

## THE SECOND LAW OF THERMODYNAMICS

**20.1. IDENTIFY:** For a heat engine,  $W = |Q_H| - |Q_C|$ .  $e = \frac{W}{Q_H}$ .  $Q_H > 0$ ,  $Q_C < 0$ .

**SET UP:**  $W = 2200 \text{ J}$ .  $|Q_C| = 4300 \text{ J}$ .

**EXECUTE:** (a)  $Q_H = W + |Q_C| = 6500 \text{ J}$ .

(b)  $e = \frac{2200 \text{ J}}{6500 \text{ J}} = 0.34 = 34\%$ .

**EVALUATE:** Since the engine operates on a cycle, the net  $Q$  equal the net  $W$ . But to calculate the efficiency we use the heat energy input,  $Q_H$ .

**20.2. IDENTIFY:** For a heat engine,  $W = |Q_H| - |Q_C|$ .  $e = \frac{W}{Q_H}$ .  $Q_H > 0$ ,  $Q_C < 0$ .

**SET UP:**  $|Q_H| = 9000 \text{ J}$ .  $|Q_C| = 6400 \text{ J}$ .

**EXECUTE:** (a)  $W = 9000 \text{ J} - 6400 \text{ J} = 2600 \text{ J}$ .

(b)  $e = \frac{W}{Q_H} = \frac{2600 \text{ J}}{9000 \text{ J}} = 0.29 = 29\%$ .

**EVALUATE:** Since the engine operates on a cycle, the net  $Q$  equal the net  $W$ . But to calculate the efficiency we use the heat energy input,  $Q_H$ .

**20.3. IDENTIFY and SET UP:** The problem deals with a heat engine.  $W = +3700 \text{ W}$  and  $Q_H = +16,100 \text{ J}$ . Use

$e = \frac{W}{Q_H} = 1 - \left| \frac{Q_C}{Q_H} \right|$  to calculate the efficiency  $e$  and  $W = |Q_H| - |Q_C|$  to calculate  $|Q_C|$ . Power =  $W/t$ .

**EXECUTE:** (a)  $e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_H} = \frac{3700 \text{ J}}{16,100 \text{ J}} = 0.23 = 23\%$ .

(b)  $W = Q = |Q_H| - |Q_C|$

Heat discarded is  $|Q_C| = |Q_H| - W = 16,100 \text{ J} - 3700 \text{ J} = 12,400 \text{ J}$ .

(c)  $Q_H$  is supplied by burning fuel;  $Q_H = mL_c$  where  $L_c$  is the heat of combustion.

$m = \frac{Q_H}{L_c} = \frac{16,100 \text{ J}}{4.60 \times 10^4 \text{ J/g}} = 0.350 \text{ g}$ .

(d)  $W = 3700 \text{ J per cycle}$

In  $t = 1.00 \text{ s}$  the engine goes through 60.0 cycles.

$P = W/t = 60.0(3700 \text{ J})/1.00 \text{ s} = 222 \text{ kW}$

$P = (2.22 \times 10^5 \text{ W})(1 \text{ hp}/746 \text{ W}) = 298 \text{ hp}$

**EVALUATE:**  $Q_C = -12,400 \text{ J}$ . In one cycle  $Q_{\text{tot}} = Q_C + Q_H = 3700 \text{ J}$ . This equals  $W_{\text{tot}}$  for one cycle.

**20.4. IDENTIFY:**  $W = |Q_H| - |Q_C|$ .  $e = \frac{W}{Q_H}$ .  $Q_H > 0$ ,  $Q_C < 0$ .

**SET UP:** For 1.00 s,  $W = 180 \times 10^3$  J.

**EXECUTE:** (a)  $Q_H = \frac{W}{e} = \frac{180 \times 10^3 \text{ J}}{0.280} = 6.43 \times 10^5 \text{ J}$ .

(b)  $|Q_C| = |Q_H| - W = 6.43 \times 10^5 \text{ J} - 1.80 \times 10^5 \text{ J} = 4.63 \times 10^5 \text{ J}$ .

**EVALUATE:** Of the  $6.43 \times 10^5$  J of heat energy supplied to the engine each second,  $1.80 \times 10^5$  J is converted to mechanical work and the remaining  $4.63 \times 10^5$  J is discarded into the low temperature reservoir.

**20.5. IDENTIFY:** This cycle involves adiabatic ( $ab$ ), isobaric ( $bc$ ), and isochoric ( $ca$ ) processes.

**SET UP:**  $ca$  is at constant volume,  $ab$  has  $Q = 0$ , and  $bc$  is at constant pressure. For a constant pressure

process  $W = p\Delta V$  and  $Q = nC_p \Delta T$ .  $pV = nRT$  gives  $n\Delta T = \frac{p\Delta V}{R}$ , so  $Q = \left(\frac{C_p}{R}\right)p\Delta V$ . If  $\gamma = 1.40$  the

gas is diatomic and  $C_p = \frac{7}{2}R$ . For a constant volume process  $W = 0$  and  $Q = nC_V \Delta T$ .  $pV = nRT$  gives

$n\Delta T = \frac{V\Delta p}{R}$ , so  $Q = \left(\frac{C_V}{R}\right)V\Delta p$ . For a diatomic ideal gas  $C_V = \frac{5}{2}R$ .  $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ .

**EXECUTE:** (a)  $V_b = 9.0 \times 10^{-3} \text{ m}^3$ ,  $p_b = 1.5 \text{ atm}$  and  $V_a = 2.0 \times 10^{-3} \text{ m}^3$ . For an adiabatic process

$$p_a V_a^\gamma = p_b V_b^\gamma. \quad p_a = p_b \left(\frac{V_b}{V_a}\right)^\gamma = (1.5 \text{ atm}) \left(\frac{9.0 \times 10^{-3} \text{ m}^3}{2.0 \times 10^{-3} \text{ m}^3}\right)^{1.4} = 12.3 \text{ atm}.$$

(b) Heat enters the gas in process  $ca$ , since  $T$  increases.

$$Q = \left(\frac{C_V}{R}\right)V\Delta p = \left(\frac{5}{2}\right)(2.0 \times 10^{-3} \text{ m}^3)(12.3 \text{ atm} - 1.5 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm}) = 5470 \text{ J}. \quad Q_H = 5470 \text{ J}.$$

(c) Heat leaves the gas in process  $bc$ , since  $T$  decreases.

$$Q = \left(\frac{C_p}{R}\right)p\Delta V = \left(\frac{7}{2}\right)(1.5 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(-7.0 \times 10^{-3} \text{ m}^3) = -3723 \text{ J}. \quad Q_C = -3723 \text{ J}.$$

(d)  $W = Q_H + Q_C = +5470 \text{ J} + (-3723 \text{ J}) = 1747 \text{ J}$ .

(e)  $e = \frac{W}{Q_H} = \frac{1747 \text{ J}}{5470 \text{ J}} = 0.319 = 31.9\%$ .

**EVALUATE:** We did not use the number of moles of the gas.

**20.6. IDENTIFY:** Apply  $e = 1 - \frac{1}{r^{\gamma-1}}$ .  $e = 1 - \frac{|Q_C|}{|Q_H|}$ .

**SET UP:** In part (b),  $Q_H = 10,000 \text{ J}$ . The heat discarded is  $|Q_C|$ .

**EXECUTE:** (a)  $e = 1 - \frac{1}{9.50^{0.40}} = 0.594 = 59.4\%$ .

(b)  $|Q_C| = |Q_H|(1 - e) = (10,000 \text{ J})(1 - 0.594) = 4060 \text{ J}$ .

**EVALUATE:** The work output of the engine is  $W = |Q_H| - |Q_C| = 10,000 \text{ J} - 4060 \text{ J} = 5940 \text{ J}$ .

**20.7. IDENTIFY:** For the Otto-cycle engine,  $e = 1 - r^{1-\gamma}$ .

**SET UP:**  $r$  is the compression ratio.

**EXECUTE:** (a)  $e = 1 - (8.8)^{-0.40} = 0.581$ , which rounds to 58%.

(b)  $e = 1 - (9.6)^{-0.40} = 0.595$  an increase of 1.4%.

**EVALUATE:** An increase in  $r$  gives an increase in  $e$ .

**20.8. IDENTIFY:** Convert coefficient of performance ( $K$ ) to energy efficiency rating (EER).

**SET UP:**  $K = \frac{H_{\text{watts}}}{P_{\text{watts}}}$  and  $\text{EER} = \frac{H_{\text{Btu/h}}}{P_{\text{watts}}}$ .

**EXECUTE:**  $1 \text{ Btu/h} = 0.293 \text{ W}$  so  $H_{\text{watts}} = H_{\text{Btu/h}} (0.293)$ .  $K = 0.293 \frac{H_{\text{Btu/h}}}{P_{\text{watts}}} = (0.293)\text{EER}$  and

$\text{EER} = 3.41K$ . For  $K = 3.0$ ,  $\text{EER} = (3.41)(3.0) = 10.2$ .

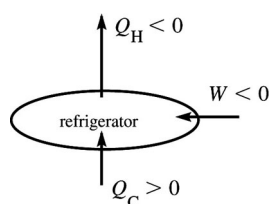
**EVALUATE:** The EER is larger than  $K$ , but this does not mean that the air conditioner is suddenly better at cooling!

**20.9. IDENTIFY and SET UP:** For the refrigerator  $K = 2.10$  and  $Q_C = +3.1 \times 10^4 \text{ J}$ . Use  $K = Q_C/|W|$  to calculate  $|W|$  and then  $W = Q_C + Q_H$  to calculate  $Q_H$ .

**EXECUTE:** (a) Coefficient of performance:  $K = Q_C/|W|$ .

$|W| = Q_C/K = 3.10 \times 10^4 \text{ J} / 2.10 = 1.48 \times 10^4 \text{ J}$ .

(b) The operation of the device is illustrated in Figure 20.9.



$$W = Q_C + Q_H$$

$$Q_H = W - Q_C$$

$$Q_H = -1.48 \times 10^4 \text{ J} - 3.10 \times 10^4 \text{ J} = -4.58 \times 10^4 \text{ J}$$

(negative because heat goes out of the system)

**Figure 20.9**

**EVALUATE:**  $|Q_H| = |W| + |Q_C|$ . The heat  $|Q_H|$  delivered to the high temperature reservoir is greater than the heat taken in from the low temperature reservoir.

**20.10. IDENTIFY:**  $|Q_H| = |Q_C| + |W|$ .  $K = \frac{|Q_C|}{|W|}$ .

**SET UP:** For water,  $c_w = 4190 \text{ J/kg} \cdot \text{K}$  and  $L_f = 3.34 \times 10^5 \text{ J/kg}$ . For ice,  $c_{\text{ice}} = 2100 \text{ J/kg} \cdot \text{K}$ .

**EXECUTE:** (a)  $Q = mc_{\text{ice}}\Delta T_{\text{ice}} - mL_f + mc_w\Delta T_w$ .

$$Q = (1.80 \text{ kg}) \left[ (2100 \text{ J/kg} \cdot \text{K})(-5.0 \text{ C}^\circ) - 3.34 \times 10^5 \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(-25.0 \text{ C}^\circ) \right] = -8.09 \times 10^5 \text{ J}$$

$Q = -8.09 \times 10^5 \text{ J}$ .  $Q$  is negative for the water since heat is removed from it.

(b)  $|Q_C| = 8.09 \times 10^5 \text{ J}$ .  $W = \frac{|Q_C|}{K} = \frac{8.09 \times 10^5 \text{ J}}{2.40} = 3.37 \times 10^5 \text{ J}$ .

(c)  $|Q_H| = 8.09 \times 10^5 \text{ J} + 3.37 \times 10^5 \text{ J} = 1.15 \times 10^6 \text{ J}$ .

**EVALUATE:** For this device,  $Q_C > 0$  and  $Q_H < 0$ . More heat is rejected to the room than is removed from the water.

**20.11. IDENTIFY:** The heat  $Q = mc\Delta T$  that comes out of the water to cool it to  $5.0^\circ\text{C}$  is  $Q_C$  for the refrigerator.

**SET UP:** For water  $1.0 \text{ L}$  has a mass of  $1.0 \text{ kg}$  and  $c = 4.19 \times 10^3 \text{ J/kg} \cdot \text{C}^\circ$ .  $P = \frac{|W|}{t}$ . The coefficient of

performance is  $K = \frac{|Q_C|}{|W|}$ .

**EXECUTE:**  $Q = mc\Delta T = (12.0 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{C}^\circ)(5.0^\circ\text{C} - 31^\circ\text{C}) = -1.31 \times 10^6 \text{ J}$ .  $|Q_C| = 1.31 \times 10^6 \text{ J}$ .

$$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{Pt} \text{ so } t = \frac{|Q_C|}{PK} = \frac{1.31 \times 10^6 \text{ J}}{(135 \text{ W})(2.25)} = 4313 \text{ s} = 71.88 \text{ min} = 1.20 \text{ h}.$$

**EVALUATE:**  $1.2 \text{ h}$  seems like a reasonable time to cool down the dozen bottles.

**20.12. IDENTIFY:**  $|W| = |Q_H| - |Q_C|$ .  $Q_C < 0$ ,  $Q_H > 0$ .  $e = \frac{W}{Q_H}$ . For a Carnot cycle,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ .

**SET UP:**  $T_C = 300 \text{ K}$ ,  $T_H = 520 \text{ K}$ .  $|Q_H| = 6.45 \times 10^3 \text{ J}$ .

**EXECUTE: (a)**  $Q_C = -Q_H \left( \frac{T_C}{T_H} \right) = -(6.45 \times 10^3 \text{ J}) \left( \frac{300 \text{ K}}{520 \text{ K}} \right) = -3.72 \times 10^3 \text{ J}$ .

