32

ELECTROMAGNETIC WAVES

32.1. IDENTIFY: Since the speed is constant, distance $x = ct$.

SET UP: The speed of light is $c = 3.00\times10^8$ m/s. $1\text{ y} = 3.156\times10^7$ s.

EXECUTE: **(a)** $t = \dfrac{x}{c} = \dfrac{3.84\times10^8 \text{ m}}{3.00\times10^8 \text{ m/s}} = 1.28 \text{ s}.$

(b) $x = ct = (3.00\times10^8 \text{ m/s})(8.61 \text{ y})(3.156\times10^7 \text{ s/y}) = 8.15\times10^{16} \text{ m} = 8.15\times10^{13} \text{ km}.$

EVALUATE: The speed of light is very great. The distance between stars is very large compared to terrestrial distances.

32.2. IDENTIFY: Find the direction of propagation of an electromagnetic wave if we know the directions of the electric and magnetic fields.

SET UP: The direction of propagation of an electromagnetic wave is in the direction of $\vec{E}\times\vec{B}$, which is related to the directions of $\vec{E}$ and $\vec{B}$ according to the right-hand rule for the cross product. The directions of $\vec{E}$ and $\vec{B}$ in each case are shown in Figure 32.2.

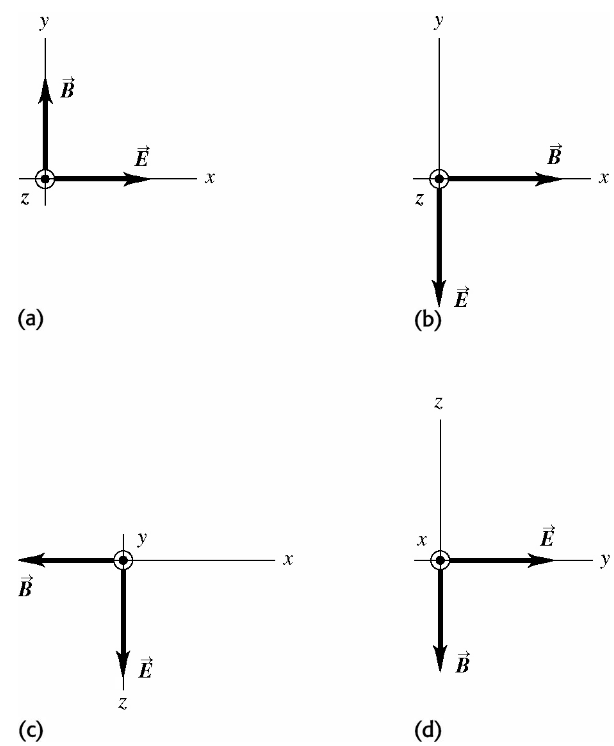


Figure 32.2

EXECUTE: **(a)** The wave is propagating in the $+z$-direction.

(b) $+z$-direction.

(c) $-y$-direction.

(d) $-x$-direction.

EVALUATE: In each case, the direction of propagation is perpendicular to the plane of $\vec{E}$ and $\vec{B}$.

32.3. **IDENTIFY:** $E_{max} = cB_{max}$. $\vec{E} \times \vec{B}$ is in the direction of propagation.

SET UP: $c = 3.00 \times 10^8$ m/s. $E_{max} = 4.00$ V/m.

EXECUTE: $B_{max} = E_{max}/c = 1.33 \times 10^{-8}$ T. For $\vec{E}$ in the $+x$-direction, $\vec{E} \times \vec{B}$ is in the $+z$-direction when $\vec{B}$ is in the $+y$-direction.

EVALUATE: $\vec{E}$, $\vec{B}$, and the direction of propagation are all mutually perpendicular.

32.4. **IDENTIFY** and **SET UP:** The direction of propagation is given by $\vec{E} \times \vec{B}$.

EXECUTE: **(a)** $\hat{S} = \hat{i} \times (-\hat{j}) = -\hat{k}$.

(b) $\hat{S} = \hat{j} \times \hat{i} = -\hat{k}$.

(c) $\hat{S} = (-\hat{k}) \times (-\hat{i}) = \hat{j}$.

(d) $\hat{S} = \hat{i} \times (-\hat{k}) = \hat{j}$.

EVALUATE: In each case the directions of $\vec{E}$, $\vec{B}$, and the direction of propagation are all mutually perpendicular.

32.5. **IDENTIFY:** Knowing the wavelength and speed of x rays, find their frequency, period, and wave number. All electromagnetic waves travel through vacuum at the speed of light.

SET UP: $c = 3.00 \times 10^8$ m/s. $c = f\lambda$. $T = \dfrac{1}{f}$. $k = \dfrac{2\pi}{\lambda}$.

EXECUTE: $f = \dfrac{c}{\lambda} = \dfrac{3.0 \times 10^8 \text{ m/s}}{0.10 \times 10^{-9} \text{ m}} = 3.0 \times 10^{18}$ Hz,

$$T = \frac{1}{f} = \frac{1}{3.0 \times 10^{18} \text{ Hz}} = 3.3 \times 10^{-19} \text{ s}, \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.10 \times 10^{-9} \text{ m}} = 6.3 \times 10^{10} \text{ m}^{-1}.$$

EVALUATE: The frequency of the x rays is much higher than the frequency of visible light, so their period is much shorter.

32.6. **IDENTIFY:** $c = f\lambda$ and $k = \dfrac{2\pi}{\lambda}$.

SET UP: $c = 3.00 \times 10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda}$. UVA: 7.50×10^{14} Hz to 9.38×10^{14} Hz. UVB: 9.38×10^{14} Hz to 1.07×10^{15} Hz.

(b) $k = \dfrac{2\pi}{\lambda}$. UVA: 1.57×10^7 rad/m to 1.96×10^7 rad/m. UVB: 1.96×10^7 rad/m to 2.24×10^7 rad/m.

EVALUATE: Larger λ corresponds to smaller f and k.

32.7. **IDENTIFY:** $c = f\lambda$. $E_{max} = cB_{max}$. $k = 2\pi/\lambda$. $\omega = 2\pi f$.

SET UP: Since the wave is traveling in empty space, its wave speed is $c = 3.00 \times 10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00 \times 10^8 \text{ m/s}}{432 \times 10^{-9} \text{ m}} = 6.94 \times 10^{14}$ Hz.

(b) $E_{max} = cB_{max} = (3.00 \times 10^8 \text{ m/s})(1.25 \times 10^{-6} \text{ T}) = 375$ V/m.

(c) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi \text{ rad}}{432 \times 10^{-9} \text{ m}} = 1.45 \times 10^7$ rad/m. $\omega = (2\pi \text{ rad})(6.94 \times 10^{14} \text{ Hz}) = 4.36 \times 10^{15}$ rad/s.

$$E = E_{max} \cos(kx - \omega t) = (375 \text{ V/m}) \cos[(1.45 \times 10^7 \text{ rad/m})x - (4.36 \times 10^{15} \text{ rad/s})t].$$

$$B = B_{max} \cos(kx - \omega t) = (1.25 \times 10^{-6} \text{ T}) \cos[(1.45 \times 10^7 \text{ rad/m})x - (4.36 \times 10^{15} \text{ rad/s})t].$$

EVALUATE: The $\cos(kx-\omega t)$ factor is common to both the electric and magnetic field expressions, since these two fields are in phase.

32.8. **IDENTIFY:** $c = f\lambda$. $E_{max} = cB_{max}$. Apply Eqs. (32.17) and (32.19).

SET UP: The speed of the wave is $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{435\times10^{-9} \text{ m}} = 6.90\times10^{14}$ Hz.

(b) $B_{max} = \dfrac{E_{max}}{c} = \dfrac{2.70\times10^{-3} \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 9.00\times10^{-12}$ T.

(c) $k = \dfrac{2\pi}{\lambda} = 1.44\times10^7$ rad/m. $\omega = 2\pi f = 4.34\times10^{15}$ rad/s. If $\vec{\boldsymbol{E}}(z,t) = \hat{\boldsymbol{\imath}}E_{max}\cos(kz+\omega t)$, then $\vec{\boldsymbol{B}}(z,t) = -\hat{\boldsymbol{\jmath}}B_{max}\cos(kz+\omega t)$, so that $\vec{\boldsymbol{E}}\times\vec{\boldsymbol{B}}$ will be in the $-\hat{\boldsymbol{k}}$-direction.

$\vec{\boldsymbol{E}}(z,t) = \hat{\boldsymbol{\imath}}(2.70\times10^{-3} \text{ V/m})\cos[(1.44\times10^7 \text{ rad/m})z + (4.34\times10^{15} \text{ rad/s})t]$ and

$\vec{\boldsymbol{B}}(z,t) = -\hat{\boldsymbol{\jmath}}(9.00\times10^{-12} \text{ T})\cos[(1.44\times10^7 \text{ rad/m})z + (4.34\times10^{15} \text{ rad/s})t]$.

EVALUATE: The directions of $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ and of the propagation of the wave are all mutually perpendicular. The argument of the cosine is $kz+\omega t$ since the wave is traveling in the $-z$-direction. Waves for visible light have very high frequencies.

32.9. **IDENTIFY:** Electromagnetic waves propagate through air at essentially the speed of light. Therefore, if we know their wavelength, we can calculate their frequency or vice versa.

SET UP: The wave speed is $c = 3.00\times10^8$ m/s. $c = f\lambda$.

