Chapter 7 – Magnetic Field and Magnetic Forces

UM-SJTU Joint Institute
Physics II (Fall 2020)
Mateusz Krzyzosiak

Agenda

Magnetic Field

Introduction

- permanent magnets (e.g. a bar magnet, the Earth)

(no magnetic monopoles!)

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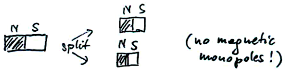
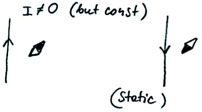


- Oersted's experiment (H.Ch. Oersted, 1820)



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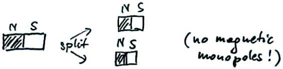

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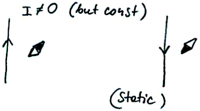

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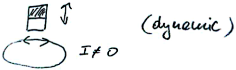

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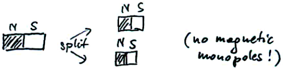

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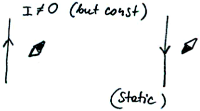

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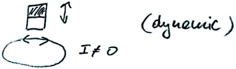

- two wires carrying electric current attract/repel each other

Features of Magnetic Field

MAGNETIC FIELD

(vector field)

static

magnetostatics

dynamic

electrodynamics

(Maxwell's eqns. – four equations unifying electric and magnetic interactions)

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dynamic → electrodynamics
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Magnetic Field vs. Electric Field

electric field (static)

- static electric charges ⇒ static electric field

Features of Magnetic Field

MAGNETIC FIELD (vector field)

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dynamic → electrodynamics (Maxwell's eqns. – four equations unifying electric and magnetic interactions)

Magnetic Field vs. Electric Field

electric field (static)

- static electric charges $\Rightarrow$ static electric field
- electric force $\overline{F} = q\overline{E}$ acts on any charge placed in the electric field

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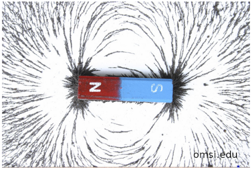
magnetic field

- moving charges (currents) $\Rightarrow$ magnetic field
- a magnetic force acts on a charge moving in the magnetic field

(a) permanent magnet – bar magnet



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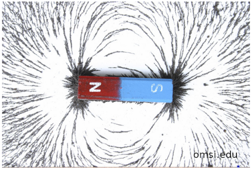

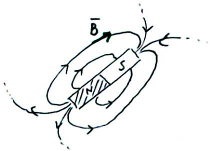

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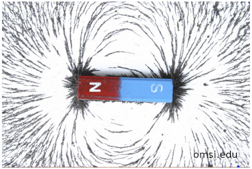

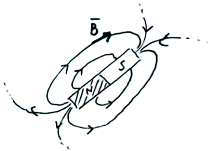

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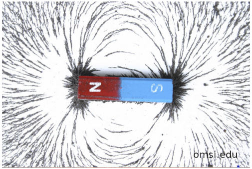

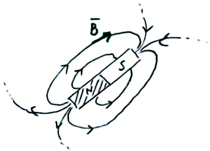

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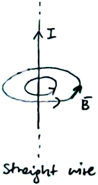
(b) electric current



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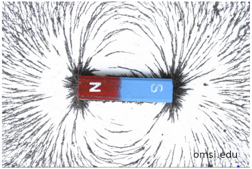

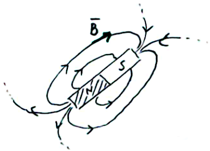

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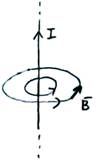
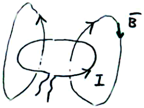


Straight wire



Current loop

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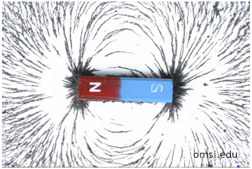

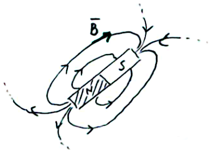

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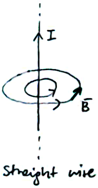

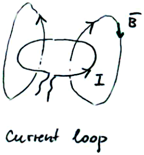

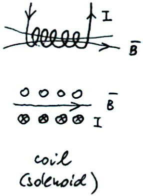

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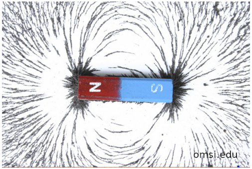

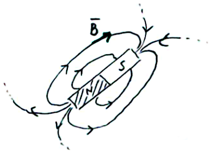

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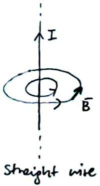

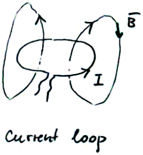

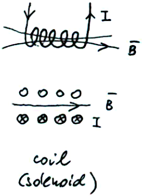

Observation. Magnetic field lines always form loops!

