

## EQUILIBRIUM AND ELASTICITY

- 11.1. IDENTIFY:** Use  $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$  to calculate  $x_{\text{cm}}$ . The center of gravity of the bar is at its

center and it can be treated as a point mass at that point.

**SET UP:** Use coordinates with the origin at the left end of the bar and the  $+x$ -axis along the bar.

$m_1 = 0.120 \text{ kg}$ ,  $m_2 = 0.055 \text{ kg}$ ,  $m_3 = 0.110 \text{ kg}$ .

**EXECUTE:**  $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(0.120 \text{ kg})(0.250 \text{ m}) + 0 + (0.110 \text{ kg})(0.500 \text{ m})}{0.120 \text{ kg} + 0.055 \text{ kg} + 0.110 \text{ kg}} = 0.298 \text{ m}$ . The

fulcrum should be placed 29.8 cm to the right of the left-hand end.

**EVALUATE:** The mass at the right-hand end is greater than the mass at the left-hand end. So the center of gravity is to the right of the center of the bar.

- 11.2. IDENTIFY:** Use  $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$  to calculate  $x_{\text{cm}}$  of the composite object.

**SET UP:** Use coordinates where the origin is at the original center of gravity of the object and  $+x$  is to the right. With the 1.50 kg mass added,  $x_{\text{cm}} = -2.20 \text{ cm}$ ,  $m_1 = 5.00 \text{ kg}$  and  $m_2 = 1.50 \text{ kg}$ .  $x_1 = 0$ .

**EXECUTE:**  $x_{\text{cm}} = \frac{m_2x_2}{m_1 + m_2}$ .  $x_2 = \left( \frac{m_1 + m_2}{m_2} \right) x_{\text{cm}} = \left( \frac{5.00 \text{ kg} + 1.50 \text{ kg}}{1.50 \text{ kg}} \right) (-2.20 \text{ cm}) = -9.53 \text{ cm}$ .

The additional mass should be attached 9.53 cm to the left of the original center of gravity.

**EVALUATE:** The new center of gravity is somewhere between the added mass and the original center of gravity.

- 11.3. IDENTIFY:** Treat the rod and clamp as point masses. The center of gravity of the rod is at its midpoint, and we know the location of the center of gravity of the rod-clamp system.

**SET UP:**  $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$ .

**EXECUTE:**  $1.20 \text{ m} = \frac{(1.80 \text{ kg})(1.00 \text{ m}) + (2.40 \text{ kg})x_2}{1.80 \text{ kg} + 2.40 \text{ kg}}$ .

$x_2 = \frac{(1.20 \text{ m})(1.80 \text{ kg} + 2.40 \text{ kg}) - (1.80 \text{ kg})(1.00 \text{ m})}{2.40 \text{ kg}} = 1.35 \text{ m}$

**EVALUATE:** The clamp is to the right of the center of gravity of the system, so the center of gravity of the system lies between that of the rod and the clamp, which is reasonable.

- 11.4. IDENTIFY:** Apply the first and second conditions for equilibrium to the trap door.

**SET UP:** For  $\sum \tau_z = 0$  take the axis at the hinge. Then the torque due to the applied force must balance the torque due to the weight of the door.

**EXECUTE:** (a) The force is applied at the center of gravity, so the applied force must have the same magnitude as the weight of the door, or 300 N. In this case the hinge exerts no force.

(b) With respect to the hinges, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or 150 N.

The hinges supply an upward force of  $300\text{ N} - 150\text{ N} = 150\text{ N}$ .

**EVALUATE:** Less force must be applied when it is applied farther from the hinges.

**11.5. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the ladder.

**SET UP:** Take the axis to be at point  $A$ . The free-body diagram for the ladder is given in Figure 11.5. The torque due to  $F$  must balance the torque due to the weight of the ladder.

**EXECUTE:**  $F(8.0\text{ m})\sin 40^\circ = (3400\text{ N})(10.0\text{ m})$ , so  $F = 6.6\text{ kN}$ .

**EVALUATE:** The force required is greater than the weight of the ladder, because the moment arm for  $F$  is less than the moment arm for  $w$ .

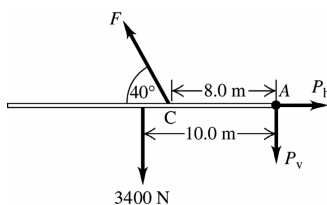


Figure 11.5

**11.6. IDENTIFY:** Apply the first and second conditions of equilibrium to the board.

**SET UP:** The free-body diagram for the board is given in Figure 11.6. Since the board is uniform its center of gravity is  $1.50\text{ m}$  from each end. Apply  $\sum F_y = 0$ , with  $+y$  upward. Apply  $\sum \tau_z = 0$  with the axis at the end where the first person applies a force and with counterclockwise torques positive.

**EXECUTE:**  $\sum F_y = 0$  gives  $F_1 + F_2 - w = 0$  and  $F_2 = w - F_1 = 160\text{ N} - 60\text{ N} = 100\text{ N}$ .  $\sum \tau_z = 0$  gives

$$F_2 x - w(1.50\text{ m}) = 0 \text{ and } x = \left(\frac{w}{F_2}\right)(1.50\text{ m}) = \left(\frac{160\text{ N}}{100\text{ N}}\right)(1.50\text{ m}) = 2.40\text{ m. The other person lifts with a}$$

force of  $100\text{ N}$  at a point  $2.40\text{ m}$  from the end where the other person lifts.

**EVALUATE:** By considering the axis at the center of gravity we can see that a larger force is applied by the person who pushes closer to the center of gravity.

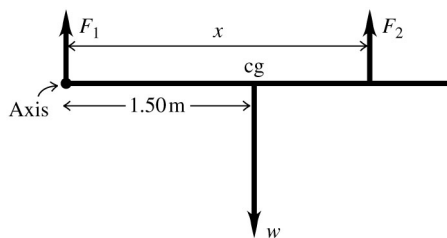


Figure 11.6

**11.7. IDENTIFY:** Apply  $\sum F_y = 0$  and  $\sum \tau_z = 0$  to the board.

**SET UP:** Let  $+y$  be upward. Let  $x$  be the distance of the center of gravity of the motor from the end of the board where the  $400\text{ N}$  force is applied.

**EXECUTE:** (a) If the board is taken to be massless, the weight of the motor is the sum of the applied forces,  $1000\text{ N}$ . The motor is a distance  $\frac{(2.00\text{ m})(600\text{ N})}{(1000\text{ N})} = 1.20\text{ m}$  from the end where the  $400\text{ N}$  force is

applied, and so is  $0.800\text{ m}$  from the end where the  $600\text{ N}$  force is applied.

(b) The weight of the motor is  $400\text{ N} + 600\text{ N} - 200\text{ N} = 800\text{ N}$ . Applying  $\sum \tau_z = 0$  with the axis at the end of the board where the  $400\text{ N}$  acts gives  $(600\text{ N})(2.00\text{ m}) = (200\text{ N})(1.00\text{ m}) + (800\text{ N})x$  and  $x = 1.25\text{ m}$ .

The center of gravity of the motor is  $0.75\text{ m}$  from the end of the board where the  $600\text{ N}$  force is applied.

**EVALUATE:** The motor is closest to the end of the board where the larger force is applied.

**11.8. IDENTIFY:** Apply the first and second conditions of equilibrium to the shelf.

**SET UP:** The free-body diagram for the shelf is given in Figure 11.8. Take the axis at the left-hand end of the shelf and let counterclockwise torque be positive. The center of gravity of the uniform shelf is at its center.

**EXECUTE:** (a)  $\sum \tau_z = 0$  gives  $-w_t(0.200 \text{ m}) - w_s(0.300 \text{ m}) + T_2(0.400 \text{ m}) = 0$ .

$$T_2 = \frac{(25.0 \text{ N})(0.200 \text{ m}) + (50.0 \text{ N})(0.300 \text{ m})}{0.400 \text{ m}} = 50.0 \text{ N}$$

$\sum F_y = 0$  gives  $T_1 + T_2 - w_t - w_s = 0$  and  $T_1 = 25.0 \text{ N}$ . The tension in the left-hand wire is 25.0 N and the tension in the right-hand wire is 50.0 N.

**EVALUATE:** We can verify that  $\sum \tau_z = 0$  is zero for any axis, for example for an axis at the right-hand end of the shelf.

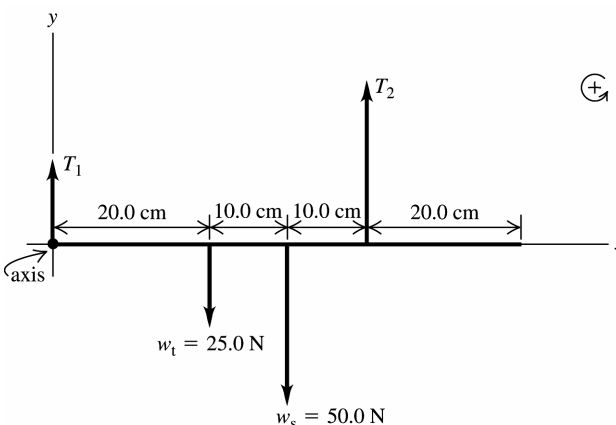


Figure 11.8

**11.9. IDENTIFY:** Apply the conditions for equilibrium to the bar. Set each tension equal to its maximum value.

**SET UP:** Let cable A be at the left-hand end. Take the axis to be at the left-hand end of the bar and  $x$  be the distance of the weight  $w$  from this end. The free-body diagram for the bar is given in Figure 11.9.

**EXECUTE:** (a)  $\sum F_y = 0$  gives  $T_A + T_B - w - w_{\text{bar}} = 0$  and

$$w = T_A + T_B - w_{\text{bar}} = 500.0 \text{ N} + 400.0 \text{ N} - 350.0 \text{ N} = 550 \text{ N}.$$

(b)  $\sum \tau_z = 0$  gives  $T_B(1.50 \text{ m}) - wx - w_{\text{bar}}(0.750 \text{ m}) = 0$ .

$$x = \frac{T_B(1.50 \text{ m}) - w_{\text{bar}}(0.750 \text{ m})}{w} = \frac{(400.0 \text{ N})(1.50 \text{ m}) - (350 \text{ N})(0.750 \text{ m})}{550 \text{ N}} = 0.614 \text{ m}.$$

The weight should be placed 0.614 m from the left-hand end of the bar (cable A).

**EVALUATE:** If the weight is moved to the left,  $T_A$  exceeds 500.0 N and if it is moved to the right  $T_B$  exceeds 400.0 N.

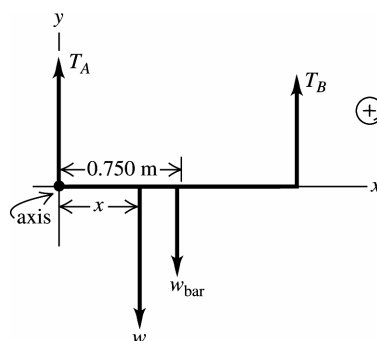


Figure 11.9

**11.10. IDENTIFY:** Apply the first and second conditions for equilibrium to the ladder.

**SET UP:** Let  $n_2$  be the upward normal force exerted by the ground and let  $n_1$  be the horizontal normal force exerted by the wall. The maximum possible static friction force that can be exerted by the ground is  $\mu_s n_2$ .

**EXECUTE:** (a) Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force  $n_2$ . For the vertical forces to balance,  $n_2 = w_l + w_m = 160 \text{ N} + 740 \text{ N} = 900 \text{ N}$ , and the maximum frictional force is  $\mu_s n_2 = (0.40)(900 \text{ N}) = 360 \text{ N}$ .

(b) Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground,

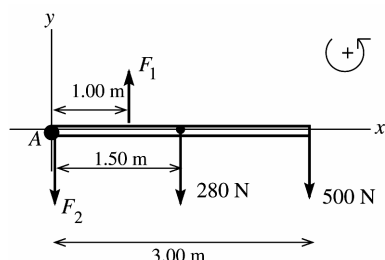
$(4.0 \text{ m})n_1 = (1.5 \text{ m})(160 \text{ N}) + (1.0 \text{ m})(3/5)(740 \text{ N}) = 684 \text{ N} \cdot \text{m}$ , so  $n_1 = 171.0 \text{ N}$ . This horizontal force must be balanced by the friction force, which must then be 170 N to two figures.

(c) Setting the friction force, and hence  $n_1$ , equal to the maximum of 360 N and solving for the distance  $x$  along the ladder,  $(4.0 \text{ m})(360 \text{ N}) = (1.50 \text{ m})(160 \text{ N}) + x(3/5)(740 \text{ N})$ , so  $x = 2.7 \text{ m}$ .

**EVALUATE:** The normal force exerted by the ground doesn't change as the man climbs up the ladder. But the normal force exerted by the wall and the friction force exerted by the ground both increase as he moves up the ladder.

**11.11. IDENTIFY:** The system of the person and diving board is at rest so the two conditions of equilibrium apply.

(a) **SET UP:** The free-body diagram for the diving board is given in Figure 11.11. Take the origin of coordinates at the left-hand end of the board (point A).



$\vec{F}_1$  is the force applied at the support point and  $\vec{F}_2$  is the force at the end that is held down.

Figure 11.11

**EXECUTE:**  $\sum \tau_A = 0$  gives  $+F_1(1.00 \text{ m}) - (500 \text{ N})(3.00 \text{ m}) - (280 \text{ N})(1.50 \text{ m}) = 0$

$$F_1 = \frac{(500 \text{ N})(3.00 \text{ m}) + (280 \text{ N})(1.50 \text{ m})}{1.00 \text{ m}} = 1920 \text{ N}$$

(b)  $\sum F_y = ma_y$

$$F_1 - F_2 - 280 \text{ N} - 500 \text{ N} = 0$$

$$F_2 = F_1 - 280 \text{ N} - 500 \text{ N} = 1920 \text{ N} - 280 \text{ N} - 500 \text{ N} = 1140 \text{ N}$$

**EVALUATE:** We can check our answers by calculating the net torque about some point and checking that  $\sum \tau_z = 0$  for that point also. Net torque about the right-hand end of the board:

$$(1140 \text{ N})(3.00 \text{ m}) + (280 \text{ N})(1.50 \text{ m}) - (1920 \text{ N})(2.00 \text{ m}) = 3420 \text{ N} \cdot \text{m} + 420 \text{ N} \cdot \text{m} - 3840 \text{ N} \cdot \text{m} = 0, \text{ which checks.}$$

**11.12. IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

**SET UP:** The boy exerts a downward force on the beam that is equal to his weight.

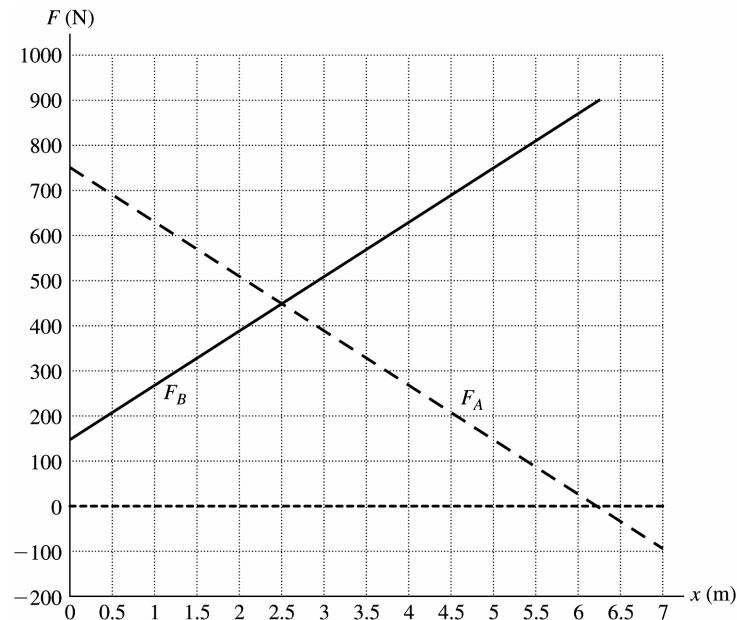
**EXECUTE:** (a) The graphs are given in Figure 11.12.

(b)  $x = 6.25 \text{ m}$  when  $F_A = 0$ , which is 1.25 m beyond point B.

(c) Take torques about the right end. When the beam is just balanced,  $F_A = 0$ , so  $F_B = 900 \text{ N}$ .

$$\text{The distance that point B must be from the right end is then } \frac{(300 \text{ N})(4.50 \text{ m})}{(900 \text{ N})} = 1.50 \text{ m.}$$

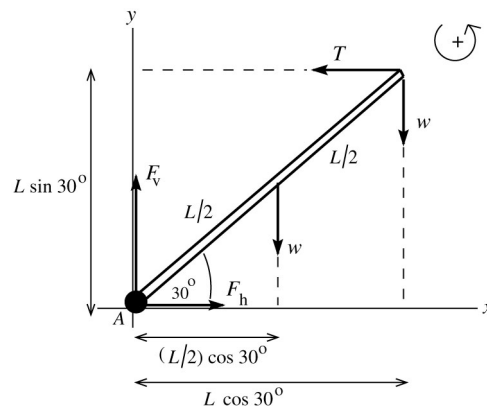
**EVALUATE:** When the beam is on the verge of tipping it starts to lift off the support  $A$  and the normal force  $F_A$  exerted by the support goes to zero.



**Figure 11.12**

**11.13. IDENTIFY:** Apply the first and second conditions of equilibrium to the strut.

**(a) SET UP:** The free-body diagram for the strut is given in Figure 11.13a. Take the origin of coordinates at the hinge (point A) and  $+y$  upward. Let  $F_h$  and  $F_v$  be the horizontal and vertical components of the force  $\vec{F}$  exerted on the strut by the pivot. The tension in the vertical cable is the weight  $w$  of the suspended object. The weight  $w$  of the strut can be taken to act at the center of the strut. Let  $L$  be the length of the strut.



**EXECUTE:**

$$\sum F_y = ma_y$$

$$F_v - w - w = 0$$

$$F_v = 2w$$

**Figure 11.13a**

Sum torques about point A. The pivot force has zero moment arm for this axis and so doesn't enter into the torque equation.

$$\tau_A = 0$$

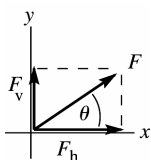
$$TL \sin 30.0^\circ - w((L/2) \cos 30.0^\circ) - w(L \cos 30.0^\circ) = 0$$

$$T \sin 30.0^\circ - (3w/2) \cos 30.0^\circ = 0$$

$$T = \frac{3w \cos 30.0^\circ}{2 \sin 30.0^\circ} = 2.60w$$

Then  $\Sigma F_x = ma_x$  implies  $T - F_h = 0$  and  $F_h = 2.60w$ .

We now have the components of  $\vec{F}$  so can find its magnitude and direction (Figure 11.13b).



$$F = \sqrt{F_h^2 + F_v^2}$$

$$F = \sqrt{(2.60w)^2 + (2.00w)^2}$$

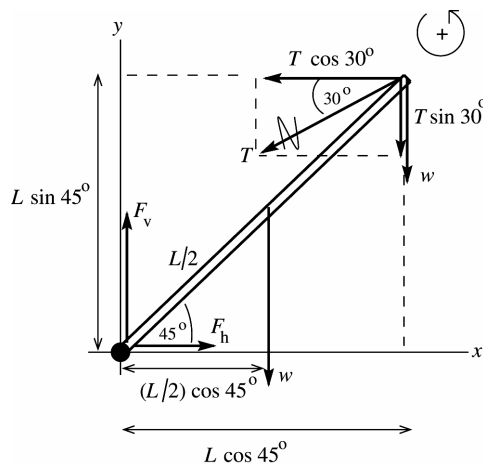
$$F = 3.28w$$

$$\tan \theta = \frac{F_v}{F_h} = \frac{2.00w}{2.60w}$$

$$\theta = 37.6^\circ$$

**Figure 11.13b**

**(b) SET UP:** The free-body diagram for the strut is given in Figure 11.13c.



**Figure 11.13c**

The tension  $T$  has been replaced by its  $x$  and  $y$  components. The torque due to  $T$  equals the sum of the torques of its components, and the latter are easier to calculate.

$$\text{EXECUTE: } \Sigma \tau_A = 0 + (T \cos 30.0^\circ)(L \sin 45.0^\circ) - (T \sin 30.0^\circ)(L \cos 45.0^\circ) - w[(L/2) \cos 45.0^\circ] - w(L \cos 45.0^\circ) = 0$$

The length  $L$  divides out of the equation. The equation can also be simplified by noting that  $\sin 45.0^\circ = \cos 45.0^\circ$ .

$$\text{Then } T(\cos 30.0^\circ - \sin 30.0^\circ) = 3w/2.$$

$$T = \frac{3w}{2(\cos 30.0^\circ - \sin 30.0^\circ)} = 4.10w$$

$$\Sigma F_x = ma_x$$

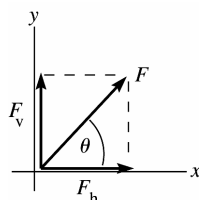
$$F_h - T \cos 30.0^\circ = 0$$

$$F_h = T \cos 30.0^\circ = (4.10w)(\cos 30.0^\circ) = 3.55w$$

$$\Sigma F_y = ma_y$$

$$F_v - w - w - T \sin 30.0^\circ = 0$$

$$F_v = 2w + (4.10w)\sin 30.0^\circ = 4.05w$$



From Figure 11.13d,

$$F = \sqrt{F_h^2 + F_v^2}$$

$$F = \sqrt{(3.55w)^2 + (4.05w)^2} = 5.39w$$

$$\tan \theta = \frac{F_v}{F_h} = \frac{4.05w}{3.55w}$$

$$\theta = 48.8^\circ$$

**Figure 11.13d**

**EVALUATE:** In each case the force exerted by the pivot does not act along the strut. Consider the net torque about the upper end of the strut. If the pivot force acted along the strut, it would have zero torque about this point. The two forces acting at this point also have zero torque and there would be one nonzero torque, due to the weight of the strut. The net torque about this point would then not be zero, violating the second condition of equilibrium.

