Vp250 Problem Set 2

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Problem 1

(a) Suppose the Gauss surface is a sphere with radius r. Then

$$\int_\Sigma E_G \mathrm{d}\bar{A} = E_G(r)\int_\Sigma \mathrm{d}A = E_G(r)4\pi r^2 = -4\pi GM \Rightarrow \bar{E}_G(r) = -\frac{GM}{r^2}\hat{n}_r$$

$\hat{n}_r$ is the unit vector in the direction of r.

(b) r>R: Suppose the Gauss surface as a sphere with radius of r centered at the center of the ball.

$$\int_\Sigma E_G \mathrm{d}\bar{A} = E_G(r)\int_\Sigma \mathrm{d}A = E_G(r)4\pi r^2 = -4\pi GM \Rightarrow \bar{E}_G(r) = -\frac{GM}{r^2}\hat{n}_r$$

r<R: Suppose a Gauss surface as a sphere with radius of r centered at the center of the ball.
The mass enclosed by the sphere is $\frac{3M}{4\pi R^3}\cdot\frac{4\pi r^3}{3} = \frac{Mr^3}{R^3}$

$$\int_\Sigma E_G \mathrm{d}\bar{A} = E_G(r)\int_\Sigma \mathrm{d}A = E_G(r)4\pi r^2 = -4\pi G\frac{Mr^3}{R^3} \Rightarrow \bar{E}_G(r) = -\frac{GMr}{R^3}\hat{n}_r$$

(c) r>R: Suppose the Gauss surface as a sphere with radius of r centered at the center of the ball.

$$\int_\Sigma E_G \mathrm{d}\bar{A} = E_G(r)\int_\Sigma \mathrm{d}A = E_G(r)4\pi r^2 = -4\pi GM \Rightarrow \bar{E}_G(r) = -\frac{GM}{r^2}\hat{n}_r$$

r<R: Suppose a Gauss surface as a sphere with radius of r centered at the center of the ball.
The mass enclosed by the sphere is 0, so by the equation $E_G(r) = 0$.

Problem 2

(a) Define a Gauss surface as a box with 4 side-walls parallel to the electric field. Then for the left two sides, which are both perpendicular to the electric field, the flux through them will be canceled. Hence, we can see that the net flux through the box is 0.
According to the Gauss's law:

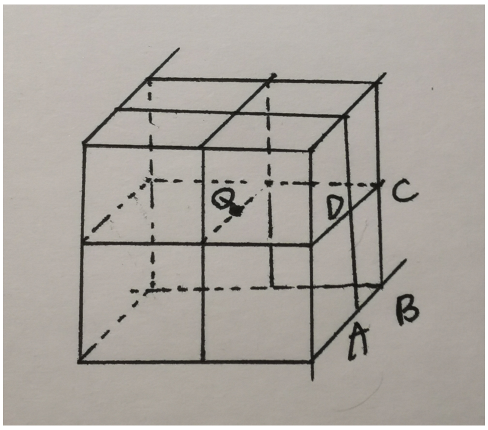
$$\int_\Sigma E\mathrm{d}\bar{A} = 0 = \frac{Q_{encl}}{\epsilon_0} \Rightarrow Q_{encl} = 0$$

If the volume charge density is not 0, then $Q_{encl} = \int_\Omega \rho \mathrm{dV} \neq 0$, which leads to contradiction. So the density must be 0.

(b) Wrong.
Suppose there is a space near a point charge that does not include the charge itself. So the net charge in such space is 0 but obviously the electric field in it is not uniform.

Problem 3

The figure using symmetry is shown below:



The flux induced by the charge Q on the surface is $\frac{Q}{\varepsilon_0}$ Since the area of $ABCD$ is $\frac{1}{6\times 4} = \frac{1}{24}$ of the total surface area. So by symmetry, the flux on $ABCD$ is $\frac{1}{24}\cdot\frac{Q}{\varepsilon_0} = \frac{Q}{24\varepsilon_0}$.