Magnetic Force

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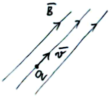

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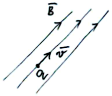

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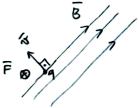

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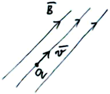

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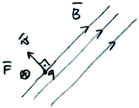

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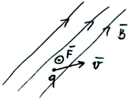

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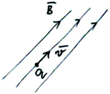

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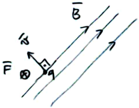

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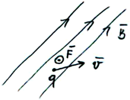

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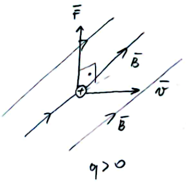
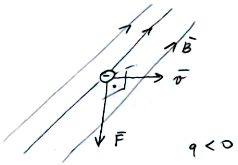
$$|\overline{F}| = |q|\,|\overline{v}|\,|\overline{B}|\,\sin\measuredangle(\overline{v},\overline{B}) = |q|\,\underbrace{|\overline{v}_\perp|}_{\text{component} \perp \overline{B}}|\overline{B}| = |q|\,|\overline{v}|\cdot\underbrace{|\overline{B}_\perp|}_{\text{component} \perp \text{ to } \overline{v}}$$

$\measuredangle(\overline{v},\overline{B})$: angle between $\overline{v}, \overline{B}$ (the smaller one)

The direction of $\overline{F}$ depends on the sign of the charge.





(left-hand rule for $\overline{v}, \overline{B}, \overline{F}$)

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Hence, the magnetic force on a moving charge is

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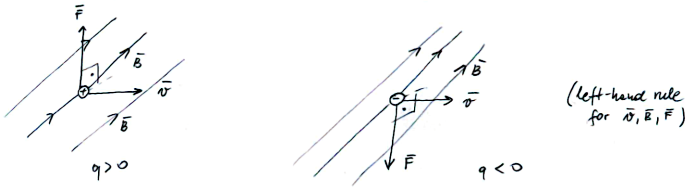

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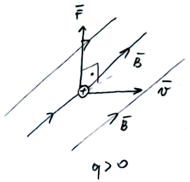

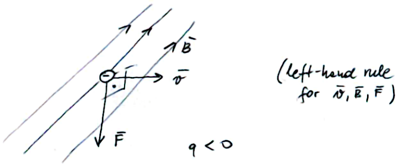

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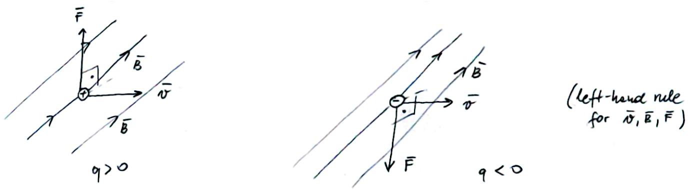

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Magnetic Force. Comments (assume $v \ll c$)

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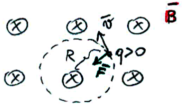
and the force is referred to as the **Lorentz force**.

Examples

(a) cyclotron motion

$\vec{B} \neq 0$ and $\vec{v} \perp \vec{B}$



trajectory – circle

Lorentz force $\equiv$ centripetal force

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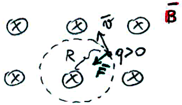

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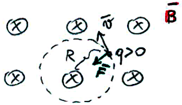

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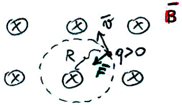

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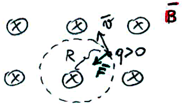

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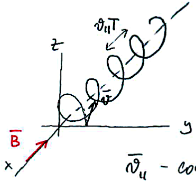
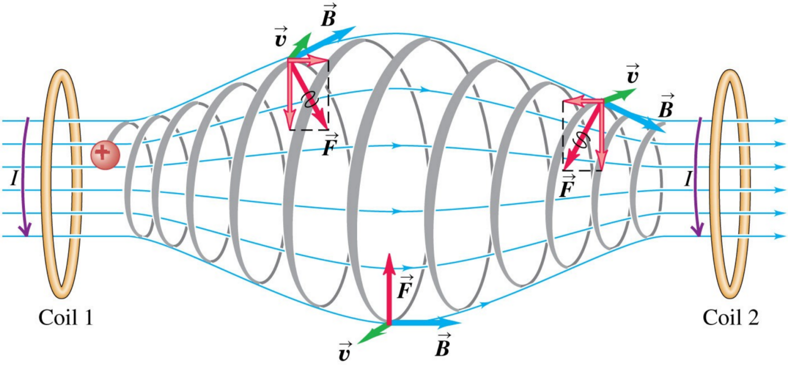
$\bar{B} \neq 0$ and $\bar{v} \not\perp \bar{B}$, $\bar{v} \nparallel \bar{B}$



