Vp250 Problem Set 8

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Problem 1

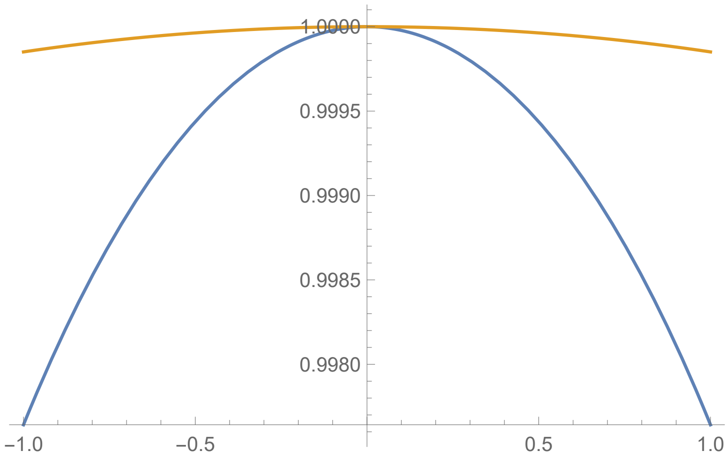
For a point x, the magnetic field due to a infinitesimal solenoid at x_1 is

$$\mathrm{d}B = \frac{\mu_0\,\mathrm{d}I}{2}\frac{R^2}{((x_1-x)^2+R^2)^{3/2}}$$

$$\mathrm{d}I = nI\,\mathrm{d}x_1$$

So

$$B(x) = \frac{\mu_0 n I R^2}{2}\int_{-l/2}^{l/2}\frac{\mathrm{d}x_1}{[(x-x_1)^2+R^2]^{3/2}} = \frac{\mu_0 n I}{2}\left[\frac{x+l/2}{\sqrt{R^2+(x+l/2)^2}} - \frac{x-l/2}{\sqrt{R^2+(x-l/2)^2}}\right]$$



Problem 2

For an infinitesimal point in the wire:

$$\begin{aligned}\mathrm{d}V &= E(x)\,\mathrm{d}x\\ &= vB(x)\,\mathrm{d}x\\ &= v\frac{\mu_0 I}{2\pi x}\,\mathrm{d}x\end{aligned}$$

So

$$\begin{aligned}V &= \int_d^{d+L} v\frac{\mu_0 I}{2\pi x}\,\mathrm{d}x\\ &= \frac{v\mu_0 I}{2\pi}\ln\frac{d+L}{d}\end{aligned}$$

Since V is greater than 0, A has higher potential.

Problem 3

(a)

$$\begin{cases} mg\sin\varphi = ILB\cos\varphi \\ I = \frac{\varepsilon}{R} \\ \varepsilon = vBL\cos\varphi \end{cases} \Rightarrow v = \frac{mgR\sin\varphi}{B^2L^2\cos^2\varphi}$$

(b) From A to B.

(c)

$$I = \frac{mg\tan\varphi}{BL}$$

(d)

$$P = I^2 R = \frac{m^2 g^2 R \tan^2\varphi}{B^2 L^2}$$

(e)

$$P_G = mgv\sin\varphi = \frac{m^2 g^2 R \tan^2\varphi}{B^2 L^2}$$

It is the same as in part d.

Problem 4

Divide the square into infinitesimal rectangulars with length of a and width of $\mathrm{d}y$. Then

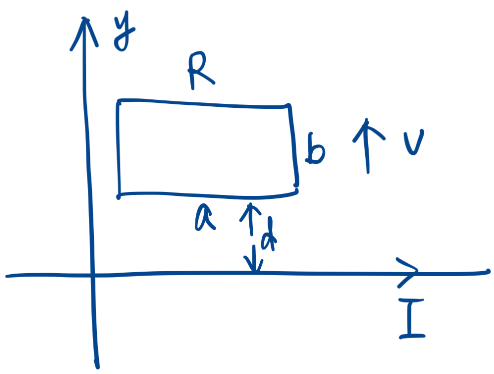
$$\mathrm{d}\varepsilon = -\frac{a\,\mathrm{d}y \cdot \mathrm{d}B}{\mathrm{d}t} = -a\,\mathrm{d}y\frac{\mathrm{d}(4t^2 y)}{\mathrm{d}t} = -8aty\,\mathrm{d}y$$

$$\varepsilon(t) = \int_0^a -8aty\,\mathrm{d}y = -4a^3 t$$

Since $\varepsilon < 0$, the current direction is clockwise.