**11.14. IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

**SET UP:** The free-body diagram for the beam is given in Figure 11.14.  $H_v$  and  $H_h$  are the vertical and horizontal components of the force exerted on the beam at the wall (by the hinge). Since the beam is uniform, its center of gravity is 2.00 m from each end. The angle  $\theta$  has  $\cos \theta = 0.800$  and  $\sin \theta = 0.600$ . The tension  $T$  has been replaced by its  $x$ - and  $y$ -components.

**EXECUTE: (a)**  $H_v$ ,  $H_h$  and  $T_x = T \cos \theta$  all produce zero torque.  $\sum \tau_z = 0$  gives

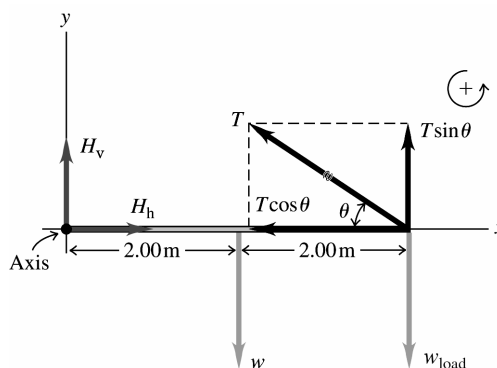
$$-w(2.00 \text{ m}) - w_{\text{load}}(4.00 \text{ m}) + T \sin \theta(4.00 \text{ m}) = 0 \text{ and}$$

$$T = \frac{(190 \text{ N})(2.00 \text{ m}) + (300 \text{ N})(4.00 \text{ m})}{(4.00 \text{ m})(0.600)} = 658.3 \text{ N, which rounds to } 658 \text{ N.}$$

**(b)**  $\sum F_x = 0$  gives  $H_h - T \cos \theta = 0$  and  $H_h = (658.3 \text{ N})(0.800) = 527 \text{ N}$ .  $\sum F_y = 0$  gives

$$H_v - w - w_{\text{load}} + T \sin \theta = 0 \text{ and } H_v = w + w_{\text{load}} - T \sin \theta = 190 \text{ N} + 300 \text{ N} - (658 \text{ N})(0.600) = 95 \text{ N.}$$

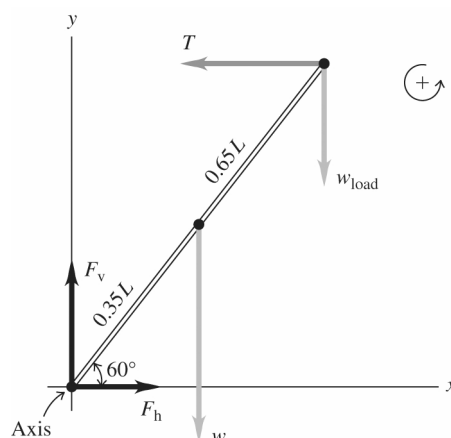
**EVALUATE:** For an axis at the right-hand end of the beam, only  $w$  and  $H_v$  produce torque. The torque due to  $w$  is counterclockwise so the torque due to  $H_v$  must be clockwise. To produce a clockwise torque,  $H_v$  must be upward, in agreement with our result from  $\sum F_y = 0$ .



**Figure 11.14**

**11.15. IDENTIFY:** The boom is at rest, so the forces and torques on it must each balance.

**SET UP:**  $\sum \tau = 0$ ,  $\sum F_x = 0$ ,  $\sum F_y = 0$ . The free-body is shown in Figure 11.15 (next page). Call  $L$  the length of the boom.

**Figure 11.15**

**EXECUTE:** (a)  $\sum \tau = 0$  gives  $T(L \sin 60.0^\circ) - w_{\text{load}}(L \cos 60.0^\circ) - w(0.35L \cos 60.0^\circ) = 0$  and

$$T = \frac{w_{\text{load}} \cos 60.0^\circ + w(0.35 \cos 60.0^\circ)}{\sin 60.0^\circ} = \frac{(5000 \text{ N}) \cos 60.0^\circ + (2600 \text{ N})(0.35 \cos 60.0^\circ)}{\sin 60.0^\circ} = 3.41 \times 10^3 \text{ N}.$$

(b)  $\sum F_x = 0$  gives  $F_h - T = 0$  and  $F_h = 3410 \text{ N}$ .

$\sum F_y = 0$  gives  $F_v - w - w_{\text{load}} = 0$  and  $F_v = 5000 \text{ N} + 2600 \text{ N} = 7600 \text{ N}$

**EVALUATE:** The bottom of the boom is the best point about which to take torques because only one unknown (the tension) appears in our equation. Using the top (or the center of mass) would give a torque equation with two (or three) unknowns.

**11.16. IDENTIFY:** Apply the conditions of equilibrium to the wheelbarrow plus its contents. The upward force applied by the person is 650 N.

**SET UP:** The free-body diagram for the wheelbarrow is given in Figure 11.16.  $F = 650 \text{ N}$ ,  $w_{\text{wb}} = 80.0 \text{ N}$  and  $w$  is the weight of the load placed in the wheelbarrow.

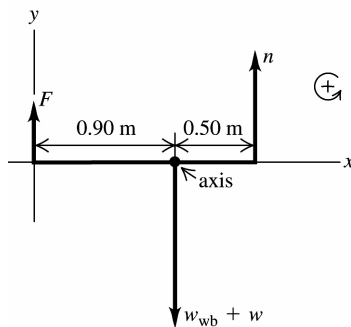
**EXECUTE:** (a)  $\sum \tau_z = 0$  with the axis at the center of gravity gives  $n(0.50 \text{ m}) - F(0.90 \text{ m}) = 0$  and

$$n = F \left( \frac{0.90 \text{ m}}{0.50 \text{ m}} \right) = 1170 \text{ N}. \quad \sum F_y = 0 \text{ gives } F + n - w_{\text{wb}} - w = 0 \text{ and}$$

$$w = F + n - w_{\text{wb}} = 650 \text{ N} + 1170 \text{ N} - 80.0 \text{ N} = 1740 \text{ N}.$$

(b) The extra force is applied by the ground pushing up on the wheel.

**EVALUATE:** You can verify that  $\sum \tau_z = 0$  for any axis, for example for an axis where the wheel contacts the ground.

**Figure 11.16**



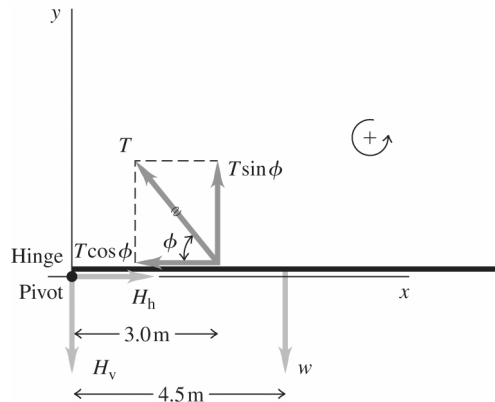
**11.17. IDENTIFY:** The beam is at rest so the forces and torques on it must each balance.

**SET UP:**  $\sum \tau = 0$ ,  $\sum F_x = 0$ ,  $\sum F_y = 0$ . The distance along the beam from the hinge to where the cable is

attached is 3.0 m. The angle  $\phi$  that the cable makes with the beam is given by  $\sin \phi = \frac{4.0 \text{ m}}{5.0 \text{ m}}$ , so

$\phi = 53.1^\circ$ . The center of gravity of the beam is 4.5 m from the hinge. Use coordinates with  $+y$  upward and  $+x$  to the right. Take the pivot at the hinge and let counterclockwise torque be positive. Express the hinge force as components  $H_v$  and  $H_h$ . Assume  $H_v$  is downward and that  $H_h$  is to the right. If one of these components is actually in the opposite direction we will get a negative value for it. Set the tension in the cable equal to its maximum possible value,  $T = 1.00 \text{ kN}$ .

**EXECUTE: (a)** The free-body diagram is shown in Figure 11.17, with  $\vec{T}$  resolved into its  $x$ - and  $y$ -components.



**Figure 11.17**

**(b)**  $\sum \tau = 0$  gives  $(T \sin \phi)(3.0 \text{ m}) - w(4.5 \text{ m}) = 0$

$$w = \frac{(T \sin \phi)(3.00 \text{ m})}{4.50 \text{ m}} = \frac{(1000 \text{ N})(\sin 53.1^\circ)(3.00 \text{ m})}{4.50 \text{ m}} = 533 \text{ N}$$

**(c)**  $\sum F_x = 0$  gives  $H_h - T \cos \phi = 0$  and  $H_h = (1.00 \text{ kN})(\cos 53.1^\circ) = 600 \text{ N}$

$\sum F_y = 0$  gives  $T \sin \phi - H_v - w = 0$  and  $H_v = (1.00 \text{ kN})(\sin 53.1^\circ) - 533 \text{ N} = 267 \text{ N}$ .

**EVALUATE:**  $T \cos \phi$ ,  $H_v$  and  $H_h$  all have zero moment arms for a pivot at the hinge and therefore produce zero torque. If we consider a pivot at the point where the cable is attached we can see that  $H_v$  must be downward to produce a torque that opposes the torque due to  $w$ .

**11.18. IDENTIFY:** Apply the conditions for equilibrium to the crane.

**SET UP:** The free-body diagram for the crane is sketched in Figure 11.18 (next page).  $F_h$  and  $F_v$  are the components of the force exerted by the axle.  $\vec{T}$  pulls to the left so  $F_h$  is to the right.  $\vec{T}$  also pulls downward and the two weights are downward, so  $F_v$  is upward.

**EXECUTE: (a)**  $\sum \tau_z = 0$  gives  $T([13 \text{ m}]\sin 25^\circ) - w_c([7.0 \text{ m}]\cos 55^\circ) - w_b([16.0 \text{ m}]\cos 55^\circ) = 0$ .

$$T = \frac{(11,000 \text{ N})([16.0 \text{ m}]\cos 55^\circ) + (15,000 \text{ N})([7.0 \text{ m}]\cos 55^\circ)}{(13.0 \text{ m})\sin 25^\circ} = 2.93 \times 10^4 \text{ N}.$$

**(b)**  $\sum F_x = 0$  gives  $F_h - T \cos 30^\circ = 0$  and  $F_h = 2.54 \times 10^4 \text{ N}$ .

$\sum F_y = 0$  gives  $F_v - T \sin 30^\circ - w_c - w_b = 0$  and  $F_v = 4.06 \times 10^4 \text{ N}$ .

**EVALUATE:**  $\tan \theta = \frac{F_v}{F_h} = \frac{4.06 \times 10^4 \text{ N}}{2.54 \times 10^4 \text{ N}}$  and  $\theta = 58^\circ$ . The force exerted by the axle is not directed along the crane.

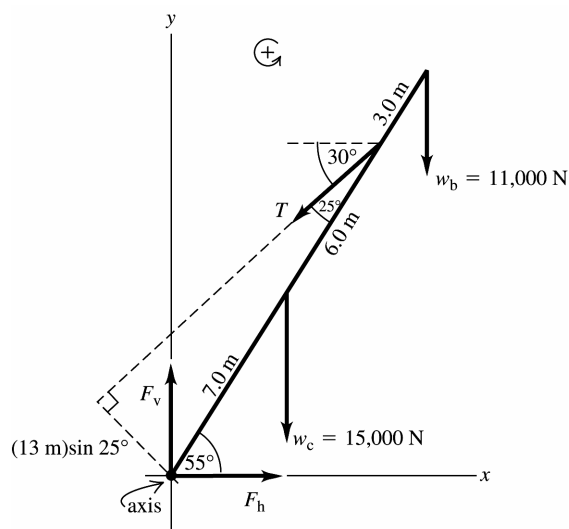


Figure 11.18

**11.19. IDENTIFY:** Apply the first and second conditions of equilibrium to the rod.

**SET UP:** The force diagram for the rod is given in Figure 11.19.

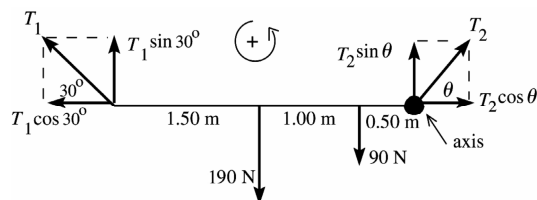


Figure 11.19

**EXECUTE:**  $\sum \tau_z = 0$ , axis at right end of rod, counterclockwise torque is positive

$$(190 \text{ N})(1.50 \text{ m}) + (90 \text{ N})(0.50 \text{ m}) - (T_1 \sin 30.0^\circ)(3.00 \text{ m}) = 0$$

$$T_1 = \frac{285 \text{ N} \cdot \text{m} + 45 \text{ N} \cdot \text{m}}{1.50 \text{ m}} = 220 \text{ N}$$

$$\sum F_x = ma_x$$

$$T_2 \cos \theta - T_1 \cos 30^\circ = 0 \text{ and } T_2 \cos \theta = (220 \text{ N})(\cos 30^\circ) = 190.5 \text{ N}$$

$$\sum F_y = ma_y$$

$$T_1 \sin 30^\circ + T_2 \sin \theta - 190 \text{ N} - 90 \text{ N} = 0$$

$$T_2 \sin \theta = 280 \text{ N} - (220 \text{ N})\sin 30^\circ = 170 \text{ N}$$

$$\text{Then } \frac{T_2 \sin \theta}{T_2 \cos \theta} = \frac{170 \text{ N}}{190.5 \text{ N}} \text{ gives } \tan \theta = 0.89239 \text{ and } \theta = 41.7^\circ$$

$$\text{And } T_2 = \frac{170 \text{ N}}{\sin 41.7^\circ} = 255 \text{ N}.$$

**EVALUATE:** The monkey is closer to the right rope than to the left one, so the tension is larger in the right rope. The horizontal components of the tensions must be equal in magnitude and opposite in direction.

Since  $T_2 > T_1$ , the rope on the right must be at a greater angle above the horizontal to have the same horizontal component as the tension in the other rope.

**11.20. IDENTIFY:** Apply the first and second conditions for equilibrium to the beam.

**SET UP:** The free-body diagram for the beam is given in Figure 11.20.

**EXECUTE:** The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point;  $T(3.00 \text{ m}) = (1.40 \text{ kN})(2.00 \text{ m})\cos 25.0^\circ + (5.00 \text{ kN})(4.50 \text{ m})\cos 25.0^\circ$ , and

$T = 7.64 \text{ kN}$ . The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of  $T$ ,  $6.00 \text{ kN} - T \cos 25.0^\circ = -0.53 \text{ kN}$ . The vertical component is downward. The horizontal force is  $T \sin 25.0^\circ = 3.23 \text{ kN}$ .

**EVALUATE:** The vertical component of the tension is nearly the same magnitude as the total weight of the object and the vertical component of the force exerted by the pivot is much less than its horizontal component.

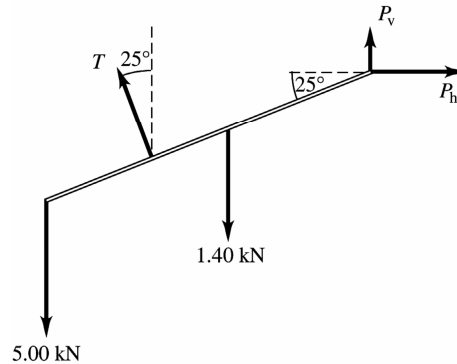
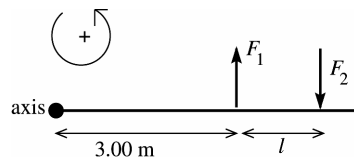


Figure 11.20

**11.21. (a) IDENTIFY and SET UP:** Use  $\tau = Fl$  to calculate the torque (magnitude and direction) for each force and add the torques as vectors. See Figure 11.21a.



**EXECUTE:**

$$\tau_1 = F_1 l_1 = +(8.00 \text{ N})(3.00 \text{ m})$$

$$\tau_1 = +24.0 \text{ N} \cdot \text{m}$$

$$\tau_2 = -F_2 l_2 = -(8.00 \text{ N})(l + 3.00 \text{ m})$$

$$\tau_2 = -24.0 \text{ N} \cdot \text{m} - (8.00 \text{ N})l$$

Figure 11.21a

$$\Sigma \tau_z = \tau_1 + \tau_2 = +24.0 \text{ N} \cdot \text{m} - 24.0 \text{ N} \cdot \text{m} - (8.00 \text{ N})l = -(8.00 \text{ N})l$$

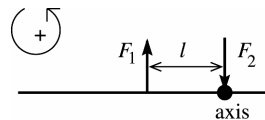
Want  $l$  that makes  $\Sigma \tau_z = -6.40 \text{ N} \cdot \text{m}$  (net torque must be clockwise)

$$-(8.00 \text{ N})l = -6.40 \text{ N} \cdot \text{m}$$

$$l = (6.40 \text{ N} \cdot \text{m}) / 8.00 \text{ N} = 0.800 \text{ m}$$

**(b)**  $|\tau_2| > |\tau_1|$  since  $F_2$  has a larger moment arm; the net torque is clockwise.

**(c)** See Figure 11.21b.



$$\tau_1 = -F_1 l_1 = -(8.00 \text{ N})l$$

$$\tau_2 = 0 \text{ since } \vec{F}_2 \text{ is at the axis}$$

Figure 11.21b

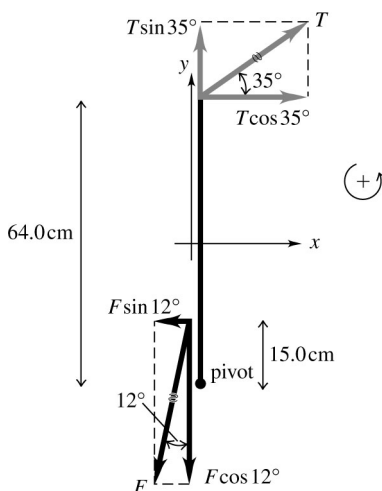
$$\Sigma \tau_z = -6.40 \text{ N} \cdot \text{m} \text{ gives } -(8.00 \text{ N})l = -6.40 \text{ N} \cdot \text{m}$$

$$l = 0.800 \text{ m, same as in part (a).}$$

**EVALUATE:** The force couple gives the same magnitude of torque for the pivot at any point.

**11.22. IDENTIFY:** The person is in equilibrium, so the torques on him must balance. The target variable is the force exerted by the deltoid muscle.

**SET UP:** The free-body diagram for the arm is given in Figure 11.22. Take the pivot at the shoulder joint and let counterclockwise torques be positive. Use coordinates as shown. Let  $F$  be the force exerted by the deltoid muscle. There are also the weight of the arm and forces at the shoulder joint, but none of these forces produce any torque when the arm is in this position. The forces  $F$  and  $T$  have been replaced by their  $x$ - and  $y$ -components.  $\sum \tau_z = 0$ .



**Figure 11.22**

**EXECUTE:**  $\sum \tau_z = 0$  gives  $(F \sin 12.0^\circ)(15.0 \text{ cm}) - (T \cos 35^\circ)(64.0 \text{ cm}) = 0$ .

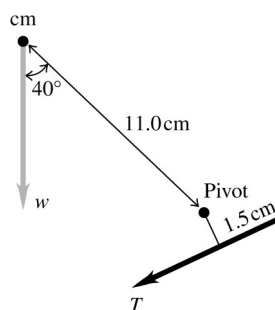
$$F = \frac{(36.0 \text{ N})(\cos 35^\circ)(64.0 \text{ cm})}{(\sin 12.0^\circ)(15.0 \text{ cm})} = 605 \text{ N}.$$

**EVALUATE:** The force exerted by the deltoid muscle is much larger than the tension in the cable because the deltoid muscle makes a small angle (only  $12.0^\circ$ ) with the humerus.

