Vp250 Problem Set 7

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Problem 1

Set the positive direction as inside the paper:
For I_1:

$$B_1 = \frac{\mu_0}{4\pi}\int_\Gamma \frac{I_1 \mathrm{d}l}{R^2} = \frac{\mu_0 I_1}{4\pi R^2}\pi R = \frac{\mu_0 I_1}{4R}$$

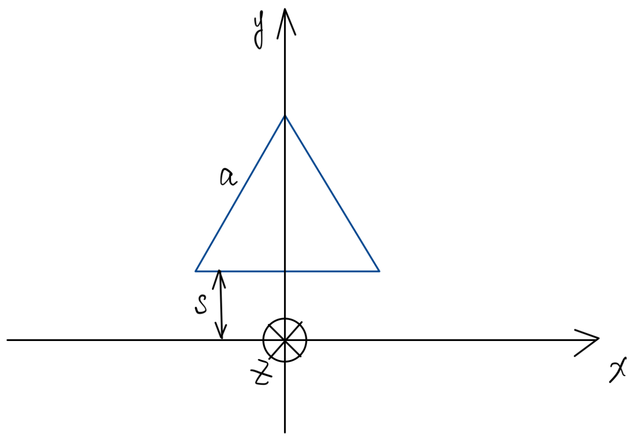
For I_2:

$$B_2 = \frac{\mu_0}{4\pi}\int_\Gamma -\frac{I_2 \mathrm{d}l}{R^2} = -\frac{\mu_0 I_2}{4\pi R^2}\pi R = -\frac{\mu_0 I_2}{4R}$$

The total magnitude: $B_P = \frac{\mu_0}{4R}(I_1 - I_2)$ If $I_1 = I_2$, $B_P = 0$.

Problem 2

Graph:



The magnetic field acting on the triangle with respect to the distance y is

$$B(y) = \frac{\mu_0 I}{2\pi y}$$

direction: positive z-axis For the lower part of this triangle, $y = s$

$$\bar{F}_1 = I \cdot \bar{l} \times \bar{B}(s) \quad \bar{l} = (-a, 0, 0) \quad \bar{B}(s) = (0, 0, \frac{\mu_0 I}{2\pi s})$$

So $\bar{F}_1 = \dfrac{a\mu_0 I^2}{2\pi s}\hat{n}_y$.
For left part,

$$\mathrm{d}\bar{l} = (\mathrm{d}x, \mathrm{d}y, 0)$$

$$\bar{B}(y) = (0, 0, \frac{\mu_0 I}{2\pi y})$$

$$\mathrm{d}\bar{F}_2 = I\mathrm{d}\bar{l} \times \bar{B}(y) = (\frac{\mu_0 I^2}{2\pi y}\mathrm{d}y, -\frac{\mu_0 I^2}{2\sqrt{3}\pi y}\mathrm{d}y)$$

$$\bar{F}_2 = \int_s^{s+\frac{\sqrt{3}}{2}a} \mathrm{d}\bar{F} = (\frac{\mu_0 I^2}{2\pi}\ln\frac{2s+\sqrt{3}a}{2s}, -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\frac{2s+\sqrt{3}a}{2s}, 0)$$

For the right part,

$$\mathrm{d}\bar{l} = (\mathrm{d}x, \mathrm{d}y, 0)$$

$$\bar{B}(y) = (0, 0, \frac{\mu_0 I}{2\pi y})$$

$$\mathrm{d}\bar{F}_3 = I\mathrm{d}\bar{l} \times \bar{B}(y) = (\frac{\mu_0 I^2}{2\pi y}\mathrm{d}y, \frac{\mu_0 I^2}{2\sqrt{3}\pi y}\mathrm{d}y)$$

$$\bar{F}_3 = \int_{s+\frac{\sqrt{3}}{2}a}^{s} \mathrm{d}\bar{F} = (\frac{\mu_0 I^2}{2\pi} \ln \frac{2s}{2s+\sqrt{3}a}, \frac{\mu_0 I^2}{2\sqrt{3}\pi} \ln \frac{2s}{2s+\sqrt{3}a}, 0)$$

Then the net force:

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = (0, \frac{a\mu_0 I^2}{2\pi s} + \frac{\mu_0 I^2}{\sqrt{3}\pi} \ln \frac{2s}{2s+\sqrt{3}a}, 0)$$

