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Question I.

① $\lambda^2-3\lambda+2=0.$ $(\lambda-2)(\lambda-1)=0 \Rightarrow \lambda=2, \lambda=1 \Rightarrow y_c: C_1e^x+C_2e^{2x}.$

y_p: $A\sin x+B\cos x+Cx\cos x+Dx\sin x.$

$\Rightarrow 2A\sin x+2B\cos x+2Cx\cos x+2Dx\sin x-3A\cos x-A\sin x+3B\sin[x]-B\cos x$

$-3C\cos x+3Cx\sin x-Cx\cos x-2C\sin x-3Dx\cos x-3D\sin x+2D\cos x-Dx\sin x=$

$\cancel{A\sin x+B\cos x+Cx\cos x+Dx\sin x}$ $x\cos x$

$\Rightarrow A=0.34$ $B=-0.12$ $C=0.1$ $D=-0.3$

$\Rightarrow y(x)=C_1\cdot e^x+C_2e^{2x}+0.34\sin x-0.12\cos x-0.3x\sin x+0.1x\cos x.$

②. $\lambda^2+3\lambda-4\lambda=0.$ $\rightarrow \lambda_1=-4, \lambda_2=1.$ $\Rightarrow y_c=C_1\cdot e^x+C_2\cdot e^{-4x}$

y_p: $A\cdot e^{-4x}+Bxe^{-x}+C\cdot e^{-x}$

$-4Ae^{-4x}-12A\cdot e^{-4x}+16\cdot e^{-4x}-4B\cdot x\cdot e^{-x}+3Be^{-x}-3Be^{-x}\cdot x-2B\cdot e^{-x}+2Be^{-x}\cdot x$

$-4Ce^{-x}-3Ce^{-x}+Ce^{-x}=e^{-4x}+xe^{-x}$

$\Rightarrow A=-\frac{1}{4}$ $B=-\frac{1}{6}$ $C=-\frac{1}{36}$

$\Rightarrow y=C_1\cdot e^x+C_2\cdot e^{-4x}-\frac{x}{4}\cdot e^{-4x}-(\frac{x}{6}+\frac{1}{36})\cdot e^{-x}.$

③. $\lambda^2-9\lambda=0.$ $\Rightarrow \lambda=3$ / $\lambda=-3$ $\Rightarrow y_c=C_1e^{3x}+C_2e^{-3x}.$

$y_p=A\cdot e^{3x}\cdot\sin x+B\cdot e^{3x}\cdot\cos x$

$\Rightarrow -9A\cdot e^{3x}\cdot\sin x+6Ae^{3x}\cdot\cos x+8Ae^{3x}\sin x-9Be^{3x}\cos x+8Be^{3x}\cos x-6Be^{3x}\sin x$

$\cancel{=Ae^{3x}\sin x+}=e^{3x}\cos x$ $\Rightarrow A=\frac{6}{37}$ $B=-\frac{1}{37}$

$\Rightarrow y=C_1e^{3x}+C_2e^{-3x}(\frac{6}{37}\sin x-\frac{1}{37}\cos x).$

④ $\lambda^2-2\lambda+1=0 \Rightarrow \lambda_1=\lambda_2=1$ $\Rightarrow y=(C_1+C_2x)\cdot e^x.$

y_p: $Ax^3e^x+Bx^2e^x+Cxe^x\cancel{+De^x}$

$\Rightarrow Ax^3e^x-6Ae^xx^2-2Ae^xx^3+6Ae^x\cdot x+6Ae^xx^2+Ae^xx^3+Bx^2e^x$

$-4Be^xx-2e^xx^2+2Be^x+4Be^xx+Be^xx^2+Ce^xx-2Ce^x-2Ce^x\cdot x$

$+2e^x+e^x\cdot x\cancel{+De^x-2De^x+De^x}=6xe^x$

$\Rightarrow A=1$ $B=0$ $C=0$

$\Rightarrow y=(C_1+C_2x)\cdot e^x+x^3e^x$

Han Yibei 5957oP1012)

Question 2.

a. $3xe^{-3x} - 2e^{3x}\cos x.$

$\Rightarrow \cancel{A_1 \cdot x^3 e^{-3x} + A_2 x^2 e^{-3x}} + A_1 x \cdot e^{-3x} + A_2 e^{-3x} + B_1 e^{3x} \cdot \cos x + B_2 \cdot e^{3x} \cdot \sin x$

b. $2xe^{x} + e^{x}\sin 2x$

$\Rightarrow A_1 \cdot x^3 e^{x} + A_2 x^2 e^{x} + A_3 \cdot x \cdot e^{x} + A_4 \cdot e^{x} + B_1 \cdot e^{x} \cdot \sin 2x + B_2 \cdot e^{x}\cos 2x$

c. $2^{x} \Rightarrow A 2^{x}.$

d. $\cosh x = \frac{e^{x}+e^{-x}}{2} \Rightarrow A \cdot e^{x} + Be^{-x}$

Question 3.