- 11.23. IDENTIFY:** The student's head is at rest, so the torques on it must balance. The target variable is the tension in her neck muscles.

**SET UP:** Let the pivot be at point  $P$  and let counterclockwise torques be positive.  $\sum \tau_z = 0$ .

**EXECUTE:** (a) The free-body diagram is given in Figure 11.23.



**Figure 11.23**

(b)  $\sum \tau_z = 0$  gives  $w(11.0 \text{ cm})(\sin 40.0^\circ) - T(1.50 \text{ cm}) = 0$ .

$$T = \frac{(4.50 \text{ kg})(9.80 \text{ m/s}^2)(11.0 \text{ cm})\sin 40.0^\circ}{1.50 \text{ cm}} = 208 \text{ N}.$$

**EVALUATE:** Her head weighs about 45 N but the tension in her neck muscles must be much larger because the tension has a small moment arm.

- 11.24. IDENTIFY:** Use  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$ .

**SET UP:**  $A = 50.0 \text{ cm}^2 = 50.0 \times 10^{-4} \text{ m}^2$ .

**EXECUTE:** relaxed:  $Y = \frac{(0.200 \text{ m})(25.0 \text{ N})}{(50.0 \times 10^{-4} \text{ m}^2)(3.0 \times 10^{-2} \text{ m})} = 3.33 \times 10^4 \text{ Pa}$

maximum tension:  $Y = \frac{(0.200 \text{ m})(500 \text{ N})}{(50.0 \times 10^{-4} \text{ m}^2)(3.0 \times 10^{-2} \text{ m})} = 6.67 \times 10^5 \text{ Pa}$

**EVALUATE:** The muscle tissue is much more difficult to stretch when it is under maximum tension.

- 11.25. IDENTIFY and SET UP:** Apply  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$  and solve for  $A$  and then use  $A = \pi r^2$  to get the radius and  $d = 2r$  to calculate the diameter.

**EXECUTE:**  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$  so  $A = \frac{l_0 F_{\perp}}{Y \Delta l}$  ( $A$  is the cross-section area of the wire)

For steel,  $Y = 2.0 \times 10^{11} \text{ Pa}$  (Table 11.1)

Thus  $A = \frac{(2.00 \text{ m})(700 \text{ N})}{(2.0 \times 10^{11} \text{ Pa})(0.25 \times 10^{-2} \text{ m})} = 2.8 \times 10^{-6} \text{ m}^2$ .

$A = \pi r^2$ , so  $r = \sqrt{A/\pi} = \sqrt{2.8 \times 10^{-6} \text{ m}^2 / \pi} = 9.44 \times 10^{-4} \text{ m}$

$d = 2r = 1.9 \times 10^{-3} \text{ m} = 1.9 \text{ mm}$ .

**EVALUATE:** Steel wire of this diameter doesn't stretch much;  $\Delta l/l_0 = 0.12\%$ .

- 11.26. IDENTIFY:** Apply  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$ .

**SET UP:** From Table 11.1, for steel,  $Y = 2.0 \times 10^{11} \text{ Pa}$  and for copper,  $Y = 1.1 \times 10^{11} \text{ Pa}$ .

$A = \pi(d^2/4) = 1.77 \times 10^{-4} \text{ m}^2$ .  $F_{\perp} = 4000 \text{ N}$  for each rod.

**EXECUTE:** (a) The strain is  $\frac{\Delta l}{l_0} = \frac{F}{YA}$ . For steel  $\frac{\Delta l}{l_0} = \frac{(4000 \text{ N})}{(2.0 \times 10^{11} \text{ Pa})(1.77 \times 10^{-4} \text{ m}^2)} = 1.1 \times 10^{-4}$ .

Similarly, the strain for copper is  $2.1 \times 10^{-4}$ .

(b) Steel:  $(1.1 \times 10^{-4})(0.750 \text{ m}) = 8.3 \times 10^{-5} \text{ m}$ . Copper:  $(2.1 \times 10^{-4})(0.750 \text{ m}) = 1.6 \times 10^{-4} \text{ m}$ .

**EVALUATE:** Copper has a smaller  $Y$  and therefore a greater elongation.

- 11.27. IDENTIFY:** Apply  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$ .

**SET UP:**  $A = 0.50 \text{ cm}^2 = 0.50 \times 10^{-4} \text{ m}^2$

**EXECUTE:**  $Y = \frac{(4.00 \text{ m})(5000 \text{ N})}{(0.50 \times 10^{-4} \text{ m}^2)(0.20 \times 10^{-2} \text{ m})} = 2.0 \times 10^{11} \text{ Pa}$

**EVALUATE:** Our result is the same as that given for steel in Table 11.1.

- 11.28. IDENTIFY:** Apply  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$ .

**SET UP:**  $A = \pi r^2 = \pi(3.5 \times 10^{-3} \text{ m})^2 = 3.85 \times 10^{-5} \text{ m}^2$ . The force applied to the end of the rope is the weight of the climber:  $F_{\perp} = (65.0 \text{ kg})(9.80 \text{ m/s}^2) = 637 \text{ N}$ .

**EXECUTE:**  $Y = \frac{(45.0 \text{ m})(637 \text{ N})}{(3.85 \times 10^{-5} \text{ m}^2)(1.10 \text{ m})} = 6.77 \times 10^8 \text{ Pa}$

**EVALUATE:** Our result is a lot smaller than the values given in Table 11.1. An object made of rope material is much easier to stretch than if the object were made of metal.

- 11.29. IDENTIFY:** Use the first condition of equilibrium to calculate the tensions  $T_1$  and  $T_2$  in the wires (Figure 11.29a, next page). Then use Eq. (11.10) to calculate the strain and elongation of each wire.

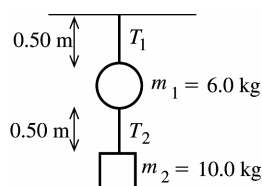
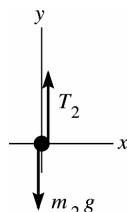


Figure 11.29a

**SET UP:** The free-body diagram for  $m_2$  is given in Figure 11.27b.



**EXECUTE:**

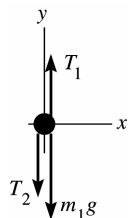
$$\sum F_y = ma_y$$

$$T_2 - m_2g = 0$$

$$T_2 = 98.0 \text{ N}$$

Figure 11.29b

**SET UP:** The free-body-diagram for  $m_1$  is given in Figure 11.29c.



**EXECUTE:**

$$\sum F_y = ma_y$$

$$T_1 - T_2 - m_1g = 0$$

$$T_1 = T_2 + m_1g$$

$$T_1 = 98.0 \text{ N} + 58.8 \text{ N} = 157 \text{ N}$$

Figure 11.29c

(a)  $Y = \frac{\text{stress}}{\text{strain}}$  so  $\text{strain} = \frac{\text{stress}}{Y} = \frac{F_{\perp}}{AY}$

upper wire:  $\text{strain} = \frac{T_1}{AY} = \frac{157 \text{ N}}{(2.5 \times 10^{-7} \text{ m}^2)(2.0 \times 10^{11} \text{ Pa})} = 3.1 \times 10^{-3}$

lower wire:  $\text{strain} = \frac{T_2}{AY} = \frac{98 \text{ N}}{(2.5 \times 10^{-7} \text{ m}^2)(2.0 \times 10^{11} \text{ Pa})} = 2.0 \times 10^{-3}$

(b)  $\text{strain} = \Delta l / l_0$  so  $\Delta l = l_0(\text{strain})$

upper wire:  $\Delta l = (0.50 \text{ m})(3.1 \times 10^{-3}) = 1.6 \times 10^{-3} \text{ m} = 1.6 \text{ mm}$

lower wire:  $\Delta l = (0.50 \text{ m})(2.0 \times 10^{-3}) = 1.0 \times 10^{-3} \text{ m} = 1.0 \text{ mm}$

**EVALUATE:** The tension is greater in the upper wire because it must support both objects. The wires have the same length and diameter, so the one with the greater tension has the greater strain and elongation.

**11.30. IDENTIFY:** Apply  $\text{stress} = \frac{F_{\perp}}{A}$ ,  $\text{strain} = \frac{\text{stress}}{Y}$ ,  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$ .

**SET UP:** The cross-sectional area of the post is  $A = \pi r^2 = \pi(0.125 \text{ m})^2 = 0.0491 \text{ m}^2$ . The force applied to the end of the post is  $F_{\perp} = (8000 \text{ kg})(9.80 \text{ m/s}^2) = 7.84 \times 10^4 \text{ N}$ . The Young's modulus of steel is  $Y = 2.0 \times 10^{11} \text{ Pa}$ .

**EXECUTE:** (a)  $\text{stress} = \frac{F_{\perp}}{A} = -\frac{7.84 \times 10^4 \text{ N}}{0.0491 \text{ m}^2} = -1.60 \times 10^6 \text{ Pa}$ . The minus sign indicates that the stress is compressive.

(b)  $\text{strain} = \frac{\text{stress}}{Y} = -\frac{1.60 \times 10^6 \text{ Pa}}{2.0 \times 10^{11} \text{ Pa}} = -8.0 \times 10^{-6}$ . The minus sign indicates that the length decreases.

(c)  $\Delta l = l_0(\text{strain}) = (2.50 \text{ m})(-8.0 \times 10^{-6}) = -2.0 \times 10^{-5} \text{ m}$

**EVALUATE:** The fractional change in length of the post is very small.

**11.31. IDENTIFY:** The amount of compression depends on the bulk modulus of the bone.

**SET UP:**  $\frac{\Delta V}{V_0} = -\frac{\Delta p}{B}$  and  $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$ .

**EXECUTE:** (a)  $\Delta p = -B \frac{\Delta V}{V_0} = -(15 \times 10^9 \text{ Pa})(-0.0010) = 1.5 \times 10^7 \text{ Pa} = 150 \text{ atm}$ .

(b) The depth for a pressure increase of  $1.5 \times 10^7 \text{ Pa}$  is 1.5 km.

**EVALUATE:** An extremely large pressure increase is needed for just a 0.10% bone compression, so pressure changes do not appreciably affect the bones. Unprotected dives do not approach a depth of 1.5 km, so bone compression is not a concern for divers.

**11.32. IDENTIFY:** Apply  $\frac{\Delta V}{V_0} = -\frac{\Delta p}{B}$ .

**SET UP:**  $\Delta V = -\frac{V_0 \Delta p}{B}$ .  $\Delta p$  is positive when the pressure increases.

**EXECUTE:** (a) The volume would increase slightly.

(b) The volume change would be twice as great.

(c) The volume change is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

**EVALUATE:** For lead,  $B = 4.1 \times 10^{10} \text{ Pa}$ , so  $\Delta p/B$  is very small and the fractional change in volume is very small.

**11.33. IDENTIFY and SET UP:** Use  $\frac{\Delta V}{V_0} = -\frac{\Delta p}{B}$  and  $k = 1/B$  to calculate  $B$  and  $k$ .

**EXECUTE:**  $B = -\frac{\Delta p}{\Delta V/V_0} = -\frac{(3.6 \times 10^6 \text{ Pa})(600 \text{ cm}^3)}{(-0.45 \text{ cm}^3)} = +4.8 \times 10^9 \text{ Pa}$

$k = 1/B = 1/4.8 \times 10^9 \text{ Pa} = 2.1 \times 10^{-10} \text{ Pa}^{-1}$

**EVALUATE:**  $k$  is the same as for glycerine (Table 11.2).

**11.34. IDENTIFY:** Apply  $\frac{\Delta V}{V_0} = -\frac{\Delta p}{B}$ . Density  $= m/V$ .

**SET UP:** At the surface the pressure is  $1.0 \times 10^5 \text{ Pa}$ , so  $\Delta p = 1.16 \times 10^8 \text{ Pa}$ .  $V_0 = 1.00 \text{ m}^3$ . At the surface  $1.00 \text{ m}^3$  of water has mass  $1.03 \times 10^3 \text{ kg}$ .

**EXECUTE:** (a)  $B = -\frac{(\Delta p)V_0}{\Delta V}$  gives  $\Delta V = -\frac{(\Delta p)V_0}{B} = -\frac{(1.16 \times 10^8 \text{ Pa})(1.00 \text{ m}^3)}{2.2 \times 10^9 \text{ Pa}} = -0.0527 \text{ m}^3$

(b) At this depth  $1.03 \times 10^3 \text{ kg}$  of seawater has volume  $V_0 + \Delta V = 0.9473 \text{ m}^3$ . The density is

$\frac{1.03 \times 10^3 \text{ kg}}{0.9473 \text{ m}^3} = 1.09 \times 10^3 \text{ kg/m}^3$ .

**EVALUATE:** The density is increased because the volume is compressed due to the increased pressure.

**11.35. IDENTIFY:** The forces on the cube must balance. The deformation  $x$  is related to the force by  $S = \frac{F_{\parallel}}{A} \frac{h}{x}$ .

$F_{\parallel} = F$  since  $F$  is applied parallel to the upper face.

**SET UP:**  $A = (0.0600 \text{ m})^2$  and  $h = 0.0600 \text{ m}$ . Table 11.1 gives  $S = 4.4 \times 10^{10} \text{ Pa}$  for copper and  $0.6 \times 10^{10} \text{ Pa}$  for lead.

**EXECUTE:** (a) Since the horizontal forces balance, the glue exerts a force  $F$  in the opposite direction.

$$(b) F = \frac{AxS}{h} = \frac{(0.0600 \text{ m})^2 (0.250 \times 10^{-3} \text{ m})(4.4 \times 10^{10} \text{ Pa})}{0.0600 \text{ m}} = 6.6 \times 10^5 \text{ N}$$

$$(c) x = \frac{Fh}{AS} = \frac{(6.6 \times 10^5 \text{ N})(0.0600 \text{ m})}{(0.0600 \text{ m})^2 (0.6 \times 10^{10} \text{ Pa})} = 1.8 \text{ mm}$$

EVALUATE: Lead has a smaller  $S$  than copper, so the lead cube has a greater deformation than the copper cube.

11.36. IDENTIFY: Apply  $S = \frac{F_{\parallel}}{A} \frac{h}{x}$ .

SET UP:  $F_{\parallel} = 9.0 \times 10^5 \text{ N}$ .  $A = (0.100 \text{ m})(0.500 \times 10^{-2} \text{ m})$ .  $h = 0.100 \text{ m}$ . From Table 11.1,  $S = 7.5 \times 10^{10} \text{ Pa}$  for steel.

EXECUTE: (a) Shear strain  $= \frac{F_{\parallel}}{AS} = \frac{(9 \times 10^5 \text{ N})}{[(0.100 \text{ m})(0.500 \times 10^{-2} \text{ m})][7.5 \times 10^{10} \text{ Pa}]} = 2.4 \times 10^{-2}$ .

(b) Since shear strain  $= x/h$ ,  $x = (\text{Shear strain}) \cdot h = (0.024)(0.100 \text{ m}) = 2.4 \times 10^{-3} \text{ m}$ .

EVALUATE: This very large force produces a small displacement;  $x/h = 2.4\%$ .

11.37. IDENTIFY: The force components parallel to the face of the cube produce a shear which can deform the cube.

SET UP:  $S = \frac{F_{\parallel}}{A\phi}$ , where  $\phi = x/h$ .  $F_{\parallel}$  is the component of the force tangent to the surface, so

$$F_{\parallel} = (1375 \text{ N}) \cos 8.50^\circ = 1360 \text{ N}. \quad \phi \text{ must be in radians, } \phi = 1.24^\circ = 0.0216 \text{ rad}.$$

EXECUTE:  $S = \frac{1360 \text{ N}}{(0.0925 \text{ m})^2 (0.0216 \text{ rad})} = 7.36 \times 10^6 \text{ Pa}$ .

EVALUATE: The shear modulus of this material is much less than the values for metals given in Table 11.1 in the text.

11.38. IDENTIFY: The breaking stress of the wire is the value of  $F_{\perp}/A$  at which the wire breaks.

SET UP: From Table 11.3, the breaking stress of brass is  $4.7 \times 10^8 \text{ Pa}$ . The area  $A$  of the wire is related to its diameter by  $A = \pi d^2/4$ .

EXECUTE:  $A = \frac{350 \text{ N}}{4.7 \times 10^8 \text{ Pa}} = 7.45 \times 10^{-7} \text{ m}^2$ , so  $d = \sqrt{4A/\pi} = 0.97 \text{ mm}$ .

EVALUATE: The maximum force a wire can withstand without breaking is proportional to the square of its diameter.

11.39. IDENTIFY and SET UP: Use stress  $= \frac{F_{\perp}}{A}$ .

EXECUTE: Tensile stress  $= \frac{F_{\perp}}{A} = \frac{F_{\perp}}{\pi r^2} = \frac{90.8 \text{ N}}{\pi (0.92 \times 10^{-3} \text{ m})^2} = 3.41 \times 10^7 \text{ Pa}$

EVALUATE: A modest force produces a very large stress because the cross-sectional area is small.

11.40. IDENTIFY: The proportional limit and breaking stress are values of the stress,  $F_{\perp}/A$ . Use  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$  to calculate  $\Delta l$ .

SET UP: For steel,  $Y = 20 \times 10^{10} \text{ Pa}$ .  $F_{\perp} = w$ .

EXECUTE: (a)  $w = (1.6 \times 10^{-3})(20 \times 10^{10} \text{ Pa})(5 \times 10^{-6} \text{ m}^2) = 1.60 \times 10^3 \text{ N}$ .

(b)  $\Delta l = \left( \frac{F_{\perp}}{A} \right) \frac{l_0}{Y} = (1.6 \times 10^{-3})(4.0 \text{ m}) = 6.4 \text{ mm}$

(c)  $(6.5 \times 10^{-3})(20 \times 10^{10} \text{ Pa})(5 \times 10^{-6} \text{ m}^2) = 6.5 \times 10^3 \text{ N}$ .

EVALUATE: At the proportional limit, the fractional change in the length of the wire is 0.16%.

11.41. IDENTIFY: The elastic limit is a value of the stress,  $F_{\perp}/A$ . Apply  $\sum \vec{F} = m\vec{a}$  to the elevator in order to find the tension in the cable.



**SET UP:**  $\frac{F_{\perp}}{A} = \frac{1}{3}(2.40 \times 10^8 \text{ Pa}) = 0.80 \times 10^8 \text{ Pa}$ . The free-body diagram for the elevator is given in

Figure 11.41.  $F_{\perp}$  is the tension in the cable.

**EXECUTE:**  $F_{\perp} = A(0.80 \times 10^8 \text{ Pa}) = (3.00 \times 10^{-4} \text{ m}^2)(0.80 \times 10^8 \text{ Pa}) = 2.40 \times 10^4 \text{ N}$ .  $\sum F_y = ma_y$  applied to the elevator gives  $F_{\perp} - mg = ma$  and  $a = \frac{F_{\perp}}{m} - g = \frac{2.40 \times 10^4 \text{ N}}{1200 \text{ kg}} - 9.80 \text{ m/s}^2 = 10.2 \text{ m/s}^2$

**EVALUATE:** The tension in the cable is about twice the weight of the elevator.

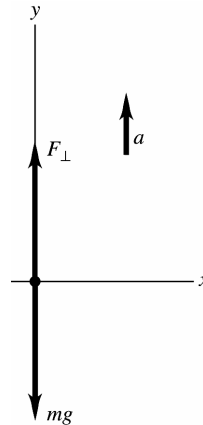
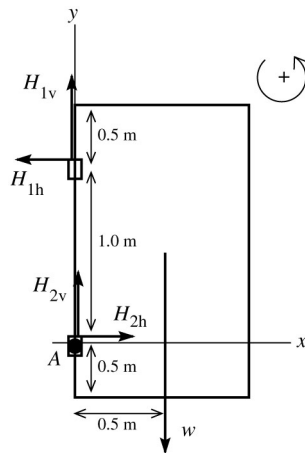


Figure 11.41

**11.42. IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

**SET UP:** The free-body diagram for the door is given in Figure 11.42. Let  $\vec{H}_1$  and  $\vec{H}_2$  be the forces exerted by the upper and lower hinges. Take the origin of coordinates at the bottom hinge (point  $A$ ) and  $+y$  upward.



**EXECUTE:**

We are given that

$$H_{1v} = H_{2v} = w/2 = 165 \text{ N}.$$

$$\sum F_x = ma_x$$

$$H_{2h} - H_{1h} = 0$$

$$H_{1h} = H_{2h}$$

The horizontal components of the hinge forces are equal in magnitude and opposite in direction.

