The image of the first lens serves as the object for the second lens.

SET UP: For a distant object, $s\to\infty$.

EXECUTE: **(a)** $s_1=\infty\Rightarrow s_1'=f_1=12\text{ cm}.$

(b) $s_2=4.0\text{ cm}-12\text{ cm}=-8\text{ cm}.$

(c) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{-8\text{ cm}}+\frac{1}{s_2'}=\frac{1}{-12\text{ cm}}\Rightarrow s_2'=24\text{ cm},$ to the right.

(d) $s_1=\infty\Rightarrow s_1'=f_1=12\text{ cm}.\;\; s_2=8.0\text{ cm}-12\text{ cm}=-4\text{ cm}.$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{-4\text{ cm}}+\frac{1}{s_2'}=\frac{1}{-12\text{ cm}}\Rightarrow s_2'=6\text{ cm}.$

EVALUATE: In each case the image of the first lens serves as a virtual object for the second lens, and $s_2<0$.

34.49. **IDENTIFY:** The *f*-number of a lens is the ratio of its focal length to its diameter. To maintain the same exposure, the amount of light passing through the lens during the exposure must remain the same.

SET UP: The *f*-number is f/D.

EXECUTE: **(a)** $f\text{-number}=\frac{f}{D}\Rightarrow f\text{-number}=\frac{180.0\text{ mm}}{16.36\text{ mm}}\Rightarrow f\text{-number}=f/11.$ (The *f*-number is an integer.)

(b) $f/11$ to $f/2.8$ is four steps of 2 in intensity, so one needs $1/16^{\text{th}}$ the exposure. The exposure should be $1/480\text{ s}=2.1\times10^{-3}\text{ s}=2.1\text{ ms}.$

EVALUATE: When opening the lens from *f*/11 to *f*/2.8, the area increases by a factor of 16, so 16 times as much light is allowed in. Therefore the exposure time must be decreased by a factor of 1/16 to maintain the same exposure on the film or light receptors of a digital camera.

34.50. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}.$

SET UP: $n_a=1.00,\;\; n_b=1.40.\;\; s=40.0\text{ cm},\;\; s'=2.60\text{ cm}.$

EXECUTE: $\frac{1}{40.0\text{ cm}}+\frac{1.40}{2.60\text{ cm}}=\frac{0.40}{R}$ and $R=0.710\text{ cm}.$

EVALUATE: The cornea presents a convex surface to the object, so $R>0$.

34.51. **(a) IDENTIFY:** The purpose of the corrective lens is to take an object 25 cm from the eye and form a virtual image at the eye's near point. Use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to solve for the image distance when the object distance is 25 cm.

SET UP: $\frac{1}{f}=+2.75$ diopters means $f=+\frac{1}{2.75}\text{ m}=+0.3636\text{ m}$ (converging lens)

$f=36.36\text{ cm};\; s=25\text{ cm};\; s'=?$

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ so

$$s'=\frac{sf}{s-f}=\frac{(25\text{ cm})(36.36\text{ cm})}{25\text{ cm}-36.36\text{ cm}}=-80.0\text{ cm}.$$

The eye's near point is 80.0 cm from the eye.

(b) IDENTIFY: The purpose of the corrective lens is to take an object at infinity and form a virtual image of it at the eye's far point. Use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to solve for the image distance when the object is at infinity.

SET UP: $\dfrac{1}{f} = -1.30$ diopters means $f = -\dfrac{1}{1.30}$ m $= -0.7692$ m (diverging lens).

$f = -76.92$ cm; $s = \infty$; $s' = ?$

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $s = \infty$ says $\dfrac{1}{s'} = \dfrac{1}{f}$ and $s' = f = -76.9$ cm. The eye's far point is 76.9 cm from the eye.

EVALUATE: In each case a virtual image is formed by the lens. The eye views this virtual image instead of the object. The object is at a distance where the eye can't focus on it, but the virtual image is at a distance where the eye can focus.

34.52. IDENTIFY and **SET UP:** For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. $P(\text{in diopters}) = 1/f\,(\text{in m})$.

EXECUTE: **(a)** The person is farsighted.

(b) A converging lens is needed.

(c) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. $f = \dfrac{ss'}{s+s'} = \dfrac{(25.0\text{ cm})(-45.0\text{ cm})}{25.0\text{ cm} - 45.0\text{ cm}} = +56.2$ cm. The power is $\dfrac{1}{0.562\text{ m}} = +1.78$ diopters.

EVALUATE: The object is inside the focal point of the lens, so it forms a virtual image.

34.53. IDENTIFY and **SET UP:** For an object 25.0 cm from the eye, the corrective lens forms a virtual image at the near point of the eye. The distances from the corrective lens are $s = 23.0$ cm and $s' = -43.0$ cm. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. $P(\text{in diopters}) = 1/f$ (in m).

EXECUTE: Solving $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ for f gives $f = \dfrac{ss'}{s+s'} = \dfrac{(23.0\text{ cm})(-43.0\text{ cm})}{23.0\text{ cm} - 43.0\text{ cm}} = +49.4$ cm. The power is $\dfrac{1}{0.494\text{ m}} = 2.02$ diopters.

EVALUATE: In Problem 34.52 the contact lenses have power 1.78 diopters. The power of the lenses is different for ordinary glasses versus contact lenses.

34.54. IDENTIFY and **SET UP:** For an object very far from the eye, the corrective lens forms a virtual image at the far point of the eye. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. $P(\text{in diopters}) = 1/f\,(\text{in m})$.

EXECUTE: **(a)** The person is nearsighted.

(b) A diverging lens is needed.

(c) In $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, $s \to \infty$, so $f = s' = -75.0$ cm. The power is $\dfrac{1}{-0.750\text{ m}} = -1.33$ diopters.

EVALUATE: A diverging lens is needed to form a virtual image of a distant object. A converging lens could not do this since distant objects cannot be inside its focal point.

34.55. IDENTIFY and **SET UP:** For an object very far from the eye, the corrective lens forms a virtual image at the far point of the eye. The distances from the lens are $s \to \infty$ and $s' = -73.0$ cm. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. $P(\text{in diopters}) = 1/f\,(\text{in m})$.

EXECUTE: In $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, $s \to \infty$, so $f = s' = -73.0$ cm. The power is $\dfrac{1}{-0.730\text{ m}} = -1.37$ diopters.

EVALUATE: A diverging lens is needed to form a virtual image of a distant object. A converging lens could not do this since distant objects cannot be inside its focal point.

34.56. IDENTIFY: When the object is at the focal point, $M = \dfrac{25.0\text{ cm}}{f}$. In part (b), apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate s for $s' = -25.0$ cm.

SET UP: Our calculation assumes the near point is 25.0 cm from the eye.

EXECUTE: **(a)** Angular magnification $M = \dfrac{25.0\text{ cm}}{f} = \dfrac{25.0\text{ cm}}{6.00\text{ cm}} = 4.17.$

(b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} + \dfrac{1}{-25.0\text{ cm}} = \dfrac{1}{6.00\text{ cm}} \Rightarrow s = 4.84\text{ cm}.$

EVALUATE: In part (b), $\theta' = \dfrac{y}{s}$, $\theta = \dfrac{y}{25.0\text{ cm}}$, and $M = \dfrac{25.0\text{ cm}}{s} = \dfrac{25.0\text{ cm}}{4.84\text{ cm}} = 5.17.$ M is greater when the image is at the near point than when the image is at infinity.

34.57. **IDENTIFY:** Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$ to calculate s and y'.

SET UP: $f = 8.00$ cm; $s' = -25.0$ cm; $s = ?$

EXECUTE: **(a)** $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, so $\dfrac{1}{s} = \dfrac{1}{f} - \dfrac{1}{s'} = \dfrac{s' - f}{s'f}.$

$$s = \frac{s'f}{s' - f} = \frac{(-25.0\text{ cm})(+8.00\text{ cm})}{-25.0\text{ cm} - 8.00\text{ cm}} = +6.06\text{ cm}.$$

(b) $m = -\dfrac{s'}{s} = -\dfrac{-25.0\text{ cm}}{6.06\text{ cm}} = +4.125.$

$|m| = \dfrac{|y'|}{|y|}$ so $|y'| = |m||y| = (4.125)(1.00\text{ mm}) = 4.12\text{ mm}.$

EVALUATE: The lens allows the object to be much closer to the eye than the near point. The lens allows the eye to view an image at the near point rather than the object.

34.58. **IDENTIFY:** For a thin lens, $-\dfrac{s'}{s} = \dfrac{y'}{y}$, so $\left|\dfrac{y'}{s'}\right| = \left|\dfrac{y}{s}\right|$, and the angular size of the image equals the angular size of the object.

SET UP: The object has angular size $\theta = \dfrac{y}{f}$, with θ in radians.

EXECUTE: $\theta = \dfrac{y}{f} \Rightarrow f = \dfrac{y}{\theta} = \dfrac{2.00\text{ mm}}{0.032\text{ rad}} = 62.5\text{ mm} = 6.25\text{ cm},$ which rounds to 6.3 cm.

EVALUATE: If the insect were at the near point of a normal human eye, its angular size would be $\dfrac{2.00\text{ mm}}{250\text{ mm}} = 0.0080\text{ rad}.$

34.59. **(a) IDENTIFY** and **SET UP:**

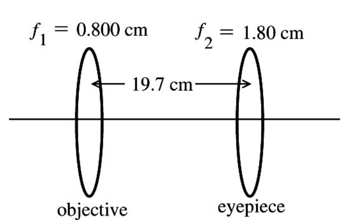


Figure 34.59

Final image is at ∞ so the object for the eyepiece is at its focal point. But the object for the eyepiece is the image of the objective so the image formed by the objective is $19.7\text{ cm} - 1.80\text{ cm} = 17.9\text{ cm}$ to the right of the lens. Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to the image formation by the objective, solve for the object distance s.

$f = 0.800$ cm; $s' = 17.9$ cm; $s = ?$

$\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$, so $\dfrac{1}{s} = \dfrac{1}{f} - \dfrac{1}{s'} = \dfrac{s' - f}{s'f}.$

EXECUTE: $s = \dfrac{s'f}{s'-f} = \dfrac{(17.9\text{ cm})(+0.800\text{ cm})}{17.9\text{ cm} - 0.800\text{ cm}} = +8.37\text{ mm}.$

(b) SET UP: Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}.$

EXECUTE: $m_1 = -\dfrac{s'}{s} = -\dfrac{17.9\text{ cm}}{0.837\text{ cm}} = -21.4.$

The magnitude of the linear magnification of the objective is 21.4.

