21

ELECTRIC CHARGE AND ELECTRIC FIELD

21.1. (a) IDENTIFY and SET UP: Use the charge of one electron $(-1.602\times10^{-19}\text{ C})$ to find the number of electrons required to produce the net charge.

EXECUTE: The number of excess electrons needed to produce net charge q is

$$\frac{q}{-e}=\frac{-3.20\times10^{-9}\text{ C}}{-1.602\times10^{-19}\text{ C/electron}}=2.00\times10^{10}\text{ electrons.}$$

(b) IDENTIFY and SET UP: Use the atomic mass of lead to find the number of lead atoms in 8.00×10^{-3} kg of lead. From this and the total number of excess electrons, find the number of excess electrons per lead atom.

EXECUTE: The atomic mass of lead is 207×10^{-3} kg/mol, so the number of moles in 8.00×10^{-3} kg is

$n=\dfrac{m_{\text{tot}}}{M}=\dfrac{8.00\times10^{-3}\text{ kg}}{207\times10^{-3}\text{ kg/mol}}=0.03865\text{ mol.}$ N_A (Avogadro's number) is the number of atoms in 1 mole,

so the number of lead atoms is $N=nN_A=(0.03865\text{ mol})(6.022\times10^{23}\text{ atoms/mol})=2.328\times10^{22}$ atoms.

The number of excess electrons per lead atom is $\dfrac{2.00\times10^{10}\text{ electrons}}{2.328\times10^{22}\text{ atoms}}=8.59\times10^{-13}.$

EVALUATE: Even this small net charge corresponds to a large number of excess electrons. But the number of atoms in the sphere is much larger still, so the number of excess electrons per lead atom is very small.

21.2. IDENTIFY: The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e=1.60\times10^{-19}$ C.

EXECUTE: The rate of charge flow is 20,000 C/s and $t=100\ \mu\text{s}=1.00\times10^{-4}$ s.

$Q=(20{,}000\text{ C/s})(1.00\times10^{-4}\text{ s})=2.00\text{ C}.$ The number of electrons is $n_e=\dfrac{Q}{1.60\times10^{-19}\text{ C}}=1.25\times10^{19}.$

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.3. IDENTIFY and SET UP: A proton has charge $+e$ and an electron has charge $-e$, with $e=1.60\times10^{-19}$ C.

The force between them has magnitude $F=k\dfrac{|q_1q_2|}{r^2}=k\dfrac{e^2}{r^2}$ and is attractive since the charges have opposite sign. A proton has mass $m_p=1.67\times10^{-27}$ kg and an electron has mass 9.11×10^{-31} kg. The acceleration is related to the net force $\vec{F}$ by $\vec{F}=m\vec{a}$.

EXECUTE: $F=k\dfrac{e^2}{r^2}=(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\text{ C})^2}{(2.0\times10^{-10}\text{ m})^2}=5.75\times10^{-9}\text{ N}.$

proton: $a_p=\dfrac{F}{m_p}=\dfrac{5.75\times10^{-9}\text{ N}}{1.67\times10^{-27}\text{ kg}}=3.4\times10^{18}\text{ m/s}^2.$

electron: $a_e = \dfrac{F}{m_e} = \dfrac{5.75\times10^{-9}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = 6.3\times10^{21}\ \text{m/s}^2$

The proton has an initial acceleration of $3.4\times10^{18}\ \text{m/s}^2$ toward the electron and the electron has an initial acceleration of $6.3\times10^{21}\ \text{m/s}^2$ toward the proton.

EVALUATE: The force the electron exerts on the proton is equal in magnitude to the force the proton exerts on the electron, but the accelerations of the two particles are very different because their masses are very different.

21.4. **IDENTIFY:** Use the mass *m* of the ring and the atomic mass *M* of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: $N_A = 6.02\times10^{23}$ atoms/mol. A proton has charge $+e$.

EXECUTE: The mass of gold is 10.8 g and the atomic weight of gold is 197 g/mol. So the number of atoms is

$N_A n = (6.02\times10^{23}\ \text{atoms/mol})\left(\dfrac{10.8\ \text{g}}{197\ \text{g/mol}}\right) = 3.300\times10^{22}$ atoms. The number of protons is

$n_p = (79\ \text{protons/atom})(3.300\times10^{22}\ \text{atoms}) = 2.61\times10^{24}$ protons.

$Q = (n_p)(1.60\times10^{-19}\ \text{C/proton}) = 4.18\times10^{5}\ \text{C}.$

(b) The number of electrons is $n_e = n_p = 2.61\times10^{24}$.

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.

21.5. **IDENTIFY:** Each ion carries charge as it enters the axon.

SET UP: The total charge *Q* is the number *N* of ions times the charge of each one, which is *e*. So $Q = Ne$, where $e = 1.60\times10^{-19}$ C.

EXECUTE: The number *N* of ions is $N = (5.6\times10^{11}\ \text{ions/m})(1.5\times10^{-2}\ \text{m}) = 8.4\times10^{9}$ ions. The total charge *Q* carried by these ions is $Q = Ne = (8.4\times10^{9})(1.60\times10^{-19}\ \text{C}) = 1.3\times10^{-9}\ \text{C} = 1.3\ \text{nC}$.

EVALUATE: The amount of charge is small, but these charges are close enough together to exert large forces on nearby charges.

21.6. **IDENTIFY:** Apply Coulomb's law and calculate the net charge *q* on each sphere.

SET UP: The magnitude of the charge of an electron is $e = 1.60\times10^{-19}$ C.

EXECUTE: $F = k\dfrac{|q_1 q_2|}{r^2}$ gives

$|q| = \sqrt{4\pi\epsilon_0 F r^2} = \sqrt{4\pi\epsilon_0 (3.33\times10^{-21}\ \text{N})(0.200\ \text{m})^2} = 1.217\times10^{-16}$ C. Therefore, the total number of electrons required is $n = |q|/e = (1.217\times10^{-16}\ \text{C})/(1.60\times10^{-19}\ \text{C/electron}) = 760$ electrons.

EVALUATE: Each sphere has 760 excess electrons and each sphere has a net negative charge. The two like charges repel.

21.7. **IDENTIFY:** Apply $F = \dfrac{k|q_1 q_2|}{r^2}$ and solve for *r*.

SET UP: $F = 650$ N.

EXECUTE: $r = \sqrt{\dfrac{k|q_1 q_2|}{F}} = \sqrt{\dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\ \text{C})^2}{650\ \text{N}}} = 3.7\times10^{3}\ \text{m} = 3.7\ \text{km}$

EVALUATE: Charged objects typically have net charges much less than 1 C.

21.8. **IDENTIFY:** Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_A = 6.02 \times 10^{23}$ atoms/mol. $|q| = n'_e e$, where n'_e is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$$n_e = n_p = (13)(N_A)\left(\frac{0.0250 \text{ kg}}{0.026982 \text{ kg/mol}}\right) = 7.25 \times 10^{24} \text{ electrons.}$$

(b) For a force of 1.00×10^4 N to act between the spheres, $F = 1.00 \times 10^4 \text{ N} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$. This gives

$|q| = \sqrt{4\pi\epsilon_0 (1.00 \times 10^4 \text{ N})(0.800 \text{ m})^2} = 8.43 \times 10^{-4}$ C. The number of electrons removed from one sphere and added to the other is $n'_e = |q|/e = 5.27 \times 10^{15}$ electrons.

(c) $n'_e/n_e = 7.27 \times 10^{-10}$.

EVALUATE: When ordinary objects receive a net charge, the fractional change in the total number of electrons in the object is very small.

21.9. IDENTIFY: Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

EXECUTE: **(a)** $q_1 = q_2 = q$ and $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2}$, so

$$q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150 \text{ m}\sqrt{\frac{0.220 \text{ N}}{8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}} = 7.42 \times 10^{-7} \text{ C (on each).}$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \text{ so } q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \frac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \frac{1}{2}(7.42 \times 10^{-7} \text{ C}) = 3.71 \times 10^{-7} \text{ C.}$$

And then $q_2 = 4q_1 = 1.48 \times 10^{-6}$ C.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the spheres have equal charges or not.

21.10. IDENTIFY: We need to determine the number of protons in each box and then use Coulomb's law to calculate the force each box would exert on the other.

SET UP: The mass of a proton is 1.67×10^{-27} kg and the charge of a proton is 1.60×10^{-19} C. The distance from the earth to the moon is 3.84×10^8 m. The electrical force has magnitude $F_e = k\frac{|q_1 q_2|}{r^2}$, where $k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$. The gravitational force has magnitude $F_{grav} = G\frac{m_1 m_2}{r^2}$, where $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** The number of protons in each box is $N = \frac{1.0 \times 10^{-3} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 5.99 \times 10^{23}$. The total charge of each box is $q = Ne = (5.99 \times 10^{23})(1.60 \times 10^{-19} \text{ C}) = 9.58 \times 10^4$ C. The electrical force on each box is $F_e = k\frac{q^2}{r^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(9.58 \times 10^4 \text{ C})^2}{(3.84 \times 10^8 \text{ m})^2} = 560 \text{ N} = 130$ lb. The tension in the string must equal this repulsive electrical force. The weight of the box on earth is $w = mg = 9.8 \times 10^{-3}$ N and the weight of the box on the moon is even less, since g is less on the moon. The gravitational forces exerted on the boxes by the earth and by the moon are much less than the electrical force and can be neglected.

(b) $F_{\text{grav}} = G\dfrac{m_1 m_2}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1.0\times10^{-3}\ \text{kg})^2}{(3.84\times10^{8}\ \text{m})^2} = 4.5\times10^{-34}\ \text{N}.$

EVALUATE: Both the electrical force and the gravitational force are proportional to $1/r^2$. But in SI units the coefficient k in the electrical force is much greater than the coefficient G in the gravitational force. And a small mass of protons contains a large amount of charge. It would be impossible to put 1.0 g of protons into a small box, because of the very large repulsive electrical forces the protons would exert on each other.

21.11. IDENTIFY: In a space satellite, the only force accelerating the free proton is the electrical repulsion of the other proton.

SET UP: Coulomb's law gives the force, and Newton's second law gives the acceleration:

$a = F/m = (1/4\pi\epsilon_0)(e^2/r^2)/m.$

EXECUTE:

(a) $a = (9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2/[(0.00250\ \text{m})^2(1.67\times10^{-27}\ \text{kg})] = 2.21\times10^{4}\ \text{m/s}^2.$

(b) The graphs are sketched in Figure 21.11.

EVALUATE: The electrical force of a single stationary proton gives the moving proton an initial acceleration about 20,000 times as great as the acceleration caused by the gravity of the entire earth. As the protons move farther apart, the electrical force gets weaker, so the acceleration decreases. Since the protons continue to repel, the velocity keeps increasing, but at a decreasing rate.

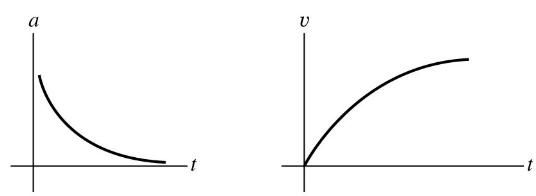


Figure 21.11

21.12. IDENTIFY: Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: **(a)** $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2}$ gives $0.600\ \text{N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(0.550\times10^{-6}\ \text{C})|q_2|}{(0.30\ \text{m})^2}$ and $|q_2| = +1.09\times10^{-5}\ \text{C} =$ $10.9\ \mu\text{C}$. The force is attractive and $q_1 < 0$, so $q_2 = +1.09\times10^{-5}\ \text{C} = +10.9\mu\text{C}$.

(b) $F = 0.600\ \text{N}$. The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.13. IDENTIFY: Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k\dfrac{|q_2 q_3|}{r_2^2}$ and is in the $+x$-direction.

$\vec{F}_1$ must be in the $-x$-direction, so q_1 must be positive. $F_1 = F_2$ gives $k\dfrac{|q_1||q_3|}{r_1^2} = k\dfrac{|q_2||q_3|}{r_2^2}$.

$$|q_1| = |q_2|\left(\frac{r_1}{r_2}\right)^2 = (3.00\ \text{nC})\left(\frac{2.00\ \text{cm}}{4.00\ \text{cm}}\right)^2 = 0.750\ \text{nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_3.

21.14. IDENTIFY: Apply Coulomb's law and find the vector sum of the two forces on Q.

SET UP: The force that q_1 exerts on Q is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The x-components cancel. We only need the y-components, and each charge contributes equally. $F_{1y} = F_{2y} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{(2.0\times10^{-6}\text{ C})(4.0\times10^{-6}\text{ C})}{(0.500\text{ m})^2}\sin\alpha = -0.173\text{ N}$ (since $\sin\alpha = 0.600$). Therefore, the total force is $2F = 0.35\text{ N}$, in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\,\mu\text{C}$ and q_2 is $+2.0\,\mu\text{C}$, then the net force is in the $+y$-direction.

21.15. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_1.

SET UP: Like charges repel and unlike charges attract, so $\vec{F}_2$ and $\vec{F}_3$ are both in the $+x$-direction.

EXECUTE: $F_2 = k\dfrac{|q_1q_2|}{r_{12}^2} = 6.749\times10^{-5}\text{ N}$, $F_3 = k\dfrac{|q_1q_3|}{r_{13}^2} = 1.124\times10^{-4}\text{ N}$. $F = F_2 + F_3 = 1.8\times10^{-4}\text{ N}$.

$F = 1.8\times10^{-4}\text{ N}$ and is in the $+x$-direction.

EVALUATE: Comparing our results to those in Example 21.3, we see that $\vec{F}_{1\text{ on }3} = -\vec{F}_{3\text{ on }1}$, as required by Newton's third law.

21.16. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_2.

SET UP: $\vec{F}_{2\text{ on }1}$ is in the $+y$-direction.

EXECUTE: $F_{2\text{ on }1} = \dfrac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{(0.60\text{ m})^2} = 0.100\text{ N}$. $(F_{2\text{ on }1})_x = 0$ and $(F_{2\text{ on }1})_y = +0.100\text{ N}$. $F_{Q\text{ on }1}$ is equal and opposite to $F_{1\text{ on }Q}$ (Example 21.4), so $(F_{Q\text{ on }1})_x = -0.23\text{ N}$ and $(F_{Q\text{ on }1})_y = 0.17\text{ N}$. $F_x = (F_{2\text{ on }1})_x + (F_{Q\text{ on }1})_x = -0.23\text{ N}$.

$F_y = (F_{2\text{ on }1})_y + (F_{Q\text{ on }1})_y = 0.100\text{ N} + 0.17\text{ N} = 0.27\text{ N}$. The magnitude of the total force is

$F = \sqrt{(0.23\text{ N})^2 + (0.27\text{ N})^2} = 0.35\text{ N}$. $\tan^{-1}\dfrac{0.23}{0.27} = 40°$, so $\vec{F}$ is $40°$ counterclockwise from the $+y$-axis, or $130°$ counterclockwise from the $+x$-axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.17. **IDENTIFY** and **SET UP:** Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1. Add these forces as vectors to get the net force. The target variable is the x-coordinate of q_3.

EXECUTE: $\vec{F}_2$ is in the x-direction.

$F_2 = k\dfrac{|q_1q_2|}{r_{12}^2} = 3.37\text{ N}$, so $F_{2x} = +3.37\text{ N}$

$F_x = F_{2x} + F_{3x}$ and $F_x = -7.00\text{ N}$

$F_{3x} = F_x - F_{2x} = -7.00\text{ N} - 3.37\text{ N} = -10.37\text{ N}$

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$F_3 = k\dfrac{|q_1q_3|}{x^2}$, so $|x| = \sqrt{\dfrac{k|q_1q_3|}{F_3}} = 0.144\text{ m}$, so $x = -0.144\text{ m}$

EVALUATE: q_2 attracts q_1 in the $+x$-direction so q_3 must attract q_1 in the $-x$-direction, and q_3 is at negative x.

21.18. **IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract. Let $\vec{F}_{21}$ be the force that q_2 exerts on q_1 and let $\vec{F}_{31}$ be the force that q_3 exerts on q_1.

EXECUTE: The charge q_3 must be to the right of the origin; otherwise both q_2 and q_3 would exert forces in the $+x$-direction. Calculating the two forces:

$$F_{21} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r_{12}^2} = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{ C})(5.00\times10^{-6}\text{ C})}{(0.200\text{ m})^2} = 3.375\text{ N, in the } +x\text{-direction.}$$

$$F_{31} = \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\ \text{C})(8.00\times10^{-6}\ \text{C})}{r_{13}^2} = \frac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2},\ \text{in the } -x\text{-direction.}$$

We need $F_x = F_{21} - F_{31} = -7.00\ \text{N}$, so $3.375\ \text{N} - \dfrac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2} = -7.00\ \text{N}$.

$r_{13} = \sqrt{\dfrac{0.216\ \text{N}\cdot\text{m}^2}{3.375\ \text{N} + 7.00\ \text{N}}} = 0.144\ \text{m}$. q_3 is at $x = 0.144$ m.

EVALUATE: $F_{31} = 10.4$ N. F_{31} is larger than F_{21}, because $|q_3|$ is larger than $|q_2|$ and also because r_{13} is less than r_{12}.

21.19. IDENTIFY: Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.19a.

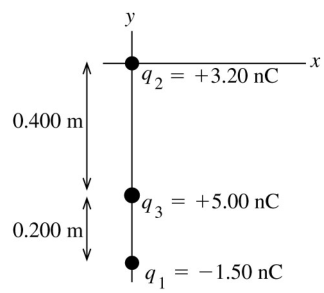
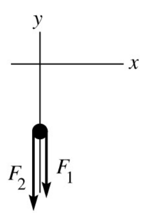


Figure 21.19a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.19b.



$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r_{23}^2}$$

Figure 21.19b

$$F_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.50\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 1.685\times10^{-6}\ \text{N}$$

$$F_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.400\ \text{m})^2} = 8.988\times10^{-7}\ \text{N}$$

The resultant force is $\vec{R} = \vec{F}_1 + \vec{F}_2$.