**(b)**  $|W| = |Q_H| - |Q_C| = 6.45 \times 10^3 \text{ J} - 3.72 \times 10^3 \text{ J} = 2.73 \times 10^3 \text{ J}$ .

**(c)**  $e = \frac{W}{Q_H} = \frac{2.73 \times 10^3 \text{ J}}{6.45 \times 10^3 \text{ J}} = 0.423 = 42.3\%$ .

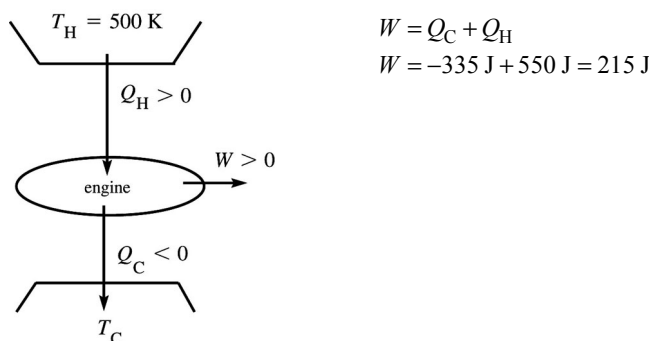
**EVALUATE:** We can verify that  $e = 1 - T_C/T_H$  also gives  $e = 42.3\%$ .

**20.13. IDENTIFY and SET UP:** Use  $W = Q_C + Q_H$  to calculate  $|W|$ . Since it is a Carnot device we can use

$\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}$  to relate the heat flows out of the reservoirs. The reservoir temperatures can be used in

$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$  to calculate  $e$ .

**EXECUTE: (a)** The operation of the device is sketched in Figure 20.13.



**Figure 20.13**

**(b)** For a Carnot cycle,  $\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}$ , which gives

$$T_C = T_H \frac{|Q_C|}{|Q_H|} = 620 \text{ K} \left( \frac{335 \text{ J}}{550 \text{ J}} \right) = 378 \text{ K}.$$

**(c)**  $e_{\text{Carnot}} = 1 - T_C/T_H = 1 - 378 \text{ K}/620 \text{ K} = 0.390 = 39.0\%$ .

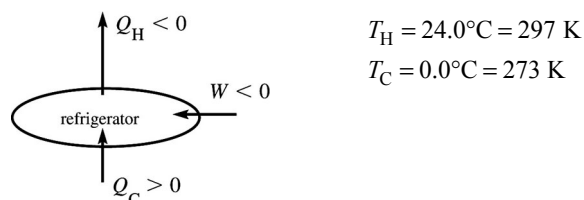
**EVALUATE:** We could use the fundamental definition of  $e$ ,  $e = \frac{W}{Q_H}$ :

$$e = W/Q_H = (215 \text{ J})/(550 \text{ J}) = 39\%, \text{ which checks.}$$

**20.14. IDENTIFY and SET UP:** The device is a Carnot refrigerator.

We can use  $W = Q_C + Q_H$  and  $|Q_C|/|Q_H| = T_C/T_H$ .

**(a)** The operation of the device is sketched in Figure 20.14.



**Figure 20.14**

The amount of heat taken out of the water to make the liquid  $\rightarrow$  solid phase change is

$Q = -mL_f = -(85.0 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -2.84 \times 10^7 \text{ J}$ . This amount of heat must go into the working substance of the refrigerator, so  $Q_C = +2.84 \times 10^7 \text{ J}$ . For Carnot cycle  $|Q_C|/|Q_H| = T_C/T_H$ .

**EXECUTE:**  $|Q_H| = |Q_C|(T_H/T_C) = 2.84 \times 10^7 \text{ J}(297 \text{ K}/273 \text{ K}) = 3.09 \times 10^7 \text{ J}$ .

**(b)**  $W = Q_C + Q_H = +2.84 \times 10^7 \text{ J} - 3.09 \times 10^7 \text{ J} = -2.5 \times 10^6 \text{ J}$

**EVALUATE:**  $W$  is negative because this much energy must be supplied to the refrigerator rather than obtained from it. Note that in  $W = Q_C + Q_H$  we must use Kelvin temperatures.

**20.15. IDENTIFY:**  $e = \frac{W}{Q_H}$  for any engine. For the Carnot cycle,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ .

**SET UP:**  $T_C = 20.0^\circ\text{C} + 273.15 \text{ K} = 293.15 \text{ K}$

**EXECUTE:** **(a)**  $Q_H = \frac{W}{e} = \frac{2.5 \times 10^4 \text{ J}}{0.66} = 3.79 \times 10^4 \text{ J}$ , which rounds to  $3.8 \times 10^4 \text{ J}$ .

**(b)**  $W = Q_H + Q_C$  so  $Q_C = W - Q_H = 2.5 \times 10^4 \text{ J} - 3.79 \times 10^4 \text{ J} = -1.29 \times 10^4 \text{ J}$ .

$T_H = -T_C \frac{Q_H}{Q_C} = -(293.15 \text{ K}) \left( \frac{3.79 \times 10^4 \text{ J}}{-1.29 \times 10^4 \text{ J}} \right) = 861 \text{ K} = 590^\circ\text{C}$ .

**EVALUATE:** For a heat engine,  $W > 0$ ,  $Q_H > 0$  and  $Q_C < 0$ .

**20.16. IDENTIFY:** The theoretical maximum performance coefficient is  $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$ .  $K = \frac{|Q_C|}{|W|}$ .  $|Q_C|$  is the

heat removed from the water to convert it to ice. For the water,  $|Q| = mc_w \Delta T + mL_f$ .

**SET UP:**  $T_C = -5.0^\circ\text{C} = 268 \text{ K}$ .  $T_H = 20.0^\circ\text{C} = 293 \text{ K}$ .  $c_w = 4190 \text{ J/kg} \cdot \text{K}$  and  $L_f = 334 \times 10^3 \text{ J/kg}$ .

**EXECUTE:** **(a)** In one year the freezer operates  $(5 \text{ h/day})(365 \text{ days}) = 1825 \text{ h}$ .

$P = \frac{730 \text{ kWh}}{1825 \text{ h}} = 0.400 \text{ kW} = 400 \text{ W}$ .

**(b)**  $K_{\text{Carnot}} = \frac{268 \text{ K}}{293 \text{ K} - 268 \text{ K}} = 10.7$ .

**(c)**  $|W| = Pt = (400 \text{ W})(3600 \text{ s}) = 1.44 \times 10^6 \text{ J}$ .  $|Q_C| = K|W| = 1.54 \times 10^7 \text{ J}$ .  $|Q| = mc_w \Delta T + mL_f$  gives

$m = \frac{|Q_C|}{c_w \Delta T + L_f} = \frac{1.54 \times 10^7 \text{ J}}{(4190 \text{ J/kg} \cdot \text{K})(20.0 \text{ K}) + 334 \times 10^3 \text{ J/kg}} = 36.9 \text{ kg}$ .

**EVALUATE:** For any actual device,  $K < K_{\text{Carnot}}$ ,  $|Q_C|$  is less than we calculated and the freezer makes less ice in one hour than the mass we calculated in part (c).

**20.17. IDENTIFY:**  $|Q_H| = |W| + |Q_C|$ .  $Q_H < 0$ ,  $Q_C > 0$ .  $K = \frac{|Q_C|}{|W|}$ . For a Carnot cycle,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ .

**SET UP:**  $T_C = 270 \text{ K}$ ,  $T_H = 320 \text{ K}$ .  $|Q_C| = 415 \text{ J}$ .

**EXECUTE:** **(a)**  $Q_H = -\left(\frac{T_H}{T_C}\right)Q_C = -\left(\frac{320 \text{ K}}{270 \text{ K}}\right)(415 \text{ J}) = -492 \text{ J}$ .

**(b)** For one cycle,  $|W| = |Q_H| - |Q_C| = 492 \text{ J} - 415 \text{ J} = 77 \text{ J}$ .  $P = \frac{(165)(77 \text{ J})}{60 \text{ s}} = 212 \text{ W}$ .

**(c)**  $K = \frac{|Q_C|}{|W|} = \frac{415 \text{ J}}{77 \text{ J}} = 5.4$ .

**EVALUATE:** The amount of heat energy  $|Q_H|$  delivered to the high-temperature reservoir is greater than the amount of heat energy  $|Q_C|$  removed from the low-temperature reservoir.

**20.18. IDENTIFY:**  $W = Q_C + Q_H$ . For a Carnot cycle,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ . For the ice to liquid water phase transition,

$$Q = mL_f.$$

**SET UP:** For water,  $L_f = 334 \times 10^3 \text{ J/kg}$ .

**EXECUTE:**  $Q_C = -mL_f = -(0.0400 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -1.336 \times 10^4 \text{ J}$ .  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$  gives

$$Q_H = -(T_H/T_C)Q_C = -(-1.336 \times 10^4 \text{ J})[(373.15 \text{ K})/(273.15 \text{ K})] = +1.825 \times 10^4 \text{ J}.$$

$$W = Q_C + Q_H = 4.89 \times 10^3 \text{ J}.$$

**EVALUATE:** For a heat engine,  $Q_C$  is negative and  $Q_H$  is positive. The heat that comes out of the engine ( $Q < 0$ ) goes into the ice ( $Q > 0$ ).

**20.19. IDENTIFY:** The power output is  $P = \frac{W}{t}$ . The theoretical maximum efficiency is  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ . In

$$\text{general, } e = \frac{W}{Q_H}.$$

**SET UP:**  $Q_H = 1.50 \times 10^4 \text{ J}$ .  $T_C = 290 \text{ K}$ .  $T_H = 650 \text{ K}$ .  $1 \text{ hp} = 746 \text{ W}$ .

**EXECUTE:**  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{290 \text{ K}}{650 \text{ K}} = 0.5538$ .  $W = eQ_H = (0.5538)(1.50 \times 10^4 \text{ J}) = 8.307 \times 10^3 \text{ J}$ ;

this is the work output in one cycle.  $P = \frac{W}{t} = \frac{(240)(8.307 \times 10^3 \text{ J})}{60.0 \text{ s}} = 3.323 \times 10^4 \text{ W} = 44.5 \text{ hp}$ .

**EVALUATE:** We could also use  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$  to calculate

$Q_C = -\left(\frac{T_C}{T_H}\right)Q_H = -\left(\frac{290 \text{ K}}{650 \text{ K}}\right)(1.50 \times 10^4 \text{ J}) = -6.69 \times 10^3 \text{ J}$ . Then  $W = Q_C + Q_H = 8.31 \times 10^3 \text{ J}$ , the same as previously calculated.

**20.20. IDENTIFY:** The immense ocean does not change temperature, but it does lose some entropy because it gives up heat to melt the ice and warm it to  $3.50^\circ\text{C}$ . The ice does not change temperature as it melts, but it gains entropy by absorbing heat from the ocean. It also gains entropy as it warms from  $0.00^\circ\text{C}$  to  $3.50^\circ\text{C}$ .

**SET UP:** For a reversible isothermal process  $\Delta S = \frac{Q}{T}$ , where  $T$  is the Kelvin temperature at which the heat flow occurs. The heat flows in this problem are irreversible, but since  $\Delta S$  is path-independent, the entropy change is the same as for a reversible heat flow. The heat flow when the ice melts is  $Q = mL_f$ , with  $L_f = 334 \times 10^3 \text{ J/kg}$ . The entropy change of the melted ice as it warms from  $0.00^\circ\text{C}$  to  $3.50^\circ\text{C}$  is given by  $\Delta S = mc \ln(T_2/T_1)$ . Heat flows out of the ocean ( $Q < 0$ ) and into the ice ( $Q > 0$ ). The heat flow for the melting ice occurs at  $T = 0^\circ\text{C} = 273.15 \text{ K}$ . The heat flow for the ocean occurs at  $T = 3.50^\circ\text{C} = 276.65 \text{ K}$ .

**EXECUTE:** *Melt the ice:*  $Q = mL_f = (4.50 \text{ kg})(334 \times 10^3 \text{ J/kg}) = 1.503 \times 10^6 \text{ J}$ . The entropy change for the melting ice is  $\Delta S = \frac{Q}{T} = \frac{+1.503 \times 10^6 \text{ J}}{273.15 \text{ K}} = 5.502 \times 10^3 \text{ J/K}$ .

*Warm the melted ice to  $3.50^\circ\text{C}$ :*  $Q = mc \Delta T = (4.50 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(3.50 \text{ K}) = 6.599 \times 10^4 \text{ J}$ .

$$\Delta S = mc \ln(T_2/T_1) = (4.50 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{276.65 \text{ K}}{273.15 \text{ K}}\right) = 240.1 \text{ J/K}.$$

For the ocean,  $\Delta S = \frac{Q}{T} = \frac{-6.599 \times 10^4 \text{ J} - 1.50 \times 10^6 \text{ J}}{276.65 \text{ K}} = -5.671 \times 10^3 \text{ J/K}$ .

The net entropy change is

$\Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{ocean}} = 5.502 \times 10^3 \text{ J/K} + 240.1 \text{ J/K} + (-5.671 \times 10^3 \text{ J/K}) = +71 \text{ J/K}$ , which rounds to +70 J/K. The entropy of the world increases by 70 J/K.

**EVALUATE:** Since this process is irreversible, we expect the entropy of the world to increase, as we have found.

- 20.21. IDENTIFY:**  $\Delta S = \frac{Q}{T}$  for each object, where  $T$  must be in kelvins. The temperature of each object remains constant.

**SET UP:** For water,  $L_f = 3.34 \times 10^5 \text{ J/kg}$ .

**EXECUTE: (a)** The heat flow into the ice is  $Q = mL_f = (0.350 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) = 1.17 \times 10^5 \text{ J}$ . The heat flow occurs at  $T = 273 \text{ K}$ , so  $\Delta S = \frac{Q}{T} = \frac{1.17 \times 10^5 \text{ J}}{273 \text{ K}} = 429 \text{ J/K}$ .  $Q$  is positive and  $\Delta S$  is positive.

**(b)**  $Q = -1.17 \times 10^5 \text{ J}$  flows out of the heat source, at  $T = 298 \text{ K}$ .  $\Delta S = \frac{Q}{T} = \frac{-1.17 \times 10^5 \text{ J}}{298 \text{ K}} = -393 \text{ J/K}$ .