EXECUTE: **(a)** (i) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^3 \text{ m}} = 6.0\times10^4$ Hz.

(ii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-6} \text{ m}} = 6.0\times10^{13}$ Hz.

(iii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-9} \text{ m}} = 6.0\times10^{16}$ Hz.

(b) (i) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{6.50\times10^{21} \text{ Hz}} = 4.62\times10^{-14} \text{ m} = 4.62\times10^{-5}$ nm.

(ii) $\lambda = \dfrac{3.00\times10^8 \text{ m/s}}{590\times10^3 \text{ Hz}} = 508 \text{ m} = 5.08\times10^{11}$ nm.

EVALUATE: Electromagnetic waves cover a huge range in frequency and wavelength.

32.10. **IDENTIFY:** For an electromagnetic wave propagating in the negative x-direction, $E = E_{max}\cos(kx+\omega t)$. $\omega = 2\pi f$ and $k = \dfrac{2\pi}{\lambda}$. $T = \dfrac{1}{f}$. $E_{max} = cB_{max}$.

SET UP: $E_{max} = 375$ V/m, $k = 1.99\times10^7$ rad/m, and $\omega = 5.97\times10^{15}$ rad/s.

EXECUTE: **(a)** $c = \omega/k = (5.97\times10^{15} \text{ rad/s})/(1.99\times10^7 \text{ rad/m}) = 3.00\times10^8$ m/s. This is what the wave speed should be for an electromagnetic wave propagating in vacuum.

(b) E_{max} = 375 V/m, the amplitude of the given cosine function for E. $B_{max} = \dfrac{E_{max}}{c} = 1.25\ \mu$T.

(c) $f = \dfrac{\omega}{2\pi} = 9.50\times10^{14}$ Hz. $\lambda = \dfrac{2\pi}{k} = 3.16\times10^{-7} \text{ m} = 316$ nm. $T = \dfrac{1}{f} = 1.05\times10^{-15}$ s. This wavelength is too short to be visible.

EVALUATE: $c = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\left(\dfrac{2\pi}{k}\right) = \dfrac{\omega}{k}$ is an alternative expression for the wave speed.

32.11. **IDENTIFY** and **SET UP:** Compare the $\vec{E}(y,t)$ given in the problem to the general form given by Eq. (32.17). Use the direction of propagation and of $\vec{E}$ to find the direction of $\vec{B}$.

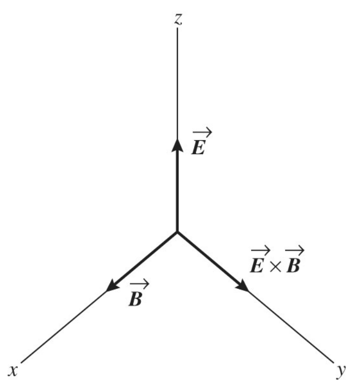
EXECUTE: **(a)** The equation for the electric field contains the factor $\cos(ky-\omega t)$ so the wave is traveling in the $+y$-direction.

(b) $\vec{E}(y,t) = (3.10\times10^5 \text{ V/m})\hat{k}\cos[ky-(12.65\times10^{12} \text{ rad/s})t]$.

Comparing to Eq. (32.17) gives $\omega = 12.65\times10^{12}$ rad/s

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} \text{ so } \lambda = \frac{2\pi c}{\omega} = \frac{2\pi(2.998\times10^8 \text{ m/s})}{(12.65\times10^{12} \text{ rad/s})} = 1.49\times10^{-4} \text{ m}.$$

(c)



$\vec{E}\times\vec{B}$ must be in the $+y$-direction (the direction in which the wave is traveling). When $\vec{E}$ is in the $+z$-direction then $\vec{B}$ must be in the $+x$-direction, as shown in Figure 32.11.

Figure 32.11

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{12.65\times10^{12} \text{ rad/s}}{2.998\times10^8 \text{ m/s}} = 4.22\times10^4 \text{ rad/m}.$$

$E_{max} = 3.10\times10^5$ V/m.

Then $B_{max} = \frac{E_{max}}{c} = \frac{3.10\times10^5 \text{ V/m}}{2.998\times10^8 \text{ m/s}} = 1.03\times10^{-3}$ T.

Using Eq. (32.17) and the fact that $\vec{B}$ is in the $+\hat{i}$-direction when $\vec{E}$ is in the $+\hat{k}$-direction,

$\vec{B} = +(1.03\times10^{-3} \text{ T})\hat{i}\cos[(4.22\times10^4 \text{ rad/m})y-(12.65\times10^{12} \text{ rad/s})t]$.

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.12. **IDENTIFY:** Apply Eqs. (32.17) and (32.19). $f = c/\lambda$ and $k = 2\pi/\lambda$.

SET UP: $B_y(x,t) = -B_{max}\cos(kx+\omega t)$.

EXECUTE: **(a)** The phase of the wave is given by $kx+\omega t$, so the wave is traveling in the $-x$-direction.

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$. $f = \frac{kc}{2\pi} = \frac{(1.38\times10^4 \text{ rad/m})(3.0\times10^8 \text{ m/s})}{2\pi} = 6.59\times10^{11}$ Hz.

(c) Since the magnetic field is in the $-y$-direction, and the wave is propagating in the $-x$-direction, then the electric field is in the $-z$-direction so that $\vec{E}\times\vec{B}$ will be in the $-x$-direction.

$\vec{E}(x,t) = +cB(x,t)\hat{k} = -cB_{max}\cos(kx+\omega t)\hat{k}$.

$\vec{E}(x,t) = -c(8.25\times10^{-9} \text{ T})\cos\left[(1.38\times10^4 \text{ rad/m})x+(4.14\times10^{12} \text{ rad/s})t\right]\hat{k}$.

$\vec{E}(x,t) = -(2.48 \text{ V/m})\cos\left[(1.38\times10^4 \text{ rad/m})x+(4.14\times10^{12} \text{ rad/s})t\right]\hat{k}$.

EVALUATE: $\vec{E}$ and $\vec{B}$ have the same phase and are in perpendicular directions.

32.13. **IDENTIFY** and **SET UP:** $c = f\lambda$ allows calculation of λ. $k = 2\pi/\lambda$ and $\omega = 2\pi f$. $E_{max} = cB_{max}$ relates the electric and magnetic field amplitudes.

EXECUTE: **(a)** $c = f\lambda$ so $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{830\times10^{3}\ \text{Hz}} = 361\ \text{m}.$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi\ \text{rad}}{361\ \text{m}} = 0.0174\ \text{rad/m}.$

(c) $\omega = 2\pi f = (2\pi)(830\times10^{3}\ \text{Hz}) = 5.22\times10^{6}\ \text{rad/s}.$

(d) Eq. (32.18): $E_{\text{max}} = cB_{\text{max}} = (2.998\times10^{8}\ \text{m/s})(4.82\times10^{-11}\ \text{T}) = 0.0144\ \text{V/m}.$

EVALUATE: This wave has a very long wavelength; its frequency is in the AM radio broadcast band. The electric and magnetic fields in the wave are very weak.

32.14. **IDENTIFY:** Apply $v = \dfrac{c}{\sqrt{KK_{\text{m}}}}$. $E_{\text{max}} = cB_{\text{max}}$. $v = f\lambda$.

SET UP: $K = 3.64$. $K_{\text{m}} = 5.18$.

EXECUTE: **(a)** $v = \dfrac{c}{\sqrt{KK_{\text{m}}}} = \dfrac{(3.00\times10^{8}\text{m/s})}{\sqrt{(3.64)(5.18)}} = 6.91\times10^{7}\ \text{m/s}.$

(b) $\lambda = \dfrac{v}{f} = \dfrac{6.91\times10^{7}\,\text{m/s}}{65.0\,\text{Hz}} = 1.06\times10^{6}\ \text{m}.$

(c) $B_{\text{max}} = \dfrac{E_{\text{max}}}{v} = \dfrac{7.20\times10^{-3}\ \text{V/m}}{6.91\times10^{7}\ \text{m/s}} = 1.04\times10^{-10}\ \text{T}.$

EVALUATE: The wave travels slower in this material than in air.

32.15. **IDENTIFY** and **SET UP:** $v = f\lambda$ relates frequency and wavelength to the speed of the wave. Use $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ to calculate n and K.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{2.17\times10^{8}\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 3.81\times10^{-7}\ \text{m}.$

(b) $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 5.26\times10^{-7}\ \text{m}.$

(c) $n = \dfrac{c}{v} = \dfrac{2.998\times10^{8}\ \text{m/s}}{2.17\times10^{8}\ \text{m/s}} = 1.38.$

(d) $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ so $K = n^{2} = (1.38)^{2} = 1.90.$

EVALUATE: In the material $v < c$ and f is the same, so λ is less in the material than in air. $v < c$ always, so n is always greater than unity.

32.16. **IDENTIFY:** We want to find the amount of energy given to each receptor cell and the amplitude of the magnetic field at the cell.

SET UP: Intensity is average power per unit area and power is energy per unit time. $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$, $I = P/A$, and $E_{\text{max}} = cB_{\text{max}}$.

EXECUTE: **(a)** For the beam, the energy is $U = Pt = (2.0\times10^{12}\ \text{W})(4.0\times10^{-9}\ \text{s}) = 8.0\times10^{3}\ \text{J} = 8.0\ \text{kJ}.$ This energy is spread uniformly over 100 cells, so the energy given to each cell is 80 J.

