6

WORK AND KINETIC ENERGY

6.1. **IDENTIFY** and **SET UP:** For parts (a) through (d), identify the appropriate value of ϕ and use the relation $W = F_P s = (F\cos\phi)s$. In part (e), apply the relation $W_{net} = W_{student} + W_{grav} + W_n + W_f$.

EXECUTE: **(a)** Since you are applying a horizontal force, $\phi = 0°$. Thus,

$W_{student} = (2.40\text{ N})(\cos 0°)(1.50\text{ m}) = 3.60\text{ J}$.

(b) The friction force acts in the horizontal direction, opposite to the motion, so $\phi = 180°$.

$W_f = (F_f \cos\phi)s = (0.600\text{ N})(\cos 180°)(1.50\text{ m}) = -0.900\text{ J}$.

(c) Since the normal force acts upward and perpendicular to the tabletop, $\phi = 90°$.

$W_n = (n\cos\phi)s = (ns)(\cos 90°) = 0.0\text{ J}$.

(d) Since gravity acts downward and perpendicular to the tabletop, $\phi = 270°$.

$W_{grav} = (mg\cos\phi)s = (mgs)(\cos 270°) = 0.0\text{ J}$.

(e) $W_{net} = W_{student} + W_{grav} + W_n + W_f = 3.60\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 0.900\text{ J} = 2.70\text{ J}$.

EVALUATE: Whenever a force acts perpendicular to the direction of motion, its contribution to the net work is zero.

6.2. **IDENTIFY:** In each case the forces are constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: In part **(a)**, when the cable pulls horizontally $\phi = 0°$ and when it pulls at $35.0°$ above the horizontal $\phi = 35.0°$. In part **(b)**, if the cable pulls horizontally $\phi = 180°$. If the cable pulls on the car at $35.0°$ above the horizontal it pulls on the truck at $35.0°$ below the horizontal and ϕ $145.0°$. For the gravity force $\phi = 90°$, since the force is vertical and the displacement is horizontal.

EXECUTE: **(a)** When the cable is horizontal, $W = (1350\text{ N})(5.00\times10^3\text{ m})\cos 0° = 6.75\times10^6\text{ J}$. When the cable is $35.0°$ above the horizontal, $W = (1350\text{ N})(5.00\times10^3\text{ m})\cos 35.0° = 5.53\times10^6\text{ J}$.

(b) $\cos 180° = -\cos 0°$ and $\cos 145.0° = -\cos 35.0°$, so the answers are $-6.75\times10^6\text{ J}$ and $-5.53\times10^6\text{ J}$.

(c) Since $\cos\phi = \cos 90° = 0$, $W = 0$ in both cases.

EVALUATE: If the car and truck are taken together as the system, the tension in the cable does no net work.

6.3. **IDENTIFY:** Each force can be used in the relation $W = F_{\|}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{net} = W_{worker} + W_{grav} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{worker} = f_k = \mu_k n$.

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{worker} = f_k = \mu_k n = \mu_k mg = (0.25)(30.0\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}$$

(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{worker} = (F_{worker}\cos\phi)s = [(74\text{ N})(\cos 0°)](4.5\text{ m}) = +333\text{ J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k \cos\phi)s = [(74 \text{ N})(\cos 180°)](4.5 \text{ m}) = -333 \text{ J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{grav} = W_n = 0.0$ J.

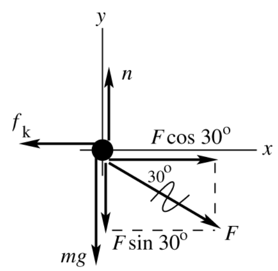
(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{net} = W_{worker} + W_{grav} + W_n + W_f = +333 \text{ J} + 0.0 \text{ J} + 0.0 \text{ J} - 333 \text{ J} = 0.0 \text{ J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6.4. **IDENTIFY:** The forces are constant so Eq. (6.2) can be used to calculate the work. Constant speed implies $a = 0$. We must use $\Sigma\vec{F} = m\vec{a}$ applied to the crate to find the forces acting on it.

(a) SET UP: The free-body diagram for the crate is given in Figure 6.4.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg - F\sin 30° = 0$

$n = mg + F\sin 30°$

$f_k = \mu_k n = \mu_k mg + F\mu_k \sin 30°$

Figure 6.4

$\Sigma F_x = ma_x$

$F\cos 30° - f_k = 0$

$F\cos 30° - \mu_k mg - \mu_k \sin 30° F = 0$

$$F = \frac{\mu_k mg}{\cos 30° - \mu_k \sin 30°} = \frac{0.25(30.0 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2 \text{ N}$$

(b) $W_F = (F\cos\phi)s = (99.2 \text{ N})(\cos 30°)(4.5 \text{ m}) = 387$ J

($F\cos 30°$ is the horizontal component of $\vec{F}$; the work done by $\vec{F}$ is the displacement times the component of $\vec{F}$ in the direction of the displacement.)

(c) We have an expression for f_k from part (a):

$$f_k = \mu_k(mg + F\sin 30°) = (0.250)[(30.0 \text{ kg})(9.80 \text{ m/s}^2) + (99.2 \text{ N})(\sin 30°)] = 85.9 \text{ N}$$

$\phi = 180°$ since f_k is opposite to the displacement. Thus $W_f = (f_k \cos\phi)s = (85.9 \text{ N})(\cos 180°)(4.5 \text{ m}) = -387$ J.

(d) The normal force is perpendicular to the displacement so $\phi = 90°$ and $W_n = 0$. The gravity force (the weight) is perpendicular to the displacement so $\phi = 90°$ and $W_w = 0$.

(e) $W_{tot} = W_F + W_f + W_n + W_w = +387 \text{ J} + (-387 \text{ J}) = 0$

EVALUATE: Forces with a component in the direction of the displacement do positive work, forces opposite to the displacement do negative work, and forces perpendicular to the displacement do zero work. The total work, obtained as the sum of the work done by each force, equals the work done by the net force. In this problem, $F_{net} = 0$ since $a = 0$ and $W_{tot} = 0$, which agrees with the sum calculated in part (e).

6.5. **IDENTIFY:** The gravity force is constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

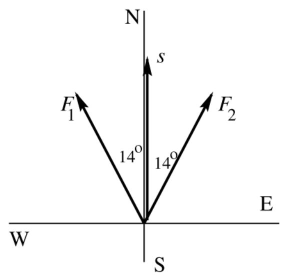
SET UP: The displacement is upward along the ladder and the gravity force is downward, so $\phi = 180.0° - 30.0° = 150.0°$. $w = mg = 735$ N.

EXECUTE: **(a)** $W = (735 \text{ N})(2.75 \text{ m})\cos 150.0° = -1750$ J.

(b) No, the gravity force is independent of the motion of the painter.

EVALUATE: Gravity is downward and the vertical component of the displacement is upward, so the gravity force does negative work.

6.6. **IDENTIFY** and **SET UP:** $W_F = (F\cos\phi)s$, since the forces are constant. We can calculate the total work by summing the work done by each force. The forces are sketched in Figure 6.6.



EXECUTE: $W_1 = F_1 s\cos\phi_1$

$W_1 = (1.80\times10^6\ \text{N})(0.75\times10^3\ \text{m})\cos14°$

$W_1 = 1.31\times10^9\ \text{J}$

$W_2 = F_2 s\cos\phi_2 = W_1$

Figure 6.6

$W_{\text{tot}} = W_1 + W_2 = 2(1.31\times10^9\ \text{J}) = 2.62\times10^9\ \text{J}$

EVALUATE: Only the component $F\cos\phi$ of force in the direction of the displacement does work. These components are in the direction of $\vec{s}$ so the forces do positive work.

6.7. **IDENTIFY:** All forces are constant and each block moves in a straight line, so $W = Fs\cos\phi$. The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

SET UP: Since the 12.0 N block moves at constant speed, $a = 0$ for it and the tension T in the string is $T = 12.0$ N. Since the 20.0 N block moves to the right at constant speed, the friction force f_k on it is to the left and $f_k = T = 12.0$ N.

EXECUTE: **(a)** (i) $\phi = 0°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos0° = 9.00$ J. (ii) $\phi = 180°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos180° = -9.00$ J.

(b) (i) $\phi = 90°$ and $W = 0$. (ii) $\phi = 0°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos0° = 9.00$ J. (iii) $\phi = 180°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos180° = -9.00$ J. (iv) $\phi = 90°$ and $W = 0$.

(c) $W_{\text{tot}} = 0$ for each block.

EVALUATE: For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

6.8. **IDENTIFY:** Apply Eq. (6.5).

SET UP: $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{j}} = 1$ and $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{j}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{i}} = 0$

EXECUTE: The work you do is $\vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{s}} = \left[(30\ \text{N})\hat{\boldsymbol{i}} - (40\ \text{N})\hat{\boldsymbol{j}}\right]\cdot\left[(-9.0\ \text{m})\hat{\boldsymbol{i}} - (3.0\ \text{m})\hat{\boldsymbol{j}}\right]$

$\vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{s}} = (30\ \text{N})(-9.0\ \text{m}) + (-40\ \text{N})(-3.0\ \text{m}) = -270\ \text{N}\cdot\text{m} + 120\ \text{N}\cdot\text{m} = -150$ J.

EVALUATE: The x-component of $\vec{\boldsymbol{F}}$ does negative work and the y-component of $\vec{\boldsymbol{F}}$ does positive work. The total work done by $\vec{\boldsymbol{F}}$ is the sum of the work done by each of its components.

6.9. **IDENTIFY:** Apply Eq. (6.2) or (6.3).

SET UP: The gravity force is in the $-y$-direction, so $\vec{\boldsymbol{F}}_{mg}\cdot\vec{\boldsymbol{s}} = -mg(y_2 - y_1)$

EXECUTE: **(a)** (i) Tension force is always perpendicular to the displacement and does no work.
(ii) Work done by gravity is $-mg(y_2 - y_1)$. When $y_1 = y_2$, $W_{mg} = 0$.

(b) (i) Tension does no work. (ii) Let l be the length of the string. $W_{mg} = -mg(y_2 - y_1) = -mg(2l) = -25.1$ J

EVALUATE: In part (b) the displacement is upward and the gravity force is downward, so the gravity force does negative work.

6.10. **IDENTIFY** and **SET UP:** Use $W = F_p s = (F\cos\phi)s$ to calculate the work done in each of parts (a) through (c). In part (d), the net work consists of the contributions due to all three forces, or $w_{net} = w_{grav} + w_n + w_f$.

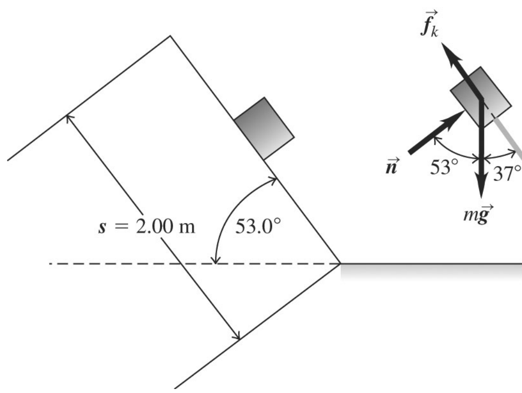


Figure 6.10

EXECUTE: **(a)** As the package slides, work is done by the frictional force which acts at $\phi = 180°$ to the displacement. The normal force is $mg\cos 53.0°$. Thus for $\mu_k = 0.40$,

$W_f = F_p s = (f_k \cos\phi)s = (\mu_k n \cos\phi)s = [\mu_k(mg\cos 53.0°)](\cos 180°)s.$

$W_f = (0.40)[(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 53.0°)](\cos 180°)(2.00\text{ m}) = -57\text{ J}.$

(b) Work is done by the component of the gravitational force parallel to the displacement. $\phi = 90° - 53° = 37°$ and the work of gravity is

$W_{grav} = (mg\cos\phi)s = [(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 37.0°)](2.00\text{ m}) = +188\text{ J}.$

(c) $W_n = 0$ since the normal force is perpendicular to the displacement.

(d) The net work done on the package is $W_{net} = W_{grav} + W_n + W_f = 188\text{ J} + 0.0\text{ J} - 57\text{ J} = 131\text{ J}.$

EVALUATE: The net work is positive because gravity does more positive work than the magnitude of the negative work done by friction.

6.11. **IDENTIFY:** As the carton is pulled up the ramp, the forces acting on it are gravity, the tension in the rope, and the normal force. Each of these forces may do work on the carton.

SET UP: Use $W = F_{\parallel} s = (F\cos\phi)s$. Calculate the work done by each force. In each case, identify the angle ϕ. In part (d), the net work is the algebraic sum of the work done by each force.

EXECUTE: **(a)** Since the force exerted by the rope and the displacement are in the same direction, $\phi = 0°$ and $W_{rope} = (72.0\text{ N})(\cos 0°)(5.20\text{ m}) = +374\text{ J}.$

(b) Gravity is downward and the displacement is at $30.0°$ above the horizontal, so $\phi = 90.0° + 30.0° = 120.0°$. $W_{grav} = (128.0\text{ N})(\cos 120°)(5.20\text{ m}) = -333\text{ J}.$

(c) The normal force n is perpendicular to the surface of the ramp while the displacement is parallel to the surface of the ramp, so $\phi = 90°$ and $W_n = 0$.

(d) $W_{net} = W_{rope} + W_{grav} + W_n = +374\text{ J} - 333\text{ J} + 0 = +41\text{ J}$

(e) Now $\phi = 50.0° - 30.0° = 20.0°$ and $W_{rope} = (72.0\text{ N})(\cos 20.0°)(5.20\text{ m}) = +352\text{ J}$

EVALUATE: In part (b), gravity does negative work since the gravity force acts downward and the carton moves upward. Less work is done by the rope in part (e), but the net work is still positive.

6.12. **IDENTIFY:** Since the speed is constant, the acceleration and the net force on the monitor are zero.

SET UP: Use the fact that the net force on the monitor is zero to develop expressions for the friction force, f_k, and the normal force, n. Then use $W = F_p s = (F\cos\phi)s$ to calculate W.