$Q$  is negative and  $\Delta S$  is negative.

**(c)**  $\Delta S_{\text{tot}} = 429 \text{ J/K} + (-393 \text{ J/K}) = +36 \text{ J/K}$ .

**EVALUATE:** For the total isolated system,  $\Delta S > 0$  and the process is irreversible.

- 20.22. IDENTIFY:** Apply  $Q_{\text{system}} = 0$  to calculate the final temperature.  $Q = mc\Delta T$ .  $\Delta S = mc \ln(T_2/T_1)$  when an object undergoes a temperature change.

**SET UP:** For water  $c = 4190 \text{ J/kg} \cdot \text{K}$ . Boiling water has  $T = 100.0^\circ\text{C} = 373 \text{ K}$ .

**EXECUTE: (a)** The heat transfer between  $100^\circ\text{C}$  water and  $30^\circ\text{C}$  water occurs over a finite temperature difference and the process is irreversible.

**(b)**  $(195 \text{ kg})c(T_2 - 30.0^\circ\text{C}) + (5.00 \text{ kg})c(T_2 - 100^\circ\text{C}) = 0$ .  $T_2 = 31.75^\circ\text{C} = 304.90 \text{ K}$ .

**(c)**  $\Delta S = (195 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{304.90 \text{ K}}{303.15 \text{ K}}\right) + (5.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{304.90 \text{ K}}{373.15 \text{ K}}\right) = 471 \text{ J/K}$ .

**EVALUATE:**  $\Delta S_{\text{system}} > 0$ , as it should for an irreversible process.

- 20.23. IDENTIFY:** Both the ice and the room are at a constant temperature, so  $\Delta S = \frac{Q}{T}$ . For the melting phase

transition,  $Q = mL_f$ . Conservation of energy requires that the quantity of heat that goes into the ice is the amount of heat that comes out of the room.

**SET UP:** For ice,  $L_f = 334 \times 10^3 \text{ J/kg}$ . When heat flows into an object,  $Q > 0$ , and when heat flows out of an object,  $Q < 0$ .

**EXECUTE: (a)** Irreversible because heat will not spontaneously flow out of  $15 \text{ kg}$  of water into a warm room to freeze the water.

**(b)**  $\Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{room}} = \frac{mL_f}{T_{\text{ice}}} + \frac{-mL_f}{T_{\text{room}}} = \frac{(15.0 \text{ kg})(334 \times 10^3 \text{ J/kg})}{273 \text{ K}} + \frac{-(15.0 \text{ kg})(334 \times 10^3 \text{ J/kg})}{293 \text{ K}}$ .

$\Delta S = +1250 \text{ J/K}$ .

**EVALUATE:** This result is consistent with the answer in (a) because  $\Delta S > 0$  for irreversible processes.

- 20.24. IDENTIFY:**  $Q = mc\Delta T$  for the water.  $\Delta S = mc \ln(T_2/T_1)$  when an object undergoes a temperature change.  $\Delta S = Q/T$  for an isothermal process.

**SET UP:** For water,  $c = 4190 \text{ J/kg} \cdot \text{K}$ .  $85.0^\circ\text{C} = 358.2 \text{ K}$ .  $20.0^\circ\text{C} = 293.2 \text{ K}$ .

**EXECUTE: (a)**  $\Delta S = mc \ln\left(\frac{T_2}{T_1}\right) = (0.250 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{293.2 \text{ K}}{358.2 \text{ K}}\right) = -210 \text{ J/K}$ . Heat comes out of the water and its entropy decreases.

(b)  $Q = mc\Delta T = (0.250)(4190 \text{ J/kg} \cdot \text{K})(-65.0 \text{ K}) = -6.81 \times 10^4 \text{ J}$ . The amount of heat that goes into the air

is  $+6.81 \times 10^4 \text{ J}$ . For the air,  $\Delta S = \frac{Q}{T} = \frac{+6.81 \times 10^4 \text{ J}}{293.1 \text{ K}} = +232 \text{ J/K}$ .

$$\Delta S_{\text{system}} = -210 \text{ J/K} + 232 \text{ J/K} = +22 \text{ J/K}.$$

**EVALUATE:**  $\Delta S_{\text{system}} > 0$  and the process is irreversible.

**20.25. IDENTIFY:** The process is at constant temperature, so  $\Delta S = \frac{Q}{T}$ .  $\Delta U = Q - W$ .

**SET UP:** For an isothermal process of an ideal gas,  $\Delta U = 0$  and  $Q = W$ . For a compression,  $\Delta V < 0$  and  $W < 0$ .

**EXECUTE:**  $Q = W = -1850 \text{ J}$ .  $\Delta S = \frac{-1850 \text{ J}}{293 \text{ K}} = -6.31 \text{ J/K}$ .

**EVALUATE:** The entropy change of the gas is negative. Heat must be removed from the gas during the compression to keep its temperature constant and therefore the gas is not an isolated system.

**20.26. IDENTIFY and SET UP:** The initial and final states are at the same temperature, at the normal boiling point of 4.216 K. Calculate the entropy change for the irreversible process by considering a reversible isothermal process that connects the same two states, since  $\Delta S$  is path independent and depends only on the initial and final states. For the reversible isothermal process we can use  $\Delta S = Q/T$ .

The heat flow for the helium is  $Q = -mL_v$ , negative since in condensation heat flows out of the helium.

The heat of vaporization  $L_v$  is given in Table 17.4 and is  $L_v = 20.9 \times 10^3 \text{ J/kg}$ .

**EXECUTE:**  $Q = -mL_v = -(0.130 \text{ kg})(20.9 \times 10^3 \text{ J/kg}) = -2717 \text{ J}$

$$\Delta S = Q/T = -2717 \text{ J}/4.216 \text{ K} = -644 \text{ J/K}.$$

**EVALUATE:** The system we considered is the 0.130 kg of helium;  $\Delta S$  is the entropy change of the helium. This is not an isolated system since heat must flow out of it into some other material. Our result that  $\Delta S < 0$  doesn't violate the second law since it is not an isolated system. The material that receives the heat that flows out of the helium would have a positive entropy change and the total entropy change would be positive.

**20.27. IDENTIFY:** Each phase transition occurs at constant temperature and  $\Delta S = \frac{Q}{T}$ .  $Q = mL_v$ .

**SET UP:** For vaporization of water,  $L_v = 2256 \times 10^3 \text{ J/kg}$ .

**EXECUTE: (a)**  $\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(1.00 \text{ kg})(2256 \times 10^3 \text{ J/kg})}{(373.15 \text{ K})} = 6.05 \times 10^3 \text{ J/K}$ . Note that this is the change

of entropy of the water as it changes to steam.

**(b)** The magnitude of the entropy change is roughly five times the value found in Example 20.5.

**EVALUATE:** Water is less ordered (more random) than ice, but water is far less random than steam; a consideration of the density changes indicates why this should be so.

**20.28. IDENTIFY and SET UP:** An amount of heat  $|Q_H|$  leaves the environment at  $T_H = 400^\circ\text{C} = 673 \text{ K}$ . An

amount  $|Q_C|$  enters the environment at  $T_C = 20^\circ\text{C} = 293 \text{ K}$ .  $\Delta S = \frac{Q}{T}$  at constant temperature.

**EXECUTE:** For a distance of 1.0 mi the car uses  $(1.0 \text{ mi}/35 \text{ mi/gal}) = 0.0286 \text{ gal}$ . For 0.0286 gal,

$$Q_H = (0.0286 \text{ gal}) \times (1.23 \times 10^8 \text{ J/gal}) = 3.52 \times 10^6 \text{ J}. \quad Q_C = \frac{3}{4}(Q_H) = 2.64 \times 10^6 \text{ J}.$$

$$\Delta S = \frac{-3.52 \times 10^6 \text{ J}}{673 \text{ K}} + \frac{+2.64 \times 10^6 \text{ J}}{293 \text{ K}} = 3780 \text{ J/K}.$$

**EVALUATE:** The net entropy change of the environment is positive. This is reasonable since the process is irreversible. Heat will not spontaneously flow from the cool air at  $20^\circ\text{C}$  into the hot engine at  $400^\circ\text{C}$ .

**20.29. IDENTIFY and SET UP: (a)** The velocity distribution of the Maxwell-Boltzmann distribution,

$$f(v) = 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}, \text{ depends only on } T, \text{ so in an isothermal process it does not change.}$$



**(b) EXECUTE:** Calculate the change in the number of available microscopic states and apply Eq. (20.23). Following the reasoning of Example 20.11, the number of possible positions available to each molecule is altered by a factor of 3 (becomes larger). Hence the number of microscopic states the gas occupies at volume  $3V$  is  $w_2 = (3)^N w_1$ , where  $N$  is the number of molecules and  $w_1$  is the number of possible microscopic states at the start of the process, where the volume is  $V$ . Then, using  $\Delta S = k \ln(w_2/w_1)$ , we have  $\Delta S = k \ln(w_2/w_1) = k \ln(3)^N = Nk \ln(3) = nN_A k \ln(3) = nR \ln(3)$ .  
 $\Delta S = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K}) \ln(3) = +18.3 \text{ J/K}$ .

**(c) IDENTIFY and SET UP:** For an isothermal reversible process  $\Delta S = Q/T$ .

**EXECUTE:** Calculate  $W$  and then use the first law to calculate  $Q$ .

$\Delta T = 0$  implies that  $\Delta U = 0$ , since system is an ideal gas.

Then by  $\Delta U = Q - W$ ,  $Q = W$ .

For an isothermal process,  $W = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} (nRT/V) \, dV = nRT \ln(V_2/V_1)$ .

Thus  $Q = nRT \ln(V_2/V_1)$  and  $\Delta S = Q/T = nR \ln(V_2/V_1)$ .

$\Delta S = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K}) \ln(3V_1/V_1) = +18.3 \text{ J/K}$ .

**EVALUATE:** This is the same result as obtained in part (b).

**20.30. IDENTIFY:** Apply  $\Delta S = k \ln(w_2/w_1)$  and follow the procedure used in Example 20.11.

**SET UP:** After the partition is punctured each molecule has equal probability of being on each side of the box. The probability of two independent events occurring simultaneously is the product of the probabilities of each separate event.

**EXECUTE: (a)** On the average, each half of the box will contain half of each type of molecule, 250 of nitrogen and 50 of oxygen.

**(b)** See Example 20.11. The total change in entropy is

$\Delta S = kN_1 \ln(2) + kN_2 \ln(2) = (N_1 + N_2)k \ln(2) = (600)(1.381 \times 10^{-23} \text{ J/K}) \ln(2) = 5.74 \times 10^{-21} \text{ J/K}$ .

**(c)** The probability is  $(1/2)^{500} \times (1/2)^{100} = (1/2)^{600}$ . To express this answer in scientific notation, most standard calculators cannot handle the calculation. However we can use the following method. We want to express  $(1/2)^{600}$  as a power of 10, which we can write as  $10^x$ . Therefore  $(1/2)^{600} = 10^x$ . Taking logarithms of both sides of this equation gives

$$\log[(1/2)^{600}] = \log(10^x)$$

$$-600 \log 2 = x \log 10 = x$$

Numerically we have

$$x = -600 \log 2 = -180.6179 = -180 - 0.6179$$

$$(1/2)^{600} = 10^{-180-0.6179} = 10^{-180} 10^{-0.6179} = 10^{-180} (0.24) = 2.4 \times 10^{-181}. \text{ Therefore this is not likely to happen.}$$

**EVALUATE:** The contents of the box constitutes an isolated system.  $\Delta S > 0$  and the process is irreversible.

**20.31. IDENTIFY:** For a free expansion,  $\Delta S = nR \ln(V_2/V_1)$ .

**SET UP:**  $V_1 = 2.40 \text{ L} = 2.40 \times 10^{-3} \text{ m}^3$ .

**EXECUTE:**  $\Delta S = (0.100 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K}) \ln\left(\frac{425 \text{ m}^3}{2.40 \times 10^{-3} \text{ m}^3}\right) = 10.0 \text{ J/K}$ .

**EVALUATE:**  $\Delta S_{\text{system}} > 0$  and the free expansion is irreversible.

**20.32. IDENTIFY:**  $W = Q_C + Q_H$ . Since it is a Carnot cycle,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ . The heat required to melt the ice is

$$Q = mL_f.$$

**SET UP:** For water,  $L_f = 334 \times 10^3 \text{ J/kg}$ .  $Q_H > 0$ ,  $Q_C < 0$ .  $Q_C = -mL_f$ .  $T_H = 527^\circ\text{C} = 800.15 \text{ K}$ .

**EXECUTE: (a)**  $Q_H = +400 \text{ J}$ ,  $W = +300 \text{ J}$ .  $Q_C = W - Q_H = -100 \text{ J}$ .

$$T_C = -T_H(Q_C/Q_H) = -(800.15 \text{ K})[(-100 \text{ J})/(400 \text{ J})] = +200 \text{ K} = -73^\circ\text{C}.$$

(b) The total  $Q_C$  required is  $-mL_f = -(10.0 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -3.34 \times 10^6 \text{ J}$ .  $Q_C$  for one cycle is  $-100 \text{ J}$ , so the number of cycles required is  $\frac{-3.34 \times 10^6 \text{ J}}{-100 \text{ J/cycle}} = 3.34 \times 10^4$  cycles.

**EVALUATE:** The results depend only on the maximum temperature of the gas, not on the number of moles or the maximum pressure.

**20.33. IDENTIFY:** The total work that must be done is  $W_{\text{tot}} = mg\Delta y$ .  $|W| = |Q_H| - |Q_C|$ .  $Q_H > 0$ ,  $W > 0$  and

$Q_C < 0$ . For a Carnot cycle,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ .

**SET UP:**  $T_C = 373 \text{ K}$ ,  $T_H = 773 \text{ K}$ .  $|Q_H| = 250 \text{ J}$ .

**EXECUTE:** (a)  $Q_C = -Q_H \left( \frac{T_C}{T_H} \right) = -(250 \text{ J}) \left( \frac{373 \text{ K}}{773 \text{ K}} \right) = -121 \text{ J}$ .

