Chapter 1 – Electric Charge and Electric Field

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Agenda

Introduction

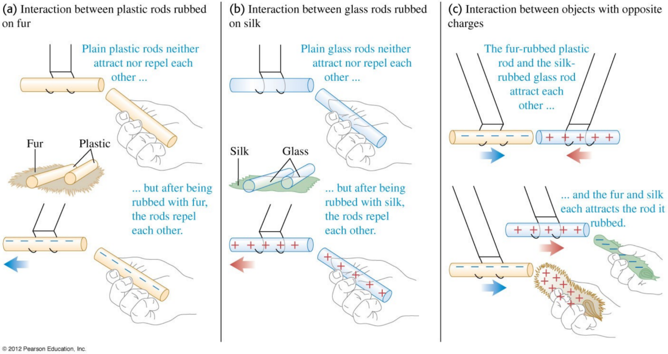
Four Fundamental Interactions

Interaction	Particles	Relative Strength	Range
Gravitational always attractive holds planets in their orbits around Sun	any massive particle	$\sim 10^{-38}$	long
Electromagnetic attractive/repulsive fundamental in optics, chemistry, biology; source of friction	electrically charged	$\sim 10^{-2}$	long
Weak necessary for buildup of heavy nuclei; responsible for radioactive decay (beta decay)	quarks, leptons	$\sim 10^{-6}$	short $\sim 10^{-18}$ m (0.1% of the diameter of the proton)
Strong holds protons and neutrons together in the nucleus	hadrons (protons, neutrons, mesons)	1	short $\sim 10^{-15}$ m (diameter of a medium sized nucleus)

Electric Charge

Electric Charge

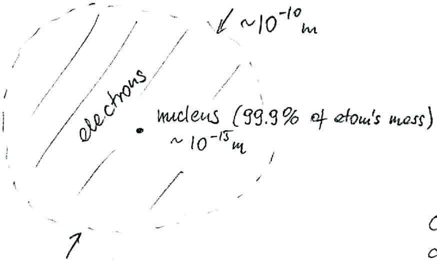
First (qualitative) observations: amber rubbed with a piece of wool (ancient Greece 600 B.C.)



- Two kinds of the electric charge: (+) and (-).
- Charges of the same kind repel each other.
- Charges of different kinds attract each other.

Electric Charge and the Structure of Matter

No visible change in the external appearance of charged objects
$\Longrightarrow$ atomic-scale property



nucleus : protons (+)
neutrons (0)

around nucleus : electrons (−)

Charge of proton $+e = +1.6 \times 10^{-19}$ C
charge of electron $-e = -1.6 \times 10^{-19}$ C

NEUTRAL ATOM
(# protons = # electrons) $\longrightarrow$

positive ion
(# protons > # electrons)

negative ion
(# protons < # electrons)

Conservation of Charge and Quantization of Charge

CONSERVATION OF CHARGE

The algebraic sum of all the electric charges in any closed system is zero.

(Electric charges do not disappear/are created, but are moved between objects.)

QUANTIZATION OF CHARGE

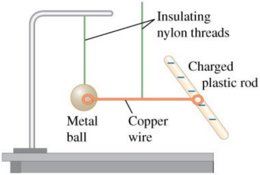
Any electric charge is a multiple of the electron's (or proton's) charge

$$Q = n \cdot (\pm e), \qquad \textit{where } n = 0, 1, 2, \ldots$$

(The electron's charge in a natural unit of the electric charge.)

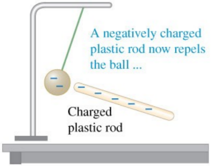
Conductors and Insulators

(a)

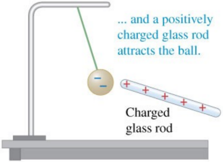


The wire conducts charge from the negatively charged plastic rod to the metal ball.