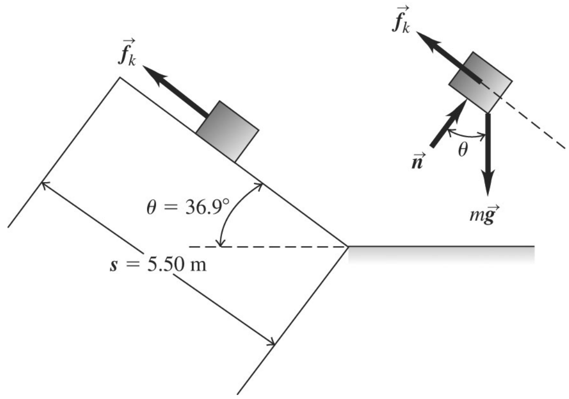


Figure 6.12

EXECUTE: **(a)** Summing forces along the incline, $\Sigma F = ma = 0 = f_k - mg\sin\theta$, giving $f_k = mg\sin\theta$, directed up the incline. Substituting gives $W_f = (f_k \cos\phi)s = [(mg\sin\theta)\cos\phi]s$.

$W_f = [(10.0 \text{ kg})(9.80 \text{ m/s}^2)(\sin 36.9°)](\cos 0°)(5.50 \text{ m}) = +324 \text{ J}.$

(b) The gravity force is downward and the displacement is directed up the incline so $\phi = 126.9°$.

$W_{grav} = (10.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 126.9°)(5.50 \text{ m}) = -324 \text{ J}.$

(c) The normal force, n, is perpendicular to the displacement and thus does zero work.

EVALUATE: Friction does positive work and gravity does negative work. The net work done is zero.

6.13. **IDENTIFY:** We want the work done by a known force acting through a known displacement.

SET UP: $W = Fs\cos\phi$

EXECUTE: $W = (48.0 \text{ N})(12.0 \text{ m})\cos(173°) = -572 \text{ J}.$

EVALUATE: The force has a component opposite to the displacement, so it does negative work.

6.14. **IDENTIFY:** We want to find the work done by a known force acting through a known displacement.

SET UP: $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$. We know the components of $\vec{F}$ but need to find the components of the displacement $\vec{s}$.

EXECUTE: Using the magnitude and direction of $\vec{s}$, its components are $x = (48.0 \text{ m})\cos 240.0^{\circ} = -24.0$ m and $y = (48.0 \text{ m})\sin 240.0^{\circ} = -41.57$ m. Therefore, $\vec{s} = (-24.0 \text{ m})\hat{i} + (-41.57 \text{ m})\hat{j}$. The definition of work gives $W = \vec{F}\cdot\vec{s} = (-68.0 \text{ N})(-24.0 \text{ m}) + (36.0 \text{ N})(-41.57 \text{ m}) = +1632 \text{ J} - 1497 \text{ J} = +135 \text{ J}.$

EVALUATE: The mass of the car is not needed since it is the given force that is doing the work.

6.15. **IDENTIFY:** We want the work done by the force, and we know the force and the displacement in terms of their components.

SET UP: We can use either $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$ or $W = Fs\cos\phi$, depending on what we know.

EXECUTE: (a) We know the magnitudes of the two given vectors and the angle between them, so $W = Fs\cos\phi = (30.0 \text{ N})(5.00 \text{ m})(\cos 37°) = 120 \text{ J}.$

(b) As in (a), we have $W = Fs\cos\phi = (30.0 \text{ N})(6.00 \text{ m})(\cos 127°) = -108 \text{ J}.$

(c) We know the components of both vectors, so we use $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$.

$W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y = (30.0 \text{ N})(\cos 37°)(-2.00 \text{ m}) + (30.00 \text{ N})(\sin 37°)(4.00 \text{ m}) = 24.3 \text{ J}.$

EVALUATE: We could check parts (a) and (b) using the method from part (c).

6.16. **IDENTIFY:** The book changes its speed and hence its kinetic energy, so work must have been done on it.

SET UP: Use the work-kinetic energy theorem $W_{net} = K_f - K_i$, with $K = \frac{1}{2}mv^2$. In part (a) use K_i and K_f to calculate W. In parts (b) and (c) use K_i and W to calculate K_f.

EXECUTE: **(a)** Substituting the notation $i = A$ and $f = B$,

$W_{net} = K_B - K_A = \frac{1}{2}(1.50 \text{ kg})[(1.25 \text{ m/s})^2 - (3.21 \text{ m/s})^2] = -6.56 \text{ J}.$

(b) Noting $\text{i} = B$ and $\text{f} = C$, $K_C = K_B + W_{\text{net}} = \frac{1}{2}(1.50 \text{ kg})(1.25 \text{ m/s})^2 - 0.750 \text{ J} = +0.422 \text{ J}$. $K_C = \frac{1}{2}mv_C^2$ so $v_C = \sqrt{2K_C/m} = 0.750 \text{ m/s}$.

(c) Similarly, $K_C = \frac{1}{2}(1.50 \text{ kg})(1.25 \text{ m/s})^2 + 0.750 \text{ J} = 1.922 \text{ J}$ and $v_C = 1.60 \text{ m/s}$.

EVALUATE: Negative W_{net} corresponds to a decrease in kinetic energy (slowing down) and positive W_{net} corresponds to an increase in kinetic energy (speeding up).

6.17. **IDENTIFY:** Find the kinetic energy of the cheetah knowing its mass and speed.

SET UP: Use $K = \frac{1}{2}mv^2$ to relate v and K.

EXECUTE: **(a)** $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(70 \text{ kg})(32 \text{ m/s})^2 = 3.6 \times 10^4 \text{ J}$.

(b) K is proportional to v^2, so K increases by a factor of 4 when v doubles.

EVALUATE: A running person, even with a mass of 70 kg, would have only 1/100 of the cheetah's kinetic energy since a person's top speed is only about 1/10 that of the cheetah.

6.18. **IDENTIFY:** Use the equations for free-fall to find the speed of the weight when it reaches the ground and use the formula for kinetic energy.

SET UP: Kinetic energy is $K = \frac{1}{2}mv^2$. The mass of an electron is 9.11×10^{-31} kg. In part (b) take $+y$ downward, so $a_y = +9.80 \text{ m/s}^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.

EXECUTE: **(a)** $K = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(2.19 \times 10^6 \text{ m/s})^2 = 2.18 \times 10^{-18} \text{ J}$.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(9.80 \text{ m/s}^2)(1.0 \text{m})} = 4.43 \text{ m/s}$. $K = \frac{1}{2}(1.0 \text{ kg})(4.43 \text{ m/s})^2 = 9.8 \text{ J}$.

(c) Solving $K = \frac{1}{2}mv^2$ for v gives $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(100 \text{ J})}{30 \text{ kg}}} = 2.6 \text{ m/s}$. Yes, this is reasonable.

EVALUATE: A running speed of 6 m/s corresponds to running a 100-m dash in about 17 s, so 2.6 m/s is reasonable for a running child.

6.19. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Since the meteor comes to rest the energy it delivers to the ground equals its original kinetic energy.

SET UP: $v = 12 \text{ km/s} = 1.2 \times 10^4 \text{ m/s}$. A 1.0 megaton bomb releases 4.184×10^{15} J of energy.

EXECUTE: **(a)** $K = \frac{1}{2}(1.4 \times 10^8 \text{ kg})(1.2 \times 10^4 \text{ m/s})^2 = 1.0 \times 10^{16} \text{ J}$.

(b) $\dfrac{1.0 \times 10^{16} \text{ J}}{4.184 \times 10^{15} \text{ J}} = 2.4$. The energy is equivalent to 2.4 one-megaton bombs.

EVALUATE: Part of the energy transferred to the ground lifts soil and rocks into the air and creates a large crater.

6.20. **IDENTIFY:** Only gravity does work on the watermelon, so $W_{\text{tot}} = W_{\text{grav}}$. $W_{\text{tot}} = \Delta K$ and $K = \frac{1}{2}mv^2$.

SET UP: Since the watermelon is dropped from rest, $K_1 = 0$.

EXECUTE: **(a)** $W_{\text{grav}} = mgs = (4.80 \text{ kg})(9.80 \text{ m/s}^2)(18.0 \text{ m}) = 847 \text{ J}$.

(b) (i) $W_{\text{tot}} = K_2 - K_1$ so $K_2 = 847$ J. **(ii)** $v = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(847 \text{ J})}{4.80 \text{ kg}}} = 18.8 \text{ m/s}$.

(c) The work done by gravity would be the same. Air resistance would do negative work and W_{tot} would be less than W_{grav}. The answer in (a) would be unchanged and both answers in (b) would decrease.

EVALUATE: The gravity force is downward and the displacement is downward, so gravity does positive work.

6.21. **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$. In each case calculate W_{tot} from what we know about the force and the displacement.

SET UP: The gravity force is mg, downward. The mass of the object isn't given, so we expect that it will divide out in the calculation.

EXECUTE: **(a)** $K_1 = 0$. $W_{tot} = W_{grav} = mgs$. $mgs = \frac{1}{2}mv_2^2$ and

$v_2 = \sqrt{2gs} = \sqrt{2(9.80 \text{ m/s}^2)(95.0 \text{ m})} = 43.2$ m/s.

(b) $K_2 = 0$ (at the maximum height). $W_{tot} = W_{grav} = -mgs$. $-mgs = -\frac{1}{2}mv_1^2$ and

$v_1 = \sqrt{2gs} = \sqrt{2(9.80 \text{ m/s}^2)(525 \text{ m})} = 101$ m/s.

EVALUATE: In part (a), gravity does positive work and the speed increases. In part (b), gravity does negative work and the speed decreases.

6.22. **IDENTIFY:** $W_{tot} = K_2 - K_1$. In each case calculate W_{tot} from what we know about the force and the displacement.

SET UP: The gravity force is *mg*, downward. The friction force is $f_k = \mu_k n = \mu_k mg$ and is directed opposite to the displacement. The mass of the object isn't given, so we expect that it will divide out in the calculation.

EXECUTE: **(a)** $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{tot} = W_f = -\mu_k mgs$. $-\mu_k mgs = -\frac{1}{2}mv_1^2$.

$$s = \frac{v_1^2}{2\mu_k g} = \frac{(5.00 \text{ m/s})^2}{2(0.220)(9.80 \text{ m/s}^2)} = 5.80 \text{ m}.$$

(b) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = \frac{1}{2}mv_2^2$. $W_{tot} = W_f = -\mu_k mgs$. $K_2 = W_{tot} + K_1$. $\frac{1}{2}mv_2^2 = -\mu_k mgs + \frac{1}{2}mv_1^2$.

$v_2 = \sqrt{v_1^2 - 2\mu_k gs} = \sqrt{(5.00 \text{ m/s})^2 - 2(0.220)(9.80 \text{ m/s}^2)(2.90 \text{ m})} = 3.53$ m/s.

(c) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{grav} = -mgy_2$, where y_2 is the vertical height. $-mgy_2 = -\frac{1}{2}mv_1^2$ and

$$y_2 = \frac{v_1^2}{2g} = \frac{(12.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 7.35 \text{ m}.$$

EVALUATE: In parts (a) and (b), friction does negative work and the kinetic energy is reduced. In part (c), gravity does negative work and the speed decreases. The vertical height in part (c) is independent of the slope angle of the hill.

6.23. **IDENTIFY** and **SET UP:** Apply Eq. (6.6) to the box. Let point 1 be at the bottom of the incline and let point 2 be at the skier. Work is done by gravity and by friction. Solve for K_1 and from that obtain the required initial speed.

EXECUTE: $W_{tot} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2$, $K_2 = 0$

Work is done by gravity and friction, so $W_{tot} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs$. The normal force is $n = mg\cos\alpha$ and $s = h/\sin\alpha$, where *s* is the distance the box travels along the incline.

$W_f = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem gives $-mgh - \mu_k mgh/\tan\alpha = -\frac{1}{2}mv_0^2$.

Solving for v_0 then gives $v_0 = \sqrt{2gh(1 + \mu_k/\tan\alpha)}$.

EVALUATE: The result is independent of the mass of the box. As $\alpha \to 90°$, $h = s$ and $v_0 = \sqrt{2gh}$, the same as throwing the box straight up into the air. For $\alpha = 90°$ the normal force is zero so there is no friction.

6.24. **IDENTIFY:** From the work-energy relation, $W = W_{grav} = \Delta K_{rock}$.

SET UP: As the rock rises, the gravitational force, $F = mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, *s*, the work is negative. Let *h* be the vertical distance the rock travels.

EXECUTE: **(a)** Applying $W_{\text{grav}} = K_2 - K_1$ we obtain $-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. Dividing by m and solving for v_1, $v_1 = \sqrt{v_2^2 + 2gh}$. Substituting $h = 15.0$ m and $v_2 = 25.0$ m/s,

$$v_1 = \sqrt{(25.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(15.0 \text{ m})} = 30.3 \text{ m/s}$$

(b) Solve the same work-energy relation for h. At the maximum height $v_2 = 0$.

$$-mgh = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \text{ and } h = \frac{v_1^2 - v_2^2}{2g} = \frac{(30.3 \text{ m/s})^2 - (0.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 46.8 \text{ m.}$$

EVALUATE: Note that the weight of the rock was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, m. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

6.25. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

SET UP: $\phi = 0°$

EXECUTE: From Eqs. (6.1), (6.5) and (6.6), and solving for F,

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(12.0 \text{ kg})\left[(6.00 \text{ m/s})^2 - (4.00 \text{ m/s})^2\right]}{(2.50 \text{ m})} = 48.0 \text{ N}$$

EVALUATE: The force is in the direction of the displacement, so the force does positive work and the kinetic energy of the object increases.

6.26. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

SET UP: Parallel to incline: force component $W_{\parallel} = mg\sin\alpha$, down incline; displacement $s = h/\sin\alpha$, down incline. Perpendicular to the incline: $s = 0$.

EXECUTE: **(a)** $W_{\parallel} = (mg\sin\alpha)(h/\sin\alpha) = mgh$. $W_{\perp} = 0$, since there is no displacement in this direction.

$W_{mg} = W_{\parallel} + W_{\perp} = mgh$, same as falling height h.

