Chapter 5 – Electric Current, Resistance, and Electromotive Force

UM-SJTU Joint Institute
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Mateusz Krzyzosiak

Agenda

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   - Electric Current Density
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   - Ohm's Law (Microscopic and Macroscopic Form)
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5. Introduction to Electric Circuits
   - Electric Potential Change Along a Circuit
   - Energy and Power in Circuits

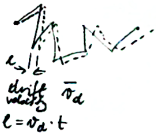
Introduction

Introduction

Electric current — macroscopic motion of electric charge.

(needs mobile charges!)

Motion of charges: due to electric field



chaotic motion $v \sim 10^6$ m/s

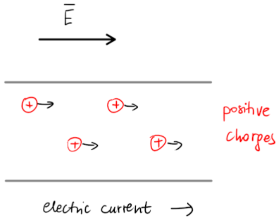
drift speed $v_\mathrm{d} \sim 10^{-4}$ m/s
(e.g. electron in copper wire at room temperature)

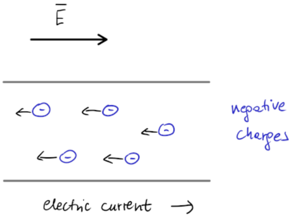
Why don't electrons in solids keep moving faster and faster?
Why do electric wires get warm?

work done by the electric field on electrons $\longrightarrow$ acceleration of electrons $\longrightarrow$ collisions with other electrons and ions $\longrightarrow$ (vibrational) kinetic energy of ions increases $\longrightarrow$ temperature of the material increases

Electric Current as a Physical Quantity

Electric Current. Definition





Convention (regarding the flow direction): *The electric current flows in the direction of motion of positive charge carriers.*

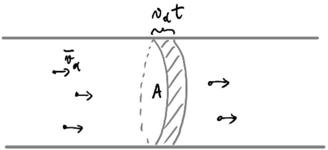
Current magnitude

$$\boxed{I \stackrel{\text{def}}{=} \frac{\mathrm{d}Q}{\mathrm{d}t}}$$

where $\mathrm{d}Q$ is the magnitude of the net charge flowing through a cross-section of the conductor in time $\mathrm{d}t$.
SI unit: ampere [$1\ \mathrm{A} = 1\ \mathrm{C}/1\ \mathrm{s}$].

Examples. Starter motor in a car $I \sim 200$ A, computer circuits $10^{-9} A$.

Electric Current Density



n – concentration of charges $[m^{-3}]$
(# of charges per unit volume)

Electric charge flowing through the area A in time $\mathrm{d}t$

$$\mathrm{d}Q = |q|(nAv_\mathrm{d}\,\mathrm{d}t) \qquad \Longrightarrow \qquad I = \frac{\mathrm{d}Q}{\mathrm{d}t} = n|q|v_\mathrm{d}A$$

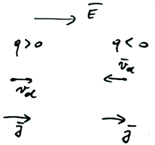
Electric current density

- magnitude (electric current per unit cross-sectional area)

$$J = \frac{I}{A} = n|q|v_\mathrm{d} \qquad\qquad \text{SI units: } [\mathrm{A}/\mathrm{m}^2]$$

- direction — always in the direction of positive charge flow

$$\bar{J} = nq\bar{v}_\mathrm{d}$$



Drude's Model of Conductivity. Ohm's Law

Drude's Model of Conductivity

Microscopic model of conductivity was formulated in 1900 by Paul Drude (and is often referred to as the *Drude's model*). It is a classical model, in the sense that it does not include any quantum–mechanical effects.