(b) The cross-sectional area of each cell is $A = \pi r^2$, with $r = 2.5\times10^{-6}$ m.

$$I = \frac{P}{A} = \frac{2.0\times10^{12}\ \text{W}}{(100)\pi(2.5\times10^{-6}\ \text{m})^2} = 1.0\times10^{21}\ \text{W/m}^2.$$

(c) $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.0\times10^{21}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{8}\ \text{m/s})}} = 8.7\times10^{11}\ \text{V/m}.$

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = 2.9\times10^{3}\ \text{T}.$$

EVALUATE: Both the electric field and magnetic field are very strong compared to ordinary fields.

32.17. **IDENTIFY:** $I = P/A$. $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2$. $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$.

EXECUTE: **(a)** $I = \dfrac{P}{A} = \dfrac{(0.05)(75\ \text{W})}{4\pi(3.0\times10^{-2}\ \text{m})^2} = 330\ \text{W/m}^2$.

(b) $E_{\max} = \sqrt{\dfrac{2I}{\varepsilon_0 c}} = \sqrt{\dfrac{2(330\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 500\ \text{V/m}$.

$B_{\max} = \dfrac{E_{\max}}{c} = 1.7\times10^{-6}\ \text{T} = 1.7\ \mu\text{T}$.

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.18. **IDENTIFY:** The intensity of the electromagnetic wave is given by $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2 = \varepsilon_0 c E_{\text{rms}}^2$. The total energy passing through a window of area A during a time t is IAt.

SET UP: $\varepsilon_0 = 8.85\times10^{-12}\ \text{F/m}$.

EXECUTE: Use the fact that energy $= \varepsilon_0 c E_{\text{rms}}^2 At$.

$\text{Energy} = (8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})(0.0400\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 6.37\times10^{-5}\ \text{J} = 63.7\ \mu\text{J}$.

EVALUATE: The intensity is proportional to the square of the electric field amplitude.

32.19. **IDENTIFY:** $I = P_{\text{av}}/A$.

SET UP: At a distance r from the star, the radiation from the star is spread over a spherical surface of area $A = 4\pi r^2$.

EXECUTE: $P_{\text{av}} = I(4\pi r^2) = (5.0\times10^3\ \text{W/m}^2)(4\pi)(2.0\times10^{10}\ \text{m})^2 = 2.5\times10^{25}\ \text{W}$.

EVALUATE: The intensity decreases with distance from the star as $1/r^2$.

32.20. **IDENTIFY** and **SET UP:** $I = \frac{1}{2}\varepsilon_0 c {E_{\max}}^2$. $E_{\max} = cB_{\max}$. At the earth the power radiated by the sun is spread over an area of $4\pi r^2$, where $r = 1.50\times10^{11}\ \text{m}$ is the distance from the earth to the sun. $P = IA$.

EXECUTE: **(a)** $E_{\max} = \sqrt{\dfrac{2I}{\varepsilon_0 c}} = \sqrt{\dfrac{2(1.4\times10^3\ \text{W/m}^2)}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 1.03\times10^3\ \text{N/C}$.

$B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{1.03\times10^3\ \text{N/C}}{3.00\times10^8\ \text{m/s}} = 3.43\times10^{-6}\ \text{T}$.

(b) $P = I(4\pi r^2) = (1.4\times10^3\ \text{W/m}^2)(4\pi)(1.50\times10^{11}\ \text{m})^2 = 4.0\times10^{26}\ \text{W}$.

EVALUATE: The intensity of the magnetic field of the light waves from the sun is about 1/10 the earth's magnetic field.

32.21. **IDENTIFY** and **SET UP:** $I = P_{\text{av}}/A$ and $I = \varepsilon_0 c E_{\text{rms}}^2$.

EXECUTE: **(a)** The average power from the beam is

$P_{\text{av}} = IA = (0.800\ \text{W/m}^2)(3.0\times10^{-4}\ \text{m}^2) = 2.4\times10^{-4}\ \text{W}$.

(b) $E_{\text{rms}} = \sqrt{\dfrac{I}{\varepsilon_0 c}} = \sqrt{\dfrac{0.800\ \text{W/m}^2}{(8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})}} = 17.4\ \text{V/m}$.

EVALUATE: The laser emits radiation only in the direction of the beam.

32.22. **IDENTIFY** and **SET UP**: $c = f\lambda$, $E_{\max} = cB_{\max}$ and $I = E_{\max}B_{\max}/2\mu_0$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\ \text{m/s}}{0.354\ \text{m}} = 8.47\times10^8\ \text{Hz}$.

(b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{0.0540\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 1.80\times10^{-10}\ \text{T}$.

(c) $I = S_{av} = \dfrac{E_{max}B_{max}}{2\mu_0} = \dfrac{(0.0540\text{ V/m})(1.80\times10^{-10}\text{ T})}{2\mu_0} = 3.87\times10^{-6}\text{ W/m}^2.$

EVALUATE: Alternatively, $I = \frac{1}{2}\varepsilon_0 c E_{max}^2$.

32.23. **IDENTIFY:** $P_{av} = IA$ and $I = E_{max}^2/2\mu_0 c$

SET UP: The surface area of a sphere is $A = 4\pi r^2$.

EXECUTE: $P_{av} = S_{av}A = \left(\dfrac{E_{max}^2}{2c\mu_0}\right)(4\pi r^2)$. $E_{max} = \sqrt{\dfrac{P_{av}c\mu_0}{2\pi r^2}} = \sqrt{\dfrac{(60.0\text{ W})(3.00\times10^8\text{ m/s})\mu_0}{2\pi(5.00\text{ m})^2}} = 12.0\text{ V/m}.$

$B_{max} = \dfrac{E_{max}}{c} = \dfrac{12.0\text{ V/m}}{3.00\times10^8\text{ m/s}} = 4.00\times10^{-8}\text{ T}.$

EVALUATE: E_{max} and B_{max} are both inversely proportional to the distance from the source.

32.24. **IDENTIFY:** The intensity and the energy density of an electromagnetic wave depend on the amplitudes of the electric and magnetic fields.

SET UP: Intensity is $I = P_{av}/A$, and the average radiation pressure is $P_{av} = 2I/c$, where $I = \frac{1}{2}\varepsilon_0 c E_{max}^2$. The energy density is $u = \varepsilon_0 E^2$.

EXECUTE: **(a)** $I = P_{av}/A = \dfrac{777{,}000\text{ W}}{2\pi(5000\text{ m})^2} = 0.004947\text{ W/m}^2.$

$p_{rad} = 2I/c = \dfrac{2(0.004947\text{ W/m}^2)}{3.00\times10^8\text{ m/s}} = 3.30\times10^{-11}\text{ Pa}.$

(b) $I = \frac{1}{2}\varepsilon_0 c E_{max}^2$ gives

$$E_{max} = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(0.004947\text{ W/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\text{ m/s})}} = 1.93\text{ N/C}.$$

$$B_{max} = E_{max}/c = (1.93\text{ N/C})/(3.00\times10^8\text{ m/s}) = 6.43\times10^{-9}\text{ T}.$$

(c) $u = \varepsilon_0 E^2$, so $u_{av} = \varepsilon_0 (E_{rms})^2$ and $E_{rms} = \dfrac{E_{max}}{\sqrt{2}}$, so

$$u_{av} = \frac{\varepsilon_0 E_{max}^2}{2} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.93\text{ N/C})^2}{2} = 1.65\times10^{-11}\text{ J/m}^3.$$

(d) As was shown in Section 32.4, the energy density is the same for the electric and magnetic fields, so each one has 50% of the energy density.

EVALUATE: Compared to most laboratory fields, the electric and magnetic fields in ordinary radiowaves are extremely weak and carry very little energy.

32.25. **IDENTIFY:** Use the radiation pressure to find the intensity, and then $P_{av} = I(4\pi r^2)$.

SET UP: For a perfectly absorbing surface, $p_{rad} = \dfrac{I}{c}$.

EXECUTE: $p_{rad} = I/c$ so $I = cp_{rad} = 2.70\times10^3\text{ W/m}^2$. Then

$P_{av} = I(4\pi r^2) = (2.70\times10^3\text{ W/m}^2)(4\pi)(5.0\text{ m})^2 = 8.5\times10^5\text{ W}.$

EVALUATE: Even though the source is very intense the radiation pressure 5.0 m from the surface is very small.

32.26. **IDENTIFY:** Apply $p_{rad} = \dfrac{I}{c}$ and $p_{rad} = \dfrac{2I}{c}$. The average momentum density is given by $\dfrac{dp}{dV} = \dfrac{S_{av}}{c^2}$ with S replaced by $S_{av} = I$.

SET UP: $1\text{ atm} = 1.013\times10^5\text{ Pa}$.

EXECUTE: **(a)** Absorbed light: $p_{\text{rad}} = \frac{I}{c} = \frac{2500 \text{ W/m}^2}{3.0\times10^8 \text{ m/s}} = 8.33\times10^{-6} \text{ Pa}$. Then

$$p_{\text{rad}} = \frac{8.33\times10^{-6} \text{ Pa}}{1.013\times10^5 \text{ Pa/atm}} = 8.23\times10^{-11} \text{ atm}.$$

(b) Reflecting light: $p_{\text{rad}} = \frac{2I}{c} = \frac{2(2500 \text{ W/m}^2)}{3.0\times10^8 \text{ m/s}} = 1.67\times10^{-5} \text{ Pa}$. Then

$$p_{\text{rad}} = \frac{1.67\times10^{-5} \text{ Pa}}{1.013\times10^5 \text{ Pa/atm}} = 1.65\times10^{-10} \text{ atm}.$$

(c) The momentum density is $\frac{dp}{dV} = \frac{S_{\text{av}}}{c^2} = \frac{2500 \text{ W/m}^2}{(3.0\times10^8 \text{ m/s})^2} = 2.78\times10^{-14} \text{ kg/m}^2\cdot\text{s}$.