(b)



(c)



Copper – charges move freely (example of a conductor)
Nylon – no mobile charges (example of an insulator)

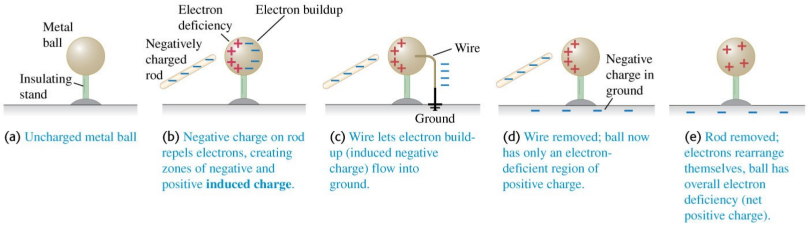
Matter and Electric Charge Transport

Classification of solids with respect to electric charge transport properties

1. **Conductors** – mostly metals (outer electrons are weakly bound within the atom and become itinerant).
2. **Insulators** – most non-metals; no mobile charges.
3. **Semiconductors** – become conducting above a certain temperature or after introducing impurities/defects; (e.g. Si, Ge, InGaAs).
4. **Superconductors** – charge moves without any resistance; perfect conductors; (e.g. Hg below 4.2 K; high-temperature superconducting ceramics below 180 K).

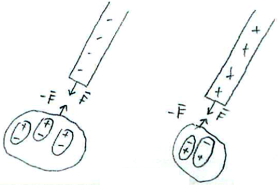
Induced Charges

Example: Charging a metal ball by induction.



Example: Electric force on an uncharged object.

polarized insulator

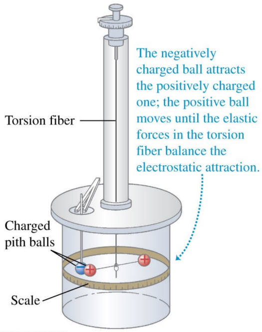


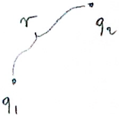
polarization = rearrangement of charge

Coulomb's Law

Coulomb's Law. Experimental Discovery (late 18th century)

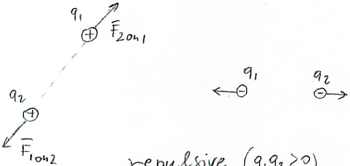
(a) A torsion balance of the type used by Coulomb to measure the electric force



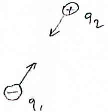


electric force between two point charges $\nearrow \propto \frac{1}{r^2}$, $\searrow \propto q_1 q_2$

$$\boxed{|\vec{F}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}}$$

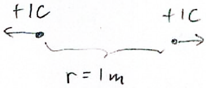


repulsive $(q_1 q_2 > 0)$



attractive $(q_1 q_2 < 0)$

- Fundamental constant $\varepsilon_0 = 8.854 \times 10^{-12}\ \left[\frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}}\right]$.
- 1 C is a huge charge.



$|F| \sim 9 \times 10^9\ \mathrm{N}$ (!)

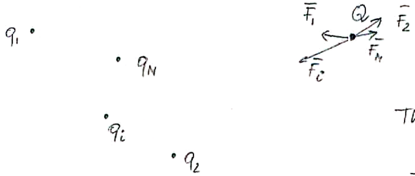
- Compare the force of electric repulsion between two α particles with the force of their gravitational attraction.

$m_\alpha = 6.64 \times 10^{-27}\ \mathrm{kg}$; $\quad q_\alpha = +2e = 3{,}2 \times 10^{-19}\ \mathrm{C}$

$$\frac{F_{el}}{F_{gr}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha^2}{r^2}}{G\frac{m_\alpha^2}{r^2}} \approx 3.1 \times 10^{35}$$

can safely neglect gravitational attraction

Principle of Superposition of Forces



The net force on Q:

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \ldots + \bar{F}_i + \ldots + \bar{F}_N$$

force exerted on Q by q_i
(as if the other charges were not present)

IMPORTANT! For the time being we will consider **static** (fixed-position) charges — **ELECTROSTATICS**.