(c) SET UP: Use $M = m_1 M_2.$

EXECUTE: $M_2 = \dfrac{25\text{ cm}}{f_2} = \dfrac{25\text{ cm}}{1.80\text{ cm}} = 13.9.$

$M = m_1 M_2 = (-21.4)(13.9) = -297.$

EVALUATE: M is not accurately given by $(25\text{ cm})s_1'/f_1 f_2 = 311,$ because the object is not quite at the focal point of the objective $(s_1 = 0.837\text{ cm and } f_1 = 0.800\text{ cm}).$

34.60. IDENTIFY: Apply $M = m_1 M_2 = \dfrac{(25\text{ cm})s_1'}{s_1 f_2}.$

SET UP: $s_1' = 160\text{ mm} + 5.0\text{ mm} = 165\text{ mm}.$

EXECUTE: (a) $s_1 = \dfrac{s_1' f_1}{s_1' - f_1} = \dfrac{(165\text{ mm})(5.00\text{ mm})}{165\text{ mm} - 5.00\text{ mm}} = 5.16\text{ mm}.$

$M = \dfrac{(250\text{ mm})s_1'}{s_1 f_2} = \dfrac{(250\text{ mm})(165\text{ mm})}{(5.16\text{ mm})(26.0\text{ mm})} = 307.$

(b) The minimum separation is $\dfrac{0.10\text{ mm}}{M} = \dfrac{0.10\text{ mm}}{307} = 3.26\times 10^{-4}\text{ mm}.$

EVALUATE: The angular size of the image viewed by the eye when looking through the microscope is 307 times larger than if the object is viewed at the near-point of the unaided eye.

34.61. (a) IDENTIFY and SET UP: Use $M = -\dfrac{f_1}{f_2},$ with $f_1 = 95.0\text{ cm}$ (objective) and $f_2 = 15.0\text{ cm}$ (eyepiece).

EXECUTE: $M = -\dfrac{f_1}{f_2} = -\dfrac{95.0\text{ cm}}{15.0\text{ cm}} = -6.33.$

(b) IDENTIFY: Use $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$ to calculate $y'.$

SET UP: $s = 3.00\times 10^3\text{ m}.$

$s' = f_1 = 95.0\text{ cm}$ (since s is very large, $s' \approx f$).

EXECUTE: $m = -\dfrac{s'}{s} = -\dfrac{0.950\text{ m}}{3.00\times 10^3\text{ m}} = -3.167\times 10^{-4}.$

$|y'| = |m||y| = (3.167\times 10^{-4})(60.0\text{ m}) = 0.0190\text{ m} = 1.90\text{ cm}.$

(c) IDENTIFY and SET UP: Use $M = \dfrac{\theta'}{\theta}$ and the angular magnification M obtained in part (a) to calculate $\theta'.$ The angular size θ of the image formed by the objective (object for the eyepiece) is its height divided by its distance from the objective.

EXECUTE: The angular size of the object for the eyepiece is $\theta = \dfrac{0.0190\text{ m}}{0.950\text{ m}} = 0.0200\text{ rad}.$

(Note that this is also the angular size of the object for the objective: $\theta = \dfrac{60.0\text{ m}}{3.00\times 10^3\text{ m}} = 0.0200\text{ rad}.$ For a thin lens the object and image have the same angular size and the image of the objective is the object for the eyepiece.) $M = \dfrac{\theta'}{\theta}$, so the angular size of the image is $\theta' = M\theta = -(6.33)(0.0200 \text{ rad}) = -0.127 \text{ rad}$. (The minus sign shows that the final image is inverted.)

EVALUATE: The lateral magnification of the objective is small; the image it forms is much smaller than the object. But the total angular magnification is larger than 1.00; the angular size of the final image viewed by the eye is 6.33 times larger than the angular size of the original object, as viewed by the unaided eye.

34.62. **IDENTIFY:** For a telescope, $M = -\dfrac{f_1}{f_2}$.

SET UP: $f_2 = 9.0 \text{ cm}$. The distance between the two lenses equals $f_1 + f_2$.

EXECUTE: $f_1 + f_2 = 1.20 \text{ m} \Rightarrow f_1 = 1.20 \text{ m} - 0.0900 \text{ m} = 1.11 \text{ m}$. $M = -\dfrac{f_1}{f_2} = -\dfrac{111 \text{ cm}}{9.00 \text{ cm}} = -12.3$.

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.63. **IDENTIFY:** $f = R/2$ and $M = -\dfrac{f_1}{f_2}$.

SET UP: For object and image both at infinity, $f_1 + f_2$ equals the distance d between the eyepiece and the mirror vertex. $f_2 = 1.10 \text{ cm}$. $R_1 = 1.30 \text{ m}$.

EXECUTE: **(a)** $f_1 = \dfrac{R_1}{2} = 0.650 \text{ m} \Rightarrow d = f_1 + f_2 = 0.661 \text{ m}$.

(b) $|M| = \dfrac{f_1}{f_2} = \dfrac{0.650 \text{ m}}{0.011 \text{ m}} = 59.1$.

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.64. **IDENTIFY:** Apply the law of reflection for rays from the feet to the eyes and from the top of the head to the eyes.

SET UP: In Figure 34.64 (next page), ray 1 travels from the feet of the woman to her eyes and ray 2 travels from the top of her head to her eyes. The total height of the woman is h.

EXECUTE: The two angles labeled θ_1 are equal because of the law of reflection, as are the two angles labeled θ_2. Since these angles are equal, the two distances labeled y_1 are equal and the two distances labeled y_2 are equal. The height of the woman is $h_\text{w} = 2y_1 + 2y_2$. As the drawing shows, the height of the mirror is $h_\text{m} = y_1 + y_2$. Comparing, we find that $h_\text{m} = h_\text{w}/2$. The minimum height required is half the height of the woman.

EVALUATE: The height of the image is the same as the height of the woman, so the height of the image is twice the height of the mirror.

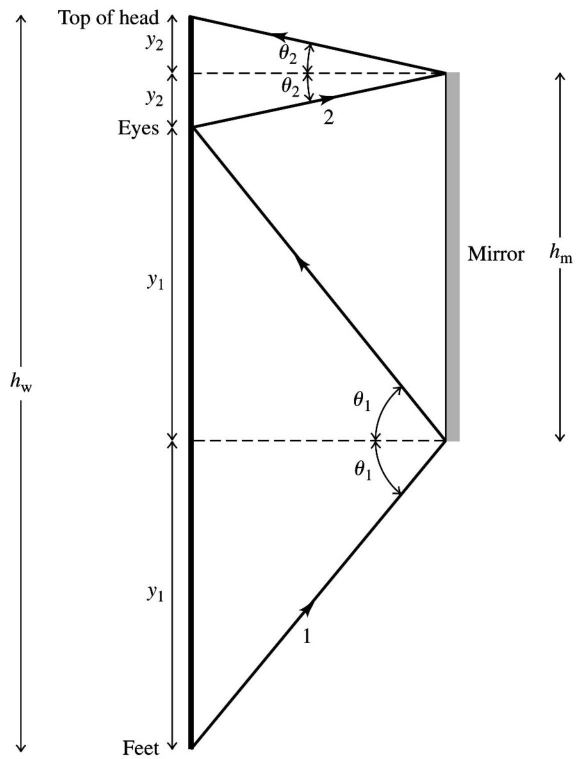


Figure 34.64

34.65. **IDENTIFY** and **SET UP:** For a plane mirror $s' = -s$. $v = \dfrac{ds}{dt}$ and $v' = \dfrac{ds'}{dt}$, so $v' = -v$.

EXECUTE: The velocities of the object and image relative to the mirror are equal in magnitude and opposite in direction. Thus both you and your image are receding from the mirror surface at 3.60 m/s, in opposite directions. Your image is therefore moving at 7.20 m/s relative to you.

EVALUATE: The result derives from the fact that for a plane mirror the image is the same distance behind the mirror as the object is in front of the mirror.

34.66. **IDENTIFY:** Combine $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$ and $m = -\dfrac{s'}{s}$.

SET UP: $m = +2.50$. $R > 0$.

EXECUTE: $m = -\dfrac{s'}{s} = +2.50$. $s' = -2.50s$. $\dfrac{1}{s} + \dfrac{1}{-2.50s} = \dfrac{2}{R}$. $\dfrac{0.600}{s} = \dfrac{2}{R}$ and $s = 0.300R$.

$s' = -2.50s = (-2.50)(0.300R) = -0.750R$. The object is a distance of $0.300R$ in front of the mirror and the image is a distance of $0.750R$ behind the mirror.

EVALUATE: For a single mirror an erect image is always virtual.

34.67. **IDENTIFY:** We are given the image distance, the image height, and the object height. Use $m = -\dfrac{s'}{s}$ to calculate the object distance s. Then use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$ to calculate R.

SET UP: The image is to be formed on screen so it is a real image; $s' > 0$. The mirror-to-screen distance is 8.00 m, so $s' = +800$ cm. $m = -\dfrac{s'}{s} < 0$ since both s and s' are positive.

EXECUTE: **(a)** $|m| = \dfrac{|y'|}{|y|} = \dfrac{24.0 \text{ cm}}{0.600 \text{ cm}} = 40.0,$ so $m = -40.0.$ Then $m = -\dfrac{s'}{s}$ gives

$s = -\dfrac{s'}{m} = -\dfrac{800 \text{ cm}}{-40.0} = +20.0 \text{ cm}.$

(b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R},$ so $\dfrac{2}{R} = \dfrac{s+s'}{ss'}.$ $R = 2\left(\dfrac{ss'}{s+s'}\right) = 2\left(\dfrac{(20.0 \text{ cm})(800 \text{ cm})}{20.0 \text{ cm} + 800 \text{ cm}}\right) = 39.0 \text{ cm}.$

EVALUATE: R is calculated to be positive, which is correct for a concave mirror. Also, in part (a) s is calculated to be positive, as it should be for a real object.

34.68. **IDENTIFY:** Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}$ and $m = -\dfrac{s'}{s}.$

SET UP: Since the image is projected onto the wall it is real and $s' > 0.$ $m = -\dfrac{s'}{s}$ so m is negative and $m = -3.50.$ The object, mirror and wall are sketched in Figure 34.68. This sketch shows that $s' - s = 3.00$ m = 300 cm.

EXECUTE: $m = -3.50 = -\dfrac{s'}{s}$ so $s' = 3.50s.$ $s' - s = 3.50s - s = 300 \text{ cm}$ so $s = 120 \text{ cm}.$

$s' = 300 \text{ cm} + 120 \text{ cm} = 420 \text{ cm}.$ The mirror should be 4.20 m from the wall. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{2}{R}.$

$\dfrac{1}{120 \text{ cm}} + \dfrac{1}{420 \text{ cm}} = \dfrac{2}{R}.$ $R = 187 \text{ cm} = 1.87 \text{ m}.$

EVALUATE: The focal length of the mirror is $f = R/2 = 93.5 \text{ cm}$ and $s > f$, as it must if the image is to be real.

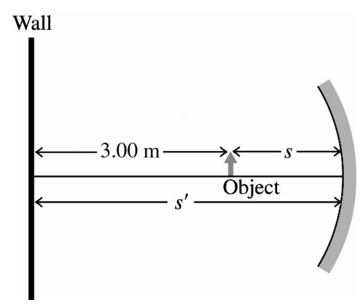


Figure 34.68

34.69. **IDENTIFY:** Since the truck is moving toward the mirror, its image will also be moving toward the mirror.
SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f},$ where $f = R/2.$

EXECUTE: Since the mirror is convex, $f = R/2 = (-1.50 \text{ m})/2 = -0.75 \text{ m}.$ Applying the equation for a spherical mirror gives $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow s' = \dfrac{fs}{s-f}.$ Using the chain rule from calculus and the fact that $v = ds/dt,$ we have $v' = \dfrac{ds'}{dt} = \dfrac{ds'}{ds}\dfrac{ds}{dt} = v\dfrac{f^2}{(s-f)^2}.$ Solving for v gives

$v = v'\left(\dfrac{s-f}{f}\right)^2 = (1.9 \text{ m/s})\left[\dfrac{2.0 \text{ m} - (-0.75 \text{ m})}{-0.75 \text{ m}}\right]^2 = 25.5 \text{ m/s}.$ This is the velocity of the truck relative to the mirror, so the truck is approaching the mirror at 25.5 m/s. You are traveling at 25 m/s, so the truck must be traveling at $25 \text{ m/s} + 25.5 \text{ m/s} = 51 \text{ m/s}$ relative to the highway.