The electric current sets in as a result of the following subsequent processes the charge carriers undergo

acceleration (by $\overline{E}$*)* $\longrightarrow$ *collision* $\longrightarrow$ *acceleration* $\longrightarrow$ *collision...*

$$\boxed{\overline{J} = nq\overline{v}_\mathrm{d}}$$

Initially, in the absence of electric field,

velocity of single electron	*velocity averaged over all electrons*
$\overline{v}_0$	$(\overline{v}_0)_\mathrm{av} = 0$ (chaotic motion; no macroscopic current)

In a non-zero electric field, electrons are acted upon by the electric force and, from Newton's 2^{nd} law of dynamics,

$$\overline{a} = \frac{q\overline{E}}{m} \qquad \Longrightarrow \qquad \overline{v} = \overline{v}_0 + \underbrace{\frac{q}{m}\overline{E}}_{\overline{a}}\, t.$$

Averaging over all electrons

$$\overline{v}_{\text{av}} = \underbrace{(\overline{v}_0)_{\text{av}}}_{=0} + \frac{q}{m}\overline{E}t_{\text{av}},$$

where $t_{\text{av}} \stackrel{\text{def}}{=} \tau$ is the average time until a collision occurs (averaged over all electrons). Hence

$$\boxed{\overline{v}_{\text{av}} = \frac{q}{m}\overline{E}\tau} \stackrel{\text{def}}{=} \overline{v}_{\text{d}}$$

Ohm's Law (microscopic form)

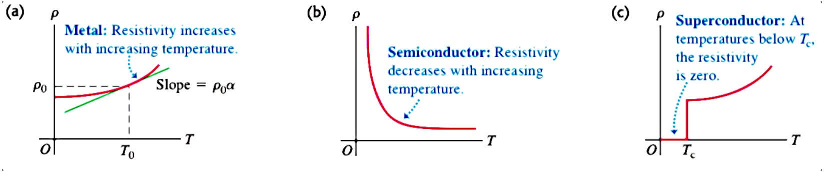
Consequently, the current density

$$\overline{J} = nq\overline{v}_{\mathrm{d}} = \frac{nq^2\tau}{m}\overline{E}.$$

So that

$$\boxed{\overline{J} = \frac{\overline{E}}{\rho}} \qquad \text{where the resisitivity } \rho = \frac{m}{nq^2\tau} \quad \left[\frac{\mathrm{V}\cdot\mathrm{m}}{\mathrm{A}} = \Omega\cdot\mathrm{m}\right]$$

This relationship is known as the **Ohm's Law** in the microscopic form. The inverse of conductivity $\sigma = \rho^{-1} = |\overline{J}|/|\overline{E}|$ is known as the conductivity.



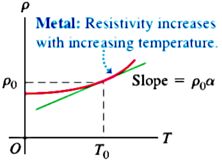
Room–Temperature Resistivity of Various Materials

Table 25.1 Resistivities at Room Temperature (20°C)

	Substance	$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	$10^{10}-10^{14}$
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	$10^{11}-10^{15}$
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	$10^{8}-10^{11}$

Temperature Coefficient of Resistivity

For metals, in a relatively small range of temperatures (ca. $0 \sim 100^o$ C), resistivity ρ is a linear function of the temperature T



$$\rho(T) \approx \rho_0 \left[1 + \alpha(T - T_0)\right],$$

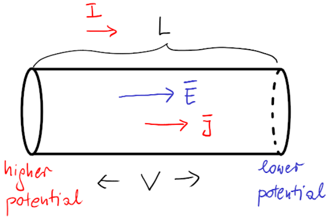
where α is the temperature coefficient of resistivity.

Table 25.2 Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

Material	α [$(^\circ C)^{-1}$]	Material	α [$(^\circ C)^{-1}$]
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

Ohm's Law (macroscopic form)

Ohm's law, that is the linear relationship between the electric current I and the potential difference (voltage) V was discovered experimentally in 1826. It is actually not a universal law of physics, as it holds under certain conditions, outlined by the so-called linear response theory.