(1) $y_c(t) = (C_1 + C_2 t) \cdot e^{t}.$ $y_1 = e^{t}$ $y_2(t) = t \cdot e^{t}.$

$W = \begin{vmatrix} e^{t} & t \cdot e^{t} \\ e^{t} & e^{t} + te^{t} \end{vmatrix} = e^{2t} + \cancel{e^{2t} t} - \cancel{t \cdot e^{2t}} = e^{2t}.$

$Y_p(t) = -e^{t}\int \frac{t e^{t} \cdot \frac{e^{t}}{t}}{e^{2t}} dt + t \cdot e^{t} \cdot \int \frac{e^{t} \cdot \frac{e^{t}}{t}}{e^{2t}} dt + (-cy_1 + ky_2)$

$= e^{t} \cdot t(\ln|t| - 1). = e^{t} \cdot t \cdot \ln|t|.$

$\Rightarrow Y = C_1 e^{t} + C_2 t \cdot e^{t} + te^{t} \cdot t\ln t$

(2) $y_c = C_1 \cos 2x + C_2 \sin 2x$ $y_1 = \cos 2x$ $y_2 = \sin 2x$

$W = \begin{vmatrix} \cos 2t & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2\cos^2 2x + 2\sin^2 2x = 2.$

$\Rightarrow Y_p(x) = -\cos 2t \cdot \int \frac{\cancel{\cos}\ \sin 2t \cdot 2\tan x}{2} dt + \sin 2t \cdot \int \frac{\cos 2t \cdot 2\tan t}{2} dt + (-cy_1 + ky_2).$

$= -x\cos x \cancel{= \cos\ldots \sin 2x \ldots} + \sin 2x \ln|x|$

$\Rightarrow Y = y_c + y_p = C_1\cos 2x + C_2 \sin 2x - x\cos x + \sin 2x \ln|x|.$

(3) $y_c = C_1 \cdot e^{-x} + C_2 x e^{-x}$ $y_1 = e^{-x}$ $y_2 = xe^{-x}$

$W = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x} - xe^{-x} \end{vmatrix} = e^{-2x} - \cancel{xe^{-2x}} + \cancel{xe^{-2x}} = e^{-2x}$

$\Rightarrow Y_p = -e^{-x}\int \frac{x \cdot e^{-x} \cdot 3e^{-x}\sqrt{x+1}}{e^{-2x}} dx + e^{-x} \cdot x \int \frac{e^{-x} \cdot 3e^{-x}\sqrt{x+1}}{e^{-2x}} dx$

$= \left[\frac{6(x+1)^{\frac{5}{2}}}{5} - 2(x+1)^{\frac{3}{2}}\right] \cdot -e^{-x} + e^{-x} \cdot + 2(x+1)^{\frac{3}{2}} \cdot x \cdot e^{-x} = \frac{4}{5} \cdot e^{-x}(1+x)^{\frac{5}{2}}$

$\Rightarrow Y = Y_c + Y_p = C_1 \cdot e^{-x} + C_2 \cdot x \cdot e^{-x} + \frac{4}{5} \cdot e^{-x}(1+x)^{\frac{5}{2}}.$

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Question 4.

① $\cancel{(\lambda x+1)}$. $(\lambda-1)\cdot\lambda-4\lambda+6=0$ $\therefore \lambda^2-5\lambda+6=0$. $(\lambda-2)(\lambda-3)=0$ $\lambda=2/3$

$\Rightarrow y=C_1x^2+C_2x^3$

②. $x^2\cdot y''-2y=\frac{6\ln x}{x}$ $\Rightarrow (\lambda-1)\cdot\lambda-2=0$. $\lambda^2-\lambda-2=(\lambda-2)(\lambda+1)=0$ $\Rightarrow y_c=C_1x^2+C_2\frac{1}{x}$

$y_1=x^2$ $y_2=\frac{1}{x}$ $W=\begin{vmatrix} x^2 & \frac{1}{x} \\ 2x & -\frac{1}{x^2}\end{vmatrix}=-2-1=-3$

$y_p=C_1(x)x^2+C_2(x)\cdot\frac{1}{x}$. $C_1'(x)x^2+C_2'(x)\frac{1}{x}=0$. $2x\cdot C_1'(x)+\ln|x|\cdot C_2'(x)=\frac{6\ln x}{x^3}$.