EVALUATE: Even though the truck and car are moving at constant speed, the image of the truck is *not* moving at constant speed because its location depends on the distance from the mirror to the truck.

34.70. IDENTIFY: Apply $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}$, with $R\to\infty$ since the surfaces are flat.

SET UP: The image formed by the first interface serves as the object for the second interface.

EXECUTE: For the water-benzene interface, we get the apparent water depth:
$\dfrac{n_a}{s}+\dfrac{n_b}{s'}=0 \Rightarrow \dfrac{1.33}{5.70 \text{ cm}}+\dfrac{1.50}{s'}=0 \Rightarrow s'=-6.429 \text{ cm}$. For the benzene-air interface, we get the total apparent distance to the bottom: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=0 \Rightarrow \dfrac{1.50}{(6.429 \text{ cm}+4.20 \text{ cm})}+\dfrac{1}{s'}=0 \Rightarrow s'=-7.09 \text{ cm}$.

EVALUATE: At the water-benzene interface the light refracts into material of greater refractive index but at the benzene-air interface it refracts into material of smaller refractive index. The overall effect is that the apparent depth is less than the actual depth.

34.71. IDENTIFY: Apply $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ to calculate s' and then use $m=-\dfrac{s'}{s}=\dfrac{y'}{y}$ to find the height of the image.

SET UP: For a convex mirror, $R<0$, so $R=-18.0 \text{ cm}$ and $f=\dfrac{R}{2}=-9.00 \text{ cm}$.

EXECUTE: **(a)** $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$. $s'=\dfrac{sf}{s-f}=\dfrac{(900 \text{ cm})(-9.00 \text{ cm})}{900 \text{ cm}-(-9.00 \text{ cm})}=-8.91 \text{ cm}$.

$m=-\dfrac{s'}{s}=-\dfrac{-8.91 \text{ cm}}{900 \text{ cm}}=9.90\times10^{-3}$. $|y'|=|m|y=(9.90\times10^{-3})(1.5 \text{ m})=0.0149 \text{ m}=1.49 \text{ cm}$.

(b) The height of the image is much less than the height of the car, so the car appears to be farther away than its actual distance.

EVALUATE: A plane mirror would form an image the same size as the car. Since the image formed by the convex mirror is smaller than the car, the car appears to be farther away compared to what it would appear using a plane mirror.

34.72. IDENTIFY: Apply $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ and the concept of principal rays.

SET UP: $s=10.0 \text{ cm}$. If extended backward the ray comes from a point on the optic axis 18.0 cm from the lens and the ray is parallel to the optic axis after it passes through the lens.

EXECUTE: **(a)** The ray is bent toward the optic axis by the lens so the lens is converging.

(b) The ray is parallel to the optic axis after it passes through the lens so it comes from the focal point; $f=18.0 \text{ cm}$.

(c) The principal-ray diagram is drawn in Figure 34.72. The diagram shows that the image is 22.5 cm to the left of the lens.

(d) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ gives $s'=\dfrac{sf}{s-f}=\dfrac{(10.0 \text{ cm})(18.0 \text{ cm})}{10.0 \text{ cm}-18.0 \text{ cm}}=-22.5 \text{ cm}$. The calculated image position agrees with the principal-ray diagram.

EVALUATE: The image is virtual. A converging lens produces a virtual image when the object is inside the focal point.

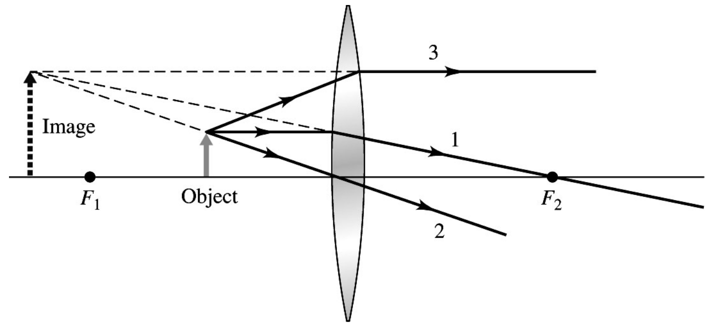


Figure 34.72

34.73. IDENTIFY and **SET UP:** Rays that pass through the hole are undeflected. All other rays are blocked. $m=-\frac{s'}{s}$.

EXECUTE: (a) The ray diagram is drawn in Figure 34.73. The ray shown is the only ray from the top of the object that reaches the film, so this ray passes through the top of the image. An inverted image is formed on the far side of the box, no matter how far this side is from the pinhole and no matter how far the object is from the pinhole.

(b) $s=1.5$ m. $s'=20.0$ cm. $m=-\frac{s'}{s}=-\frac{20.0\text{ cm}}{150\text{ cm}}=-0.133$. $y'=my=(-0.133)(18\text{ cm})=-2.4$ cm.

The image is 2.4 cm tall.

EVALUATE: A defect of this camera is that not much light energy passes through the small hole each second, so long exposure times are required.

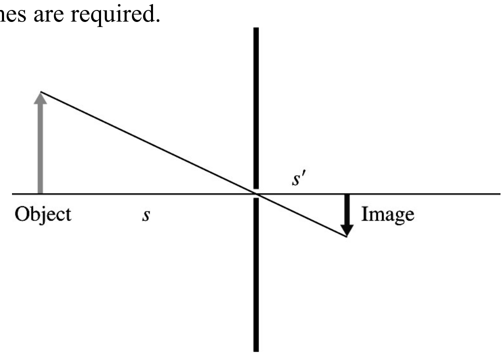


Figure 34.73

34.74. IDENTIFY: In this context, the microscope just looks at an image or object. Apply $\frac{n_a}{s}+\frac{n_b}{s'}=0$ to the image formed by refraction at the top surface of the second plate. In this calculation the object is the bottom surface of the second plate.

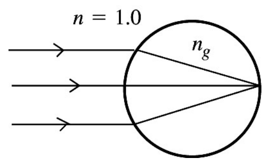
SET UP: The thickness of the second plate is $2.10\text{ mm}+0.78\text{ mm}=2.88\text{ mm}$, and this is s. The image is 2.10 mm below the top surface, so $s'=-2.10$ mm.

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{n}{s}+\frac{1}{s'}=0 \Rightarrow n=-\frac{s}{s'}=-\frac{2.88\text{ mm}}{-2.10\text{ mm}}=1.37.$

EVALUATE: The object and image distances are measured from the front surface of the second plate, and the image is virtual.

34.75. IDENTIFY: Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ to the image formed by refraction at the front surface of the sphere.

SET UP: Let n_g be the index of refraction of the glass. The image formation is shown in Figure 34.75.



$s = \infty$.
$s' = +2r$, where r is the radius of the sphere.
$n_a = 1.00, n_b = n_g, R = +r$.

Figure 34.75

EXECUTE: $\frac{1}{\infty} + \frac{n_g}{2r} = \frac{n_g - 1.00}{r}$.

$\frac{n_g}{2r} = \frac{n_g}{r} - \frac{1}{r}; \frac{n_g}{2r} = \frac{1}{r}$ and $n_g = 2.00$.

EVALUATE: The required refractive index of the glass does not depend on the radius of the sphere.

34.76. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ and $m = -\frac{n_a s'}{n_b s}$ to each refraction. The overall magnification is $m_{\text{tot}} = m_1 m_2$.

SET UP: For the first refraction, $R = +6.0$ cm, $n_a = 1.00$ and $n_b = 1.60$. For the second refraction, $R = -12.0$ cm, $n_a = 1.60$, and $n_b = 1.00$.

EXECUTE: **(a)** The image from the left end acts as the object for the right end of the rod.

(b) $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{23.0\text{ cm}} + \frac{1.60}{s'} = \frac{0.60}{6.0\text{ cm}} \Rightarrow s' = 28.3$ cm.

So the second object distance is $s_2 = 40.0\text{ cm} - 28.3\text{ cm} = 11.7$ cm. $m_1 = -\frac{n_a s'}{n_b s} = -\frac{28.3}{(1.60)(23.0)} = -0.769$.

(c) The object is real and inverted.

(d) $\frac{n_a}{s_2} + \frac{n_b}{s_2'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1.60}{11.7\text{ cm}} + \frac{1}{s_2'} = \frac{-0.60}{-12.0\text{ cm}} \Rightarrow s' = -11.5$ cm.

$m_2 = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(-11.5)}{11.7} = 1.57 \Rightarrow m_{\text{tot}} = m_1 m_2 = (-0.769)(1.57) = -1.21$.

(e) The final image is virtual, and inverted.

(f) $y' = (1.50\text{ mm})(-1.21) = -1.82$ mm.

EVALUATE: The first image is to the left of the second surface, so it serves as a real object for the second surface, with positive object distance.

34.77. IDENTIFY: We know the magnitude of the focal length is 35.0 cm and that it produces an image that is twice the height of the object. In part (a) the image is real, and in part (b) it is virtual. In each case we want to know the distance from the object to the lens and if the lens is converging or diverging. The thin-lens formula applies in both cases.

SET UP: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ with $f = \pm 35.0$ cm. We know that the magnification is $m = -\frac{s'}{s}$.

EXECUTE: **(a)** We want the size of the image to be twice that of the object, so we must have $m = \pm 2$. Since the image is real we know that $s' > 0$, which implies that $m = -2 = -\frac{s'}{s}$. Thus we conclude that $s' = 2s$. Now we can determine the location of the object: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{2s} = \frac{3}{2s} = \frac{1}{f}$. Solving for s we get $s = \frac{3}{2}f$. Since we know that $s > 0$ we must have that $f = +35.0$ cm, and thus $s = \frac{3}{2}f = \frac{3}{2}(35.0\text{ cm}) =$ 52.5 cm. The lens is a converging lens, and the object must be placed 52.5 cm in front of the lens.

(b) We again want the image to be twice the size as the object; however, in this case we have a virtual image so $s' < 0$ and $m = +2 = -\frac{s'}{s}$. Thus, we have $s' = -2s$. Now we can determine the location of the object: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{-2s} = \frac{1}{2s} = \frac{1}{f}$. Solving for s we obtain $s = \frac{1}{2}f$. Since we know that $s > 0$ we must have that $f = +35.0 \text{ cm}$ and thus $s = \frac{1}{2}(35.0 \text{ cm}) = 17.5 \text{ cm}$. The lens is a converging lens, and the object must be placed 17.5 cm in front of the lens.