$(\bar{E} = -\text{grad}\, V)$

If $\overline{J}$ and $\overline{E}$ are uniform, then $V = EL$ and

$$\rho = \frac{|\overline{E}|}{|\overline{J}|} = \frac{\frac{V}{L}}{\frac{I}{A}} = \frac{VA}{LI} \qquad \Longrightarrow \qquad \frac{V}{I} = \underbrace{\rho \frac{L}{A}}_{R \text{ (resistance)}}$$

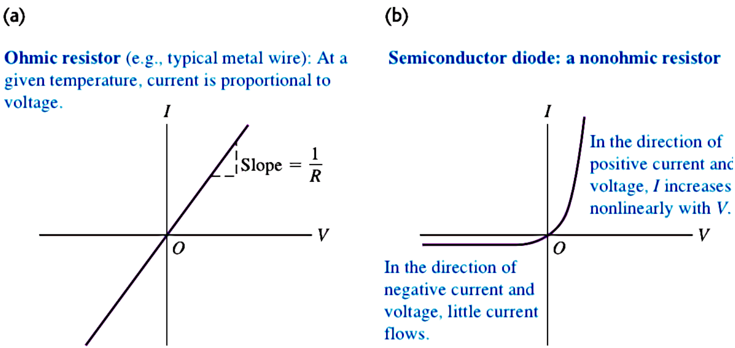
Resistance $\boxed{R = \dfrac{\rho L}{A}}$. The SI units of resistance are ohms $[\Omega = \mathrm{V/A}]$.

For metals, in a small temperature range, $R(T) = R_0[1 + \alpha(T - T_0)]$.

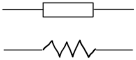
Ohm's law (macroscopic form)

$$\boxed{V = IR}$$

25.10 Current–voltage relationships for two devices. Only for a resistor that obeys Ohm's law as in **(a)** is current I proportional to voltage V.



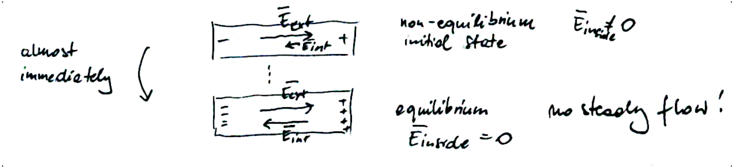
Symbols in circuits

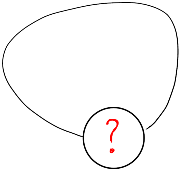


Electromotive Force

Motivation

Why, in order to have steady sustainable current flowing through a conductor, does the conductor need to be part of a loop?

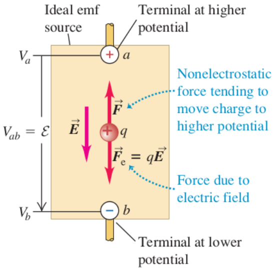




? – generates a potential difference ⇒ electric field

electromotive force (emf)

Ideal Electromotive Force

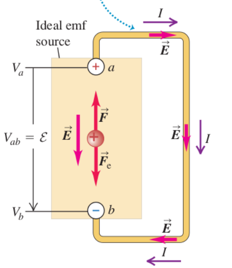


When the emf source is not part of a closed circuit, $F = F_e$ and there is no net motion of charge between the terminals.

The nature of the force $\overline{F}$, which does work $qV_{ab} = q\mathcal{E}$ on a charge q, depends on the nature of the emf (i.e. the mechanism that generated V_{ab}). For an ideal emf $V_{ab} = \mathcal{E}$.

For an ideal emf $\mathcal{E} = V_{ab} = IR$, where R is the resistance of the wire (or, in general, any elements in the loop outside of the emf).

Potential across terminals creates electric field in circuit, causing charges to move.



Real Electromotive Force

Charges moving inside a real emf experience *internal resistance*, and if the emf is in a closed circuit

$$V_{ab} = \mathcal{E} - Ir,$$

where the term Ir accounts for the drop in the electric potential due to the internal resistance. Only if the circuit is open ($I = 0$), we have $V_{ab} = \mathcal{E}$.

If the resistance of the loop (circuit) outside of the emf is R, then for a closed circuit

$$\mathcal{E} - Ir = IR \qquad \text{or} \qquad I = \frac{\mathcal{E}}{R + r}.$$

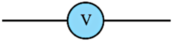
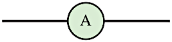
Note. If $R \to \infty$, then $I \to 0$.