Figure 11.42

Sum torques about point  $A$ .  $H_{1v}$ ,  $H_{2v}$ , and  $H_{2h}$  all have zero moment arm and hence zero torque about an axis at this point. Thus  $\sum \tau_A = 0$  gives  $H_{1h}(1.00 \text{ m}) - w(0.50 \text{ m}) = 0$

$$H_{1h} = w \left( \frac{0.50 \text{ m}}{1.00 \text{ m}} \right) = \frac{1}{2}(330 \text{ N}) = 165 \text{ N}.$$

The horizontal component of each hinge force is 165 N.

**EVALUATE:** The horizontal components of the force exerted by each hinge are the only horizontal forces so must be equal in magnitude and opposite in direction. With an axis at  $A$ , the torque due to the horizontal force exerted by the upper hinge must be counterclockwise to oppose the clockwise torque exerted by the weight of the door. So, the horizontal force exerted by the upper hinge must be to the left. You can also verify that the net torque is also zero if the axis is at the upper hinge.

- 11.43. IDENTIFY:** The center of gravity of the combined object must be at the fulcrum. Use

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots} \text{ to calculate } x_{\text{cm}}.$$

**SET UP:** The center of gravity of the sand is at the middle of the box. Use coordinates with the origin at the fulcrum and  $+x$  to the right. Let  $m_1 = 25.0 \text{ kg}$ , so  $x_1 = 0.500 \text{ m}$ . Let  $m_2 = m_{\text{sand}}$ , so  $x_2 = -0.625 \text{ m}$ .  $x_{\text{cm}} = 0$ .

**EXECUTE:**  $x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0$  and  $m_2 = -m_1 \frac{x_1}{x_2} = -(25.0 \text{ kg}) \left( \frac{0.500 \text{ m}}{-0.625 \text{ m}} \right) = 20.0 \text{ kg}.$

**EVALUATE:** The mass of sand required is less than the mass of the plank since the center of the box is farther from the fulcrum than the center of gravity of the plank is.

- 11.44. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the bridge.

**SET UP:** Let the axis of rotation be at the left end of the bridge and let counterclockwise torques be positive.

**EXECUTE:** If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from

$$T(12.0 \text{ m}) = (600 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m}) + (200 \text{ kg})(9.80 \text{ m/s}^2)(6.0 \text{ m}), \text{ so } T = 6860 \text{ N}.$$

This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance  $x$  Lancelot can ride, replace the  $12.0 \text{ m}$  multiplying Lancelot's weight by  $x$  and the tension

$T$  by  $T_{\text{max}} = 5.80 \times 10^3 \text{ N}$  and solve for  $x$ ;

$$x = \frac{(5.80 \times 10^3 \text{ N})(12.0 \text{ m}) - (200 \text{ kg})(9.80 \text{ m/s}^2)(6.0 \text{ m})}{(600 \text{ kg})(9.80 \text{ m/s}^2)} = 9.84 \text{ m}.$$

**EVALUATE:** Before Lancelot goes onto the bridge, the tension in the supporting cable is

$$T = \frac{(6.0 \text{ m})(200 \text{ kg})(9.80 \text{ m/s}^2)}{12.0 \text{ m}} = 980 \text{ N}, \text{ well below the breaking strength of the cable. As he moves}$$

along the bridge, the increase in tension is proportional to  $x$ , the distance he has moved along the bridge.

- 11.45. IDENTIFY:** Apply the conditions of equilibrium to the climber. For the minimum coefficient of friction the static friction force has the value  $f_s = \mu_s n$ .

**SET UP:** The free-body diagram for the climber is given in Figure 11.45.  $f_s$  and  $n$  are the vertical and horizontal components of the force exerted by the cliff face on the climber. The moment arm for the force  $T$  is  $(1.4 \text{ m}) \cos 10^\circ$ .

**EXECUTE:** (a)  $\sum \tau_z = 0$  gives  $T(1.4 \text{ m}) \cos 10^\circ - w(1.1 \text{ m}) \cos 35.0^\circ = 0$ .

$$T = \frac{(1.1 \text{ m}) \cos 35.0^\circ}{(1.4 \text{ m}) \cos 10^\circ} (82.0 \text{ kg})(9.80 \text{ m/s}^2) = 525 \text{ N}$$

(b)  $\sum F_x = 0$  gives  $n = T \sin 25.0^\circ = 222 \text{ N}$ .  $\sum F_y = 0$  gives  $f_s + T \cos 25^\circ - w = 0$  and

$$f_s = (82.0 \text{ kg})(9.80 \text{ m/s}^2) - (525 \text{ N}) \cos 25^\circ = 328 \text{ N}.$$

(c)  $\mu_s = \frac{f_s}{n} = \frac{328 \text{ N}}{222 \text{ N}} = 1.48$

**EVALUATE:** To achieve this large value of  $\mu_s$  the climber must wear special rough-soled shoes.

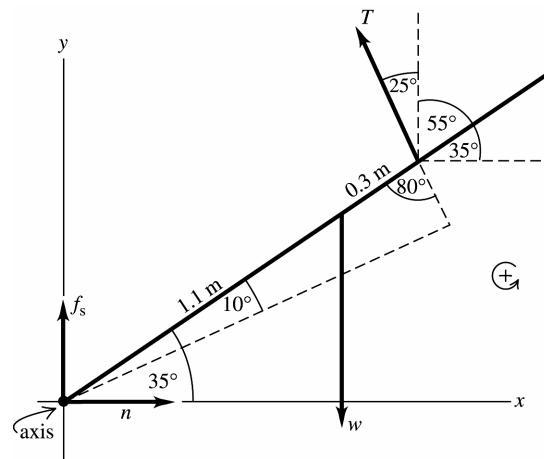
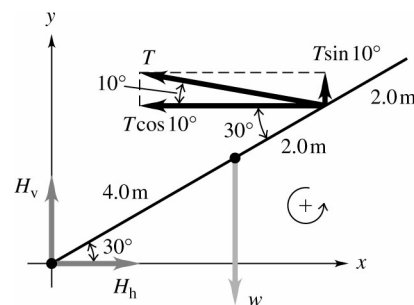


Figure 11.45

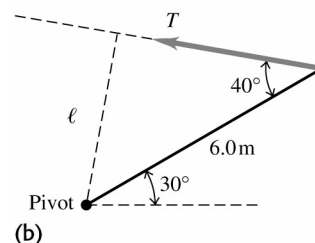
**11.46. IDENTIFY:** The beam is at rest, so the forces and torques on it must balance.

**SET UP:** The weight of the beam acts 4.0 m from each end. Take the pivot at the hinge and let counterclockwise torques be positive. Represent the force exerted by the hinge by its horizontal and vertical components,  $H_h$  and  $H_v$ .  $\sum F_x = 0$ ,  $\sum F_y = 0$  and  $\sum \tau_z = 0$ .

**EXECUTE: (a)** The free-body diagram for the beam is given in Figure 11.46a.



(a)



(b)

Figure 11.46

**(b)** The moment arm for  $T$  is sketched in Figure 11.46b and is equal to  $(6.0 \text{ m})\sin 40.0^\circ$ .  $\sum \tau_z = 0$  gives

$$T(6.0 \text{ m})(\sin 40.0^\circ) - w(4.0 \text{ m})(\cos 30.0^\circ) = 0. \quad T = \frac{(1150 \text{ kg})(9.80 \text{ m/s}^2)(4.0 \text{ m})(\cos 30.0^\circ)}{(6.0 \text{ m})(\sin 40.0^\circ)} = 1.01 \times 10^4 \text{ N}.$$

**(c)**  $\sum F_x = 0$  gives  $H_h - T \cos 10.0^\circ = 0$  and  $H_h = T \cos 10.0^\circ = 9.97 \times 10^3 \text{ N}$ .

**EVALUATE:** The tension is less than the weight of the beam because it has a larger moment arm than the weight force has.

**11.47. IDENTIFY:** In each case, to achieve balance the center of gravity of the system must be at the fulcrum. Use

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots} \text{ to locate } x_{\text{cm}}, \text{ with } m_i \text{ replaced by } w_i.$$

**SET UP:** Let the origin be at the left-hand end of the rod and take the  $+x$ -axis to lie along the rod. Let  $w_1 = 255 \text{ N}$  (the rod) so  $x_1 = 1.00 \text{ m}$ , let  $w_2 = 225 \text{ N}$  so  $x_2 = 2.00 \text{ m}$  and let  $w_3 = W$ . In part (a)  $x_3 = 0.500 \text{ m}$  and in part (b)  $x_3 = 0.750 \text{ m}$ .

**EXECUTE:** (a)  $x_{\text{cm}} = 1.25 \text{ m}$ .  $x_{\text{cm}} = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3}{w_1 + w_2 + w_3}$  gives  $w_3 = \frac{(w_1 + w_2)x_{\text{cm}} - w_1 x_1 - w_2 x_2}{x_3 - x_{\text{cm}}}$  and

$$W = \frac{(480 \text{ N})(1.25 \text{ m}) - (255 \text{ N})(1.00 \text{ m}) - (225 \text{ N})(2.00 \text{ m})}{0.500 \text{ m} - 1.25 \text{ m}} = 140 \text{ N}.$$

(b) Now  $w_3 = W = 140 \text{ N}$  and  $x_3 = 0.750 \text{ m}$ .

$$x_{\text{cm}} = \frac{(255 \text{ N})(1.00 \text{ m}) + (225 \text{ N})(2.00 \text{ m}) + (140 \text{ N})(0.750 \text{ m})}{255 \text{ N} + 225 \text{ N} + 140 \text{ N}} = 1.31 \text{ m}. W \text{ must be moved}$$

$1.31 \text{ m} - 1.25 \text{ m} = 6 \text{ cm}$  to the right.

**EVALUATE:** Moving  $W$  to the right means  $x_{\text{cm}}$  for the system moves to the right.

**11.48. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the hammer.

**SET UP:** Take the axis of rotation to be at point  $A$ .

**EXECUTE:** The force  $\vec{F}_1$  is directed along the length of the nail, and so has a moment arm of  $(0.080 \text{ m})\sin 60^\circ$ . The moment arm of  $\vec{F}_2$  is  $0.300 \text{ m}$ , so

$$F_2 = F_1 \frac{(0.0800 \text{ m})\sin 60^\circ}{(0.300 \text{ m})} = (400 \text{ N})(0.231) = 92.4 \text{ N}.$$

**EVALUATE:** The force  $F_2$  that must be applied to the hammer handle is much less than the force that the hammer applies to the nail, because of the large difference in the lengths of the moment arms.

**11.49. IDENTIFY:** Apply the conditions of equilibrium to the horizontal beam. Since the two wires are symmetrically placed on either side of the middle of the sign, their tensions are equal and are each equal to  $T_w = mg/2 = 137 \text{ N}$ .

**SET UP:** The free-body diagram for the beam is given in Figure 11.49.  $F_v$  and  $F_h$  are the vertical and horizontal forces exerted by the hinge on the beam. Since the cable is  $2.00 \text{ m}$  long and the beam is  $1.50 \text{ m}$  long,  $\cos \theta = \frac{1.50 \text{ m}}{2.00 \text{ m}}$  and  $\theta = 41.4^\circ$ . The tension  $T_c$  in the cable has been replaced by its horizontal and vertical components.

**EXECUTE:** (a)  $\sum \tau_z = 0$  gives  $T_c(\sin 41.4^\circ)(1.50 \text{ m}) - w_{\text{beam}}(0.750 \text{ m}) - T_w(1.50 \text{ m}) - T_w(0.60 \text{ m}) = 0$ .

$$T_c = \frac{(16.0 \text{ kg})(9.80 \text{ m/s}^2)(0.750 \text{ m}) + (137 \text{ N})(1.50 \text{ m} + 0.60 \text{ m})}{(1.50 \text{ m})(\sin 41.4^\circ)} = 408.6 \text{ N}, \text{ which rounds to } 409 \text{ N}.$$

(b)  $\sum F_y = 0$  gives  $F_v + T_c \sin 41.4^\circ - w_{\text{beam}} - 2T_w = 0$  and

$F_v = 2T_w + w_{\text{beam}} - T_c \sin 41.4^\circ = 2(137 \text{ N}) + (16.0 \text{ kg})(9.80 \text{ m/s}^2) - (408.6 \text{ N})(\sin 41.4^\circ) = 161 \text{ N}$ . The hinge must be able to supply a vertical force of  $161 \text{ N}$ .

**EVALUATE:** The force from the two wires could be replaced by the weight of the sign acting at a point  $0.60 \text{ m}$  to the left of the right-hand edge of the sign.

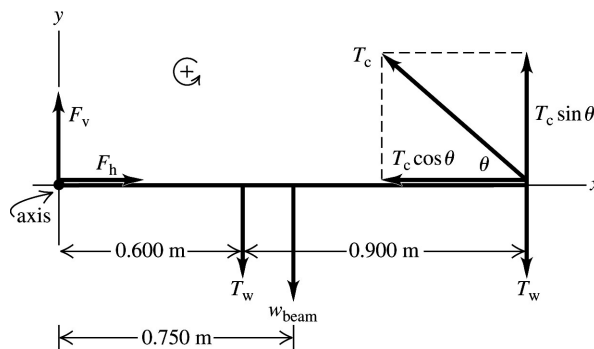


Figure 11.49

**11.50. IDENTIFY:** Apply the first and second conditions of equilibrium to the bar.

**SET UP:** The free-body diagram for the bar is given in Figure 11.50.  $n$  is the normal force exerted on the bar by the surface. There is no friction force at this surface.  $H_h$  and  $H_v$  are the components of the force exerted on the bar by the hinge. The components of the force of the bar on the hinge will be equal in magnitude and opposite in direction.

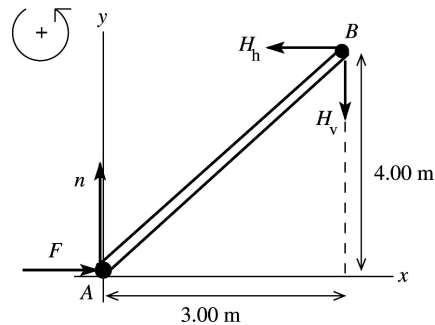


Figure 11.50

**EXECUTE:**

$$\sum F_x = ma_x$$

$$F = H_h = 220 \text{ N}$$

$$\sum F_y = ma_y$$

$$n - H_v = 0$$

$H_v = n$ , but we don't know either of these forces.

$$\sum \tau_B = 0 \text{ gives } F(4.00 \text{ m}) - n(3.00 \text{ m}) = 0.$$

$$n = (4.00 \text{ m}/3.00 \text{ m})F = \frac{4}{3}(220 \text{ N}) = 293 \text{ N} \text{ and then } H_v = 293 \text{ N}.$$

Force of bar on hinge:

horizontal component 220 N, to right

vertical component 293 N, upward

**EVALUATE:**  $H_h/H_v = 220/293 = 0.75 = 3.00/4.00$ , so the force the hinge exerts on the bar is directed along the bar.  $\vec{n}$  and  $\vec{F}$  have zero torque about point A, so the line of action of the hinge force  $\vec{H}$  must pass through this point also if the net torque is to be zero.

**11.51. IDENTIFY:** We want to locate the center of mass of the leg-cast system. We can treat each segment of the leg and cast as a point-mass located at its center of mass.

**SET UP:** The force diagram for the leg is given in Figure 11.51 (next page). The weight of each piece acts at the center of mass of that piece. The mass of the upper leg is  $m_{ul} = (0.215)(37 \text{ kg}) = 7.955 \text{ kg}$ . The mass of the lower leg is  $m_{ll} = (0.140)(37 \text{ kg}) = 5.18 \text{ kg}$ . Use the coordinates shown, with the origin at the hip

and the  $x$ -axis along the leg, and use 
$$x_{cm} = \frac{x_{ul}m_{ul} + x_{ll}m_{ll} + x_{cast}m_{cast}}{m_{ul} + m_{ll} + m_{cast}}.$$

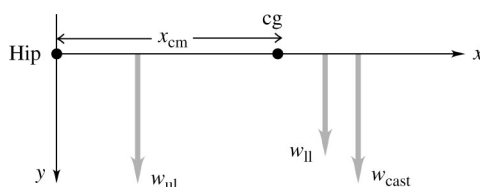


Figure 11.51

**EXECUTE:** Using  $x_{\text{cm}} = \frac{x_{\text{ul}}m_{\text{ul}} + x_{\text{l}}m_{\text{l}} + x_{\text{cast}}m_{\text{cast}}}{m_{\text{ul}} + m_{\text{l}} + m_{\text{cast}}}$ , we have

$$x_{\text{cm}} = \frac{(18.0 \text{ cm})(7.955 \text{ kg}) + (69.0 \text{ cm})(5.18 \text{ kg}) + (78.0 \text{ cm})(5.50 \text{ kg})}{7.955 \text{ kg} + 5.18 \text{ kg} + 5.50 \text{ kg}} = 49.9 \text{ cm}$$

**EVALUATE:** The strap is attached to the left of the center of mass of the cast, but it is still supported by the rigid cast since the cast extends beyond its center of mass.

**11.52. IDENTIFY:** Apply the first and second conditions for equilibrium to the bridge.

**SET UP:** Find torques about the hinge. Use  $L$  as the length of the bridge and  $w_{\text{T}}$  and  $w_{\text{B}}$  for the weights of the truck and the raised section of the bridge. Take  $+y$  to be upward and  $+x$  to be to the right.

**EXECUTE:** (a)  $TL \sin 70^\circ = w_{\text{T}}(\frac{3}{4}L)\cos 30^\circ + w_{\text{B}}(\frac{1}{2}L)\cos 30^\circ$ , so

$$T = \frac{(\frac{3}{4}m_{\text{T}} + \frac{1}{2}m_{\text{B}})(9.80 \text{ m/s}^2)\cos 30^\circ}{\sin 70^\circ} = 2.84 \times 10^5 \text{ N.}$$

(b) Horizontal:  $T \cos(70^\circ - 30^\circ) = 2.18 \times 10^5 \text{ N}$  (to the right).

Vertical:  $w_{\text{T}} + w_{\text{B}} - T \sin 40^\circ = 2.88 \times 10^5 \text{ N}$  (upward).

**EVALUATE:** If  $\phi$  is the angle of the hinge force above the horizontal,

$$\tan \phi = \frac{2.88 \times 10^5 \text{ N}}{2.18 \times 10^5 \text{ N}} \text{ and } \phi = 52.9^\circ. \text{ The hinge force is not directed along the bridge.}$$

**11.53. IDENTIFY:** The leg is not rotating, so the external torques on it must balance.

**SET UP:** The free-body diagram for the leg is given in Figure 11.53. Take the pivot at the hip joint and let counterclockwise torque be positive. There are also forces on the leg exerted by the hip joint but these forces produce no torque and aren't shown.  $\sum \tau_z = 0$  for no rotation.

**EXECUTE:** (a)  $\sum \tau_z = 0$  gives  $T(10 \text{ cm})(\sin \theta) - w(44 \text{ cm})(\cos \theta) = 0$ .

$$T = \frac{4.4w \cos \theta}{\sin \theta} = \frac{4.4w}{\tan \theta} \text{ and for } \theta = 60^\circ, T = \frac{4.4(15 \text{ kg})(9.80 \text{ m/s}^2)}{\tan 60^\circ} = 370 \text{ N.}$$

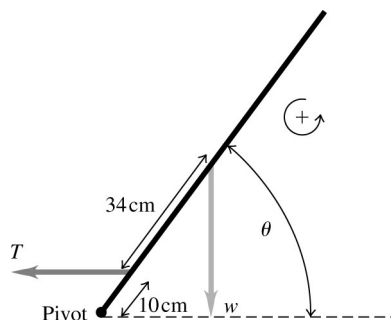


Figure 11.53

(b) For  $\theta = 5^\circ$ ,  $T = 7400 \text{ N}$ . The tension is much greater when he just starts to raise his leg off the ground.

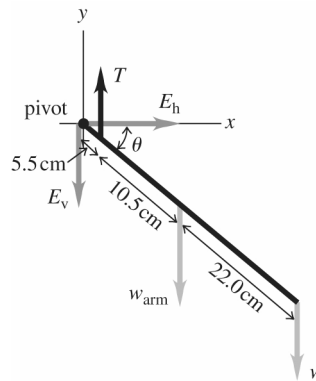
(c)  $T \rightarrow \infty$  as  $\theta \rightarrow 0$ . The person could not raise his leg. If the leg is horizontal so  $\theta$  is zero, the moment arm for  $T$  is zero and  $T$  produces no torque to rotate the leg against the torque due to its weight.

**EVALUATE:** Most of the exercise benefit of leg-raises occurs when the person just starts to raise his legs off the ground.