EVALUATE: For a diverging lens we have $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} < 0$. This can only occur if $\frac{1}{s'}$ is negative and larger in magnitude than $\frac{1}{s}$. Thus we have $|m| = \left|-\frac{s'}{s}\right| < 1$. It follows that the image is always smaller than the object for a diverging lens. In this exercise $|m| = 2 > 1$, so only a converging lens will work.

34.78. IDENTIFY: The lens forms an image of the object. That image (I_1) is reflected in the plane mirror, and its image (I_2) is just as far behind the mirror as I_1 is in front of the mirror. The image I_2 in the mirror then acts as the object for the lens which forms an image I_3 on the screen.

SET UP: The thin-lens equation, $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, applies to the lens.

EXECUTE: **(a)** Figure 34.78 shows the arrangement of the screen, object, lens, and mirror.

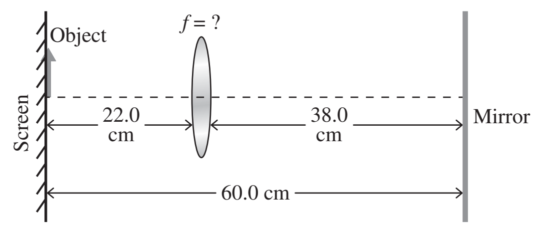


Figure 34.78

(b) First image formed by the lens (I_1): Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ at the lens. The object distance s is 22.0 cm.

$$\frac{1}{22.0 \text{ cm}} + \frac{1}{s'} = \frac{1}{f}. \qquad \text{Eq. (1)}$$

Image I_2 in the mirror: The image I_1 is a distance s' from the lens, so its distance from the mirror is 38.0 cm $-$ s'. So its image I_2 in the mirror is a distance 38.0 cm $-$ s' behind the mirror.

Second image formed by the lens (I_3): I_2 serves as the object for the lens, and its distance s from the lens is s = 38.0 cm + (38.0 cm $-$ s') = 76.0 cm $-$ s'. The lens forms the image I_3 on the screen, so the image distance is s' = 22.0 cm. Applying the thin-lens equation again gives

$$\frac{1}{76.0 \text{ cm} - s'} + \frac{1}{22.0 \text{ cm}} = \frac{1}{f}. \qquad \text{Eq. (2)}$$

Equating the two expressions for $1/f$ from Equations (1) and (2) gives

$\frac{1}{22.0 \text{ cm}} + \frac{1}{s'} = \frac{1}{76.0 \text{ cm} - s'} + \frac{1}{22.0 \text{ cm}}$, which simplifies to $\frac{1}{s'} = \frac{1}{76.0 \text{ cm} - s'}$. Solving for s' gives s' = 38.0 cm. Putting this into Eq. (1) gives $\frac{1}{f} = \frac{1}{22.0 \text{ cm}} + \frac{1}{38.0 \text{ cm}}$, so f = 13.9 cm.

EVALUATE: The image I_1 is *at* the mirror.

34.79. **IDENTIFY:** We know that the image is real, is 214 cm from the *object* (*not* from the lens), and is 5/3 times the height of the object. We want to find the type of lens and its focal length. The thin-lens equation applies.

SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ with the conditions that $s + s' = \pm 214$ cm and $m=-\frac{s'}{s}$.

EXECUTE: Since the size of the image is greater than the size of the object, we know that the image must be farther from the lens than the object. This implies that the focal length of the lens is positive and the lens is converging. We know that the image is real, so $s'>0$. In this case we have $s+s'=+214\text{ cm}$ and $m=-\frac{s'}{s}=-\frac{5}{3}$. Thus, we may write $s'=\frac{5}{3}s$ and $s+s'=\frac{8}{3}s=+214\text{ cm}$. Solving for s and s' we obtain $s=80.25\text{ cm}$ and $s'=133.75\text{ cm}$. This gives $f=\frac{ss'}{s+s'}=\frac{(80.25\text{ cm})(133.75\text{ cm})}{214\text{ cm}}=50.2\text{ cm}$.

EVALUATE: For a diverging lens we have $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}<0$. This can only occur if $\frac{1}{s'}$ is negative and larger in magnitude than $\frac{1}{s}$. Thus we have $|m|=\left|-\frac{s'}{s}\right|<1$. It follows that the image is always smaller than the object for a diverging lens. In this exercise $|m|=\frac{5}{3}>1$, so only a converging lens will work.

34.80. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$. The type of lens determines the sign of f. The sign of s' determines whether the image is real or virtual.

SET UP: $s=+8.00\text{ cm}$. $s'=-3.00\text{ cm}$. s' is negative because the image is on the same side of the lens as the object.

EXECUTE: **(a)** $\frac{1}{f}=\frac{s+s'}{ss'}$ and $f=\frac{ss'}{s+s'}=\frac{(8.00\text{ cm})(-3.00\text{ cm})}{8.00\text{ cm}-3.00\text{ cm}}=-4.80\text{ cm}$. f is negative so the lens is diverging.

(b) $m=-\frac{s'}{s}=-\frac{-3.00\text{ cm}}{8.00\text{ cm}}=+0.375$. $y'=my=(0.375)(6.50\text{ mm})=2.44\text{ mm}$. $s'<0$ and the image is virtual.

EVALUATE: A converging lens can also form a virtual image, if the object distance is less than the focal length. But in that case $|s'|>s$ and the image would be farther from the lens than the object is.

34.81. **IDENTIFY:** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. The type of lens determines the sign of f. $m=\frac{y'}{y}=-\frac{s'}{s}$. The sign of s' depends on whether the image is real or virtual. $s=16.0\text{ cm}$.

SET UP: $s'=-22.0\text{ cm}$; s' is negative because the image is on the same side of the lens as the object.

EXECUTE: **(a)** $\frac{1}{f}=\frac{s+s'}{ss'}$ and $f=\frac{ss'}{s+s'}=\frac{(16.0\text{ cm})(-22.0\text{ cm})}{16.0\text{ cm}-22.0\text{ cm}}=+58.7\text{ cm}$. f is positive so the lens is converging.

(b) $m=-\frac{s'}{s}=-\frac{-22.0\text{ cm}}{16.0\text{ cm}}=1.38$. $y'=my=(1.38)(3.25\text{ mm})=4.48\text{ mm}$. $s'<0$ and the image is virtual.

EVALUATE: A converging lens forms a virtual image when the object is closer to the lens than the focal point.

34.82. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$. Use the image distance when viewed from the flat end to determine the refractive index n of the rod.

SET UP: When viewing from the flat end, $n_a=n$, $n_b=1.00$ and $R\to\infty$. When viewing from the curved end, $n_a=n$, $n_b=1.00$, and $R=-10.0\text{ cm}$.

EXECUTE: When viewed from the flat end of the rod:

$$\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{n}{15.0\text{ cm}}+\frac{1}{-8.20\text{ cm}}=0 \Rightarrow n=\frac{15.0}{8.20}=1.829.$$

When viewed from the curved end of the rod:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{n}{s}+\frac{1}{s'}=\frac{1-n}{R} \Rightarrow \frac{1.829}{15.0\text{ cm}}+\frac{1}{s'}=\frac{-0.829}{-10.0\text{ cm}},\ \text{so } s'=-25.6\text{ cm}.$$

The image is 25.6 cm within the rod from the curved end.

EVALUATE: In each case the image is virtual and on the same side of the surface as the object.

34.83. IDENTIFY: The image formed by refraction at the surface of the eye is located by $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

SET UP: $n_a=1.00$, $n_b=1.35$. $R>0$. For a distant object, $s\approx\infty$ and $\frac{1}{s}\approx 0$.

EXECUTE: **(a)** $s\approx\infty$ and $s'=2.5$ cm: $\frac{1.35}{2.5\text{ cm}}=\frac{1.35-1.00}{R}$ and $R=0.648\text{ cm}=6.48\text{ mm}$.

(b) $R=0.648$ cm and $s=25$ cm: $\frac{1.00}{25\text{ cm}}+\frac{1.35}{s'}=\frac{1.35-1.00}{0.648}$. $\frac{1.35}{s'}=0.500$ and $s'=2.70\text{ cm}=27.0\text{ mm}$. The image is formed behind the retina.

(c) Calculate s' for $s\approx\infty$ and $R=0.50$ cm: $\frac{1.35}{s'}=\frac{1.35-1.00}{0.50\text{ cm}}$. $s'=1.93\text{ cm}=19.3\text{ mm}$. The image is formed in front of the retina.

EVALUATE: The cornea alone cannot achieve focus of both close and distant objects.

34.84. IDENTIFY and SET UP: Use the lensmaker's equation $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate the focal length of the lenses. The image formed by the first lens serves at the object for the second lens. $m_{\text{tot}}=m_1m_2$. The thin-lens formula $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}$.

EXECUTE: **(a)** $\frac{1}{f}=(0.60)\left(\frac{1}{12.0\text{ cm}}-\frac{1}{28.0\text{ cm}}\right)$ and $f=+35.0$ cm.

<u>Lens 1:</u> $f_1=+35.0$ cm. $s_1=+45.0$ cm. $s_1'=\frac{s_1f_1}{s_1-f_1}=\frac{(45.0\text{ cm})(35.0\text{ cm})}{45.0\text{ cm}-35.0\text{ cm}}=+158\text{ cm}.$

$m_1=-\frac{s_1'}{s_1}=-\frac{158\text{ cm}}{45.0\text{ cm}}=-3.51$. $|y_1'|=|m_1|y_1=(3.51)(5.00\text{ mm})=17.6\text{ mm}$. The image of the first lens is 158 cm to the right of lens 1 and is 17.6 mm tall.

(b) The image of lens 1 is $315\text{ cm}-158\text{ cm}=157\text{ cm}$ to the left of lens 2. $f_2=+35.0$ cm. $s_2=+157$ cm.

$s_2'=\frac{s_2f_2}{s_2-f_2}=\frac{(157\text{ cm})(35.0\text{ cm})}{157\text{ cm}-35.0\text{ cm}}=+45.0\text{ cm}$. $m_2=-\frac{s_2'}{s_2}=-\frac{45.0\text{ cm}}{157\text{ cm}}=-0.287$.

$m_{\text{tot}}=m_1m_2=(-3.51)(-0.287)=+1.00$. The final image is 45.0 cm to the right of lens 2. The final image is 5.00 mm tall. $m_{\text{tot}}>0$ and the final image is erect.

EVALUATE: The final image is real. It is erect because each lens produces an inversion of the image, and two inversions return the image to the orientation of the object.

34.85. IDENTIFY and SET UP: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ for each lens position. The lens to screen distance in each case is the image distance. There are two unknowns, the original object distance x and the focal length f of the lens. But each lens position gives an equation, so there are two equations for these two unknowns. The object, lens and screen before and after the lens is moved are shown in Figure 34.85 (next page).