Problem 4

(a)

$$Q_{total} = \int_0^{2\pi}\int_0^{\pi}\int_a^b \frac{k}{r} r^2 \sin\theta \mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi = 2\pi\int_0^{\pi}\int_a^b kr\sin\theta \mathrm{d}r\mathrm{d}\theta = \pi\int_0^{\pi} k\sin\theta(b^2-a^2)\mathrm{d}\theta = 2\pi k(b^2-a^2)$$

(b) (i) Q_{encl} in the region enclosed by the sphere with radius $r < a$ is 0. So

$$\frac{Q_{encl}}{\varepsilon_0} = \int_\Sigma E\mathrm{d}\bar{A} = 0 \Rightarrow E = 0$$

(ii) Q_{encl} in the region enclosed by the sphere with radius $a < r < b$ is

$$Q_{encl} = \int_0^{2\pi}\int_0^{\pi}\int_a^r \frac{k}{r} r^2 \sin\theta \mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi = 2\pi k(r^2 - a^2)$$

By Gauss' law,

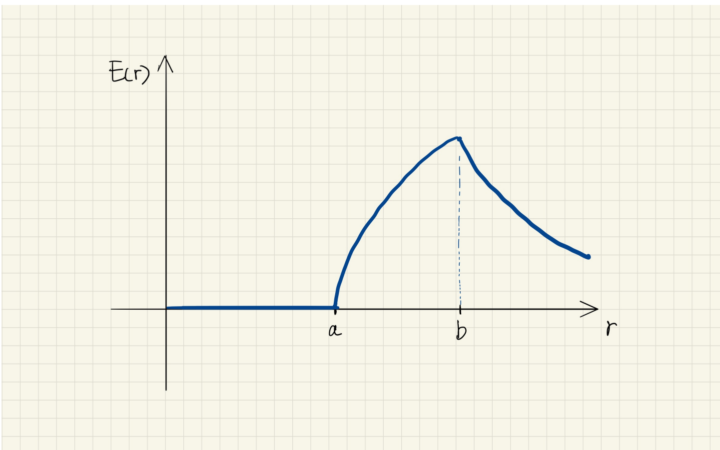
$$\frac{Q_{encl}}{\varepsilon_0} = \int_\Sigma E\mathrm{d}\bar{A} = E(r)\int_\Sigma \mathrm{d}A = E(r)4\pi r^2 \Rightarrow \bar{E}(r) = \frac{Q_{encl}}{4\pi r^2\varepsilon_0} = \frac{k(r^2-a^2)}{2r^2\varepsilon_0}\hat{n}_r$$

($\hat{n}_r$ is the unit vector in the direction of r)

(iii) Q_{encl} in the region enclosed by the sphere with radius $r > b$ is Q_{total}. So

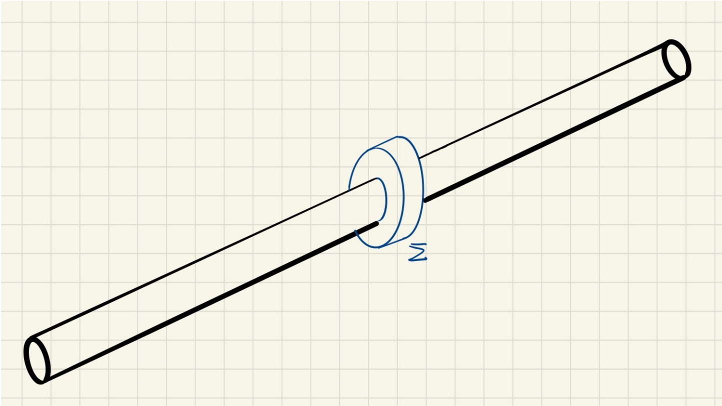
$$\frac{Q_{encl}}{\varepsilon_0} = \int_\Sigma E\mathrm{d}\bar{A} = E(r)4\pi r^2 \Rightarrow \bar{E}(r) = \frac{Q_{encl}}{4\pi r^2\varepsilon_0} = \frac{k(b^2-a^2)}{2r^2\varepsilon_0}\hat{n}_r$$

(c) The plot is shown below



Problem 5

The Gauss surface (a small cylinder) constructed is shown below:



So that we know the radius of the cylinder is r. And suppose the height of the small cylinder is $\mathrm{d}h$ When $r > R$, the charge enclosed by the small cylinder is

$$Q_{encl} = \pi\rho R^2 \mathrm{d}h$$

By Gauss's law,

$$\frac{Q_{encl}}{\varepsilon_0} = \int_{\Sigma} E\mathrm{d}\bar{A} = \int_{side-wall} E\mathrm{d}\bar{A} + \underbrace{\int_{top-downwall} E\mathrm{d}\bar{A}}_{0} = E(r)2\pi r\mathrm{d}h \Rightarrow \bar{E}(r) = \frac{\rho R^2}{2\varepsilon_0 r}\hat{n}_r$$