$R_x = 0$.

$R_y = -(F_1 + F_2) = -(1.685\times10^{-6}\ \text{N} + 8.988\times10^{-7}\ \text{N}) = -2.58\times10^{-6}\ \text{N}$.

The resultant force has magnitude 2.58×10^{-6} N and is in the $-y$-direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is replusive.

21.20. IDENTIFY: Apply $F = k\dfrac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.20.

EXECUTE: $F_1 = k\dfrac{|q_1 q_3|}{r_1^2} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(4.00\times 10^{-9}\ \text{C})(6.00\times 10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times 10^{-6}\ \text{N}.$

$F_2 = k\dfrac{|q_2 q_3|}{r_2^2} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times 10^{-9}\ \text{C})(6.00\times 10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times 10^{-6}\ \text{N}.$

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times 10^{-6}\ \text{N}.$ The net force has magnitude $2.40\times 10^{-6}\ \text{N}$ and is in the $+x$-direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

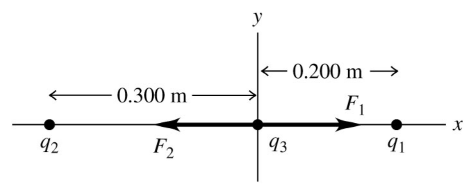


Figure 21.20

21.21. IDENTIFY: We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-N combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}.$

EXECUTE: **(a)** $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}.$

O-H-N:

O^- - H^+: $F = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times 10^{-19}\ \text{C})^2}{(0.170\times 10^{-9}\ \text{m})^2} = 7.96\times 10^{-9}\ \text{N}$, attractive

O^- - N^-: $F = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times 10^{-19}\ \text{C})^2}{(0.280\times 10^{-9}\ \text{m})^2} = 2.94\times 10^{-9}\ \text{N}$, repulsive

N-H-N:

N^- - H^+: $F = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times 10^{-19}\ \text{C})^2}{(0.190\times 10^{-9}\ \text{m})^2} = 6.38\times 10^{-9}\ \text{N}$, attractive

N^- - N^-: $F = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times 10^{-19}\ \text{C})^2}{(0.300\times 10^{-9}\ \text{m})^2} = 2.56\times 10^{-9}\ \text{N}$, repulsive

The total attractive force is $1.43\times 10^{-8}\ \text{N}$ and the total repulsive force is $5.50\times 10^{-9}\ \text{N}$. The net force is attractive and has magnitude $1.43\times 10^{-8}\ \text{N} - 5.50\times 10^{-9}\ \text{N} = 8.80\times 10^{-9}\ \text{N}$.

(b) $F = k\dfrac{e^2}{r^2} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times 10^{-19}\ \text{C})^2}{(0.0529\times 10^{-9}\ \text{m})^2} = 8.22\times 10^{-8}\ \text{N}.$

EVALUATE: The bonding force of the electron in the hydrogen atom is a factor of 10 larger than the bonding force of the adenine-thymine molecules.

21.22. IDENTIFY: We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-O combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other O^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. In the O-H-N combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}$.

EXECUTE: Using $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}$, we find that the attractive forces are: O^- - H^+, 7.10×10^{-9} N; N^- - H^+, 6.37×10^{-9} N; O^- - H^+, 7.10×10^{-9} N. The total attractive force is 2.06×10^{-8} N. The repulsive forces are: O^- - O^-, 2.74×10^{-9} N; N^- - N^-, 2.56×10^{-9} N; O^- - N^-, 2.74×10^{-9} N. The total repulsive force is 8.04×10^{-9} N. The net force is attractive and has magnitude 1.26×10^{-8} N.

EVALUATE: The net force is attractive, as it should be if the molecule is to stay together.

21.23. IDENTIFY: $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge $+e$ and mass 1.67×10^{-27} kg.

EXECUTE: **(a)** $F = (1.60\times10^{-19}\ \text{C})(2.75\times10^{3}\ \text{N/C}) = 4.40\times10^{-16}\ \text{N}$.

(b) $a = \dfrac{F}{m} = \dfrac{4.40\times10^{-16}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 2.63\times10^{11}\ \text{m/s}^2$.

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63\times10^{11}\ \text{m/s}^2)(1.00\times10^{-6}\ \text{s}) = 2.63\times10^{5}\ \text{m/s}$.

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

21.24. IDENTIFY: For a point charge, $E = k\dfrac{|q|}{r^2}$.

SET UP: $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: **(a)** The field is toward the negative charge so is downward.

$$E = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.250\ \text{m})^2} = 719\ \text{N/C}.$$

(b) $r = \sqrt{\dfrac{k|q|}{E}} = \sqrt{\dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\ \text{C})}{12.0\ \text{N/C}}} = 1.94\ \text{m}$.

EVALUATE: At different points the electric field has different directions, but it is always directed toward the negative point charge.

21.25. IDENTIFY: The acceleration that stops the charge is produced by the force that the electric field exerts on it. Since the field and the acceleration are constant, we can use the standard kinematics formulas to find acceleration and time.

(a) SET UP: First use kinematics to find the proton's acceleration. $v_x = 0$ when it stops. Then find the electric field needed to cause this acceleration using the fact that $F = qE$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. $0 = (4.50\times10^{6}\ \text{m/s})^2 + 2a(0.0320\ \text{m})$ and $a = 3.16\times10^{14}\ \text{m/s}^2$. Now find the electric field, with $q = e$. $eE = ma$ and

$E = ma/e = (1.67\times10^{-27}\ \text{kg})(3.16\times10^{14}\ \text{m/s}^2)/(1.60\times10^{-19}\ \text{C}) = 3.30\times10^{6}\ \text{N/C}$, to the left.

(b) SET UP: Kinematics gives $v = v_0 + at$, and $v = 0$ when the electron stops, so $t = v_0/a$.

EXECUTE: $t = v_0/a = (4.50\times10^{6}\ \text{m/s})/(3.16\times10^{14}\ \text{m/s}^2) = 1.42\times10^{-8}\ \text{s} = 14.2\ \text{ns}$.

(c) SET UP: In part (a) we saw that the electric field is proportional to m, so we can use the ratio of the electric fields. $E_e/E_p = m_e/m_p$ and $E_e = (m_e/m_p)E_p$.

EXECUTE: $E_e = [(9.11\times10^{-31}\text{ kg})/(1.67\times10^{-27}\text{ kg})](3.30\times10^6\text{ N/C}) = 1.80\times10^3\text{ N/C}$, to the right.

EVALUATE: Even a modest electric field, such as the ones in this situation, can produce enormous accelerations for electrons and protons.

21.26. IDENTIFY: Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y-y_0)}{t^2} = \frac{2(4.50\text{ m})}{(3.00\times10^{-6}\text{ s})^2} = 1.00\times10^{12}\text{ m/s}^2.$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11\times10^{-31}\text{ kg})(1.00\times10^{12}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 5.69\text{ N/C}$$

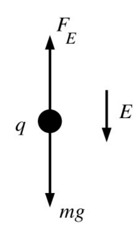
The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim10^{11}g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.27. IDENTIFY: The equation $\vec{F} = q\vec{E}$ relates the electric field, charge of the particle, and the force on the particle. If the particle is to remain stationary the net force on it must be zero.

SET UP: The free-body diagram for the particle is sketched in Figure 21.27. The weight is mg, downward. For the net force to be zero the force exerted by the electric field must be upward. The electric field is downward. Since the electric field and the electric force are in opposite directions the charge of the particle is negative.



$mg = |q|E$

Figure 21.27

EXECUTE: **(a)** $|q| = \frac{mg}{E} = \frac{(1.45\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{650\text{ N/C}} = 2.19\times10^{-5}\text{ C}$ and $q = -21.9\ \mu\text{C}$.

(b) SET UP: The electrical force has magnitude $F_E = |q|E = eE$. The weight of a proton is $w = mg$. $F_E = w$ so $eE = mg$.

EXECUTE: $E = \frac{mg}{e} = \frac{(1.673\times10^{-27}\text{ kg})(9.80\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 1.02\times10^{-7}\text{ N/C}.$

This is a very small electric field.

EVALUATE: In both cases $|q|E = mg$ and $E = (m/|q|)g$. In part (b) the $m/|q|$ ratio is much smaller $(\sim10^{-8})$ than in part (a) $(\sim10^{2})$ so E is much smaller in (b). For subatomic particles gravity can usually be ignored compared to electric forces.

21.28. IDENTIFY: The electric force is $\vec{F} = q\vec{E}$.

SET UP: The gravity force (weight) has magnitude $w = mg$ and is downward.

EXECUTE: **(a)** To balance the weight the electric force must be upward. The electric field is downward, so for an upward force the charge q of the person must be negative. $w = F$ gives $mg = |q|E$ and

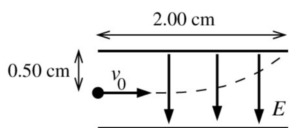
$$|q| = \frac{mg}{E} = \frac{(60\text{ kg})(9.80\text{ m/s}^2)}{150\text{ N/C}} = 3.9\text{ C}.$$

(b) $F = k\frac{|qq'|}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(3.9\text{ C})^2}{(100\text{ m})^2} = 1.4\times10^7\text{ N}.$ The repulsive force is immense and this is not a feasible means of flight.

EVALUATE: The net charge of charged objects is typically much less than 1 C.

21.29. **IDENTIFY:** The equation $\vec{F} = q\vec{E}$ gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

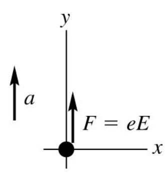
SET UP: The motion is sketched in Figure 21.29a.



For an electron $q = -e$.

Figure 21.29a

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.29b.



EXECUTE: **(a)** $\sum F_y = ma_y$

$eE = ma$

Figure 21.29b

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00\text{ cm}$ when $y - y_0 = +0.500\text{ cm}$.

<u>x-component</u>:

$v_{0x} = v_0 = 1.60\times10^6\text{ m/s},\ a_x = 0,\ x - x_0 = 0.0200\text{ m},\ t = ?$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}} = 1.25\times10^{-8}\text{ s}$$

In this same time t the electron travels 0.0050 m vertically.

<u>y-component</u>:

$t = 1.25\times10^{-8}\text{ s},\ v_{0y} = 0,\ y - y_0 = +0.0050\text{ m},\ a_y = ?$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.0050\text{ m})}{(1.25\times10^{-8}\text{ s})^2} = 6.40\times10^{13}\text{ m/s}^2.$$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma$.

$$E = \frac{ma}{e} = \frac{(9.109\times10^{-31}\ \text{kg})(6.40\times10^{13}\ \text{m/s}^2)}{1.602\times10^{-19}\ \text{C}} = 364\ \text{N/C}$$

Note that the acceleration produced by the electric field is much larger than g, the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the electron in this problem.

(b) $a = \dfrac{eE}{m_\text{p}} = \dfrac{(1.602\times10^{-19}\ \text{C})(364\ \text{N/C})}{1.673\times10^{-27}\ \text{kg}} = 3.49\times10^{10}\ \text{m/s}^2.$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25\times10^{-8}\ \text{s}$ to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49\times10^{10}\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = \frac{1}{2}(-3.49\times10^{10}\ \text{m/s}^2)(1.25\times10^{-8}\ \text{s})^2 = -2.73\times10^{-6}\ \text{m}.$ The displacement is $2.73\times10^{-6}\ \text{m},$ downward.

EVALUATE: **(c)** The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

(d) In each case $a \gg g$ and it is reasonable to ignore the effects of gravity.

21.30. **IDENTIFY:** Use the components of $\vec{E}$ from Example 21.6 to calculate the magnitude and direction of $\vec{E}$. Use $\vec{F} = q\vec{E}$ to calculate the force on the -2.5 nC charge and use Newton's third law for the force on the -8.0 nC charge.

SET UP: From Example 21.6, $\vec{E} = (-11\ \text{N/C})\hat{i} + (14\ \text{N/C})\hat{j}$.

EXECUTE: **(a)** $E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-11\ \text{N/C})^2 + (14\ \text{N/C})^2} = 17.8\ \text{N/C}.$

$\tan^{-1}\left(\dfrac{|E_y|}{|E_x|}\right) = \tan^{-1}(14/11) = 51.8°,$ so $\theta = 128°$ counterclockwise from the $+x$-axis.

(b) (i) $\vec{F} = \vec{E}q$ so $F = (17.8\ \text{N/C})(2.5\times10^{-9}\ \text{C}) = 4.45\times10^{-8}\ \text{N},$ at $52°$ below the $+x$-axis.

(ii) 4.45×10^{-8} N at $128°$ counterclockwise from the $+x$-axis.

EVALUATE: The forces in part (b) are repulsive so they are along the line connecting the two charges and in each case the force is directed away from the charge that exerts it.

21.31. **IDENTIFY:** Apply constant acceleration equations to the motion of the electron.

SET UP: Let $+x$ be to the right and let $+y$ be downward. The electron moves 2.00 cm to the right and 0.50 cm downward.

EXECUTE: Use the horizontal motion to find the time when the electron emerges from the field.

$x - x_0 = 0.0200\ \text{m}, a_x = 0, v_{0x} = 1.60\times10^6\ \text{m/s}. x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives $t = 1.25\times10^{-8}$ s. Since $a_x = 0,\ v_x = 1.60\times10^6\ \text{m/s}.\ y - y_0 = 0.0050\ \text{m}, v_{0y} = 0,\ t = 1.25\times10^{-8}\ \text{s}.\ y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times10^5\ \text{m/s}.$ Then $v = \sqrt{v_x^2 + v_y^2} = 1.79\times10^6\ \text{m/s}.$

EVALUATE: $v_y = v_{0y} + a_yt$ gives $a_y = 6.4\times10^{13}\ \text{m/s}^2.$ The electric field between the plates is

$E = \dfrac{ma_y}{e} = \dfrac{(9.11\times10^{-31}\ \text{kg})(6.4\times10^{13}\ \text{m/s}^2)}{1.60\times10^{-19}\ \text{C}} = 364\ \text{N/C}.$ This is not a very large field.

21.32. **IDENTIFY:** Apply constant acceleration equations to the motion of the proton. $E = F/|q|$.

SET UP: A proton has mass $m_\text{p} = 1.67\times10^{-27}$ kg and charge $+e$. Let $+x$ be in the direction of motion of the proton.

EXECUTE: **(a)** $v_{0x} = 0$. $a = \dfrac{eE}{m_p}$. $x - x_0 = v_{0x}t + \dfrac{1}{2}a_x t^2$ gives $x - x_0 = \dfrac{1}{2}a_x t^2 = \dfrac{1}{2}\dfrac{eE}{m_p}t^2$. Solving for E gives

$$E = \frac{2(0.0160\text{ m})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(3.20\times10^{-6}\text{ s})^2} = 32.6\text{ N/C}.$$

(b) $v_x = v_{0x} + a_x t = \dfrac{eE}{m_p}t = \dfrac{e}{m_p}\left(\dfrac{2(x-x_0)m_p}{et^2}\right)t = \dfrac{2(x-x_0)}{t} = \dfrac{2(0.0160\text{ m})}{3.20\times10^{-6}\text{ s}} = 1.00\times10^4\text{ m/s}.$

EVALUATE: The electric field is directed from the positively charged plate toward the negatively charged plate and the force on the proton is also in this direction.

21.33. **IDENTIFY:** Find the angle θ that $\hat{\boldsymbol{r}}$ makes with the $+x$-axis. Then $\hat{\boldsymbol{r}} = (\cos\theta)\hat{\boldsymbol{i}} + (\sin\theta)\hat{\boldsymbol{j}}$.

SET UP: $\tan\theta = y/x$.

EXECUTE: **(a)** $\tan^{-1}\left(\dfrac{-1.35}{0}\right) = -\dfrac{\pi}{2}$ rad. $\hat{\boldsymbol{r}} = -\hat{\boldsymbol{j}}$.

(b) $\tan^{-1}\left(\dfrac{12}{12}\right) = \dfrac{\pi}{4}$ rad. $\hat{\boldsymbol{r}} = \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{i}} + \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{j}}$.

(c) $\tan^{-1}\left(\dfrac{2.6}{+1.10}\right) = 1.97\text{ rad} = 112.9°$. $\hat{\boldsymbol{r}} = -0.39\hat{\boldsymbol{i}} + 0.92\hat{\boldsymbol{j}}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{\boldsymbol{r}}$ is a unit vector, because $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}} = 1$.

21.34. **IDENTIFY:** The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50\ \mu$C charge is given in Figure 21.34. FE is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the $+x$-axis to be to the right, as shown in the figure.

EXECUTE: **(a)** $F_E = |q|E = (6.50\times10^{-6}\text{ C})(1.85\times10^8\text{ N/C}) = 1.20\times10^3\text{ N}$

$$F_q = k\frac{|q_1q_2|}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(6.50\times10^{26}\text{ C})(8.75\times10^{-6}\text{ C})}{(0.0250\text{ m})^2} = 8.18\times10^2\text{ N}$$

$\sum F_x = 0$ gives $T + F_q - F_E = 0$ and $T = F_E - F_q = 382$ N.

(b) Now F_q is to the left, since like charges repel.

$\sum F_x = 0$ gives $T - F_q - F_E = 0$ and $T = F_E + F_q = 2.02\times10^3$ N.

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

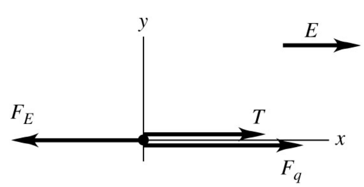


Figure 21.34

21.35. **IDENTIFY** and **SET UP:** Use $\vec{E}$ in $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate $\vec{F}$, $\vec{F} = m\vec{a}$ to calculate $\vec{a}$, and a constant acceleration equation to calculate the final velocity. Let $+x$ be east.