$C_1'=2\ln x\cdot x^{-4}$ $C_2=-2\ln x\cdot x^{-1}$ $\Rightarrow C_1=-\frac{2}{9}x^{-3}-\frac{2}{6}\ln x\, x^{-3}+r_1$. $C_2=-\ln^2|x|+r_2$.

$\Rightarrow y=C_1x^2+C_2\frac{1}{x}-\frac{\ln^2|x|}{x}-\frac{2}{3}\frac{\ln|x|}{x}$

③. $\lambda(\lambda-1)-3\lambda+4=0$ $\lambda^2-4\lambda+4=0$ $\Rightarrow \lambda_1=\lambda_2=2$. $\Rightarrow y_c=C_1(x-2)^2+C_2(x-2)^2\cdot\ln(x-2)$

$\therefore y_1=(x-2)^2$ $y_2=(x-2)^2\ln x$ $W=\begin{vmatrix}(x-2)^2 & (x-2)^2\ln x \\ 2x-4 & -4+\frac{4}{x}+x-4\ln x+2x\ln x\end{vmatrix}=\frac{(x-2)^4}{x}$

$\Rightarrow$ $y=C_1(x-2)^2+C_2\cdot(x-2)^2\ln x$. $C_1'(x-2)^2+C_2'(x-2)^2\cdot\ln|x-2|=0$.

$C_1'(2x-4)+C_2'(x-2.+(2x-4)\cdot\ln|x-2|)=\frac{x}{(x-2)^2}$.

$\Rightarrow$. $C_1'=-\frac{x}{(x-2)^3}\ln|x-2|$ $\Rightarrow$ $\begin{cases} C_1=\frac{2(\ln|x-2|+1)x-\ln(x-2)-3}{2(x-2)^2}+c \\ C_2=\frac{x-1}{(x-2)^2}+c\end{cases}$

$C_2'=\frac{x}{(x-2)^3}$

$\Rightarrow$. $y_p=x-\frac{3}{2}$ $\Rightarrow y=C_1\cdot(x-2)^2+C_2\cdot\ln|x-2|(x-2)^2+x-\frac{3}{2}$.

Question 5

(1) $e^{\alpha t}$. $\Rightarrow$ $\alpha^2\cdot 2x+\alpha^2+4\alpha x-4=0$. $2\alpha^2+4\alpha=0$ $\alpha^2-4=0$ $\Rightarrow \alpha=-2$. $\Rightarrow y_1=e^{-2x}$.

$y_2(t)=e^{-2t}\left(C_1\int\frac{e^{-\int\frac{4t}{2t+1}dt}}{(e^{-2t})^2}dt+C_2\right)=C_1t+C_2\cdot e^{-2t}$.

(2) $y_2=\frac{e^x}{x}\left(C_1\int\frac{e^{-\int\frac{2}{x}dx}}{(\frac{e^x}{x})^2}dx+C_2\right)=C_1\frac{(-\frac{1}{2})e^{-x}}{x}+C_2\frac{e^x}{x}$ $\Rightarrow$. $xy=C_1\cdot e^{-x}+C_2\cdot e^x$.

(3) guess $y=x$ $\Rightarrow y=x\cdot\left(C_1\cdot\int\frac{e^{-\int(-\frac{1}{x\ln x})dx}}{x^2}dx+C_2\right)=C_1(-1-\ln x)+C_2x$

$\Rightarrow y=C_1(1+\ln x)+C_2x$.

(4). guess $y=(x^2+1)$.

$y_2=(x^2+1)\cdot\left(C_1\cdot\int\frac{e^{-\int x\,dx}}{(x^2+1)^2}+C_2\right)=C_1\cdot\frac{1}{2}(x+(1+x^2)\arctan x)+C_2(x^2+1)$

$\Rightarrow y=C_2(x^2+1)+C_1\cdot(x+(x^2+1)\,\cancel{\tan}\arctan x)$.

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Question 6.