Problem 5

(a) Let the current lies in the x-axis. And let the y-axis be along one width.
Divide the rectangle into infinitesimal rectangles with length of a and width of $\mathrm{d}y$.



$$\mathrm{d}\phi = a\,\mathrm{d}y\frac{\mu_0 I}{2\pi(y+vt)}$$

$$\phi(t) = \int_d^{d+b} \frac{a\mu_0 I}{2\pi(y+vt)}\,\mathrm{d}y = \frac{a\mu_0 I}{2\pi}\ln\frac{d+b+vt}{d+vt}$$

(b)

$$\varepsilon(t) = -\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{a\mu_0 I v}{2\pi}\left(\frac{1}{d+vt} - \frac{1}{d+b+vt}\right)$$

$$I(t) = \frac{\varepsilon(t)}{R} = \frac{a\mu_0 I v}{2\pi R}\left(\frac{1}{d+vt} - \frac{1}{d+b+vt}\right)$$

Problem 6

(a) The capacitance is $C = \varepsilon_0 \varepsilon_r \dfrac{A}{d}$.

The resistance is $R = \rho \dfrac{d}{A}$.

The time constant of the capacitor is $\tau = RC = \rho \varepsilon_0 \varepsilon_r$.

So the voltage between the capacitor when discharging is $V(t) = \dfrac{Q_0}{C} e^{-t/\tau} = \dfrac{dQ_0}{\varepsilon_0 \varepsilon_r A} e^{-\frac{t}{\rho \varepsilon_0 \varepsilon_r}}$.

The current is $I(t) = \dfrac{V(t)}{R} = \dfrac{Q_0}{RC} e^{-\frac{t}{RC}} = \dfrac{Q_0}{\rho \varepsilon_0 \varepsilon_r} e^{-\frac{t}{\rho \varepsilon_0 \varepsilon_r}}$.

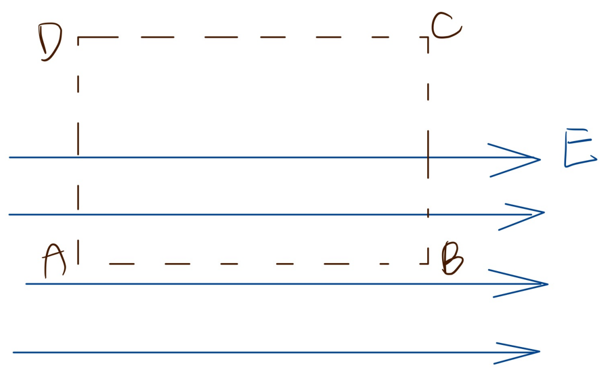
The current density is $J_c(t) = \dfrac{I(t)}{A} = \dfrac{Q_0}{A \rho \varepsilon_0 \varepsilon_r} e^{-\frac{t}{\rho \varepsilon_0 \varepsilon_r}}$.

(b) The electric field between the capacitor is $E(t) = \dfrac{V(t)}{d}$.

Then the displacement current is $i_D(t) = \varepsilon_0 \varepsilon_r \dfrac{\mathrm{d}\phi_E}{\mathrm{d}t} = \varepsilon_r \varepsilon_0 A \dfrac{\mathrm{d}E}{\mathrm{d}t} = \dfrac{\varepsilon_0 \varepsilon_r A}{d} \cdot \dfrac{\mathrm{d}V}{\mathrm{d}t} = -\dfrac{Q_0}{\rho \varepsilon_0 \varepsilon_r} e^{-\frac{t}{\rho \varepsilon_0 \varepsilon_r}}$, which has the same magnitude as $I(t)$ but opposite direction.

Problem 7

(a) The graph is shown below:



(b) $\dfrac{\mathrm{d}\phi}{\mathrm{d}t} = 0$ because the magnetic field is constant and the area of the rectangle is static.

But

$$\oint_\Gamma \bar{E} \mathrm{d}\bar{l} = |AB| E \neq 0$$

which contradicts Faraday's Law.