EVALUATE: The factor of 2 in p_{rad} for the reflecting surface arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

32.27. IDENTIFY: We know the greatest intensity that the eye can safely receive.

SET UP: $I = \frac{P}{A}$. $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$.

EXECUTE: **(a)** $P = IA = (1.0\times10^2 \text{ W/m}^2)\pi(0.75\times10^{-3} \text{ m})^2 = 1.8\times10^{-4} \text{ W} = 0.18 \text{ mW}$.

(b) $E = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(1.0\times10^2 \text{ W/m}^2)}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8 \text{ m/s})}} = 274 \text{ V/m}$. $B_{\max} = \frac{E_{\max}}{c} = 9.13\times10^{-7} \text{ T}$.

(c) $P = 0.18 \text{ mW} = 0.18 \text{ mJ/s}$.

(d) $I = (1.0\times10^2 \text{ W/m}^2)\left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right)^2 = 0.010 \text{ W/cm}^2$.

EVALUATE: Both the electric and magnetic fields are quite weak compared to normal laboratory fields.

32.28. IDENTIFY and SET UP: For a totally reflected beam of intensity I, the radiation pressure is $p_{\text{rad}} = 2I/c$. The intensity of the beam is $I = \frac{E_{\max}^2}{2\mu_0\varepsilon_0}$, and the pressure is $p = F/A$.

EXECUTE: $p_{\text{rad}} = 2I/c = F/A$, which gives $I = Fc/2A$. Using $I = \frac{E_{\max}^2}{2\mu_0\varepsilon_0}$, we have $\frac{E_{\max}^2}{2\mu_0\varepsilon_0} = \frac{Fc}{2A}$. Solving for $E_{\max}$ gives $E_{\max} = c\sqrt{\frac{F\mu_0}{A}}$. Putting in the numbers gives

$$E_{\max} = c\sqrt{\frac{F\mu_0}{A}} = (3.00\times10^8 \text{ m/s})\sqrt{\frac{(3.8\times10^{-9} \text{ N})(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})}{\pi(0.600\times10^{-3} \text{ m})^2}} = 1.9\times10^4 \text{ V/m}.$$

EVALUATE: The amplitude of the magnetic field is $B_{\max} = E_{\max}/c = 65\ \mu\text{T}$, which is a very small field.

32.29. IDENTIFY: We know the wavelength and power of the laser beam, as well as the area over which it acts.

SET UP: $P = IA$. $A = \pi r^2$. $E_{\max} = cB_{\max}$. The intensity $I = S_{\text{av}}$ is related to the maximum electric field by $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$. The average energy density u_{av} is related to the intensity I by $I = u_{\text{av}} c$.

EXECUTE: **(a)** $I = \frac{P}{A} = \frac{0.500\times10^{-3} \text{ W}}{\pi(0.500\times10^{-3} \text{ m})^2} = 637 \text{ W/m}^2$.

(b) $E_{\max} = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(637 \text{ W/m}^2)}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8 \text{ m/s})}} = 693 \text{ V/m}$. $B_{\max} = \frac{E_{\max}}{c} = 2.31\ \mu\text{T}$.

(c) $u_{\text{av}} = \frac{I}{c} = \frac{637 \text{ W/m}^2}{3.00\times10^8 \text{ m/s}} = 2.12\times10^{-6} \text{ J/m}^3$.

EVALUATE: The fields are very weak, so a cubic meter of space contains only about $2\ \mu\text{J}$ of energy.

32.30. IDENTIFY and SET UP: The nodal planes of the electric field are $\lambda/2$ apart, and the nodal planes of the magnetic field are also $\lambda/2$ apart.

EXECUTE: **(a)** The nodal planes of the $\vec{B}$ field are a distance $\lambda/2$ apart, so $\lambda/2 = 4.65\text{ mm}$ and $\lambda = 9.30\text{ mm}$.

(b) The nodal planes of the $\vec{E}$ field are also a distance $\lambda/2 = 4.65\text{ mm}$ apart.

(c) $v = f\lambda = (2.20\times10^{10}\text{ Hz})(9.30\times10^{-3}\text{ m}) = 2.05\times10^{8}\text{ m/s}$.

EVALUATE: The spacing between the nodes of $\vec{E}$ is the same as the spacing between the nodes of $\vec{B}$. Note that $v < c$, as it must.

32.31. IDENTIFY: The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $(2\frac{1}{2})\lambda = L$, and the frequency of these waves obeys the equation $f\lambda = c$.

EXECUTE: **(a)** Since $(2\frac{1}{2})\lambda = L$, we have $L = (5/2)(12.2\text{ cm}) = 30.5\text{ cm}$.

(b) Solving for the frequency gives $f = c/\lambda = (3.00\times10^{8}\text{ m/s})/(0.122\text{ m}) = 2.46\times10^{9}\text{ Hz}$.

(c) $L = 35.5\text{ cm}$ in this case. $(2\frac{1}{2})\lambda = L$, so $\lambda = 2L/5 = 2(35.5\text{ cm})/5 = 14.2\text{ cm}$.

$$f = c/\lambda = (3.00\times10^{8}\text{ m/s})/(0.142\text{ m}) = 2.11\times10^{9}\text{ Hz}.$$

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.32. IDENTIFY: The nodal planes of $\vec{E}$ and $\vec{B}$ are located by Eqs. (32.26) and (32.27).

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{75.0\times10^{6}\text{ Hz}} = 4.00\text{ m}$.

EXECUTE: **(a)** $\Delta x = \dfrac{\lambda}{2} = 2.00\text{ m}$.

(b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength, so is $\dfrac{\lambda}{4} = \dfrac{\Delta x}{2} = \dfrac{2.00\text{ m}}{2} = 1.00\text{ m}$.

EVALUATE: The nodal planes of $\vec{B}$ are separated by a distance $\lambda/2$ and are midway between the nodal planes of $\vec{E}$.

32.33. IDENTIFY: We know the wavelength and power of a laser beam as well as the area over which it acts and the duration of a pulse.

SET UP: The energy is $U = Pt$. For absorption the radiation pressure is $\dfrac{I}{c}$, where $I = \dfrac{P}{A}$. The wavelength in the eye is $\lambda = \dfrac{\lambda_0}{n}$. $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$ and $E_{\max} = cB_{\max}$.

EXECUTE: **(a)** $U = Pt = (250\times10^{-3}\text{ W})(1.50\times10^{-3}\text{ s}) = 3.75\times10^{-4}\text{ J} = 0.375\text{ mJ}$.

(b) $I = \dfrac{P}{A} = \dfrac{250\times10^{-3}\text{ W}}{\pi(255\times10^{-6}\text{ m})^2} = 1.22\times10^{6}\text{ W/m}^2$. The average pressure is $\dfrac{I}{c} = \dfrac{1.22\times10^{6}\text{ W/m}^2}{3.00\times10^{8}\text{ m/s}} = 4.08\times10^{-3}\text{ Pa}$.

(c) $\lambda = \dfrac{\lambda_0}{n} = \dfrac{810\text{ nm}}{1.34} = 604\text{ nm}$. $f = \dfrac{v}{\lambda} = \dfrac{c}{\lambda_0} = \dfrac{3.00\times10^{8}\text{ m/s}}{810\times10^{-9}\text{ m}} = 3.70\times10^{14}\text{ Hz}$; f is the same in the air and in the vitreous humor.

(d) $E_{\max} = \sqrt{\dfrac{2I}{\varepsilon_0 c}} = \sqrt{\dfrac{2(1.22\times10^6\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 3.03\times10^4\ \text{V/m}.$

$B_{\max} = \dfrac{E_{\max}}{c} = 1.01\times10^{-4}\ \text{T}.$

EVALUATE: The intensity of the beam is high, as it must be to weld tissue, but the pressure it exerts on the retina is only around 10^{-8} that of atmospheric pressure. The magnetic field in the beam is about twice that of the earth's magnetic field.

32.34. IDENTIFY: Evaluate the partial derivatives of the expressions for $E_y(x,t)$ and $B_z(x,t)$.

SET UP: $\dfrac{\partial}{\partial x}\cos(kx-\omega t) = -k\sin(kx-\omega t)$, $\dfrac{\partial}{\partial t}\cos(kx-\omega t) = \omega\sin(kx-\omega t)\cdot$

$\dfrac{\partial}{\partial x}\sin(kx-\omega t) = k\cos(kx-\omega t)$, $\dfrac{\partial}{\partial t}\sin(kx-\omega t) = -\omega\cos(kx-\omega t)$.