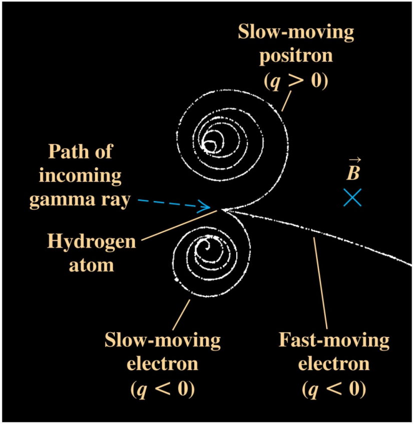
$\bar{v}_{\parallel}$ – component of $\bar{v}$ parallel to $\bar{B}$

Applications

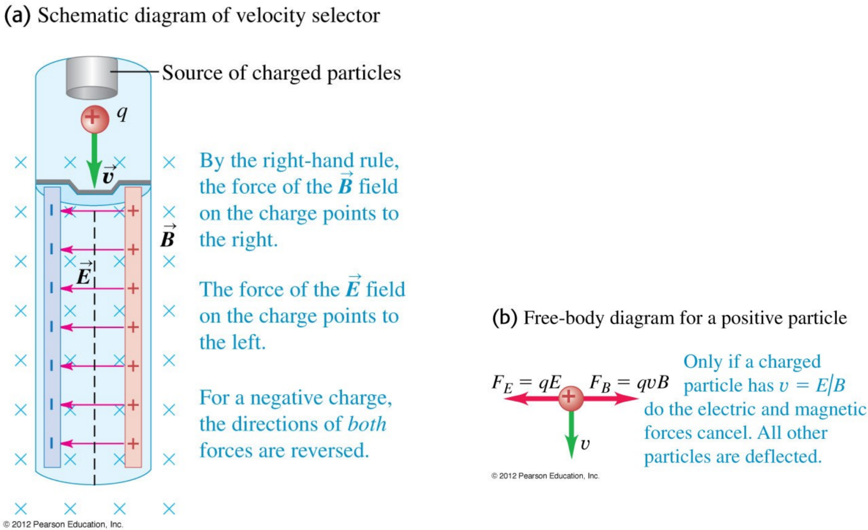
Magnetic Trap



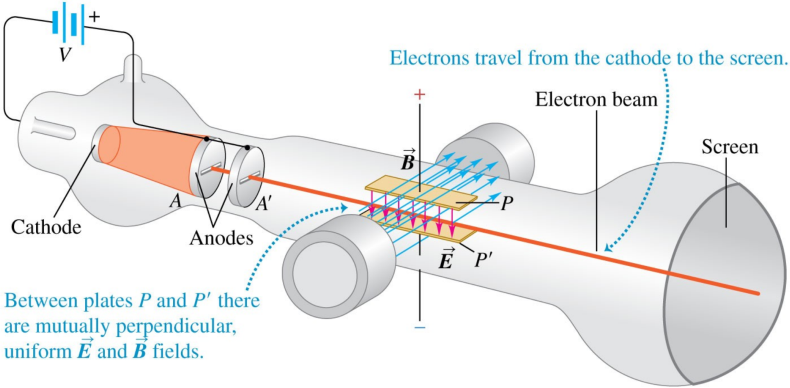
Bubble Chamber



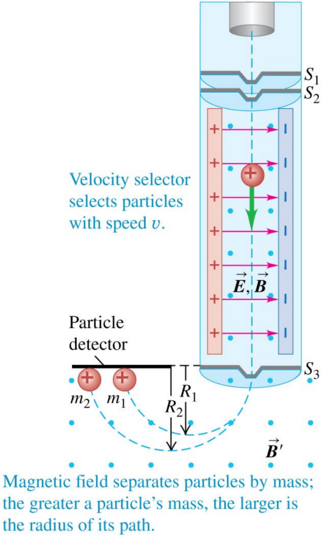
Velocity Selector



Thomson's Experiment (e/m Ratio)



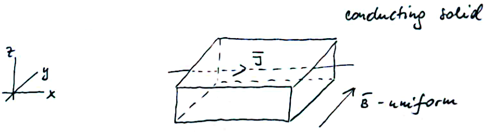
Mass Spectrometer



The Hall Effect

The Hall Effect

Consider electric current flowing though a conducting slab placed in a uniform magnetic field.