(a) EXECUTE: $F_x = |q|E = (1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = 2.403\times10^{-19}$ N.

$a_x = F_x/m = (2.403\times10^{-19}\text{ N})/(9.109\times10^{-31}\text{ kg}) = +2.638\times10^{11}\text{ m/s}^2$.

$v_{0x} = +4.50\times10^5$ m/s, $a_x = +2.638\times10^{11}$ m/s^2, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33\times10^5$ m/s.

EVALUATE: $\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

(b) EXECUTE: $F_x = -|q|E = -(1.602\times10^{-19}\ \text{C})(1.50\ \text{N/C}) = -2.403\times10^{-19}$ N.

$a_x = F_x/m = (-2.403\times10^{-19}\ \text{N})/(1.673\times10^{-27}\ \text{kg}) = -1.436\times10^8\ \text{m/s}^2$.

$v_{0x} = +1.90\times10^4$ m/s, $a_x = -1.436\times10^8\ \text{m/s}^2$, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 1.59\times10^4$ m/s.

EVALUATE: $q > 0$ so $\vec{F}$ is west and the proton slows down.

21.36. IDENTIFY: The net electric field is the vector sum of the fields due to the individual charges.

SET UP: The electric field points toward negative charge and away from positive charge.

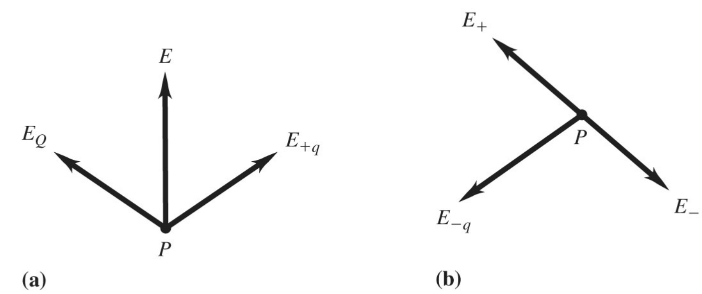


Figure 21.36

EXECUTE: **(a)** Figure 21.36a shows $\vec{E}_Q$ and $\vec{E}_{+q}$ at point P. $\vec{E}_Q$ must have the direction shown, to produce a resultant field in the specified direction. $\vec{E}_Q$ is toward Q, so Q is negative. In order for the horizontal components of the two fields to cancel, Q and q must have the same magnitude.

(b) No. If the lower charge were negative, its field would be in the direction shown in Figure 21.36b. The two possible directions for the field of the upper charge, when it is positive ($\vec{E}_+$) or negative ($\vec{E}_-$), are shown. In neither case is the resultant field in the direction shown in the figure in the problem.

EVALUATE: When combining electric fields, it is always essential to pay attention to their directions.

21.37. IDENTIFY: Calculate the electric field due to each charge and find the vector sum of these two fields.

SET UP: At points on the x-axis only the x-component of each field is nonzero. The electric field of a point charge points away from the charge if it is positive and toward it if it is negative.

EXECUTE: **(a)** Halfway between the two charges, $E = 0$.

(b) For $|x| < a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right) = -\dfrac{4q}{4\pi\epsilon_0}\dfrac{ax}{(x^2-a^2)^2}$.

For $x > a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = \dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

For $x < -a$, $E_x = \dfrac{-1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = -\dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

The graph of E_x versus x is sketched in Figure 21.37 (next page).

EVALUATE: The magnitude of the field approaches infinity at the location of one of the point charges.

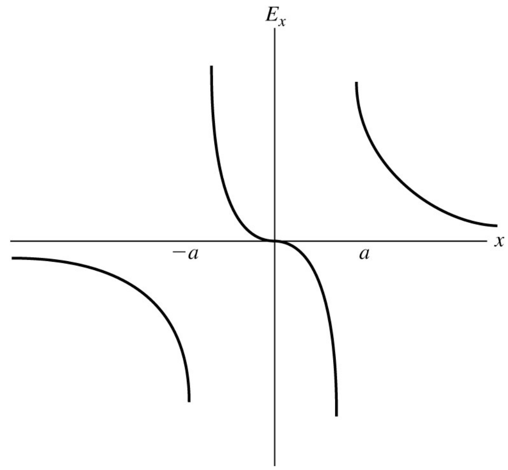


Figure 21.37

21.38. **IDENTIFY:** Add the individual electric fields to obtain the net field.
SET UP: The electric field points away from positive charge and toward negative charge. The electric fields $\vec{E}_1$ and $\vec{E}_2$ add to form the net field $\vec{E}$.

EXECUTE: **(a)** The electric field is toward *A* at points *B* and *C* and the field is zero at *A*.
(b) The electric field is away from *A* at *B* and *C*. The field is zero at *A*.
(c) The field is horizontal and to the right at points *A*, *B*, and *C*.
EVALUATE: Compare your results to the field lines shown in Figure 21.28a and b in the textbook.

21.39. **IDENTIFY:** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).
SET UP: The placement of charges is sketched in Figure 21.39a.

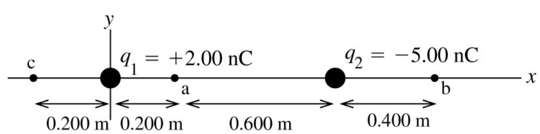


Figure 21.39a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$, where *r* is the distance between the point where the field is calculated and the point charge.
(a) EXECUTE: (i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.39b.

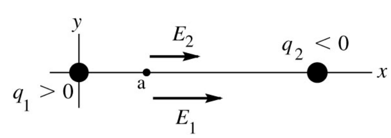


Figure 21.39b

$E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$

$E_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}.$

$E_{1x} = 449.4\ \text{N/C}, E_{1y} = 0.$

$E_{2x} = 124.8\ \text{N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +449.4\ \text{N/C} + 124.8\ \text{N/C} = +574.2\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 574 N/C and is in the $+x$-direction.

(ii) At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.39c.

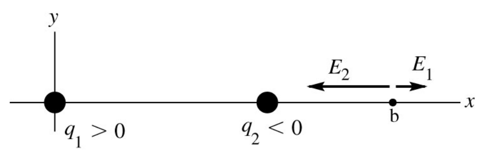


Figure 21.39c

$E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 12.5\ \text{N/C}.$

$E_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$

$E_{1x} = 12.5\ \text{N/C}, E_{1y} = 0.$

$E_{2x} = -280.9\ \text{N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +12.5\ \text{N/C} - 280.9\ \text{N/C} = -268.4\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 268 N/C and is in the $-x$-direction.

(iii) At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.39d.

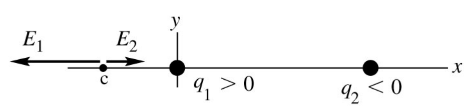


Figure 21.39d

$E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$

$E_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$

$E_{1x} = -449.4\ \text{N / C}, E_{1y} = 0.$

$E_{2x} = +44.9\ \text{N / C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -449.4\ \text{N/C}\ + 44.9\ \text{N/C} = -404.5\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 404 N/C and is in the $-x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\ \text{C})(574.2\ \text{N/C}) = 9.20\times10^{-17}\ \text{N},\ -x$-direction.

(ii) $F = (1.602\times10^{-19}\ \text{C})(268.4\ \text{N/C}) = 4.30\times10^{-17}\ \text{N},\ +x$-direction.

(iii) $F = (1.602\times10^{-19}\ \text{C})(404.5\ \text{N/C}) = 6.48\times10^{-17}\ \text{N},\ +x$-direction.

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the +*x*- or –*x*-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were added because the fields were in the same direction and in (ii) and (iii) the field magnitudes were subtracted because the two fields were in opposite directions. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.40. IDENTIFY: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).

(a) SET UP: The placement of charges is sketched in Figure 21.40a.

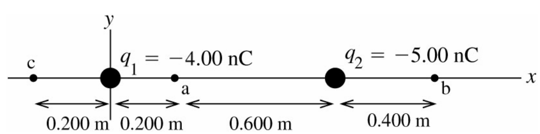


Figure 21.40a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$, where *r* is the distance between the point where the field is calculated and the point charge.

(i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.40b.

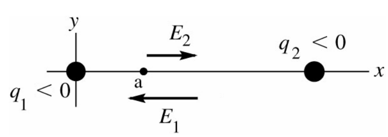


Figure 21.40b

EXECUTE: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 898.8\ \text{N/C}.$

$E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}.$

$E_{1x} = 898.8\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = 124.8\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -898.8\ \text{N/C} + 124.8\ \text{N/C} = -774\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 774 N/C and is in the –*x*-direction.

(ii) **SET UP:** At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.40c.

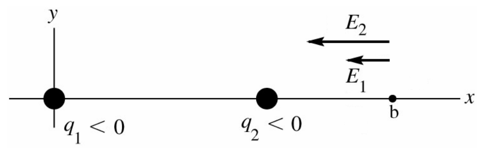


Figure 21.40c

EXECUTE: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 24.97\ \text{N/C}.$

$E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$

$E_{1x} = -24.97\ \text{N/C}, E_{1y} = 0.$

$E_{2x} = -280.9\ \text{N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -24.97\ \text{N/C} - 280.9\ \text{N/C} = -305.9\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 306 N/C and is in the $-x$-direction.

(iii) **SET UP:** At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.40d.

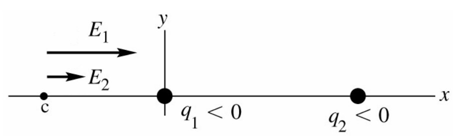


Figure 21.40d

EXECUTE: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 898.8\ \text{N/C}.$

$E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$

$E_{1x} = +898.8\ \text{N/C}, E_{1y} = 0.$

$E_{2x} = +44.9\ \text{N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +898.8\ \text{N/C} + 44.9\ \text{N/C} = +943.7\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 944 N/C and is in the $+x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\ \text{C})(774\ \text{N/C}) = 1.24\times10^{-16}\ \text{N}$, $+x$-direction.

(ii) $F = (1.602\times10^{-19}\ \text{C})(305.9\ \text{N/C}) = 4.90\times10^{-17}\ \text{N}$, $+x$-direction.

(iii) $F = (1.602\times10^{-19}\ \text{C})(943.7\ \text{N/C}) = 1.51\times10^{-16}\ \text{N}$, $-x$-direction.

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the +*x*- or –*x*-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were subtracted because the fields were in opposite directions and in (ii) and (iii) the field magnitudes were added because the two fields were in the same direction. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.41. IDENTIFY: $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges q_1, q_2, and q_3, as shown in Figure 21.41a. This figure also shows additional distances and angles. The electric fields at point *P* are shown in Figure 21.41b. This figure also shows the *xy*-coordinates we will use and the *x*- and *y*-components of the fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-6}\ \text{C}}{(0.100\ \text{m})^2} = 4.49\times10^6\ \text{N/C}.$

$E_2 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-6}\ \text{C}}{(0.0600\ \text{m})^2} = 4.99\times10^6\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} + E_{3y} = 0$ and $E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1\cos 53.1° = 1.04\times10^7$ N/C.

$E = 1.04\times10^7$ N/C, toward the $-2.00\ \mu\text{C}$ charge.

EVALUATE: The *x*-components of the fields of all three charges are in the same direction.

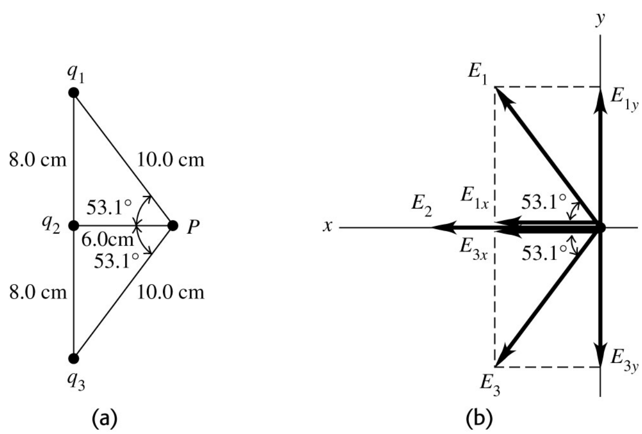


Figure 21.41

21.42. IDENTIFY: The net electric field is the vector sum of the individual fields.

SET UP: The distance from a corner to the center of the square is $r = \sqrt{(a/2)^2 + (a/2)^2} = a/\sqrt{2}$. The magnitude of the electric field due to each charge is the same and equal to $E_q = \dfrac{kq}{r^2} = 2\dfrac{kq}{a^2}$. All four *y*-components add and the *x*-components cancel.

EXECUTE: Each *y*-component is equal to $E_{qy} = -E_q\cos 45° = -\dfrac{E_q}{\sqrt{2}} = \dfrac{-2kq}{\sqrt{2}a^2} = -\dfrac{\sqrt{2}kq}{a^2}$. The resultant field is $\dfrac{4\sqrt{2}kq}{a^2}$, in the $-y$-direction.

EVALUATE: We must add the *y*-components of the fields, not their magnitudes.

21.43. IDENTIFY: For a point charge, $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields produced by each charge. A charge *q* in an electric field $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$.

SET UP: The electric field of a negative charge is directed toward the charge. Point A is 0.100 m from q_2 and 0.150 m from q_1. Point B is 0.100 m from q_1 and 0.350 m from q_2.

EXECUTE: **(a)** The electric fields at point A due to the charges are shown in Figure 21.43a.

$$E_1 = k\frac{|q_1|}{r_{A1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2.50\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{A2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 1.124\times10^4\ \text{N/C}.$$

Since the two fields are in opposite directions, we subtract their magnitudes to find the net field.

$E = E_2 - E_1 = 8.74\times10^3$ N/C, to the right.

(b) The electric fields at point B are shown in Figure 21.43b.

$$E_1 = k\frac{|q_1|}{r_{B1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 5.619\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{B2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.350\ \text{m})^2} = 9.17\times10^2\ \text{N/C}.$$

Since the fields are in the same direction, we add their magnitudes to find the net field.

$E = E_1 + E_2 = 6.54\times10^3$ N/C, to the right.

(c) At A, $E = 8.74\times10^3$ N/C, to the right. The force on a proton placed at this point would be

$F = qE = (1.60\times10^{-19}\ \text{C})(8.74\times10^3\ \text{N/C}) = 1.40\times10^{-15}$ N, to the right.

EVALUATE: A proton has positive charge so the force that an electric field exerts on it is in the same direction as the field.

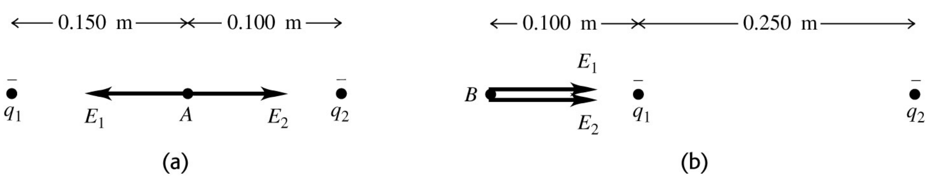


Figure 21.43

21.44. IDENTIFY: Apply $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$ to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1, $\hat{\boldsymbol{r}} = \hat{\boldsymbol{j}}$. For q_2, $\hat{\boldsymbol{r}} = \cos\theta\hat{\boldsymbol{i}} + \sin\theta\hat{\boldsymbol{j}}$, where θ is the angle between $\vec{\boldsymbol{E}}_2$ and the $+x$-axis.

EXECUTE: **(a)** $\vec{\boldsymbol{E}}_1 = \frac{q_1}{4\pi\epsilon_0 r_1^2}\hat{\boldsymbol{j}} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-5.00\times10^{-9}\ \text{C})}{(0.0400\ \text{m})^2}\hat{\boldsymbol{j}} = (-2.813\times10^4\ \text{N/C})\ \hat{\boldsymbol{j}}.$

$\left|\vec{\boldsymbol{E}}_2\right| = \frac{q_2}{4\pi\epsilon_0 r_2^2} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2 + (0.0400\ \text{m})^2} = 1.080\times10^4\ \text{N/C}.$ The angle of $\vec{\boldsymbol{E}}_2$, measured from the x-axis, is $180° - \tan^{-1}\left(\frac{4.00\ \text{cm}}{3.00\ \text{cm}}\right) = 126.9°$ Thus

$\vec{\boldsymbol{E}}_2 = (1.080\times10^4\ \text{N/C})(\hat{\boldsymbol{i}}\cos126.9° + \hat{\boldsymbol{j}}\sin126.9°) = (-6.485\times10^3\ \text{N/C})\hat{\boldsymbol{i}} + (8.64\times10^3\ \text{N/C})\hat{\boldsymbol{j}}.$

(b) The resultant field is $\vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 = (-6.485\times10^3\ \text{N/C})\hat{\boldsymbol{i}} + (-2.813\times10^4\ \text{N/C} + 8.64\times10^3\ \text{N/C})\hat{\boldsymbol{j}}.$

$$\vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 = (-6.485\times10^3\ \text{N/C})\hat{\boldsymbol{i}} - (1.95\times10^4\ \text{N/C})\hat{\boldsymbol{j}}.$$

EVALUATE: $\vec{\boldsymbol{E}}_1$ is toward q_1 since q_1 is negative. $\vec{\boldsymbol{E}}_2$ is directed away from q_2, since q_2 is positive.

21.45. **IDENTIFY:** The forces the charges exert on each other are given by Coulomb's law. The net force on the proton is the vector sum of the forces due to the electrons.

SET UP: $q_e = -1.60\times10^{-19}$ C. $q_p = +1.60\times10^{-19}$ C. The net force is the vector sum of the forces exerted by each electron. Each force has magnitude $F = k\dfrac{|q_1q_2|}{r^2} = k\dfrac{e^2}{r^2}$ and is attractive so is directed toward the electron that exerts it.

EXECUTE: Each force has magnitude

$F_1 = F_2 = k\dfrac{|q_1q_2|}{r^2} = k\dfrac{e^2}{r^2} = \dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.50\times10^{-10}\ \text{m})^2} = 1.023\times10^{-8}$ N. The vector force diagram is shown in Figure 21.45.