EXECUTE: Assume $\vec{E} = E_{\max}\hat{\boldsymbol{j}}\cos(kx-\omega t)$ and $\vec{B} = B_{\max}\hat{\boldsymbol{k}}\cos(kx-\omega t+\phi)$, with $-\pi<\phi<\pi$. Eq. (32.12) is $\dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t}$. This gives $kE_{\max}\sin(kx-\omega t) = +\omega B_{\max}\sin(kx-\omega t+\phi)$, so $\phi=0$, and $kE_{\max} = \omega B_{\max}$, so $E_{\max} = \dfrac{\omega}{k}B_{\max} = \dfrac{2\pi f}{2\pi/\lambda}B_{\max} = f\lambda B_{\max} = cB_{\max}$. Similarly for Eq. (32.14), $-\dfrac{\partial B_z}{\partial x} = \varepsilon_0\mu_0\dfrac{\partial E_y}{\partial t}$ gives $kB_{\max}\sin(kx-\omega t+\phi) = \varepsilon_0\mu_0\omega E_{\max}\sin(kx-\omega t)$, so $\phi=0$ and $kB_{\max} = \varepsilon_0\mu_0\omega E_{\max}$, so

$B_{\max} = \dfrac{\varepsilon_0\mu_0\omega}{k}E_{\max} = \dfrac{2\pi f}{c^2 2\pi/\lambda}E_{\max} = \dfrac{f\lambda}{c^2}E_{\max} = \dfrac{1}{c}E_{\max}.$

EVALUATE: The $\vec{E}$ and $\vec{B}$ fields must oscillate in phase.

32.35. IDENTIFY: The intensity of an electromagnetic wave depends on the amplitude of the electric and magnetic fields. Such a wave exerts a force because it carries energy.

SET UP: The intensity of the wave is $I = P_{\text{av}}/A = \frac{1}{2}\varepsilon_0 c E_{\max}^2$, and the force is $F = p_{\text{rad}}A$ where $p_{\text{rad}} = I/c$.

EXECUTE: **(a)** $I = P_{\text{av}}/A = (25{,}000\ \text{W})/[4\pi(5.75\times10^5\ \text{m})^2] = 6.02\times10^{-9}\ \text{W/m}^2$.

(b) $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$, so $E_{\max} = \sqrt{\dfrac{2I}{\varepsilon_0 c}} = \sqrt{\dfrac{2(6.02\times10^{-9}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 2.13\times10^{-3}\ \text{N/C}.$

$$B_{\max} = E_{\max}/c = (2.13\times10^{-3}\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 7.10\times10^{-12}\ \text{T}.$$

(c) $F = p_{\text{rad}}A = (I/c)A = (6.02\times10^{-9}\ \text{W/m}^2)(0.150\ \text{m})(0.400\ \text{m})/(3.00\times10^8\ \text{m/s}) = 1.20\times10^{-18}\ \text{N}.$

EVALUATE: The fields are very weak compared to ordinary laboratory fields, and the force is hardly worth worrying about!

32.36. IDENTIFY: The average energy density in the electric field is $u_{E,\text{av}} = \frac{1}{2}\varepsilon_0(E^2)_{\text{av}}$ and the average energy density in the magnetic field is $u_{B,\text{av}} = \dfrac{1}{2\mu_0}(B^2)_{\text{av}}$.

SET UP: $(\cos^2(kx-\omega t))_{\text{av}} = \frac{1}{2}$.

EXECUTE: $E_y(x,t) = E_{\max}\cos(kx-\omega t)$. $u_E = \frac{1}{2}\varepsilon_0 E_y^2 = \frac{1}{2}\varepsilon_0 E_{\max}^2\cos^2(kx-\omega t)$ and $u_{E,\text{av}} = \frac{1}{4}\varepsilon_0 E_{\max}^2$.

$B_z(x,t) = B_{\max}\cos(kx-\omega t)$, so $u_B = \dfrac{1}{2\mu_0}B_z^2 = \dfrac{1}{2\mu_0}B_{\max}^2\cos^2(kx-\omega t)$ and $u_{B,\text{av}} = \dfrac{1}{4\mu_0}B_{\max}^2$.

$E_{\max} = cB_{\max}$, so $u_{E,\text{av}} = \frac{1}{4}\varepsilon_0 c^2 B_{\max}^2$. $c = \dfrac{1}{\sqrt{\varepsilon_0\mu_0}}$, so $u_{E,\text{av}} = \dfrac{1}{4\mu_0}B_{\max}^2$, which equals $u_{B,\text{av}}$.

EVALUATE: Our result allows us to write $u_{\text{av}} = 2u_{E,\text{av}} = \frac{1}{2}\varepsilon_0 E_{\max}^2$ and $u_{\text{av}} = 2u_{B,\text{av}} = \dfrac{1}{2\mu_0}B_{\max}^2$.

32.37. IDENTIFY: $I = P_{av}/A$. For an absorbing surface, the radiation pressure is $p_{rad} = \frac{I}{c}$.

SET UP: Assume the electromagnetic waves are formed at the center of the sun, so at a distance r from the center of the sun $I = P_{av}/(4\pi r^2)$.

EXECUTE: **(a)** At the sun's surface: $I = \frac{P_{av}}{4\pi R^2} = \frac{3.9\times10^{26}\ \text{W}}{4\pi(6.96\times10^8\ \text{m})^2} = 6.4\times10^7\ \text{W/m}^2$ and

$p_{rad} = \frac{I}{c} = \frac{6.4\times10^7\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}} = 0.21\ \text{Pa}.$

Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $I = 2.6\times10^8\ \text{W/m}^2$ and $p_{rad} = 0.85\ \text{Pa}$. At the top of the earth's atmosphere, the measured sunlight intensity is $1400\ \text{W/m}^2$ and $p_{rad} = 5\times10^{-6}\ \text{Pa}$, which is about 100,000 times less than the values above.

EVALUATE: **(b)** The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.38. **(a) IDENTIFY** and **SET UP:** Calculate I and then use $I = \frac{E_{max}^2}{2\mu_0 c}$ to calculate E_{max} and $E_{max} = cB_{max}$ to calculate B_{max}.

EXECUTE: The intensity is power per unit area: $I = \frac{P}{A} = \frac{5.80\times10^{-3}\ \text{W}}{\pi(1.25\times10^{-3}\ \text{m})^2} = 1182\ \text{W/m}^2$.

$I = \frac{E_{max}^2}{2\mu_0 c}$, so $E_{max} = \sqrt{2\mu_0 cI}$. $E_{max} = \sqrt{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.998\times10^8\ \text{m/s})(1182\ \text{W/m}^2)} = 943.5\ \text{V/m}$, which rounds to 943 V/m.

$B_{max} = \frac{E_{max}}{c} = \frac{943.5\ \text{V/m}}{2.998\times10^8\ \text{m/s}} = 3.148\times10^{-6}\ \text{T}$, which rounds to 3.15 μT.

EVALUATE: The magnetic field amplitude is quite small compared to laboratory fields.

(b) IDENTIFY and **SET UP:** $u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = \frac{B^2}{2\mu_0}$ give the energy density in terms of the electric and magnetic field values at any time. For sinusoidal fields average over E^2 and B^2 to get the average energy densities.

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{max}\cos(kx - \omega t)$ and the average value of $\cos^2(kx - \omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is

$u_{E,av} = \frac{1}{4}\epsilon_0 E_{max}^2 = \frac{1}{4}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(943.5\ \text{V/m})^2 = 1.97\times10^{-6}\ \text{J/m}^3 = 1.97\ \mu\text{J/m}^3$. The energy density in the magnetic field is $u_B = \frac{B^2}{2\mu_0}$. The average value is

$u_{B,av} = \frac{B_{max}^2}{4\mu_0} = \frac{(3.148\times10^{-6}\ \text{T})^2}{4(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 1.97\times10^{-6}\ \text{J/m}^3 = 1.97\ \mu\text{J/m}^3$.

EVALUATE: Our result agrees with the statement in Section 32.4 that the average energy density for the electric field is the same as the average energy density for the magnetic field.

(c) IDENTIFY and **SET UP:** The total energy in this length of beam is the average energy density $(u_{av} = u_{E,av} + u_{B,av} = 3.94\times10^{-6}\ \text{J/m}^3)$ times the volume of this part of the beam.

EXECUTE: $U = u_{av}LA = (3.94\times10^{-6}\ \text{J/m}^3)(1.00\ \text{m})\pi(1.25\times10^{-3}\ \text{m})^2 = 1.93\times10^{-11}\ \text{J}$.

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel $L = 1.00$ m: $U = P\left(\frac{L}{c}\right) = (5.80\times10^{-3}\ \text{W})\left(\frac{1.00\ \text{m}}{2.998\times10^{8}\ \text{m/s}}\right) = 1.93\times10^{-11}$ J, which checks.

32.39. IDENTIFY: The same intensity light falls on both reflectors, but the force on the reflecting surface will be twice as great as the force on the absorbing surface. Therefore there will be a net torque about the rotation axis.

SET UP: For a totally absorbing surface, $F = p_{\text{rad}}A = (I/c)A,$ while for a totally reflecting surface the force will be twice as great. The intensity of the wave is $I = \frac{1}{2}\varepsilon_0 cE_{\text{max}}^2.$ Once we have the torque, we can use the rotational form of Newton's second law, $\tau_{\text{net}} = I\alpha,$ to find the angular acceleration.