Electric Field

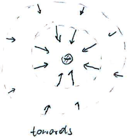
Electric Field of a Point Charge

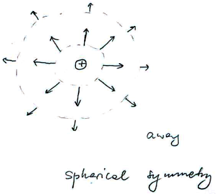
$\hat{r} = \frac{\overline{r}}{|\overline{r}|} = \frac{\overline{r}}{r}$ — the unit vector in the direction of $\overline{r}$

The magnitude of the Coulomb force, due to the source charge q on a test charge $q_0 > 0$, is $F_0 = \frac{1}{4\pi\varepsilon_0}\frac{|qq_0|}{r^2}$. The magnitude of the corresponding **electric field** is $E = \frac{F_0}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$.

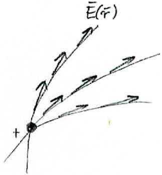
In the vector notation

$$\boxed{\overline{E}(\overline{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}.}$$





Electric Field Lines



$\bar{E}(\bar{r})$ always tangential to the field line at $\bar{r}$

How is the magnitude visualised?

weak field

sparse

strong field

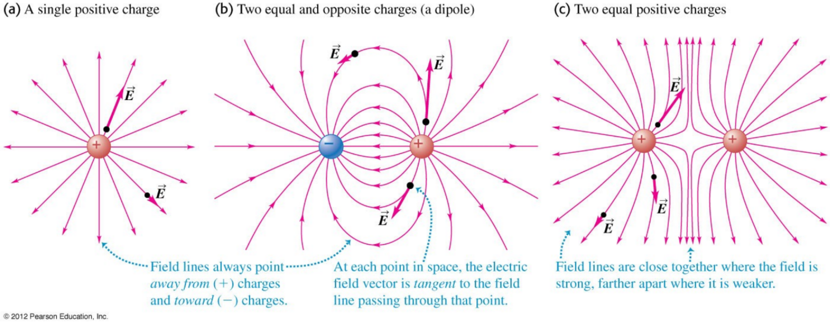
dense

(magnitude indicated by density of lines)

↳ caution 2D vs. 3D

$\frac{\#\text{ lines}}{2\pi r}$ $\quad$ $\frac{\#\text{ lines}}{4\pi r^2}$

Electric Field Lines. Illustration



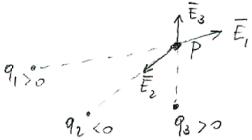
Note.

- Field lines *are not* trajectories of charged particles.
- When can two electric lines cross?

Superposition Principle for Electric Field

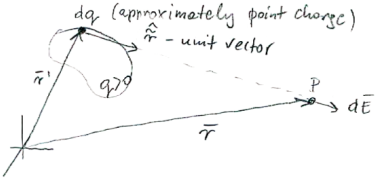
(1) single charge

(2) discrete distribution (e.g. three charges)



net electric field $\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3$

(3) continuous distribution



Contribution due to dq $\hat{\bar{r}}$

$$d\bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{|\bar{r}-\bar{r}'|^2}\frac{(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|}$$

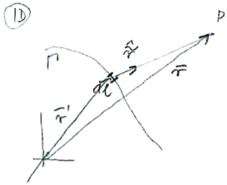
Sum of all contributions (integral)

$$\bar{E} = \int_{\text{object } A} d\bar{E}$$

Electric Field Due to Charged Lines, Surfaces and Solids

dimension	object	charge density	infinitesimal charge
1D	line	λ [C/m]	$dq = \lambda \cdot dl$
2D	surface	σ [C/m^2]	$dq = \sigma\, dS$
3D	solid	ρ [C/m^3]	$dq = \rho\, dV$

1D



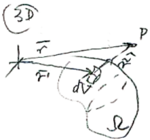
may depend on position

$$\bar E = \frac{1}{4\pi\varepsilon_0}\int_\Gamma \frac{\lambda(\bar r')}{|\bar r-\bar r'|^2}\,\hat{\tilde r}\, dl$$