(b)  $|W| = 250 \text{ J} - 121 \text{ J} = 129 \text{ J}$ . This is the work done in one cycle.

$W_{\text{tot}} = (500 \text{ kg})(9.80 \text{ m/s}^2)(100 \text{ m}) = 4.90 \times 10^5 \text{ J}$ . The number of cycles required is

$\frac{W_{\text{tot}}}{|W|} = \frac{4.90 \times 10^5 \text{ J}}{129 \text{ J/cycle}} = 3.80 \times 10^3$  cycles.

**EVALUATE:** In  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ , the temperatures must be in kelvins.

**20.34. IDENTIFY:** The same amount of heat that enters the person's body also leaves the body, but these transfers of heat occur at different temperatures, so the person's entropy changes.

**SET UP:** We are asked to find the entropy change of the person. The person is not an isolated system.

In  $1.0 \text{ s}$ ,  $0.80(80 \text{ J}) = 64 \text{ J}$  of heat enters the person's body at  $37^\circ\text{C} = 310 \text{ K}$ . This amount of heat leaves

the person at a temperature of  $30^\circ\text{C} = 303 \text{ K}$ .  $\Delta S = \frac{Q}{T}$ .

**EXECUTE:** For the person,  $\Delta S = \frac{+64 \text{ J}}{310 \text{ K}} + \frac{-64 \text{ J}}{303 \text{ K}} = -4.8 \times 10^{-3} \text{ J/K}$ .

**EVALUATE:** The entropy of the person can decrease without violating the second law of thermodynamics because the person isn't an isolated system.

**20.35. IDENTIFY:** We know the efficiency of this Carnot engine, the heat it absorbs at the hot reservoir and the temperature of the hot reservoir.

**SET UP:** For a heat engine  $e = \frac{W}{|Q_H|}$  and  $Q_H + Q_C = W$ . For a Carnot cycle,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ .  $Q_C < 0$ ,  $W > 0$ ,

and  $Q_H > 0$ .  $T_H = 135^\circ\text{C} = 408 \text{ K}$ . In each cycle,  $|Q_H|$  leaves the hot reservoir and  $|Q_C|$  enters the cold reservoir. The work done on the water equals its increase in gravitational potential energy,  $mgh$ .

**EXECUTE:** (a)  $e = \frac{W}{Q_H}$  so  $W = eQ_H = (0.220)(410 \text{ J}) = 90.2 \text{ J}$ .

(b)  $Q_C = W - Q_H = 90.2 \text{ J} - 410 \text{ J} = -319.8 \text{ J}$ , which rounds to  $-320 \text{ J}$ .

(c)  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$  so  $T_C = -T_H \left( \frac{Q_C}{Q_H} \right) = -(408 \text{ K}) \left( \frac{-319.8 \text{ J}}{410 \text{ J}} \right) = 318 \text{ K} = 45^\circ\text{C}$ .

(d)  $\Delta S = \frac{-|Q_H|}{T_H} + \frac{|Q_C|}{T_C} = \frac{-410 \text{ J}}{408 \text{ K}} + \frac{319.8 \text{ J}}{318 \text{ K}} = 0$ . The Carnot cycle is reversible and  $\Delta S = 0$ .

(e)  $W = mgh$  so  $m = \frac{W}{gh} = \frac{90.2 \text{ J}}{(9.80 \text{ m/s}^2)(35.0 \text{ m})} = 0.263 \text{ kg} = 263 \text{ g}$ .

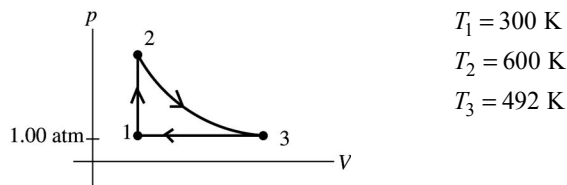
**EVALUATE:** The Carnot cycle is reversible so  $\Delta S = 0$  for the world. However some parts of the world gain entropy while other parts lose it, making the sum equal to zero.

**20.36. IDENTIFY:** Use the ideal gas law to calculate  $p$  and  $V$  for each state. Use the first law and specific expressions

for  $Q$ ,  $W$ , and  $\Delta U$  for each process. Use  $e = \frac{W}{Q_H} = 1 - \left| \frac{Q_C}{Q_H} \right|$  to calculate  $e$ .  $Q_H$  is the net heat flow into the gas.

**SET UP:**  $\gamma = 1.40$

$C_V = R/(\gamma - 1) = 20.79 \text{ J/mol} \cdot \text{K}$ ;  $C_p = C_V + R = 29.10 \text{ J/mol} \cdot \text{K}$ . The cycle is sketched in Figure 20.36.



**Figure 20.36**

**EXECUTE: (a) point 1:**

$p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$  (given);  $pV = nRT$ ;

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.350 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 8.62 \times 10^{-3} \text{ m}^3$$

point 2:

process  $1 \rightarrow 2$  at constant volume so  $V_2 = V_1 = 8.62 \times 10^{-3} \text{ m}^3$

$pV = nRT$  and  $n, R, V$  constant implies  $p_1/T_1 = p_2/T_2$

$$p_2 = p_1(T_2/T_1) = (1.00 \text{ atm})(600 \text{ K}/300 \text{ K}) = 2.00 \text{ atm} = 2.03 \times 10^5 \text{ Pa}$$

point 3:

Consider the process  $3 \rightarrow 1$ , since it is simpler than  $2 \rightarrow 3$ .

Process  $3 \rightarrow 1$  is at constant pressure so  $p_3 = p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

$pV = nRT$  and  $n, R, p$  constant implies  $V_1/T_1 = V_3/T_3$

$$V_3 = V_1(T_3/T_1) = (8.62 \times 10^{-3} \text{ m}^3)(492 \text{ K}/300 \text{ K}) = 14.1 \times 10^{-3} \text{ m}^3$$

**(b) process  $1 \rightarrow 2$ :**

constant volume ( $\Delta V = 0$ )

$$Q = nC_V\Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 300 \text{ K}) = 2180 \text{ J}$$

$\Delta V = 0$  and  $W = 0$ . Then  $\Delta U = Q - W = 2180 \text{ J}$

process  $2 \rightarrow 3$ :

Adiabatic means  $Q = 0$ .

$\Delta U = nC_V\Delta T$  (any process), so

$$\Delta U = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(492 \text{ K} - 600 \text{ K}) = -780 \text{ J}$$

Then  $\Delta U = Q - W$  gives  $W = Q - \Delta U = +780 \text{ J}$ . (It is correct for  $W$  to be positive since  $\Delta V$  is positive.)

process  $3 \rightarrow 1$ :

For constant pressure

$$W = p\Delta V = (1.013 \times 10^5 \text{ Pa})(8.62 \times 10^{-3} \text{ m}^3 - 14.1 \times 10^{-3} \text{ m}^3) = -560 \text{ J}$$

or  $W = nR\Delta T = (0.350 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -560 \text{ J}$ , which checks. (It is correct for  $W$  to be negative, since  $\Delta V$  is negative for this process.)

$$Q = nC_p\Delta T = (0.350 \text{ mol})(29.10 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1960 \text{ J}$$

$$\Delta U = Q - W = -1960 \text{ J} - (-560 \text{ J}) = -1400 \text{ J}$$

or  $\Delta U = nC_V\Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1400 \text{ J}$ , which checks.

(c)  $W_{\text{net}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 1} = 0 + 780 \text{ J} - 560 \text{ J} = +220 \text{ J}$

(d)  $Q_{\text{net}} = Q_{1 \rightarrow 2} + Q_{2 \rightarrow 3} + Q_{3 \rightarrow 1} = 2180 \text{ J} + 0 - 1960 \text{ J} = +220 \text{ J}$

(e)  $e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_H} = \frac{220 \text{ J}}{2180 \text{ J}} = 0.101 = 10.1\%$ .

$e(\text{Carnot}) = 1 - T_C/T_H = 1 - 300 \text{ K}/600 \text{ K} = 0.500$ .

**EVALUATE:** For a cycle  $\Delta U = 0$ , so by  $\Delta U = Q - W$  it must be that  $Q_{\text{net}} = W_{\text{net}}$  for a cycle. We can also check that  $\Delta U_{\text{net}} = 0$ :  $\Delta U_{\text{net}} = \Delta U_{1 \rightarrow 2} + \Delta U_{2 \rightarrow 3} + \Delta U_{3 \rightarrow 1} = 2180 \text{ J} - 1050 \text{ J} - 1130 \text{ J} = 0$

$e < e(\text{Carnot})$ , as it must.

**20.37. IDENTIFY:** The same amount of heat that enters the person's body also leaves the body, but these transfers of heat occur at different temperatures, so the person's entropy changes.

**SET UP:** 1 food calorie = 1000 cal = 4186 J. The heat enters the person's body at  $37^\circ\text{C} = 310 \text{ K}$  and leaves at a temperature of  $30^\circ\text{C} = 303 \text{ K}$ .  $\Delta S = \frac{Q}{T}$ .

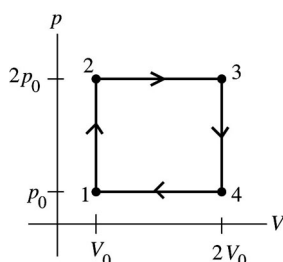
**EXECUTE:**  $|Q| = (0.80)(2.50 \text{ g})(9.3 \text{ food calorie/g})\left(\frac{4186 \text{ J}}{1 \text{ food calorie}}\right) = 7.79 \times 10^4 \text{ J}$ .

$\Delta S = \frac{+7.79 \times 10^4 \text{ J}}{310 \text{ K}} + \frac{-7.79 \times 10^4 \text{ J}}{303 \text{ K}} = -5.8 \text{ J/K}$ . Your body's entropy decreases.

**EVALUATE:** The entropy of your body can decrease without violating the second law of thermodynamics because you are not an isolated system.

**20.38. IDENTIFY:** Use  $e = \frac{W}{Q_H}$  to calculate  $e$ .

**SET UP:** The cycle is sketched in Figure 20.38.



$C_V = 5R/2$

for an ideal gas  $C_p = C_V + R = 7R/2$

**Figure 20.38**

**SET UP:** Calculate  $Q$  and  $W$  for each process.

process 1  $\rightarrow$  2:

$\Delta V = 0$  implies  $W = 0$

$\Delta V = 0$  implies  $Q = nC_V\Delta T = nC_V(T_2 - T_1)$

But  $pV = nRT$  and  $V$  constant says  $p_1V = nRT_1$  and  $p_2V = nRT_2$ .

Thus  $(p_2 - p_1)V = nR(T_2 - T_1)$ ;  $V\Delta p = nR\Delta T$  (true when  $V$  is constant).

Then  $Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)V\Delta p = (C_V/R)V_0(2p_0 - p_0) = (C_V/R)p_0V_0$ . ( $Q > 0$ ; heat is absorbed by the gas.)

process 2  $\rightarrow$  3:

$\Delta p = 0$  so  $W = p\Delta V = p(V_3 - V_2) = 2p_0(2V_0 - V_0) = 2p_0V_0$  ( $W$  is positive since  $V$  increases.)

$\Delta p = 0$  implies  $Q = nC_p\Delta T = nC_p(T_2 - T_1)$ .

But  $pV = nRT$  and  $p$  constant says  $pV_1 = nRT_1$  and  $pV_2 = nRT_2$ .

Thus  $p(V_2 - V_1) = nR(T_2 - T_1)$ ;  $p\Delta V = nR\Delta T$  (true when  $p$  is constant).

Then  $Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)2p_0(2V_0 - V_0) = (C_p/R)2p_0V_0$ . ( $Q > 0$ ; heat is absorbed by the gas.)

process 3  $\rightarrow$  4:

$\Delta V = 0$  implies  $W = 0$

$\Delta V = 0$  so

$Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)(2V_0)(p_0 - 2p_0) = -2(C_V/R)p_0V_0$

( $Q < 0$  so heat is rejected by the gas.)

process 4  $\rightarrow$  1:

$\Delta p = 0$  so  $W = p\Delta V = p(V_1 - V_4) = p_0(V_0 - 2V_0) = -p_0V_0$  ( $W$  is negative since  $V$  decreases)

$\Delta p = 0$  so  $Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)p_0(V_0 - 2V_0) = -(C_p/R)p_0V_0$  ( $Q < 0$  so heat is rejected by the gas.)

total work performed by the gas during the cycle:

$W_{\text{tot}} = W_{1\rightarrow 2} + W_{2\rightarrow 3} + W_{3\rightarrow 4} + W_{4\rightarrow 1} = 0 + 2p_0V_0 + 0 - p_0V_0 = p_0V_0$

(Note that  $W_{\text{tot}}$  equals the area enclosed by the cycle in the  $pV$ -diagram.)

total heat absorbed by the gas during the cycle ( $Q_H$ ):

Heat is absorbed in processes 1  $\rightarrow$  2 and 2  $\rightarrow$  3.

$Q_H = Q_{1\rightarrow 2} + Q_{2\rightarrow 3} = \frac{C_V}{R}p_0V_0 + 2\frac{C_p}{R}p_0V_0 = \left(\frac{C_V + 2C_p}{R}\right)p_0V_0$

But  $C_p = C_V + R$  so  $Q_H = \frac{C_V + 2(C_V + R)}{R}p_0V_0 = \left(\frac{3C_V + 2R}{R}\right)p_0V_0$ .

total heat rejected by the gas during the cycle ( $Q_C$ ):

Heat is rejected in processes 3  $\rightarrow$  4 and 4  $\rightarrow$  1.