EXECUTE: The force on the absorbing reflector is $F_{\text{abs}} = p_{\text{rad}}A = (I/c)A = \frac{\frac{1}{2}\varepsilon_0 cE_{\text{max}}^2 A}{c} = \frac{1}{2}\varepsilon_0 AE_{\text{max}}^2.$

For a totally reflecting surface, the force will be twice as great, which is $\varepsilon_0 cE_{\text{max}}^2.$ The net torque is therefore $\tau_{\text{net}} = F_{\text{refl}}(L/2) - F_{\text{abs}}(L/2) = \varepsilon_0 AE_{\text{max}}^2 L/4.$

Newton's second law for rotation gives $\tau_{\text{net}} = I\alpha.$ $\varepsilon_0 AE_{\text{max}}^2 L/4 = 2m(L/2)^2\alpha.$

Solving for α gives

$$\alpha = \varepsilon_0 AE_{\text{max}}^2/(2mL) = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0150\ \text{m})^2(1.25\ \text{N/C})^2}{(2)(0.00400\ \text{kg})(1.00\ \text{m})} = 3.89\times10^{-13}\ \text{rad/s}^2.$$

EVALUATE: This is an extremely small angular acceleration. To achieve a larger value, we would have to greatly increase the intensity of the light wave or decrease the mass of the reflectors.

32.40. IDENTIFY: The intensity of the wave, not the electric field strength, obeys an inverse-square distance law.

SET UP: The intensity is inversely proportional to the distance from the source, and it depends on the amplitude of the electric field by $I = S_{\text{av}} = \frac{1}{2}\varepsilon_0 cE_{\text{max}}^2.$

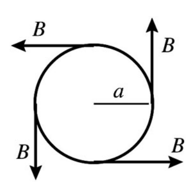
EXECUTE: Since $I = \frac{1}{2}\varepsilon_0 cE_{\text{max}}^2,$ $E_{\text{max}} \propto \sqrt{I}.$ A point at 20.0 cm (0.200 m) from the source is 50 times closer to the source than a point that is 10.0 m from it. Since $I \propto 1/r^2$ and $(0.200\ \text{m})/(10.0\ \text{m}) = 1/50,$ we have $I_{0.20} = 50^2 I_{10}.$ Since $E_{\text{max}} \propto \sqrt{I},$ we have $E_{0.20} = 50E_{10} = (50)(3.50\ \text{N/C}) = 175\ \text{N/C}.$

EVALUATE: While the intensity increases by a factor of $50^2 = 2500,$ the amplitude of the wave only increases by a factor of 50. Recall that the intensity of *any* wave is proportional to the *square* of its amplitude.

32.41. IDENTIFY and SET UP: In the wire the electric field is related to the current density by $\vec{E} = \rho\vec{J}.$ Use Ampere's law to calculate $\vec{B}$. The Poynting vector is given by $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ and $\vec{P} = \oint\vec{S}\cdot d\vec{A}$ relates the energy flow through a surface to $\vec{S}$.

EXECUTE: **(a)** The direction of $\vec{E}$ is parallel to the axis of the cylinder, in the direction of the current. $E = \rho J = \rho I/\pi a^2.$ (E is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.41a; take the current to be coming out of the page.



Apply Ampere's law to a circle of radius a.

$\oint\vec{B}\cdot d\vec{l} = B(2\pi ra)$

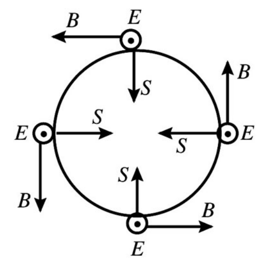
$I_{\text{encl}} = I.$

Figure 32.41a

$\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi a) = \mu_0 I$ and $B = \frac{\mu_0 I}{2\pi a}.$

The direction of $\vec{B}$ is counterclockwise around the circle.

(c) The directions of $\vec{E}$ and $\vec{B}$ are shown in Figure 32.41b.



The direction of $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}.$

is radially inward.

$S = \dfrac{1}{\mu_0}EB = \dfrac{1}{\mu_0}\left(\dfrac{\rho I}{\pi a^2}\right)\left(\dfrac{\mu_0 I}{2\pi a}\right).$

$S = \dfrac{\rho I^2}{2\pi^2 a^3}.$

Figure 32.41b

EVALUATE: **(d)** Since S is constant over the surface of the conductor, the rate of energy flow P is given by S times the surface of a length l of the conductor: $P = SA = S(2\pi al) = \dfrac{\rho I^2}{2\pi^2 a^3}(2\pi al) = \dfrac{\rho lI^2}{\pi a^2}$. But $R = \dfrac{\rho l}{\pi a^2}$, so the result from the Poynting vector is $P = RI^2$. This agrees with $P_R = I^2 R$, the rate at which electrical energy is being dissipated by the resistance of the wire. Since $\vec{S}$ is radially inward at the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

32.42. **IDENTIFY:** The changing magnetic field of the electromagnetic wave produces a changing flux through the wire loop, which induces an emf in the loop. The wavelength of the wave is much greater than the diameter of the loop, so we can treat the magnetic field as being uniform over the area of the loop.

SET UP: $\Phi_B = B\pi r^2 = \pi r^2 B_{\text{max}}\cos(kx-\omega t)$, taking x for the direction of propagation of the wave. Faraday's law says $|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right|$. The intensity of the wave is $I = \dfrac{E_{\text{max}}B_{\text{max}}}{2\mu_0} = \dfrac{c}{2\mu_0}B_{\text{max}}^2$, and $f = \dfrac{c}{\lambda}$.

EXECUTE: $|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right| = \omega B_{\text{max}}\sin(kx-\omega t)\pi r^2$. $|\varepsilon|_{\text{max}} = 2\pi f B_{\text{max}}\pi r^2$.

$f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{6.90\text{ m}} = 4.348\times10^7\text{ Hz}$. Solving $I = \dfrac{E_{\text{max}}B_{\text{max}}}{2\mu_0} = \dfrac{c}{2\mu_0}B_{\text{max}}^2$ for B_{max} gives

$B_{\text{max}} = \sqrt{\dfrac{2\mu_0 I}{c}} = \sqrt{\dfrac{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.0275\text{ W/m}^2)}{3.00\times10^8\text{ m/s}}} = 1.518\times10^{-8}\text{ T}.$

$|\varepsilon|_{\text{max}} = 2\pi(4.348\times10^7\text{ Hz})(1.518\times10^{-8}\text{ T})\pi(0.075\text{ m})^2 = 7.33\times10^{-2}\text{ V} = 73.3\text{ mV}.$

EVALUATE: This voltage is quite small compared to everyday voltages, so it normally would not be noticed. But in very delicate laboratory work, it could be large enough to take into consideration.

32.43. **IDENTIFY:** The nodal planes are one-half wavelength apart.

SET UP: The nodal planes of B are at $x = \lambda/4,\ 3\lambda/4,\ 5\lambda/4,\ \ldots,$ which are $\lambda/2$ apart.

EXECUTE: **(a)** The wavelength is $\lambda = c/f = (2.998\times10^8\text{ m/s})/(110.0\times10^6\text{ Hz}) = 2.725\text{ m}$. So the nodal planes are at $(2.725\text{ m})/2 = 1.363\text{ m}$ apart.

(b) For the nodal planes of E, we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.725\text{ m})/2 = 10.90\text{ m}$.

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.

32.44. **IDENTIFY:** $P_{\text{av}} = IA$ and $I = \frac{1}{2}\varepsilon_0 c E_{\text{max}}^2$. $E_{\text{max}} = cB_{\text{max}}$.

SET UP: The power carried by the current i is $P = Vi$.

EXECUTE: $I=\dfrac{P_{av}}{A}=\tfrac{1}{2}\epsilon_0 cE_{max}^2$ and

$$E_{max}=\sqrt{\frac{2P_{av}}{A\epsilon_0 c}}=\sqrt{\frac{2Vi}{A\epsilon_0 c}}=\sqrt{\frac{2(5.00\times 10^5\ \text{V})(1000\ \text{A})}{(100\ \text{m}^2)\epsilon_0(3.00\times 10^8\ \text{m/s})}}=6.14\times 10^4\ \text{V/m}.$$

$$B_{max}=\frac{E_{max}}{c}=\frac{6.14\times 10^4\ \text{V/m}}{3.00\times 10^8\ \text{m/s}}=2.05\times 10^{-4}\ \text{T}.$$

EVALUATE: $I=Vi/A=\dfrac{(5.00\times 10^5\ \text{V})(1000\ \text{A})}{100\ \text{m}^2}=5.00\times 10^6\ \text{W/m}^2$. This is a very intense beam spread over a large area.

32.45. IDENTIFY: The orbiting satellite obeys Newton's second law of motion. The intensity of the electromagnetic waves it transmits obeys the inverse-square distance law, and the intensity of the waves depends on the amplitude of the electric and magnetic fields.

SET UP: Newton's second law applied to the satellite gives $mv^2/r=GmM/r^2$, where M is the mass of the earth and m is the mass of the satellite. The intensity I of the wave is $I=S_{av}=\tfrac{1}{2}\epsilon_0 cE_{max}{}^2$, and by definition, $I=P_{av}/A$.