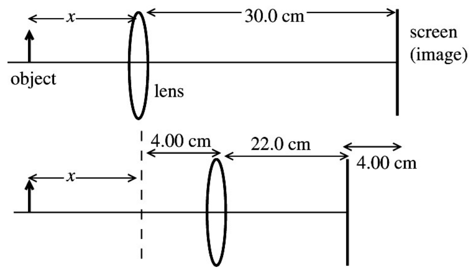


Figure 34.85

$s = x;\ s' = 30.0$ cm.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}.$

$\frac{1}{x}+\frac{1}{30.0\text{ cm}}=\frac{1}{f}.$

$s = x + 4.00$ cm; $s' = 22.0$ cm.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $\frac{1}{x+4.00\text{ cm}}+\frac{1}{22.0\text{ cm}}=\frac{1}{f}.$

EXECUTE: Equate these two expressions for $1/f$:

$$\frac{1}{x}+\frac{1}{30.0\text{ cm}}=\frac{1}{x+4.00\text{ cm}}+\frac{1}{22.0\text{ cm}}.$$

$$\frac{1}{x}-\frac{1}{x+4.00\text{ cm}}=\frac{1}{22.0\text{ cm}}-\frac{1}{30.0\text{ cm}}.$$

$$\frac{x+4.00\text{ cm}-x}{x(x+4.00\text{ cm})}=\frac{30.0-22.0}{660\text{ cm}} \text{ and } \frac{4.00\text{ cm}}{x(x+4.00\text{ cm})}=\frac{8}{660\text{ cm}}.$$

$x^2+(4.00\text{ cm})x-330\text{ cm}^2=0$ and $x=\frac{1}{2}(-4.00\pm\sqrt{16.0+4(330)})$ cm.

x must be positive so $x=\frac{1}{2}(-4.00+36.55)\text{ cm}=16.28$ cm.

Then $\frac{1}{x}+\frac{1}{30.0\text{ cm}}=\frac{1}{f}$ and $\frac{1}{f}=\frac{1}{16.28\text{ cm}}+\frac{1}{30.0\text{ cm}}.$

$f=+10.55$ cm, which rounds to 10.6 cm. $f>0$; the lens is converging.

EVALUATE: We can check that $s=16.28$ cm and $f=10.55$ cm gives $s'=30.0$ cm and that $s=(16.28+4.0)\text{ cm}=20.28$ cm and $f=10.55$ cm gives $s'=22.0$ cm.

34.86. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}.$

SET UP: $s+s'=22.0$ cm.

EXECUTE: **(a)** $\frac{1}{22.0\text{ cm}-s'}+\frac{1}{s'}=\frac{1}{3.00\text{ cm}}.$ $(s')^2-(22.0\text{ cm})s'+66.0\text{ cm}^2=0$ so

$s'=18.42$ cm or 3.58 cm. $s=3.58$ cm or 18.42 cm, so the lens must either be 3.58 cm or 18.4 cm from the object.

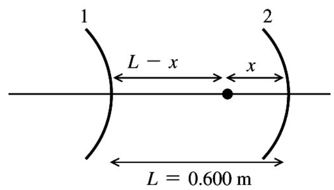
(b) $s=3.58$ cm and s' = 18.42 cm gives $m=-\frac{s'}{s}=-\frac{18.42}{3.58}=-5.15.$

$s=18.42$ cm and s' = 3.58 cm gives $m=-\frac{s'}{s}=-\frac{3.58}{18.42}=-0.914.$

EVALUATE: Since the image is projected onto the screen, the image is real and s' is positive. We assumed this when we wrote the condition $s+s'=22.0$ cm.

34.87. (a) IDENTIFY: Use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to locate the image formed by each mirror. The image formed by the first mirror serves as the object for the second mirror.

SET UP: The positions of the object and the two mirrors are shown in Figure 34.87a.



$|R| = 0.360 \text{ m}.$

$|f| = |R|/2 = 0.180 \text{ m}.$

Figure 34.87a

EXECUTE: Image formed by convex mirror (mirror #1):
convex means $f_1 = -0.180$ m; $s_1 = L - x$.

$$s_1' = \frac{s_1 f_1}{s_1 - f_1} = \frac{(L-x)(-0.180 \text{ m})}{L - x + 0.180 \text{ m}} = -(0.180 \text{ m})\left(\frac{0.600 \text{ m} - x}{0.780 \text{ m} - x}\right) < 0.$$

The image is $(0.180 \text{ m})\left(\frac{0.600 \text{ m} - x}{0.780 \text{ m} - x}\right)$ to the left of mirror #1 so is

$$0.600 \text{ m} + (0.180 \text{ m})\left(\frac{0.600 \text{ m} - x}{0.780 \text{ m} - x}\right) = \frac{0.576 \text{ m}^2 - (0.780 \text{ m})x}{0.780 \text{ m} - x}$$ to the left of mirror #2.

Image formed by concave mirror (mirror #2):
concave implies $f_2 = +0.180$ m.

$$s_2 = \frac{0.576 \text{ m}^2 - (0.780 \text{ m})x}{0.780 \text{ m} - x}.$$

Rays return to the source implies $s_2' = x$. Using these expressions in $s_2 = \frac{s_2' f_2}{s_2' - f_2}$ gives

$$\frac{0.576 \text{ m}^2 - (0.780 \text{ m})x}{0.780 \text{ m} - x} = \frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}}.$$

$$0.600x^2 - (0.576 \text{ m})x + 0.10368 \text{ m}^2 = 0.$$

$$x = \frac{1}{1.20}\left(0.576 \pm \sqrt{(0.576)^2 - 4(0.600)(0.10368)}\right) \text{ m} = \frac{1}{1.20}(0.576 \pm 0.288) \text{ m}.$$

$x = 0.72$ m (impossible; can't have $x > L = 0.600$ m) or $x = 0.24$ m.

(b) SET UP: Which mirror is #1 and which is #2 is now reversed form part (a). This is shown in Figure 34.87b.

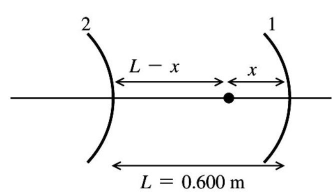


Figure 34.87b

EXECUTE: Image formed by concave mirror (mirror #1):
concave means $f_1 = +0.180$ m; $s_1 = x$.

$$s_1' = \frac{s_1 f_1}{s_1 - f_1} = \frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}}.$$

The image is $\frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}}$ to the left of mirror #1, so

$$s_2 = 0.600 \text{ m} - \frac{(0.180 \text{ m})x}{x - 0.180 \text{ m}} = \frac{(0.420 \text{ m})x - 0.180 \text{ m}^2}{x - 0.180 \text{ m}}.$$

Image formed by convex mirror (mirror #2):

convex means $f_2 = -0.180$ m.

rays return to the source means $s_2' = L - x = 0.600 \text{ m} - x$.

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives

$$\frac{x - 0.180 \text{ m}}{(0.420 \text{ m})x - 0.180 \text{ m}^2} + \frac{1}{0.600 \text{ m} - x} = -\frac{1}{0.180 \text{ m}}.$$

$$\frac{x - 0.180 \text{ m}}{(0.420 \text{ m})x - 0.180 \text{ m}^2} = -\left(\frac{0.780 \text{ m} - x}{0.180 \text{ m}^2 - (0.180 \text{ m})x}\right).$$

$0.600x^2 - (0.576 \text{ m})x + 0.1036 \text{ m}^2 = 0.$

This is the same quadratic equation as obtained in part (a), so again $x = 0.24$ m.

EVALUATE: For $x = 0.24$ m the image is at the location of the source, both for rays that initially travel from the source toward the left and for rays that travel from the source toward the right.

34.88. IDENTIFY and SET UP: The thin-lens equation, $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, applies. The lens forms an image of the object on the screen, so the distance from the lens to the screen is the image distance s'. The distance from the object to the lens is s, so $s + s' = d$.

EXECUTE: We combine $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $s + s' = d$ to solve for d.

$s + s' = d \quad \rightarrow \quad s' = d - s.$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \rightarrow \quad s' = \frac{sf}{s - f} \quad \rightarrow \quad d - s = \frac{sf}{s - f}.$

$ds - s^2 - df + sf = sf \quad \rightarrow \quad s^2 - ds + df = 0 \quad \rightarrow \quad s = \frac{1}{2}(d \pm \sqrt{d^2 - 4df}).$

If $4df > d^2$, there is no real solution, so we must have $d^2 \geq 4df$. The smallest that d can be is if $d^2 = 4df$, in which case $d = 4f$.

EVALUATE: Larger values of d are possible, but we want only the smallest one.

34.89. IDENTIFY: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $s' = \frac{sf}{s - f}$, for both the mirror and the lens.

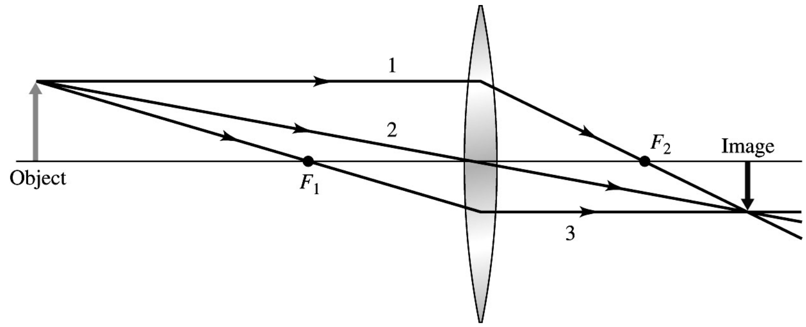
SET UP: For the second image, the image formed by the mirror serves as the object for the lens. For the mirror, $f_m = +10.0$ cm. For the lens, $f = 32.0$ cm. The center of curvature of the mirror is $R = 2f_m = 20.0$ cm to the right of the mirror vertex.

EXECUTE: (a) The principal-ray diagrams from the two images are sketched in Figure 34.89. In Figure 34.89b, only the image formed by the mirror is shown. This image is at the location of the candle so the principal-ray diagram that shows the image formation when the image of the mirror serves as the object for the lens is analogous to that in Figure 34.89a and is not drawn.

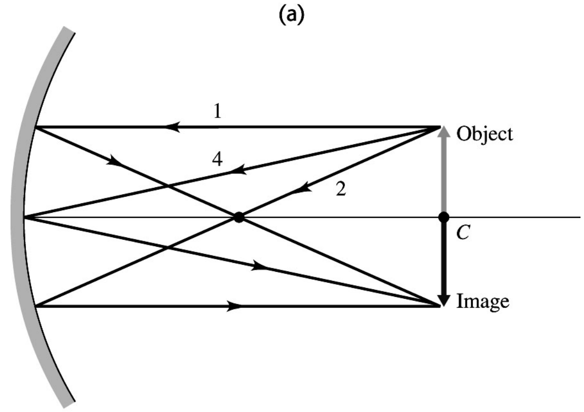
(b) Image formed by the light that passes directly through the lens: The candle is 85.0 cm to the left of the lens. $s' = \frac{sf}{s - f} = \frac{(85.0 \text{ cm})(32.0 \text{ cm})}{85.0 \text{ cm} - 32.0 \text{ cm}} = +51.3$ cm. $m = -\frac{s'}{s} = -\frac{51.3 \text{ cm}}{85.0 \text{ cm}} = -0.604.$ This image is 51.3 cm to the right of the lens. $s' > 0$ so the image is real. $m < 0$ so the image is inverted. Image formed by the light that first reflects off the mirror: First consider the image formed by the mirror. The candle is 20.0 cm to the right of the mirror, so $s = +20.0$ cm. $s' = \frac{sf}{s - f} = \frac{(20.0 \text{ cm})(10.0 \text{ cm})}{20.0 \text{ cm} - 10.0 \text{ cm}} = 20.0$ cm.