(b) $W_{\text{tot}} = K_2 - K_1$ gives $mgh = \frac{1}{2}mv^2$ and $v = \sqrt{2gh}$, same as if had been dropped from height h. The work done by gravity depends only on the vertical displacement of the object. When the slope angle is small, there is a small force component in the direction of the displacement but a large displacement in this direction. When the slope angle is large, the force component in the direction of the displacement along the incline is larger but the displacement in this direction is smaller.

(c) $h = 15.0$ m, so $v = \sqrt{2gh} = 17.1$ s.

EVALUATE: The acceleration and time of travel are different for an object sliding down an incline and an object in free-fall, but the final velocity is the same in these two cases.

6.27. **IDENTIFY:** Apply $W_{\text{tot}} = \Delta K$.

SET UP: $v_1 = 0$, $v_2 = v$. $f_k = \mu_k mg$ and f_k does negative work. The force $F = 36.0$ N is in the direction of the motion and does positive work.

EXECUTE: **(a)** If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0 \text{ N})(1.20 \text{ m})}{(4.30 \text{ kg})}} = 4.48 \text{ m/s.}$$

(b) The net work is $Fs - f_k s = (F - \mu_k mg)s$, so

$$v = \sqrt{\frac{2(F - \mu_k mg)s}{m}} = \sqrt{\frac{2(36.0 \text{ N} - (0.30)(4.30 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m})}{(4.30 \text{ kg})}} = 3.61 \text{ m/s}$$

EVALUATE: The total work done is larger in the absence of friction and the final speed is larger in that case.

6.28. **IDENTIFY** and **SET UP:** Use Eq. (6.6) to calculate the work done by the foot on the ball. Then use Eq. (6.2) to find the distance over which this force acts.

EXECUTE: $W_{tot} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420 \text{ kg})(2.00 \text{ m/s})^2 = 0.84 \text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420 \text{ kg})(6.00 \text{ m/s})^2 = 7.56 \text{ J}$

$W_{tot} = K_2 - K_1 = 7.56 \text{ J} - 0.84 \text{ J} = 6.72 \text{ J}$

The 40.0 N force is the only force doing work on the ball, so it must do 6.72 J of work. $W_F = (F\cos\phi)s$ gives that $s = \dfrac{W}{F\cos\phi} = \dfrac{6.72 \text{ J}}{(40.0 \text{ N})(\cos 0)} = 0.168 \text{ m}.$

EVALUATE: The force is in the direction of the motion so positive work is done and this is consistent with an increase in kinetic energy.

6.29. **(a) IDENTIFY** and **SET UP:** Use $W_F = (F\cos\phi)s$ to find the work done by the force. Then use $W_{tot} = K_2 - K_1$ to find the final kinetic energy, and then $K_2 = \frac{1}{2}mv_2^2$ gives the final speed.

EXECUTE: $W_{tot} = K_2 - K_1$, so $K_2 = W_{tot} + K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(7.00 \text{ kg})(4.00 \text{ m/s})^2 = 56.0 \text{ J}$

The only force that does work on the wagon is the 10.0 N force. This force is in the direction of the displacement so $\phi = 0°$ and the force does positive work:

$$W_F = (F\cos\phi)s = (10.0 \text{ N})(\cos 0)(3.0 \text{ m}) = 30.0 \text{ J}$$

Then $K_2 = W_{tot} + K_1 = 30.0 \text{ J} + 56.0 \text{ J} = 86.0 \text{ J}.$

$$K_2 = \tfrac{1}{2}mv_2^2;\ v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(86.0 \text{ J})}{7.00 \text{ kg}}} = 4.96 \text{ m/s}$$

(b) IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the wagon to calculate a. Then use a constant acceleration equation to calculate the final speed. The free-body diagram is given in Figure 6.29.

SET UP:

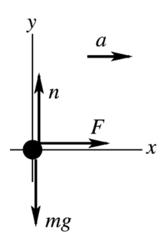


Figure 6.29

EXECUTE: $\Sigma F_x = ma_x$

$F = ma_x$

$a_x = \dfrac{F}{m} = \dfrac{10.0 \text{ N}}{7.00 \text{ kg}} = 1.43 \text{ m/s}^2$

$v_{2x}^2 = v_{1x}^2 + 2a_2(x - x_0)$

$v_{2x} = \sqrt{v_{1x}^2 + 2a_x(x - x_0)} = \sqrt{(4.00 \text{ m/s})^2 + 2(1.43 \text{ m/s}^2)(3.0 \text{ m})} = 4.96 \text{ m/s}$

EVALUATE: This agrees with the result calculated in part (a). The force in the direction of the motion does positive work and the kinetic energy and speed increase. In part (b), the equivalent statement is that the force produces an acceleration in the direction of the velocity and this causes the magnitude of the velocity to increase.

6.30. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$.

SET UP: $K_1 = 0$. The normal force does no work. The work W done by gravity is $W = mgh$, where $h = L\sin\theta$ is the vertical distance the block has dropped when it has traveled a distance L down the incline and θ is the angle the plane makes with the horizontal.

EXECUTE: The work-energy theorem gives $v=\sqrt{\dfrac{2K}{m}}=\sqrt{\dfrac{2W}{m}}=\sqrt{2gh}=\sqrt{2gL\sin\theta}$. Using the given numbers, $v=\sqrt{2(9.80\text{ m/s}^2)(1.35\text{ m})\sin 36.9°}=3.99\text{ m/s}$.

EVALUATE: The final speed of the block is the same as if it had been dropped from a height h.

6.31. IDENTIFY: $W_{\text{tot}}=K_2-K_1$. Only friction does work.

SET UP: $W_{\text{tot}}=W_{f_{\text{k}}}=-\mu_{\text{k}}mgs$. $K_2=0$ (car stops). $K_1=\frac{1}{2}mv_0^2$.

EXECUTE: **(a)** $W_{\text{tot}}=K_2-K_1$ gives $-\mu_{\text{k}}mgs=-\frac{1}{2}mv_0^2$. $s=\dfrac{v_0^2}{2\mu_{\text{k}}g}$.

(b) (i) $\mu_{\text{k}b}=2\mu_{\text{k}a}$. $s\mu_{\text{k}}=\dfrac{v_0^2}{2g}=$ constant so $s_a\mu_{\text{k}a}=s_b\mu_{\text{k}b}$. $s_b=\left(\dfrac{\mu_{\text{k}a}}{\mu_{\text{k}b}}\right)s_a=s_a/2$. The minimum stopping distance would be halved. (ii) $v_{0b}=2v_{0a}$. $\dfrac{s}{v_0^2}=\dfrac{1}{2\mu_{\text{k}}g}=$ constant, so $\dfrac{s_a}{v_{0a}^2}=\dfrac{s_b}{v_{0b}^2}$. $s_b=s_a\left(\dfrac{v_{0b}}{v_{0a}}\right)^2=4s_a$. The stopping distance would become 4 times as great. (iii) $v_{0b}=2v_{0a}$, $\mu_{\text{k}b}=2\mu_{\text{k}a}$. $\dfrac{s\mu_{\text{k}}}{v_0^2}=\dfrac{1}{2g}=$ constant, so $\dfrac{s_a\mu_{\text{k}a}}{v_{0a}^2}=\dfrac{s_b\mu_{\text{k}b}}{v_{0b}^2}$. $s_b=s_a\left(\dfrac{\mu_{\text{k}a}}{\mu_{\text{k}b}}\right)\left(\dfrac{v_{0b}}{v_{0a}}\right)^2=s_a\left(\dfrac{1}{2}\right)(2)^2=2s_a$. The stopping distance would double.

EVALUATE: The stopping distance is directly proportional to the square of the initial speed and indirectly proportional to the coefficient of kinetic friction.

6.32. IDENTIFY: We know (or can calculate) the change in the kinetic energy of the crate and want to find the work needed to cause this change, so the work-energy theorem applies.

SET UP: $W_{\text{tot}}=\Delta K=K_{\text{f}}-K_{\text{i}}=\frac{1}{2}mv_{\text{f}}^2-\frac{1}{2}mv_{\text{i}}^2$.

EXECUTE: $W_{\text{tot}}=K_{\text{f}}-K_{\text{i}}=\frac{1}{2}(30.0\text{ kg})(5.62\text{ m/s})^2-\frac{1}{2}(30.0\text{ kg})(3.90\text{ m/s})^2$.

$W_{\text{tot}}=473.8\text{ J}-228.2\text{ J}=246\text{ J}$.

EVALUATE: Kinetic energy is a scalar and does not depend on direction, so only the initial and final speeds are relevant.

6.33. IDENTIFY: The elastic aortal material behaves like a spring, so we can apply Hooke's law to it.

SET UP: $|F_{\text{spr}}|=F$, where F is the pull on the strip or the force the strip exerts, and $F=kx$.

EXECUTE: **(a)** Solving $F=kx$ for k gives $k=\dfrac{F}{x}=\dfrac{1.50\text{ N}}{0.0375\text{ m}}=40.0\text{ N/m}$.

(b) $F=kx=(40.0\text{ N/m})(0.0114\text{ m})=0.456\text{ N}$.

EVALUATE: It takes 0.40 N to stretch this material by 1.0 cm, so it is not as stiff as many laboratory springs.

6.34. IDENTIFY: The work that must be done to move the end of a spring from x_1 to x_2 is $W=\frac{1}{2}kx_2^2-\frac{1}{2}kx_1^2$. The force required to hold the end of the spring at displacement x is $F_x=kx$.

SET UP: When the spring is at its unstretched length, $x=0$. When the spring is stretched, $x>0$, and when the spring is compressed, $x<0$.

EXECUTE: **(a)** $x_1=0$ and $W=\frac{1}{2}kx_2^2$. $k=\dfrac{2W}{x_2^2}=\dfrac{2(12.0\text{ J})}{(0.0300\text{ m})^2}=2.67\times10^4\text{ N/m}$.

(b) $F_x=kx=(2.67\times10^4\text{ N/m})(0.0300\text{ m})=801\text{ N}$.

(c) $x_1=0$, $x_2=-0.0400\text{ m}$. $W=\frac{1}{2}(2.67\times10^4\text{ N/m})(-0.0400\text{ m})^2=21.4\text{ J}$.

$F_x=kx=(2.67\times10^4\text{ N/m})(0.0400\text{ m})=1070\text{ N}$.

EVALUATE: When a spring, initially unstretched, is either compressed or stretched, positive work is done by the force that moves the end of the spring.

6.35. **IDENTIFY:** The springs obey Hooke's law and balance the downward force of gravity.

SET UP: Use coordinates with $+y$ upward. Label the masses 1, 2, and 3, with 1 the top mass and 3 the bottom mass, and call the amounts the springs are stretched x_1, x_2, and x_3. Each spring force is kx.

EXECUTE: **(a)** The three free-body diagrams are shown in Figure 6.35.

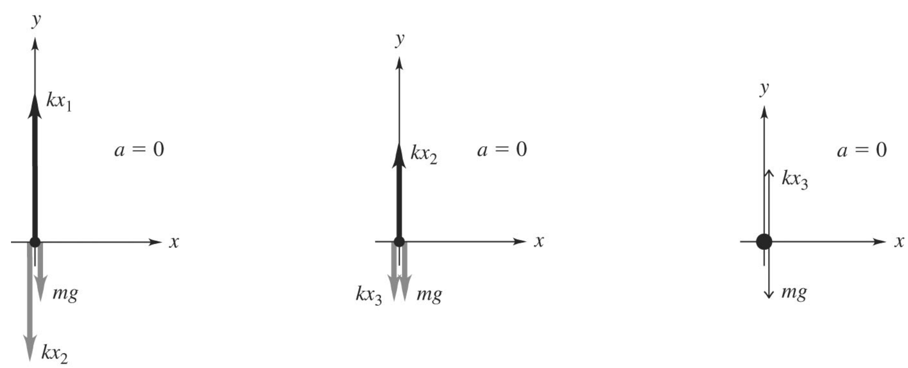


Figure 6.35

(b) Balancing forces on each of the masses and using $F = kx$ gives $kx_3 = mg$ so

$$x_3 = \frac{mg}{k} = \frac{(8.50\text{ kg})(9.80\text{ m/s}^2)}{7.80\times 10^3\text{ N/m}} = 1.068\text{ cm}.\quad kx_2 = mg + kx_3 = 2mg \text{ so } x_2 = 2\left(\frac{mg}{k}\right) = 2.136\text{ cm}.$$

$kx_1 = mg + kx_2 = 3mg$ so $x_3 = 3\left(\frac{mg}{k}\right) = 3.204$ cm. Adding the original lengths to the distance stretched, the lengths of the springs, starting from the bottom one, are 13.1 cm, 14.1 cm, and 15.2 cm.

EVALUATE: The top spring stretches most because it supports the most weight, while the bottom spring stretches least because it supports the least weight.

6.36. **IDENTIFY:** The magnitude of the work can be found by finding the area under the graph.

SET UP: The area under each triangle is $1/2$ base$\times$height. $F_x > 0$, so the work done is positive when x increases during the displacement.

EXECUTE: **(a)** $1/2\ (8\text{ m})(10\text{ N}) = 40\text{ J}$.

(b) $1/2\ (4\text{ m})(10\text{ N}) = 20\text{ J}$.

(c) $1/2\ (12\text{ m})(10\text{ N}) = 60\text{ J}$.

EVALUATE: The sum of the answers to parts (a) and (b) equals the answer to part (c).

6.37. **IDENTIFY:** Use the work-energy theorem and the results of Problem 6.36.

SET UP: For $x = 0$ to $x = 8.0$ m, $W_{\text{tot}} = 40$ J. For $x = 0$ to $x = 12.0$ m, $W_{\text{tot}} = 60$ J.

EXECUTE: **(a)** $v = \sqrt{\frac{(2)(40\text{ J})}{10\text{ kg}}} = 2.83\text{ m/s}$

(b) $v = \sqrt{\frac{(2)(60\text{ J})}{10\text{ kg}}} = 3.46\text{ m/s}$.

EVALUATE: $\vec{F}$ is always in the $+x$-direction. For this motion $\vec{F}$ does positive work and the speed continually increases during the motion.