EXECUTE: **(a)** The period of the orbit is 12 hr. Applying Newton's second law to the satellite gives $mv^2/r=GmM/r^2$, which gives $\dfrac{m(2\pi r/T)^2}{r}=\dfrac{GmM}{r^2}$. Solving for r, we get

$$r=\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}=\left[\frac{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24}\ \text{kg})(12\times 3600\ \text{s})^2}{4\pi^2}\right]^{1/3}=2.66\times 10^7\ \text{m}.$$

The height above the surface is $h=2.66\times 10^7\ \text{m}-6.37\times 10^6\ \text{m}=2.02\times 10^7\ \text{m}$. The satellite only radiates its energy to the lower hemisphere, so the area is 1/2 that of a sphere. Thus, from the definition of intensity, the intensity at the ground is

$$I=P_{av}/A=P_{av}/(2\pi h^2)=(25.0\ \text{W})/[2\pi(2.02\times 10^7\ \text{m})^2]=9.75\times 10^{-15}\ \text{W/m}^2$$

(b) $I=S_{av}=\tfrac{1}{2}\epsilon_0 cE_{max}^2$, so $E_{max}=\sqrt{\dfrac{2I}{\epsilon_0 c}}=\sqrt{\dfrac{2(9.75\times 10^{-15}\ \text{W/m}^2)}{(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times 10^8\ \text{m/s})}}=2.71\times 10^{-6}\ \text{N/C}.$

$B_{max}=E_{max}/c=(2.71\times 10^{-6}\ \text{N/C})/(3.00\times 10^8\ \text{m/s})=9.03\times 10^{-15}\ \text{T}.$

$t=d/c=(2.02\times 10^7\ \text{m})/(3.00\times 10^8\ \text{m/s})=0.0673\ \text{s}.$

(c) $p_{rad}=I/c=(9.75\times 10^{-15}\ \text{W/m}^2)/(3.00\times 10^8\ \text{m/s})=3.25\times 10^{-23}\ \text{Pa}.$

(d) $\lambda=c/f=(3.00\times 10^8\ \text{m/s})/(1575.42\times 10^6\ \text{Hz})=0.190\ \text{m}.$

EVALUATE: The fields and pressures due to these waves are very small compared to typical laboratory quantities.

32.46. IDENTIFY: For a totally reflective surface the radiation pressure is $\dfrac{2I}{c}$. Find the force due to this pressure and express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_g=G\dfrac{mM_{sun}}{r^2}$.

SET UP: The mass of the sun is $M_{sun}=1.99\times 10^{30}\ \text{kg}$. $G=6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{rad}=\left(\dfrac{2I}{c}\right)A$, where A is the area of the sail. $I=\dfrac{P}{4\pi r^2}$, where r is the distance of the sail from the sun. $F_{rad}=\left(\dfrac{2A}{c}\right)\left(\dfrac{P}{4\pi r^2}\right)=\dfrac{PA}{2\pi r^2 c}\cdot F_{rad}=F_g$ so $\dfrac{PA}{2\pi r^2 c}=G\dfrac{mM_{sun}}{r^2}$.

$$A = \frac{2\pi c G m M_{\text{sun}}}{P} = \frac{2\pi(3.00\times10^{8}\ \text{m/s})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(10{,}000\ \text{kg})(1.99\times10^{30}\ \text{kg})}{3.9\times10^{26}\ \text{W}}.$$

$A = 6.42\times10^{6}\ \text{m}^2 = 6.42\ \text{km}^2$.

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{\text{rad}} = F_{\text{g}}$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.47. **IDENTIFY and SET UP:** The gravitational force is given by $F_{\text{g}} = G\dfrac{mM}{r^2}$. Express the mass of the particle in terms of its density and volume. The radiation pressure is given by $p_{\text{rad}} = \dfrac{I}{c}$; relate the power output L of the sun to the intensity at a distance r. The radiation force is the pressure times the cross-sectional area of the particle.

EXECUTE: **(a)** The gravitational force is $F_{\text{g}} = G\dfrac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho \frac{4}{3}\pi R^3$.

Thus $F_{\text{g}} = \dfrac{4\rho G\pi M R^3}{3r^2}$.

(b) For a totally absorbing surface $p_{\text{rad}} = \dfrac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation a distance r from the sun is $I = \dfrac{L}{4\pi r^2}$. Thus $p_{\text{rad}} = \dfrac{L}{4\pi c r^2}$. The force F_{rad} that corresponds to p_{rad} is in the direction of propagation of the radiation, so $F_{\text{rad}} = p_{\text{rad}} A_{\perp}$, where $A_{\perp} = \pi R^2$ is the component of area of the particle perpendicular to the radiation direction. Thus

$$F_{\text{rad}} = \left(\frac{L}{4\pi c r^2}\right)(\pi R^2) = \frac{LR^2}{4cr^2}.$$

(c) $F_g = F_{\text{rad}}$.

$$\frac{4\rho G\pi M R^3}{3r^2} = \frac{LR^2}{4cr^2}.$$

$$\left(\frac{4\rho G\pi M}{3}\right)R = \frac{L}{4c} \text{ and } R = \frac{3L}{16c\rho G\pi M}.$$

$$R = \frac{3(3.9\times10^{26}\ \text{W})}{16(2.998\times10^{8}\ \text{m/s})(3000\ \text{kg/m}^3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\pi(1.99\times10^{30}\ \text{kg})}.$$

$R = 1.9\times10^{-7}\ \text{m} = 0.19\ \mu\text{m}$.

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R.

(d) $\dfrac{F_{\text{rad}}}{F_{\text{g}}} = \left(\dfrac{LR^2}{4cr^2}\right)\left(\dfrac{3r^2}{4\rho G\pi M R^3}\right) = \dfrac{3L}{16c\rho G\pi M R}$. F_{rad} is proportional to R^2 and F_{g} is proportional to R^3, so this ratio is proportional to $1/R$. If $R < 0.20\ \mu\text{m}$ then $F_{\text{rad}} > F_{\text{g}}$ and the radiation force will drive the particles out of the solar system.

32.48. **IDENTIFY and SET UP:** The intensity of an electromagnetic wave can be expressed in many ways, including $I = \dfrac{P}{A} = \frac{1}{2}\epsilon_0 c E_{\max}^2 = \dfrac{c p_{\text{rad}}}{2}$, with the last way valid at a totally reflecting surface. In addition, the average energy density u in a wave is $u = \frac{1}{2}\epsilon_0 E_{\max}^2$. Also, $B = E/c$ and $p = \dfrac{F_{\perp}}{A}$.

EXECUTE: For each laser, we calculate the beam intensity using formula that is appropriate for the information we know about the beam.

Laser A: $I = P/A = (2.6\text{ W})/[\pi(1.3\times10^{-3}\text{ m})^2] = 4.9\times10^5\text{ W/m}^2$.
Laser B: $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2 = (1/2)(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\text{ m/s})(480\text{ V/m})^2 = 310\text{ W/m}^2$.

Laser C: Combining $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$ and $B_{\max} = E_{\max}/c$, we get

$I = \frac{1}{2}\varepsilon_0 c^3 B_{\max}^2 = (1/2)(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\text{ m/s})^3(8.7\times10^{-6}\text{ T})^2 = 9000\text{ W/m}^2$.

Laser D: The surface is totally reflecting, so

$I = \dfrac{cp_{\text{rad}}}{2} = \dfrac{cF_\perp}{2A} = (3.00\times10^8\text{ m/s})(6.0\times10^{-8}\text{ N})/[2\pi(0.90\times10^{-3}\text{ m})^2] = 3.5\times10^7\text{ W/m}^2$.

Laser E: Combining $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$ and $u = \frac{1}{2}\varepsilon_0 E_{\max}^2$ gives $I = \frac{1}{2}\varepsilon_0 c\left(\dfrac{2u}{\varepsilon_0}\right) = cu$, so

$I = (3.0\times10^8\text{ m/s})(3.0\times10^{-7}\text{ J/m}^3) = 90\text{ W/m}^2$.
In order of increasing intensity, we have E, B, C, A, D.
EVALUATE: The laser intensities vary a great deal. But even the least intense one is around 10 times as intense as a 100-W lightbulb viewed at 1 m, if the 100 W all went into light (which it certainly does *not*).

32.49. IDENTIFY and SET UP: The intensity of the light beam is $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2$.

EXECUTE: **(a)** A graph of I versus $E_{\max}^2$ should be a straight line having slope equal to $\frac{1}{2}\varepsilon_0 c$.

(b) Using the slope of the graph given with the problem, we have $\frac{1}{2}\varepsilon_0 c = 1.33\times10^{-3}\text{ J/(V}^2\cdot\text{s)}$. Solving for c gives $c = 2[1.33\times10^{-3}\text{ J/(V}^2\cdot\text{s)}]/(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 3.00\times10^8\text{ m/s}$.

EVALUATE: This result is nearly identical to the speed of light in vacuum.

32.50. IDENTIFY and SET UP: The spacing between antinodes is $\lambda/2$, and $f\lambda = c$.

EXECUTE: The antinode spacing is $d = \lambda/2 = \dfrac{c}{2}\cdot\dfrac{1}{f}$. Therefore a graph of d versus $1/f$ should be a straight line having a slope equal to $c/2$. Figure 32.50 shows the graph of d versus $1/f$.