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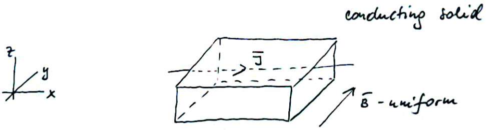

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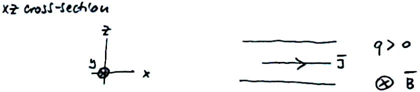

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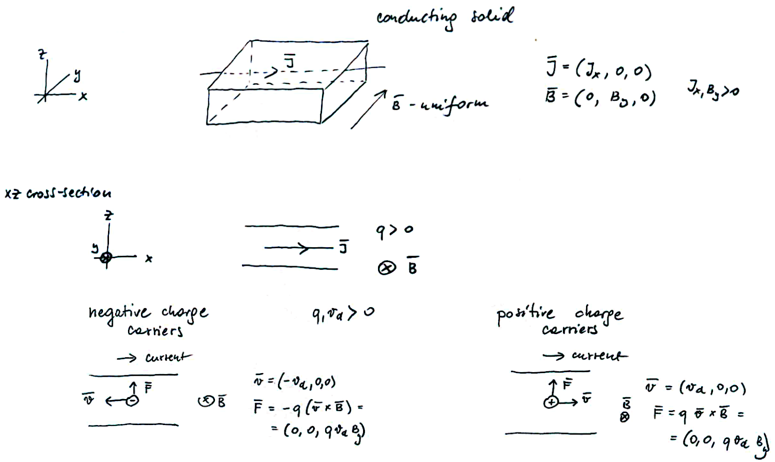

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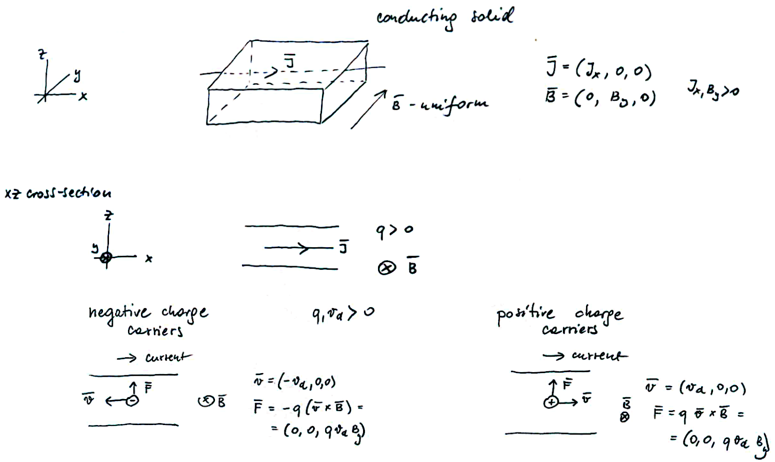
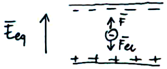
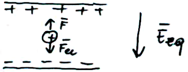


Observation. Charges are deflected towards the surface of the slab $\Longrightarrow$ electric field in the z direction builds up.

Equilibrium FBD



note the direction of $\bar{E}_{eq}$ in both cases!



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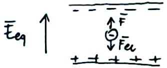

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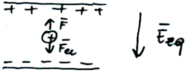

Equilibrium condition

$$\bar{F}_{el} + \bar{F} = 0$$

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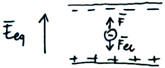

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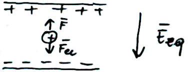

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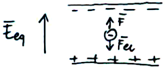

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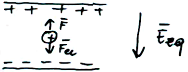

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in the positive z-direction for negative charges

in the negative z-direction for positive charges

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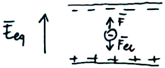

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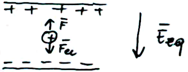

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Comments

- Can be used to determine the sign of charge carriers.
- If the concentration of charge carries is known, can be used to measure the magnetic field (Hall probe).