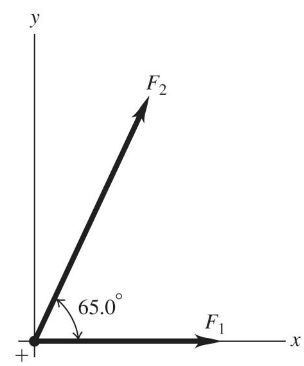


Figure 21. 45

Taking components, we get $F_{1x} = 1.023\times10^{-8}$ N; $F_{1y} = 0$. $F_{2x} = F_2\cos65.0° = 4.32\times10^{-9}$ N;

$F_{2y} = F_2\sin65.0° = 9.27\times10^{-9}$ N. $F_x = F_{1x} + F_{2x} = 1.46\times10^{-8}$ N; $F_y = F_{1y} + F_{2y} = 9.27\times10^{-9}$ N.

$F = \sqrt{F_x^2 + F_y^2} = 1.73\times10^{-8}$ N. $\tan\theta = \dfrac{F_y}{F_x} = \dfrac{9.27\times10^{-9}\ \text{N}}{1.46\times10^{-8}\ \text{N}} = 0.6349$ which gives

$\theta = 32.4°$. The net force is 1.73×10^{-8} N and is directed toward a point midway between the two electrons.

EVALUATE: Note that the net force is less than the algebraic sum of the individual forces.

21.46. **IDENTIFY:** We can model a segment of the axon as a point charge.

SET UP: If the axon segment is modeled as a point charge, its electric field is $E = k\dfrac{q}{r^2}$. The electric field of a point charge is directed away from the charge if it is positive.

EXECUTE: **(a)** 5.6×10^{11} Na^+ ions enter per meter so in a 0.10 mm $= 1.0\times10^{-4}$ m section, 5.6×10^7 Na^+ ions enter. This number of ions has charge $q = (5.6\times10^7)(1.60\times10^{-19}\ \text{C}) = 9.0\times10^{-12}$ C.

(b) $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{9.0\times10^{-12}\ \text{C}}{(5.00\times10^{-2}\ \text{m})^2} = 32$ N/C, directed away from the axon.

(c) $r = \sqrt{\dfrac{k|q|}{E}} = \sqrt{\dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(9.0\times10^{-12}\ \text{C})}{1.0\times10^{-6}\ \text{N/C}}} = 280$ m.

EVALUATE: The field in (b) is considerably smaller than ordinary laboratory electric fields.

21.47. **IDENTIFY:** The electric field of a positive charge is directed radially outward from the charge and has magnitude $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2}$. The resultant electric field is the vector sum of the fields of the individual charges.

SET UP: The placement of the charges is shown in Figure 21.47a.

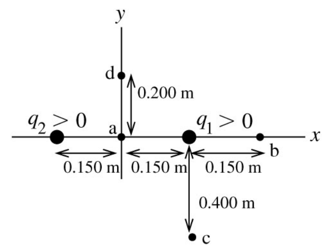
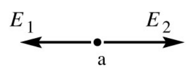


Figure 21.47a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.47b.

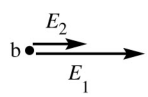


$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \text{ with } r = 0.150 \text{ m}.$$

$$E = E_2 - E_1 = 0;\ E_x = 0,\ E_y = 0.$$

Figure 21. 47b

(b) The two fields have the directions shown in Figure 21.47c.



$E = E_1 + E_2$, in the $+x$-direction.

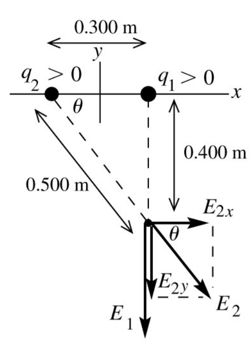
Figure 21. 47c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9} \text{ C}}{(0.150 \text{ m})^2} = 2396.8 \text{ N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9} \text{ C}}{(0.450 \text{ m})^2} = 266.3 \text{ N/C}.$$

$E = E_1 + E_2 = 2396.8 \text{ N/C} + 266.3 \text{ N/C} = 2660 \text{ N/C};\ E_x = +2660 \text{ N/C},\ E_y = 0.$

(c) The two fields have the directions shown in Figure 21.47d.



$$\sin\theta = \frac{0.400 \text{ m}}{0.500 \text{ m}} = 0.800.$$

$$\cos\theta = \frac{0.300 \text{ m}}{0.500 \text{ m}} = 0.600.$$

Figure 21. 47d

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9} \text{ C}}{(0.400 \text{ m})^2} = 337.1 \text{ N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.500\ \text{m})^2} = 215.7\ \text{N/C}.$$

$E_{1x} = 0,\ E_{1y} = -E_1 = -337.1$ N/C.

$E_{2x} = +E_2\cos\theta = +(215.7\ \text{N/C})(0.600) = +129.4$ N/C.

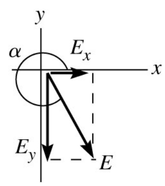
$E_{2y} = -E_2\sin\theta = -(215.7\ \text{N/C})(0.800) = -172.6$ N/C.

$E_x = E_{1x} + E_{2x} = +129$ N/C.

$E_y = E_{1y} + E_{2y} = -337.1\ \text{N/C} - 172.6\ \text{N/C} = -510$ N/C.

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(129\ \text{N/C})^2 + (-510\ \text{N/C})^2} = 526$ N/C.

$\vec{E}$ and its components are shown in Figure 21.47e.



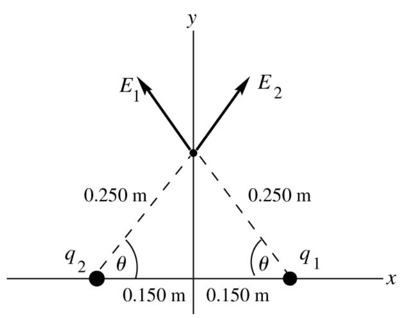
$$\tan\alpha = \frac{E_y}{E_x}.$$

$$\tan\alpha = \frac{-510\ \text{N/C}}{+129\ \text{N/C}} = -3.953.$$

$\alpha = 284°$, counterclockwise from $+x$-axis.

Figure 21. 47e

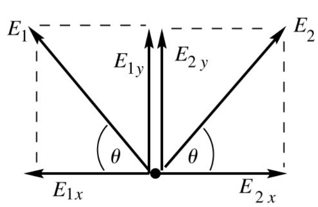
(d) The two fields have the directions shown in Figure 21.47f.



$$\sin\theta = \frac{0.200\ \text{m}}{0.250\ \text{m}} = 0.800.$$

Figure 21. 47f

The components of the two fields are shown in Figure 21.47g.



$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}.$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.250\ \text{m})^2}.$$

$E_1 = E_2 = 862.8$ N/C.

Figure 21. 47g

$E_{1x} = -E_1\cos\theta,\ E_{2x} = +E_2\cos\theta.$

$E_x = E_{1x} + E_{2x} = 0.$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = +E_2\sin\theta.$

$E_y = E_{1y} + E_{2y} = 2E_{1y} = 2E_1 \sin\theta = 2(862.8\text{ N/C})(0.800) = 1380\text{ N/C}.$

$E = 1380\text{ N/C}$, in the $+y$-direction.

EVALUATE: Point *a* is symmetrically placed between identical charges, so symmetry tells us the electric field must be zero. Point *b* is to the right of both charges and both electric fields are in the +*x*-direction and the resultant field is in this direction. At point *c* both fields have a downward component and the field of q_2 has a component to the right, so the net $\vec{E}$ is in the fourth quadrant. At point *d* both fields have an upward component but by symmetry they have equal and opposite *x*-components so the net field is in the +*y*-direction. We can use this sort of reasoning to deduce the general direction of the net field before doing any calculations.

21.48. IDENTIFY: Apply $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.48.

EXECUTE: $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{(6.00\times10^{-9}\text{ C})}{(0.6\text{ m})^2}(-\hat{i}) = -150\hat{i}\text{ N/C}.$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times10^{-9}\text{ C})\left(\frac{1}{(1.00\text{ m})^2}(0.600)\hat{i} + \frac{1}{(1.00\text{ m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\text{N/C}.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\text{ N/C})\hat{i} + (28.8\text{ N/C})\hat{j}.$ $E = \sqrt{(128.4\text{ N/C})^2 + (28.8\text{ N/C})^2} = 131.6\text{ N/C}$ at

$\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6°$ above the $-x$-axis and therefore $167.4°$ counterclockwise from the +*x*-axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

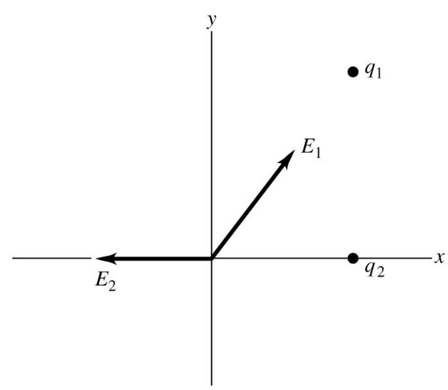


Figure 21.48

21.49. IDENTIFY: We must use the appropriate electric field formula: a uniform disk in (a), a ring in (b) because all the charge is along the rim of the disk, and a point-charge in (c).

(a) SET UP: First find the surface charge density (*Q*/*A*), then use the formula for the field due to a disk of charge, $E_x = \dfrac{\sigma}{2\epsilon_0}\left[1 - \dfrac{1}{\sqrt{(R/x)^2 + 1}}\right].$

EXECUTE: The surface charge density is $\sigma = \dfrac{Q}{A} = \dfrac{Q}{\pi r^2} = \dfrac{6.50\times10^{-9}\text{ C}}{\pi(0.0125\text{ m})^2} = 1.324\times10^{-5}\text{ C/m}^2.$

The electric field is

$$E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R/x)^2+1}}\right] = \frac{1.324\times 10^{-5}\ \text{C/m}^2}{2(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[1 - \frac{1}{\sqrt{\left(\frac{1.25\ \text{cm}}{2.00\ \text{cm}}\right)^2+1}}\right]$$

$$E_x = 1.14\times 10^5\ \text{N/C, toward the center of the disk.}$$

(b) SET UP: For a ring of charge, the field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$.

EXECUTE: Substituting into the electric field formula gives

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}} = \frac{(9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.50\times 10^{-9}\ \text{C})(0.0200\ \text{m})}{[(0.0200\ \text{m})^2+(0.0125\ \text{m})^2]^{3/2}}$$

$$E = 8.92\times 10^4\ \text{N/C, toward the center of the disk.}$$

(c) SET UP: For a point charge, $E = (1/4\pi\epsilon_0)q/r^2$.

EXECUTE: $E = (9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.50\times 10^{-9}\ \text{C})/(0.0200\ \text{m})^2 = 1.46\times 10^5\ \text{N/C}$.

(d) EVALUATE: With the ring, more of the charge is farther from P than with the disk. Also with the ring the component of the electric field parallel to the plane of the ring is greater than with the disk, and this component cancels. With the point charge in (c), all the field vectors add with no cancellation, and all the charge is closer to point P than in the other two cases.

21.50. IDENTIFY: For a long straight wire, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$.

SET UP: $\dfrac{1}{2\pi\epsilon_0} = 1.80\times 10^{10}\ \text{N}\cdot\text{m}^2/\text{C}^2$.

EXECUTE: Solve $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for r: $r = \dfrac{3.20\times 10^{-10}\ \text{C/m}}{2\pi\epsilon_0(2.50\ \text{N/C})} = 2.30\ \text{m}$.

EVALUATE: For a point charge, E is proportional to $1/r^2$. For a long straight line of charge, E is proportional to $1/r$.

21.51. IDENTIFY: For a ring of charge, the magnitude of the electric field is given by $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$.

Use $\vec{F} = q\vec{E}$. In part (b) use Newton's third law to relate the force on the ring to the force exerted by the ring.

SET UP: $Q = 0.125\times 10^{-9}\ \text{C}$, $a = 0.025\ \text{m}$ and $x = 0.400\ \text{m}$.

EXECUTE: **(a)** $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}\hat{i} = (7.0\ \text{N/C})\hat{i}$.

(b) $\vec{F}_{\text{on ring}} = -\vec{F}_{\text{on q}} = -q\vec{E} = -(-2.50\times 10^{-6}\ \text{C})(7.0\ \text{N/C})\hat{i} = (1.75\times 10^{-5}\ \text{N})\hat{i}$.

EVALUATE: Charges q and Q have opposite sign, so the force that q exerts on the ring is attractive.

21.52. (a) IDENTIFY: The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{x\sqrt{(x/a)^2+1}}.$$

EXECUTE: $E = \dfrac{(18.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(175\times 10^{-9}\ \text{C/m})}{(0.0600\ \text{m})\sqrt{\left(\frac{6.00\ \text{cm}}{4.25\ \text{cm}}\right)^2+1}} = 3.03\times 10^4\ \text{N/C}$, directed upward.

(b) IDENTIFY: The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. We first find the radius a of the circle using $2\pi a = l$.

EXECUTE: Solving for a gives $a = l/2\pi = (8.50\ \text{cm})/2\pi = 1.353\ \text{cm}$.

The charge on this circle is $Q = \lambda l = (175\ \text{nC/m})(0.0850\ \text{m}) = 14.88\ \text{nC}$.

The electric field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}} = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(14.88\times10^{-9}\ \text{C/m})(0.0600\ \text{m})}{\left[(0.0600\ \text{m})^2+(0.01353\ \text{m})^2\right]^{3/2}}$$

$$E = 3.45\times10^4\ \text{N/C, upward.}$$

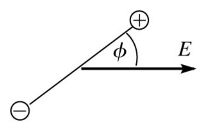
EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

21.53. **(a) IDENTIFY** and **SET UP:** Use $p = qd$ to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.

EXECUTE: $p = qd = (4.5\times10^{-9}\ \text{C})(3.1\times10^{-3}\ \text{m}) = 1.4\times10^{-11}\ \text{C}\cdot\text{m}$. The direction of $\vec{p}$ is from q_1 toward q_2.

(b) IDENTIFY and **SET UP:** Use $\tau = pE\sin\phi$ to relate the magnitudes of the torque and field.

EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.53, so



$$E = \frac{\tau}{p\sin\phi}.$$

$$E = \frac{7.2\times10^{-9}\ \text{N}\cdot\text{m}}{(1.4\times10^{-11}\ \text{C}\cdot\text{m})\sin36.9^\circ} = 860\ \text{N/C}.$$

Figure 21. 53

EVALUATE: The equation $\tau = pE\sin\phi$ gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2\times10^{-8}\ \text{N}\cdot\text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

21.54. **(a) IDENTIFY:** The potential energy is given by $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$.

SET UP: $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.

EXECUTE: parallel: $\phi = 0$ and $U(0^\circ) = -pE$.

perpendicular: $\phi = 90^\circ$ and $U(90^\circ) = 0$.

$\Delta U = U(90^\circ) - U(0^\circ) = pE = (5.0\times10^{-30}\ \text{C}\cdot\text{m})(1.6\times10^6\ \text{N/C}) = 8.0\times10^{-24}\ \text{J}$.

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0\times10^{-24}\ \text{J})}{3(1.381\times10^{-23}\ \text{J/K})} = 0.39\ \text{K}$.

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.55. **IDENTIFY:** The torque on a dipole in an electric field is given by $\vec{\tau} = \vec{p}\times\vec{E}$.

SET UP: $\tau = pE\sin\phi$, where ϕ is the angle between the direction of $\vec{p}$ and the direction of $\vec{E}$.

EXECUTE: **(a)** The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* direction, as shown in Figure 21.55a (next page).

(b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{p}$ back into alignment with $\vec{E}$. When $\vec{p}$ is directed opposite to $\vec{E}$, a small displacement results in a torque that takes $\vec{p}$ farther from alignment with $\vec{E}$.
(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.55b.
EVALUATE: The field of the dipole is directed from the + charge toward the − charge.

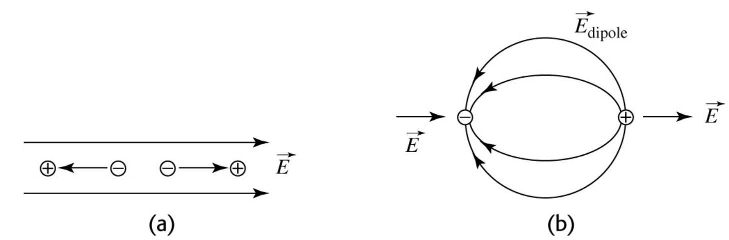


Figure 21. 55

21.56. **IDENTIFY:** Calculate the electric field due to the dipole and then apply $\vec{F} = q\vec{E}$.

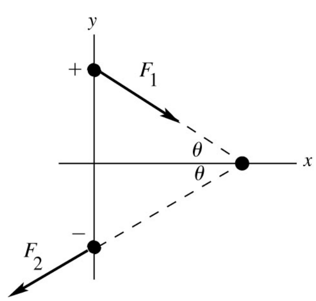
SET UP: The field of a dipole is $E_{\text{dipole}}(x) = \dfrac{p}{2\pi\epsilon_0 x^3}$.

EXECUTE: $E_{\text{dipole}} = \dfrac{6.17\times 10^{-30}\ \text{C}\cdot\text{m}}{2\pi\epsilon_0(3.0\times 10^{-9}\ \text{m})^3} = 4.11\times 10^{6}\ \text{N/C}$. The electric force is

$F = qE = (1.60\times 10^{-19}\ \text{C})(4.11\times 10^{6}\ \text{N/C}) = 6.58\times 10^{-13}\ \text{N}$ and is toward the water molecule (negative x-direction).

EVALUATE: $\vec{E}_{\text{dipole}}$ is in the direction of $\vec{p}$, so is in the $+x$-direction. The charge q of the ion is negative, so $\vec{F}$ is directed opposite to $\vec{E}$ and is therefore in the $-x$-direction.