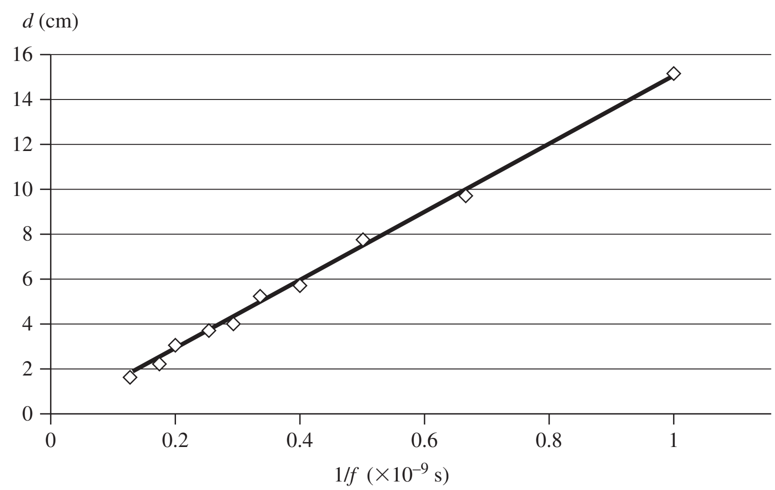


Figure 32.50

The slope of the best-fit line is 15.204×10^9 cm/s = 15.204×10^7 m/s, so $c/2 = 15.204\times10^7$ m/s, which gives $c = 3.0\times10^8$ m/s.
EVALUATE: This result is *very* close to the well-established value for the speed of light in vacuum.

32.51. IDENTIFY: The orbiting particle has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11\times10^{-31}$ kg and a proton has mass $m_p = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\left[\dfrac{q^2a^2}{6\pi\varepsilon_0 c^3}\right] = \dfrac{C^2(m/s^2)^2}{(C^2/N\cdot m^2)(m/s)^3} = \dfrac{N\cdot m}{s} = \dfrac{J}{s} = W = \left[\dfrac{dE}{dt}\right]$.

(b) For a proton moving in a circle, the acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(6.00\times10^6\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{(1.67\times10^{-27}\text{ kg})(0.75\text{ m})} = 1.53\times10^{15}\text{ m/s}^2$. The rate at which it emits energy because of its acceleration is

$$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\varepsilon_0 c^3} = \frac{(1.6\times10^{-19}\text{ C})^2(1.53\times10^{15}\text{ m/s}^2)^2}{6\pi\varepsilon_0(3.0\times10^8\text{ m/s})^3} = 1.33\times10^{-23}\text{ J/s} = 8.32\times10^{-5}\text{ eV/s}.$$

Therefore, the fraction of its energy that it radiates every second is

$$\frac{(dE/dt)(1\text{ s})}{E} = \frac{8.32\times10^{-5}\text{ eV}}{6.00\times10^6\text{ eV}} = 1.39\times10^{-11}.$$

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses: $E_e = E_p\dfrac{m_e}{m_p} = (6.00\times10^6\text{ eV})\dfrac{(9.11\times10^{-31}\text{ kg})}{(1.67\times10^{-27}\text{ kg})} = 3273\text{ eV}$. Therefore, the fraction of its energy that it radiates every second is $\dfrac{(dE/dt)(1\text{ s})}{E} = \dfrac{8.32\times10^{-5}\text{ eV}}{3273\text{ eV}} = 2.54\times10^{-8}$.

EVALUATE: The proton has speed $v = \sqrt{\dfrac{2E}{m_p}} = \sqrt{\dfrac{2(6.0\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{1.67\times10^{-27}\text{ kg}}} = 3.39\times10^7\text{ m/s}$. The electron has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.52 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.52. IDENTIFY: The electron has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $1\text{ eV} = 1.60\times10^{-19}$ C. An electron has $|q| = e = 1.60\times10^{-19}$ C.

EXECUTE: For the electron in the classical hydrogen atom, its acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(13.6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(9.11\times10^{-31}\text{ kg})(5.29\times10^{-11}\text{ m})} = 9.03\times10^{22}\text{ m/s}^2$. Then using the formula for the rate of energy emission given in Problem 32.51:

$\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\varepsilon_0 c^3} = \dfrac{(1.60\times10^{-19}\text{ C})^2(9.03\times10^{22}\text{ m/s}^2)^2}{6\pi\varepsilon_0(3.00\times10^8\text{ m/s})^3} = 4.64\times10^{-8}\text{ J/s} = 2.89\times10^{11}\text{ eV/s}$. This large value of $\dfrac{dE}{dt}$ would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.51 must not apply to electrons in atoms.

32.53. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) $E_y(x,t) = E_{\max} e^{-k_C x} \cos(k_C x - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max}(-k_C)e^{-k_C x}\cos(k_C x - \omega t) + E_{\max}(-k_C)e^{-k_C x}\sin(k_C x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max}(+k_C^2)e^{-k_C x}\cos(k_C x - \omega t) + E_{\max}(+k_C^2)e^{-k_C x}\sin(k_C x - \omega t).$$

$$+E_{\max}(+k_C^2)e^{-k_C x}\sin(k_C x - \omega t) + E_{\max}(-k_C^2)e^{-k_C x}\cos(k_C x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = +2E_{\max}k_C^2 e^{-k_C x}\sin(k_C x - \omega t). \quad \frac{\partial E_y}{\partial t} = +E_{\max}e^{-k_C x}\,\omega\sin(k_C x - \omega t).$$

Setting $\frac{\partial^2 E_y}{\partial x^2} = \frac{\mu \partial E_y}{\rho \partial t}$ gives $2E_{\max}k_C^2 e^{-k_C x}\sin(k_C x - \omega t) = \mu/\rho E_{\max} e^{-k_C x}\,\omega\sin(k_C x - \omega t)$. This will only be true if $\frac{2k_C^2}{\omega} = \frac{\mu}{\rho}$, or $k_C = \sqrt{\frac{\omega\mu}{2\rho}}$.

(b) The energy in the wave is dissipated by the i^2R heating of the conductor.

(c) $E_y = \frac{E_{y0}}{e} \Rightarrow k_C x = 1$, $x = \frac{1}{k_C} = \sqrt{\frac{2\rho}{\omega\mu}} = \sqrt{\frac{2(1.72\times10^{-8}\ \Omega\cdot\text{m})}{2\pi(1.0\times10^{6}\ \text{Hz})\mu_0}} = 6.60\times10^{-5}\ \text{m}$.

EVALUATE: The lower the frequency of the waves, the greater is the distance they can penetrate into a conductor. A dielectric (insulator) has a much larger resistivity and these waves can penetrate a greater distance in these materials.

32.54. IDENTIFY and SET UP: Since 60 Hz is in the range 25 Hz to 3 kHz, we use the formula $E_{\max} = \frac{350}{f}$ V/m, where f is in kHz. The intensity is $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$.

EXECUTE: The maximum electric field is $E_{\max} = \frac{350}{f}$ V/m $= \frac{350}{0.060}$ V/m = 5800 V/m. Now find the intensity for the maximum field.

$I = \frac{1}{2}\epsilon_0 c E_{\max}^2 = (1/2)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\ \text{m/s})(5800\ \text{V/m})^2 = 4.5\times10^4\ \text{W/m}^2 = 45\ \text{kW/m}^2$, which is choice (c).

EVALUATE: At higher frequencies the intensity would be less because the maximum electric field, which is inversely proportional to the frequency, would be smaller.

32.55. IDENTIFY and SET UP: The maximum electric field is proportional to $1/f$, and the intensity is proportional to $E_{\max}^2$.

EXECUTE: Since $E_{\max}$ is proportional to $1/f$, doubling f decreases the maximum field by $\frac{1}{2}$. Because the intensity is proportional to $E_{\max}^2$, decreasing $E_{\max}$ by a factor of $\frac{1}{2}$ will decrease the intensity by a factor of $(\frac{1}{2})^2 = \frac{1}{4}$, which is choice (d).

EVALUATE: Higher frequencies could be more harmful, so we tolerate lower fields at higher frequency.

32.56. IDENTIFY and SET UP: In the frequency range 25 Hz to 3 kHz, for a given frequency the maximum electric field is $E_{\max} = 350/f$ and the maximum electric field is $B_{\max} = 5/f$. $B = cE$.

EXECUTE: For the electric field, the maximum intensity at a frequency f is $I_{\max} = \frac{1}{2}\epsilon_0 c E_{\max}^2$. Since $E_{\max} = 350/f$, the intensity is $I_{\max}^E = \frac{1}{2}\epsilon_0 c\left(\frac{350}{f}\right)^2$.

The intensity in terms of the magnetic field is $I = \frac{1}{2}\varepsilon_0 cE_{\max}^2 = \frac{1}{2}\varepsilon_0 c(B_{\max}c)^2 = \frac{1}{2}\varepsilon_0 c^3 B_{\max}^2$, where we have used $E_{\max} = cB_{\max}$. The maximum magnetic field is $B_{\max} = 5/f$, so the maximum intensity for this magnetic field is $I_{\max}^B = \frac{1}{2}\varepsilon_0 c^3 \left(\frac{5}{f}\right)^2$. Taking the ratio of the two intensities gives

$\dfrac{I_{\max}^E}{I_{\max}^B} = \dfrac{\frac{1}{2}\varepsilon_0 c\left(\frac{350}{f}\right)^2}{\frac{1}{2}\varepsilon_0 c^3\left(\frac{5}{f}\right)^2} = \dfrac{1}{c^2}\left(\dfrac{350}{5}\right)^2 = 5.4\times 10^{-14}$. The allowed intensity using the electric field limitation is *much* less than the allowed intensity using the magnetic field limitation, which is choice (b).

EVALUATE: The magnetic force on a charge due to an electromagnetic wave is normally much less than the electric force, so the intensity allowed for the electric field is much less than for the magnetic field.