**11.54. IDENTIFY:** The arm is stationary, so the forces and torques must each balance.

**SET UP:**  $\sum \tau = 0$ ,  $\sum F_x = 0$ ,  $\sum F_y = 0$ . Let the forearm be at an angle  $\phi$  below the horizontal. Take the pivot at the elbow joint and let counterclockwise torques be positive. Let  $+y$  be upward and let  $+x$  be to the right. Each forearm has mass  $m_{\text{arm}} = \frac{1}{2}(0.0600)(72 \text{ kg}) = 2.16 \text{ kg}$ . The weight held in each hand is  $w = mg$ , with  $m = 7.50 \text{ kg}$ .  $\vec{T}$  is the force the biceps muscle exerts on the forearm.  $\vec{E}$  is the force exerted by the elbow and has components  $E_v$  and  $E_h$ .

**EXECUTE: (a)** The free-body diagram is shown in Figure 11.54.



**Figure 11.54**

**(b)**  $\sum \tau = 0$  gives  $T(5.5 \text{ cm})(\cos \theta) - w_{\text{arm}}(16.0 \text{ cm})(\cos \theta) - w(38.0 \text{ cm})(\cos \theta) = 0$

$$T = \frac{16.0w_{\text{arm}} + 38.0w}{5.5} = \frac{16.0(2.16 \text{ kg})(9.80 \text{ m/s}^2) + 38.0(7.50 \text{ kg})(9.80 \text{ m/s}^2)}{5.5} = 569 \text{ N}$$

**(c)**  $\sum F_x = 0$  gives  $E_h = 0$ .  $\sum F_y = 0$  gives  $T - E_v - w_{\text{arm}} - w = 0$ , so

$$E_v = T - w_{\text{arm}} - w = 569 \text{ N} - (2.16 \text{ kg})(9.80 \text{ m/s}^2) - (7.50 \text{ kg})(9.80 \text{ m/s}^2) = 474 \text{ N}$$

Since we calculate  $E_v$  to be positive, we correctly assumed that it was downward when we drew the free-body diagram.

**(d)** The weight and the pull of the biceps are both always vertical in this situation, so the factor  $\cos \theta$  divides out of the  $\sum \tau = 0$  equation in part (b). Therefore the force  $T$  stays the same as she raises her arm.

**EVALUATE:** The biceps force must be much greater than the weight of the forearm and the weight in her hand because it has such a small lever arm compared to those two forces.

**11.55. IDENTIFY:** The presence of the fetus causes the woman's center of mass to shift forward. Figure 11.55 (next page) shows the cylinder and sphere model suggested in the problem.

**SET UP:**  $x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ . The mass of each object can be considered as located at its center

of mass, at its geometrical center. Use coordinates that have the origin at the center of the cylinder and the  $x$ -axis horizontal.

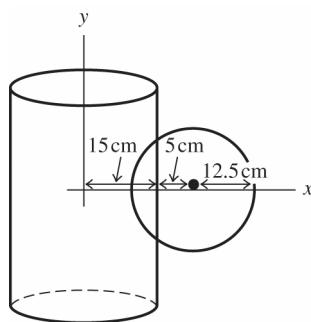


Figure 11.55

**EXECUTE:** (a) Before pregnancy,  $x_{\text{cm},i} = 0$ . The center of mass of the pregnant woman is at

$$x_{\text{cm},f} = \frac{0 + (20 \text{ cm})(10 \text{ kg})}{60 \text{ kg} + 10 \text{ kg}} = 2.9 \text{ cm. Her center of mass moves a horizontal distance of 2.9 cm forward.}$$

(b) The woman must shift her upper body backward to keep her center of mass from extending past her feet.

(c) The unnatural posture and curved back strains the back muscles.

**EVALUATE:** Observation of a pregnant woman walking should confirm the results found here.

**11.56. IDENTIFY:** Apply the first and second conditions of equilibrium to each rod.

**SET UP:** Apply  $\sum F_y = 0$  with  $+y$  upward and apply  $\sum \tau_z = 0$  with the pivot at the point of suspension for each rod.

**EXECUTE:** (a) The free-body diagram for each rod is given in Figure 11.56.

(b)  $\sum \tau_z = 0$  for the lower rod:  $(6.0 \text{ N})(4.0 \text{ cm}) = w_A(8.0 \text{ cm})$  and  $w_A = 3.0 \text{ N}$ .

$\sum F_y = 0$  for the lower rod:  $S_3 = 6.0 \text{ N} + w_A = 9.0 \text{ N}$ .

$\sum \tau_z = 0$  for the middle rod:  $w_B(3.0 \text{ cm}) = (5.0 \text{ cm})S_3$  and  $w_B = \left(\frac{5.0}{3.0}\right)(9.0 \text{ N}) = 15.0 \text{ N}$ .

$\sum F_y = 0$  for the middle rod:  $S_2 = 9.0 \text{ N} + S_3 = 24.0 \text{ N}$ .

$\sum \tau_z = 0$  for the upper rod:  $S_2(2.0 \text{ cm}) = w_C(6.0 \text{ cm})$  and  $w_C = \left(\frac{2.0}{6.0}\right)(24.0 \text{ N}) = 8.0 \text{ N}$ .

$\sum F_y = 0$  for the upper rod:  $S_1 = S_2 + w_C = 32.0 \text{ N}$ .

In summary,  $w_A = 3.0 \text{ N}$ ,  $w_B = 15.0 \text{ N}$ ,  $w_C = 8.0 \text{ N}$ .  $S_1 = 32.0 \text{ N}$ ,  $S_2 = 24.0 \text{ N}$ ,  $S_3 = 9.0 \text{ N}$ .

(c) The center of gravity of the entire mobile must lie along a vertical line that passes through the point where  $S_1$  is located.

**EVALUATE:** For the mobile as a whole the vertical forces must balance, so  $S_1 = w_A + w_B + w_C + 6.0 \text{ N}$ .

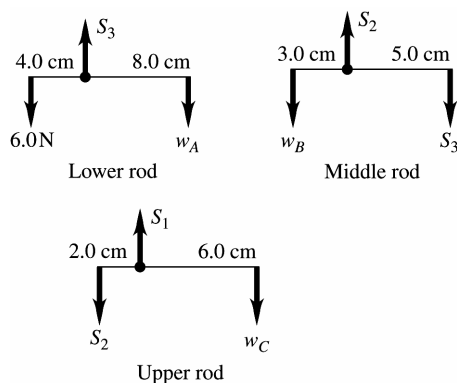


Figure 11.56



**11.57. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the beam.

**SET UP:** The free-body diagram for the beam is given in Figure 11.57.

**EXECUTE:**  $\sum \tau_z = 0$ , axis at hinge, gives  $T(6.0 \text{ m})(\sin 40^\circ) - (6490 \text{ N})(3.75 \text{ m})(\cos 30^\circ) = 0$  and  $T = 5500 \text{ N}$ .

**EVALUATE:** The tension in the cable is less than the weight of the beam.  $T \sin 40^\circ$  is the component of  $T$  that is perpendicular to the beam.

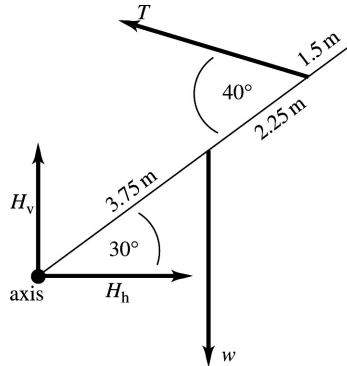


Figure 11.57

**11.58. IDENTIFY:** Apply the first and second conditions of equilibrium to the drawbridge.

**SET UP:** The free-body diagram for the drawbridge is given in Figure 11.58 (next page).  $H_v$  and  $H_h$  are the components of the force the hinge exerts on the bridge. In part (c), apply  $\sum \tau_z = I\alpha$  to the rotating bridge and in part (d) apply energy conservation to the bridge.

**EXECUTE:** (a)  $\sum \tau_z = 0$  with the axis at the hinge gives  $-w(7.0 \text{ m})(\cos 37^\circ) + T(3.5 \text{ m})(\sin 37^\circ) = 0$  and

$$T = 2w \frac{\cos 37^\circ}{\sin 37^\circ} = 2 \frac{(45,000 \text{ N})}{\tan 37^\circ} = 1.19 \times 10^5 \text{ N}.$$

(b)  $\sum F_x = 0$  gives  $H_h = T = 1.19 \times 10^5 \text{ N}$ .  $\sum F_y = 0$  gives  $H_v = w = 4.50 \times 10^4 \text{ N}$ .

$$H = \sqrt{H_h^2 + H_v^2} = 1.27 \times 10^5 \text{ N}. \quad \tan \theta = \frac{H_v}{H_h} \quad \text{and} \quad \theta = 20.7^\circ. \quad \text{The hinge force has magnitude } 1.27 \times 10^5 \text{ N}$$

and is directed at  $20.7^\circ$  above the horizontal.

(c) We can treat the bridge as a uniform bar rotating around one end, so  $I = 1/3 mL^2$ .  $\sum \tau_z = I\alpha_z$  gives

$$mg(L/2)\cos 37^\circ = 1/3 mL^2\alpha. \quad \text{Solving for } \alpha \text{ gives } \alpha = \frac{3g\cos 37^\circ}{2L} = \frac{3(9.80 \text{ m/s}^2)\cos 37^\circ}{2(14.0 \text{ m})} = 0.839 \text{ rad/s}^2.$$

(d) Energy conservation gives  $U_1 = K_2$ , giving  $mgh = 1/2 I\omega^2 = (1/2)(1/3 mL^2)\omega^2$ . Trigonometry gives

$$h = L/2 \sin 37^\circ. \quad \text{Canceling } m, \text{ the energy conservation equation gives } g(L/2) \sin 37^\circ = (1/6)L^2\omega^2. \quad \text{Solving}$$

$$\text{for } \omega \text{ gives } \omega = \sqrt{\frac{3g \sin 37^\circ}{L}} = \sqrt{\frac{3(9.80 \text{ m/s}^2) \sin 37^\circ}{14.0 \text{ m}}} = 1.12 \text{ rad/s}.$$

**EVALUATE:** The hinge force is not directed along the bridge. If it were, it would have zero torque for an axis at the center of gravity of the bridge and for that axis the tension in the cable would produce a single, unbalanced torque.

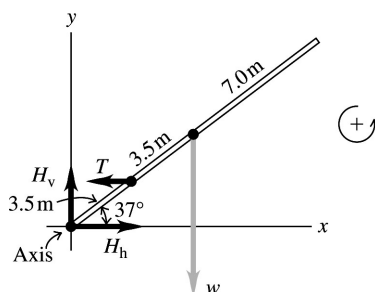


Figure 11.58

- 11.59. IDENTIFY:** The amount the tendon stretches depends on Young's modulus for the tendon material. The foot is in rotational equilibrium, so the torques on it balance.

**SET UP:**  $Y = \frac{F_T/A}{\Delta l/l_0}$ . The foot is in rotational equilibrium, so  $\sum \tau_z = 0$ .

**EXECUTE:** (a) The free-body diagram for the foot is given in Figure 11.59.  $T$  is the tension in the tendon and  $A$  is the force exerted on the foot by the ankle.  $n = (75 \text{ kg})g$ , the weight of the person.

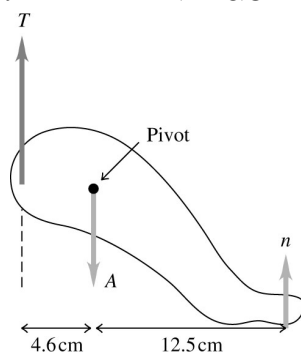


Figure 11.59

- (b) Apply  $\sum \tau_z = 0$ , letting counterclockwise torques be positive and with the pivot at the ankle:

$$T(4.6 \text{ cm}) - n(12.5 \text{ cm}) = 0. \quad T = \left( \frac{12.5 \text{ cm}}{4.6 \text{ cm}} \right) (75 \text{ kg})(9.80 \text{ m/s}^2) = 2000 \text{ N}, \text{ which is 2.72 times his weight.}$$

- (c) The foot pulls downward on the tendon with a force of 2000 N.

$$\Delta l = \left( \frac{F_T}{YA} \right) l_0 = \frac{2000 \text{ N}}{(1470 \times 10^6 \text{ Pa})(78 \times 10^{-6} \text{ m}^2)} (25 \text{ cm}) = 4.4 \text{ mm.}$$

**EVALUATE:** The tension is quite large, but the Achilles tendon stretches about 4.4 mm, which is only about 1/6 of an inch, so it must be a strong tendon.

- 11.60. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the beam.

**SET UP:** The center of mass of the beam is 1.0 m from the suspension point.

**EXECUTE:** (a) Taking torques about the suspension point,  
 $w(4.00 \text{ m})\sin 30^\circ + (140.0 \text{ N})(1.00 \text{ m})\sin 30^\circ = (100 \text{ N})(2.00 \text{ m})\sin 30^\circ$ .

The common factor of  $\sin 30^\circ$  divides out, from which  $w = 15.0 \text{ N}$ .

(b) In this case, a common factor of  $\sin 45^\circ$  would be factored out, and the result would be the same.

**EVALUATE:** All the forces are vertical, so the moments are all horizontal and all contain the factor  $\sin \theta$ , where  $\theta$  is the angle the beam makes with the horizontal.

- 11.61. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the flagpole.

**SET UP:** The free-body diagram for the flagpole is given in Figure 11.61. Let clockwise torques be positive.  $\theta$  is the angle the cable makes with the horizontal pole.

**EXECUTE:** (a) Taking torques about the hinged end of the pole

$$(200 \text{ N})(2.50 \text{ m}) + (600 \text{ N})(5.00 \text{ m}) - T_y(5.00 \text{ m}) = 0. \quad T_y = 700 \text{ N.} \text{ The } x\text{-component of the tension is then}$$

$T_x = \sqrt{(1000 \text{ N})^2 - (700 \text{ N})^2} = 714 \text{ N}$ .  $\tan \theta = \frac{h}{5.00 \text{ m}} = \frac{T_y}{T_x}$ . The height above the pole that the wire must

be attached is  $(5.00 \text{ m}) \frac{700}{714} = 4.90 \text{ m}$ .

(b) The  $y$ -component of the tension remains 700 N. Now  $\tan \theta = \frac{4.40 \text{ m}}{5.00 \text{ m}}$  and  $\theta = 41.35^\circ$ , so

$$T = \frac{T_y}{\sin \theta} = \frac{700 \text{ N}}{\sin 41.35^\circ} = 1060 \text{ N}, \text{ an increase of } 60 \text{ N}.$$

**EVALUATE:** As the wire is fastened closer to the hinged end of the pole, the moment arm for  $T$  decreases and  $T$  must increase to produce the same torque about that end.

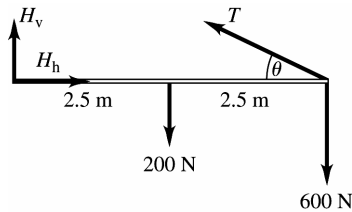


Figure 11.61

- 11.62. IDENTIFY:** Apply  $\sum \vec{F} = 0$  to each object, including the point where  $D$ ,  $C$ , and  $B$  are joined. Apply  $\sum \tau_z = 0$  to the rod.

**SET UP:** To find  $T_C$  and  $T_D$ , use a coordinate system with axes parallel to the cords.

**EXECUTE:**  $A$  and  $B$  are straightforward, the tensions being the weights suspended:

$$T_A = (0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.353 \text{ N} \text{ and } T_B = (0.0240 \text{ kg} + 0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.588 \text{ N}.$$

Applying  $\sum F_x = 0$  and  $\sum F_y = 0$  to the point where the cords are joined,  $T_C = T_B \cos 36.9^\circ = 0.470 \text{ N}$  and

$T_D = T_B \sin 36.9^\circ = 0.353 \text{ N}$ . To find  $T_E$ , take torques about the point where string  $F$  is attached.

$$T_E(1.00 \text{ m}) = T_D \sin 36.9^\circ(0.800 \text{ m}) + T_C \sin 53.1^\circ(0.200 \text{ m}) + (0.120 \text{ kg})(9.80 \text{ m/s}^2)(0.500 \text{ m}) \text{ and}$$

$$T_E = 0.833 \text{ N}.$$

$T_F$  may be found similarly, or from the fact that  $T_E + T_F$  must be the total weight of the ornament.

$$(0.180 \text{ kg})(9.80 \text{ m/s}^2) = 1.76 \text{ N}, \text{ from which } T_F = 0.931 \text{ N}.$$

**EVALUATE:** The vertical line through the spheres is closer to  $F$  than to  $E$ , so we expect  $T_F > T_E$ , and this is indeed the case.

- 11.63. IDENTIFY:** The torques must balance since the person is not rotating.

**SET UP:** Figure 11.63a (next page) shows the distances and angles.  $\theta + \phi = 90^\circ$ .  $\theta = 56.3^\circ$  and

$\phi = 33.7^\circ$ . The distances  $x_1$  and  $x_2$  are  $x_1 = (90 \text{ cm}) \cos \theta = 50.0 \text{ cm}$  and  $x_2 = (135 \text{ cm}) \cos \phi = 112 \text{ cm}$ .

The free-body diagram for the person is given in Figure 11.63b.  $w_1 = 277 \text{ N}$  is the weight of his feet and

legs, and  $w_t = 473 \text{ N}$  is the weight of his trunk.  $n_f$  and  $f_f$  are the total normal and friction forces exerted

on his feet and  $n_h$  and  $f_h$  are those forces on his hands. The free-body diagram for his legs is given in

Figure 11.63c.  $F$  is the force exerted on his legs by his hip joints. For balance,  $\sum \tau_z = 0$ .

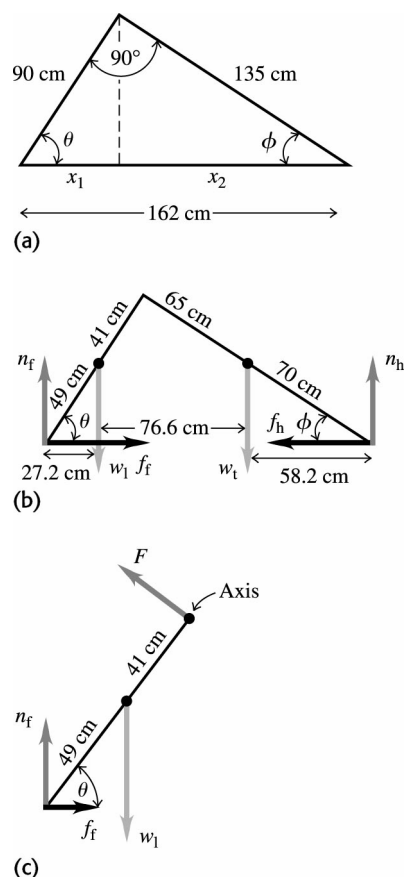


Figure 11.63

**EXECUTE:** (a) Consider the force diagram of Figure 11.63b.  $\Sigma \tau_z = 0$  with the pivot at his feet and counterclockwise torques positive gives  $n_h(162 \text{ cm}) - (277 \text{ N})(27.2 \text{ cm}) - (473 \text{ N})(103.8 \text{ cm}) = 0$ .

$n_h = 350 \text{ N}$ , so there is a normal force of 175 N at each hand.  $n_f + n_h - w_l - w_t = 0$  so

$n_f = w_l + w_t - n_h = 750 \text{ N} - 350 \text{ N} = 400 \text{ N}$ , so there is a normal force of 200 N at each foot.

(b) Consider the force diagram of Figure 11.63c.  $\Sigma \tau_z = 0$  with the pivot at his hips and counterclockwise torques positive gives  $f_f(74.9 \text{ cm}) + w_l(22.8 \text{ cm}) - n_f(50.0 \text{ cm}) = 0$ .

$f_f = \frac{(400 \text{ N})(50.0 \text{ cm}) - (277 \text{ N})(22.8 \text{ cm})}{74.9 \text{ cm}} = 182.7 \text{ N}$ . There is a friction force of 91 N at each foot.

$\Sigma F_x = 0$  in Figure 11.63b gives  $f_h = f_f$ , so there is a friction force of 91 N at each hand.

**EVALUATE:** In this position the normal forces at his feet and at his hands don't differ very much.

**11.64. IDENTIFY:** The bar is in equilibrium until the cable breaks, so the forces and torques on it must all balance.

**SET UP:** Look at the bar when the cable is just ready to break. At that time, the tension in it is 455 N.

$\Sigma F_x = 0$ ,  $\Sigma F_y = 0$ ,  $\Sigma \tau_z = 0$ .