21.57. **(a) IDENTIFY:** Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.
SET UP: The two forces on each charge in the dipole are shown in Figure 21.57a.



$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$.

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0$.

Figure 21. 57a

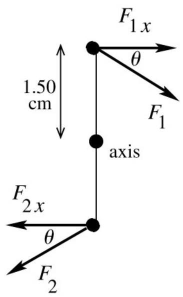
EXECUTE: $F_1 = k\dfrac{|qq'|}{r^2} = k\dfrac{(5.00\times 10^{-6}\ \text{C})(10.0\times 10^{-6}\ \text{C})}{(0.0200\ \text{m})^2} = 1.124\times 10^{3}\ \text{N}$.

$F_{1y} = -F_1\sin\theta = -842.6\ \text{N}$.

$F_{2y} = -842.6\ \text{N}$ so $F_y = F_{1y} + F_{2y} = -1680\ \text{N}$ (in the direction from the $+5.00\text{-}\mu\text{C}$ charge toward the $-5.00\text{-}\mu\text{C}$ charge).

EVALUATE: The x-components cancel and the y-components add.

(b) SET UP: Refer to Figure 21.57b.



The y-components have zero moment arm and therefore zero torque.
F_{1x} and F_{2x} both produce clockwise torques.

Figure 21. 57b

EXECUTE: $F_{1x} = F_1 \cos\theta = 743.1$ N.

$\tau = 2(F_{1x})(0.0150 \text{ m}) = 22.3 \text{ N}\cdot\text{m}$, clockwise.

EVALUATE: The electric field produced by the $-10.00\ \mu$C charge is not uniform so $\tau = pE\sin\phi$ does not apply.

21.58. IDENTIFY: Find the vector sum of the fields due to each charge in the dipole.

SET UP: A point on the x-axis with coordinate x is a distance $r = \sqrt{(d/2)^2 + x^2}$ from each charge.

EXECUTE: **(a)** The magnitude of the field due to each charge is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} = \dfrac{q}{4\pi\epsilon_0}\left(\dfrac{1}{(d/2)^2 + x^2}\right)$,

where d is the distance between the two charges. The x-components of the forces due to the two charges are equal and oppositely directed and so cancel each other. The two fields have equal y-components,

so $E = 2E_y = \dfrac{2q}{4\pi\epsilon_0}\left(\dfrac{1}{(d/2)^2 + x^2}\right)\sin\theta$, where θ is the angle below the x-axis for both fields.

$\sin\theta = \dfrac{d/2}{\sqrt{(d/2)^2 + x^2}}$ and $E_{\text{dipole}} = \left(\dfrac{2q}{4\pi\epsilon_0}\right)\left(\dfrac{1}{(d/2)^2 + x^2}\right)\left(\dfrac{d/2}{\sqrt{(d/2)^2 + x^2}}\right) = \dfrac{qd}{4\pi\epsilon_0\left[(d/2)^2 + x^2\right]^{3/2}}$. The field is the $-y$-direction.

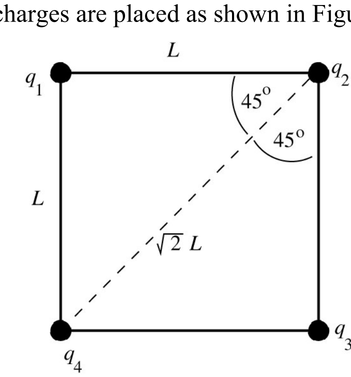
(b) At large x, $x^2 \gg (d/2)^2$, so the expression in part (a) reduces to the approximation $E_{\text{dipole}} \approx \dfrac{qd}{4\pi\epsilon_0 x^3}$.

EVALUATE: Example 21.14 shows that at points on the $+y$-axis far from the dipole, $E_{\text{dipole}} \approx \dfrac{qd}{2\pi\epsilon_0 y^3}$.

The expression in part (b) for points on the x-axis has a similar form.

21.59. IDENTIFY: Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

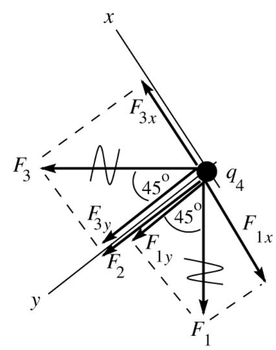
SET UP: The charges are placed as shown in Figure 21.59a.



$q_1 = q_2 = q_3 = q_4 = Q$

Figure 21.59a

Consider forces on q_4. The free-body diagram is given in Figure 21.59b. Take the y-axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2. Then $\vec{F}_2$ is in the $+y$-direction.



EXECUTE: **(a)** $F_3 = F_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{L^2}$.

$F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{2L^2}$.

$F_{1x} = -F_1 \sin 45° = -F_1/\sqrt{2}$.

$F_{1y} = +F_1 \cos 45° = +F_1/\sqrt{2}$.

$F_{3x} = +F_3 \sin 45° = +F_3/\sqrt{2}$.

$F_{3y} = +F_3 \cos 45° = +F_3/\sqrt{2}$.

$F_{2x} = 0, F_{2y} = F_2$.

Figure 21.59b

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0$.

$$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2})\frac{1}{4\pi\epsilon_0}\frac{Q^2}{L^2} + \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2L^2} = \frac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2}).$$

$R = \dfrac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$. Same for all four charges.

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

21.60. **IDENTIFY:** Apply $F = \dfrac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance d to the left of q_1, so it is a distance $d + 0.600$ m from q_2.

EXECUTE: $F_1 = F_2$ gives $\dfrac{kq|q_1|}{d^2} = \dfrac{kq|q_2|}{(d+0.600\text{ m})^2}$. $d = \pm\sqrt{\dfrac{|q_1|}{|q_2|}}(d + 0.600\text{ m}) = \pm(0.8452)(d + 0.600\text{ m})$.

d must be positive, so $d = \dfrac{(0.8452)(0.600\text{ m})}{1 - 0.8452} = 3.27$ m. The net force would be zero when $+q$ is at $x = -3.27$ m.

EVALUATE: When $+q$ is at $x = -3.27$ m, $\vec{F}_1$ is in the $-x$-direction and $\vec{F}_2$ is in the $+x$-direction.

21.61. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ for each pair of charges and find the vector sum of the forces that q_1 and q_2 exert on q_3.

SET UP: Like charges repel and unlike charges attract. The three charges and the forces on q_3 are shown in Figure 21.61.

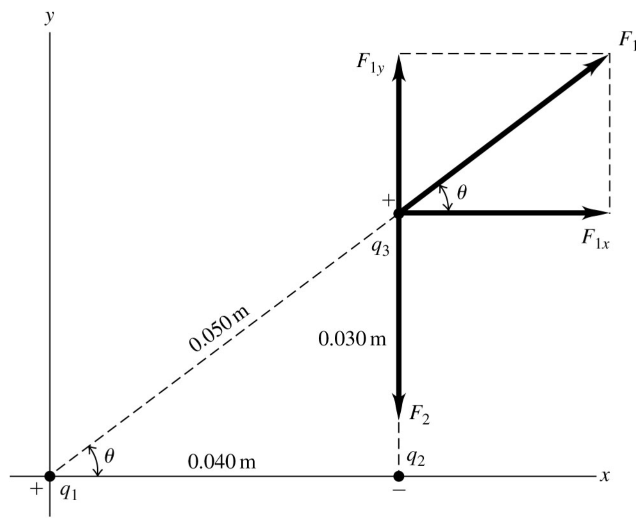


Figure 21.61

EXECUTE: **(a)** $F_1 = k\frac{|q_1 q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0500\ \text{m})^2} = 1.079\times10^{-4}\ \text{N}.$

$\theta = 36.9°$. $F_{1x} = +F_1\cos\theta = 8.63\times10^{-5}$ N. $F_{1y} = +F_1\sin\theta = 6.48\times10^{-5}$ N.

$$F_2 = k\frac{|q_2 q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2} = 1.20\times10^{-4}\ \text{N}.$$

$F_{2x} = 0$, $F_{2y} = -F_2 = -1.20\times10^{-4}$ N. $F_x = F_{1x} + F_{2x} = 8.63\times10^{-5}$ N.

$F_y = F_{1y} + F_{2y} = 6.48\times10^{-5}\ \text{N} + (-1.20\times10^{-4}\ \text{N}) = -5.52\times10^{-5}$ N.

(b) $F = \sqrt{F_x^2 + F_y^2} = 1.02\times10^{-4}$ N. $\tan\phi = \left|\frac{F_y}{F_x}\right| = 0.640$. $\phi = 32.6°$, below the $+x$-axis.

EVALUATE: The individual forces on q_3 are computed from Coulomb's law and then added as vectors, using components.

21.62. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.62 (next page). F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\frac{mg\sin\theta}{\cos\theta} = F_e = \frac{kq^2}{d^2}$. But

$$\tan\theta \approx \sin\theta = \frac{d}{2L},\ \text{so}\ d^3 = \frac{2kq^2L}{mg}\ \text{and}\ d = \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

EVALUATE: d increases when q increases.

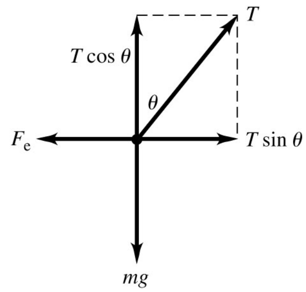


Figure 21.62

21.63. IDENTIFY: Use Coulomb's law for the force that one sphere exerts on the other and apply the first condition of equilibrium to one of the spheres.

SET UP: The placement of the spheres is sketched in Figure 21.63a.

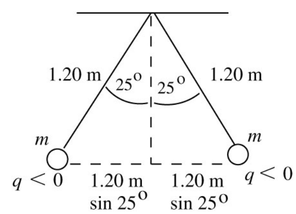


Figure 21.63a

EXECUTE: (a) The free-body diagrams for each sphere are given in Figure 21.63b.

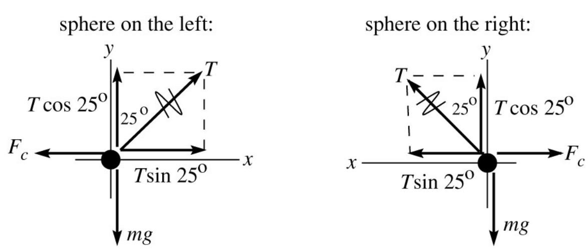


Figure 21.63b

F_c is the repulsive Coulomb force exerted by one sphere on the other.

(b) From either force diagram in part (a): $\sum F_y = ma_y$.

$T\cos 25.0° - mg = 0$ and $T = \dfrac{mg}{\cos 25.0°}$.

$\sum F_x = ma_x$.

$T\sin 25.0° - F_c = 0$ and $F_c = T\sin 25.0°$.

Use the first equation to eliminate T in the second: $F_c = (mg/\cos 25.0°)(\sin 25.0°) = mg\tan 25.0°$.

$$F_c = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[2(1.20\ \text{m})\sin 25.0°]^2}.$$

Combine this with $F_c = mg\tan 25.0°$ and get $mg\tan 25.0° = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{[2(1.20\ \text{m})\sin 25.0°]^2}$.

$$q = (2.40\text{ m})\sin 25.0°\sqrt{\frac{mg\tan 25.0°}{(1/4\pi\epsilon_0)}}.$$

$$q = (2.40\text{ m})\sin 25.0°\sqrt{\frac{(15.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 25.0°}{8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 2.80\times10^{-6}\text{ C}.$$

(c) The separation between the two spheres is given by $2L\sin\theta$. $q = 2.80\,\mu\text{C}$ as found in part (b).

$F_c = (1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2$ and $F_c = mg\tan\theta$. Thus $(1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2 = mg\tan\theta$.

$$(\sin\theta)^2\tan\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4L^2mg} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(2.80\times10^{-6}\text{ C})^2}{4(0.600\text{ m})^2(15.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)} = 0.3328.$$

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of θ that solves the equation. For θ small, $\tan\theta \approx \sin\theta$. With this approximation the equation becomes $\sin^3\theta = 0.3328$ and $\sin\theta = 0.6930$, so $\theta = 43.9°$. Now refine this guess:

θ	$\sin^2\theta\tan\theta$	
45.0°	0.5000	
40.0°	0.3467	
39.6°	0.3361	
39.5°	0.3335	
39.4°	0.3309	so $\theta = 39.5°$.

EVALUATE: The expression in part (c) says $\theta \to 0$ as $L \to \infty$ and $\theta \to 90°$ as $L \to 0$. When L is decreased from the value in part (a), θ increases.

21.64. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to each sphere.

SET UP: **(a)** Free body diagrams are given in Figure 21.64 (next page). F_e is the repulsive electric force that one sphere exerts on the other.

EXECUTE: **(b)** $T = mg/\cos 20° = 0.0834$ N, so $F_e = T\sin 20° = 0.0285\text{ N} = \dfrac{kq_1q_2}{r_1^2}$.

(Note: $r_1 = 2(0.500\text{ m})\sin 20° = 0.342$ m.)

(c) From part (b), $q_1q_2 = 3.71\times10^{-13}\text{ C}^2$.

(d) The charges on the spheres are made equal by connecting them with a wire, but we still have

$F_e = mg\tan\theta = 0.0453\text{ N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{r_2^2}$, where $Q = \dfrac{q_1+q_2}{2}$. But the separation r_2 is known:

$r_2 = 2(0.500\text{ m})\sin 30° = 0.500$ m. Hence: $Q = \dfrac{q_1+q_2}{2} = \sqrt{4\pi\epsilon_0 F_e r_2^2} = 1.12\times10^{-6}$ C. This equation, along with that from part (c), gives us two equations in q_1 and q_2: $q_1 + q_2 = 2.24\times10^{-6}$ C and $q_1q_2 = 3.71\times10^{-13}\text{ C}^2$. By elimination, substitution and after solving the resulting quadratic equation, we find: $q_1 = 2.06\times10^{-6}$ C and $q_2 = 1.80\times10^{-7}$ C.

EVALUATE: After the spheres are connected by the wire, the charge on sphere 1 decreases and the charge on sphere 2 increases. The product of the charges on the sphere increases and the thread makes a larger angle with the vertical.

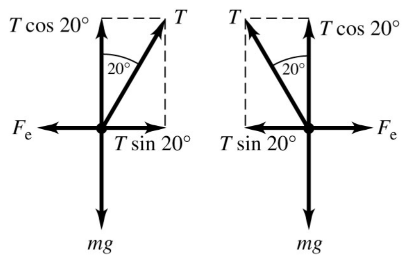


Figure 21.64

21.65. **IDENTIFY:** The electric field exerts a horizontal force away from the wall on the ball. When the ball hangs at rest, the forces on it (gravity, the tension in the string, and the electric force due to the field) add to zero.

SET UP: The ball is in equilibrium, so for it $\sum F_x = 0$ and $\sum F_y = 0$. The force diagram for the ball is given in Figure 21.65. F_E is the force exerted by the electric field. $\vec{F} = q\vec{E}$. Since the electric field is horizontal, $\vec{F}_E$ is horizontal. Use the coordinates shown in the figure. The tension in the string has been replaced by its x- and y-components.

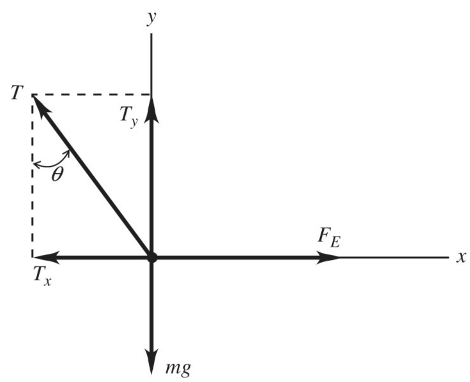


Figure 21.65

EXECUTE: $\sum F_y = 0$ gives $T_y - mg = 0$. $T\cos\theta - mg = 0$ and $T = \dfrac{mg}{\cos\theta}$. $\sum F_x = 0$ gives $F_E - T_x = 0$. $F_E - T\sin\theta = 0$. Combing the equations and solving for F_E gives

$F_E = \left(\dfrac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = (12.3\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(\tan 17.4°) = 3.78\times10^{-2}\text{ N}$. $F_E = |q|E$ so

$E = \dfrac{F_E}{|q|} = \dfrac{3.78\times10^{-2}\text{ N}}{1.11\times10^{-6}\text{ C}} = 3.41\times10^{4}\text{ N/C}$. Since q is negative and $\vec{F}_E$ is to the right, $\vec{E}$ is to the left in the figure.

EVALUATE: The larger the electric field E the greater the angle the string makes with the wall.

21.66. **IDENTIFY:** The net force on q_3 is the vector sum of the individual forces. Coulomb's law gives the force between any two point-charges.

SET UP: Use $F = k\dfrac{|q_1 q_2|}{r^2}$. The force on q_3 due to q_1 is in the $-x$-direction, so q_2 must be negative to make the net force on q_3 in the $+x$-direction. We know that the x-component of the net force on q_3 is $F_{3x} = +6.00$ N.

(a) EXECUTE: The net force on q_3 is the sum of the two forces: $F_{3x} = F_{1x} + F_{2x} = +6.00$ N. Applying Coulomb's law gives

$6.00\text{ N} = k[-(6.00\ \mu\text{C})(3.00\ \mu\text{C})/(0.200\text{ m})^2 + (3.00\ \mu\text{C})q_2/(0.400\text{ m})^2]$, $q_2 = -5.96\times10^{-6}\text{ C} = -59.6\ \mu\text{C}$.

(b) Now $F_{3x} = -6.00$ N. In this case, assume that q_2 is positive, so the x-components all add. Using the same approach as in (a), we have

$-6.00\text{ N} = k[-(6.00\ \mu\text{C})(3.00\ \mu\text{C})/(0.200\text{ m})^2 - (3.00\ \mu\text{C})q_2/(0.400\text{ m})^2] = +1.16\times10^{-5}\text{ C} = +11.6\ \mu\text{C}$.