$m_1 = -\frac{s_1'}{s_1} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = -1.00.$ The image formed by the mirror is at the location of the candle, so $s_2 = +85.0$ cm and $s_2' = 51.3$ cm. $m_2 = -0.604.$ $m_{tot} = m_1 m_2 = (-1.00)(-0.604) = 0.604.$ The second image is 51.3 cm to the right of the lens. $s_2' > 0,$ so the final image is real. $m_{tot} > 0,$ so the final image is erect.

EVALUATE: The two images are at the same place. They are the same size. One is erect and one is inverted.



(a)



(b)

Figure 34.89

34.90. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. The image formed by the first lens serves as the object for the second lens. The focal length of the lens combination is defined by $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}$. In part (b) use $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate *f* for the meniscus lens and for the CCl_4, treated as a thin lens.

SET UP: With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens.

EXECUTE: **(a)** $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{s_1'}=\frac{1}{f_1}-\frac{1}{s_1}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{-s_1'}+\frac{1}{s_2'}=\left(\frac{1}{s_1}-\frac{1}{f_1}\right)+\frac{1}{s_2'}=\frac{1}{f_2}$. But overall for the lens system, $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}$.

(b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a). For the meniscus lens

$$\frac{1}{f_{\mathrm{m}}}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.55)\left(\frac{1}{4.50\text{ cm}}-\frac{1}{9.00\text{ cm}}\right)=0.061\text{ cm}^{-1}\ \text{and}\ f_{\mathrm{m}}=16.4\text{ cm}.$$

For the CCl_4: $\frac{1}{f_w} = (n_b - n_a)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (0.46)\left(\frac{1}{9.00\text{ cm}} - \frac{1}{\infty}\right) = 0.051\text{ cm}^{-1}$ and $f_w = 19.6$ cm.

$\frac{1}{f} = \frac{1}{f_w} + \frac{1}{f_m} = 0.112\text{ cm}^{-1}$ and $f = 8.93$ cm.

EVALUATE: $f = \frac{f_1 f_2}{f_1 + f_2}$, so f for the combination is less than either f_1 or f_2.

34.91. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$.

SET UP: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected $15\text{ cm} + 19.2\text{ cm} = 34.2\text{ cm}$ from the diverging lens.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-15.0\text{ cm}} + \frac{1}{34.2\text{ cm}} = \frac{1}{f} \Rightarrow f = -26.7\text{ cm}.$

EVALUATE: Our calculation yields a negative value of f, which should be the case for a diverging lens.

34.92. IDENTIFY: Start with the two formulas right after the beginning of the section in the textbook on the lensmaker's equation: $\frac{n_a}{s_1} + \frac{n_b}{s_1'} = \frac{n_b - n_a}{R_1}$ and $\frac{n_b}{s_2} + \frac{n_c}{s_2'} = \frac{n_c - n_b}{R_2}$.

SET UP: The lens is surrounded by a liquid, so $n_a = n_c = n_{\text{liq}}$ and $n_b = n$ (for the lens), and $s_2 = -s_1'$.

EXECUTE: **(a)** Putting in the quantities indicated above, the two starting equations become $\frac{n_{\text{liq}}}{s_1} + \frac{n}{-s_2} = \frac{n - n_{\text{liq}}}{R_1}$ and $\frac{n}{s_2} + \frac{n_{\text{liq}}}{s_2'} = \frac{n_{\text{liq}} - n}{R_2}$. Add these two equations to eliminate n/s_2, giving

$\frac{n_{\text{liq}}}{s_1} + \frac{n_{\text{liq}}}{s_2'} = (n - n_{\text{liq}})\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Dividing by n_{liq} gives $\frac{1}{s_1} + \frac{1}{s_2'} = \left(\frac{n}{n_{\text{liq}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, which gives

$\frac{1}{s_1} + \frac{1}{s_2'} = \left(\frac{n}{n_{\text{liq}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Therefore $\frac{1}{f_{\text{liq}}} = \left(\frac{n}{n_{\text{liq}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, where f_{liq} is the focal length of the lens when it is immersed in the liquid.

(b) Take the ratio of $1/f_{\text{liq}}$ to $1/f_{\text{air}}$: $\dfrac{\frac{1}{f_{\text{liq}}}}{\frac{1}{f_{\text{air}}}} = \dfrac{\left(\frac{n}{n_{\text{liq}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$. Now solve for f_{liq},

giving $f_{\text{liq}} = f_{\text{air}}\left(\frac{n-1}{n/n_{\text{liq}} - 1}\right) = (18.0\text{ cm})\left(\frac{1.60 - 1}{1.60/1.42 - 1}\right) = +85.2\text{ cm}.$

EVALUATE: In part (b) we saw that immersing a lens in a liquid can change its focal length considerably. But even more extreme behavior can result. If the "liquid" is air, $\frac{1}{f_{\text{air}}} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, and the factor $(n - 1)$ is always positive. But if the liquid has an index of refraction greater than that of the lens material, then $n/n_{\text{liq}} < 1$, so the factor $(n/n_{\text{liq}} - 1)$ is negative. This means that f changes sign from what it was in air. In other words, submerging a converging lens in a liquid can turn it into a diverging lens, and vice versa!

34.93. IDENTIFY: The spherical mirror forms an image of the object. It forms another image when the image of the plane mirror serves as an object.

SET UP: For the convex mirror $f = -24.0$ cm. The image formed by the plane mirror is 10.0 cm to the right of the plane mirror, so is $20.0\text{ cm} + 10.0\text{ cm} = 30.0\text{ cm}$ from the vertex of the spherical mirror.

EXECUTE: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{10.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}} \Rightarrow s'=-7.06$ cm, and the image height is

$y'=-\frac{s'}{s}y=-\frac{-7.06}{10.0}(0.250\text{ cm})=0.177\text{ cm}.$

The image of the object formed by the plane mirror is located 30.0 cm from the vertex of the spherical mirror. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{30.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}} \Rightarrow s'=-13.3$ cm and the image height is

$y'=-\frac{s'}{s}y=-\frac{-13.3}{30.0}(0.250\text{ cm})=0.111\text{ cm}.$

EVALUATE: Other images are formed by additional reflections from the two mirrors.

34.94. **IDENTIFY:** The smallest image we can resolve occurs when the image is the size of a retinal cell.

SET UP: $m=-\frac{s'}{s}=\frac{y'}{y}$. $s'=2.50$ cm.

$|y'|=5.0\ \mu\text{m}$. The angle subtended (in radians) is height divided by distance from the eye.

EXECUTE: **(a)** $m=-\frac{s'}{s}=-\frac{2.50\text{ cm}}{25\text{ cm}}=-0.10$. $y=\left|\frac{y'}{m}\right|=\frac{5.0\ \mu\text{m}}{0.10}=50\ \mu\text{m}.$

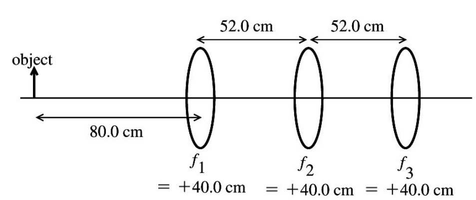
(b) $\theta=\frac{y}{s}=\frac{50\ \mu\text{m}}{25\text{ cm}}=\frac{50\times10^{-6}\text{ m}}{25\times10^{-2}\text{ m}}=2.0\times10^{-4}\text{ rad}=0.0115°=0.69\text{ min}$. This is only a bit smaller than the typical experimental value of 1.0 min.

EVALUATE: The angle subtended by the object equals the angular size of the image,

$\frac{|y'|}{s'}=\frac{5.0\times10^{-6}\text{ m}}{2.50\times10^{-2}\text{ m}}=2.0\times10^{-4}\text{ rad}.$

34.95. **IDENTIFY:** Apply the thin-lens equation to calculate the image distance for each lens. The image formed by the first lens serves as the object for the second lens, and the image formed by the second lens serves as the object for the third lens.

SET UP: The positions of the object and lenses are shown in Figure 34.95.



$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}.$

$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}.$

$s'=\frac{sf}{s-f}.$

Figure 34.95

EXECUTE: Lens #1:

$s=+80.0$ cm; $f=+40.0$ cm.

$s'=\frac{sf}{s-f}=\frac{(+80.0\text{ cm})(+40.0\text{ cm})}{+80.0\text{ cm}-40.0\text{ cm}}=+80.0\text{ cm}.$

The image formed by the first lens is 80.0 cm to the right of the first lens, so it is $80.0\text{ cm}-52.0\text{ cm}=28.0\text{ cm}$ to the right of the second lens.

Lens #2:

$s=-28.0$ cm; $f=+40.0$ cm.

$s'=\frac{sf}{s-f}=\frac{(-28.0\text{ cm})(+40.0\text{ cm})}{-28.0\text{ cm}-40.0\text{ cm}}=+16.47\text{ cm}.$

The image formed by the second lens is 16.47 cm to the right of the second lens, so it is $52.0\text{ cm}-16.47\text{ cm}=35.53\text{ cm}$ to the left of the third lens.

Lens #3:
$s = +35.53\text{ cm}; f = +40.0\text{ cm}.$

$$s' = \frac{sf}{s-f} = \frac{(+35.53\text{ cm})(+40.0\text{ cm})}{+35.53\text{ cm} - 40.0\text{ cm}} = -318\text{ cm}.$$

The final image is 318 cm to the left of the third lens, so it is $318\text{ cm} - 52\text{ cm} - 52\text{ cm} - 80\text{ cm} = 134\text{ cm}$ to the left of the object.

EVALUATE: We used the separation between the lenses and the sign conventions for s and s' to determine the object distances for the second and third lenses. The final image is virtual since the final s' is negative.

34.96. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and calculate s' for each s.

SET UP: $f = 90$ mm.

EXECUTE: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{1300\text{ mm}} + \frac{1}{s'} = \frac{1}{90\text{ mm}} \Rightarrow s' = 96.7\text{ mm}.$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{6500\text{ mm}} + \frac{1}{s'} = \frac{1}{90\text{ mm}} \Rightarrow s' = 91.3\text{ mm}.$

$\Rightarrow \Delta s' = 96.7\text{ mm} - 91.3\text{ mm} = 5.4\text{ mm}$ toward the sensor.

EVALUATE: $s' = \frac{sf}{s-f}$. For $f > 0$ and $s > f$, s' decreases as s increases.

34.97. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. The near point is at infinity, so that is where the image must be formed for any objects that are close.

SET UP: The power in diopters equals $\frac{1}{f}$, with f in meters.

EXECUTE: $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{24\text{ cm}} + \frac{1}{-\infty} = \frac{1}{0.24\text{ m}} = 4.17$ diopters.

EVALUATE: To focus on closer objects, the power must be increased.

34.98. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: $n_a = 1.00$, $n_b = 1.40$.