$Q_C = Q_{3\rightarrow 4} + Q_{4\rightarrow 1} = -2\frac{C_V}{R}p_0V_0 - \frac{C_p}{R}p_0V_0 = -\left(\frac{2C_V + C_p}{R}\right)p_0V_0$

But  $C_p = C_V + R$  so  $Q_C = -\frac{2C_V + (C_V + R)}{R}p_0V_0 = -\left(\frac{3C_V + R}{R}\right)p_0V_0$ .

efficiency:

$e = \frac{W}{Q_H} = \frac{p_0V_0}{\left[\frac{(3C_V + 2R)}{R}\right](p_0V_0)} = \frac{R}{3C_V + 2R} = \frac{R}{3(5R/2) + 2R} = \frac{2}{19}$ .

$e = 0.105 = 10.5\%$ .

**EVALUATE:** As a check on the calculations note that

$Q_C + Q_H = -\left(\frac{3C_V + R}{R}\right)p_0V_0 + \left(\frac{3C_V + 2R}{R}\right)p_0V_0 = p_0V_0 = W$ , as it should.

**20.39. IDENTIFY:**  $pV = nRT$ , so  $pV$  is constant when  $T$  is constant. Use the appropriate expression to calculate

$Q$  and  $W$  for each process in the cycle.  $e = \frac{W}{Q_H}$ .

**SET UP:** For an ideal diatomic gas,  $C_V = \frac{5}{2}R$  and  $C_p = \frac{7}{2}R$ .

**EXECUTE: (a)**  $p_aV_a = 2.0 \times 10^3 \text{ J}$ .  $p_bV_b = 2.0 \times 10^3 \text{ J}$ .  $pV = nRT$  so  $p_aV_a = p_bV_b$  says  $T_a = T_b$ .

(b) For an isothermal process,  $Q = W = nRT \ln(V_2/V_1)$ .  $ab$  is a compression, with  $V_b < V_a$ , so  $Q < 0$  and heat is rejected.  $bc$  is at constant pressure, so  $Q = nC_p \Delta T = \frac{C_p}{R} p \Delta V$ .  $\Delta V$  is positive, so  $Q > 0$  and heat is absorbed.  $ca$  is at constant volume, so  $Q = nC_V \Delta T = \frac{C_V}{R} V \Delta p$ .  $\Delta p$  is negative, so  $Q < 0$  and heat is rejected.

$$(c) T_a = \frac{p_a V_a}{nR} = \frac{2.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 241 \text{ K}. \quad T_b = \frac{p_b V_b}{nR} = T_a = 241 \text{ K}.$$

$$T_c = \frac{p_c V_c}{nR} = \frac{4.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 481 \text{ K}.$$

$$(d) Q_{ab} = nRT \ln\left(\frac{V_b}{V_a}\right) = (1.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) \ln\left(\frac{0.0050 \text{ m}^3}{0.010 \text{ m}^3}\right) = -1.39 \times 10^3 \text{ J}.$$

$$Q_{bc} = nC_p \Delta T = (1.00) \left(\frac{7}{2}\right) (8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) = 7.01 \times 10^3 \text{ J}.$$

$$Q_{ca} = nC_V \Delta T = (1.00) \left(\frac{5}{2}\right) (8.314 \text{ J/mol} \cdot \text{K})(-241 \text{ K}) = -5.01 \times 10^3 \text{ J}. \quad Q_{\text{net}} = Q_{ab} + Q_{bc} + Q_{ca} = 610 \text{ J}.$$

$$W_{\text{net}} = Q_{\text{net}} = 610 \text{ J}.$$

$$(e) e = \frac{W}{Q_H} = \frac{610 \text{ J}}{7.01 \times 10^3 \text{ J}} = 0.087 = 8.7\%.$$

**EVALUATE:** We can calculate  $W$  for each process in the cycle.  $W_{ab} = Q_{ab} = -1.39 \times 10^3 \text{ J}$ .

$W_{bc} = p \Delta V = (4.0 \times 10^5 \text{ Pa})(0.0050 \text{ m}^3) = 2.00 \times 10^3 \text{ J}$ .  $W_{ca} = 0$ .  $W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = 610 \text{ J}$ , which does equal  $Q_{\text{net}}$ .

**20.40. IDENTIFY:** For a Carnot engine,  $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ .  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ .  $|W| = |Q_H| - |Q_C|$ .  $Q_H > 0$ ,  $Q_C < 0$ .

$$pV = nRT.$$

**SET UP:** The work done by the engine each cycle is  $mg\Delta y$ , with  $m = 15.0 \text{ kg}$  and  $\Delta y = 2.00 \text{ m}$ .

$$T_H = 773 \text{ K}. \quad Q_H = 500 \text{ J}.$$

**EXECUTE:** (a) The  $pV$  diagram is sketched in Figure 20.40.

$$(b) W = mg\Delta y = (15.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) = 294 \text{ J}. \quad |Q_C| = |Q_H| - |W| = 500 \text{ J} - 294 \text{ J} = 206 \text{ J}, \text{ and } Q_C = -206 \text{ J}.$$

$$T_C = -T_H \left(\frac{Q_C}{Q_H}\right) = -(773 \text{ K}) \left(\frac{-206 \text{ J}}{500 \text{ J}}\right) = 318 \text{ K} = 45^\circ\text{C}.$$

$$(c) e = 1 - \frac{T_C}{T_H} = 1 - \frac{318 \text{ K}}{773 \text{ K}} = 0.589 = 58.9\%.$$

$$(d) |Q_C| = 206 \text{ J}.$$

(e) The maximum pressure is for state  $a$ . This is also where the volume is a minimum, so

$$V_a = 5.00 \text{ L} = 5.00 \times 10^{-3} \text{ m}^3. \quad T_a = T_H = 773 \text{ K}.$$

$$p_a = \frac{nRT_a}{V_a} = \frac{(2.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(773 \text{ K})}{5.00 \times 10^{-3} \text{ m}^3} = 2.57 \times 10^6 \text{ Pa}.$$

**EVALUATE:** We can verify that  $e = \frac{W}{Q_H}$  gives the same value for  $e$  as calculated in part (c).

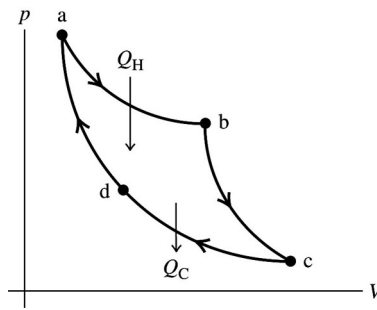


Figure 20.40

**20.41. IDENTIFY:**  $T_b = T_c$  and is equal to the maximum temperature. Use the ideal gas law to calculate  $T_a$ . Apply the appropriate expression to calculate  $Q$  for each process.  $e = \frac{W}{Q_H}$ .  $\Delta U = 0$  for a complete cycle and for an isothermal process of an ideal gas.

**SET UP:** For helium,  $C_V = 3R/2$  and  $C_p = 5R/2$ . The maximum efficiency is for a Carnot cycle, and

$$e_{\text{Carnot}} = 1 - T_C/T_H.$$

**EXECUTE: (a)**  $Q_{\text{in}} = Q_{ab} + Q_{bc}$ .  $Q_{\text{out}} = Q_{ca}$ .  $T_{\text{max}} = T_b = T_c = 327^\circ\text{C} = 600\text{ K}$ .

$$\frac{p_a V_a}{T_a} = \frac{p_b V_b}{T_b} \rightarrow T_a = \frac{p_a}{p_b} T_b = \frac{1}{3}(600\text{ K}) = 200\text{ K}.$$

$$p_b V_b = nRT_b \rightarrow V_b = \frac{nRT_b}{p_b} = \frac{(2\text{ moles})(8.31\text{ J/mol} \cdot \text{K})(600\text{ K})}{3.0 \times 10^5\text{ Pa}} = 0.0332\text{ m}^3.$$

$$\frac{p_b V_b}{T_b} = \frac{p_c V_c}{T_c} \rightarrow V_c = V_b \frac{p_b}{p_c} = (0.0332\text{ m}^3) \left( \frac{3}{1} \right) = 0.0997\text{ m}^3 = V_a.$$

$$Q_{ab} = nC_V \Delta T_{ab} = (2\text{ mol}) \left( \frac{3}{2} \right) (8.31\text{ J/mol} \cdot \text{K})(400\text{ K}) = 9.97 \times 10^3\text{ J}.$$

$$Q_{bc} = W_{bc} = \int_b^c p dV = \int_b^c \frac{nRT_b}{V} dV = nRT_b \ln \frac{V_c}{V_b} = nRT_b \ln 3.$$

$$Q_{bc} = (2.00\text{ mol})(8.31\text{ J/mol} \cdot \text{K})(600\text{ K}) \ln 3 = 1.10 \times 10^4\text{ J}. \quad Q_{\text{in}} = Q_{ab} + Q_{bc} = 2.10 \times 10^4\text{ J}.$$

$$Q_{\text{out}} = Q_{ca} = nC_p \Delta T_{ca} = (2.00\text{ mol}) \left( \frac{5}{2} \right) (8.31\text{ J/mol} \cdot \text{K})(400\text{ K}) = 1.66 \times 10^4\text{ J}.$$

$$\text{(b)} \quad Q = \Delta U + W = 0 + W \rightarrow W = Q_{\text{in}} - Q_{\text{out}} = 2.10 \times 10^4\text{ J} - 1.66 \times 10^4\text{ J} = 4.4 \times 10^3\text{ J}.$$

$$e = W/Q_{\text{in}} = \frac{4.4 \times 10^3\text{ J}}{2.10 \times 10^4\text{ J}} = 0.21 = 21\%.$$

$$\text{(c)} \quad e_{\text{max}} = e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{200\text{ K}}{600\text{ K}} = 0.67 = 67\%.$$

**EVALUATE:** The thermal efficiency of this cycle is about one-third of the efficiency of a Carnot cycle that operates between the same two temperatures.

**20.42. IDENTIFY:** The air in the room receives heat radiated from the person at  $30.0^\circ\text{C}$  but radiates part of it back to the person at  $20.0^\circ\text{C}$ , so it undergoes an entropy change.

**SET UP:** A person with surface area  $A$  and surface temperature  $T = 303\text{ K}$  radiates at a rate  $H = Ae\sigma T^4$ .

The person absorbs heat from the room at a rate  $H_s = Ae\sigma T_s^4$ , where  $T_s = 293\text{ K}$  is the temperature of the room. In  $t = 1.0\text{ s}$ , heat  $Ae\sigma t T^4$  flows into the room and heat  $Ae\sigma t T_s^4$  flows out of the room. The heat flows into and out of the room occur at a temperature of  $T_s$ .

**EXECUTE:** For the room,  $\Delta S = \frac{Ae\sigma T^4}{T_s} - \frac{Ae\sigma T_s^4}{T_s} = \frac{Ae\sigma(T^4 - T_s^4)}{T_s}$ . Putting in the numbers gives

$$\Delta S = \frac{(1.85 \text{ m}^2)(1.00)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.0 \text{ s})[(303 \text{ K})^4 - (293 \text{ K})^4]}{293 \text{ K}} = 0.379 \text{ J/K}.$$

**EVALUATE:** The room gains entropy because its disorder increases.

**20.43. IDENTIFY:**  $e_{\max} = e_{\text{Carnot}} = 1 - T_C/T_H$ .  $e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t}$ .  $W = Q_H + Q_C$  so  $\frac{W}{t} = \frac{Q_C}{t} + \frac{Q_H}{t}$ . For a temperature change  $Q = mc\Delta T$ .

**SET UP:**  $T_H = 300.15 \text{ K}$ ,  $T_C = 279.15 \text{ K}$ . For water,  $\rho = 1000 \text{ kg/m}^3$ , so a mass of 1 kg has a volume of 1 L. For water,  $c = 4190 \text{ J/kg} \cdot \text{K}$ .

**EXECUTE: (a)**  $e = 1 - \frac{279.15 \text{ K}}{300.15 \text{ K}} = 7.0\%$ .

**(b)**  $\frac{Q_H}{t} = \frac{P_{\text{out}}}{e} = \frac{210 \text{ kW}}{0.070} = 3.0 \text{ MW}$ .  $\frac{|Q_C|}{t} = \frac{Q_H}{t} - \frac{W}{t} = 3.0 \text{ MW} - 210 \text{ kW} = 2.8 \text{ MW}$ .

**(c)**  $\frac{m}{t} = \frac{|Q_C|/t}{c\Delta T} = \frac{(2.8 \times 10^6 \text{ W})(3600 \text{ s/h})}{(4190 \text{ J/kg} \cdot \text{K})(4 \text{ K})} = 6 \times 10^5 \text{ kg/h} = 6 \times 10^5 \text{ L/h}$ .

**EVALUATE:** The efficiency is small since  $T_C$  and  $T_H$  don't differ greatly.

**20.44. IDENTIFY:** Since there is temperature difference between the inside and outside of your body, you can use it as a heat engine.

**SET UP:** For a heat engine  $e = \frac{W}{Q_H}$ . For a Carnot engine  $e = 1 - \frac{T_C}{T_H}$ . Gravitational potential energy is

$$U_{\text{grav}} = mgh. \text{ 1 food calorie} = 1000 \text{ cal} = 4186 \text{ J}.$$

**EXECUTE: (a)**  $e = 1 - \frac{T_C}{T_H} = 1 - \frac{303 \text{ K}}{310 \text{ K}} = 0.0226 = 2.26\%$ . This engine has a very low thermal efficiency.