EVALUATE: Is is tempting to think that the answer to (b) should be just the negative of the answer to (a), but that is not the case. In (a) the two forces on q_3 were in opposite directions, but in (b) they are in the same direction.

21.67. IDENTIFY: For a point charge, $E = k\dfrac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500\text{ nC}$ and $q_2 = +8.00\text{ nC}$. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.67. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a distance $1.20\text{ m} - x$ from q_2. $E_1 = E_2$ gives $k\dfrac{0.500\text{ nC}}{x^2} = k\dfrac{8.00\text{ nC}}{(1.20\text{ m} - x)^2}$. $16x^2 = (1.20\text{ m} - x)^2$. $4x = \pm(1.20\text{ m} - x)$ and $x = 0.24\text{ m}$ is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

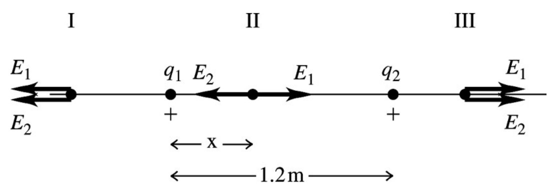


Figure 21.67

21.68. IDENTIFY: The net electric field at the origin is the vector sum of the fields due to the two charges.

SET UP: $E = k\dfrac{|q|}{r^2}$. $\vec{E}$ is toward a negative charge and away from a positive charge. At the origin, $\vec{E}_1$ due to the -3.00 nC charge is in the $+x$-direction, toward the charge.

EXECUTE: **(a)** $E_1 = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.00\times10^{-9}\text{ C})}{(1.20\text{ m})^2} = 18.73\text{ N/C}$, so $E_{1x} = +18.73\text{ N/C}$.

$E_x = E_{1x} + E_{2x}$. $E_x = +45.0\text{ N/C}$, so $E_{2x} = E_x - E_{1x} = +45.0\text{ N/C} - 18.73\text{ N/C} = 26.27\text{ N/C}$. $\vec{E}$ is away from Q so Q is positive. Using $E_2 = k\dfrac{|Q|}{r^2}$ gives

$|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(26.27\text{ N/C})(0.600\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 1.05\times10^{-9}\text{ C} = 1.05\text{ nC}$. Since Q is positive, $Q = +1.05\text{ nC}$.

(b) $E_x = -45.0\text{ N/C}$, so $E_{2x} = E_x - E_{1x} = -45.0\text{ N/C} - 18.73\text{ N/C} = -63.73\text{ N/C}$. $\vec{E}$ is toward Q so Q is negative. $|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(63.73\text{ N/C})(0.600\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.55\times10^{-9}\text{ C} = 2.55\text{ nC}$. Since Q is negative, we have $Q = -2.55\text{ nC}$.

EVALUATE: The equation $E = k\dfrac{|q|}{r^2}$ gives only the *magnitude* of the electric field. When combining fields, you still must figure out whether to add or subtract the magnitudes depending on the direction in which the fields point.

21.69. **IDENTIFY:** For equilibrium, the forces must balance. The electrical force is given by Coulomb's law.
SET UP: Set up axes so that the charge $+Q$ is located at $x = 0$, the charge $+4Q$ is located at $x = d$, and the unknown charge that is required to produce equilibrium, q, is located at a position $x = a$. Apply $F = k\frac{|q_1 q_2|}{r^2}$ to each pair of charges to obtain eqilibrium.
EXECUTE: For a charge q to be in equilbrium, it must be placed between the two given positive charges $(0 < a < d)$ and the magnitude of the force between q and $+Q$ must be equal to the magnitude of the force between q and $+4Q$: $k\frac{|q|Q}{a^2} = k\frac{4|q|Q}{(d-a)^2}$. Solving for a we obtain $(d-a) = \pm 4a$, which has $a = \frac{d}{3}$ as its only root in the required interval $(0 < a < d)$. Furthermore, to conteract the repulsive force between $+Q$ and $+4Q$ the charge q must be negative $(q = -|q|)$. The condition that $+Q$ is in equilibrium gives us $k\frac{-qQ}{(d/3)^2} = k\frac{4Q^2}{d^2}$. Solving for q we obtain $q = -\frac{4}{9}Q$.
EVALUATE: We have shown that both q and $+Q$ are in equilibrium provided that $a = \frac{d}{3}$ and $q = -\frac{4}{9}Q$. To make sure that the problem is well posed, we should check that these conditions also place the charge $+4Q$ is in equilbrium. We can do this by showing that $k\frac{-4qQ}{(d-a)^2}$ is equal to $k\frac{4Q^2}{d^2}$ when the given values for both a and q are substituted.

21.70. **IDENTIFY** and **SET UP:** Like charges repel and unlike charges attract, and Coulomb's law applies. The positions of the three charges are sketched in Figure 21.70a, and each force acting on q_3 is shown. The distance between q_1 and q_3 is 5.00 cm.

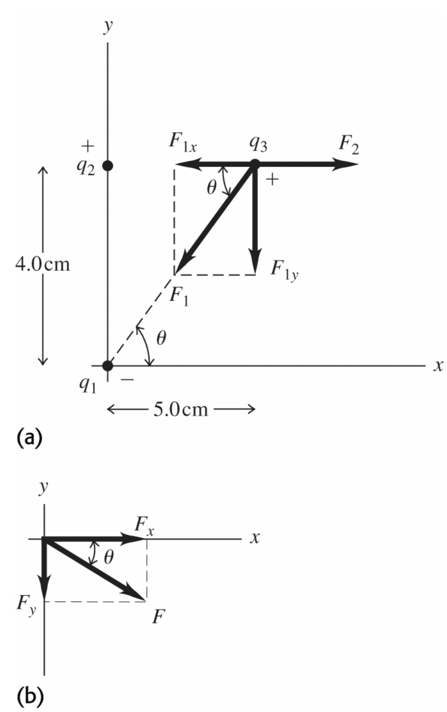


Figure 21.70

EXECUTE: **(a)** $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(5.00\times10^{-2}\ \text{m})^2} = 5.394\times10^{-5}\ \text{N}.$

$F_{1x} = -F_1\cos\theta = -(5.394\times10^{-5}\ \text{N})(0.600) = -3.236\times10^{-5}\ \text{N}.$

$F_{1y} = -F_1\sin\theta = -(5.394\times10^{-5}\ \text{N})(0.800) = -4.315\times10^{-5}\ \text{N}.$

$F_2 = k\dfrac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(3.00\times10^{-2}\ \text{m})^2} = 9.989\times10^{-5}\ \text{N}.$

$F_{2x} = 9.989\times10^{-5}\ \text{N};\ F_{2y} = 0.$

$F_x = F_{1x} + F_{2x} = 9.989\times10^{-5}\ \text{N} + (-3.236\times10^{-5}\ \text{N}) = 6.75\times10^{-5}\ \text{N};$

$F_y = F_{1y} + F_{2y} = -4.32\times10^{-5}\ \text{N}.$

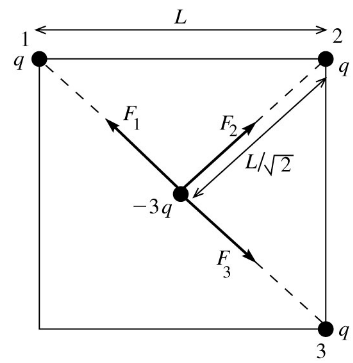
(b) $\vec{F}$ and its components are shown in Figure 21.70b.

$F = \sqrt{F_x^2 + F_y^2} = 8.01\times10^{-5}\ \text{N}.$ $\tan\theta = \left|\dfrac{F_y}{F_x}\right| = 0.640$ and $\theta = 32.6°$. $\vec{F}$ is $327°$ counterclockwise from the $+x$-axis.

EVALUATE: The equation $F = k\dfrac{|q_1q_2|}{r^2}$ gives only the magnitude of the force. We must find the direction by deciding if the force between the charges is attractive or repulsive.

21.71. **IDENTIFY:** Use Coulomb's law to calculate the forces between pairs of charges and sum these forces as vectors to find the net charge.

(a) SET UP: The forces are sketched in Figure 21.71a.

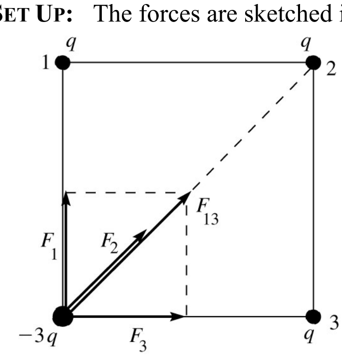


EXECUTE: $\vec{F}_1 + \vec{F}_3 = 0$, so the net force is $\vec{F} = \vec{F}_2$.

$F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{(L/\sqrt{2})^2} = \dfrac{6q^2}{4\pi\epsilon_0 L^2}$, away from the vacant corner.

Figure 21. 71a

(b) SET UP: The forces are sketched in Figure 21.71b.



EXECUTE: $F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{(\sqrt{2}L)^2} = \dfrac{3q^2}{4\pi\epsilon_0(2L^2)}.$

$F_1 = F_3 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{L^2} = \dfrac{3q^2}{4\pi\epsilon_0 L^2}.$

The vector sum of F_1 and F_3 is $F_{13} = \sqrt{F_1^2 + F_3^2}$.

Figure 21. 71b

$F_{13} = \sqrt{2}F_1 = \dfrac{3\sqrt{2}q^2}{4\pi\epsilon_0 L^2}$; $\vec{F}_{13}$ and $\vec{F}_2$ are in the same direction.

$F = F_{13} + F_2 = \dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(\sqrt{2} + \dfrac{1}{2}\right)$, and is directed toward the center of the square.

EVALUATE: By symmetry the net force is along the diagonal of the square. The net force is only slightly larger when the $-3q$ charge is at the center. Here it is closer to the charge at point 2 but the other two forces cancel.

21.72. IDENTIFY: For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Qq_1|}{r^2}$ (for q_1) and likewise for q_2.

EXECUTE: First find the net force: $F = ma =$ (0.00500 kg)(324 m/s^2) $= 1.62$ N. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q.

$F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \dfrac{F}{2\cos\theta} = \dfrac{1.62\text{ N}}{2\left(\dfrac{2.25\text{ cm}}{3.00\text{ cm}}\right)} = 1.08\text{ N}$. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs. $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|q_1}{r^2}$ and $q_1 = \dfrac{r^2 F_{Qq_1}}{\dfrac{1}{4\pi\epsilon_0}|Q|} = \dfrac{(0.0300\text{ m})^2(1.08\text{ N})}{(9.00\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.75\times 10^{-6}\text{ C})} = 6.17\times 10^{-8}$ C.

$q_2 = -q_1 = -6.17\times 10^{-8}$ C.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

21.73. IDENTIFY: The small bags of protons behave like point-masses and point-charges since they are extremely far apart.

SET UP: For point-particles, we use Newton's formula for universal gravitation $(F = Gm_1m_2/r^2)$ and Coulomb's law. The number of protons is the mass of protons in the bag divided by the mass of a single proton.

EXECUTE: **(a)** $(0.0010\text{ kg})/(1.67\times 10^{-27}\text{ kg}) = 6.0\times 10^{23}$ protons.

(b) Using Coulomb's law, where the separation is twice the radius of the earth, we have

$F_{\text{electrical}} = (9.00\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.0\times 10^{23}\times 1.60\times 10^{-19}\text{ C})^2/(2\times 6.37\times 10^6\text{ m})^2 = 5.1\times 10^5$ N.

$F_{\text{grav}} = (6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.0010\text{ kg})^2/(2\times 6.37\times 10^6\text{ m})^2 = 4.1\times 10^{-31}$ N.

EVALUATE: **(c)** The electrical force (≈200,000 lb!) is certainly large enough to feel, but the gravitational force clearly is not since it is about 10^{36} times weaker.

21.74. IDENTIFY: The positive sphere will be deflected in the direction of the electric field but the negative sphere will be deflected in the direction opposite to the electric field. Since the spheres hang at rest, they are in equilibrium so the forces on them must balance. The external forces on each sphere are gravity, the tension in the string, the force due to the uniform electric field and the electric force due to the other sphere.

SET UP: The electric force on one sphere due to the other is $F_C = k\dfrac{|q^2|}{r^2}$ in the horizontal direction, the force on it due to the uniform electric field is $F_E = qE$ in the horizontal direction, the gravitational force is mg vertically downward and the force due to the string is T directed along the string. For equilibrium $\sum F_x = 0$ and $\sum F_y = 0$.

EXECUTE: **(a)** The positive sphere is deflected in the same direction as the electric field, so the one that is deflected to the left is positive.

(b) The separation between the two spheres is $2(0.530\text{ m})\sin 29.0^\circ = 0.5139\text{ m}$.

$F_C = k\dfrac{|q^2|}{r^2} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(72.0\times10^{-9}\text{ C})^2}{(0.5139\text{ m})^2} = 1.765\times10^{-4}\text{ N}$. $F_E = qE$. $\sum F_y = 0$ gives

$T\cos 29.0^\circ - mg = 0$ so $T = \dfrac{mg}{\cos 29.0^\circ}$. $\sum F_x = 0$ gives $T\sin 29.0^\circ + F_C - F_E = 0$.

$mg\tan 29.0^\circ + F_C = qE$. Combining the equations and solving for E gives

$$E = \frac{mg\tan 29.0^\circ + F_C}{q} = \frac{(6.80\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)\tan 29.0^\circ + 1.765\times10^{-4}\text{ N}}{72.0\times10^{-9}\text{ C}} = 2.96\times10^3\text{ N/C}.$$

EVALUATE: Since the charges have opposite signs, they attract each other, which tends to reduce the angle between the strings. Therefore if their charges were negligibly small, the angle between the strings would be greater than 58.0°.

21.75. **IDENTIFY:** The only external force acting on the electron is the electrical attraction of the proton, and its acceleration is toward the center of its circular path (that is, toward the proton). Newton's second law applies to the electron and Coulomb's law gives the electrical force on it due to the proton.

SET UP: Newton's second law gives $F_C = m\dfrac{v^2}{r}$. Using the electrical force for F_C gives $k\dfrac{e^2}{r^2} = m\dfrac{v^2}{r}$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{ke^2}{mr}} = \sqrt{\dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(9.109\times10^{-31}\text{ kg})(5.29\times10^{-11}\text{ m})}} = 2.19\times10^6\text{ m/s}$.

EVALUATE: This speed is less than 1% the speed of light, so it is reasonably safe to use Newtonian physics.

21.76. **IDENTIFY:** To be suspended, the electric force on the raindrop due to the earth's electric field must be equal to the weight of the drop.

SET UP: The weight of the raindrop is $w = mg$ and is downward. We can calculate the mass of the raindrop from the known density of water: $m = \rho V$, where $\rho = 10^3\text{ kg/m}^3$ and $V = \frac{4}{3}\pi r^3$. The electric force is $\vec{F} = q\vec{E}$, where $E = 150\text{ N/C}$.

EXECUTE: To balance the weight of the raindrop the electric force must be upward. Since the electric field is downward the net charge on the raindrop must be negative. For equilibrium we must have $w = mg = |q|E$. Therefore

$$|q| = \frac{mg}{E} = \left(\frac{4}{3}\pi r^3\rho g\right)/E = \frac{4}{3}\pi(1.0\times10^{-5}\text{ m})^3(10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)/(150\text{ N/C}) = 2.7\times10^{-13}\text{ C}.$$

The number of excess electrons is $\dfrac{|q|}{e} = \dfrac{2.7\times10^{-13}\text{ C}}{1.60\times10^{-19}\text{ C}} = 1.7\times10^6$.

EVALUATE: Although this may appear to be a large number in absolute terms, the excess number of electrons represents only about 10^{-7} % of the total number of electrons in the raindrop.

21.77. IDENTIFY: $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ gives the force exerted by the electric field. This force is constant since the electric field is uniform and gives the proton a constant acceleration. Apply the constant acceleration equations for the *x*- and *y*-components of the motion, just as for projectile motion.

SET UP: The electric field is upward so the electric force on the positively charged proton is upward and has magnitude $F = eE$. Use coordinates where positive *y* is downward. Then applying $\sum \vec{F} = m\vec{a}$ to the proton gives that $a_x = 0$ and $a_y = -eE/m$. In these coordinates the initial velocity has components $v_x = +v_0 \cos\alpha$ and $v_y = +v_0 \sin\alpha$, as shown in Figure 21.77a.

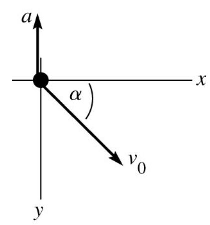


Figure 21.77a

EXECUTE: **(a)** Finding $h_{\max}$: At $y = h_{\max}$ the *y*-component of the velocity is zero.

$v_y = 0, v_{0y} = v_0 \sin\alpha, a_y = -eE/m, y - y_0 = h_{\max} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0).$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y}.$$

$$h_{\max} = \frac{-v_0^2 \sin^2\alpha}{2(-eE/m)} = \frac{mv_0^2 \sin^2\alpha}{2eE}.$$

(b) Use the vertical motion to find the time *t*: $y - y_0 = 0, v_{0y} = v_0 \sin\alpha, a_y = -eE/m, t = ?$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2.$$

With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(v_0 \sin\alpha)}{-eE/m} = \dfrac{2mv_0 \sin\alpha}{eE}.$

Then use the *x*-component motion to find *d*: $a_x = 0, v_{0x} = v_0 \cos\alpha, t = 2mv_0 \sin\alpha/eE, x - x_0 = d = ?$

$x - x_0 = v_{0x}t + \dfrac{1}{2}a_x t^2$ gives $d = v_0 \cos\alpha\left(\dfrac{2mv_0 \sin\alpha}{eE}\right) = \dfrac{mv_0^2\, 2\sin\alpha\cos\alpha}{eE} = \dfrac{mv_0^2 \sin 2\alpha}{eE}.$

(c) The trajectory of the proton is sketched in Figure 21.77b.