6.38. **IDENTIFY:** The spring obeys Hooke's law.

SET UP: Solve $F = kx$ for x to determine the length of stretch and use $W = +\frac{1}{2}kx^2$ to assess the corresponding work.

EXECUTE: $x = \dfrac{F}{k} = \dfrac{15.0\text{ N}}{300.0\text{ N/m}} = 0.0500\text{ m}.$ The new length will be $0.240\text{ m} + 0.0500\text{ m} = 0.290\text{ m}.$

The corresponding work done is $W = \dfrac{1}{2}(300.0\text{ N/m})(0.0500\text{ m})^2 = 0.375\text{ J}.$

EVALUATE: In $F = kx$, F is always the force applied to one end of the spring, thus we did not need to double the 15.0 N force. Consider a free-body diagram of a spring at rest; forces of equal magnitude and opposite direction are always applied to both ends of every section of the spring examined.

6.39. **IDENTIFY:** Apply Eq. (6.6) to the box.

SET UP: Let point 1 be just before the box reaches the end of the spring and let point 2 be where the spring has maximum compression and the box has momentarily come to rest.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\ \ K_2 = 0$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx_2^2,$ where x_2 is the amount the spring is compressed.

$-\frac{1}{2}kx_2^2 = -\frac{1}{2}mv_0^2$ and $x_2 = v_0\sqrt{m/k} = (3.0\text{ m/s})\sqrt{(6.0\text{ kg})/(7500\text{ N/m})} = 8.5\text{ cm}$

EVALUATE: The compression of the spring increases when either v_0 or m increases and decreases when k increases (stiffer spring).

6.40. **IDENTIFY:** The force applied to the springs is $F_x = kx.$ The work done on a spring to move its end from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2.$ Use the information that is given to calculate k.

SET UP: When the springs are compressed 0.200 m from their uncompressed length, $x_1 = 0$ and $x_2 = -0.200\text{ m}.$ When the platform is moved 0.200 m farther, x_2 becomes $-0.400\text{ m}.$

EXECUTE: **(a)** $k = \dfrac{2W}{x_2^2 - x_1^2} = \dfrac{2(80.0\text{ J})}{(0.200\text{ m})^2 - 0} = 4000\text{ N/m}.$ $F_x = kx = (4000\text{ N/m})(-0.200\text{ m}) = -800\text{ N}.$

The magnitude of force that is required is 800 N.

(b) To compress the springs from $x_1 = 0$ to $x_2 = -0.400\text{ m},$ the work required is

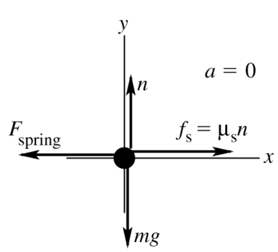
$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}(4000\text{ N/m})(-0.400\text{ m})^2 = 320\text{ J}.$ The additional work required is

$320\text{ J} - 80\text{ J} = 240\text{ J}.$ For $x = -0.400\text{ m},$ $F_x = kx = -1600\text{ N}.$ The magnitude of force required is 1600 N.

EVALUATE: More work is required to move the end of the spring from $x = -0.200\text{ m}$ to $x = -0.400\text{ m}$ than to move it from $x = 0$ to $x = -0.200\text{ m},$ even though the displacement of the platform is the same in each case. The magnitude of the force increases as the compression of the spring increases.

6.41. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to calculate the μ_s required for the static friction force to equal the spring force.

SET UP: **(a)** The free-body diagram for the glider is given in Figure 6.41.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s mg$

Figure 6.41

$$\Sigma F_x = ma_x$$
$$f_s - F_{spring} = 0$$
$$\mu_s mg - kd = 0$$
$$\mu_s = \frac{kd}{mg} = \frac{(20.0\text{ N/m})(0.086\text{ m})}{(0.100\text{ kg})(9.80\text{ m/s}^2)} = 1.76$$

(b) IDENTIFY and **SET UP:** Apply $\Sigma\vec{F} = m\vec{a}$ to find the maximum amount the spring can be compressed and still have the spring force balanced by friction. Then use $W_{tot} = K_2 - K_1$ to find the initial speed that results in this compression of the spring when the glider stops.

EXECUTE: $\mu_s mg = kd$

$$d = \frac{\mu_s mg}{k} = \frac{(0.60)(0.100\text{ kg})(9.80\text{ m/s}^2)}{20.0\text{ N/m}} = 0.0294\text{ m}$$

Now apply the work-energy theorem to the motion of the glider:

$$W_{tot} = K_2 - K_1$$
$$K_1 = \tfrac{1}{2}mv_1^2,\quad K_2 = 0\ \text{(instantaneously stops)}$$
$$W_{tot} = W_{spring} + W_{fric} = -\tfrac{1}{2}kd^2 - \mu_k mgd\ \text{(as in Example 6.7)}$$
$$W_{tot} = -\tfrac{1}{2}(20.0\text{ N/m})(0.0294\text{ m})^2 - 0.47(0.100\text{ kg})(9.80\text{ m/s}^2)(0.0294\text{ m}) = -0.02218\text{ J}$$

Then $W_{tot} = K_2 - K_1$ gives $-0.02218\text{ J} = -\tfrac{1}{2}mv_1^2.$

$$v_1 = \sqrt{\frac{2(0.02218\text{ J})}{0.100\text{ kg}}} = 0.67\text{ m/s}.$$

EVALUATE: In Example 6.7 an initial speed of 1.50 m/s compresses the spring 0.086 m and in part (a) of this problem we found that the glider doesn't stay at rest. In part (b) we found that a smaller displacement of 0.0294 m when the glider stops is required if it is to stay at rest. And we calculate a smaller initial speed (0.67 m/s) to produce this smaller displacement.

6.42. **IDENTIFY:** For the spring, $W = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$. Apply $W_{tot} = K_2 - K_1$.

SET UP: $x_1 = -0.025$ m and $x_2 = 0$.

EXECUTE: **(a)** $W = \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(200\text{ N/m})(-0.025\text{ m})^2 = 0.0625\text{ J}$, which rounds to 0.063 J.

(b) The work-energy theorem gives $v_2 = \sqrt{\dfrac{2W}{m}} = \sqrt{\dfrac{2(0.0625\text{ J})}{(4.0\text{ kg})}} = 0.18\text{ m/s}.$

EVALUATE: The block moves in the direction of the spring force, the spring does positive work and the kinetic energy of the block increases.

6.43. **IDENTIFY** and **SET UP:** The magnitude of the work done by F_x equals the area under the F_x versus x curve. The work is positive when F_x and the displacement are in the same direction; it is negative when they are in opposite directions.

EXECUTE: **(a)** F_x is positive and the displacement Δx is positive, so $W > 0$.

$$W = \tfrac{1}{2}(2.0\text{ N})(2.0\text{ m}) + (2.0\text{ N})(1.0\text{ m}) = +4.0\text{ J}$$

(b) During this displacement $F_x = 0$, so $W = 0$.

(c) F_x is negative, Δx is positive, so $W < 0$. $W = -\tfrac{1}{2}(1.0\text{ N})(2.0\text{ m}) = -1.0\text{ J}$

(d) The work is the sum of the answers to parts (a), (b), and (c), so $W = 4.0\text{ J} + 0 - 1.0\text{ J} = +3.0\text{ J}$.

(e) The work done for $x = 7.0$ m to $x = 3.0$ m is $+1.0$ J. This work is positive since the displacement and the force are both in the $-x$-direction. The magnitude of the work done for $x = 3.0$ m to $x = 2.0$ m is 2.0 J, the area under F_x versus x. This work is negative since the displacement is in the $-x$-direction and the force is in the $+x$-direction. Thus $W = +1.0\text{ J} - 2.0\text{ J} = -1.0\text{ J}$.

EVALUATE: The work done when the car moves from $x = 2.0\text{ m}$ to $x = 0$ is $-\frac{1}{2}(2.0\text{ N})(2.0\text{ m}) = -2.0\text{ J}.$ Adding this to the work for $x = 7.0\text{ m}$ to $x = 2.0\text{ m}$ gives a total of $W = -3.0\text{ J}$ for $x = 7.0\text{ m}$ to $x = 0.$ The work for $x = 7.0\text{ m}$ to $x = 0$ is the negative of the work for $x = 0$ to $x = 7.0\text{ m}.$

6.44. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1.$

SET UP: $K_1 = 0.$ From Exercise 6.43, the work for $x = 0$ to $x = 3.0\text{ m}$ is 4.0 J. W for $x = 0$ to $x = 4.0\text{ m}$ is also 4.0 J. For $x = 0$ to $x = 7.0\text{ m},$ $W = 3.0\text{ J}.$

EXECUTE: **(a)** $K = 4.0\text{ J},$ so $v = \sqrt{2K/m} = \sqrt{2(4.0\text{ J})/(2.0\text{ kg})} = 2.00\text{ m/s}.$

(b) No work is done between $x = 3.0\text{ m}$ and $x = 4.0\text{ m},$ so the speed is the same, 2.00 m/s.

(c) $K = 3.0\text{ J},$ so $v = \sqrt{2K/m} = \sqrt{2(3.0\text{ J})/(2.0\text{ kg})} = 1.73\text{ m/s}.$

EVALUATE: In each case the work done by F is positive and the car gains kinetic energy.

6.45. **IDENTIFY** and **SET UP:** Apply Eq. (6.6). Let point 1 be where the sled is released and point 2 be at $x = 0$ for part (a) and at $x = -0.200\text{ m}$ for part (b). Use Eq. (6.10) for the work done by the spring and calculate $K_2.$ Then $K_2 = \frac{1}{2}mv_2^2$ gives $v_2.$

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$ (released with no initial velocity), $K_2 = \frac{1}{2}mv_2^2$

The only force doing work is the spring force. Eq. (6.10) gives the work done *on* the spring to move its end from x_1 to $x_2.$ The force the spring exerts on an object attached to it is $F = -kx,$ so the work the spring does is

$W_{\text{spr}} = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2.$ Here $x_1 = -0.375\text{ m}$ and $x_2 = 0.$ Thus

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - 0 = 281\text{ J}.$

$K_2 = K_1 + W_{\text{tot}} = 0 + 281\text{ J} = 281\text{ J}.$

Then $K_2 = \frac{1}{2}mv_2^2$ implies $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(281\text{ J})}{70.0\text{ kg}}} = 2.83\text{ m/s}.$

(b) $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2.$ Now $x_2 = -0.200\text{ m},$ so

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - \frac{1}{2}(4000\text{ N/m})(-0.200\text{ m})^2 = 281\text{ J} - 80\text{ J} = 201\text{ J}$

Thus $K_2 = 0 + 201\text{ J} = 201\text{ J}$ and $K_2 = \frac{1}{2}mv_2^2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(201\text{ J})}{70.0\text{ kg}}} = 2.40\text{ m/s}.$

EVALUATE: The spring does positive work and the sled gains speed as it returns to $x = 0.$ More work is done during the larger displacement in part (a), so the speed there is larger than in part (b).

6.46. **IDENTIFY:** $F_x = kx$

SET UP: When the spring is in equilibrium, the same force is applied to both ends of any segment of the spring.

EXECUTE: **(a)** When a force F is applied to each end of the original spring, the end of the spring is displaced a distance x. Each half of the spring elongates a distance $x_{\text{h}},$ where $x_{\text{h}} = x/2.$ Since F is also the force applied to each half of the spring, $F = kx$ and $F = k_{\text{h}}x_{\text{h}}.$ $kx = k_{\text{h}}x_{\text{h}}$ and $k_{\text{h}} = k\left(\dfrac{x}{x_{\text{h}}}\right) = 2k.$

(b) The same reasoning as in part (a) gives $k_{\text{seg}} = 3k,$ where k_{seg} is the force constant of each segment.

EVALUATE: For half of the spring the same force produces less displacement than for the original spring. Since $k = F/x,$ smaller x for the same F means larger k.

6.47. **IDENTIFY** and **SET UP:** Apply Eq. (6.6) to the glider. Work is done by the spring and by gravity. Take point 1 to be where the glider is released. In part (a) point 2 is where the glider has traveled 1.80 m and $K_2 = 0$. There are two points shown in Figure 6.47a. In part (b) point 2 is where the glider has traveled 0.80 m.

EXECUTE: (a) $W_{\text{tot}} = K_2 - K_1 = 0$. Solve for x_1, the amount the spring is initially compressed.

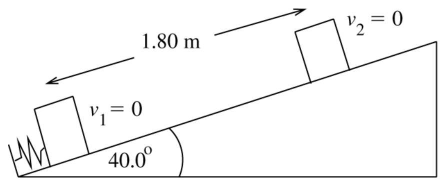


Figure 6.47a

$W_{\text{tot}} = W_{\text{spr}} + W_w = 0$

So $W_{\text{spr}} = -W_w$

(The spring does positive work on the glider since the spring force is directed up the incline, the same as the direction of the displacement.)

The directions of the displacement and of the gravity force are shown in Figure 6.47b.

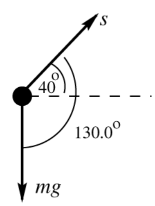


Figure 6.47b

$W_w = (w\cos\phi)s = (mg\cos 130.0°)s$

$W_w = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(1.80\text{ m}) = -1.020\text{ J}$

(The component of w parallel to the incline is directed down the incline, opposite to the displacement, so gravity does negative work.)

$W_{\text{spr}} = -W_w = +1.020\text{ J}$

$W_{\text{spr}} = \frac{1}{2}kx_1^2$ so $x_1 = \sqrt{\dfrac{2W_{\text{spr}}}{k}} = \sqrt{\dfrac{2(1.020\text{ J})}{640\text{ N/m}}} = 0.0565\text{ m}$

(b) The spring was compressed only 0.0565 m so at this point in the motion the glider is no longer in contact with the spring. Points 1 and 2 are shown in Figure 6.47c.