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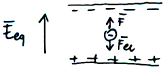

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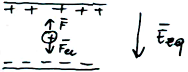

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$$nq = \frac{J_x}{v_d} = \frac{J_x}{E_{eq}} B_y \Rightarrow \boxed{B_y = \frac{nq E_{eq}}{J_x}}$$

Current–Carrying Conductor In Magnetic Field

Magnetic Force On a Current–Carrying Conductor

Starting point

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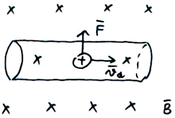
$\downarrow$

uniform



force on a single charge

$\downarrow$ add all contributions

total force

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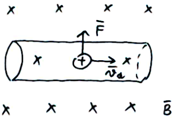

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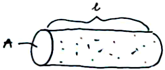

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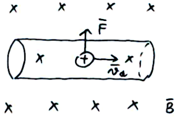

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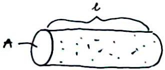

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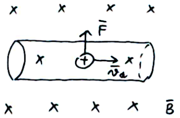

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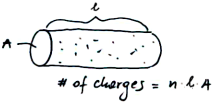

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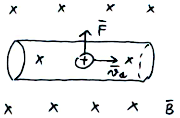

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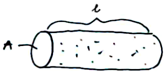

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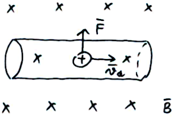
$\downarrow$

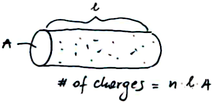
uniform



force on a single charge

$\Big\downarrow$ add all contributions

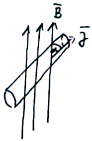
total force



total force = # of charges · force on a single charge

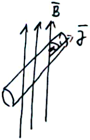
$$F = (n\ell A)qv_\mathrm{d}B = \underbrace{(nqv_\mathrm{d})A}_{I}\,\ell B = I\ell B$$

$\bar{B} \perp \bar{J}$



$$F = (n\ell A) q v_{\mathrm{d}} B \sin \measuredangle(\overline{J}, \overline{B})$$

$\overline{B} \measuredangle \overline{J}$



$$F = (n\ell A)qv_{\mathrm{d}}B\sin\measuredangle(\overline{J},\overline{B}) = \underbrace{(nqv_{\mathrm{d}})A}_{I}\,\ell B\sin\measuredangle(\overline{J},\overline{B})$$

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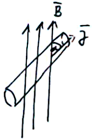

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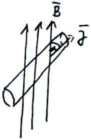

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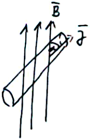

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Let $\overline{\ell}$ point in the current's direction. Then

$$\boxed{\overline{F} = I \overline{\ell} \times \overline{B}.}$$

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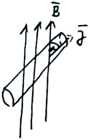

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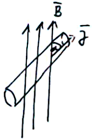

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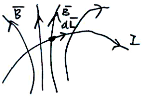
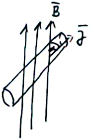


$\overline{B} \measuredangle \overline{J}$



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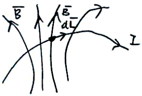
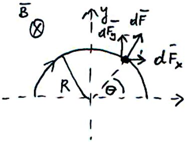


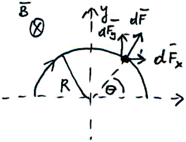
$$\boxed{\overline{F} = \int_{\text{conductor}} I \, \mathrm{d}\overline{\ell} \times \overline{B}.}$$

Example



$\otimes$ $\bar{B}$ - uniform

Example



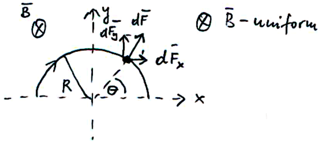
$\otimes$ $\bar{B}$ - uniform

$$dF_x = \cos\theta \, dF$$

$$dF_y = \sin\theta \, dF$$

$$|d\bar{F}| = I \, |d\bar{l} \times \bar{B}| \overset{d\bar{l} \perp \bar{B}}{=\!=} I B \, d\bar{l}$$

Example



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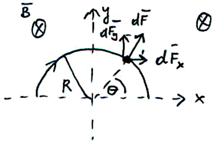
$$F_y = \int_{\text{Conductor}} dF_y = \int_{\text{Conductor}} \sin\theta \, I B \, dL$$

Example



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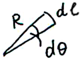
$dF_y = \sin\theta \; dF$

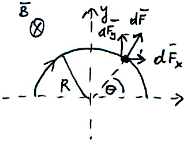
$|d\bar{F}| = I \, |d\bar{l} \times \bar{B}| \overset{d\bar{l} \perp \bar{B}}{=\!=} I B \, d\bar{l}$



$d\theta = \frac{dl}{R}$

$dl = R \, d\theta$

$$F_y = \int_{\text{Conductor}} dF_y = \int_{\text{Conductor}} \sin\theta \, I B \, dL$$