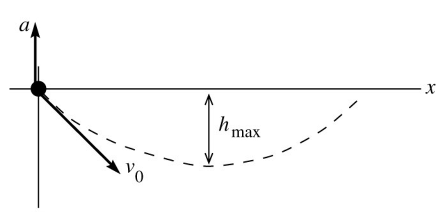


Figure 21.77b

(d) Use the expression in part (a): $h_{\max} = \dfrac{[(4.00\times 10^5 \text{ m/s})(\sin 30.0°)]^2 (1.673\times 10^{-27} \text{ kg})}{2(1.602\times 10^{-19} \text{ C})(500 \text{ N/C})} = 0.418 \text{ m}.$

Use the expression in part (b): $d = \dfrac{(1.673\times10^{-27}\ \text{kg})(4.00\times10^{5}\ \text{m/s})^2 \sin 60.0^\circ}{(1.602\times10^{-19}\ \text{C})(500\ \text{N/C})} = 2.89\ \text{m}.$

EVALUATE: In part (a), $a_y = -eE/m = -4.8\times10^{10}\ \text{m/s}^2$. This is much larger in magnitude than g, the acceleration due to gravity, so it is reasonable to ignore gravity. The motion is just like projectile motion, except that the acceleration is upward rather than downward and has a much different magnitude. h_{max} and d increase when α or v_0 increase and decrease when E increases.

21.78. IDENTIFY: The electric field is vertically downward and the charged object is deflected downward, so it must be positively charged. While the object is between the plates, it is accelerated downward by the electric field. Once it is past the plates, it moves downward with a constant vertical velocity which is the same downward velocity it acquired while between the plates. Its horizontal velocity remains constant at v_0 throughout its motion. The forces on the object are all constant, so its acceleration is constant; therefore we can use the standard kinematics equations. Newton's second law applies to the object.

SET UP: Call the x-axis positive to the right and the y-axis positive downward. The equations $\vec{E} = \dfrac{\vec{F}_0}{q_0}$, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, $x = v_x t$, and $\Sigma F_y = ma_y$ all apply. $v_x = v_0 = \text{constant}$.

EXECUTE: *Time through the plates:* $t = x/v_x = x/v_0 = (0.260\ \text{m})/(5000\ \text{m/s}) = 5.20\times10^{-5}\ \text{s}$.

Vertical deflection between the plates: $\Delta y_1 = y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}ayt^2 = \frac{1}{2}(qE/m)t^2$

$\Delta y_1 = \frac{1}{2}(800\ \text{N/C})(5.20\times10^{-5}\ \text{s})^2(q/m) = (1.0816\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m)$.

v_y as the object just emerges from the plates:

$v_y = v_{0y} + a_y t = (qE/m)t = (q/m)(800\ \text{N/C})(5.20\times10^{-5}\ \text{s}) = (0.04160\ \text{kg}\cdot\text{m/C}\cdot\text{s})(q/m)$. (This is the initial vertical velocity for the next step.)

Time to travel 56.0 cm: $t = x/v_x = (0.560\ \text{m})/(5000\ \text{m/s}) = 1.120\times10^{-4}\ \text{s}$.

Vertical deflection after leaving the plates:

$\Delta y_2 = v_{0y}\,t = (0.04160\ \text{kg}\cdot\text{m/C}\cdot\text{s})\,(q/m)(1.120\times10^{-4}\ \text{s}) = (4.6592\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m)$.

Total vertical deflection:

$d = \Delta y_1 + \Delta y_2$.

$1.25\ \text{cm} = 0.0125\ \text{m} = (1.0816\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m) + (4.6592\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m)$.

$q/m = 2180\ \text{C/kg}$.

EVALUATE: The charge on 1.0 kg is so huge that it could not be dealt with in a laboratory. But this is a tiny object, more likely with a mass in the range of 1.0 μg, so its charge would be $(2180\ \text{C/kg})(10^{-9}\ \text{kg}) = 2.18\times10^{-6}\ \text{C} \approx 2\ \mu\text{C}$. That amount of charge could be used in an experiment.

21.79. IDENTIFY: Divide the charge distribution into infinitesimal segments of length dx'. Calculate E_x and E_y due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx' is $dQ = (Q/a)dx'$. The distance between a segment at x' and a point at x on the x-axis is $x - x'$ since $x > a$.

EXECUTE: **(a)** $dE_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{dQ}{(x-x')^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/a)dx'}{(x-x')^2}$. Integrating with respect to x' over the length of the charge distribution gives

$$E_x = \frac{1}{4\pi\epsilon_0}\int_0^a \frac{(Q/a)dx'}{(x-x')^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{a}{x(x-a)} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x(x-a)}.\ \ Ey = 0.$$

(b) At the location of the charge, $x = r + a$, so $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{(r+a)(r+a-a)} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r(r+a)}$.

Using $\vec{F} = q\vec{E}$, we have $\vec{F} = q\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r(r+a)}\hat{\boldsymbol{i}}$.

Evaluate: **(c)** For $r \gg a$, $r + a \to r$, so the magnitude of the force becomes $F = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}$. The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.80. **Identify:** The electric field is upward, but whether it exerts an upward or downward force on the object depends on the sign of the charge on the object, so we should first find the sign of this charge. Then apply Newton's second law. The forces (gravity and the electric force) are both constant, so the acceleration is constant. Therefore the standard kinematics formulas apply.

Set Up: Call the $+y$-axis upward. The equations $\vec{E} = \frac{\vec{F}_0}{q_0}$, $\Sigma F_y = ma_y$, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ all apply.

Execute: First find the sign of the charge of the object. If no electric field were present, only gravity would be acting, so the distance the object would travel in 0.200 s would be

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.92\text{ m/s})(0.200\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(0.200\text{ s})^2 = 0.1880\text{ m} = 18.8\text{ cm}.$

Since the object travels only 6.98 cm in 0.200 s, the force due to the electric field must be opposing its motion, so this force must be downward. Since the electric field is upward, the charge must be negative. Now look at the motion with the electric field present. Newton's second law gives

$\Sigma F_y = ma_y$: $mg + qE = ma_y$. We get a_y using kinematics.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$: $0.0698\text{ m} = (1.92\text{ m/s})(0.200\text{ s}) + \frac{1}{2}a_y(0.200\text{ s})^2.$

$a_y = -15.71\text{ m/s}^2$, with the minus sign telling us it is downward. Now use this value in Newton's second law. Solve $mg + qE = ma_y$ for q/m:

$q/m = (a_y - g)/E = (15.71\text{ m/s}^2 - 9.80\text{ m/s}^2)/(3.60\times10^4\text{ N/C}) = 1.64\times10^{-4}\text{ C/kg}.$

Evaluate: A kilogram of the material of this object would have a charge of $1.64\times10^{-4}\text{ C} = 164\ \mu\text{C}$.

21.81. **Identify:** $E_x = E_{1x} + E_{2x}$. Use $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$ for the electric field due to each point charge.

Set Up: $\vec{E}$ is directed away from positive charges and toward negative charges.

Execute: **(a)** $E_x = +50.0\text{ N/C}$. $E_{1x} = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{4.00\times10^{-9}\text{ C}}{(0.60\text{ m})^2} = +99.9\text{ N/C}.$

$E_x = E_{1x} + E_{2x}$, so $E_{2x} = E_x - E_{1x} = +50.0\text{ N/C} - 99.9\text{ N/C} = -49.9\text{ N/C}$. Since E_{2x} is negative, q_2 must be negative. $|q_2| = \frac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \frac{(49.9\text{ N/C})(1.20\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 7.99\times10^{-9}\text{ C}$. $q_2 = -7.99\times10^{-9}\text{ C}$.

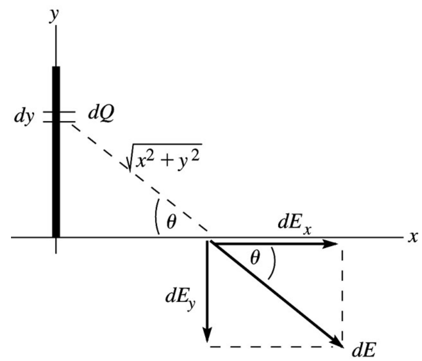
(b) $E_x = -50.0\text{ N/C}$. $E_{1x} = +99.9\text{ N/C}$, as in part (a). $E_{2x} = E_x - E_{1x} = -149.9\text{ N/C}$. q_2 is negative.

$|q_2| = \frac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \frac{(149.9\text{ N/C})(1.20\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.40\times10^{-8}\text{ C}$. $q_2 = -2.40\times10^{-8}\text{ C}$.

Evaluate: q_2 would be positive if E_{2x} were positive.

21.82. **Identify:** Use $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$ to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.10. Use $\vec{E} = \frac{\vec{F}}{q_0}$ to calculate $\vec{F}$.

Set Up: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.82.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}.$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}.$$

Figure 21.82

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right).$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right).$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right).$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}.$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right).$$

(b) $\vec{F} = q_0\vec{E}.$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\ F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right).$$

(c) For $x \gg a$, $\dfrac{1}{\sqrt{x^2+a^2}} = \dfrac{1}{x}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} = \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right) = \dfrac{1}{x}-\dfrac{a^2}{2x^3}.$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\ F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{x}+\frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}.$$

EVALUATE: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x \gg a$ the charge distribution Q acts like a point charge.

21.83. IDENTIFY: Apply $E = \dfrac{\sigma}{2\epsilon_0}[1-(R^2/x^2+1)^{-1/2}].$

SET UP: $\sigma = Q/A = Q/\pi R^2$. $(1+y^2)^{-1/2} \approx 1-y^2/2$, when $y^2 \ll 1$.

EXECUTE: **(a)** $E = \dfrac{\sigma}{2\varepsilon_0}[1-(R^2/x^2+1)^{-1/2}]$ gives

$$E = \frac{7.00\ \text{pC}/\pi(0.025\ \text{m})^2}{2\varepsilon_0}\left[1-\left(\frac{(0.025\ \text{m})^2}{(0.200\ \text{m})^2}+1\right)^{-1/2}\right] = 1.56\ \text{N/C},\ \text{in the } +x\text{-direction.}$$

(b) For $x \gg R$, $E = \dfrac{\sigma}{2\varepsilon_0}[1-(1-R^2/2x^2+\cdots)] \approx \dfrac{\sigma}{2\varepsilon_0}\dfrac{R^2}{2x^2} = \dfrac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} = \dfrac{Q}{4\pi\varepsilon_0 x^2}$.

(c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the first correction term to the point charge result is negative.

(d) For $x = 0.200$ m, the percent difference is $\dfrac{(1.58-1.56)}{1.56} = 0.01 = 1\%$. For $x = 0.100$ m,

$E_{\text{disk}} = 6.00$ N/C and $E_{\text{point}} = 6.30$ N/C, so the percent difference is $\dfrac{(6.30-6.00)}{6.30} = 0.047 \approx 5\%$.

EVALUATE: The field of a disk becomes closer to the field of a point charge as the distance from the disk increases. At $x = 10.0$ cm, $R/x = 25\%$ and the percent difference between the field of the disk and the field of a point charge is 5%.

21.84. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere, with x horizontal and y vertical.

SET UP: The free-body diagram for the sphere is given in Figure 21.84. The electric field $\vec{E}$ of the sheet is directed away from the sheet and has magnitude $E = \dfrac{\sigma}{2\varepsilon_0}$.

EXECUTE: $\sum F_y = 0$ gives $T\cos\alpha = mg$ and $T = \dfrac{mg}{\cos\alpha}$. $\sum F_x = 0$ gives $T\sin\alpha = \dfrac{q\sigma}{2\varepsilon_0}$ and

$T = \dfrac{q\sigma}{2\varepsilon_0 \sin\alpha}$. Combining these two equations we have $\dfrac{mg}{\cos\alpha} = \dfrac{q\sigma}{2\varepsilon_0\sin\alpha}$ and $\tan\alpha = \dfrac{q\sigma}{2\varepsilon_0 mg}$. Therefore,

$\alpha = \arctan\left(\dfrac{q\sigma}{2\varepsilon_0 mg}\right)$.

EVALUATE: The electric field of the sheet, and hence the force it exerts on the sphere, is independent of the distance of the sphere from the sheet.

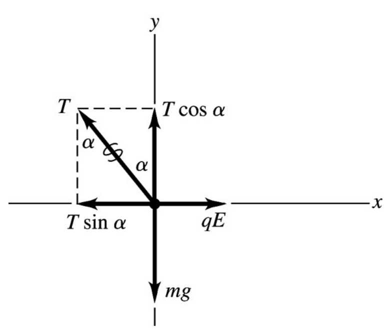


Figure 21.84

21.85. **IDENTIFY:** Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the x- and y-components of the total field.

SET UP: Consider the small segment shown in Figure 21.85a.

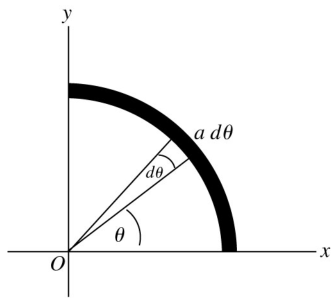
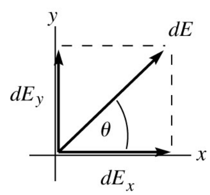


Figure 21.85a

EXECUTE: A small segment that subtends angle $d\theta$ has length $a\,d\theta$ and contains charge $dQ = \left(\dfrac{a d\theta}{\frac{1}{2}\pi a}\right) Q = \dfrac{2Q}{\pi} d\theta.$ ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.85b, along with its components.



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{|dQ|}{a^2}.$$

$$dE = \frac{Q}{2\pi^2\varepsilon_0 a^2} d\theta.$$

Figure 21.85b

$dE_x = dE\cos\theta = (Q/2\pi^2\varepsilon_0 a^2)\cos\theta d\theta.$

$E_x = \int dE_x = \dfrac{Q}{2\pi^2\varepsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \dfrac{Q}{2\pi^2\varepsilon_0 a^2}(\sin\theta\Big|_0^{\pi/2}) = \dfrac{Q}{2\pi^2\varepsilon_0 a^2}.$

$dE_y = dE\sin\theta = (Q/2\pi^2\varepsilon_0 a^2)\sin\theta d\theta.$

$E_y = \int dE_y = \dfrac{Q}{2\pi^2\varepsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \dfrac{Q}{2\pi^2\varepsilon_0 a^2}(-\cos\theta\Big|_0^{\pi/2}) = \dfrac{Q}{2\pi^2\varepsilon_0 a^2}.$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.86. IDENTIFY: We must add the electric field components of the positive half and the negative half.

SET UP: From Problem 21.85, the electric field due to the quarter-circle section of positive charge has components $E_x = +\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$, $E_y = -\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$. The field due to the quarter-circle section of negative charge has components $E_x = +\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$, $E_y = +\dfrac{Q}{2\pi^2\varepsilon_0 a^2}$.

EXECUTE: The components of the resultant field is the sum of the *x*- and *y*-components of the fields due to each half of the semicircle. The *y*-components cancel, but the *x*-components add, giving

$E_x = +\dfrac{Q}{\pi^2\varepsilon_0 a^2}$, in the $+x$-direction.

EVALUATE: Even though the net charge on the semicircle is zero, the field it produces is *not* zero because of the way the charge is arranged.

21.87. **IDENTIFY:** Each wire produces an electric field at P due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is $E_{\text{net}} = 2E_1 \cos 45° = 2\frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at P is eE.

EXECUTE: **(a)** The net field is $E_{\text{net}} = 2\frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$.

$$E_{\text{net}} = \frac{2(9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.50\times 10^{-6}\ \text{C})\cos 45°}{(0.600\ \text{m})\sqrt{(0.600\ \text{m})^2+(0.600\ \text{m})^2}} = 6.25\times 10^4\ \text{N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right at point P.

(b) $F = eE = (1.60\times 10^{-19}\ \text{C})(6.25\times 10^4\ \text{N/C}) = 1.00\times 10^{-14}\ \text{N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

21.88. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\varepsilon_0$, and the net field is the vector sum of these, $E_{\text{net}} = \frac{\sigma_B}{2\varepsilon_0} \pm \frac{\sigma_A}{2\varepsilon_0} = \frac{\sigma_B \pm \sigma_A}{2\varepsilon_0}$, where we use the + or − sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The two fields oppose and the field of B is stronger than that of A, so

$$E_{\text{net}} = \frac{\sigma_B}{2\varepsilon_0} - \frac{\sigma_A}{2\varepsilon_0} = \frac{\sigma_B - \sigma_A}{2\varepsilon_0} = \frac{11.6\ \mu\text{C/m}^2 - 8.80\ \mu\text{C/m}^2}{2(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.58\times 10^5\ \text{N / C},\ \text{to the right.}$$

(b) The fields are now in the same direction, so their magnitudes add.

$$E_{\text{net}} = (11.6\ \mu\text{C/m}^2 + 8.80\ \mu\text{C/m}^2)/2\varepsilon_0 = 1.15\times 10^6\ \text{N/C},\ \text{to the right.}$$

(c) The fields add but now point to the left, so $E_{\text{net}} = 1.15\times 10^6$ N/C, to the left.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.89. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\varepsilon_0$, and the net field is the vector sum of these. $E_{\text{net}} = \frac{\sigma_B}{2\varepsilon_0} \pm \frac{\sigma_A}{2\varepsilon_0} = \frac{\sigma_B \pm \sigma_A}{2\varepsilon_0}$, where we use the + or − sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The fields add and point to the left, giving $E_{\text{net}} = 1.15\times 10^6$ N/C.

(b) The fields oppose and point to the left, so $E_{\text{net}} = 1.58\times 10^5$ N/C.

(c) The fields oppose but now point to the right, giving $E_{\text{net}} = 1.58\times 10^5$ N/C.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.90. **IDENTIFY:** The sheets produce an electric field in the region between them which is the vector sum of the fields from the two sheets.