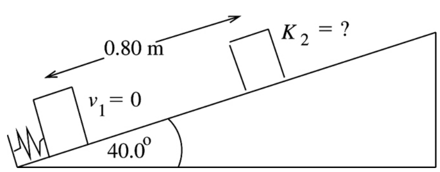


Figure 6.47c

$W_{\text{tot}} = K_2 - K_1$

$K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} + W_w$

From part (a), $W_{\text{spr}} = 1.020\text{ J}$ and

$W_w = (mg\cos 130.0°)s = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(0.80\text{ m}) = -0.454\text{ J}$

Then $K_2 = W_{\text{spr}} + W_w = +1.020\text{ J} - 0.454\text{ J} = +0.57\text{ J}$.

EVALUATE: The kinetic energy in part (b) is positive, as it must be. In part (a), $x_2 = 0$ since the spring force is no longer applied past this point. In computing the work done by gravity we use the full 0.80 m the glider moves.

6.48. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the brick. Work is done by the spring force and by gravity.

SET UP: At the maximum height, $v = 0$. Gravity does negative work, $W_{\text{grav}} = -mgh$. The work done by the spring is $\frac{1}{2}kd^2$, where d is the distance the spring is compressed initially.

EXECUTE: The initial and final kinetic energies of the brick are both zero, so the net work done on the brick by the spring and gravity is zero, so $(1/2)kd^2 - mgh = 0$, or $d = \sqrt{2mgh/k} = \sqrt{2(1.80\text{ kg})(9.80\text{ m/s}^2)(3.6\text{ m})/(450\text{ N/m})} = 0.53\text{ m}$. The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length. But when the spring reaches its uncompressed length the brick has an upward velocity and leaves the spring.

EVALUATE: Gravity does negative work because the gravity force is downward and the brick moves upward. The spring force does positive work on the brick because the spring force is upward and the brick moves upward.

6.49. **IDENTIFY:** The force does work on the box, which gives it kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the box.

SET UP: $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$ and $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx$.

EXECUTE: $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx = \int_0^{14.0\text{m}} [18.0\text{ N} - (0.530\text{ N/m})x]dx$

$W_{\text{tot}} = (18.0\text{ N})(14.0\text{ m}) - (0.265\text{ N/m})(14.0\text{ m})^2 = 252.0\text{ J} - 51.94\text{ J} = 200.1\text{ J}$. The initial kinetic energy is zero, so $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2$. Solving for v_{f} gives $v_{\text{f}} = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(200.1\text{ J})}{6.00\text{ kg}}} = 8.17\text{ m/s}$.

EVALUATE: We could not readily do this problem by integrating the acceleration over time because we know the force as a function of x, not of t. The work-energy theorem provides a much simpler method.

6.50. **IDENTIFY:** The force acts through a distance over time, so it does work on the crate and hence supplies power to it. The force exerted by the worker is variable but the acceleration of the cart is constant.

SET UP: Use $P = Fv$ to find the power, and we can use $v = v_0 + at$ to find the instantaneous velocity.

EXECUTE: First find the instantaneous force and velocity: $F = (5.40\text{ N/s})(5.00\text{ s}) = 27.0\text{ N}$ and $v = v_0 + at = (2.80\text{ m/s}^2)(5.00\text{ s}) = 14.0\text{ m/s}$. Now find the power: $P = (27.0\text{ N})(14.0\text{ m/s}) = 378\text{ W}$.

EVALUATE: The instantaneous power will increase as the worker pushes harder and harder.

6.51. **IDENTIFY:** Apply the relation between energy and power.

SET UP: Use $P = \dfrac{W}{\Delta t}$ to solve for W, the energy the bulb uses. Then set this value equal to $\frac{1}{2}mv^2$ and solve for the speed.

EXECUTE: $W = P\Delta t = (100\text{ W})(3600\text{ s}) = 3.6\times 10^5\text{ J}$

$K = 3.6\times 10^5\text{ J}$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.6\times 10^5\text{ J})}{70\text{ kg}}} = 100\text{ m/s}$

EVALUATE: Olympic runners achieve speeds up to approximately 10 m/s, or roughly one-tenth the result calculated.

6.52. **IDENTIFY:** Knowing the rate at which energy is consumed, we want to find out the total energy used.

SET UP: Find the elapsed time Δt in each case by dividing the distance by the speed, $\Delta t = d/v$. Then calculate the energy as $W = P\Delta t$.

EXECUTE: *Running:* $\Delta t = (5.0\text{ km})/(10\text{ km/h}) = 0.50\text{ h} = 1.8\times 10^3\text{ s}$. The energy used is $W = (700\text{ W})(1.8\times 10^3\text{ s}) = 1.3\times 10^6\text{ J}$.

Walking: $\Delta t = \dfrac{5.0\text{ km}}{3.0\text{ km/h}}\left(\dfrac{3600\text{ s}}{\text{h}}\right) = 6.0\times 10^3\text{ s}$. The energy used is $W = (290\text{ W})(6.0\times 10^3\text{ s}) = 1.7\times 10^6\text{ J}$.

EVALUATE: The less intense exercise lasts longer and therefore burns up more energy than the intense exercise.

6.53. **IDENTIFY:** $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$. ΔW is the energy released.

SET UP: ΔW is to be the same. $1\text{ y} = 3.156 \times 10^7\text{ s}$.

EXECUTE: $P_{\text{av}} \Delta t = \Delta W = \text{constant}$, so $P_{\text{av-sun}} \Delta t_{\text{sun}} = P_{\text{av-m}} \Delta t_{\text{m}}$.

$$P_{\text{av-m}} = P_{\text{av-sun}} \left(\frac{\Delta t_{\text{sun}}}{\Delta t_{\text{m}}} \right) = P \left(\frac{(2.5 \times 10^5\text{ y})(3.156 \times 10^7\text{ s/y})}{0.20\text{ s}} \right) = 3.9 \times 10^{13} P.$$

EVALUATE: Since the power output of the magnetar is so much larger than that of our sun, the mechanism by which it radiates energy must be quite different.

6.54. **IDENTIFY:** The thermal energy is produced as a result of the force of friction, $F = \mu_{\text{k}} mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|} v_{\text{av}}$.

SET UP: $v_{\text{av}} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00\text{ m/s} + 0}{2} \right) = 4.00\text{ m/s}$

EXECUTE: $P = F v_{\text{av}} = [(0.200)(20.0\text{ kg})(9.80\text{ m/s}^2)](4.00\text{ m/s}) = 157\text{ W}$

EVALUATE: The power could also be determined as the rate of change of kinetic energy, $\Delta K/t$, where the time is calculated from $v_{\text{f}} = v_{\text{i}} + at$ and a is calculated from a force balance, $\Sigma F = ma = \mu_{\text{k}} mg$.

6.55. **IDENTIFY:** Use the relation $P = F_{\|} v$ to relate the given force and velocity to the total power developed.

SET UP: $1\text{ hp} = 746\text{ W}$

EXECUTE: The total power is $P = F_{\|} v = (165\text{ N})(9.00\text{ m/s}) = 1.49 \times 10^3\text{ W}$. Each rider therefore contributes $P_{\text{each rider}} = (1.49 \times 10^3\text{ W})/2 = 745\text{ W} \approx 1\text{ hp}$.

EVALUATE: The result of one horsepower is very large; a rider could not sustain this output for long periods of time.

6.56. **IDENTIFY** and **SET UP:** Calculate the power used to make the plane climb against gravity. Consider the vertical motion since gravity is vertical.

EXECUTE: The rate at which work is being done against gravity is

$P = Fv = mgv = (700\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m/s}) = 17.15\text{ kW}$.

This is the part of the engine power that is being used to make the airplane climb. The fraction this is of the total is $17.15\text{ kW}/75\text{ kW} = 0.23$.

EVALUATE: The power we calculate for making the airplane climb is considerably less than the power output of the engine.

6.57. **IDENTIFY:** $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$. The work you do in lifting mass *m* a height *h* is *mgh*.

SET UP: $1\text{ hp} = 746\text{ W}$

EXECUTE: **(a)** The number per minute would be the average power divided by the work (*mgh*) required to lift one box, $\dfrac{(0.50\text{ hp})(746\text{ W/hp})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 1.41/\text{s}$, or $84.6/\text{min}$.

(b) Similarly, $\dfrac{(100\text{ W})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 0.378/\text{s}$, or $22.7/\text{min}$.

EVALUATE: A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

6.58. **IDENTIFY** and **SET UP:** Use Eq. (6.15) to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.

EXECUTE: Find the total mass that can be lifted:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t}, \text{ so } m = \frac{P_{\text{av}} t}{gh}$$

$$P_{\text{av}} = (40\text{ hp}) \left(\frac{746\text{ W}}{1\text{ hp}} \right) = 2.984 \times 10^4\text{ W}$$

$$m = \frac{P_{\text{av}}t}{gh} = \frac{(2.984\times10^4\text{ W})(16.0\text{ s})}{(9.80\text{ m/s}^2)(20.0\text{ m})} = 2.436\times10^3\text{ kg}$$

This is the total mass of elevator plus passengers. The mass of the passengers is

$2.436\times10^3\text{ kg} - 600\text{ kg} = 1.836\times10^3\text{ kg}$. The number of passengers is $\dfrac{1.836\times10^3\text{ kg}}{65.0\text{ kg}} = 28.2.$

28 passengers can ride.

EVALUATE: Typical elevator capacities are about half this, in order to have a margin of safety.

6.59. **IDENTIFY:** To lift the skiers, the rope must do positive work to counteract the negative work developed by the component of the gravitational force acting on the total number of skiers, $F_{\text{rope}} = Nmg\sin\alpha.$

SET UP: $P = F_{\|}v = F_{\text{rope}}v$

EXECUTE: $P_{\text{rope}} = F_{\text{rope}}v = [+Nmg(\cos\phi)]v.$

$$P_{\text{rope}} = [(50\text{ riders})(70.0\text{ kg})(9.80\text{ m/s}^2)(\cos 75.0)]\left[(12.0\text{ km/h})\left(\frac{1\text{ m/s}}{3.60\text{ km/h}}\right)\right].$$

$P_{\text{rope}} = 2.96\times10^4\text{ W} = 29.6\text{ kW}.$

EVALUATE: Some additional power would be needed to give the riders kinetic energy as they are accelerated from rest.

6.60. **IDENTIFY:** We want to find the power supplied by a known force acting on a crate at a known velocity.

SET UP: We know the vector components, so we use $P = \vec{F}\cdot\vec{v} = F_xv_x + F_yv_y$

EXECUTE: $P = F_xv_x + F_yv_y = (-8.00\text{ N})(3.20\text{ m/s}) + (3.00\text{ N})(2.20\text{ m/s}) = -19.0\text{ W}.$

EVALUATE: The power is negative because the x-component of the force is opposite to the x-component of the velocity and hence opposes the motion of the crate.

6.61. **IDENTIFY:** Relate power, work, and time.

SET UP: Work done in each stroke is $W = Fs$ and $P_{\text{av}} = W/t.$

EXECUTE: 100 strokes per second means $P_{\text{av}} = 100Fs/t$ with $t = 1.00\text{ s}$, $F = 2mg$ and $s = 0.010\text{ m}.$

$P_{\text{av}} = 0.20\text{ W}.$

EVALUATE: For a 70-kg person to apply a force of twice his weight through a distance of 0.50 m for 100 times per second, the average power output would be $7.0\times10^4\text{ W}$. This power output is very far beyond the capability of a person.

6.62. **IDENTIFY:** The force has only an x-component and the motion is along the x-direction, so $W = \int_{x_1}^{x_2} F_x dx.$

SET UP: $x_1 = 0$ and $x_2 = 6.9\text{ m}.$

EXECUTE: The work you do with your changing force is

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2}(-20.0\text{ N})dx - \int_{x_1}^{x_2}(3.0\text{ N/m})x dx = (-20.0\text{ N})x\Big|_{x_1}^{x_2} - (3.0\text{ N/m})(x^2/2)\Big|_{x_1}^{x_2}$$

$W = -138\text{ N}\cdot\text{m} - 71.4\text{ N}\cdot\text{m} = -209\text{ J}.$

EVALUATE: The work is negative because the cow continues to move forward (in the $+x$-direction) as you vainly attempt to push her backward.

6.63. **IDENTIFY** and **SET UP:** Since the forces are constant, Eq. (6.2) can be used to calculate the work done by each force. The forces on the suitcase are shown in Figure 6.63a.

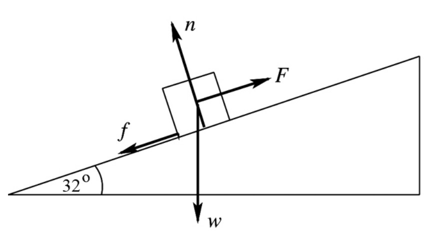


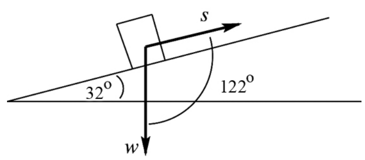
Figure 6.63a

In part (f), Eq. (6.6) is used to relate the total work to the initial and final kinetic energy.

EXECUTE: (a) $W_F = (F\cos\phi)s$

Both $\vec{F}$ and $\vec{s}$ are parallel to the incline and in the same direction, so $\phi = 90°$ and $W_F = Fs = (160\text{ N})(3.80\text{ m}) = 608\text{ J}$.

(b) The directions of the displacement and of the gravity force are shown in Figure 6.63b.



$W_w = (w\cos\phi)s$

$\phi = 122°$, so

$W_w = (196\text{ N})(\cos 122°)(3.80\text{ m})$

$W_w = -395\text{ J}$

Figure 6.63b

Alternatively, the component of w parallel to the incline is $w\sin 32°$. This component is down the incline so its angle with $\vec{s}$ is $\phi = 180°$. $W_{w\sin 32°} = (196\text{ N}\sin 32°)(\cos 180°)(3.80\text{ m}) = -395\text{ J}$. The other component of w, $w\cos 32°$, is perpendicular to $\vec{s}$ and hence does no work. Thus $W_w = W_{w\sin 25°} = -315\text{ J}$, which agrees with the above.

(c) The normal force is perpendicular to the displacement $(\phi = 90°)$, so $W_n = 0$.