Problem 3

Charge density:

$$\sigma = \frac{Q}{\pi a^2}$$

Current in one infinitesimal ring is

$$\mathrm{d}I = \frac{\sigma\,\mathrm{d}r \cdot 2\pi r}{T} = \frac{nQr\,\mathrm{d}r}{2a^2}$$

Then

$$B = \int_0^a \frac{\mu_0\,\mathrm{d}I}{2} \cdot \frac{1}{r} = \frac{nQ\mu_0}{a}$$

direction: perpendicular to the disk. And if the disk rotate clockwise, pointing inwards paper, otherwise outwards.

Problem 4

(a) The infinitesimal magnetic field exerted on the point is

$$\mathrm{d}B = \frac{\mu_0\,\mathrm{d}I}{2\pi\sqrt{x^2+y^2}} \cdot \frac{y}{\sqrt{x^2+y^2}} = \frac{\mu_0 I\,\mathrm{d}x}{2\pi L} \cdot \frac{y}{x^2+y^2}$$

Then

$$B = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\mu_0 I}{2\pi L} \cdot \frac{y}{x^2+y^2}\,\mathrm{d}x = \int_0^{\frac{L}{2}} \frac{\mu_0 I}{\pi L} \cdot \frac{y}{x^2+y^2}\,\mathrm{d}x = \frac{\mu_0 I}{\pi L} \arctan \frac{L}{2y}$$

(b) When $y >> L$,

$$B \approx \frac{\mu_0 I}{\pi L}(\arctan(0) + \frac{1}{x^2+1}\bigg|_{x=0} \cdot \frac{L}{2y}) = \frac{\mu_0 I}{2\pi y}$$

Problem 5

a:

$$\oint \bar{B}\,\mathrm{d}\bar{l} = \mu_0 \cdot 0 = 0$$

b:

$$\oint \bar{B}\,\mathrm{d}\bar{l} = -\mu_0 I_1$$

c:

$$\oint \bar{B}\,\mathrm{d}\bar{l} = \mu_0(I_2 - I_1)$$

d:

$$\oint \bar{B}\,\mathrm{d}\bar{l} = \mu_0(I_2 + I_3 - I_1)$$

Problem 6

Units: $a : m$, $b : A/m$, $\delta = m$.

(a)

$$\mathrm{d}I = J(r)2\pi r\,\mathrm{d}r = 2\pi b \exp\left(\frac{r-a}{\delta}\right)\,\mathrm{d}r$$

$$I_0 = \int_0^a 2\pi b \exp\left(\frac{r-a}{\delta}\right)\,\mathrm{d}r = 2\pi b\delta\left(1 - \exp\left(-\frac{a}{\delta}\right)\right)$$

(b)

$$\oint \bar{B}\,\mathrm{d}\bar{l} = \mu_0 I_0 \Rightarrow \bar{B}(r)2\pi r = \mu_0 I_0 \Rightarrow B(r) = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0 b\delta}{r}\left(1 - \exp\left(-\frac{a}{\delta}\right)\right)$$

direction: perpendicular to the radius and it is clockwise if the current runs inwards the paper.

(c)

$$\mathrm{d}I = J(r)2\pi r\,\mathrm{d}r = 2\pi b \exp\left(\frac{r-a}{\delta}\right)\,\mathrm{d}r$$

$$I = \int_0^r 2\pi b \exp\left(\frac{r-a}{\delta}\right)\,\mathrm{d}r = 2\pi b\delta\left(\exp\left(\frac{r-a}{\delta}\right) - \exp\left(-\frac{a}{\delta}\right)\right) = \frac{I_0}{\exp(\frac{a}{\delta}) - 1}\left(\exp\left(\frac{r}{\delta}\right) - 1\right)$$

(d)

$$\oint \bar{B}\,\mathrm{d}\bar{l} = \mu_0 I \Rightarrow \bar{B}(r)2\pi r = \mu_0 I \Rightarrow B(r) = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I_0}{2\pi r(\exp(\frac{a}{\delta}) - 1)}\left(\exp\left(\frac{r}{\delta}\right) - 1\right)$$

direction: perpendicular to the radius and it is clockwise if the current runs inwards the paper.