**(b)**  $U_{\text{grav}} = mgh = (2.50 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m}) = 29.4 \text{ J}$ . This equals the work output of the engine.

$$e = \frac{W}{Q_H} \text{ so } Q_H = \frac{W}{e} = \frac{29.4 \text{ J}}{0.0226} = 1.30 \times 10^3 \text{ J}.$$

**(c)** Since 80% of food energy goes into heat, you must eat food with a food energy of

$$\frac{1.30 \times 10^3 \text{ J}}{0.80} = 1.63 \times 10^3 \text{ J}. \text{ Each candy bar gives } (350 \text{ food calorie})(4186 \text{ J/food calorie}) = 1.47 \times 10^6 \text{ J}.$$

$$\text{The number of candy bars required is } \frac{1.63 \times 10^3 \text{ J}}{1.47 \times 10^6 \text{ J/candy bar}} = 1.11 \times 10^{-3} \text{ candy bars}.$$

**EVALUATE:** A large amount of mechanical work must be done to use up the energy from one candy bar.

**20.45. IDENTIFY:** Use  $pV = nRT$ . Apply the expressions for  $Q$  and  $W$  that apply to each type of process.

$$e = \frac{W}{Q_H}.$$

**SET UP:** For  $\text{O}_2$ ,  $C_V = 20.85 \text{ J/mol} \cdot \text{K}$  and  $C_p = 29.17 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE: (a)**  $p_1 = 2.00 \text{ atm}$ ,  $V_1 = 4.00 \text{ L}$ ,  $T_1 = 300 \text{ K}$ .

$$p_2 = 2.00 \text{ atm}. \frac{V_1}{T_1} = \frac{V_2}{T_2}. V_2 = \left(\frac{T_2}{T_1}\right)V_1 = \left(\frac{450 \text{ K}}{300 \text{ K}}\right)(4.00 \text{ L}) = 6.00 \text{ L}.$$

$$V_3 = 6.00 \text{ L}. \frac{p_2}{T_2} = \frac{p_3}{T_3}. p_3 = \left(\frac{T_3}{T_2}\right)p_2 = \left(\frac{250 \text{ K}}{450 \text{ K}}\right)(2.00 \text{ atm}) = 1.11 \text{ atm}.$$



$$V_4 = 4.00 \text{ L}. \quad p_3 V_3 = p_4 V_4. \quad p_4 = p_3 \left( \frac{V_3}{V_4} \right) = (1.11 \text{ atm}) \left( \frac{6.00 \text{ L}}{4.00 \text{ L}} \right) = 1.67 \text{ atm}.$$

These processes are shown in Figure 20.45.

$$(b) \quad n = \frac{p_1 V_1}{RT_1} = \frac{(2.00 \text{ atm})(4.00 \text{ L})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(300 \text{ K})} = 0.325 \text{ mol}$$

$$\text{process } 1 \rightarrow 2: \quad W = p \Delta V = nR \Delta T = (0.325 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(150 \text{ K}) = 405 \text{ J}.$$

$$Q = nC_p \Delta T = (0.325 \text{ mol})(29.17 \text{ J/mol} \cdot \text{K})(150 \text{ K}) = 1422 \text{ J}.$$

$$\text{process } 2 \rightarrow 3: \quad W = 0. \quad Q = nC_V \Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(-200 \text{ K}) = -1355 \text{ J}.$$

$$\text{process } 3 \rightarrow 4: \quad \Delta U = 0 \quad \text{and}$$

$$Q = W = nRT_3 \ln \left( \frac{V_4}{V_3} \right) = (0.325 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(250 \text{ K}) \ln \left( \frac{4.00 \text{ L}}{6.00 \text{ L}} \right) = -274 \text{ J}.$$

$$\text{process } 4 \rightarrow 1: \quad W = 0. \quad Q = nC_V \Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(50 \text{ K}) = 339 \text{ J}.$$

$$(c) \quad W = 405 \text{ J} - 274 \text{ J} = 131 \text{ J}.$$

$$(d) \quad e = \frac{W}{Q_H} = \frac{131 \text{ J}}{1422 \text{ J} + 339 \text{ J}} = 0.0744 = 7.44\%.$$

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{250 \text{ K}}{450 \text{ K}} = 0.444 = 44.4\%; \quad e_{\text{Carnot}} \text{ is much larger.}$$

**EVALUATE:**  $Q_{\text{tot}} = +1422 \text{ J} + (-1355 \text{ J}) + (-274 \text{ J}) + 339 \text{ J} = 132 \text{ J}$ . This is equal to  $W_{\text{tot}}$ , apart from a slight difference due to rounding. For a cycle,  $W_{\text{tot}} = Q_{\text{tot}}$ , since  $\Delta U = 0$ .

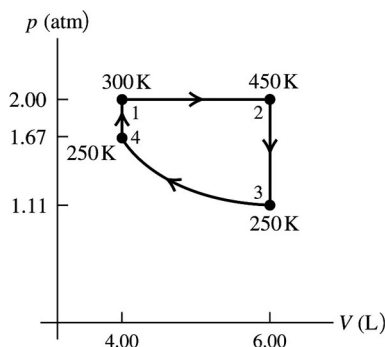


Figure 20.45

**20.46. IDENTIFY:** Use  $\Delta U = Q - W$  and the appropriate expressions for  $Q$ ,  $W$  and  $\Delta U$  for each type of process.

$$pV = nRT \text{ relates } \Delta T \text{ to } p \text{ and } V \text{ values. } e = \frac{W}{Q_H}, \text{ where } Q_H \text{ is the heat that enters the gas during the cycle.}$$

**SET UP:** For a monatomic ideal gas,  $C_p = \frac{5}{2}R$  and  $C_V = \frac{3}{2}R$ .

**(a) ab:** The temperature changes by the same factor as the volume, and so

$$Q = nC_p \Delta T = \frac{C_p}{R} p_a (V_a - V_b) = (2.5)(3.00 \times 10^5 \text{ Pa})(0.300 \text{ m}^3) = 2.25 \times 10^5 \text{ J}.$$

The work  $p \Delta V$  is the same except for the factor of  $\frac{5}{2}$ , so  $W = 0.90 \times 10^5 \text{ J}$ .

$$\Delta U = Q - W = 1.35 \times 10^5 \text{ J}.$$

**bc:** The temperature now changes in proportion to the pressure change, and

$$Q = \frac{3}{2}(p_c - p_b)V_b = (1.5)(-2.00 \times 10^5 \text{ Pa})(0.800 \text{ m}^3) = -2.40 \times 10^5 \text{ J}, \text{ and the work is zero}$$

$$(\Delta V = 0). \quad \Delta U = Q - W = -2.40 \times 10^5 \text{ J}.$$

*ca*: The easiest way to do this is to find the work done first;  $W$  will be the negative of area in the  $p$ - $V$  plane bounded by the line representing the process *ca* and the verticals from points *a* and *c*. The area of this trapezoid is  $\frac{1}{2}(3.00 \times 10^5 \text{ Pa} + 1.00 \times 10^5 \text{ Pa})(0.800 \text{ m}^3 - 0.500 \text{ m}^3) = 6.00 \times 10^4 \text{ J}$  and so the work is  $-0.60 \times 10^5 \text{ J}$ .  $\Delta U$  must be  $1.05 \times 10^5 \text{ J}$  (since  $\Delta U = 0$  for the cycle, anticipating part (b)), and so  $Q$  must be  $\Delta U + W = 0.45 \times 10^5 \text{ J}$ .

(b) See above;  $Q = W = 0.30 \times 10^5 \text{ J}$ ,  $\Delta U = 0$ .

(c) The heat added, during process *ab* and *ca*, is  $2.25 \times 10^5 \text{ J} + 0.45 \times 10^5 \text{ J} = 2.70 \times 10^5 \text{ J}$  and the efficiency is  $e = \frac{W}{Q_H} = \frac{0.30 \times 10^5}{2.70 \times 10^5} = 0.111 = 11.1\%$ .

**EVALUATE:** For any cycle,  $\Delta U = 0$  and  $Q = W$ .

**20.47. IDENTIFY:** The efficiency of the composite engine is  $e_{12} = \frac{W_1 + W_2}{Q_{H1}}$ , where  $Q_{H1}$  is the heat input to the first engine and  $W_1$  and  $W_2$  are the work outputs of the two engines. For any heat engine,  $W = Q_C + Q_H$ ,

and for a Carnot engine,  $\frac{Q_{\text{low}}}{Q_{\text{high}}} = -\frac{T_{\text{low}}}{T_{\text{high}}}$ , where  $Q_{\text{low}}$  and  $Q_{\text{high}}$  are the heat flows at the two reservoirs

that have temperatures  $T_{\text{low}}$  and  $T_{\text{high}}$ .

**SET UP:**  $Q_{\text{high},2} = -Q_{\text{low},1}$ ,  $T_{\text{low},1} = T'$ ,  $T_{\text{high},1} = T_H$ ,  $T_{\text{low},2} = T_C$  and  $T_{\text{high},2} = T'$ .

**EXECUTE:**  $e_{12} = \frac{W_1 + W_2}{Q_{H1}} = \frac{Q_{\text{high},1} + Q_{\text{low},1} + Q_{\text{high},2} + Q_{\text{low},2}}{Q_{\text{high},1}}$ . Since  $Q_{\text{high},2} = -Q_{\text{low},1}$ , this reduces to

$e_{12} = 1 + \frac{Q_{\text{low},2}}{Q_{\text{high},1}}$ .  $Q_{\text{low},2} = -Q_{\text{high},2} \frac{T_{\text{low},2}}{T_{\text{high},2}} = Q_{\text{low},1} \frac{T_C}{T'} = -Q_{\text{high},1} \left( \frac{T_{\text{low},1}}{T_{\text{high},1}} \right) \frac{T_C}{T'} = -Q_{\text{high},1} \left( \frac{T'}{T_H} \right) \frac{T_C}{T'}$ . This gives

$e_{12} = 1 - \frac{T_C}{T_H}$ . The efficiency of the composite system is the same as that of the original engine.

**EVALUATE:** The overall efficiency is independent of the value of the intermediate temperature  $T'$ .

**20.48. IDENTIFY:**  $e = \frac{W}{Q_H}$ . 1 day =  $8.64 \times 10^4 \text{ s}$ . For the river water,  $Q = mc\Delta T$ , where the heat that goes into

the water is the heat  $Q_C$  rejected by the engine. The density of water is  $1000 \text{ kg/m}^3$ . When an object undergoes a temperature change,  $\Delta S = mc \ln(T_2/T_1)$ .

**SET UP:**  $18.0^\circ\text{C} = 291.1 \text{ K}$ .  $18.5^\circ\text{C} = 291.6 \text{ K}$ .

**EXECUTE:** (a)  $Q_H = \frac{W}{e}$  so  $P_H = \frac{P_W}{e} = \frac{1000 \text{ MW}}{0.40} = 2.50 \times 10^3 \text{ MW}$ .

(b) The heat input in one day is  $(2.50 \times 10^9 \text{ W})(8.64 \times 10^4 \text{ s}) = 2.16 \times 10^{14} \text{ J}$ . The mass of coal used per day is  $\frac{2.16 \times 10^{14} \text{ J}}{2.65 \times 10^7 \text{ J/kg}} = 8.15 \times 10^6 \text{ kg}$ .

(c)  $|Q_H| = |W| + |Q_C|$ .  $|Q_C| = |Q_H| - |W|$ .  $P_C = P_H - P_W = 2.50 \times 10^3 \text{ MW} - 1000 \text{ MW} = 1.50 \times 10^3 \text{ MW}$ .

(d) The heat input to the river is  $1.50 \times 10^9 \text{ J/s}$ .  $Q = mc\Delta T$  and  $\Delta T = 0.5^\circ\text{C}$  gives

$m = \frac{Q}{c\Delta T} = \frac{1.50 \times 10^9 \text{ J}}{(4190 \text{ J/kg} \cdot \text{K})(0.5 \text{ K})} = 7.16 \times 10^5 \text{ kg}$ .  $V = \frac{m}{\rho} = 716 \text{ m}^3$ . The river flow rate must be  $716 \text{ m}^3/\text{s}$ .

(e) In one second,  $7.16 \times 10^5 \text{ kg}$  of water goes from  $291.1 \text{ K}$  to  $291.6 \text{ K}$ .

$\Delta S = mc \ln\left(\frac{T_2}{T_1}\right) = (7.16 \times 10^5 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{291.6 \text{ K}}{291.1 \text{ K}}\right) = 5.1 \times 10^6 \text{ J/K}$ .

**EVALUATE:** The entropy of the river increases because heat flows into it. The mass of coal used per second is huge.

- 20.49. (a) IDENTIFY and SET UP:** Calculate  $e$  from  $e = 1 - 1/(r^{\gamma-1})$ ,  $Q_C$  from  $e = (Q_H + Q_C)/Q_H$ , and then  $W$  from  $W = Q_C + Q_H$ .

**EXECUTE:**  $e = 1 - 1/(r^{\gamma-1}) = 1 - 1/(10.6^{0.4}) = 0.6111$

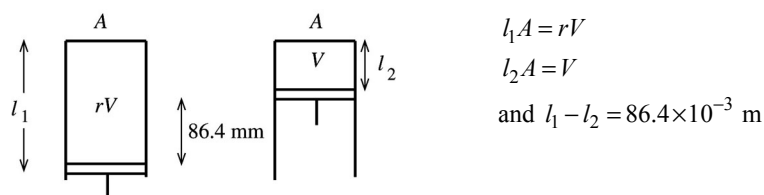
$e = (Q_H + Q_C)/Q_H$  and we are given  $Q_H = 200$  J; calculate  $Q_C$ .

$Q_C = (e - 1)Q_H = (0.6111 - 1)(200 \text{ J}) = -78 \text{ J}$ . (negative, since corresponds to heat leaving)

Then  $W = Q_C + Q_H = -78 \text{ J} + 200 \text{ J} = 122 \text{ J}$ . (positive, in agreement with Figure 20.6 in the text)

**EVALUATE:**  $Q_H$ ,  $W > 0$ , and  $Q_C < 0$  for an engine cycle.

**(b) IDENTIFY and SET UP:** The stroke times the bore equals the change in volume. The initial volume is the final volume  $V$  times the compression ratio  $r$ . Combining these two expressions gives an equation for  $V$ . For each cylinder of area  $A = \pi(d/2)^2$  the piston moves 0.0864 m and the volume changes from  $rV$  to  $V$ , as shown in Figure 20.49a.