EXECUTE: $\frac{1}{36.0\text{ cm}} + \frac{1.40}{s'} = \frac{0.40}{0.75\text{ cm}} \Rightarrow s' = 2.77\text{ cm}.$

EVALUATE: This distance is greater than for the normal eye, which has a cornea vertex to retina distance of about 2.6 cm.

34.99. IDENTIFY and **SET UP:** The person's eye cannot focus on anything closer than 85.0 cm. The problem asks us to find the location of an object such that his old lenses produce a virtual image 85.0 cm from his eye. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $P(\text{in diopters}) = 1/f\,(\text{in m})$.

EXECUTE: **(a)** $\frac{1}{f} = 2.25$ diopters so $f = 44.4$ cm. The image is 85.0 cm from his eye so is 83.0 cm from the eyeglass lens. Solving $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ for s gives $s = \frac{s'f}{s'-f} = \frac{(-83.0\text{ cm})(44.4\text{ cm})}{-83.0\text{ cm} - 44.4\text{ cm}} = +28.9\text{ cm}.$ The object is 28.9 cm from the eyeglasses so is 30.9 cm from his eyes.

(b) Now $s' = -85.0$ cm. $s = \frac{s'f}{s'-f} = \frac{(-85.0\text{ cm})(44.4\text{ cm})}{-85.0\text{ cm} - 44.4\text{ cm}} = +29.2\text{ cm}.$

EVALUATE: The old glasses allow him to focus on objects as close as about 30 cm from his eyes. This is much better than a closest distance of 85 cm with no glasses, but his current glasses probably allow him to focus as close as 25 cm.

34.100. IDENTIFY: For u and u' as defined in Figure P34.100 in the textbook, $M = \frac{u'}{u}$.

SET UP: f_2 is negative. From Figure P34.100 in the textbook, the length of the telescope is $f_1 + f_2$, since f_2 is negative.

EXECUTE: **(a)** From the figure, $u = \frac{y}{f_1}$ and $u' = \frac{y}{|f_2|} = -\frac{y}{f_2}$. The angular magnification is $M = \frac{u'}{u} = -\frac{f_1}{f_2}$.

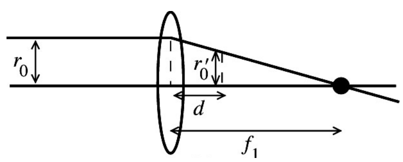
(b) $M = -\frac{f_1}{f_2} \Rightarrow f_2 = -\frac{f_1}{M} = -\frac{95.0\text{ cm}}{6.33} = -15.0\text{ cm}$.

(c) The length of the telescope is $95.0\text{ cm} - 15.0\text{ cm} = 80.0\text{ cm}$, compared to the length of 110 cm for the telescope in Exercise 34.61.

EVALUATE: An advantage of this construction is that the telescope is somewhat shorter.

34.101. IDENTIFY: Use similar triangles in Figure P34.101 in the textbook and $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to derive the expressions called for in the problem.

(a) SET UP: The effect of the converging lens on the ray bundle is sketched in Figure 34.101a.



EXECUTE: From similar triangles in Figure 34.101a, $\frac{r_0}{f_1} = \frac{r_0'}{f_1 - d}$.

Figure 34.101a

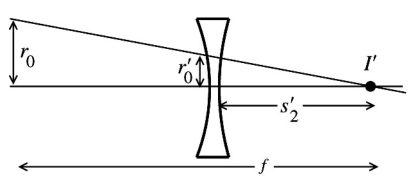
Thus $r_0' = \left(\frac{f_1 - d}{f_1}\right) r_0$, as was to be shown.

(b) SET UP: The image at the focal point of the first lens, a distance f_1 to the right of the first lens, serves as the object for the second lens. The image is a distance $f_1 - d$ to the right of the second lens, so $s_2 = -(f_1 - d) = d - f_1$.

EXECUTE: $s_2' = \frac{s_2 f_2}{s_2 - f_2} = \frac{(d - f_1) f_2}{d - f_1 - f_2}$.

$f_2 < 0$ so $|f_2| = -f_2$ and $s_2' = \frac{(f_1 - d)|f_2|}{|f_2| - f_1 + d}$, as was to be shown.

(c) SET UP: The effect of the diverging lens on the ray bundle is sketched in Figure 34.101b.



EXECUTE: From similar triangles in the sketch, $\frac{r_0}{f} = \frac{r_0'}{s_2'}$.

Thus $\frac{r_0}{r_0'} = \frac{f}{s_2'}$.

Figure 34.101b

From the results of part (a), $\frac{r_0}{r_0'} = \frac{f_1}{f_1 - d}$. Combining the two results gives $\frac{f_1}{f_1 - d} = \frac{f}{s_2'}$.

$f = s_2'\left(\frac{f_1}{f_1 - d}\right) = \frac{(f_1 - d)|f_2| f_1}{(|f_2| - f_1 + d)(f_1 - d)} = \frac{f_1 |f_2|}{|f_2| - f_1 + d}$, as was to be shown.

(d) SET UP: Put the numerical values into the expression derived in part (c).

EXECUTE: $f = \dfrac{f_1|f_2|}{|f_2| - f_1 + d}.$

$f_1 = 12.0\text{ cm},\ |f_2| = 18.0\text{ cm, so } f = \dfrac{216\text{ cm}^2}{6.0\text{ cm} + d}.$

$d = 0$ gives $f = 36.0$ cm; maximum f.

$d = 4.0$ cm gives $f = 21.6$ cm; minimum f.

$f = 30.0$ cm says $30.0\text{ cm} = \dfrac{216\text{ cm}^2}{6.0\text{ cm} + d}.$

$6.0\text{ cm} + d = 7.2$ cm and $d = 1.2$ cm.

EVALUATE: Changing d produces a range of effective focal lengths. The effective focal length can be both smaller and larger than $f_1 + |f_2|$.

34.102. IDENTIFY and **SET UP:** The formulas $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = -\dfrac{s'}{s}$ both apply for the mirror.

EXECUTE: **(a)** Combining the two formula above and eliminating s' gives $s = f - f\left(\dfrac{1}{m}\right)$. Therefore a graph of s versus $1/m$ should be a straight line having a slope equal to $-f$.

(b) Using the points (–1.8, 70 cm) and (–0.2, 30 cm) on the graph, we calculate the slope to be

slope = $\dfrac{30\text{ cm} - 70\text{ cm}}{-0.2 + 1.8} = -25\text{ cm} = -f$, so $f = 25$ cm.

(c) The image is inverted, so the magnification is negative. The image is twice as high as the object, so the magnification has magnitude 2. Combining these conditions tells us that $m = -2$, so $1/m = -1/2$. Using our equation, we have $s = 25\text{ cm} - (25\text{ cm})(-1/2) = 37.5$ cm.

(d) Since m is negative, we can write our formula for s as $s = f + f\left(\dfrac{1}{|m|}\right)$. To increase the size of the image, we must increase the magnitude of the magnification, which means we must decrease $1/|m|$. To do this, we must make s smaller, so we must move the object *closer* to the mirror. If we want m to be –3, our equation for s gives us $s = 25\text{ cm} - 24\text{ cm }(-1/3) = 33.3$ cm. This result agrees with our reasoning that we must move the object closer to the mirror.

(e) As $s \to 25$ cm, the object is approaching the focal point of the mirror, so $s' \to \infty$. Therefore $m = -\dfrac{s'}{s} \to \infty$, so $1/m \to 0$.

(f) When $s < 25$ cm and $m > 0$, the image distance is negative, so the image is virtual and therefore cannot be seen on a screen. Only real images can be focused on a screen.

EVALUATE: According to our equation $s = f - f\left(\dfrac{1}{m}\right)$ in (a), as $1/m \to 0$, $s \to f$. By extending our graph downward and to the left, we see that s does approach 25 cm as $1/m$ approaches zero, so 25 cm should be the focal length. This agrees with our result in (b).

34.103. IDENTIFY: The thin-lens formula applies. The converging lens forms a real image on its right side. This image acts as the object for the diverging lens. The image formed by the converging lens is on the right side of the diverging lens, so this image acts as a *virtual object* for the diverging lens and its object distance is *negative*.

SET UP: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ in each case. $m = -\dfrac{s'}{s}$. The total magnification of two lenses is $m_{tot} = m_1 m_2$.

EXECUTE: **(a)** For the first trial on the diverging lens, we have
$s = 20.0\text{ cm} - 29.7\text{ cm} = -9.7\text{ cm}$ and $s' = 42.8\text{ cm} - 20.0\text{ cm} = 22.8\text{ cm}$.

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'} = \frac{1}{-9.7\text{ cm}}+\frac{1}{22.8\text{ cm}} \quad \rightarrow \quad f = -16.88\text{ cm}.$$

For the second trial on the diverging lens, we have
$s = 25.0\text{ cm} - 29.7\text{ cm} = -4.7\text{ cm}$ and $s' = 31.6\text{ cm} - 25.0\text{ cm} = 6.6\text{ cm}$.

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'} = \frac{1}{-4.7\text{ cm}}+\frac{1}{6.6\text{ cm}} \quad \rightarrow \quad f = -16.33\text{ cm}.$$

Taking the average of the focal lengths, we get $f_{av} = (-16.88\text{ cm} - 16.33\text{ cm})/2 = -16.6\text{ cm}$.

(b) The total magnification is $m_{tot} = m_1 m_2$. The converging lens does not move during the two trials, so m_1 is the same for both of them. But m_2 does change.

At 20.0 cm: $m = -\dfrac{s'}{s} = -(22.8\text{ cm})/(-9.7\text{ cm}) = +2.35$.

At 25.0 cm: $m = -\dfrac{s'}{s} = -(6.6\text{ cm})/(-4.7\text{ cm}) = +1.40$.

The magnification is greater when the lens is at 20.0 cm.

EVALUATE: This is a case where a diverging lens can form a real image, but only when it is used in conjunction with one or more other lenses.

34.104. IDENTIFY and **SET UP:** We measure s and s'. The equation $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}$ applies. In this case, $n_a = 1.00$ for air and $n_b = n$.

EXECUTE: **(a)** In this case, the equation $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}$ becomes $\dfrac{1}{s}+\dfrac{n}{s'}=\dfrac{n-1}{R}$. Solving for $1/s'$ gives $\dfrac{1}{s'}=\dfrac{n-1}{nR}-\dfrac{1}{n}\cdot\dfrac{1}{s}$. Therefore a graph of $1/s'$ versus $1/s$ should be a straight line having slope equal to $-1/n$ and a y-intercept equal to $(n-1)/nR$. Figure 34.104 shows the graph of the data from the table in the problem.