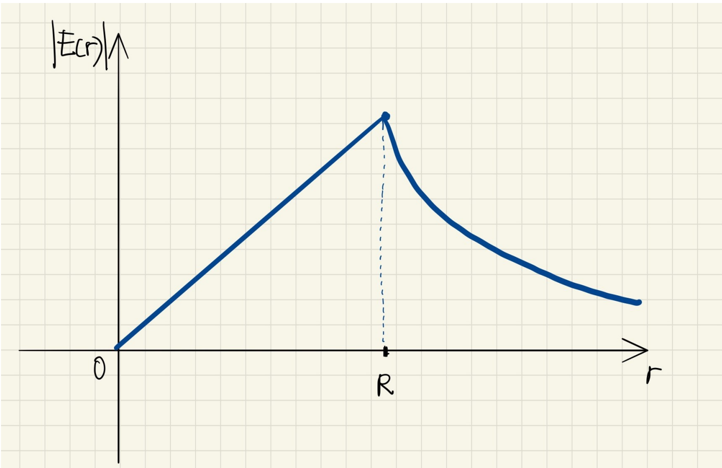
When $r < R$, the charge enclosed by the small cyliner is

$$Q_{encl} = \pi r^2 \rho \mathrm{d}h$$

By Gauss's law,

$$\frac{Q_{encl}}{\varepsilon_0} = \int_{\Sigma} E\mathrm{d}\bar{A} = \int_{side-wall} E\mathrm{d}\bar{A} + \underbrace{\int_{top-down\quad wall} E\mathrm{d}\bar{A}}_{0} = E(r)2\pi r\mathrm{d}h \Rightarrow \bar{E}(r) = \frac{\rho r}{2\varepsilon_0}\hat{n}_r$$

($\hat{n}_r$ is a unit vector in the direction of r) The sketch of $|E(r)| - r$:



Problem 6

According to Problem 5, the electric field induced by a infinite cylinder is $E(r) = \dfrac{\rho r}{2\varepsilon_0}$. So for a point inside the hole with a distance r from the axis of the big cylinder:

$$\bar{E}(r) = \frac{\rho r}{2\varepsilon_0}\hat{n}_r - \frac{\rho r_h}{2\varepsilon_0}\hat{n}_h = \frac{\rho b}{2\varepsilon_0}\hat{n}_b$$

(where $\hat{n}_r, \hat{n}_h$ and $\hat{n}_b$ are unit vectors. $\hat{n}_r$ points from origin to the point, $\hat{n}_h$ points from the axis of the hole to the point and $\hat{n}_b$ point from origin to the axis of the hole)

So that we can see that for any point inside the hole, $E(r)$ is the same magnitute $(\frac{\rho b}{2\varepsilon_0})$ with same direction (point from the origin to the axis of the hole).

Problem 7

(a) The charge residing on the surface of the cavity is $-q_a$ and $-q_b$ respectively. So the density:

$$\sigma_a = \frac{-q_a}{4\pi r_a^2} \qquad \sigma_b = \frac{-q_b}{4\pi r_b^2}$$

The total charge residing on the surface of the ball is $q_a + q_b$. So the density:

$$\sigma_R = \frac{q_a + q_b}{4\pi R^2}$$

(b) Gauss surface: a sphere centered the same as the ball with radius r $(r > R)$. Then by Gauss's law:

$$\frac{q_a + q_b}{\varepsilon_0} = \int_\Sigma E \mathrm{d}\bar{A} = E(r) 4\pi r^2 \Rightarrow \bar{E}(r) = \frac{q_a + q_b}{4\pi r^2 \varepsilon_0} \hat{n}_r$$

($\hat{n}_r$ is the unit vector in the direction of r)

(c) For a: Suppose the distance between the center of the cavity a and the point inside that cavity is r_1.

$$\frac{Q_{encl}}{\varepsilon_0} = \frac{q_a}{\varepsilon_0} = \int_\Sigma E \mathrm{d}\bar{A} = E_a(r) 4\pi r_1^2 \Rightarrow \bar{E}_a(r) = \frac{q_a}{4\pi r_1^2 \varepsilon_0} \hat{n}_1$$

($\hat{n}_1$ is the unit vector in the direction of r_1) For b: Srppose the distance between the center of the cavity a and the point inside that cabity is r_2.

$$\frac{Q_{encl}}{\varepsilon_0} = \frac{q_b}{\varepsilon_0} = \int_\Sigma E \mathrm{d}\bar{A} = E_b(r) 4\pi r_2^2 \Rightarrow \bar{E}_b(r) = \frac{q_b}{4\pi r_2^2 \varepsilon_0} \hat{n}_2$$

($\hat{n}_2$ is the unit vector in the direction of r_2)

(d) 0. Because the induced electric field has symmetry subh that the resulting forces will cancel out each other.

(e) σ_R and electric field outside the ball will change. Because they are influenced by the extra charge outside and the charge residing on the outer surface will change their position.
σ_a, σ_b, electric field within the cavity and the force on the charge will not change due to the electrostatic property of the conductor.