SET UP: The force on the negative oil droplet must be upward to balance gravity. The net electric field between the sheets is $E = \sigma/\varepsilon_0$, and the electrical force on the droplet must balance gravity, so $qE = mg$.

EXECUTE: **(a)** The electrical force on the drop must be upward, so the field should point downward since the drop is negative.

(b) The charge of the drop is $5e$, so $qE = mg$. $(5e)(\sigma/\varepsilon_0) = mg$ and

$$\sigma = \frac{mg\varepsilon_0}{5e} = \frac{(486\times10^{-9}\text{ kg})(9.80\text{ m/s}^2)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}{5(1.60\times10^{-19}\text{ C})} = 52.7\text{ C/m}^2.$$

EVALUATE: Balancing oil droplets between plates was the basis of the Milliken Oil-Drop Experiment which produced the first measurement of the mass of an electron.

21.91. **IDENTIFY:** Apply the formula for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - 1/\sqrt{(R/x)^2 + 1}\right].$$

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma.$

(b) $\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2 + 1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2 + 1}\right]\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$

$\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(1/\sqrt{(R_1/x)^2 + 1} - 1/\sqrt{(R_2/x)^2 + 1}\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$ The electric field is in the $+x$-direction at points above the disk and in the $-x$-direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{\boldsymbol{i}}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2 + 1} = \dfrac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \dfrac{|x|}{R_1}$. This gives

$\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\dfrac{|x|^2}{x}\hat{\boldsymbol{i}} = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x\hat{\boldsymbol{i}}.$ Sufficiently close means that $(x/R_1)^2 \ll 1$.

(d) $F_x = -qE_x = -\dfrac{q\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x.$ The force is in the form of Hooke's law: $F_x = -kx$, with

$$k = \frac{q\sigma}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\varepsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$$

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

21.92. **IDENTIFY:** Apply constant acceleration equations to a drop to find the acceleration. Then use $F = ma$ to find the force and $F = |q|E$ to find $|q|$.

SET UP: Let $D = 2.0$ cm be the horizontal distance the drop travels and $d = 0.30$ mm be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x = 0$ and call $a_y = a$.

EXECUTE: **(a)** Find the time of flight: $t = D/v = (0.020\text{ m})/(50\text{ m/s}) = 4.00\times10^{-4}$ s. $d = \dfrac{1}{2}at^2$.

$a = \dfrac{2d}{t^2} = \dfrac{2(3.00\times10^{-4}\text{ m})}{(4.00\times10^{-4}\text{ s})^2} = 3750\text{ m/s}^2$. Then $a = F/m = qE/m$ gives

$q = ma/E = \dfrac{(1.4\times10^{-11}\text{ kg})(3750\text{ m/s}^2)}{8.00\times10^4\text{ N/C}} = 6.56\times10^{-13}$ C, which rounds to 6.6×10^{-13} s.

(b) Use the equations and calculations above: if $v \to v/2$, then $t \to 2t$, so $a \to a/4$, which means that $q \to q/4$, so $q = (6.56\times10^{-13}\text{ s})/4 = 1.64\times10^{-13}$ s, which rounds to 1.6×10^{-13} s.

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

21.93 **IDENTIFY:** The net force on the third sphere is the vector sum of the forces due to the other two charges. Coulomb's law gives the forces.

SET UP: $F = k\dfrac{|q_1 q_2|}{r^2}$.

EXECUTE: **(a)** Between the two fixed charges, the electric forces on the third sphere q_3 are in opposite directions and have magnitude 4.50 N in the $+x$-direction. Applying Coulomb's law gives
$4.50\text{ N} = k[q_1(4.00\ \mu\text{C})/(0.200\text{ m})^2 - q_2(4.00\ \mu\text{C})/(0.200\text{ m})^2]$.
Simplifying gives $q_1 - q_2 = 5.00\ \mu\text{C}$.
With q_3 at $x = +0.600$ m, the electric forces on q_3 are all in the $+x$-direction and add to 3.50 N. As before, Coulomb's law gives
$3.50\text{ N} = k[q_1(4.00\ \mu\text{C})/(0.600\text{ m})^2 + q_2(4.00\ \mu\text{C})/(0.200\text{ m})^2]$.
Simplifying gives $q_1 + 9q_2 = 35.0\ \mu\text{C}$.
Solving the two equations simultaneously gives $q_1 = 8.00\ \mu\text{C}$ and $q_2 = 3.00\ \mu\text{C}$.
(b) Both forces on q_3 are in the $-x$-direction, so their magnitudes add. Factoring out common factors and using the values for q_1 and q_2 we just found, Coulomb's law gives
$F_{\text{net}} = kq_3\,[q_1/(0.200\text{ m})^2 + q_2/(0.600\text{ m})^2]$.
$F_{\text{net}} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,[(8.00\ \mu\text{C})/(0.200\text{ m})^2 + (3.00\ \mu\text{C})/(0.600\text{ m})^2] = 7.49\text{ N}$, and it is in the $-x$-direction.
(c) The forces are in opposite direction and add to zero, so
$0 = kq_1q_3/x^2 - kq_2q_3/(0.400\text{ m} - x)^2$.
$(0.400\text{ m} - x)^2 = (q_2/q_1)x^2$.
Taking square roots of both sides gives
$0.400\text{ m} - x = \pm x\sqrt{q_2/q_1} = \pm 0.6124x$.
Solving for x, we get two values: $x = 0.248$ m and $x = 1.03$ m. The charge q_3 must be between the other two charges for the forces on it to balance. Only the first value is between the two charges, so it is the correct one: $x = 0.248$ m.

EVALUATE: Check the answers in part (a) by substituting these values back into the original equations. $8.00\ \mu\text{C} - 3.00\ \mu\text{C} = 5.00\ \mu\text{C}$ and $8.00\ \mu\text{C} + 9(3.00\ \mu\text{C}) = 35.0\ \mu\text{C}$, so the answers check in both equations. In part (c), the second root, $x = 1.03$ m, has some meaning. The condition we imposed to solve the problem was that the magnitudes of the two forces were equal. This happens at $x = 0.248$ mn, but it also happens at $x = 1.03$ m. However at the second root the forces are both in the $+x$-direction and therefore cannot cancel.

21.94. **IDENTIFY and SET UP:** The electric field E_x produced by a uniform ring of charge, for points on an axis perpendicular to the plane of the ring at its center, is $E_x = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$, where a is the radius of the ring, x is the distance from its center along the axis, and Q is the total charge on the ring.

EXECUTE: **(a)** Far from the ring, at large values of x, the ring can be considered as a point-charge, so its electric field would be $E = kQ/x^2$. Therefore $Ex^2 = kQ$, which is a constant. From the graph (a) in the problem, we read off that at large distances $Ex^2 = 45\ \text{N}\cdot\text{m}^2/\text{C}$, which is equal to kQ, so
$Q = (45\ \text{N}\cdot\text{m}^2/\text{C})/k = 5.0\times 10^{-9}\text{ C} = 5.0\text{ nC}$.
(b) The electric field along the axis a distance x from the ring is $E_x = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$. Very close to the ring, $x^2 << a^2$, so the formula becomes $Ex = kQx/a^3$. Therefore $E/x = kQ/a^3$, which is a constant. From graph (b) in the problem, E/x approaches $700\ \text{N/C}\cdot\text{m}$ as x approaches zero. So $kQ/a^3 = 700\ \text{N/C}\cdot\text{m}$, which gives
$a = [kQ/(700\ \text{N/C}\cdot\text{m})]^{1/3} = [(45\ \text{N}\cdot\text{m}^2/\text{C})/(700\ \text{N/C}\cdot\text{m})]^{1/3} = 0.40\text{ m} = 40\text{ cm}$.

EVALUATE: It is physically reasonable that a ring 40 cm in radius could carry 5.0 nC of charge.

21.95. **IDENTIFY:** Apply Coulomb's law to calculate the forces that q_1 and q_2 exert on q_3, and add these force vectors to get the net force.

SET UP: Like charges repel and unlike charges attract. Let $+x$ be to the right and $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible force diagrams are sketched in Figure 21.95a.

Only the last picture can result in a net force in the $-x$-direction.

(b) $q_1 = -2.00\ \mu\text{C}, q_3 = +4.00\ \mu\text{C}$, and $q_2 > 0$.

(c) The forces $\vec{F}_1$ and $\vec{F}_2$ and their components are sketched in Figure 21.95b.

$F_y = 0 = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1||q_3|}{(0.0400\text{ m})^2}\sin\theta_1 + \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2||q_3|}{(0.0300\text{ m})^2}\sin\theta_2$. This gives

$q_2 = \dfrac{9}{16}|q_1|\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{9}{16}|q_1|\dfrac{3/5}{4/5} = \dfrac{27}{64}|q_1| = 0.843\ \mu\text{C}.$

(d) $F_x = F_{1x} + F_{2x}$ and $F_y = 0$, so $F = |q_3|\dfrac{1}{4\pi\epsilon_0}\left(\dfrac{|q_1|}{(0.0400\text{ m})^2}\dfrac{4}{5} + \dfrac{|q_2|}{(0.0300\text{ m})^2}\dfrac{3}{5}\right) = 56.2\text{ N}.$

EVALUATE: The net force $\vec{F}$ on q_3 is in the same direction as the resultant electric field at the location of q_3 due to q_1 and q_2.

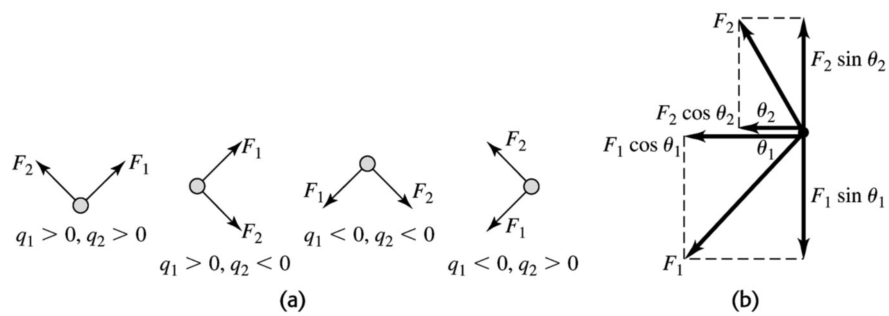


Figure 21.95

21.96. **IDENTIFY:** Calculate the electric field at P due to each charge and add these field vectors to get the net field.

SET UP: The electric field of a point charge is directed away from a positive charge and toward a negative charge. Let $+x$ be to the right and let $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible diagrams are sketched in Figure 21.96a (next page).

The first diagram is the only one in which the electric field must point in the negative y-direction.

(b) $q_1 = -3.00\ \mu\text{C}$, and $q_2 < 0$.

(c) The electric fields $\vec{E}_1$ and $\vec{E}_2$ and their components are sketched in Figure 21.96b. $\cos\theta_1 = \dfrac{5}{13}$,

$\sin\theta_1 = \dfrac{12}{13}$, $\cos\theta_2 = \dfrac{12}{13}$ and $\sin\theta_2 = \dfrac{5}{13}$. $E_x = 0 = -\dfrac{k|q_1|}{(0.050\text{ m})^2}\dfrac{5}{13} + \dfrac{k|q_2|}{(0.120\text{ m})^2}\dfrac{12}{13}$. This gives

$\dfrac{k|q_2|}{(0.120\text{ m})^2} = \dfrac{k|q_1|}{(0.050\text{ m})^2}\dfrac{5}{12}$. Solving for $|q_2|$ gives $|q_2| = 7.2\ \mu\text{C}$, so $q_2 = -7.2\ \mu\text{C}$. Then

$E_y = -\dfrac{k|q_1|}{(0.050\text{ m})^2}\dfrac{12}{13} - \dfrac{kq_2}{(0.120\text{ m})^2}\dfrac{5}{13} = -1.17\times10^7\text{ N/C}.\quad E = 1.17\times10^7\text{ N/C}.$

EVALUATE: With q_1 known, specifying the direction of $\vec{E}$ determines both q_2 and E.

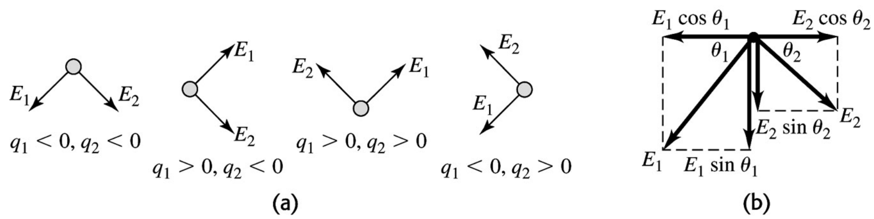


Figure 21.96

21.97. **IDENTIFY:** To find the electric field due to the second rod, divide that rod into infinitesimal segments of length dx, calculate the field dE due to each segment and integrate over the length of the rod to find the total field due to the rod. Use $d\vec{F} = dq\,\vec{E}$ to find the force the electric field of the second rod exerts on each infinitesimal segment of the first rod.

SET UP: An infinitesimal segment of the second rod is sketched in Figure 21.97. $dQ = (Q/L)dx'$.

EXECUTE: **(a)** $dE = \dfrac{k\,dQ}{(x + a/2 + L - x')^2} = \dfrac{kQ}{L}\dfrac{dx'}{(x + a/2 + L - x')^2}$.

$$E_x = \int_0^L dE_x = \frac{kQ}{L}\int_0^L \frac{dx'}{(x + a/2 + L - x')^2} = \frac{kQ}{L}\left[\frac{1}{x + a/2 + L - x'}\right]_0^L = \frac{kQ}{L}\left(\frac{1}{x + a/2} - \frac{1}{x + a/2 + L}\right).$$

$$E_x = \frac{2kQ}{L}\left(\frac{1}{2x + a} - \frac{1}{2L + 2x + a}\right).$$

(b) Now consider the force that the field of the second rod exerts on an infinitesimal segment dq of the first rod. This force is in the $+x$-direction. $dF = dq\,E$.

$$F = \int E\,dq = \int_{a/2}^{L+a/2} \frac{EQ}{L}\,dx = \frac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\left(\frac{1}{2x + a} - \frac{1}{2L + 2x + a}\right)dx.$$

$$F = \frac{2kQ^2}{L^2}\frac{1}{2}\left(\left[\ln\,(a + 2x)\right]_{a/2}^{L+a/2} - \left[\ln(2L + 2x + a)\right]_{a/2}^{L+a/2}\right) = \frac{kQ^2}{L^2}\ln\left[\left(\frac{a + 2L + a}{2a}\right)\left(\frac{2L + 2a}{4L + 2a}\right)\right].$$

$$F = \frac{kQ^2}{L^2}\ln\left(\frac{(a + L)^2}{a(a + 2L)}\right).$$

(c) For $a \gg L$, $F = \dfrac{kQ^2}{L^2}\ln\left(\dfrac{a^2(1 + L/a)^2}{a^2(1 + 2L/a)}\right) = \dfrac{kQ^2}{L^2}(2\ln\,(1 + L/a) - \ln(1 + 2L/a))$.

For small z, $\ln(1 + z) \approx z - \dfrac{z^2}{2}$. Therefore, for $a \gg L$,

$$F \approx \frac{kQ^2}{L^2}\left[2\left(\frac{L}{a} - \frac{L^2}{2a^2} + \cdots\right) - \left(\frac{2L}{a} - \frac{2L^2}{a^2} + \cdots\right)\right] \approx \frac{kQ^2}{a^2}.$$

EVALUATE: The distance between adjacent ends of the rods is a. When $a \gg L$ the distance between the rods is much greater than their lengths and they interact as point charges.

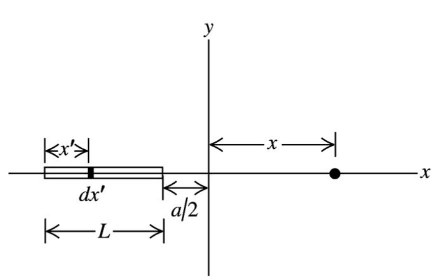


Figure 21.97

21.98. **IDENTIFY** and **SET UP:** The charge of n electrons is ne.
EXECUTE: The charge on the bee is $Q = ne$, so the number of missing electrons is
$n = Q/e = (30\text{ pC})/e = (30\times10^{-12}\text{ C})/(1.60\times10^{-19}\text{ C}) = 1.88\times10^{8} \approx 1.9\times10^{8}$ electrons, which makes choice (a) correct.
EVALUATE: This charge is due to around 190 million electrons.

21.99. **IDENTIFY** and **SET UP:** One charge exerts a force on another charge without being in contact.
EXECUTE: Even though the bee does not touch the stem, the positive charges on the bee attract negative charges (electrons normally) in the stem. This pulls electrons toward the bee, leaving positive charge at the opposite end of the stem, which polarizes it. Thus choice (c) is correct.
EVALUATE: Choice (b) cannot be correct because the bee is positive and would therefore not attract the positive charges in the stem.

21.100. **IDENTIFY** and **SET UP:** Electric field lines begin on positive charges and end on negative charges.
EXECUTE: The flower and bee are both positive, so no field lines can end on either of them. This makes the figure in choice (c) the correct one.
EVALUATE: The net electric field is the vector sum of the field due to the bee and the field due to the flower. Somewhere between the bee and flower the fields cancel, depending on the relative amounts of charge on the bee and flower.

21.101. **IDENTIFY** and **SET UP:** Assume that the charge remains at the end of the stem and that the bees approach to 15 cm from this end of the stem. The electric field is $E = k\dfrac{|q|}{r^2}$.
EXECUTE: Using the numbers given, we have
$E = k\dfrac{|q|}{r^2} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(40\times10^{-12}\text{ C})/(0.15\text{ m})^2 = 16\text{ N/C}$, which is choice (b).
EVALUATE: Even if the charge spread out a bit over the stem, the result would be in the neighborhood of the value we calculated.