**EXECUTE:** (a) Take torques about the hinge, calling  $L$  your distance from the hinge.  $\Sigma \tau_z = 0$  gives  $(455 \text{ N})(\sin 37.0^\circ)(8.00 \text{ m}) - (65.0 \text{ kg})(9.80 \text{ m/s}^2)L(\cos 64.0^\circ) - (30.0 \text{ kg})(9.80 \text{ m/s}^2)(4.00 \text{ m})(\cos 64.0^\circ) = 0$ . Solving for  $L$  gives  $L = 6.00 \text{ m}$  from the hinge, which is 2.00 m from the upper end of the bar.

(b) Calling  $H$  the magnitude of the hinge force,  $\Sigma F_x = 0$  gives  $H_x = (455 \text{ N})(\cos 27.0^\circ) = 405.4 \text{ N}$ .

$\Sigma F_y = 0$  gives  $H_y = (65.0 \text{ kg})(9.80 \text{ m/s}^2) + (30.0 \text{ kg})(9.80 \text{ m/s}^2) + (455 \text{ N})(\sin 27.0^\circ) = 1138 \text{ N}$ .

$$H = \sqrt{H_x^2 + H_y^2} = \sqrt{(405.4 \text{ N})^2 + (1138 \text{ N})^2} = 1210 \text{ N}.$$

The angle that  $H$  makes above the horizontal is  $\phi = \arctan \frac{H_y}{H_x} = \arctan \frac{1138 \text{ N}}{405.4 \text{ N}} = 70.4^\circ$ .

**EVALUATE:** The bar rises at  $64.0^\circ$  above the horizontal but the hinge force points at  $70.4^\circ$  above the horizontal, so the hinge force does not point along the bar.

- 11.65. IDENTIFY:** Apply the equilibrium conditions to the crate. When the crate is on the verge of tipping it touches the floor only at its lower left-hand corner and the normal force acts at this point. The minimum coefficient of static friction is given by the equation  $f_s = \mu_s n$ .

**SET UP:** The free-body diagram for the crate when it is ready to tip is given in Figure 11.65.

**EXECUTE:** (a)  $\sum \tau_z = 0$  gives  $P(1.50 \text{ m})\sin 53.0^\circ - w(1.10 \text{ m}) = 0$ .

$$P = w \left( \frac{1.10 \text{ m}}{[1.50 \text{ m}][\sin 53.0^\circ]} \right) = 1.15 \times 10^3 \text{ N}$$

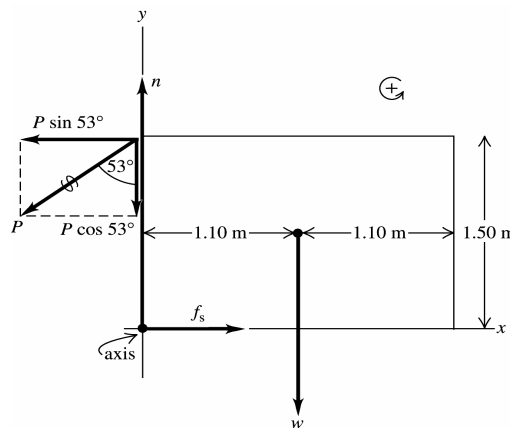
(b)  $\sum F_y = 0$  gives  $n - w - P \cos 53.0^\circ = 0$ .

$$n = w + P \cos 53.0^\circ = 1250 \text{ N} + (1.15 \times 10^3 \text{ N}) \cos 53^\circ = 1.94 \times 10^3 \text{ N}$$

(c)  $\sum F_x = 0$  gives  $f_s = P \sin 53.0^\circ = (1.15 \times 10^3 \text{ N}) \sin 53.0^\circ = 918 \text{ N}$ .

$$(d) \mu_s = \frac{f_s}{n} = \frac{918 \text{ N}}{1.94 \times 10^3 \text{ N}} = 0.473$$

**EVALUATE:** The normal force is greater than the weight because  $P$  has a downward component.



**Figure 11.65**

- 11.66. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the meterstick.

**SET UP:** The wall exerts an upward static friction force  $f$  and a horizontal normal force  $n$  on the stick. Denote the length of the stick by  $l$ .  $f = \mu_s n$ .

**EXECUTE:** (a) Taking torques about the right end of the stick, the friction force is half the weight of the stick,  $f = w/2$ . Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point), and noting that the moment arm of the normal force is  $l \tan \theta$ ,  $n \tan \theta = w/2$ . Then,  $(f/n) = \tan \theta < 0.40$ , so  $\theta < \arctan (0.40) = 22^\circ$ .

(b) Taking torques as in part (a),  $fl = w \frac{l}{2} + w(l-x)$  and  $nl \tan \theta = w \frac{l}{2} + wx$ . In terms of the coefficient of

friction  $\mu_s$ ,  $\mu_s > \frac{f}{n} = \frac{l/2 + (l-x)}{l/2 + x} \tan \theta = \frac{3l-2x}{l+2x} \tan \theta$ . Solving for  $x$ ,  $x > \frac{l}{2} \frac{3 \tan \theta - \mu_s}{\mu_s + \tan \theta} = 30.2 \text{ cm}$ .

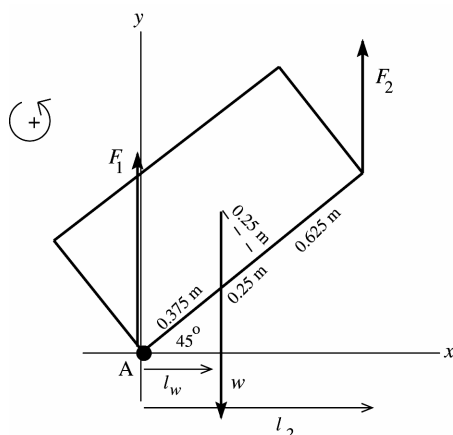
(c) In the above expression, setting  $x = 10 \text{ cm}$  and  $l = 100 \text{ cm}$  and solving for  $\mu_s$  gives

$$\mu_s > \frac{(3-20/l) \tan \theta}{1+20/l} = 0.625.$$

**EVALUATE:** For  $\theta = 15^\circ$  and without the block suspended from the stick, a value of  $\mu_s \geq 0.268$  is required to prevent slipping. Hanging the block from the stick increases the value of  $\mu_s$  that is required.

**11.67. IDENTIFY:** Apply the first and second conditions of equilibrium to the crate.

**SET UP:** The free-body diagram for the crate is given in Figure 11.67.



$$l_w = (0.375 \text{ m}) \cos 45^\circ$$

$$l_2 = (1.25 \text{ m}) \cos 45^\circ$$

Let  $\vec{F}_1$  and  $\vec{F}_2$  be the vertical forces exerted by you and your friend. Take the origin at the lower left-hand corner of the crate (point A).

**Figure 11.67**

**EXECUTE:**  $\sum F_y = ma_y$  gives  $F_1 + F_2 - w = 0$

$$F_1 + F_2 = w = (200 \text{ kg})(9.80 \text{ m/s}^2) = 1960 \text{ N}$$

$\sum \tau_A = 0$  gives  $F_2 l_2 - w l_w = 0$

$$F_2 = w \left( \frac{l_w}{l_2} \right) = 1960 \text{ N} \left( \frac{0.375 \text{ m} \cos 45^\circ}{1.25 \text{ m} \cos 45^\circ} \right) = 590 \text{ N}$$

Then  $F_1 = w - F_2 = 1960 \text{ N} - 590 \text{ N} = 1370 \text{ N}$ .

**EVALUATE:** The person below (you) applies a force of 1370 N. The person above (your friend) applies a force of 590 N. It is better to be the person above. As the sketch shows, the moment arm for  $\vec{F}_1$  is less than for  $\vec{F}_2$ , so must have  $F_1 > F_2$  to compensate.

**11.68. IDENTIFY:** Apply the first and second conditions for equilibrium to the forearm.

**SET UP:** The free-body diagram is given in Figure 11.68a, and when holding the weight in Figure 11.68b. Let  $+y$  be upward.

**EXECUTE: (a)**  $\sum \tau_{\text{Elbow}} = 0$  gives  $F_B(3.80 \text{ cm}) = (15.0 \text{ N})(15.0 \text{ cm})$  and  $F_B = 59.2 \text{ N}$ .

**(b)**  $\sum \tau_{\text{Elbow}} = 0$  gives  $F_B(3.80 \text{ cm}) = (15.0 \text{ N})(15.0 \text{ cm}) + (80.0 \text{ N})(33.0 \text{ cm})$  and  $F_B = 754 \text{ N}$ . The biceps force has a short lever arm, so it must be large to balance the torques.

**(c)**  $\sum F_y = 0$  gives  $-F_E + F_B - 15.0 \text{ N} - 80.0 \text{ N} = 0$  and  $F_E = 754 \text{ N} - 15.0 \text{ N} - 80.0 \text{ N} = 659 \text{ N}$ .

**EVALUATE: (d)** The biceps muscle acts perpendicular to the forearm, so its lever arm stays the same, but those of the other two forces decrease as the arm is raised. Therefore the tension in the biceps muscle decreases.

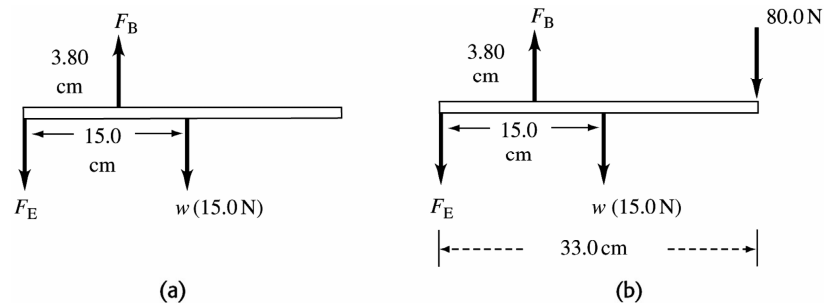


Figure 11.68

**11.69. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the forearm.

**SET UP:** The free-body diagram for the forearm is given in Figure 11.10 in the textbook.

**EXECUTE:** (a)  $\sum \tau_z = 0$ , axis at elbow gives

$$wL - (T \sin \theta)D = 0, \sin \theta = \frac{h}{\sqrt{h^2 + D^2}} \text{ so } w = T \frac{hD}{L\sqrt{h^2 + D^2}}.$$

$$w_{\max} = T_{\max} \frac{hD}{L\sqrt{h^2 + D^2}}.$$

(b)  $\frac{dw_{\max}}{dD} = \frac{T_{\max} h}{L\sqrt{h^2 + D^2}} \left( 1 - \frac{D^2}{h^2 + D^2} \right)$ ; the derivative is positive.

**EVALUATE:** (c) The result of part (b) shows that  $w_{\max}$  increases when  $D$  increases, since the derivative is positive.  $w_{\max}$  is larger for a chimp since  $D$  is larger.

**11.70. IDENTIFY:** The beam is at rest, so the forces and torques on it must all balance.

**SET UP:** The cables could point inward toward each other or outward away from each other. We shall assume they point away from each other. Call  $d$  the distance of the center of gravity from the left end, call  $w$  the weight of the beam, and call  $T$  the tension in the right-hand cable.  $\sum F_x = 0$ ,  $\sum F_y = 0$ ,  $\sum \tau_z = 0$ .

**EXECUTE:**  $\sum F_x = 0$  gives  $(620 \text{ N})(\sin 30.0^\circ) - T(\sin 50.0^\circ) = 0$ , so  $T = 404.68 \text{ N}$ .

$\sum F_y = 0$  gives  $(620 \text{ N})(\cos 30.0^\circ) + (404.68 \text{ N})(\cos 50.0^\circ) - w = 0$ , so  $w = 797 \text{ N}$ .

Taking torques about the left end,  $\sum \tau_z = 0$  gives  $(404.68 \text{ N})(\cos 50.0^\circ)(4.00 \text{ m}) - (797 \text{ N})d = 0$ , so  $d = 1.31 \text{ m}$  from the left end of the beam, or  $2.69 \text{ m}$  from the right end.

**EVALUATE:** The center of gravity is closer to the cable having the greater tension. The answer would be no different if we assumed that the cables pointed inward toward each other.

**11.71. IDENTIFY:** The beam is at rest, so the forces and torques on it must all balance.

**SET UP:**  $\sum F_x = 0$ ,  $\sum F_y = 0$ ,  $\sum \tau_z = 0$ . Look at the situation where the cable is just about to break, in which case the tension in it is  $650 \text{ N}$ .

**EXECUTE:** (a) Taking torques about the hinge, with  $L$  the length of the beam,  $\sum \tau_z = 0$  gives  $(650 \text{ N})(\sin 30.0^\circ)L - mg(L/2)(\cos 22.0^\circ) = 0$ , which gives  $m = 71.535 \text{ kg}$ , which rounds to  $71.5 \text{ kg}$ .

(b) Now  $m = 61.5 \text{ kg}$ . Taking torques about the hinge and calling  $T$  the tension, we have  $LT(\sin 30.0^\circ) = (61.5 \text{ kg})(9.80 \text{ m/s}^2)(L/2)(\cos 22.0^\circ)$ , so  $T = 559 \text{ N}$ .

Call  $H$  the magnitude of the hinge force.  $\sum F_x = 0$  gives  $H_x = (559 \text{ N})(\sin 38.0^\circ) = 344.24 \text{ N}$

$\sum F_y = 0$  gives  $H_y + (559 \text{ N})(\cos 38.0^\circ) - (61.5 \text{ kg})(9.80 \text{ m/s}^2)$ , so  $H_y = 162.1 \text{ N}$ .

$H = \sqrt{H_x^2 + H_y^2} = \sqrt{(344.24 \text{ N})^2 + (162.1 \text{ N})^2} = 380 \text{ N}$ . The angle that the hinge force makes above the

horizontal is  $\phi = \arctan \frac{H_y}{H_x} = \arctan \frac{162.1 \text{ N}}{344.24 \text{ N}} = 25.2^\circ$ .

**EVALUATE:** The hinge force points at  $25.2^\circ$  above the horizontal but the beam makes an angle of  $22.0^\circ$  below the horizontal, so the hinge force does not point along the beam. We never needed to know the length of the beam since it always canceled out in the equations.

**11.72. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the wheel.

**SET UP:** Take torques about the upper corner of the curb.

**EXECUTE:** The force  $\vec{F}$  acts at a perpendicular distance  $R - h$  and the weight acts at a perpendicular distance  $\sqrt{R^2 - (R - h)^2} = \sqrt{2Rh - h^2}$ . Setting the torques equal for the minimum necessary force,

$$F = mg \frac{\sqrt{2Rh - h^2}}{R - h}.$$

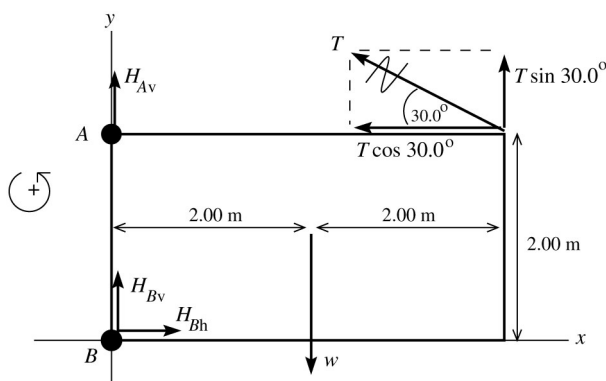
**(b)** The torque due to gravity is the same, but the force  $\vec{F}$  acts at a perpendicular distance  $2R - h$ ,

so the minimum force is  $(mg)\sqrt{2Rh - h^2}/(2R - h)$ .

**EVALUATE:** **(c)** Less force is required when the force is applied at the top of the wheel, since in this case  $\vec{F}$  has a larger moment arm.

**11.73. IDENTIFY:** Apply the first and second conditions of equilibrium to the gate.

**SET UP:** The free-body diagram for the gate is given in Figure 11.73.



**Figure 11.73**

Use coordinates with the origin at  $B$ . Let  $\vec{H}_A$  and  $\vec{H}_B$  be the forces exerted by the hinges at  $A$  and  $B$ . The problem states that  $\vec{H}_A$  has no horizontal component. Replace the tension  $\vec{T}$  by its horizontal and vertical components.

**EXECUTE:** **(a)**  $\sum \tau_B = 0$  gives  $+(T \sin 30.0^\circ)(4.00 \text{ m}) + (T \cos 30.0^\circ)(2.00 \text{ m}) - w(2.00 \text{ m}) = 0$   
 $T(2 \sin 30.0^\circ + \cos 30.0^\circ) = w$

$$T = \frac{w}{2 \sin 30.0^\circ + \cos 30.0^\circ} = \frac{700 \text{ N}}{2 \sin 30.0^\circ + \cos 30.0^\circ} = 375 \text{ N}.$$

**(b)**  $\sum F_x = ma_x$  says  $H_{Bh} - T \cos 30.0^\circ = 0$

$$H_{Bh} = T \cos 30.0^\circ = (375 \text{ N}) \cos 30.0^\circ = 325 \text{ N}.$$

**(c)**  $\sum F_y = ma_y$  says  $H_{Av} + H_{Bv} + T \sin 30.0^\circ - w = 0$

$$H_{Av} + H_{Bv} = w - T \sin 30.0^\circ = 700 \text{ N} - (375 \text{ N}) \sin 30.0^\circ = 512 \text{ N}.$$

**EVALUATE:**  $T$  has a horizontal component to the left so  $H_{Bh}$  must be to the right, as these are the only two horizontal forces. Note that we cannot determine  $H_{Av}$  and  $H_{Bv}$  separately, only their sum.

**11.74. IDENTIFY:** Use  $x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$  to locate the  $x$ -coordinate of the center of gravity of the block combinations.



**SET UP:** The center of mass and the center of gravity are the same point. For two identical blocks, the center of gravity is midway between the center of the two blocks.

**EXECUTE: (a)** The center of gravity of the top block can be as far out as the edge of the lower block. The center of gravity of this combination is then  $3L/4$  to the left of the right edge of the upper block, so the overhang is  $3L/4$ .

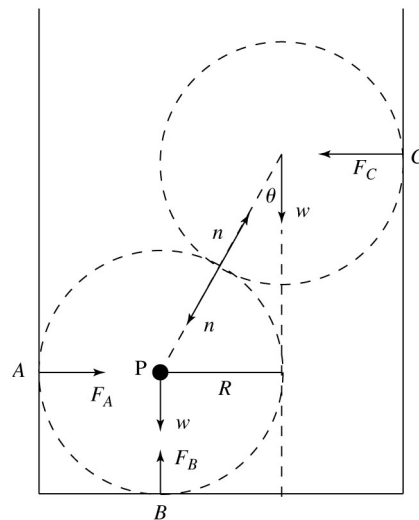
**(b)** Take the two-block combination from part (a), and place it on top of the third block such that the overhang of  $3L/4$  is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is  $-L/6$  and so the largest possible overhang is  $(3L/4) + (L/6) = 11L/12$ . Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of  $L/8$ , for a total of  $25L/24$ .

**(c)** As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

**EVALUATE:** The sequence of maximum overhangs is  $\frac{18L}{24}, \frac{22L}{24}, \frac{25L}{24}, \dots$ . The increase of overhang when one more block is added is decreasing.

**11.75. IDENTIFY:** Apply the first and second conditions of equilibrium, first to both marbles considered as a composite object and then to the bottom marble.

**(a) SET UP:** The forces on each marble are shown in Figure 11.75.



**EXECUTE:**

$$F_B = 2w = 1.47 \text{ N}$$

$$\sin \theta = R/2R \text{ so } \theta = 30^\circ$$

$$\sum \tau_z = 0, \text{ axis at } P$$

$$F_C(2R \cos \theta) - wR = 0$$

$$F_C = \frac{mg}{2 \cos 30^\circ} = 0.424 \text{ N}$$

$$F_A = F_C = 0.424 \text{ N}$$

**Figure 11.75**

**(b)** Consider the forces on the bottom marble. The horizontal forces must sum to zero, so  $F_A = n \sin \theta$ .

$$n = \frac{F_A}{\sin 30^\circ} = 0.848 \text{ N}$$

Could use instead that the vertical forces sum to zero

$$F_B - mg - n \cos \theta = 0$$

$$n = \frac{F_B - mg}{\cos 30^\circ} = 0.848 \text{ N, which checks.}$$

**EVALUATE:** If we consider each marble separately, the line of action of every force passes through the center of the marble so there is clearly no torque about that point for each marble. We can use the results we obtained to show that  $\sum F_x = 0$  and  $\sum F_y = 0$  for the top marble.

**11.76. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the right-hand beam.

**SET UP:** Use the hinge as the axis of rotation and take counterclockwise rotation as positive. If  $F_{\text{wire}}$  is the tension in each wire and  $w = 260 \text{ N}$  is the weight of each beam,  $2F_{\text{wire}} - 2w = 0$  and  $F_{\text{wire}} = w$ . Let  $L$  be the length of each beam.

