Chapter 4 – Capacitors and Dielectrics. Electric Field in Matter

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Agenda

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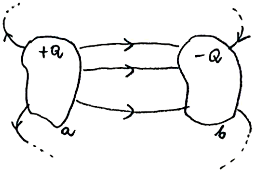
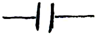
2. Capacitance
   - Definition
   - Examples of Calculation
   - Systems of Capacitors and Equivalent Capacitance
3. Capacitors as Energy Storage Systems
4. Dielectrics
   - Basic Properties
   - Polarization. Induced vs. Free Charges
   - Gauss's Law In Dielectrics
   - Fringing Effect

What Is a Capacitor?

What Is a Capacitor?



Capacitor — system of two conductors separated by an insulator (or vacuum).

Symbol representing a capacitor in diagrams: 

"Capacitor has charge Q": the conductor at a higher potential has charge $Q > 0$ and that at a lower potential has charge $-Q$.

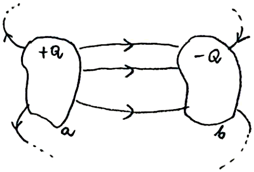
Recall that the surface of a conductor is equipotential, hence we can define the potential difference (voltage) across a capacitor as

$$\boxed{V_{ab} = V_a - V_b.}$$

How to charge a capacitor?

Capacitance

Capacitance. Definition

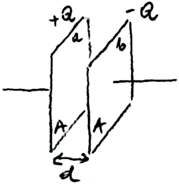


$$\boxed{C \stackrel{\text{def}}{=} \frac{Q}{V_{ab}}}$$

Comments

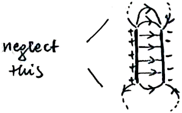
1. SI units: 1 F (farad) $=$ 1 Coulomb/ 1 Volt.
2. 1 F is a huge capacitance; usually μF, nF, pF.
3. In almost all cases, the capacitance depends only on the geometry of the conductors and poperties of the insulator between them.

Example 1. Parallel–Plate Vacuum Capacitor



$$\sigma = \frac{Q}{A} = \text{const}$$

If $d \ll \sqrt{A}$, edge effects can be ignored.



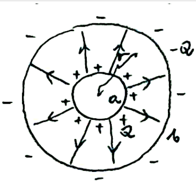
Potential difference

$$V_{ab} = V_a - V_b = \int_{a\to b} \bar{E}\, d\bar{r} \underset{}{\overset{E = \text{const} = \frac{\sigma}{\varepsilon_0}}{=\!=}} \frac{\sigma}{\varepsilon_0} \int_{a\to b} dr = \frac{\sigma}{\varepsilon_0} d = \frac{Q}{\varepsilon_0} \frac{d}{A}$$

Hence, the capacitance

$$\boxed{C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d}}$$

Example 2. Spherical Vacuum Capacitor



(cross · section)

Inner radius R_a, outer radius R_b.

$|\bar{E}|$ between the spheres

$$|\bar{E}| = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

Potential difference

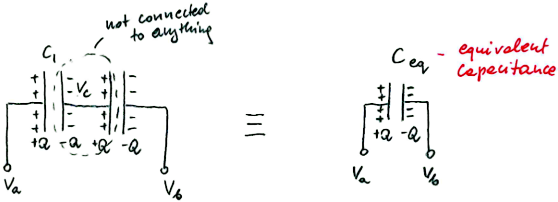
$$V_{ab} = V_a - V_b = \int_{a\to b} \bar{E}\, d\bar{r} = \ldots = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_a} - \frac{1}{R_b}\right) = \frac{Q}{4\pi\varepsilon_0}\,\frac{R_b - R_a}{R_a \cdot R_b}$$

Hence, the capacitance,

$$\boxed{C = \frac{Q}{V_{ab}} = 4\pi\varepsilon_0 \frac{R_a R_b}{R_b - R_a}}$$

Systems of Capacitors and Equivalent Capacitance

Capacitors Connected in Series



Potential difference

$$V_{ab} = V_{ac} + V_{cb}$$

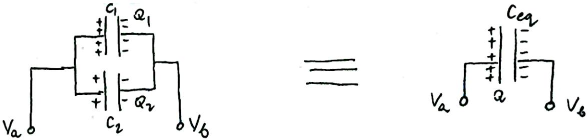
$$V_{ab} = \frac{Q}{C_1} + \frac{Q}{C_2} \quad \equiv \quad V_{ab} = \frac{Q}{C_{eq}}$$

Hence $$\frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{eq}}$$

And the equivalent capacitance for series connection

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \Longrightarrow \quad \boxed{C_{\rm eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

Capacitors Connected in Parallel



Compare the charge

$$Q_1 + Q_2 = Q$$

$$C_1 V_{ab} + C_2 V_{ab} = C_{eq} V_{ab}$$

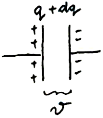
And the equivalent capacitance for parallel connection

$$\boxed{C_{\text{eq}} = C_1 + C_2}$$

Note. Both results can be easily generalized to systems of N capacitors.