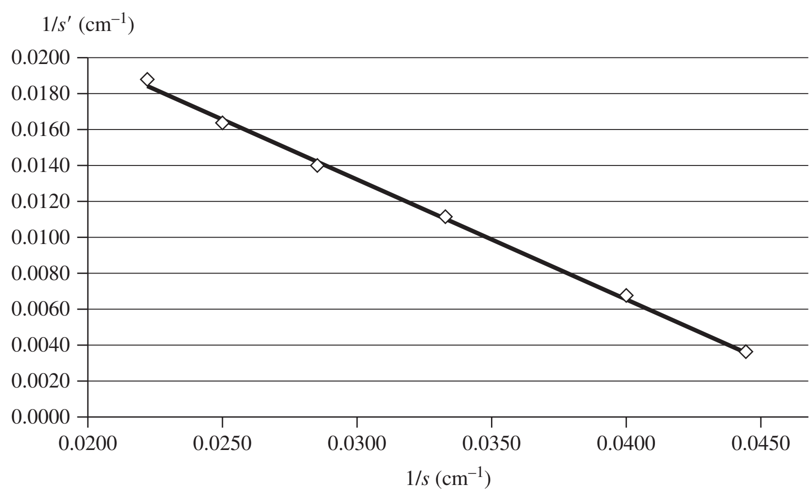


Figure 34.104

(b) The equation of the best-fit graph of the data is $\dfrac{1}{s'} = -(0.6666)\dfrac{1}{s} + 0.0333\text{ cm}^{-1}$. From this we have

slope $= -1/n \quad \rightarrow \quad n = -1/(\text{slope}) = -1/(-0.6666) = 1.50$.

Using the y-intercept, we have y-intercept = $(n-1)/nR$. Solving for R gives $R = (n-1)/[n(y\text{-intercept})] = (1.50-1)/[(1.50)(0.03333\text{ cm}^{-1})] = 10.0$ cm.

(c) Using our equation from the graph $\frac{1}{s'} = -(0.6666)\frac{1}{s} + 0.0333\text{ cm}^{-1}$, we have

$$\frac{1}{s'} = -(0.6666)\frac{1}{15.0\text{ cm}} + 0.0333\text{ cm}^{-1} \quad \rightarrow \quad s' = -90\text{ cm}.$$

The image is 90 cm in front of the glass and is virtual.

EVALUATE: An index of refraction of $n = 1.50$ for glass is very reasonable.

34.105. IDENTIFY: The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.

SET UP: For a real image $s' > 0$ and the distance between the object and the image is $D = s + s'$. For a real image must have $s > f$.

EXECUTE: **(a)** $D = s + s'$ but $s' = \frac{sf}{s-f} \Rightarrow D = s + \frac{sf}{s-f} = \frac{s^2}{s-f}$.

$\frac{dD}{ds} = \frac{d}{ds}\left(\frac{s^2}{s-f}\right) = \frac{2s}{s-f} - \frac{s^2}{(s-f)^2} = \frac{s^2-2sf}{(s-f)^2} = 0.$ $s^2 - 2sf = 0.$ $s(s-2f) = 0.$ $s = 2f$ is the solution for which $s > f$. For $s = 2f$, $s' = 2f$. Therefore, the minimum separation is $2f + 2f = 4f$.

(b) A graph of D/f versus s/f is sketched in Figure 34.105. Note that the minimum does occur for $D = 4f$.

EVALUATE: If, for example, $s = 3f/2$, then $s' = 3f$ and $D = s + s' = 4.5f$, greater than the minimum value.

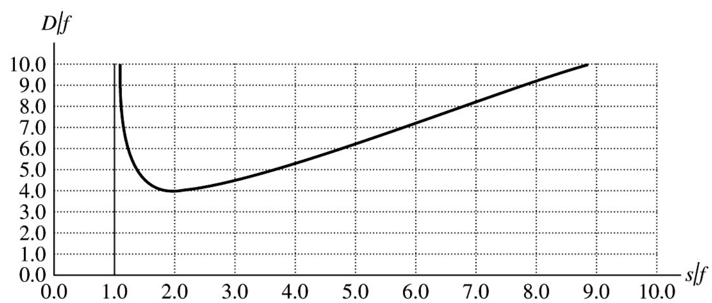


Figure 34.105

34.106. IDENTIFY: Use $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s' (the distance of each point from the lens), for points A, B, and C.

SET UP: The object and lens are shown in Figure 34.106a.

EXECUTE: **(a)** For point C: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{45.0\text{ cm}} + \frac{1}{s'} = \frac{1}{20.0\text{ cm}} \Rightarrow s' = 36.0$ cm.

$y' = -\frac{s'}{s}y = -\frac{36.0}{45.0}(15.0\text{ cm}) = -12.0$ cm, so the image of point C is 36.0 cm to the right of the lens, and 12.0 cm below the axis.

For point A: $s = 45.0\text{ cm} + (8.00\text{ cm})(\cos 45°) = 50.7$ cm.

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{50.7\text{ cm}} + \frac{1}{s'} = \frac{1}{20.0\text{ cm}} \Rightarrow s' = 33.0$ cm.

$y' = -\frac{s'}{s}y = -\frac{33.0}{45.0}[15.0\text{ cm} - (8.00\text{ cm})(\sin 45°)] = -6.10$ cm, so the image of point A is 33.0 cm to the right of the lens, and 6.10 cm below the axis.

For point B: $s = 45.0\text{ cm} - (8.00\text{ cm})(\cos 45°) = 39.3\text{ cm}.$

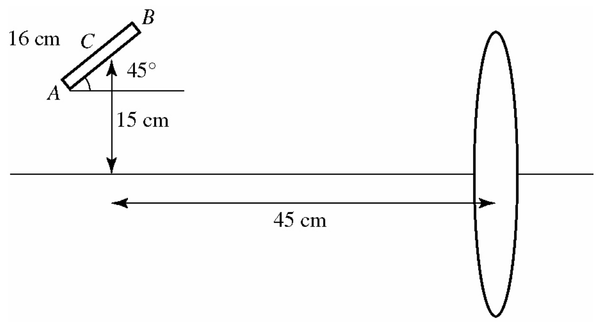
$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{39.3\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s' = 40.7\text{ cm}.$

$y' = -\frac{s'}{s}y = -\frac{40.7}{39.3}[15.0\text{ cm} + (8.00\text{ cm})(\sin 45°)] = -21.4\text{ cm},$ so the image of point B is 40.7 cm to the right of the lens, and 21.4 cm below the axis. The image is shown in Figure 34.106b.

(b) The length of the pencil is the distance from point A to B:

$L = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(33.0\text{ cm} - 40.7\text{ cm})^2 + (6.10\text{ cm} - 21.4\text{ cm})^2} = 17.1\text{ cm}$

EVALUATE: The image is below the optic axis and is larger than the object.



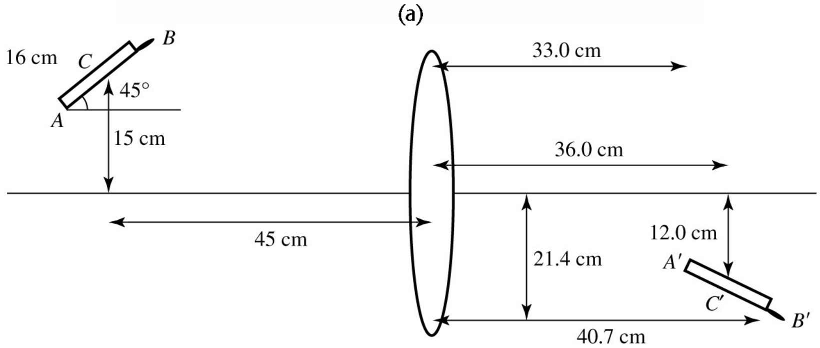


Figure 34.106

34.107. IDENTIFY: Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b - n_a}{R}$ to refraction at the cornea to find where the object for the cornea must be in order for the image to be at the retina. Then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate f so that the lens produces an image of a distant object at this point.

SET UP: For refraction at the cornea, $n_a = 1.333$ and $n_b = 1.40.$ The distance from the cornea to the retina in this model of the eye is 2.60 cm. From Problem 34.50, $R = 0.710\text{ cm}.$

EXECUTE: **(a)** People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.

(b) When introducing glasses, let's first consider what happens at the eye:

$\frac{n_a}{s_2}+\frac{n_b}{s_2'}=\frac{n_b - n_a}{R} \Rightarrow \frac{1.333}{s_2}+\frac{1.40}{2.6\text{ cm}}=\frac{0.067}{0.71\text{ cm}} \Rightarrow s_2 = -3.00\text{ cm}.$ That is, the object for the cornea must be 3.00 cm behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, so

$s_1' = 2.00\text{ cm} + |s_2| = 5.00\text{ cm}$. $\dfrac{1}{s_1} + \dfrac{1}{s_1'} = \dfrac{1}{f_1'}$ gives $\dfrac{1}{\infty} + \dfrac{1}{5.00\text{ cm}} = \dfrac{1}{f_1'}$ and $f_1' = 5.00\text{ cm}$. This is the focal length in water, but to get it in air, we use the formula from Problem 34.92:

$$f_1 = f_1'\left[\frac{n - n_{\text{liq}}}{n_{\text{liq}}(n-1)}\right] = (5.00\text{ cm})\left[\frac{1.62 - 1.333}{1.333(1.62-1)}\right] = 1.74\text{ cm}.$$

EVALUATE: A converging lens is needed.

34.108. IDENTIFY and SET UP: Apply the thin-lens formula $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to the eye and calculate s' in both cases. The focal length stays the same.

EXECUTE: At 10 cm: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{10\text{ cm}} + \dfrac{1}{0.8\text{ cm}}$. At 15 cm: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{15\text{ cm}} + \dfrac{1}{s'}$. Equate the two expressions for $1/f$ and solve for s'.

$$\frac{1}{15\text{ cm}} + \frac{1}{s'} = \frac{1}{10\text{ cm}} + \frac{1}{0.8\text{ cm}} \quad \rightarrow s' = 0.779\text{ cm}.$$

The distance the lens must move is 0.8 cm – 0.779 cm = 0.021 cm ≈ 0.02 cm, which is choice (a).

EVALUATE: This is a very small distance to move, but the eye of a frog is also very small, so the result seems plausible.

34.109. IDENTIFY and SET UP: Apply the thin-lens formula $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to the eye. The lens power in diopters is $1/f$ (in m). For the corrected eye, the image is at infinity, and it would take –6.0 D to correct the frog's vision so it could see at infinity.

EXECUTE: Using $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$ gives $-6.0\text{ m}^{-1} = 1/s + 1/\infty = 1/s$, so $s = 0.17$ m = 17 cm, which makes choice (d) the correct one.

EVALUATE: It is reasonable for a fog to see clearly up to only 17 cm since its food consists of insects, which must be fairly close to get caught.

34.110. IDENTIFY and SET UP: Apply Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, at the cornea.

EXECUTE: From $n_a \sin\theta_a = n_b \sin\theta_b$, we have $\sin\theta_b = (n_b/n_a)\sin\theta_a$. When n_a and n_b are closer to each other, θ_b is closer to θ_a, so less refraction occurs at the cornea. This will be the case when a frog goes under water, since the refractive index of water (1.33) is closer to that of the cornea than is the refractive index of air, which is 1.00. Therefore choice (b) is correct.

EVALUATE: Frogs must adapt when they go under water. They are probably better hunters there and are better able to spot predators.

34.111. IDENTIFY and SET UP: Apply the thin-lens formula $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to the eye. The lens power in diopters is D = $1/f$ (in m).

EXECUTE: Since $D = 1/f$ (in m), the larger |D| the smaller s. So the frog with the –15-D lens could focus at a shorter distance than the frog with the –9-D lens, which is choice (b).

EVALUATE: The lens would not move the same distance with the –15-D lens as with the 9-D lens.