**EXECUTE:** (a)  $\sum \tau_z = 0$  gives  $F_{\text{wire}}L \sin \frac{\theta}{2} - F_c \frac{L}{2} \cos \frac{\theta}{2} - w \frac{L}{2} \sin \frac{\theta}{2} = 0$ , where  $\theta$  is the angle between the beams and  $F_c$  is the force exerted by the cross bar. The length drops out, and all other quantities except  $F_c$  are

known, so  $F_c = \frac{F_{\text{wire}} \sin(\theta/2) - \frac{1}{2} w \sin(\theta/2)}{\frac{1}{2} \cos(\theta/2)} = (2F_{\text{wire}} - w) \tan \frac{\theta}{2}$ . Therefore

$$F_c = (260 \text{ N}) \tan \frac{53^\circ}{2} = 130 \text{ N}.$$

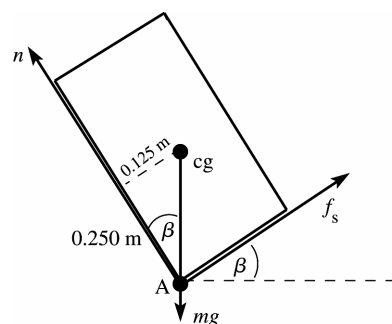
(b) The crossbar is under compression, as can be seen by imagining the behavior of the two beams if the crossbar were removed. It is the crossbar that holds them apart.

(c) The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmetry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the crossbar. The hinge exerts a force 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it's instructive to visualize what the beams would do if the hinge were removed.

**EVALUATE:** The force exerted on each beam increases as  $\theta$  increases and exceeds the weight of the beam for  $\theta \geq 90^\circ$ .

**11.77. IDENTIFY:** Apply the first and second conditions of equilibrium to the bale.

(a) **SET UP:** Find the angle where the bale starts to tip. When it starts to tip only the lower left-hand corner of the bale makes contact with the conveyor belt. Therefore the line of action of the normal force  $n$  passes through the left-hand edge of the bale. Consider  $\sum \tau_z = 0$  with point A at the lower left-hand corner. Then  $\tau_n = 0$  and  $\tau_f = 0$ , so it must be that  $\tau_{mg} = 0$  also. This means that the line of action of the gravity must pass through point A. Thus the free-body diagram must be as shown in Figure 11.77a.



$$\text{EXECUTE: } \tan \beta = \frac{0.125 \text{ m}}{0.250 \text{ m}} \\ \beta = 27^\circ, \text{ angle where tips}$$

Figure 11.77a

**SET UP:** At the angle where the bale is ready to slip down the incline  $f_s$  has its maximum possible value,  $f_s = \mu_s n$ . The free-body diagram for the bale, with the origin of coordinates at the cg is given in Figure 11.77b.

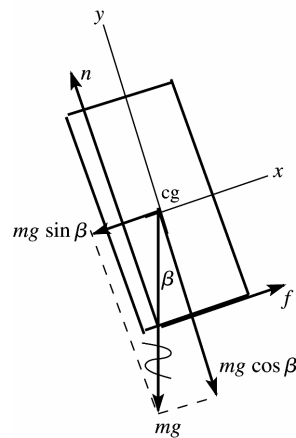


Figure 11.77b

$\beta = 27^\circ$  to tip;  $\beta = 31^\circ$  to slip, so tips first

(b) The magnitude of the friction force didn't enter into the calculation of the tipping angle; still tips at  $\beta = 27^\circ$ . For  $\mu_s = 0.40$  slips at  $\beta = \arctan(0.40) = 22^\circ$ .

Now the bale will start to slide down the incline before it tips.

**EVALUATE:** With a smaller  $\mu_s$  the slope angle  $\beta$  where the bale slips is smaller.

- 11.78. IDENTIFY:** Apply the equilibrium conditions to the pole. The horizontal component of the tension in the wire is 22.0 N.

**SET UP:** The free-body diagram for the pole is given in Figure 11.78. The tension in the cord equals the weight  $W$ .  $F_v$  and  $F_h$  are the components of the force exerted by the hinge. If either of these forces is actually in the opposite direction to what we have assumed, we will get a negative value when we solve for it.

**EXECUTE:** (a)  $T \sin 37.0^\circ = 22.0 \text{ N}$  so  $T = 36.6 \text{ N}$ .  $\sum \tau_z = 0$  gives  $(T \sin 37.0^\circ)(1.75 \text{ m}) - W(1.35 \text{ m}) = 0$ .

$$W = \frac{(22.0 \text{ N})(1.75 \text{ m})}{1.35 \text{ m}} = 28.5 \text{ N}.$$

(b)  $\sum F_y = 0$  gives  $F_v - T \cos 37.0^\circ - w = 0$  and  $F_v = (36.6 \text{ N}) \cos 37.0^\circ + 55.0 \text{ N} = 84.2 \text{ N}$ .  $\sum F_x = 0$  gives  $W - T \sin 37.0^\circ - F_h = 0$  and  $F_h = 28.5 \text{ N} - 22.0 \text{ N} = 6.5 \text{ N}$ . The magnitude of the hinge force is

$$F = \sqrt{F_h^2 + F_v^2} = 84.5 \text{ N}.$$

**EVALUATE:** If we consider torques about an axis at the top of the pole, we see that  $F_h$  must be to the left in order for its torque to oppose the torque produced by the force  $W$ .

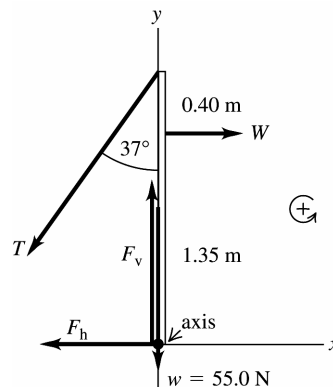
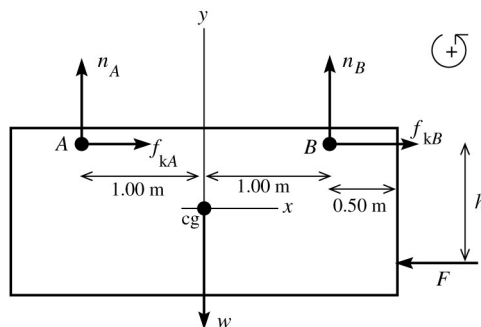


Figure 11.78

- 11.79. IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

(a) **SET UP:** The free-body diagram for the door is given in Figure 11.79.



**Figure 11.79**

Take the origin of coordinates at the center of the door (at the cg). Let  $n_A$ ,  $f_{kA}$ ,  $n_B$ , and  $f_{kB}$  be the normal and friction forces exerted on the door at each wheel.

**EXECUTE:**  $\sum F_y = ma_y$

$$n_A + n_B - w = 0$$

$$n_A + n_B = w = 950 \text{ N}$$

$$\sum F_x = ma_x$$

$$f_{kA} + f_{kB} - F = 0$$

$$F = f_{kA} + f_{kB}$$

$$f_{kA} = \mu_k n_A, \quad f_{kB} = \mu_k n_B, \quad \text{so } F = \mu_k (n_A + n_B) = \mu_k w = (0.52)(950 \text{ N}) = 494 \text{ N}$$

$$\sum \tau_B = 0$$

$n_B$ ,  $f_{kA}$ , and  $f_{kB}$  all have zero moment arms and hence zero torque about this point.

$$\text{Thus } +w(1.00 \text{ m}) - n_A(2.00 \text{ m}) - F(h) = 0$$

$$n_A = \frac{w(1.00 \text{ m}) - F(h)}{2.00 \text{ m}} = \frac{(950 \text{ N})(1.00 \text{ m}) - (494 \text{ N})(1.60 \text{ m})}{2.00 \text{ m}} = 80 \text{ N}$$

$$\text{And then } n_B = 950 \text{ N} - n_A = 950 \text{ N} - 80 \text{ N} = 870 \text{ N}.$$

(b) **SET UP:** If  $h$  is too large the torque of  $F$  will cause wheel  $A$  to leave the track. When wheel  $A$  just starts to lift off the track  $n_A$  and  $f_{kA}$  both go to zero.

**EXECUTE:** The equations in part (a) still apply.

$$n_A + n_B - w = 0 \quad \text{gives } n_B = w = 950 \text{ N}$$

$$\text{Then } f_{kB} = \mu_k n_B = 0.52(950 \text{ N}) = 494 \text{ N}$$

$$F = f_{kA} + f_{kB} = 494 \text{ N}$$

$$+w(1.00 \text{ m}) - n_A(2.00 \text{ m}) - F(h) = 0$$

$$h = \frac{w(1.00 \text{ m})}{F} = \frac{(950 \text{ N})(1.00 \text{ m})}{494 \text{ N}} = 1.92 \text{ m}$$

**EVALUATE:** The result in part (b) is larger than the value of  $h$  in part (a). Increasing  $h$  increases the clockwise torque about  $B$  due to  $F$  and therefore decreases the clockwise torque that  $n_A$  must apply.

**11.80. IDENTIFY:** Apply  $\sum \tau_z = 0$  to the slab.

**SET UP:** The free-body diagram is given in Figure 11.80a.  $\tan \beta = \frac{3.75 \text{ m}}{1.75 \text{ m}}$  so  $\beta = 65.0^\circ$ .

$20.0^\circ + \beta + \alpha = 90^\circ$  so  $\alpha = 5.0^\circ$ . The distance from the axis to the center of the block is

$$\sqrt{\left(\frac{3.75 \text{ m}}{2}\right)^2 + \left(\frac{1.75 \text{ m}}{2}\right)^2} = 2.07 \text{ m}.$$

**EXECUTE:** (a)  $w(2.07 \text{ m})\sin 5.0^\circ - T(3.75 \text{ m})\sin 52.0^\circ = 0$ .  $T = 0.061w$ . Each worker must exert a force of  $0.012w$ , where  $w$  is the weight of the slab.

(b) As  $\theta$  increases, the moment arm for  $w$  decreases and the moment arm for  $T$  increases, so the worker needs to exert less force.

(c)  $T \rightarrow 0$  when  $w$  passes through the support point. This situation is sketched in Figure 11.80b.

$\tan \theta = \frac{(1.75 \text{ m})/2}{(3.75 \text{ m})/2}$  and  $\theta = 25.0^\circ$ . If  $\theta$  exceeds this value the gravity torque causes the slab to tip over.

**EVALUATE:** The moment arm for  $T$  is much greater than the moment arm for  $w$ , so the force the workers apply is much less than the weight of the slab.

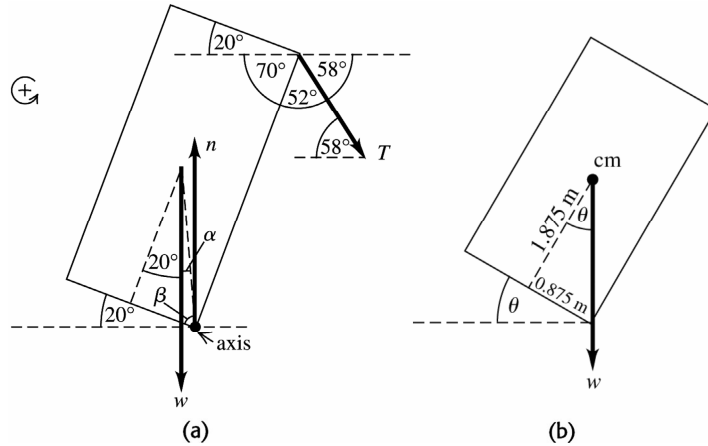
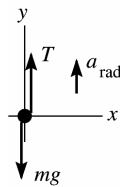


Figure 11.80

- 11.81. IDENTIFY:** Apply Newton's second law to the mass to find the tension in the wire. Then apply  $Y = \frac{l_0 F_\perp}{A \Delta l}$  to the wire to find the elongation this tensile force produces.

(a) **SET UP:** Calculate the tension in the wire as the mass passes through the lowest point. The free-body diagram for the mass is given in Figure 11.81a.



The mass moves in an arc of a circle with radius  $R = 0.70 \text{ m}$ . It has acceleration  $\vec{a}_{\text{rad}}$  directed in toward the center of the circle, so at this point  $\vec{a}_{\text{rad}}$  is upward.

Figure 11.81a

**EXECUTE:**  $\sum F_y = ma_y$

$$T - mg = mR\omega^2 \text{ so that } T = m(g + R\omega^2).$$

But  $\omega$  must be in rad/s:

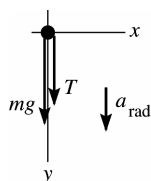
$$\omega = (120 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 12.57 \text{ rad/s}.$$

$$\text{Then } T = (12.0 \text{ kg})[9.80 \text{ m/s}^2 + (0.70 \text{ m})(12.57 \text{ rad/s})^2] = 1445 \text{ N}.$$

Now calculate the elongation  $\Delta l$  of the wire that this tensile force produces:

$$Y = \frac{F_\perp l_0}{A \Delta l} \text{ so } \Delta l = \frac{F_\perp l_0}{YA} = \frac{(1445 \text{ N})(0.70 \text{ m})}{(7.0 \times 10^{10} \text{ Pa})(0.014 \times 10^{-4} \text{ m}^2)} = 0.0103 \text{ m} = 1.0 \text{ cm}.$$

(b) **SET UP:** The acceleration  $\vec{a}_{\text{rad}}$  is directed in toward the center of the circular path, and at this point in the motion this direction is downward. The free-body diagram is given in Figure 11.81b.



**EXECUTE:**

$$\sum F_y = ma_y$$

$$mg + T = mR\omega^2$$

$$T = m(R\omega^2 - g)$$

Figure 11.81b

$$T = (12.0 \text{ kg})[(0.70 \text{ m})(12.57 \text{ rad/s})^2 - 9.80 \text{ m/s}^2] = 1210 \text{ N.}$$

$$\Delta l = \frac{F_{\perp} l_0}{YA} = \frac{(1210 \text{ N})(0.70 \text{ m})}{(7.0 \times 10^{10} \text{ Pa})(0.014 \times 10^{-4} \text{ m}^2)} = 8.6 \times 10^{-3} \text{ m} = 0.86 \text{ cm.}$$

**EVALUATE:** At the lowest point  $T$  and  $w$  are in opposite directions and at the highest point they are in the same direction, so  $T$  is greater at the lowest point and the elongation is greatest there. The elongation is at most 1.4% of the length.

**11.82. IDENTIFY:** For a spring,  $F = kx$ .  $Y = \frac{F_{\perp} l_0}{A \Delta l}$ .

**SET UP:**  $F_{\perp} = F = W$  and  $\Delta l = x$ . For copper,  $Y = 11 \times 10^{10} \text{ Pa}$ .

**EXECUTE:** (a)  $F = \left(\frac{YA}{l_0}\right) \Delta l = \left(\frac{YA}{l_0}\right) x$ . This is in the form of  $F = kx$ , with  $k = \frac{YA}{l_0}$ .

(b)  $k = \frac{YA}{l_0} = \frac{(11 \times 10^{10} \text{ Pa})\pi(6.455 \times 10^{-4} \text{ m})^2}{0.750 \text{ m}} = 1.9 \times 10^5 \text{ N/m}$

(c)  $W = kx = (1.9 \times 10^5 \text{ N/m})(1.25 \times 10^{-3} \text{ m}) = 240 \text{ N}$

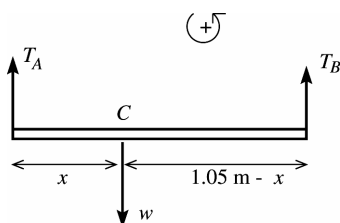
**EVALUATE:** For the wire the force constant is very large, much larger than for a typical spring.

**11.83. IDENTIFY:** Use the second condition of equilibrium to relate the tension in the two wires to the distance  $w$  is from the left end. Use  $\text{stress} = \frac{F_{\perp}}{A}$  and  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$  to relate the tension in each wire to its stress and strain.

(a) **SET UP:**  $\text{stress} = F_{\perp}/A$ , so equal stress implies  $T/A$  same for each wire.

$$T_A/2.00 \text{ mm}^2 = T_B/4.00 \text{ mm}^2 \text{ so } T_B = 2.00T_A$$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point  $C$ , a distance  $x$  to the right of wire  $A$ . The free-body diagram for the rod is given in Figure 11.83.



**EXECUTE:**

$$\sum \tau_C = 0$$

$$+T_B(1.05 \text{ m} - x) - T_A x = 0$$

Figure 11.83

But  $T_B = 2.00T_A$  so  $2.00T_A(1.05 \text{ m} - x) - T_A x = 0$

$$2.10 \text{ m} - 2.00x = x \text{ and } x = 2.10 \text{ m}/3.00 = 0.70 \text{ m (measured from A).}$$

(b) **SET UP:**  $Y = \text{stress/strain}$  gives that  $\text{strain} = \text{stress}/Y = F_{\perp}/AY$ .

**EXECUTE:** Equal strain thus implies

$$\frac{T_A}{(2.00 \text{ mm}^2)(1.80 \times 10^{11} \text{ Pa})} = \frac{T_B}{(4.00 \text{ mm}^2)(1.20 \times 10^{11} \text{ Pa})}$$

$$T_B = \left( \frac{4.00}{2.00} \right) \left( \frac{1.20}{1.80} \right) T_A = 1.333 T_A.$$

The  $\sum \tau_C = 0$  equation still gives  $T_B(1.05 \text{ m} - x) - T_A x = 0$ .

But now  $T_B = 1.333 T_A$  so  $(1.333 T_A)(1.05 \text{ m} - x) - T_A x = 0$ .

$1.40 \text{ m} = 2.33x$  and  $x = 1.40 \text{ m} / 2.33 = 0.60 \text{ m}$  (measured from  $A$ ).

**EVALUATE:** Wire  $B$  has twice the diameter so it takes twice the tension to produce the same stress. For equal stress the moment arm for  $T_B$  ( $0.35 \text{ m}$ ) is half that for  $T_A$  ( $0.70 \text{ m}$ ), since the torques must be equal.

The smaller  $Y$  for  $B$  partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

**11.84. IDENTIFY:** Apply  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$  and calculate  $\Delta l$ .

**SET UP:** When the ride is at rest the tension  $F_{\perp}$  in the rod is the weight  $1900 \text{ N}$  of the car and occupants.

When the ride is operating, the tension  $F_{\perp}$  in the rod is obtained by applying  $\sum \vec{F} = m\vec{a}$  to a car and its occupants. The free-body diagram is shown in Figure 11.84. The car travels in a circle of radius  $r = l \sin \theta$ , where  $l$  is the length of the rod and  $\theta$  is the angle the rod makes with the vertical. For steel,

$Y = 2.0 \times 10^{11} \text{ Pa}$ .  $\omega = 12.0 \text{ rev/min} = 1.2566 \text{ rad/s}$ .

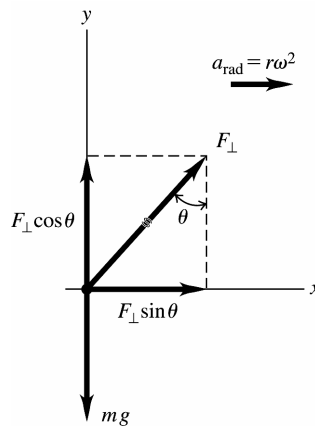
**EXECUTE: (a)**  $\Delta l = \frac{l_0 F_{\perp}}{YA} = \frac{(15.0 \text{ m})(1900 \text{ N})}{(2.0 \times 10^{11} \text{ Pa})(8.00 \times 10^{-4} \text{ m}^2)} = 1.78 \times 10^{-4} \text{ m} = 0.18 \text{ mm}$

**(b)**  $\sum F_x = ma_x$  gives  $F_{\perp} \sin \theta = mr\omega^2 = ml \sin \theta \omega^2$  and

$$F_{\perp} = ml\omega^2 = \left( \frac{1900 \text{ N}}{9.80 \text{ m/s}^2} \right) (15.0 \text{ m})(1.2566 \text{ rad/s})^2 = 4.592 \times 10^3 \text{ N}.$$

$$\Delta l = \left( \frac{4.592 \times 10^3 \text{ N}}{1900 \text{ N}} \right) (0.18 \text{ mm}) = 0.44 \text{ mm}.$$

**EVALUATE:**  $\sum F_y = ma_y$  gives  $F_{\perp} \cos \theta = mg$  and  $\cos \theta = mg/F_{\perp}$ . As  $\omega$  increases  $F_{\perp}$  increases and  $\cos \theta$  becomes small. Smaller  $\cos \theta$  means  $\theta$  increases, so the rods move toward the horizontal as  $\omega$  increases.



**Figure 11.84**

- 11.85. IDENTIFY:** Apply  $\frac{F_{\perp}}{A} = Y \left( \frac{\Delta l}{l_0} \right)$ . The height from which he jumps determines his speed at the ground.