Capacitors as Energy Storage Systems

energy stored in a capacitor $\equiv$ work needed to charge it



q - changes during the charging process

(convention: lowercase letters denote time dependent quantities)

Elementary work $\qquad W_{el} = v\, dq = \frac{q}{C}\, dq$

Total work

$$W = \int_{0}^{Q} \frac{q}{C}\, dq = \frac{1}{2}\frac{Q^2}{C}$$

Q → final charge

0 → uncharged

Defining the (potential) energy of an uncharged capacitor to be zero, we have

$$\boxed{U = \frac{Q^2}{2C} \quad \left(\text{or } = \frac{1}{2} C V^2 = \frac{1}{2} Q V\right)}$$

↳ final voltage across the capacitor

(use $C = \frac{Q}{V}$)

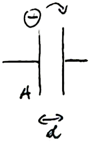
Applications of Capacitors

- Energy storage and release (flashlights, some electric cars).
- Important element in RC, LC, and RLC circuits (will be discussed soon).
- Can act as a "buffer" in circuits, preventing sudden changes of the current.

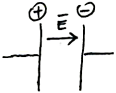
Electric Field Energy

Observation: Energy of a charged capacitor can be regarded as the energy of the electric field in the region between the capacitor's plates.

E.g. parallel-plate capacitor



charging by moving electrons
⇒ work against the electric field between the plates



Energy density (energy per unit volume)

$$u = \frac{\text{energy}}{\text{volume}} = \frac{\frac{1}{2} C V^2}{A \cdot d}$$

But $C = \varepsilon_0 A/d$ and $V = Ed$, so

$$\boxed{u = \frac{1}{2} \varepsilon_0 E^2}$$

Total energy stored

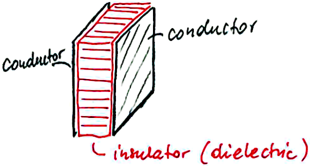
$$\boxed{U = \int_{\substack{\text{space}\\\text{between}\\\text{plates}\\\text{(or whole space)}}} u \, d\tau = \int_{\substack{\text{whole}\\\text{space}}} \frac{1}{2}\varepsilon_0 E^2 \, d\tau}$$

Comments

- Although derived for a specific geometry (parallel-plate capacitor), this result is valid for any electric field configuration in vaccum.
- Exercise (see recitation class): Find U for a spherical capacitor.

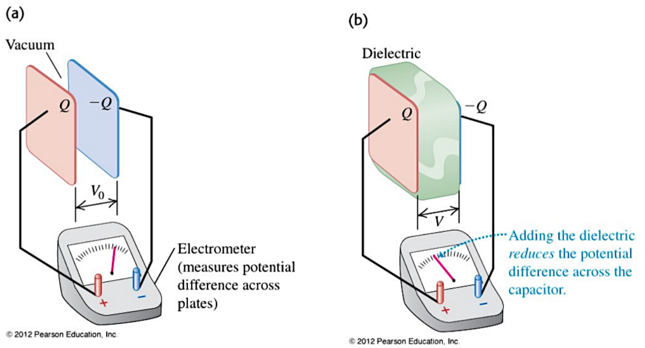
Dielectrics

Dielectrics

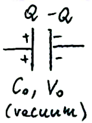


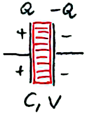
Why to place a dielectric into a capacitor?

- Separates two conductors ($\rightarrow$ miniaturization).
- Increases performance: more energy can be stored.



Dielectric Constant (Relative Permittivity)





$\varepsilon_{\mathrm{r}} = C/C_0$ — dielectric constant (relative permittivity)

If the capacitor is detached from a power source

$$Q = \text{const} = C_0 V_0 = CV,$$

and hence

$$V = \frac{V_0}{\varepsilon_r}.$$

(Reduction by the factor of ε_{r}.)