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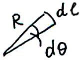


$$d\theta = \frac{dl}{R}$$
$$dl = R \, d\theta$$

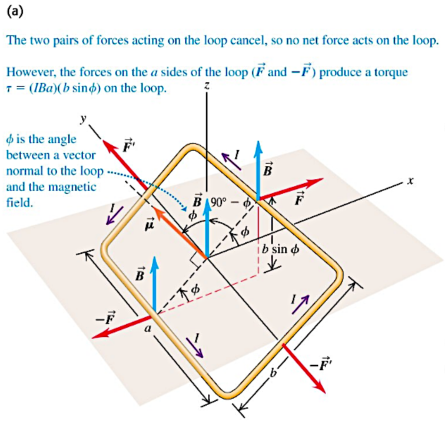
$$F_y = \int_{\text{Conductor}} dF_y = \int_{\text{Conductor}} \sin\theta \, I B \, dL$$

$$= \int_0^{\pi} I R \, B \sin\theta \, d\theta = I B R \cdot 2$$

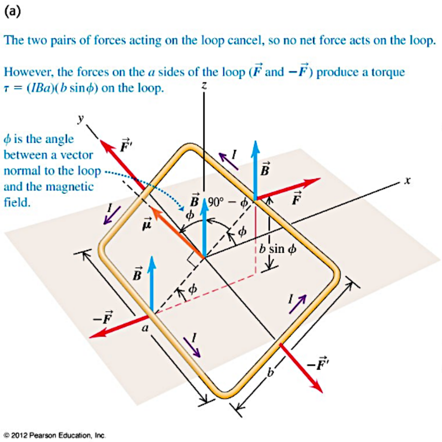
$F_x = 0$ (same method, or symmetry)

Current Loop In Uniform Magnetic Field

Current Loop in Uniform Magnetic Field

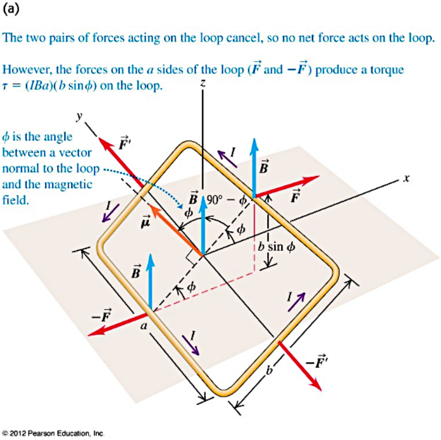


Current Loop in Uniform Magnetic Field



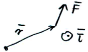
Observation. The net force on the loop is always zero.

Current Loop in Uniform Magnetic Field



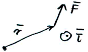
Observation. The net force on the loop is always zero. However, the net torque is not.

Torque $\overline{\tau} = \overline{r} \times \overline{F}$



(always with respect to a point/axis!)

Torque $\overline{\tau} = \overline{r} \times \overline{F}$



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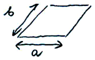
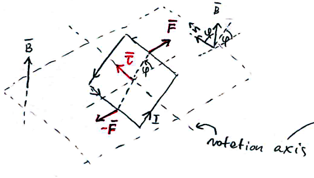
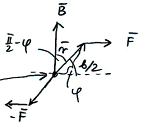




$\varphi = \sphericalangle(\bar{B}, \hat{n})$



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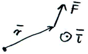

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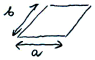
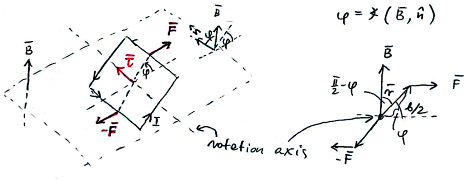
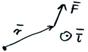




$$\tau = 2\,\frac{b}{2} F \sin\varphi$$

Torque $\overline{\tau} = \overline{r} \times \overline{F}$



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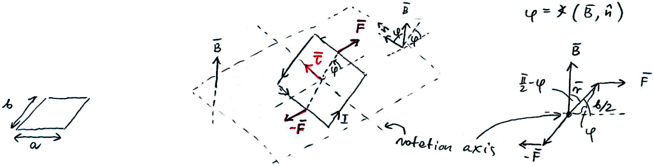

$$\tau = 2\,\frac{b}{2} F \sin\varphi = Fb \sin\varphi$$

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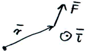

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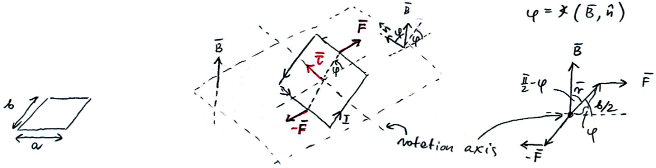

$$\tau = 2\,\frac{b}{2} F \sin\varphi = Fb\sin\varphi \overset{F=IaB}{=} IaBb\sin\varphi$$