Introduction to Electric Circuits

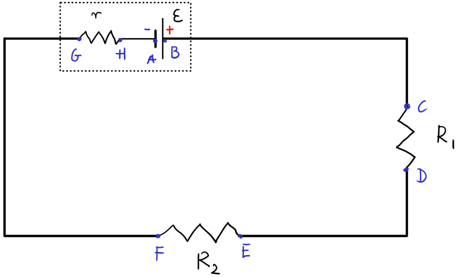
Assumptions

- Real emf with $\mathcal{E}, r =$ cont (in fact, r increases with the age of the battery).
- Only Ohmic elements in the circuit.
- Wires connecting elements in the circuit do not have resistance (are perfect conductors).

Table 25.4 Symbols for Circuit Diagrams

	Conductor with negligible resistance
	Resistor
	Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)
 or 	Source of emf with internal resistance r (r can be placed on either side)
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current through it)

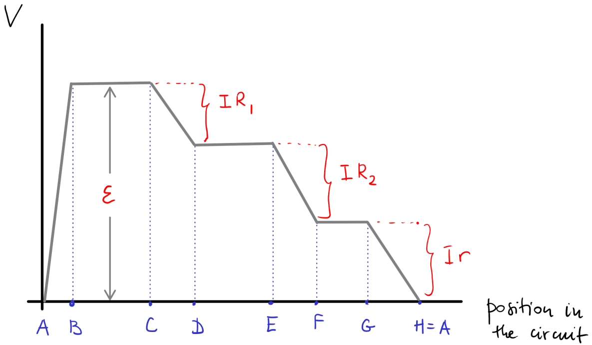
Electric Potential Change Along a Circuit



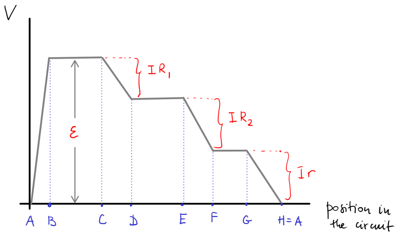
closed loop

$\Downarrow$

$V_A = V_H$



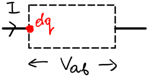
Energy in Circuits



- On CD, EF, GH — loss of potential energy, but no increase in the charges' kinetic energy ($I = \mathrm{const.}$ Hence, part of energy converted into other forms of energy (thermal — heat).
- On AB — increase of the potential energy as a result to conversion from other forms of energy (e.g. chemical) into electric potential energy.

Power in Circuits

Power — rate at which energy is being changed (dissipated/delivered).



$$dq = I dt \quad \Longrightarrow \quad V_{ab}\, dq = \underbrace{I\, V_{ab}}_{P}\, dt$$

$$\boxed{P = I\, V_{ab}}$$

SI Units $\left[\frac{\mathrm{C}}{\mathrm{s}} \cdot \mathrm{V} = \frac{\mathrm{C}}{\mathrm{s}} \cdot \frac{\mathrm{J}}{\mathrm{C}} = \frac{\mathrm{J}}{\mathrm{s}} = W\right]$ (Watt)

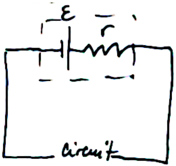
Examples

(a) resistor

$$V_{ab} = R\, I \quad \Longrightarrow \quad P = V_{ab}\, I \overset{\text{Ohm's law}}{=} I^2 R = \frac{V_{ab}^2}{R}$$

The energy is dissipated as heat (Joule heat/Ohmic heat).

(b) emf



$$P = V_{ab}\, I = (\mathcal{E} - Ir)I = \mathcal{E}I - I^2 r$$

The power output of the source (P) is the difference between the rate at which the potential energy of the charges increases ($\mathcal{E}I$) and the rate at which the energy is dissipated (Ir) on the internal resistance.

(c) car's alternator — see recitation class