(d) $n = w\cos 32°$ so $f_k = \mu_k n = \mu_k w\cos 32° = (0.30)(196\text{ N})\cos 32° = 49.87\text{ N}$

$W_f = (f_k\cos\phi)x = (49.87\text{ N})(\cos 180°)(3.80\text{ m}) = -189\text{ J}$.

(e) $W_{tot} = W_F + W_w + W_n + W_f = +608\text{ J} - 395\text{ J} + 0 - 189\text{ J} = 24\text{ J}$.

(f) $W_{tot} = K_2 - K_1$, $K_1 = 0$, so $K_2 = W_{tot}$

$\frac{1}{2}mv_2^2 = W_{tot}$ so $v_2 = \sqrt{\frac{2W_{tot}}{m}} = \sqrt{\frac{2(24\text{ J})}{20.0\text{ kg}}} = 1.5\text{ m/s}$.

EVALUATE: The total work done is positive and the kinetic energy of the suitcase increases as it moves up the incline.

6.64. IDENTIFY: The work he does to lift his body a distance h is $W = mgh$. The work per unit mass is $(W/m) = gh$.

SET UP: The quantity gh has units of N/kg.

EXECUTE: (a) The man does work, $(9.8\text{ N/kg})(0.4\text{ m}) = 3.92\text{ J/kg}$.

(b) $(3.92\text{ J/kg})/(70\text{ J/kg}) \times 100 = 5.6\%$.

(c) The child does work $(9.8\text{ N/kg})(0.2\text{ m}) = 1.96\text{ J/kg}$. $(1.96\text{ J/kg})/(70\text{ J/kg}) \times 100 = 2.8\%$.

(d) If both the man and the child can do work at the rate of 70 J/kg, and if the child only needs to use 1.96 J/kg instead of 3.92 J/kg, the child should be able to do more chin-ups.

EVALUATE: Since the child has arms half the length of his father's arms, the child must lift his body only 0.20 m to do a chin-up.

6.65. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to each block to find the tension in the string. Each force is constant and $W = Fs\cos\phi$.

SET UP: The free-body diagram for each block is given in Figure 6.65 (next page).

$m_A = \frac{20.0\text{ N}}{g} = 2.04\text{ kg}$ and $m_B = \frac{12.0\text{ N}}{g} = 1.22\text{ kg}$.

EXECUTE: $T - f_k = m_A a$. $w_B - T = m_B a$. $w_B - f_k = (m_A + m_B)a$.

(a) $f_k = 0$. $a = \left(\frac{w_B}{m_A + m_B}\right)$ and $T = w_B\left(\frac{m_A}{m_A + m_B}\right) = w_B\left(\frac{w_A}{w_A + w_B}\right) = 7.50\text{ N}$.

20.0 N block: $W_{tot} = Ts = (7.50\text{ N})(0.750\text{ m}) = 5.62\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 7.50\text{ N})(0.750\text{ m}) = 3.38\text{ J}$.

(b) $f_k = \mu_k w_A = 6.50\text{ N}$. $a = \dfrac{w_B - \mu_k w_A}{m_A + m_B}$.

$$T = f_k + (w_B - \mu_k w_A)\left(\frac{m_A}{m_A + m_B}\right) = \mu_k w_A + (w_B - \mu_k w_A)\left(\frac{w_A}{w_A + w_B}\right).\ T = 6.50\text{ N} + (5.50\text{ N})(0.625) = 9.94\text{ N}.$$

20.0 N block: $W_{tot} = (T - f_k)s = (9.94\text{ N} - 6.50\text{ N})(0.750\text{ m}) = 2.58\text{ J}.$

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 9.94\text{ N})(0.750\text{ m}) = 1.54\text{ J}.$

EVALUATE: Since the two blocks move with equal speeds, for each block $W_{tot} = K_2 - K_1$ is proportional to the mass (or weight) of that block. With friction the gain in kinetic energy is less, so the total work on each block is less.

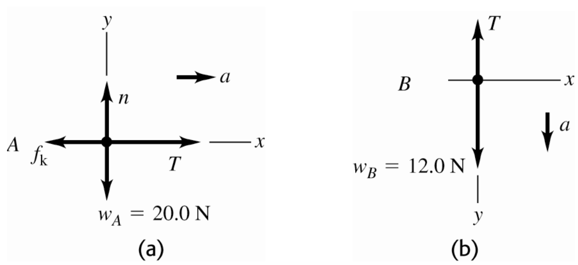


Figure 6.65

6.66. **IDENTIFY:** $W = Fs\cos\phi$ and $W_{tot} = K_2 - K_1$.

SET UP: $f_k = \mu_k n$. The normal force is $n = mg\cos\theta$, with $\theta = 24.0°$. The component of the weight parallel to the incline is $mg\sin\theta$.

EXECUTE: **(a)** $\phi = 180°$ and

$W_f = -f_k s = -(\mu_k mg\cos\theta)s = -(0.31)(5.00\text{ kg})(9.80\text{ m/s}^2)(\cos 24.0°)(2.80\text{ m}) = -38.9\text{ J}.$

(b) $(5.00\text{ kg})(9.80\text{ m/s}^2)(\sin 24.0°)(2.80\text{ m}) = 55.8\text{ J}.$

(c) The normal force does no work.

(d) $W_{tot} = 55.8\text{ J} - 38.9\text{ J} = +16.9\text{ J}.$

(e) $K_2 = K_1 + W_{tot} = (1/2)(5.00\text{ kg})(2.20\text{ m/s})^2 + 16.9\text{ J} = 29.0\text{ J}$, and so

$v_2 = \sqrt{2(29.0\text{ J})/(5.00\text{ kg})} = 3.41\text{ m/s}.$

EVALUATE: Friction does negative work and gravity does positive work. The net work is positive and the kinetic energy of the object increases.

6.67. **IDENTIFY:** The initial kinetic energy of the head is absorbed by the neck bones during a sudden stop. Newton's second law applies to the passengers as well as to their heads.

SET UP: In part (a), the initial kinetic energy of the head is absorbed by the neck bones, so $\frac{1}{2}mv_{max}^2 = 8.0\text{ J}$. For part (b), assume constant acceleration and use $v_f = v_i + at$ with $v_i = 0$, to calculate a; then apply $F_{net} = ma$ to find the net accelerating force.

Solve: **(a)** $v_{max} = \sqrt{\dfrac{2(8.0\text{ J})}{5.0\text{ kg}}} = 1.8\text{ m/s} = 4.0\text{ mph}.$

(b) $a = \dfrac{v_f - v_i}{t} = \dfrac{1.8\text{ m/s} - 0}{10.0 \times 10^{-3}\text{ s}} = 180\text{ m/s}^2 \approx 18g$, and $F_{net} = ma = (5.0\text{ kg})(180\text{ m/s}^2) = 900\text{ N}.$

EVALUATE: The acceleration is very large, but if it lasts for only 10 ms it does not do much damage.

6.68. **IDENTIFY:** The force does work on the object, which changes its kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the object.

SET UP: $W_{tot} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ and $W_{tot} = \int_{x_1}^{x_2} F(x)dx.$

EXECUTE: $W_{tot} = \int_{x_1}^{x_2} F(x)dx = \int_0^{5.00\text{ m}} [-12.0\text{ N} + (0.300\text{ N/m}^2)x^2]dx.$

$W_{tot} = -(12.0\text{ N})(5.00\text{ m}) + (0.100\text{ N/m}^2)(5.00\text{ m})^3 = -60.0\text{ J} + 12.5\text{ J} = -47.5\text{ J}.$

$W_{tot} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -47.5\text{ J},$ so the final velocity is

$$v_f = \sqrt{v_i^2 - \frac{2(47.5\text{ J})}{m}} = \sqrt{(6.00\text{ m/s})^2 - \frac{2(47.5\text{ J})}{5.00\text{ kg}}} = 4.12\text{ m/s}.$$

EVALUATE: We could not readily do this problem by integrating the acceleration over time because we know the force as a function of x, not of t. The work-energy theorem provides a much simpler method.

6.69. **IDENTIFY:** Calculate the work done by friction and apply $W_{tot} = K_2 - K_1$. Since the friction force is not constant, use Eq. (6.7) to calculate the work.

SET UP: Let x be the distance past P. Since μ_k increases linearly with x, $\mu_k = 0.100 + Ax$. When $x = 12.5$ m, $\mu_k = 0.600$, so $A = 0.500/(12.5\text{ m}) = 0.0400/\text{m}$.

EXECUTE: **(a)** $W_{tot} = \Delta K = K_2 - K_1$ gives $-\int \mu_k mgdx = 0 - \frac{1}{2}mv_1^2$. Using the above expression for μ_k,

$g\int_0^{x_2}(0.100 + Ax)dx = \frac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\frac{x_2^2}{2}\right] = \frac{1}{2}v_1^2.$

$(9.80\text{ m/s}^2)\left[(0.100)x_2 + (0.0400/\text{m})\frac{x_2^2}{2}\right] = \frac{1}{2}(4.50\text{ m/s})^2.$ Solving for x_2 gives $x_2 = 5.11$ m.

(b) $\mu_k = 0.100 + (0.0400/\text{m})(5.11\text{ m}) = 0.304$

(c) $W_{tot} = K_2 - K_1$ gives $-\mu_k mgx_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \frac{v_1^2}{2\mu_k g} = \frac{(4.50\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = 10.3$ m.

EVALUATE: The box goes farther when the friction coefficient doesn't increase.

6.70. **IDENTIFY:** Use Eq. (6.7) to calculate W.

SET UP: $x_1 = 0$. In part (a), $x_2 = 0.050$ m. In part (b), $x_2 = -0.050$ m.

EXECUTE: **(a)** $W = \int_0^{x_2} Fdx = \int_0^{x_2}(kx - bx^2 + cx^3)\,dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4.$

$W = (50.0\text{ N/m})\,x_2^2 - (233\text{ N/m}^2)\,x_2^3 + (3000\text{ N/m}^3)\,x_2^4.$ When $x_2 = 0.050$ m, $W = 0.12$ J.

(b) When $x_2 = -0.050$ m, $W = 0.17$ J.

(c) It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

EVALUATE: When $x = 0.050$ m, $F_x = 4.75$ N. When $x = -0.050$ m, $F_x = -8.25$ N.

6.71. **IDENTIFY** and **SET UP:** Use $\Sigma\vec{F} = m\vec{a}$ to find the tension force T. The block moves in uniform circular motion and $\vec{a} = \vec{a}_{rad}$.

(a) The free-body diagram for the block is given in Figure 6.71.

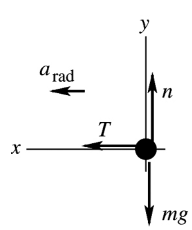


Figure 6.71

EXECUTE: $\Sigma F_x = ma_x$

$$T = m\frac{v^2}{R}$$

$$T = (0.0600\text{ kg})\frac{(0.70\text{ m/s})^2}{0.40\text{ m}} = 0.074\text{ N}.$$

(b) $T = m\dfrac{v^2}{R} = (0.0600\text{ kg})\dfrac{(2.80\text{ m/s})^2}{0.10\text{ m}} = 4.7\text{ N}.$

(c) SET UP: The tension changes as the distance of the block from the hole changes. We could use $W = \int_{x_1}^{x_2} F_x\,dx$ to calculate the work. But a much simpler approach is to use $W_{\text{tot}} = K_2 - K_1$.

EXECUTE: The only force doing work on the block is the tension in the cord, so $W_{\text{tot}} = W_T$.

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0600\text{ kg})(0.70\text{ m/s})^2 = 0.01470\text{ J}$, $K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0600\text{ kg})(2.80\text{ m/s})^2 = 0.2352\text{ J}$, so $W_{\text{tot}} = K_2 - K_1 = 0.2352\text{ J} - 0.01470\text{ J} = 0.22\text{ J}$. This is the amount of work done by the person who pulled the cord.

EVALUATE: The block moves inward, in the direction of the tension, so T does positive work and the kinetic energy increases.

6.72. **IDENTIFY:** Use Eq. (6.7) to find the work done by F. Then apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $\int \dfrac{dx}{x^2} = -\dfrac{1}{x}$.

EXECUTE: $W = \int_{x_1}^{x_2} \dfrac{\alpha}{x^2}\,dx = \alpha\left(\dfrac{1}{x_1} - \dfrac{1}{x_2}\right)$.

$W = (2.12\times10^{-26}\text{ N}\cdot\text{m}^2)\left[(0.200\text{ m}^{-1}) - (1.25\times10^{9}\text{ m}^{-1})\right] = -2.65\times10^{-17}\text{ J}.$

Note that x_1 is so large compared to x_2 that the term $1/x_1$ is negligible. Then, using Eq. (6.13) and solving for v_2,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00\times10^5\text{ m/s})^2 + \frac{2(-2.65\times10^{-17}\text{ J})}{(1.67\times10^{-27}\text{ kg})}} = 2.41\times10^5\text{ m/s}.$$

(b) With $K_2 = 0, W = -K_1$. Using $W = -\dfrac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12\times10^{-26}\text{ N}\cdot\text{m}^2)}{(1.67\times10^{-27}\text{ kg})(3.00\times10^5\text{ m/s})^2} = 2.82\times10^{-10}\text{ m}.$$

(c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is 3.00×10^5 m/s.

EVALUATE: As the proton moves toward the uranium nucleus the repulsive force does negative work and the kinetic energy of the proton decreases. As the proton moves away from the uranium nucleus the repulsive force does positive work and the kinetic energy of the proton increases.

6.73. **IDENTIFY:** The negative work done by the spring equals the change in kinetic energy of the car.

SET UP: The work done by a spring when it is compressed a distance x from equilibrium is $-\dfrac{1}{2}kx^2$. $K_2 = 0$.

EXECUTE: $-\frac{1}{2}kx^2 = K_2 - K_1$ gives $\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2$ and

$k = (mv_1^2)/x^2 = [(1200\text{ kg})(0.65\text{ m/s})^2]/(0.090\text{ m})^2 = 6.3\times10^4\text{ N/m}.$

EVALUATE: When the spring is compressed, the spring force is directed opposite to the displacement of the object and the work done by the spring is negative.