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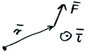

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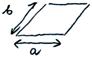
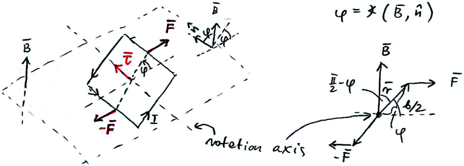




$$\tau = 2\,\frac{b}{2} F \sin\varphi = F b \sin\varphi \overset{F = IaB}{=} IaBb \sin\varphi$$

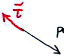
$|\tau| = |I\,a\,B\,\sin\varphi|$

max for $\varphi = \pi/2,\ \frac{3}{2}\pi$

min for $\varphi = 0,\ \pi$

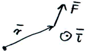
stable equilibrium (for $\varphi = 0$)

unstable equilibrium (for $\varphi = \pi$)



positive direction of $\overline{\varphi}$

Torque $\overline{\tau} = \overline{r} \times \overline{F}$



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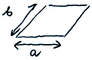
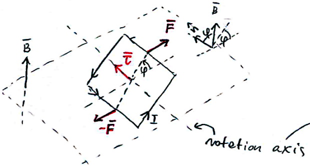
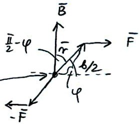




$\varphi = \sphericalangle(\bar{B}, \hat{n})$



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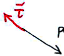
Stable equilibrium

unstable equilibrium



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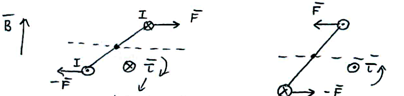

tries to restore the equilibrium $\varphi=0$

rotates out of $\varphi=\pi$ equilibrium

Magnetic Dipole Moment

Magnetic Dipole Moment

Torque on a current loop placed in uniform magnetic field

$$\tau = I \underbrace{(ab)}_{\text{area } A} B \sin\varphi$$

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Define the magnitude of the **magnetic dipole moment**

$$\mu = IA.$$

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Then

$$\tau = \mu B \sin\varphi$$

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$$\tau = \mu B \sin\varphi \qquad \text{or in the vector form} \qquad \boxed{\tau = \overline{\mu} \times \overline{B}}.$$

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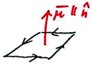
Magnetic dipole moment (vector)

$$\boxed{\overline{\mu} = (IA)\hat{n}}$$

Right-hand rule for current loop



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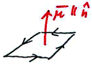

Compare: Electric dipole moment.

Potential Energy of Magnetic Dipole in Uniform Magnetic Field

Potential energy (see derivation for the electric dipole)

$$\boxed{U = -\overline{\mu} \circ \overline{B}}$$

Potential Energy of Magnetic Dipole in Uniform Magnetic Field

Potential energy (see derivation for the electric dipole)

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where we have chosen the gauge so that $U(\pi/2) = 0$, i.e. when $\overline{B} \perp \overline{\mu}$.

Potential Energy of Magnetic Dipole in Uniform Magnetic Field

Potential energy (see derivation for the electric dipole)

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where we have chosen the gauge so that $U(\pi/2) = 0$, i.e. when $\overline{B} \perp \overline{\mu}$.

Comments

- Stable equilibrium (minimum of U) if $\varphi = 0$.

Potential Energy of Magnetic Dipole in Uniform Magnetic Field

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Comments

- Stable equilibrium (minimum of U) if $\varphi = 0$.
- Unstable equilibrium (maximum of U) if $\varphi = \pi$.

Potential Energy of Magnetic Dipole in Uniform Magnetic Field

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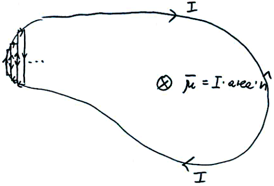

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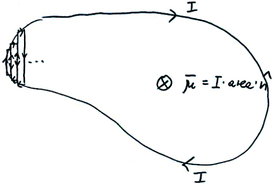

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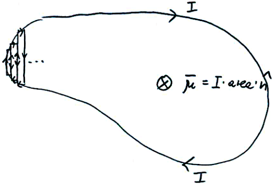

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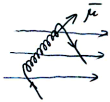
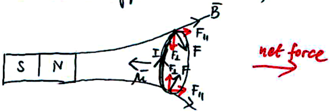


collection of N loops of area A

$\boxed{\bar{\mu} = N \cdot I A \hat{n}}$ right-hand rule

What happens if a magnetic dipole moment (current loop) is placed in a non-uniform magnetic field?