Parameters of Dielectric Materials

Table 24.1 Values of Dielectric Constant ϵ_r at 20°C

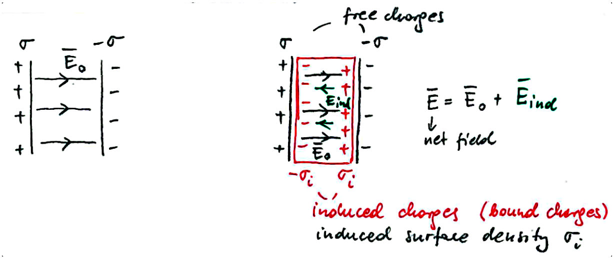
Material	ϵ_r	Material	ϵ_r
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas®	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

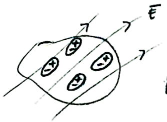
Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Dielectric Constant, ϵ_r	Dielectric Strength, E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

Polarization. Induced vs. Free Charges

What is the microscopic mechanism responsible for the observed effects? **Polarization.**

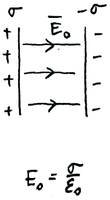


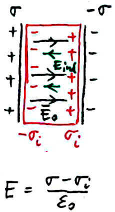


polarization = reorientation due to $\bar{E}$

$$V = \frac{V_0}{\varepsilon_r} \iff E = \frac{E_0}{\varepsilon_r}$$

Note. For weak fields, σ_i is proportional to the magnitude of the electric field inside the material.





But

$$\varepsilon_r = \frac{E_0}{E} = \frac{\sigma}{\sigma - \sigma_i} \qquad \Longrightarrow \qquad \boxed{\sigma_i = \sigma\left(1 - \frac{1}{\varepsilon_r}\right)}$$

Note. If $\varepsilon_r \to \infty$, then $\sigma_i \to \sigma$, that is the induced charge density becomes equal to the free charge density.

Relative vs. Absolute Permittivity

$$E = \frac{E_0}{\varepsilon_r} = \frac{\sigma}{\varepsilon_0 \varepsilon_r} = \frac{\sigma}{\varepsilon}$$

↳ (absolute) permittivity

$$\varepsilon = \varepsilon_r \varepsilon_0$$

↳ relative permittivity (dimensionless)

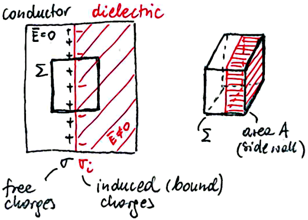
All other formulas for capacitors filled with dielectrics get modified accordingly ($\varepsilon_0 \longleftrightarrow \varepsilon_r \varepsilon_0$). For example,

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$

$$u = \frac{1}{2} \varepsilon_r \varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2$$

Gauss's Law In Dielectrics

Gauss's Law For Dielectrics



Gauss's law $\oint_\Sigma \overline{E} \circ d\overline{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$

$$EA = \frac{\overbrace{(\sigma - \sigma_\text{i})A}^{Q_{\text{encl}}}}{\varepsilon_0}$$

But $\sigma_\text{i} = \sigma \left(1 - \frac{1}{\varepsilon_\text{r}}\right)$, that is $\sigma - \sigma_\text{i} = \frac{\sigma}{\varepsilon_\text{r}}$. Hence, $EA = \sigma A/\varepsilon_\text{r}\varepsilon_0$, or

$$\boxed{\underbrace{\varepsilon_r E A}_{\text{flux of } \varepsilon_r \overline{E} \text{ through } \Sigma} = \frac{\overbrace{\sigma A}^{\text{free charge}}}{\varepsilon_0}}$$

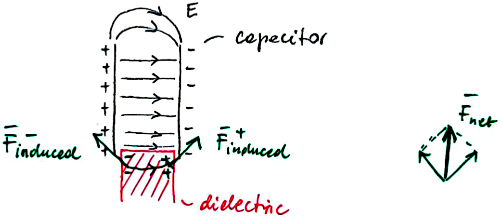
Gauss' law for dielectrics

$$\oint_\Sigma \varepsilon_\text{r} \overline{E} \circ d\overline{A} = \frac{Q_{\text{free-enclosed}}}{\varepsilon_0}$$

Final Remark. The Fringing Effect

The Fringing Effect

What happens if a dielectric is placed close to the plates of a parallel-plate capacitor and the edge effects are not ignored?



There will be a net pull exerted on the dielectric, due to the fringing electric field.

(See also Problem Set 4.)