**Figure 20.49a**

**EXECUTE:**  $l_1A - l_2A = rV - V$  and  $(l_1 - l_2)A = (r - 1)V$

$$V = \frac{(l_1 - l_2)A}{r - 1} = \frac{(86.4 \times 10^{-3} \text{ m})\pi(41.25 \times 10^{-3} \text{ m})^2}{10.6 - 1} = 4.811 \times 10^{-5} \text{ m}^3$$

At point  $a$  the volume is  $rV = 10.6(4.811 \times 10^{-5} \text{ m}^3) = 5.10 \times 10^{-4} \text{ m}^3$ .

**(c) IDENTIFY and SET UP:** The processes in the Otto cycle are either constant volume or adiabatic. Use the  $Q_H$  that is given to calculate  $\Delta T$  for process  $bc$ . Use  $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$  and  $pV = nRT$  to relate  $p$ ,  $V$  and  $T$  for the adiabatic processes  $ab$  and  $cd$ .

**EXECUTE:** point  $a$ :  $T_a = 300$  K,  $p_a = 8.50 \times 10^4$  Pa and  $V_a = 5.10 \times 10^{-4} \text{ m}^3$ .

point  $b$ :  $V_b = V_a/r = 4.81 \times 10^{-5} \text{ m}^3$ . Process  $a \rightarrow b$  is adiabatic, so  $T_aV_a^{\gamma-1} = T_bV_b^{\gamma-1}$ .

$$T_a(rV)^{\gamma-1} = T_bV^{\gamma-1}$$

$$T_b = T_a r^{\gamma-1} = 300 \text{ K}(10.6)^{0.4} = 771 \text{ K}$$

$$pV = nRT \text{ so } pV/T = nR = \text{constant, so } p_aV_a/T_a = p_bV_b/T_b$$

$$p_b = p_a(V_a/V_b)(T_b/T_a) = (8.50 \times 10^4 \text{ Pa})(rV/V)(771 \text{ K}/300 \text{ K}) = 2.32 \times 10^6 \text{ Pa}$$

point  $c$ : Process  $b \rightarrow c$  is at constant volume, so  $V_c = V_b = 4.81 \times 10^{-5} \text{ m}^3$

$Q_H = nC_V\Delta T = nC_V(T_c - T_b)$ . The problem specifies  $Q_H = 200$  J; use to calculate  $T_c$ . First use the  $p$ ,  $V$ ,  $T$  values at point  $a$  to calculate the number of moles  $n$ .

$$n = \frac{pV}{RT} = \frac{(8.50 \times 10^4 \text{ Pa})(5.10 \times 10^{-4} \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 0.01738 \text{ mol}$$

$$\text{Then } T_c - T_b = \frac{Q_H}{nC_V} = \frac{200 \text{ J}}{(0.01738 \text{ mol})(20.5 \text{ J/mol} \cdot \text{K})} = 561.3 \text{ K, and}$$

$$T_c = T_b + 561.3 \text{ K} = 771 \text{ K} + 561 \text{ K} = 1332 \text{ K}$$

$$p/T = nR/V = \text{constant} \quad \text{so} \quad p_b/T_b = p_c/T_c$$

$$p_c = p_b(T_c/T_b) = (2.32 \times 10^6 \text{ Pa})(1332 \text{ K}/771 \text{ K}) = 4.01 \times 10^6 \text{ Pa}$$

point d:  $V_d = V_a = 5.10 \times 10^{-4} \text{ m}^3$

process  $c \rightarrow d$  is adiabatic, so  $T_d V_d^{\gamma-1} = T_c V_c^{\gamma-1}$

$$T_d (rV)^{\gamma-1} = T_c V^{\gamma-1}$$

$$T_d = T_c / r^{\gamma-1} = 1332 \text{ K} / 10.6^{0.4} = 518 \text{ K}$$

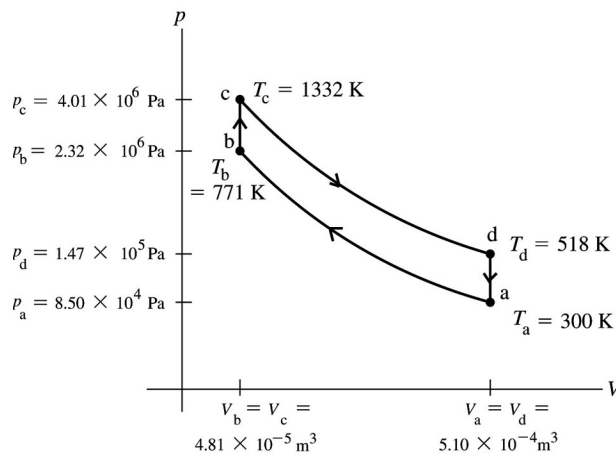
$$p_c V_c / T_c = p_d V_d / T_d$$

$$p_d = p_c (V_c / V_d) (T_d / T_c) = (4.01 \times 10^6 \text{ Pa}) (V / rV) (518 \text{ K} / 1332 \text{ K}) = 1.47 \times 10^5 \text{ Pa}$$

**EVALUATE:** Can look at process  $d \rightarrow a$  as a check.

$$Q_C = nC_V(T_a - T_d) = (0.01738 \text{ mol})(20.5 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 518 \text{ K}) = -78 \text{ J}, \text{ which agrees with part (a).}$$

The cycle is sketched in Figure 20.49b.



**Figure 20.49b**

**(d) IDENTIFY and SET UP:** The Carnot efficiency is given by  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ .  $T_H$  is the highest

temperature reached in the cycle and  $T_C$  is the lowest.

**EXECUTE:** From part (a) the efficiency of this Otto cycle is  $e = 0.611 = 61.1\%$ .

The efficiency of a Carnot cycle operating between 1332 K and 300 K is

$$e_{\text{Carnot}} = 1 - T_C / T_H = 1 - 300 \text{ K} / 1332 \text{ K} = 0.775 = 77.5\%, \text{ which is larger.}$$

**EVALUATE:** The second law of thermodynamics requires that  $e \leq e_{\text{Carnot}}$ , and our result obeys this law.

**20.50. IDENTIFY:**  $K = \frac{|Q_C|}{|W|}$ .  $|Q_H| = |Q_C| + |W|$ . The heat flows for the inside and outside air occur at constant  $T$ ,

$$\text{so } \Delta S = Q/T.$$

**SET UP:**  $21.0^\circ\text{C} = 294.1 \text{ K}$ .  $35.0^\circ\text{C} = 308.1 \text{ K}$ .

**EXECUTE: (a)**  $|Q_C| = K|W|$ .  $P_C = KP_W = (2.80)(800 \text{ W}) = 2.24 \times 10^3 \text{ W}$ .

**(b)**  $P_H = P_C + P_W = 2.24 \times 10^3 \text{ W} + 800 \text{ W} = 3.04 \times 10^3 \text{ W}$ .

**(c)** In  $1 \text{ h} = 3600 \text{ s}$ ,  $Q_H = P_H t = 1.094 \times 10^7 \text{ J}$ .  $\Delta S_{\text{out}} = \frac{Q_H}{T_H} = \frac{1.094 \times 10^7 \text{ J}}{308.1 \text{ K}} = 3.55 \times 10^4 \text{ J/K}$ .

(d)  $Q_C = P_C t = 8.064 \times 10^6 \text{ J}$ . Heat  $Q_C$  is removed from the inside air.

$$\Delta S_{\text{in}} = \frac{-Q_C}{T_C} = \frac{-8.064 \times 10^6 \text{ J}}{294.1 \text{ K}} = -2.74 \times 10^4 \text{ J/K}. \quad \Delta S_{\text{net}} = \Delta S_{\text{out}} + \Delta S_{\text{in}} = 8.1 \times 10^3 \text{ J/K}.$$

**EVALUATE:** The increase in the entropy of the outside air is greater than the entropy decrease of the air in the room.

**20.51. IDENTIFY and SET UP:** A refrigerator is like a heat engine run in reverse. In the  $pV$ -diagram shown with the figure, heat enters the gas during parts  $ab$  and  $bc$  of the cycle, and leaves during  $ca$ . Treating  $\text{H}_2$  as a diatomic gas, we know that  $C_V = \frac{5}{2}R$  and  $C_p = \frac{7}{2}R$ . Segment  $bc$  is isochoric, so  $Q_{bc} = nC_V \Delta T$ . Segment  $ca$  is isobaric, so  $Q_{ca} = nC_p \Delta T$ . Segment  $ab$  is isothermal, so  $Q_{ab} = nRT \ln(V_b/V_a)$ . The coefficient of

performance of a refrigerator is  $K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$ , and  $pV = nRT$  applies. Calculate the values for

$Q_C$  and  $Q_H$  and use the definition of  $K$ . Use  $1000 \text{ L} = 1 \text{ m}^3$  and work in units of  $\text{L} \cdot \text{atm}$ .

**EXECUTE:** Use  $pV = nRT$  to find  $p_b$ . Since  $ab$  is isothermal,  $p_a V_a = p_b V_b$ , which gives

$$p_b = (0.700 \text{ atm})(0.0300 \text{ m}^3)/(0.100 \text{ m}^3) = 0.210 \text{ atm}.$$

$Q_C = Q_{ab} + Q_{bc}$ , so we need to calculate these quantities.

$$Q_{ab} = nRT \ln(V_b/V_a) = p_a V_a \ln(V_b/V_a) = (0.700 \text{ atm})(30.0 \text{ L}) \ln[(100 \text{ L})/(30 \text{ L})] = 25.2834 \text{ L} \cdot \text{atm}.$$

$$Q_{bc} = nC_V \Delta T_{bc} = n\left(\frac{5}{2}R\right) \Delta T_{bc} = \frac{5}{2} V_b \Delta p_{bc} = (5/2)(100 \text{ L})(0.700 \text{ atm} - 0.210 \text{ atm}) = 122.5 \text{ L} \cdot \text{atm}.$$

$$\text{Therefore } Q_C = Q_{ab} + Q_{bc} = 25.2834 \text{ L} \cdot \text{atm} + 122.500 \text{ L} \cdot \text{atm} = 147.7834 \text{ L} \cdot \text{atm}.$$

$$Q_H = Q_{ca} = nC_p \Delta T_{ca} = n\left(\frac{7}{2}R\right) \Delta T_{ca} = \frac{7}{2} p_c \Delta T_{ca} = (7/2)(0.700 \text{ atm})(30.0 \text{ L} - 100.0 \text{ L}) =$$

$$-171.500 \text{ L} \cdot \text{atm}.$$

$$\text{Now get } K: K = \frac{|Q_C|}{|Q_H| - |Q_C|} = \frac{147.7834 \text{ L} \cdot \text{atm}}{(171.500 \text{ L} \cdot \text{atm} - 147.7834 \text{ L} \cdot \text{atm})} = 6.23.$$

**EVALUATE:**  $K$  is greater than 1, which it must be. Efficiencies are less than 1.

**20.52. IDENTIFY and SET UP:** A person radiates heat from the surface of her body which is at a constant temperature of  $T = 30.0^\circ\text{C}$  into air at  $T_s = 20.0^\circ\text{C}$ . In  $1.0 \text{ s}$ , heat  $Ae\sigma T^4$  flows from the person into the room and heat  $Ae\sigma T_s^4$  flows out of the room into the person. The heat flows into and out of the room occur at a temperature of  $T_s$ . At constant temperature,  $\Delta S = Q/T$ .

**EXECUTE:** For the room,

$$\Delta S = \frac{Ae\sigma T^4}{T_s} - \frac{Ae\sigma T_s^4}{T_s} = \frac{Ae\sigma(T^4 - T_s^4)}{T_s}$$

$$\Delta S = \frac{(1.85 \text{ m}^2)(1.00)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.0)[(303 \text{ K})^4 - (293 \text{ K})^4]}{293 \text{ K}} = 0.379 \text{ J/K}$$

**EVALUATE:** The entropy change is positive since the air in the room becomes more disordered. In addition, this process is irreversible; heat from the cool room will not spontaneously flow into the warmer person and increase her temperature even further. Heat flows only from hot to cold.

**20.53. IDENTIFY:** Use  $\Delta S = mc \ln(T_2/T_1)$  for an isothermal process. For the value of  $T$  for which  $\Delta S$  is a maximum,  $d(\Delta S)/dT = 0$ .

**SET UP:** The heat flow for a temperature change is  $Q = mc\Delta T$ .

**EXECUTE: (a)** As in Example 20.10, the entropy change of the first object is  $m_1 c_1 \ln(T/T_1)$  and that of the second is  $m_2 c_2 \ln(T'/T_2)$ , and so the net entropy change is as given. Neglecting heat transfer to the surroundings,  $Q_1 + Q_2 = 0$ ,  $m_1 c_1 (T - T_1) + m_2 c_2 (T' - T_2) = 0$ , which is the given expression.

**(b)** Solving the energy-conservation relation for  $T'$  and substituting into the expression for  $\Delta S$  gives

$$\Delta S = m_1 c_1 \ln\left(\frac{T}{T_1}\right) + m_2 c_2 \ln\left(1 - \frac{m_1 c_1}{m_2 c_2} \left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right). \quad \text{Differentiating with respect to } T \text{ and setting the}$$

derivative equal to 0 gives  $0 = \frac{m_1 c_1}{T} + \frac{(m_2 c_2)(m_1 c_1 / m_2 c_2)(-1/T_2)}{\left(1 - (m_1 c_1 / m_2 c_2) \left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)}$ . This may be solved for

$$T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}.$$
 Using this value for  $T$  in the conservation of energy expression in part (a) and

solving for  $T'$  gives  $T' = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$ . Therefore,  $T = T'$  when  $\Delta S$  is a maximum.

**EVALUATE:** (c) The final state of the system will be that for which no further entropy change is possible. If  $T < T'$ , it is possible for the temperatures to approach each other while increasing the total entropy, but when  $T = T'$ , no further spontaneous heat exchange is possible.

**20.54. IDENTIFY:** Use the expression derived in Example 20.6 for the entropy change in a temperature change.

**SET UP:** For water,  $c = 4190 \text{ J/kg} \cdot \text{K}$ .  $20^\circ\text{C} = 293.15 \text{ K}$ ,  $78^\circ\text{C} = 351.15 \text{ K}$  and  $120^\circ\text{C} = 393.15 \text{ K}$ .