①. $x = \dot{y} - y \Rightarrow \ddot{y} - \cancel{\dot{y}} + \dot{y} - y - 8y = 0$ $\quad \ddot{y} - 9y = 0.$ $\quad \lambda^2 - 9 = 0$ $\lambda_1 = 3$ $\lambda_2 = -3.$

$$\Rightarrow y = C_1 \cdot e^{3t} + C_2 \cdot e^{-3t} \Rightarrow x = 2C_1 \cdot e^{3t} - 4C_2 e^{-3t}$$

② $y = -\frac{\dot{x}}{3} + \frac{x}{3}$ $\qquad -\ddot{x} + \dot{x} = 9x - \dot{x} + x \Rightarrow \ddot{x} - 2\dot{x} + 10x = 0.$ $\quad \lambda = 1 \pm 3i$

$$\Rightarrow x = e^{t} \cdot (C_1 \cos 3t + C_2 \sin 3t) \Rightarrow y = e^{t} (C_1 \sin 3t - C_2 \cos 3t).$$

③ $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{pmatrix}$ $\quad \det(A - \lambda I) = 0 \Rightarrow \lambda_1 = 2,\ \lambda_2 = -1,\ \lambda_3 = 1$

① $\lambda = 2.$ $\begin{pmatrix} -1 & -1 & 1 \\ 1 & -1 & -1 \\ 2 & -1 & -2 \end{pmatrix} \begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \end{pmatrix} = 0$ $\quad 2v_{11} - 3v_{12} - 2v_{13} = 0$ $\qquad \{1, 0, 1\}$

② $\lambda = -1$ $\{-1, 3, 5\}$ $\qquad$ ③ $\lambda = 1$ $\{1, 1, 1\}.$

$$\begin{cases} x = C_1 \cdot e^{2t} - C_2 \cdot e^{-t} + C_3 \cdot e^{t} \\ y = 3C_2 \cdot e^{-t} + C_3 \cdot e^{t} \\ z = C_1 \cdot e^{2t} + 5C_2 \cdot e^{-t} + C_3 \cdot e^{t} \end{cases}$$

④. $A = \begin{pmatrix} 4 & -1 & -1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ $\Rightarrow \lambda = \{3, 3, 2\}$

① $(A - \lambda I) \cdot v_1 = 0$ $\quad v_1 = \{1, 0, 1\}$

② $(A - \lambda I) \cdot v_2 = v_1$ $\quad v_2 = \{1, 1, 0\}$

③ $(A - \lambda I) \cdot v_3 = 0$ $\quad v_3 = \{1, 1, 1\}$

$$\begin{cases} x = C_1 \cdot e^{3t} + C_2 e^{3t} + C_3 \cdot e^{2t} \\ y = C_2 \cdot e^{3t} + C_3 \cdot e^{2t} \\ z = C_1 \cdot e^{3t} + C_3 \cdot e^{2t}. \end{cases}$$

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Hanyibei 5937-910123

6. $A = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 2 & 3 & -1 \end{vmatrix} \Rightarrow \det|A-\lambda I| = 0 \Rightarrow \Rightarrow -x^3+4x^2-x-6-2-(-6+3\lambda-1-\lambda).$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 3-\lambda & -1 \\ 2 & 3 & -1-\lambda \end{vmatrix} = 0. \quad \Rightarrow 2 \begin{pmatrix}1\\0\\1\end{pmatrix} \quad 3+i \begin{pmatrix}\frac12-\frac12 i\\1\\-\frac12-\frac32 i\end{pmatrix} \quad 3-i \begin{pmatrix}\frac12+\frac12 i\\1\\-\frac12+\frac32 i\end{pmatrix}$$

$$e^{\lambda_2 t} = e^{3t}\left[\begin{pmatrix}\frac12\\1\\-\frac12\end{pmatrix}\cos t - \begin{pmatrix}-\frac12\\0\\\frac32\end{pmatrix}\sin t\right]. \quad e^{\lambda_3 t} = e^{3t}\cancel{\left[\begin{pmatrix}\frac12\\1\\-\frac12\end{pmatrix}\sin t + \begin{pmatrix}\frac12\\0\\\frac32\end{pmatrix}\cos t\right]}$$

$$[\cos t + i\sin t]\left[\begin{pmatrix}\frac12\\1\\-\frac12\end{pmatrix} + i\begin{pmatrix}\frac12\\0\\\frac32\end{pmatrix}\right]$$

$$\Rightarrow \begin{cases} x = C_1 e^{2t} + C_2 e^{3t}\cos t + C_3 e^{3t}\sin t \\ y = C_2 e^{3t}(\cos t - \sin t) + C_3 e^{3t}(\cos t + \sin t) \\ z = C_1 e^{2t} + C_2 e^{3t}(2\cos t + \sin t) + C_3 e^{3t}(-\cos t + 2\sin t) \end{cases}$$

Question 7.