6.74. **IDENTIFY** and **SET UP:** Use Eq. (6.6). You do positive work and gravity does negative work. Let point 1 be at the base of the bridge and point 2 be at the top of the bridge.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 1000\text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(80.0\text{ kg})(1.50\text{ m/s})^2 = 90\text{ J}$

$W_{\text{tot}} = 90\text{ J} - 1000\text{ J} = -910\text{ J}$

(b) Neglecting friction, work is done by you (with the force you apply to the pedals) and by gravity: $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$. The gravity force is $w = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$, downward. The displacement is 5.20 m, upward. Thus $\phi = 180°$ and

$$W_{\text{gravity}} = (F\cos\phi)s = (784\text{ N})(5.20\text{ m})\cos 180° = -4077\text{ J}$$

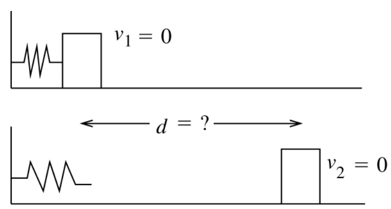
Then $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$ gives

$$W_{\text{you}} = W_{\text{tot}} - W_{\text{gravity}} = -910\text{ J} - (-4077\text{ J}) = +3170\text{ J}$$

EVALUATE: The total work done is negative and you lose kinetic energy.

6.75. **IDENTIFY** and **SET UP:** Use Eq. (6.6). Work is done by the spring and by gravity. Let point 1 be where the textbook is released and point 2 be where it stops sliding. $x_2 = 0$ since at point 2 the spring is neither stretched nor compressed. The situation is sketched in Figure 6.75.

EXECUTE:



$$W_{\text{tot}} = K_2 - K_1$$
$$K_1 = 0,\ \ K_2 = 0$$
$$W_{\text{tot}} = W_{\text{fric}} + W_{\text{spr}}$$

Figure 6.75

$W_{\text{spr}} = \frac{1}{2}kx_1^2$, where $x_1 = 0.250\text{ m}$ (Spring force is in direction of motion of block so it does positive work.)

$W_{\text{fric}} = -\mu_k mgd$

Then $W_{\text{tot}} = K_2 - K_1$ gives $\frac{1}{2}kx_1^2 - \mu_k mgd = 0$

$$d = \frac{kx_1^2}{2\mu_k mg} = \frac{(250\text{ N/m})(0.250\text{ m})^2}{2(0.30)(2.50\text{ kg})(9.80\text{ m/s}^2)} = 1.1\text{ m},$$ measured from the point where the block was released.

EVALUATE: The positive work done by the spring equals the magnitude of the negative work done by friction. The total work done during the motion between points 1 and 2 is zero, and the textbook starts and ends with zero kinetic energy.

6.76. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: Let x_0 be the initial distance the spring is compressed. The work done by the spring is $\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$, where x is the final distance the spring is compressed.

EXECUTE: **(a)** Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}}(0.060\text{ m}) = 6.93\text{ m/s}.$$

(b) W_{tot} must now include friction, so $\frac{1}{2}mv^2 = W_{\text{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where f is the magnitude of the friction force. Then,

$$v = \sqrt{\frac{k}{m}x_0^2 - \frac{2f}{m}x_0} = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}(0.060\text{ m})^2 - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m})} = 4.90\text{ m/s}.$$

(c) The greatest speed occurs when the acceleration (and the net force) are zero. Let x be the amount the spring is still compressed, so the distance the ball has moved is $x_0 - x$. $kx = f$, $x = \frac{f}{k} = \frac{6.00\text{ N}}{400\text{ N/m}} =$ 0.0150 m.

The ball is 0.0150 m from the end of the barrel, or 0.0450 m from its initial position.

To find the speed, the net work is $W_{\text{tot}} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is

$$v_{\max} = \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)}.$$

$$v_{\max} = \sqrt{\frac{400\text{ N/m}}{(0.0300\text{ kg})}\left[(0.060\text{ m})^2 - (0.0150\text{ m})^2\right] - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m} - 0.0150\text{ m})} = 5.20\text{ m/s}$$

EVALUATE: The maximum speed with friction present (part (c)) is larger than the result of part (b) but smaller than the result of part (a).

6.77. **IDENTIFY:** A constant horizontal force pushes a block against a spring on a rough floor. The work-energy theorem and Newton's second law both apply.

SET UP: In part (a), we apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the block. $f_k = \mu_k n$ and $W_{\text{spring}} = -½\, kx^2$. In part (b), we apply Newton's second law to the block.

EXECUTE: **(a)** $W_F + W_{\text{spring}} + W_f = K_2 - K_1$. $Fx - ½\, kx^2 - \mu_k mgx = ½\, mv^2 - 0$. Putting in the numbers from the problem gives (82.0 N)(0.800 m) – (130.0 N/m)(0.800 m)^2/2 – (0.400)(4.00 kg)(9.80 m/s^2)(0.800 m) = (4.00 kg)v^2/2, v = 2.39 m/s.

(b) Looking at quantities parallel to the floor, with the positive direction toward the wall, Newton's second law gives $F - f_k - F_{\text{spring}} = ma$.

$F - \mu_k mg - kx = ma$: 82.0 N – (0.400)(4.00 kg)(9.80 m/s^2) – (130.0 N/m)(0.800 m) = (4.00 kg)a

a = –9.42 m/s^2. The minus sign means that the acceleration is away from the wall.

EVALUATE: The force you apply is toward the wall but the block is accelerating away from the wall.

6.78. **IDENTIFY:** A constant horizontal force pushes a frictionless block of ice against a spring on the floor. The work-energy theorem and Newton's second law both apply.

SET UP: In part (a), we apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the ice. $W_{\text{spring}} = -½\, kx^2$. In part (b), we apply Newton's second law to the ice.

EXECUTE: **(a)** $W_F + W_{\text{spring}} = K_2 - K_1$. $Fx - ½\, kx^2 = ½\, mv^2 - 0$. Putting in the numbers from the problem gives (54.0 N)(0.400 m) – (76.0 N/m)(0.400 m)^2/2 = (2.00 kg)v^2/2, v = 3.94 m/s.

(b) Looking at quantities parallel to the floor, with the positive direction away from the post, Newton's second law gives $F - F_{\text{spring}} = ma$, so $F - kx = ma$.

54.0 N – (76.0 N/m)(0.400 m) = (2.00 kg)a, which gives a = 11.8 m/s^2. The acceleration is positive, so the block is accelerating away from the post.

EVALUATE: The given force must be greater than the spring force since the ice is accelerating away from the post.

6.79. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the blocks.

SET UP: If X is the distance the spring is compressed, the work done by the spring is $-\frac{1}{2}kX^2$. At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy.

EXECUTE: **(a)** The work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv_0^2$ and $X = \sqrt{\frac{m}{k}}v_0 = \sqrt{\frac{5.00\text{ kg}}{500\text{ N/m}}}(6.00\text{ m/s}) = 0.600\text{ m}.$

(b) Solving for v_0 in terms of a known X, $v_0 = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\text{ N/m}}{5.00\text{ kg}}}(0.150\text{ m}) = 1.50\text{ m/s}.$

EVALUATE: The negative work done by the spring removes the kinetic energy of the block.

6.80. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$. $W = Fs\cos\phi$.

SET UP: The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $120.0°$ with the chair's motion.

EXECUTE: The total work done is $W_{\text{tot}} = ((600\text{ N})\cos 30.0° + (85.0\text{ kg})(9.80\text{ m/s}^2)\cos 120.0°)(2.50\text{ m}) =$ 257.8 J, and so the speed at the top of the ramp is $v_2 = \sqrt{v_1^2 + \frac{2W_{\text{tot}}}{m}} = \sqrt{(2.00\text{ m/s})^2 + \frac{2(257.8\text{ J})}{(85.0\text{ kg})}} = 3.17\text{ m/s}.$

EVALUATE: The component of gravity down the incline is $mg\sin 30° = 417\text{ N}$ and the component of the push up the incline is $(600\text{ N})\cos 30° = 520\text{ N}$. The force component up the incline is greater than the force component down the incline; the net work done is positive and the speed increases.

6.81. **IDENTIFY** and **SET UP:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system consisting of both blocks. Since they are connected by the cord, both blocks have the same speed at every point in the motion. Also, when the 6.00-kg block has moved downward 1.50 m, the 8.00-kg block has moved 1.50 m to the right. The target variable, μ_k, will be a factor in the work done by friction. The forces on each block are shown in Figure 6.81.

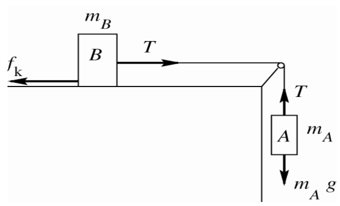


Figure 6.81

EXECUTE:

$K_1 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_1^2 = \frac{1}{2}(m_A + m_B)v_1^2$

$K_2 = 0$

The tension T in the rope does positive work on block B and the same magnitude of negative work on block A, so T does no net work on the system. Gravity does work $W_{mg} = m_A gd$ on block A, where $d = 2.00\text{ m}$. (Block B moves horizontally, so no work is done on it by gravity.) Friction does work $W_{\text{fric}} = -\mu_k m_B gd$ on block B. Thus $W_{\text{tot}} = W_{mg} + W_{\text{fric}} = m_A gd - \mu_k m_B gd$. Then $W_{\text{tot}} = K_2 - K_1$ gives $m_A gd - \mu_k m_B gd = -\frac{1}{2}(m_A + m_B)v_1^2$ and

$$\mu_k = \frac{m_A}{m_B} + \frac{\frac{1}{2}(m_A + m_B)v_1^2}{m_B gd} = \frac{6.00\text{ kg}}{8.00\text{ kg}} + \frac{(6.00\text{ kg} + 8.00\text{ kg})(0.900\text{ m/s})^2}{2(8.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})} = 0.786$$

EVALUATE: The weight of block A does positive work and the friction force on block B does negative work, so the net work is positive and the kinetic energy of the blocks increases as block A descends. Note that K_1 includes the kinetic energy of both blocks. We could have applied the work-energy theorem to block A alone, but then W_{tot} includes the work done on block A by the tension force.

6.82. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

SET UP: Let h be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)gh and the work done by friction is $-\mu_k(8.00\text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))(9.80\text{ m/s}^2)(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\frac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.

6.83. **IDENTIFY:** Apply Eq. (6.6) to the skater.

SET UP: Let point 1 be just before she reaches the rough patch and let point 2 be where she exits from the patch. Work is done by friction. We don't know the skater's mass so can't calculate either friction or the initial kinetic energy. Leave her mass m as a variable and expect that it will divide out of the final equation.

EXECUTE: $f_k = 0.25mg$ so $W_f = W_{\text{tot}} = -(0.25mg)s$, where s is the length of the rough patch.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2$, $K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.55v_0)^2 = 0.3025\left(\frac{1}{2}mv_0^2\right)$

The work-energy relation gives $-(0.25mg)s = (0.3025 - 1)\frac{1}{2}mv_0^2$.

The mass divides out, and solving gives $s = 1.3\text{ m}$.

EVALUATE: Friction does negative work and this reduces her kinetic energy.

6.84. **IDENTIFY** and **SET UP:** $W = Pt$

EXECUTE: **(a)** The hummingbird produces energy at a rate of 0.7 J/s to 1.75 J/s. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.

(b) The steady output of the athlete is $(500\text{ W})/(70\text{ kg}) = 7\text{ W/kg}$, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend $1400\text{ W}/70\text{ kg} = 20\text{ W/kg}$ for short periods of time, no human-powered aircraft could stay aloft for very long.

EVALUATE: Movies of early attempts at human-powered flight bear out our results.

6.85. **IDENTIFY:** To lift a mass m a height h requires work $W = mgh$. To accelerate mass m from rest to speed v requires $W = K_2 - K_1 = \frac{1}{2}mv^2$. $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$.

SET UP: $t = 60\text{ s}$

EXECUTE: **(a)** $(800\text{ kg})(9.80\text{ m/s}^2)(14.0\text{ m}) = 1.10\times10^5\text{ J}$.

(b) $(1/2)(800\text{ kg})(18.0\text{ m/s}^2) = 1.30\times10^5\text{ J}$.

(c) $\dfrac{1.10\times10^5\text{ J} + 1.30\times10^5\text{ J}}{60\text{ s}} = 3.99\text{ kW}$.

EVALUATE: Approximately the same amount of work is required to lift the water against gravity as to accelerate it to its final speed.

6.86. **IDENTIFY** and **SET UP:** Use Eq. (6.15). The work done on the water by gravity is mgh, where $h = 170\text{ m}$. Solve for the mass m of water for 1.00 s and then calculate the volume of water that has this mass.

EXECUTE: The power output is $P_{\text{av}} = 2000\text{ MW} = 2.00\times10^9\text{ W}$. $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$ and 92% of the work done on the water by gravity is converted to electrical power output, so in 1.00 s the amount of work done on the water by gravity is

$$W = \frac{P_{\text{av}}\Delta t}{0.92} = \frac{(2.00\times10^9\text{ W})(1.00\text{ s})}{0.92} = 2.174\times10^9\text{ J}.$$

$W = mgh$, so the mass of water flowing over the dam in 1.00 s must be

$$m = \frac{W}{gh} = \frac{2.174\times10^9\text{ J}}{(9.80\text{ m/s}^2)(170\text{ m})} = 1.30\times10^6\text{ kg}.$$

$$\text{density} = \frac{m}{V} \text{ so } V = \frac{m}{\text{density}} = \frac{1.30\times10^6\text{ kg}}{1.00\times10^3\text{ kg/m}^3} = 1.30\times10^3\text{ m}^3.$$

EVALUATE: The dam is 1270 m long, so this volume corresponds to about a m^3 flowing over each 1 m length of the dam, a reasonable amount.

6.87. **IDENTIFY** and **SET UP:** Energy is $P_{\text{av}}t$. The total energy expended in one day is the sum of the energy expended in each type of activity.

EXECUTE: $1\text{ day} = 8.64\times10^4\text{ s}$

Let t_{walk} be the time she spends walking and t_{other} be the time she spends in other activities; $t_{\text{other}} = 8.64\times10^4\text{ s} - t_{\text{walk}}$.