(1) $\bar{\mu}$ in the direction opposite to the field



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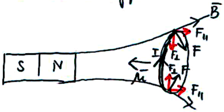

net force $\longrightarrow$

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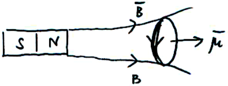

net force $\longleftarrow$

$F_{\perp} = 0$

$F_{\parallel} \neq 0$ (to the left) $\Rightarrow$ attraction

Flux of Magnetic Field (Magnetic Flux) and Gauss's Law for Magnetic Field

Magnetic Flux

Definition of the flux of the magnetic field (magnetic flux) is fully analogous to that of the electric flux.

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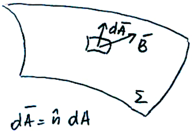

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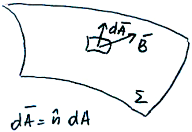

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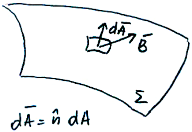

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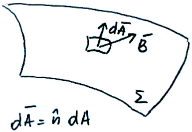

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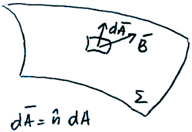

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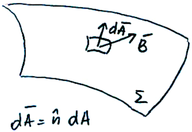

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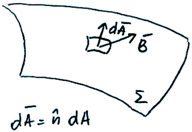

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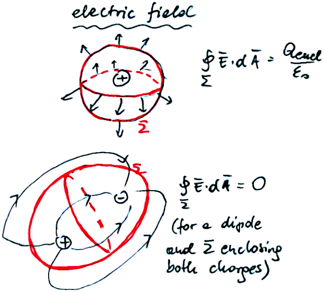

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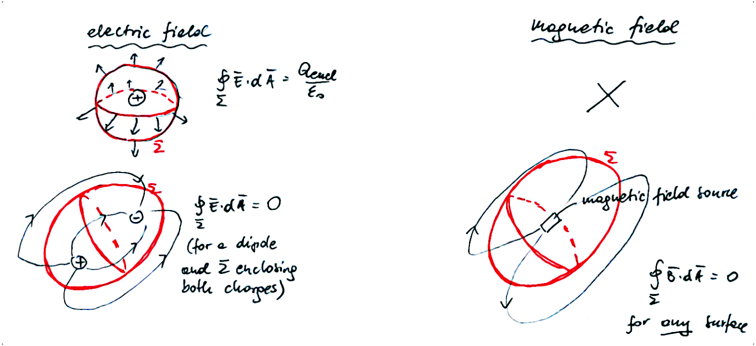

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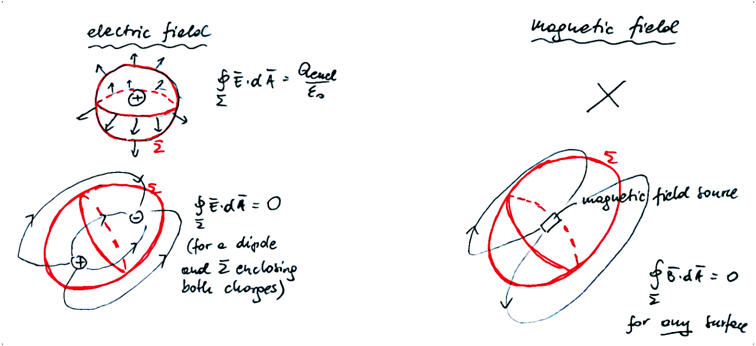

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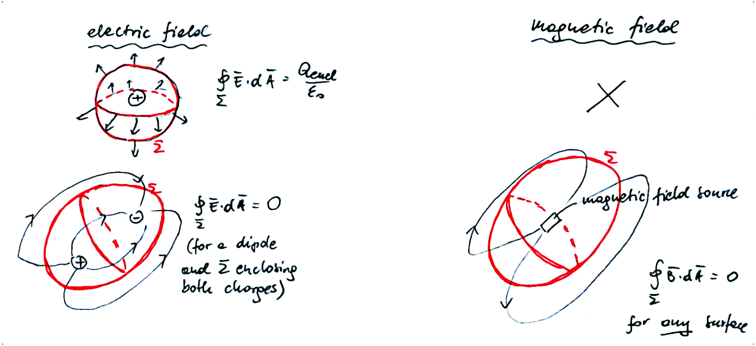

Electric field lines begin and end on electric charges; magnetic field lines <u>always</u> form loops.

Gauss's Law For Magnetic Field. Differential Form

The differential form of Gauss's Law for the magnetic field can be derived in the same manner as for the electric field

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Recall the interpretation of divergence

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Recall the interpretation of divergence; there are no magnetic monopoles (no magnetic "point charges").