2D

$$\bar E = \frac{1}{4\pi\varepsilon_0}\iint_\Sigma \frac{\sigma(\bar r')}{|\bar r-\bar r'|^2}\,\hat{\tilde r}\, dS$$

3D



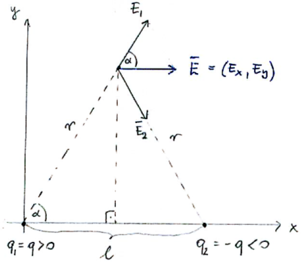
$$\bar E = \frac{1}{4\pi\varepsilon_0}\iiint_\Omega \frac{\rho(\bar r')}{|\bar r-\bar r'|^2}\,\hat{\tilde r}\, dV$$

Note. An object is said to be charged uniformly if λ, σ, ϱ are constant throughout it.

Electric Field Calculations. Examples

Example 1

Find the electric field due to a dipole on the axis of symmetry, perpendicular to the dipole. *Electric dipole* — two charges of the same magnitude, but opposite signs.



$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\cos\alpha = \frac{\frac{\ell}{2}}{r} = \frac{\ell}{2r}$$

→ "x" components: $E_{1x} = E_{2x} = E_1\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{q\ell}{2r^3} = \frac{1}{8\pi\varepsilon_0}\frac{q\ell}{r^3}$

$$E_x = E_{1x} + E_{2x} = \underline{\frac{1}{4\pi\varepsilon_0}\frac{q\ell}{r^3}} > 0$$

→ "y" components; Cancel out

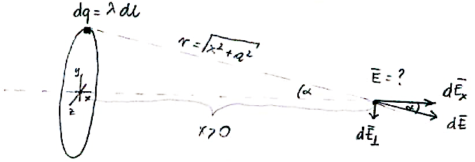
$$E_y = E_{1y} + E_{2y} = E_{1y} - E_{1y} = \underline{0}$$

Hence

$$\boxed{\bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\ell}{r^3}\hat{n}_x}$$

Example 2

Find the electric field due to a circle with radius a, charged with constant linear density $\lambda > 0$, on the axis of symmetry perpendicular to the circle's plane.



$$\cos\alpha = \frac{x}{\sqrt{x^2+a^2}}$$

The component $\bar{E}_\perp$ cancels out (symmetry of the ring; find pairs of charges)

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{a^2+x^2}$$

$$dE_x = dE \cdot \cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{dq}{a^2+x^2}\frac{x}{\sqrt{a^2+x^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\overbrace{\lambda\, dl}^{dq}}{(a^2+x^2)^{3/2}}$$

Net E_x

$$E_x = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{1}{4\pi\varepsilon_0}\frac{x\lambda\, dl}{(x^2+a^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\lambda}{(x^2+a^2)^{3/2}}\underbrace{\int_{\text{ring}} dl}_{2\pi a} =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{x \overbrace{2\pi a \lambda}^{= q \text{ (total charge on the ring)}}}{(x^2+a^2)^{3/2}}$$

Hence

$$\boxed{\overline{E} = \frac{1}{4\pi\varepsilon_0} \frac{x q}{(x^2+a^2)^{3/2}} \hat{n}_x}$$

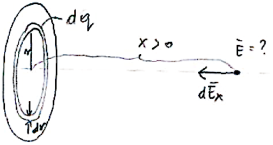
where $q = 2\pi a \lambda$

Discussion. Limit case: if $x \gg a$, then $\overline{E} \longrightarrow \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2} \hat{n}_x$ (electric field of a point charge).

Example 3

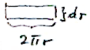
Find the electric field due to a disk with radius R, charged uniformly with surface density $\sigma < 0$, on the axis of symmetry perpendicular to the disk.

Hint. Treat the disk as composed of rings; use the results of Example 2.