**EXECUTE:**

(a)  $\Delta S = mc \ln(T_2/T_1) = (250 \times 10^{-3} \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln(351.15 \text{ K}/293.15 \text{ K}) = 189 \text{ J/K}$ .

(b)  $\Delta S = \frac{-mc\Delta T}{T_{\text{element}}} = \frac{-(250 \times 10^{-3} \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(351.15 \text{ K} - 293.15 \text{ K})}{393.15 \text{ K}} = -155 \text{ J/K}$ .

(c) The sum of the result of parts (a) and (b) is  $\Delta S_{\text{system}} = 34.6 \text{ J/K}$ . (Carry extra figures when subtraction is involved.)

**EVALUATE:** (d) Heating a liquid is not reversible. Whatever the energy source for the heating element, heat is being delivered at a higher temperature than that of the water, and the entropy loss of the source will be less in magnitude than the entropy gain of the water. The net entropy change is positive.

**20.55. IDENTIFY and SET UP:** The most efficient heat engine operating between any two given temperatures is

the Carnot engine, and its efficiency is  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ .

**EXECUTE:** (a) For prototype A,  $e_{\text{max}} = e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - (320 \text{ K})/(450 \text{ K}) = 0.289 = 28.9\%$ . By

similar calculations, we get the following:

A:  $e_{\text{max}} = 0.289 = 28.9\%$

B:  $e_{\text{max}} = 0.383 = 38.3\%$

C:  $e_{\text{max}} = 0.538 = 53.8\%$

D:  $e_{\text{max}} = 0.244 = 24.4\%$

(b) Engine C claims a maximum efficiency of 56%, which is greater than the maximum possible for its temperature range, so it is impossible.

(c) We get the following ratios:

A:  $e_{\text{claimed}}/e_{\text{max}} = 0.21/0.289 = 0.73$

B:  $e_{\text{claimed}}/e_{\text{max}} = 0.35/0.383 = 0.90$

D:  $e_{\text{claimed}}/e_{\text{max}} = 0.20/0.244 = 0.82$

In decreasing order, we have B, D, A.

**EVALUATE:** Engine B is not only the most efficient, it is also closest to its maximum possible efficiency for its temperature range. Engines A and D have nearly the same efficiency, but D comes somewhat closer to its theoretical maximum than does A.

**20.56. IDENTIFY and SET UP:** In terms of power,  $K = \frac{|Q_C|/t}{|W|/t}$ .  $1 \text{ Btu} = 1055 \text{ J}$  and  $1 \text{ W} \cdot \text{h} = 3600 \text{ J}$ .

**EXECUTE:** (a)  $1 \text{ W} \cdot \text{h} = (3600 \text{ J})(1 \text{ Btu}/1055 \text{ J}) = 3.412 \text{ Btu}$ , so  $\text{EER} = (3.412 \text{ Btu}/\text{W} \cdot \text{h})K$ .

(b)  $T_H = 95^\circ\text{F} = 35^\circ\text{C} = 308.1 \text{ K}$ ;  $T_C = 80^\circ\text{F} = 26.7^\circ\text{C} = 299.8 \text{ K}$ .

For a Carnot air conditioner,  $K = T_C/(T_H - T_C) = (299.8 \text{ K})/(308.1 \text{ K} - 299.8 \text{ K}) = 36$ . The corresponding EER is

$\text{EER} = (3.412 \text{ Btu}/\text{W} \cdot \text{h})(36) = 120 \text{ Btu}/\text{W} \cdot \text{h}$ .

(c)  $K = \text{EER}/3.412 = 10.9/3.412 = 3.195$

$|W| = |Q_C|/K = (1.9 \times 10^{10} \text{ J})/(3.195) = 5.95 \times 10^9 \text{ J} [(1 \text{ kWh})/(3.6 \times 10^6 \text{ J})] = 1650 \text{ kWh}$ . The cost is  $\$0.153(1650 \text{ kWh}) = \$253$ .

(d) Using the same approach as in (c) gives the following values:

$K = 4.279$ ,  $W = 1233 \text{ kWh}$ , cost = \$189, savings = \$253 – \$189 = \$64 per year.

**EVALUATE:** Whether it is worth it to replace your air conditioner depends on what it costs. In 10 years you would save \$640 (probably more since utility rates tend to rise over time). But would the unit last 10 years?

- 20.57. IDENTIFY and SET UP:** The cycle consists of two isochoric processes ( $ab$  and  $cd$ ) and two isobaric processes ( $bc$  and  $da$ ). Use  $Q = nC_V \Delta T$  and  $Q = nC_p \Delta T$  for these processes. For an ideal monatomic gas (argon),  $C_V = \frac{3}{2}R$  and  $C_p = \frac{5}{2}R$ . Use  $R = 8.3145 \text{ J/mol} \cdot \text{K}$ .

**EXECUTE: (a)** Using the equations listed above, the heat transfers are as follows:

$$Q_{ab} = (3/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300.0 \text{ K} - 250.0 \text{ K}) = 2.494 \text{ kJ}$$

$$Q_{bc} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(380.0 \text{ K} - 300.0 \text{ K}) = 6.652 \text{ kJ}$$

$$Q_{cd} = (3/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(316.7 \text{ K} - 380.0 \text{ K}) = -3.158 \text{ kJ}$$

$$Q_{da} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(250.0 \text{ K} - 316.7 \text{ K}) = -5.546 \text{ kJ}$$

The efficiency of this cycle is  $e = \frac{W}{|Q_{in}|} = \frac{|Q_{in}| - |Q_{out}|}{|Q_{in}|} = 1 - \frac{|Q_{out}|}{|Q_{in}|}$ . This gives

$$e = 1 - \frac{3.158 \text{ kJ} + 5.546 \text{ kJ}}{2.494 \text{ kJ} + 6.652 \text{ kJ}} = 0.0483 = 4.83\%.$$

(b) If we double the number of moles, all the values of  $Q$  will double, but the factor of 2 cancels out, so the efficiency remains the same.

(c) Using the same procedure as in (a), the revised numbers are

$$Q_{ab} = 2.494 \text{ kJ (unchanged)}$$

$$Q_{bc} = 38.247 \text{ kJ}$$

$$Q_{cd} = -6.316 \text{ kJ}$$

$$Q_{da} = -31.878 \text{ kJ}$$

As in part (a), the efficiency of this cycle is  $e = 1 - \frac{|Q_{out}|}{|Q_{in}|}$ , which gives

$$e = 1 - \frac{31.878 \text{ kJ} + 6.316 \text{ kJ}}{38.247 \text{ kJ} + 2.494 \text{ kJ}} = 0.0625 = 6.25\%.$$

(d) In symbolic form, we have  $Q_{ab} = +2.494 \text{ kJ}$  (unchanged)

$$Q_{bc} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(T_c - 300.0 \text{ K}), \text{ which is positive.}$$

$$Q_{cd} = (3/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(T_d - T_c), \text{ which is negative.}$$

$$Q_{da} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(250.0 \text{ K} - T_d), \text{ which is positive.}$$

Using these values, the efficiency becomes  $e = 1 - \frac{3(T_c - T_d) + 5(T_d - 250.0 \text{ K})}{150 + 5(T_c - 300.0 \text{ K})}$ . Using the fact that

$$T_c = 1.20T_d \text{ and simplifying, we get } e = \frac{0.40T_d - 100 \text{ K}}{6.00T_d - 1350 \text{ K}}. \text{ As } T_d \rightarrow \infty, e \rightarrow 0.40/6.00 = 0.0667 = 6.67\%.$$

**EVALUATE:** In (c), the Carnot efficiency for the temperature extremes given would be  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} =$

$1 - (250 \text{ K})/(760 \text{ K}) = 0.67 = 67\%$ , which is 10 times the maximum efficiency of your engine. Maybe you need a new design!

- 20.58. IDENTIFY:** Calculate  $Q_C$  and  $Q_H$  in terms of  $p$  and  $V$  at each point. Use the ideal gas law and the

pressure-volume relation for adiabatic processes for an ideal gas.  $e = 1 - \frac{|Q_C|}{|Q_H|}$ .

**SET UP:** For an ideal gas,  $C_p = C_V + R$ , and taking air to be diatomic,  $C_p = \frac{7}{2}R$ ,  $C_V = \frac{5}{2}R$ , and  $\gamma = \frac{7}{5}$ .

**EXECUTE: (a)** Referring to Figure 20.7 in the textbook,  $Q_H = n \frac{7}{2} R(T_c - T_b) = \frac{7}{2}(p_c V_c - p_b V_b)$ . Similarly,  $Q_C = n \frac{5}{2} R(p_a V_a - p_d V_d)$ . What needs to be done is to find the relations between the product of the pressure and the volume at the four points. For an ideal gas,  $\frac{p_c V_c}{T_c} = \frac{p_b V_b}{T_b}$  so  $p_c V_c = p_a V_a \left( \frac{T_c}{T_a} \right)$ . For a compression ratio  $r$ , and given that for the Diesel cycle the process  $ab$  is adiabatic,  $p_b V_b = p_a V_a \left( \frac{V_a}{V_b} \right)^{\gamma-1} = p_a V_a r^{\gamma-1}$ .

Similarly,  $p_d V_d = p_c V_c \left( \frac{V_c}{V_d} \right)^{\gamma-1}$ . Note that the last result uses the fact that process  $da$  is isochoric, and

$V_d = V_a$ ; also,  $p_c = p_b$  (process  $bc$  is isobaric), and so  $V_c = V_b \left( \frac{T_c}{T_a} \right)$ . Then,

$$\frac{V_c}{V_a} = \frac{T_c}{T_b} \cdot \frac{V_b}{V_a} = \frac{T_b}{T_a} \cdot \frac{T_a}{T_b} \cdot \frac{V_a}{V_b} = \frac{T_c}{T_a} \cdot \left( \frac{T_a V_a^{\gamma-1}}{T_b V_b^{\gamma-1}} \right) \left( \frac{V_a}{V_b} \right)^{-\gamma} = \frac{T_c}{T_a} r^{\gamma}$$

Combining the above results,  $p_d V_d = p_a V_a \left( \frac{T_c}{T_a} \right)^{\gamma} r^{\gamma-\gamma^2}$ . Substitution of the above results into

$$e = 1 - \frac{|Q_C|}{|Q_H|} \text{ gives } e = 1 - \frac{5}{7} \frac{\left( \frac{T_c}{T_a} \right)^{\gamma} r^{\gamma-\gamma^2} - 1}{\left( \frac{T_c}{T_a} \right) - r^{\gamma-1}}.$$

**(b)**  $e = 1 - \frac{1}{1.4} \left[ \frac{(5.002)r^{-0.56} - 1}{(3.167) - r^{0.40}} \right]$ , where  $\frac{T_c}{T_a} = 3.167$  and  $\gamma = 1.40$  have been used. Substitution of  $r = 21.0$  yields  $e = 0.708 = 70.8\%$ .

**EVALUATE:** The efficiency for an Otto cycle with  $r = 21.0$  and  $\gamma = 1.40$  is

$e = 1 - r^{1-\gamma} = 1 - (21.0)^{-0.40} = 70.4\%$ . This efficiency is very close to the value for the Diesel cycle.

**20.59. IDENTIFY and SET UP:** The Carnot efficiency is  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ . Solve for  $T_C$  to get the temperature for the desired efficiency. Then use the graph to find the depth at which the water is at that temperature.

**EXECUTE:** Solving  $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$  for  $T_C$  gives  $T_C = T_H(1 - e) = (300 \text{ K})(1 - 0.065) = 280.5 \text{ K} = 7.5^\circ\text{C}$ .

From the graph, we see that this temperature occurs at a depth of about 400 m, which is choice (b).

**EVALUATE:** This depth is over 1200 ft, so deep water is essential for such a power plant.

**20.60. IDENTIFY and SET UP:** This power plant produces energy (i.e., does work) at a rate of 10 MW and is 6.5% efficient. This means that the 10 MW is 6.5% of the heat input,  $Q_H$ , per second.

**EXECUTE:**  $W/t = 0.065 Q_H/t$ , so  $Q_H/t = (10 \text{ MW})/(0.065)$ . The entropy change per second is  $[Q/T_H]/t = [(10 \text{ MW})/(0.065)]/(300 \text{ K}) = 5.1 \times 10^5 \text{ J/K}$ , which is choice (b).

**EVALUATE:** The entropy increases because the ammonia gets more disordered as it vaporizes, so our answer is plausible.

**20.61. IDENTIFY and SET UP:** Both the warm and cold reservoirs are so large that they do not change temperature as heat is added or lost, so  $\Delta S = Q/T$ . For a Carnot engine,  $|Q_C/Q_H| = |T_C/T_H|$ .

**EXECUTE:**  $\Delta S_{\text{warm}} = Q_H/T_H$  and  $\Delta S_{\text{cold}} = Q_C/T_C$ . But for a Carnot engine,  $|Q_H/T_H| = |Q_C/T_C|$ . Therefore  $\Delta S_{\text{warm}} = -\Delta S_{\text{cold}}$ , which is choice (d).

**EVALUATE:** Our result means that the net entropy change for a cycle is zero. An ideal Carnot engine is reversible, so the entropy change in a cycle is zero, which agrees with our result.



**20.62. IDENTIFY and SET UP:** For a cycle,  $Q_H = W + Q_C$  and  $e = W/Q_H$ .

**EXECUTE:** For this engine,  $W/t = 10$  MW and  $Q_C/t = 165$  MW, so  $Q_H/t = W/t + Q_C/t = 175$  W. The efficiency is  $e = (W/t)/(Q_H/t) = (10 \text{ MW})/(175 \text{ MW}) = 0.057 = 5.7\%$ , choice (a).

**EVALUATE:** An actual heat engine is always less efficient than the theoretical limit due to friction as well as internal effects in the gas which prevent it from behaving exactly like an ideal gas.