1. $A = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \Rightarrow \det|A-\lambda I| = 0 \Rightarrow \lambda = -1/1. \Rightarrow \lambda = -1 \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = 0. \Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\lambda = 1 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} V_{21} \\ V_{22} \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

$\Rightarrow y_1 = \begin{pmatrix}1\\1\end{pmatrix}e^t \quad y_2 = \begin{pmatrix}1\\-1\end{pmatrix}e^{-t}.$

$\Rightarrow \Phi(t) = y_1 \cdot y_2 = \begin{pmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{pmatrix} \Rightarrow \Phi^{-1}(t) = \begin{pmatrix} \frac{e^{-t}}{2} & \frac{e^{-t}}{2} \\ \frac{e^t}{2} & -\frac{e^t}{2} \end{pmatrix}.$

$\int \Phi^{-1}(t)\cdot b(t)\, dt) = \int \begin{pmatrix} \frac{e^{-t}}{2} & \frac{e^{-t}}{2} \\ \frac{e^t}{2} & -\frac{e^t}{2} \end{pmatrix}\begin{pmatrix} 2e^t \\ t^2 \end{pmatrix} = \int \begin{pmatrix} 1 + \frac{t^2}{2}e^{-t} \\ e^{2t} - \frac{e^t t^2}{2} \end{pmatrix} = \begin{pmatrix} t - \frac{e^t t^4}{8} \\ e^t + \frac{e^t t}{2} + te^t - \frac{e^t t^2}{2} \end{pmatrix}$

$y(t) = \Phi(t)\left(C + \int \Phi^{-1}(t)\, b(t)\, dt\right).$

2. $\begin{cases} x = \cancel{2C_1e^{3t} - 4C_2e^{-3t}} \quad C_1e^t + C_2e^{-t} + te^t - t^2 - 2 \\ y = \cancel{C_1e^{3t} + C_2e^{-3t}} \quad C_1e^t - C_2e^{-t} + (t-1)e^t - 2t \end{cases}$

Han Yibei 5193709110123

④ $A=\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \Rightarrow \lambda = 4/1.$ $\lambda=4 \Rightarrow \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} = 0. \Rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$

$\lambda=1$ $\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} v_{11} \\ v_{21} \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

$y_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t.$ $y_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot e^{4t}.$

$b = \begin{pmatrix} 4e^{5t} \\ 0 \end{pmatrix}.$ $\Phi(t) = \begin{pmatrix} e^t & 2e^{4t} \\ -e^t & e^{4t} \end{pmatrix} \Rightarrow \Phi^{-1}(t) = \begin{pmatrix} \frac{e^{-t}}{3} & -\frac{2e^{-t}}{3} \\ \frac{e^{-t}}{3} & \frac{e^{-4t}}{3} \end{pmatrix}$

$\int \Phi^{-1}(t) \cdot b(t) = \int \begin{pmatrix} \frac{4}{3} e^{4t} \\ \frac{4}{3} \cdot e^{t} \end{pmatrix} = \begin{pmatrix} \frac{e^{4t}}{3} \\ \frac{e^{4t}}{3} \end{pmatrix}$

$y(t) = \Phi(t)\left(C + \int \Phi^{-1}(t)\, b(t)\right) dt.$

$\Rightarrow \begin{cases} x = \cancel{e^t(C_1\cos 3t + C_2 + \sin 3t)} \quad C_1 e^{t} + 2C_2 e^{4t} + 3e^{5t} \\ y = \cancel{e^t(C_1\sin 3t - C_2\cos 3t)} \quad -C_1 e^{t} + C_2 e^{4t} + e^{5t} \end{cases}$

⑤. $A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \Rightarrow \lambda = \{2, -1\} \Rightarrow$ $\lambda = 2 \Rightarrow \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$

$\lambda = -1 \Rightarrow \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

$\Rightarrow y_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot e^{2t}$ $y_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot e^{-t}.$ $b = \begin{pmatrix} 0 \\ -5\sin t \end{pmatrix}$