The acceleration as he stops depends on the force exerted on his legs by the ground.

**SET UP:** In considering his motion take  $+y$  downward. Assume constant acceleration as he is stopped by the floor.

**EXECUTE:** (a)  $F_{\perp} = YA \left( \frac{\Delta l}{l_0} \right) = (3.0 \times 10^{-4} \text{ m}^2)(14 \times 10^9 \text{ Pa})(0.010) = 4.2 \times 10^4 \text{ N}$

(b) As he is stopped by the ground, the net force on him is  $F_{\text{net}} = F_{\perp} - mg$ , where  $F_{\perp}$  is the force exerted on him by the ground. From part (a),  $F_{\perp} = 2(4.2 \times 10^4 \text{ N}) = 8.4 \times 10^4 \text{ N}$  and

$$F = 8.4 \times 10^4 \text{ N} - (70 \text{ kg})(9.80 \text{ m/s}^2) = 8.33 \times 10^4 \text{ N}. \quad F_{\text{net}} = ma \text{ gives } a = 1.19 \times 10^3 \text{ m/s}^2.$$

$$a_y = -1.19 \times 10^3 \text{ m/s}^2 \text{ since the acceleration is upward. } v_y = v_{0y} + a_y t \text{ gives}$$

$$v_{0y} = -a_y t = (-1.19 \times 10^3 \text{ m/s}^2)(0.030 \text{ s}) = 35.7 \text{ m/s. His speed at the ground therefore is } v = 35.7 \text{ m/s.}$$

This speed is related to his initial height  $h$  above the floor by  $\frac{1}{2}mv^2 = mgh$  and

$$h = \frac{v^2}{2g} = \frac{(35.7 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 65 \text{ m.}$$

**EVALUATE:** Our estimate is based solely on compressive stress; other injuries are likely at a much lower height.

- 11.86. IDENTIFY:** The graph gives the change in length of the wire as a function of the weight hanging from it, which is equal to the tension in the wire. Young's modulus  $Y$  applies to the stretching of the wire. Energy conservation and Newton's second law apply to the swinging sphere.

**SET UP:**  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$ ,  $K_1 + U_1 = K_2 + U_2$ ,  $\Sigma \vec{F} = m\vec{a}$ ,  $a_{\text{rad}} = v^2/R$ .

**EXECUTE:** (a) Solve  $Y = \frac{l_0 F_{\perp}}{A \Delta l}$  for  $\Delta l$  and realize that  $F_{\perp} = mg$ :  $\Delta l = \frac{gl_0}{AY} m$ . Therefore, in the graph of

$\Delta l$  versus  $m$ , the slope is equal to  $gl_0/AY$ . The equation of the graph is given in the problem as  $\Delta l = (0.422 \text{ mm/kg})m$ , so the slope is  $0.422 \text{ mm/kg}$ , so  $gl_0/AY = 0.422 \text{ mm/kg} = 4.22 \times 10^{-4} \text{ m/kg}$ . Solving

for  $Y$  gives  $Y = \frac{gl_0}{A(4.22 \times 10^{-4} \text{ m/kg})}$ . Using  $A = \pi r^2$  and putting in the given numbers gives

$$Y = \frac{(9.80 \text{ m/s}^2)(22.0 \text{ m})}{\pi(4.30 \times 10^{-4} \text{ m})^2(4.22 \times 10^{-4} \text{ m/kg})} = 8.80 \times 10^{11} \text{ Pa.}$$

(b) Use energy conservation to find the speed of the sphere.  $K_1 + U_1 = K_2 + U_2$  gives

$$mgL(1 - \cos \theta) = \frac{1}{2}mv^2. \text{ Solving for } v \text{ using } \theta = 36.0^\circ \text{ and } L = 22.0 \text{ m gives } v = 9.075 \text{ m/s.}$$

Now apply Newton's second law to the sphere at the bottom of the swing.  $\Sigma \vec{F} = m\vec{a}$  and  $a_{\text{rad}} = v^2/R$  give  $T - mg = mv^2/L$ , so  $T = mv^2/L + mg = (9.50 \text{ kg})(9.075 \text{ m/s})^2/(22.0 \text{ m}) + (9.50 \text{ kg})(9.80 \text{ m/s}^2) = 129 \text{ N}$ .

Using the value of  $Y$  found in part (a), we have

$$\Delta l = \frac{F_{\perp} l_0}{AY} = (129 \text{ N})(22.0 \text{ m})/[\pi(4.30 \times 10^{-4} \text{ m})^2(8.80 \times 10^{11} \text{ Pa})] = 0.00554 \text{ m} = 5.54 \text{ mm.}$$

**EVALUATE:** For a wire 22 m long, 5.5 mm is a very small stretch,  $0.0055/22 = 0.025\%$ .

- 11.87. IDENTIFY:** The bar is at rest, so the forces and torques on it must all balance.

**SET UP:**  $\Sigma F_y = 0$ ,  $\Sigma \tau_z = 0$ .

**EXECUTE:** (a) The free-body diagram is shown in Figure 11.87a, where  $F_p$  is the force due to the knife-edge pivot.



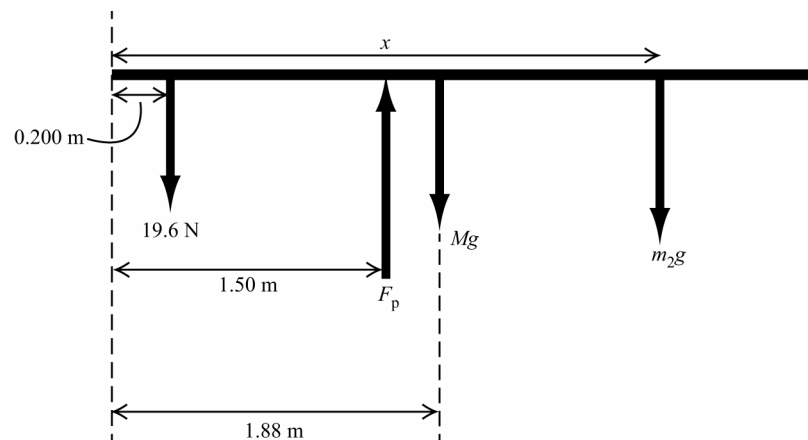


Figure 11.87a

(b)  $\sum \tau_z = 0$ , with torques taken about the location of the knife-edge pivot, gives

$$(2.00 \text{ kg})g(1.30 \text{ m}) - Mg(0.38 \text{ m}) - m_2g(x - 1.50 \text{ m}) = 0$$

Solving for  $x$  gives

$$x = [(2.00 \text{ kg})(1.30 \text{ m}) - M(0.38 \text{ m})](1/m_2) + 1.50 \text{ m}$$

The graph of this equation ( $x$  versus  $1/m_2$ ) is a straight line of slope  $[(2.00 \text{ kg})(1.30 \text{ m}) - M(0.38 \text{ m})]$ .

(c) The plot of  $x$  versus  $1/m_2$  is shown in Figure 11.87b. The equation of the best-fit line is

$$x = (1.9955 \text{ m} \cdot \text{kg})/m_2 + 1.504 \text{ m. The slope of the best-fit line is } 1.9955 \text{ m} \cdot \text{kg, so}$$

$$[(2.00 \text{ kg})(1.30 \text{ m}) - M(0.38 \text{ m})] = 1.9955 \text{ m} \cdot \text{kg, which gives } M = 1.59 \text{ kg.}$$

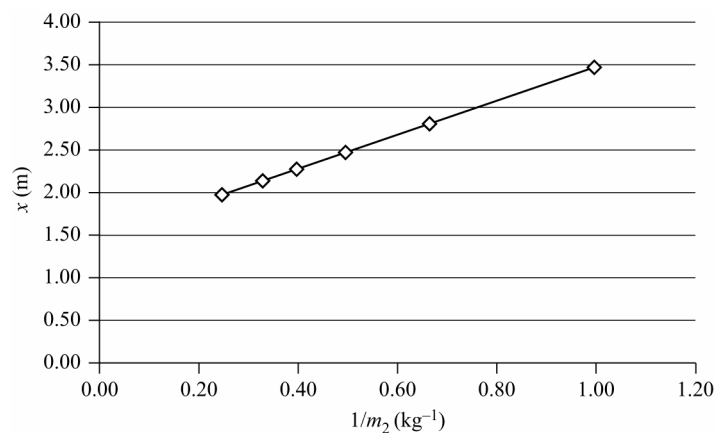


Figure 11.87b

(d) The  $y$ -intercept of the best-fit line is 1.50 m. This is plausible. As the graph approaches the  $y$ -axis,  $1/m_2$  approaches zero, which means that  $m_2$  is getting extremely large. In that case, it would be much larger than any other masses involved, so to balance the system,  $m_2$  would have to be at the knife-point pivot, which is at  $x = 1.50 \text{ m}$ .

**EVALUATE:** The fact that the graph gave a physically plausible result in part (d) suggests that this graphical analysis is reasonable.

**11.88. IDENTIFY:** The bar is at rest, so the forces and torques on it must all balance.

**SET UP:**  $\sum F_x = 0$ ,  $\sum F_y = 0$ ,  $\sum \tau_z = 0$ .

**EXECUTE: (a)** Take torques about the hinge, calling  $m$  the mass of the bar and  $L$  its length.  $\sum \tau_z = 0$  gives

$$xT \sin \theta = mg \frac{L}{2}. \text{ Solving for } T \text{ gives } T = \frac{mgL/2}{x \sin \theta}. \text{ Therefore the alternative having the largest value of}$$

$x \sin \theta$  will have the smallest tension, and the one with the smallest value of  $x \sin \theta$  will have the greatest

tension. Calculating  $x \sin \theta$  for each alternative gives the following values. A: 1.00 m, B: 1.30 m, C: 0.451 m, D: 0.483 m. Therefore alternative B gives the smallest tension and C produces the largest tension

(b) Calling  $H$  the magnitude of the hinge force,  $\Sigma F_x = 0$  gives  $H_x = T \cos \theta$ . Using the value of  $T$  from

part (a), we get  $H_x = \frac{mg L/2}{x \sin \theta} \cdot \cos \theta = \frac{mg L/2}{x \tan \theta}$ . From this result, we can see that  $H_x$  is greatest when

$x \tan \theta$  is the smallest, and  $H_x$  is least when  $x \tan \theta$  is greatest. Calculating  $x \tan \theta$  for each alternative gives A: 1.15 m, B: 2.60 m, C: 0.565 m, D: 1.87 m. Therefore alternative C gives the greatest  $H_x$  and B gives the smallest  $H_x$ .

(c) Taking torques about the point where the cable is connected to the bar,  $\Sigma \tau_z = 0$  gives

$H_y x = mg(x - L/2)$ . Solving for  $H_y$  gives  $H_y = mg(1 - L/2x)$ . Since  $H_y$  could be positive or negative, we

should calculate all four possibilities. For alternative A, we have  $H_y = mg \left( 1 - \frac{2.00 \text{ m}}{4.00 \text{ m}} \right) = 0.500mg$ . For B

we have  $H_y = mg \left( 1 - \frac{2.00 \text{ m}}{3.00 \text{ m}} \right) = 0.333mg$ , and likewise we get  $H_y = -0.333mg$  for C and  $H_y = -1.00mg$

for D. Therefore alternative D gives the largest  $H_y$  and B and C both give the smallest value.

(d) Alternative B is clearly optimal because it results in the smallest values for  $T$ ,  $H_x$ , and  $H_y$ . It might be a good idea to avoid alternative C because it has the greatest  $T$  and  $H_x$ .

**EVALUATE:** As a check, part (c) could be solved by using  $\Sigma F_y = 0$ .

**11.89. IDENTIFY:** Apply the equilibrium conditions to the ladder combination and also to each ladder.

**SET UP:** The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by  $F_L$  and  $F_R$  (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless.

**EXECUTE:** (a) Taking torques about the right end,  $F_L(5.00 \text{ m}) = (480 \text{ N})(3.40 \text{ m}) + (360 \text{ N})(0.90 \text{ m})$ ,

so  $F_L = 391 \text{ N}$ .  $F_R$  may be found in a similar manner, or from  $F_R = 840 \text{ N} - F_L = 449 \text{ N}$ .

(b) The tension in the rope may be found by finding the torque on each ladder, using the point  $A$  as the origin. The lever arm of the rope is 1.50 m. For the left ladder,

$T(1.50 \text{ m}) = F_L(3.20 \text{ m}) - (480 \text{ N})(1.60 \text{ m})$ , so  $T = 322.1 \text{ N}$  (322 N to three figures). As a check, using the torques on the right ladder,  $T(1.50 \text{ m}) = F_R(1.80 \text{ m}) - (360 \text{ N})(0.90 \text{ m})$  gives the same result.

(c) The horizontal component of the force at  $A$  must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder,  $480 \text{ N} - 391 \text{ N} = 449 \text{ N} - 360 \text{ N} = 89 \text{ N}$ . The magnitude of the force at  $A$  is then

$$\sqrt{(322.1 \text{ N})^2 + (89 \text{ N})^2} = 334 \text{ N}.$$

(d) The easiest way to do this is to see that the added load will be distributed at the floor in such a way that  $F'_L = F_L + (0.36)(800 \text{ N}) = 679 \text{ N}$ , and  $F'_R = F_R + (0.64)(800 \text{ N}) = 961 \text{ N}$ . Using these forces in the form for the tension found in part (b) gives

$$T = \frac{F'_L(3.20 \text{ m}) - (480 \text{ N})(1.60 \text{ m})}{(1.50 \text{ m})} = \frac{F'_R(1.80 \text{ m}) - (360 \text{ N})(0.90 \text{ m})}{(1.50 \text{ m})} = 937 \text{ N}.$$

**EVALUATE:** The presence of the painter increases the tension in the rope, even though his weight is vertical and the tension force is horizontal.

**11.90. IDENTIFY:** Apply  $\Sigma \tau_z = 0$  to the post, for various choices of the location of the rotation axis.

**SET UP:** When the post is on the verge of slipping,  $f_s$  has its largest possible value,  $f_s = \mu_s n$ .

**EXECUTE:** (a) Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is  $h/2$  and the lever arm of both the weight and the normal force is  $h \tan \theta$ , and so

$$F \frac{h}{2} = (n - w)h \tan \theta.$$

Taking torques about the upper point (where the rope is attached to the post),  $fh = F\frac{h}{2}$ . Using  $f \leq \mu_s n$

and solving for  $F$ ,  $F \leq 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400 \text{ N})\left(\frac{1}{0.30} - \frac{1}{\tan 36.9^\circ}\right)^{-1} = 400 \text{ N}$ .

(b) The above relations between  $F$ ,  $n$  and  $f$  become  $F\frac{3}{5}h = (n - w)h \tan\theta$ ,  $f = \frac{2}{5}F$ , and eliminating

$f$  and  $n$  and solving for  $F$  gives  $F \leq w\left(\frac{2/5}{\mu_s} - \frac{3/5}{\tan\theta}\right)^{-1}$ , and substitution of numerical values gives

750 N to two figures.

(c) If the force is applied a distance  $y$  above the ground, the above relations become

$$Fy = (n - w)h \tan\theta, F(h - y) = fh, \text{ which become, on eliminating } n \text{ and } f, w \geq F\left[\frac{(1 - y/h)}{\mu_s} - \frac{(y/h)}{\tan\theta}\right].$$

As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of  $y$  is found by setting the term in square brackets equal to zero. Solving for  $y$  gives

$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9^\circ}{0.30 + \tan 36.9^\circ} = 0.71.$$

**EVALUATE:** For the post to slip, for an axis at the top of the post the torque due to  $F$  must balance the torque due to the friction force. As the point of application of  $F$  approaches the top of the post, its moment arm for this axis approaches zero.

**11.91. IDENTIFY:** Apply  $Y = \frac{l_0 F_\perp}{A\Delta l}$  to calculate  $\Delta l$ .

**SET UP:** For steel,  $Y = 2.0 \times 10^{11} \text{ Pa}$ .

**EXECUTE: (a)** From  $Y = \frac{l_0 F_\perp}{A\Delta l}$ ,  $\Delta l = \frac{(4.50 \text{ kg})(9.80 \text{ m/s}^2)(1.50 \text{ m})}{(20 \times 10^{10} \text{ Pa})(5.00 \times 10^{-7} \text{ m}^2)} = 6.62 \times 10^{-4} \text{ m}$ , or 0.66 mm to two figures.

(b)  $(4.50 \text{ kg})(9.80 \text{ m/s}^2)(0.0500 \times 10^{-2} \text{ m}) = 0.022 \text{ J}$ .

(c) The magnitude  $F$  will vary with distance; the average force is  $YA(0.0250 \text{ cm}/l_0) = 16.7 \text{ N}$ , and so the work done by the applied force is  $(16.7 \text{ N})(0.0500 \times 10^{-2} \text{ m}) = 8.35 \times 10^{-3} \text{ J}$ .

(d) The average force the wire exerts is  $(4.50 \text{ kg})g + 16.7 \text{ N} = 60.8 \text{ N}$ . The work done is negative, and equal to  $-(60.8 \text{ N})(0.0500 \times 10^{-2} \text{ m}) = -3.04 \times 10^{-2} \text{ J}$ .

(e) The equation  $Y = \frac{l_0 F_\perp}{A\Delta l}$  can be put into the form of Hooke's law, with  $k = \frac{YA}{l_0}$ .  $U_{\text{el}} = \frac{1}{2}kx^2$ , so

$\Delta U_{\text{el}} = \frac{1}{2}k(x_2^2 - x_1^2)$ .  $x_1 = 6.62 \times 10^{-4} \text{ m}$  and  $x_2 = 0.500 \times 10^{-3} \text{ m} + x_1 = 11.62 \times 10^{-4} \text{ m}$ . The change in elastic potential energy is

$$\frac{(20 \times 10^{10} \text{ Pa})(5.00 \times 10^{-7} \text{ m}^2)}{2(1.50 \text{ m})} \left[ (11.62 \times 10^{-4} \text{ m})^2 - (6.62 \times 10^{-4} \text{ m})^2 \right] = 3.04 \times 10^{-2} \text{ J}, \text{ the negative of}$$

the result of part (d).

**EVALUATE:** The tensile force in the wire is conservative and obeys the relation  $W = -\Delta U$ .

**11.92. IDENTIFY and SET UP:** The forces and torques on the competitor must balance, so  $\Sigma F_x = 0$ ,  $\Sigma F_y = 0$ , and  $\Sigma \tau_z = 0$ .

**EXECUTE:** Take torques about his feet, giving  $(T_1 - T_2)(1.5 \text{ m})(\cos 30^\circ) = mg(1.0 \text{ m})(\sin 30^\circ)$ . Solving for  $T_2$  gives  $T_2 = 1160 \text{ N} - [(80.0 \text{ kg})(9.80 \text{ m/s}^2)/(1.5 \text{ m})]\tan 30^\circ = 858 \text{ N} \approx 860 \text{ N}$ , which is choice (c).

**EVALUATE:** We find  $T_2 < T_1$  as expected.

**11.93. IDENTIFY and SET UP:** The forces and torques on the competitor must balance, so  $\Sigma F_x = 0$ ,  $\Sigma F_y = 0$ , and  $\Sigma \tau_z = 0$ .

**EXECUTE:** As in the previous problem,  $T_1 - T_2$  is proportional to  $\tan \theta$ , so as  $\theta$  increases, so does  $\tan \theta$  and so does  $T_1 - T_2$ , which makes choice (a) correct.

**EVALUATE:** The result is physically reasonable. As he leans back, the ropes get lower, which reduces their moment arm, and his weight also gets lower, which increases its moment arm. Therefore to keep balance, the difference in the tensions must be greater than before.

**11.94. IDENTIFY and SET UP:** Apply  $\tau = Fl$ .

**EXECUTE:** The moment arm for  $T_1$  has increased, so  $T_1$  can be smaller and still produce the same torque needed to balance the torque due to gravity, so choice (c) is correct.

**EVALUATE:** If the rope is held too high, it will be hard for the competitor to hold it, so there is a limit on how much the holding height can be effectively increased.

**11.95. IDENTIFY and SET UP:** The competitor will slip if the static friction force would need to be greater than its maximum possible value.  $f_s^{\max} = \mu_s n$ .

**EXECUTE:** From earlier work, we know that  $T_1 - T_2 = 1160 \text{ N} - 858 \text{ N} = 302 \text{ N}$ . The maximum static friction force is  $f_s^{\max} = \mu_s n = (0.50)(80.0 \text{ kg})(9.80 \text{ m/s}^2) = 392 \text{ N}$ . He needs only 302 N to balance the tension difference, yet the static friction force could be as great as 392 N, so he is not even ready to slip. Therefore he will not move, choice (d).

**EVALUATE:** The friction force is 302 N, not 392 N, because he is not just ready to slip.