Charge on a single ring with radius r, width dr

$$dq = \sigma \cdot dS = \sigma \, 2\pi r \, dr$$



contribution due to a single ring

$$dE_x = \frac{1}{4\pi\varepsilon_0} \frac{x \, dq}{(x^2+r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{x \, (\sigma 2\pi r \, dr)}{(x^2+r^2)^{3/2}}$$

Total field

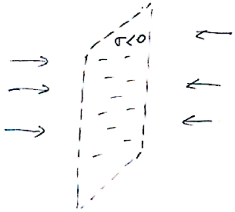
$$E_x = \int_{disk} dE_x = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{x \sigma 2\pi r \, dr}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0} \int_0^R \frac{r \, dr}{(x^2+r^2)^{3/2}} = \left\{ \begin{array}{l} \text{substitute} \\ u = x^2 + r^2 \\ du = 2r\,dr \end{array} \right\}$$

$$= \frac{\sigma x}{2\varepsilon_0} \int_{x^2}^{x^2+R^2} \frac{1}{2} \frac{du}{u^{3/2}} = \frac{\sigma x}{4\varepsilon_0} \left(-2u^{-1/2}\right)\Big|_{x^2}^{x^2+R^2} = \frac{\sigma x}{2\varepsilon_0} \left[-\frac{1}{\sqrt{x^2+R^2}} + \frac{1}{x} \right] =$$

$$= \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{1}{\sqrt{1+R^2/x^2}} \right]$$

$$\boxed{\bar{E} = \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{1}{\sqrt{1+\frac{R^2}{x^2}}} \right) \hat{n}_x}$$

Discussion. Limit case: $R \to \infty$ (infinitely large, uniformly charged sheet)



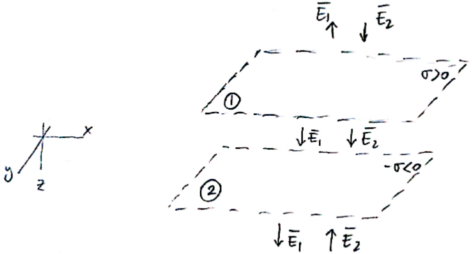
$\bar{E} \to \frac{\sigma}{2\varepsilon_0} \hat{n}_x$ for $x > 0$ (and $\bar{E} \to -\frac{\sigma}{2\varepsilon_0} \hat{n}_x$ for $x < 0$)

UNIFORM ELECTRIC FIELD

Example 4

Find the electric field due two parallel infinite sheets, charged uniformly with surface densities $+\sigma$ and $-\sigma$.

Hint. Use the results of Example 3 and the superposition principle.



$$|\bar{E}_1| = |\bar{E}_2|$$

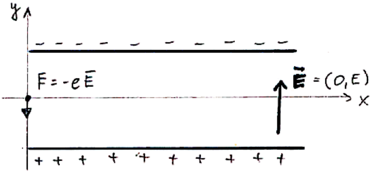
$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

$$\bar{E} = \begin{cases} \frac{\sigma}{\varepsilon_0}\hat{n}_z & \text{between sheets} \\ 0 & \text{above the upper sheet} \\ & \text{below the lower sheet} \end{cases}$$

Conclusion. The electric field is uniform in the region between the sheets, and there is no electric field in the remaining part of space.

Example 5

Discuss motion of an electron in the region between the sheets from the previous example.



initial conditions

$\bar{r}(0) = (v_0, 0)$

$\bar{v}(0) = (0, 0)$

$$\bar{F} = (0, -eE) \overset{\bar{F}=m\bar{a}}{\Longrightarrow} \bar{a} = \left(0, -\frac{e}{m}E\right) \underset{\bar{v}=\int \bar{a}\,dt}{\overset{\bar{a}=\dot{\bar{v}}}{\Longrightarrow}} \bar{v} = \left(v_0, -\frac{e}{m}Et\right)$$

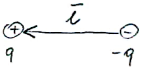
$$\Downarrow \ \dot{\bar{r}} = \bar{v}, \quad \bar{r} = \int \bar{v}\,dt$$

$$\bar{r} = \left(v_0 t, -\frac{1}{2}\frac{e}{m}Et^2\right)$$

Conclusion. The electron moves along a parabola $y(x) = -\frac{1}{2}\frac{e}{mv_0^2}Ex^2$.