$\Rightarrow \Phi(t) = \begin{pmatrix} 2e^{2t} & e^{-t} \\ e^{2t} & -e^{-t} \end{pmatrix} \Rightarrow$ $\Phi^{-1}(t) = \begin{pmatrix} \frac{e^{-2t}}{3}, & \frac{e^{-2t}}{3} \\ \frac{e^t}{3} & -\frac{2e^t}{3} \end{pmatrix}$

$\int \Phi^{-1}(t) \cdot b(t) = \int \begin{pmatrix} -\frac{5}{3}\sin t \cdot e^{-2t} \\ \frac{10}{3} \cdot \sin t \cdot e^t \end{pmatrix} dt = \begin{pmatrix} \frac{1}{3} e^{-2t}(\cos(t) + 2\sin t) \\ \frac{5}{3} e^{t} (-\cos t + \sin t). \end{pmatrix}.$

$y(t) = \Phi(t)\left(C + \int \Phi^{-1}(t)\, b(t)\, dt\right) =$

$\Rightarrow \begin{cases} x = \cancel{C_1 e^t + C_2 e^{2t} + C_3 e^{-t}} \quad C_1 e^{-t} + 2C_2 e^{2t} - \cos t + 3\sin t. \\ y = \cancel{C_1 e^t - 3C_3 e^{-t}} \quad -C_1 e^{-t} + C_2 e^{2t} + 2\cos t - \sin t. \end{cases}$

$\cancel{z} =$.

Han Yibei 5437 of 10123

① $A = \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix} \Rightarrow \lambda = (3, 0). \Rightarrow \lambda = 3 \quad \begin{pmatrix} -1 & -1 \\ -2 & -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

$\lambda = 0. \quad \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

$\Rightarrow y_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot e^{3t}. \quad y_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}. \quad b = \begin{pmatrix} 0 \\ 18t \end{pmatrix}.$

$\Rightarrow \Phi(t) = \begin{pmatrix} e^{3t} & 1 \\ -e^{3t} & 2 \end{pmatrix} \Rightarrow \Phi^{-1}(t) = \begin{pmatrix} \frac{2e^{-3t}}{3} & -\frac{1}{3} \\ -\frac{1}{3}e^{-3t} & \frac{1}{3} \end{pmatrix}.$

$\Rightarrow \int \Phi^{-1}(t) \cdot b(t)\, dt = \int \begin{pmatrix} 6t \\ 6t \end{pmatrix} dt = \begin{pmatrix} 3t^2 \\ 3t^2 \end{pmatrix}.$

$\Rightarrow y(t) = \Phi(t) \left(C + \int \Phi^{-1}(t)\, b(t)\, dt \right) =$

$\Rightarrow \begin{cases} x = c_1 e^{3t} + 3t^2 + 2t + c_2 \\ y = -c_1 e^{3t} + 6t^2 - 2t + 2c_2 - 2 \end{cases}$

Question 8.

① $A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} \quad \exp A = \left(I + A + \frac{A^2}{2} + \cdots + \frac{A^k}{k!} + \cdots \right) = \sum_{k=0}^{\infty} \frac{A^k}{k!} = \begin{pmatrix} 1 + 3 + \frac{3^2}{2!} + \cdots & 0 \\ 0 & 1 - 2 + \frac{-2^2}{2!} + \cdots \end{pmatrix}$

$= \begin{pmatrix} e^3 & 0 \\ 0 & e^{-2} \end{pmatrix}.$

② $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow e^{\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}} = e^{\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}} \cdot e^{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}$

$= \begin{pmatrix} e^2 & 0 \\ 0 & e^2 \end{pmatrix} \cdot \left[1 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{1} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right] = \begin{pmatrix} e^2 & 0 \\ 0 & e^2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} e^2 & e^2 \\ 0 & e^2 \end{pmatrix}.$

③. $A = \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} -\frac{i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix}.$

$= \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-i} & 0 \\ 0 & e^{i} \end{pmatrix} \begin{pmatrix} -\frac{i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \cos 1 - i \sin 1 & 0. \\ 0 & \cos 1 + i \sin 1 \end{pmatrix}$

$\begin{pmatrix} -\frac{i}{2} & \frac{1}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \cos 1 & \sin 1 \\ -\sin 1 & \cos 1 \end{pmatrix}.$

4. $A = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} e^{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}.$