The energy expended in each activity is the power output times the time, so

$$E = Pt = (280\text{ W})t_{\text{walk}} + (100\text{ W})t_{\text{other}} = 1.1\times10^7\text{ J}$$

$$(280\text{ W})t_{\text{walk}} + (100\text{ W})(8.64\times10^4\text{ s} - t_{\text{walk}}) = 1.1\times10^7\text{ J}$$

$$(180\text{ W})t_{\text{walk}} = 2.36\times10^6\text{ J}$$

$$t_{\text{walk}} = 1.31\times10^4\text{ s} = 218\text{ min} = 3.6\text{ h}.$$

EVALUATE: Her average power for one day is $(1.1\times10^7\text{ J})/[(24)(3600\text{ s})] = 127\text{ W}$. This is much closer to her 100 W rate than to her 280 W rate, so most of her day is spent at the 100 W rate.

6.88. **IDENTIFY:** $W = \int_{x_1}^{x_2} F_x dx$, and F_x depends on both x and y.

SET UP: In each case, use the value of y that applies to the specified path. $\int x dx = \frac{1}{2}x^2$. $\int x^2 dx = \frac{1}{3}x^3$.

EXECUTE: **(a)** Along this path, y is constant, with the value $y = 3.00$ m.

$W = \alpha y \int_{x_1}^{x_2} x dx = (2.50\ \text{N/m}^2)(3.00\ \text{m})\dfrac{(2.00\ \text{m})^2}{2} = 15.0\ \text{J}$, since $x_1 = 0$ and $x_2 = 2.00$ m.

(b) Since the force has no y-component, no work is done moving in the y-direction.

(c) Along this path, y varies with position along the path, given by $y = 1.5x$, so $F_x = \alpha(1.5x)x = 1.5\alpha x^2$, and

$$W = \int_{x_1}^{x_2} F dx = 1.5\alpha \int_{x_1}^{x_2} x^2 dx = 1.5(2.50\ \text{N/m}^2)\frac{(2.00\ \text{m})^3}{3} = 10.0\ \text{J}.$$

EVALUATE: The force depends on the position of the object along its path.

6.89. **IDENTIFY** and **SET UP:** For part (a) calculate m from the volume of blood pumped by the heart in one day. For part (b) use W calculated in part (a) in Eq. (6.15).

EXECUTE: (a) $W = mgh$, as in Example 6.10. We need the mass of blood lifted; we are given the volume

$V = (7500\ \text{L})\left(\dfrac{1\times10^{-3}\ \text{m}^3}{1\ \text{L}}\right) = 7.50\ \text{m}^3.$

$m = \text{density} \times \text{volume} = (1.05\times10^3\ \text{kg/m}^3)(7.50\ \text{m}^3) = 7.875\times10^3\ \text{kg}$

Then $W = mgh = (7.875\times10^3\ \text{kg})(9.80\ \text{m/s}^2)(1.63\ \text{m}) = 1.26\times10^5\ \text{J}.$

(b) $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{1.26\times10^5\ \text{J}}{(24\ \text{h})(3600\ \text{s/h})} = 1.46\ \text{W}.$

EVALUATE: Compared to light bulbs or common electrical devices, the power output of the heart is rather small.

6.90. **IDENTIFY:** We know information about the force exerted by a stretched rubber band and want to know if it obeys Hooke's law.

SET UP: Hooke's law is $F = kx$. The graph fits the equation $F = 33.55x^{0.4871}$, with F in newtons and x in meters.

EXECUTE: **(a)** For Hooke's law, a graph of F versus x is a straight line through the origin. This graph is not a straight line, so the rubber band does not obey Hooke's law.

(b) $k_{\text{eff}} = \dfrac{dF}{dx} = \dfrac{d}{dx}(33.55x^{0.4871}) = 16.34x^{-0.5129}$. Because of the negative exponent for x, as x increases, k_{eff} decreases.

(c) The definition of work gives $W = \int_a^b F_x dx = \int_0^{0.0400\ \text{m}} 0.3355x^{0.4871} dx = (33.55/1.4871)\ 0.0400^{1.4871}$

$W = 0.188$ J. From 0.0400 m to 0.0800 m, we follow the same procedure but with different limits of integration. The result is $W = (33.55/1.4871)\ (0.0800^{1.4871} - 0.0400^{1.4871}) = 0.339$ J.

(d) $W = K_2 - K_1 = ½\ mv^2 - 0$, which gives $0.339\ \text{J} = (0.300\ \text{kg})v^2/2$, $v = 1.50$ m/s.

EVALUATE: The rubber band does not obey Hooke's law, but it does obey the work-energy theorem.

6.91. **IDENTIFY:** We know a spring obeys Hooke's law, and we want to use observations of the motion of a block attached to this spring to determine its force constant and the coefficient of friction between the block and the surface on which it is sliding. The work-energy theorem applies.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $W_{\text{spring}} = ½\ kx^2$.

EXECUTE: **(a)** The spring force is initially greater than friction, so the block accelerates forward. But eventually the spring force decreases enough so that it is less than the force of friction, and the block then slows down (decelerates).

(b) The spring is initially compressed a distance x_0, and after the block has moved a distance d, the spring is compressed a distance $x = x_0 - d$. Therefore the work done by the spring is

$W_{\text{spring}} = \dfrac{1}{2}kx_0^2 - \dfrac{1}{2}k(x_0 - d)^2$. The work done by friction is $W_f = -\mu_k mgd$.

The work-energy theorem gives $W_{\text{spring}} + W_f = K_2 - K_1 = \frac{1}{2} mv^2$. Using our previous results, we get $\frac{1}{2}kx_0^2 - \frac{1}{2}k(x_0 - d)^2 - \mu_k mgd = \frac{1}{2}mv^2$. Solving for v^2 gives $v^2 = -\frac{k}{m}d^2 + 2d\left(\frac{k}{m}x_0 - \mu_k g\right)$, where $x_0 = 0.400$ m.

(c) Figure 6.91 shows the resulting graph of v^2 versus d. Using a graphing program and a quadratic fit gives $v^2 = -39.96d^2 + 16.31d$. The maximum speed occurs when $dv^2/dd = 0$, which gives $(-39.96)(2d) + 16.31 = 0$, so $d = 0.204$ m. For this value of d, we have $v^2 = (-39.96)(0.204\text{ m})^2 + (16.31)(0.204\text{ m})$, giving $v = 1.29$ m/s.

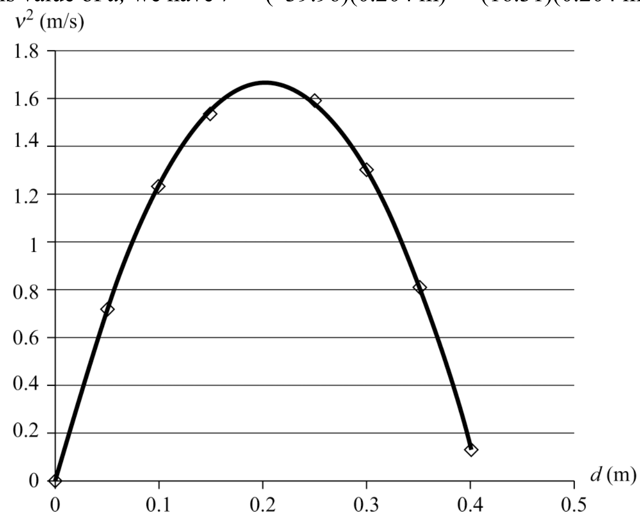


Figure 6.91

(d) From our work in (b) and (c), we know that $-k/m$ is the coefficient of d^2, so $-k/m = -39.96$, which gives $k = (39.96)(0.300\text{ kg}) = 12.0$ N/m. We also know that $2(kx_0/m - \mu_k g)$ is the coefficient of d. Solving for μ_k and putting in the numbers gives $\mu_k = 0.800$.

EVALUATE: The graphing program makes analysis of complicated behavior relatively easy.

6.92. **IDENTIFY:** The power output of the runners is the work they do in running from the basement to the top floor divided by the time it takes to make this run.

SET UP: $P = W/t$ and $W = mgh$.

EXECUTE: **(a)** For each runner, $P = mgh/t$. We must read the time of each runner from the figure shown with the problem. For example, for Tatiana we have $P = (50.2\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})/32\text{ s} = 246.0$ W, which we must round to 2 significant figures because we cannot read the times any more accurate than that using the figure in the text. Carrying out these calculations for all the runners, we get the following results. Tatiana: 250 W, Bill: 210 W, Ricardo: 290 W, Melanie: 170 W. Ricardo had the greatest power output, and Melanie had the least.

(b) Solving $P = mgh/t$ for t gives $t = mgh/P = (62.3\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})/(746\text{ W}) = 13.1$ s, where we have used the fact that 1 hp = 746 W.

EVALUATE: Even though Tatiana had the shortest time, her power output was less than Ricardo's because she weighs less than he does.

6.93. **IDENTIFY:** In part (a) follow the steps outlined in the problem. For parts (b), (c), and (d) apply the work-energy theorem.

SET UP: $\int x^2 dx = \frac{1}{3}x^3$

EXECUTE: **(a)** Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v(l/L)$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time). The mass of a piece of length dl is $dm = (M/L)dl$, and so $dK = \frac{1}{2}(dm)u^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2 dl$, and $K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}$.

(b) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)x} = \sqrt{(3200\text{ N/m})/(0.053\text{ kg})}\,(2.50\times10^{-2}\text{ m}) = 6.1$ m/s.

(c) With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for v,

$$v = \sqrt{\frac{k}{m + M/3}}\,x = \sqrt{\frac{(3200\ \text{N/m})}{(0.053\ \text{kg}) + (0.243\ \text{kg})/3}}(2.50\times10^{-2}\,\text{m}) = 3.9\ \text{m/s}.$$

(d) Algebraically, $\frac{1}{2}mv^2 = \frac{(1/2)kx^2}{(1+M/3m)} = 0.40\ \text{J}$ and $\frac{1}{6}Mv^2 = \frac{(1/2)kx^2}{(1+3m/M)} = 0.60\ \text{J}.$

EVALUATE: For this ball and spring, $\frac{K_{\text{ball}}}{K_{\text{spring}}} = \frac{3m}{M} = 3\left(\frac{0.053\ \text{kg}}{0.243\ \text{kg}}\right) = 0.65$. The percentage of the final kinetic energy that ends up with each object depends on the ratio of the masses of the two objects. As expected, when the mass of the spring is a small fraction of the mass of the ball, the fraction of the kinetic energy that ends up in the spring is small.

6.94. **IDENTIFY:** In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. Write W_0 in terms of the range R and speed v and in terms of the time of flight T and v.

SET UP: In both cases assume v is constant, so $W_0 = RF$ and $R = vT$.

EXECUTE: In terms of the range R and the constant speed v, $W_0 = RF = R\left(\alpha v^2 + \frac{\beta}{v^2}\right).$

In terms of the time of flight $T, R = vt$, so $W_0 = vTF = T\left(\alpha v^3 + \frac{\beta}{v}\right).$

(a) Rather than solve for R as a function of v, differentiate the first of these relations with respect to v, setting $\frac{dW_0}{dv} = 0$ to obtain $\frac{dR}{dv}F + R\frac{dF}{dv} = 0$. For the maximum range, $\frac{dR}{dv} = 0$, so $\frac{dF}{dv} = 0$. Performing the differentiation, $\frac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0$, which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \text{N}\cdot\text{m}^2/\text{s}^2}{0.30\ \text{N}\cdot\text{s}^2/\text{m}^2}\right)^{1/4} = 32.9\ \text{m/s} = 118\ \text{km/h}.$$

(b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv) = 0$; performing the differentiation,

$$3\alpha v^2 - \beta/v^2 = 0.\quad v = \left(\frac{\beta}{3\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \text{N}\cdot\text{m}^2/\text{s}^2}{3(0.30\ \text{N}\cdot\text{s}^2/\text{m}^2)}\right)^{1/4} = 25\ \text{m/s} = 90\ \text{km/h}.$$

EVALUATE: When $v = (\beta/\alpha)^{1/4}$, F_{air} has its minimum value $F_{\text{air}} = 2\sqrt{\alpha\beta}$. For this v, $R_1 = (0.50)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_1 = (0.50)\alpha^{-1/4}\beta^{-3/4}$. When $v = (\beta/3\alpha)^{1/4}$, $F_{\text{air}} = 2.3\sqrt{\alpha\beta}$. For this v, $R_2 = (0.43)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_2 = (0.57)\alpha^{-1/4}\beta^{-3/4}$. $R_1 > R_2$ and $T_2 > T_1$, as they should be.

6.95. **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.

SET UP: $P = W/t$, so $W = Pt$.

EXECUTE: When biking, the person travels 3 times as fast as when walking, so the bike trip takes 1/3 the time. Since $W = Pt$ and the power is the same, the energy when biking will be 1/3 of the energy when walking, which makes choice (a) the correct one.

EVALUATE: Walking is obviously a better way to burn calories than biking.

6.96. **IDENTIFY:** When walking on a grade, metabolic power is required for walking horizontally as well as the vertical climb.

SET UP: $P = W/t$, $W = mgh$.

EXECUTE: $P_{tot} = P_{horiz} + P_{vert} = P_{horiz} + mgh/t = P_{horiz} + mg(v_{vert})$. The slope is a 5% grade, so $v_{vert} = 0.05v_{horiz}$. Therefore $P_{tot} = 300\text{ W} + (70\text{ kg})(9.80\text{ m/s}^2)(0.05)(1.4\text{ m/s}) = 348\text{ W} \approx 350\text{ W}$, which makes choice (c) correct.

EVALUATE: Even a small grade of only 5% makes a difference of about 17% in power output.

6.97. **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.

SET UP: $K = \frac{1}{2}mv^2$.

EXECUTE: The speed when biking is 3 times the speed when walking. Since the kinetic energy is proportional to the square of the speed, the kinetic energy will be $3^2 = 9$ times as great when biking, making choice (d) correct.

EVALUATE: Even a small increase in speed gives a considerable increase in kinetic energy due to the v^2.