Electric Dipole

Electric Dipole Moment

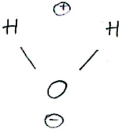


(point charges)

Electric dipole moment (units: [C·m])

$$\boxed{\bar{p} \overset{\text{def}}{=} q\bar{l}}$$

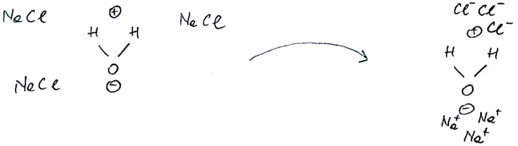
Real–world example: water molecule



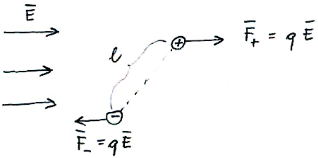
H_2O is electrically neutral, but the charge is displaced $\Rightarrow$ non-zero dipole moment

$p_{H_2O} = 6.13 \times 10^{-30}$ C·m

Consequence: H_2O is a good solvent for ionic substances

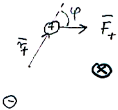


Electric Dipole Moment in Uniform Electric Field

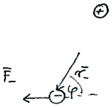


Observation

$|\bar{F}_+| = |\bar{F}_-| \implies$ the net force on the dipole is zero; however the net torque is not.



$\otimes \quad \bar{\tau}_+ = \bar{r}_+ \times \bar{F}_+$



$\otimes \quad \bar{\tau}_- = \bar{r}_- \times \bar{F}_-$

$\bar{\tau}_+, \bar{\tau}_-$ – try to rotate the dipole clockwise

Magnitude $\quad |\bar{\tau}_+| = |\bar{\tau}_-| = \frac{l}{2} qE \sin\varphi \quad$ and $\quad \bar{\tau} = \bar{\tau}_+ + \bar{\tau}_-$

$$|\bar{\tau}| = \overbrace{lq}^{=p} E \sin\varphi$$

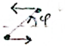
In vector notation $\quad \bar{\tau} = \bar{p} \times \bar{E}$

Potential Energy for a Dipole in Uniform Electric Field

Recall: Any uniform force field is conservative, i.e. $\delta W = -\mathrm{d}U$.
And for work done by the torque $\delta W = \tau_z \mathrm{d}\varphi$.

$$\delta W = (-)\, q\ell E \sin\varphi \, d\varphi = -pE \sin\varphi \, d\varphi$$

$\otimes$ $\odot$ positive direction
$\underline{\tau}$ (increasing φ)



and
$$W_{1\to 2} = \int_{\varphi_1}^{\varphi_2} (-pE\sin\varphi)\, d\varphi = pE\cos\varphi_2 - pE\cos\varphi_1 = U_1 - U_2$$

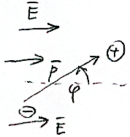
Hence the potential energy is defined as

$$U = -pE\cos\varphi + C = -\bar{p}\cdot\bar{E} + C$$

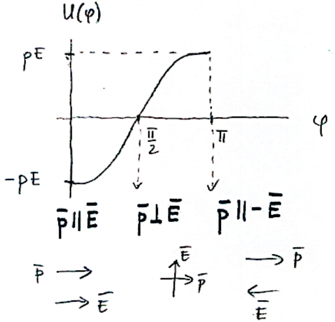
↳ arbitrary constant

Choose $C=0$

$$\boxed{U = U(\varphi) = -\bar{p}\cdot\bar{E}}$$



Discussion



$\bar{p} \parallel \bar{E} \Rightarrow \varphi = 0 \Rightarrow U = U(0) = \min$

(stable equilibrium)

$\bar{p} \parallel -\bar{E} \Rightarrow \varphi = \pi \Rightarrow U = U(\pi) = \max$

(unstable equilibrium)

Conclusion. The electric field $\overline{E}$ tries to align the electric dipole so, that the dipole moment $\overline{p}$ points in the direction of $\overline{E}$.