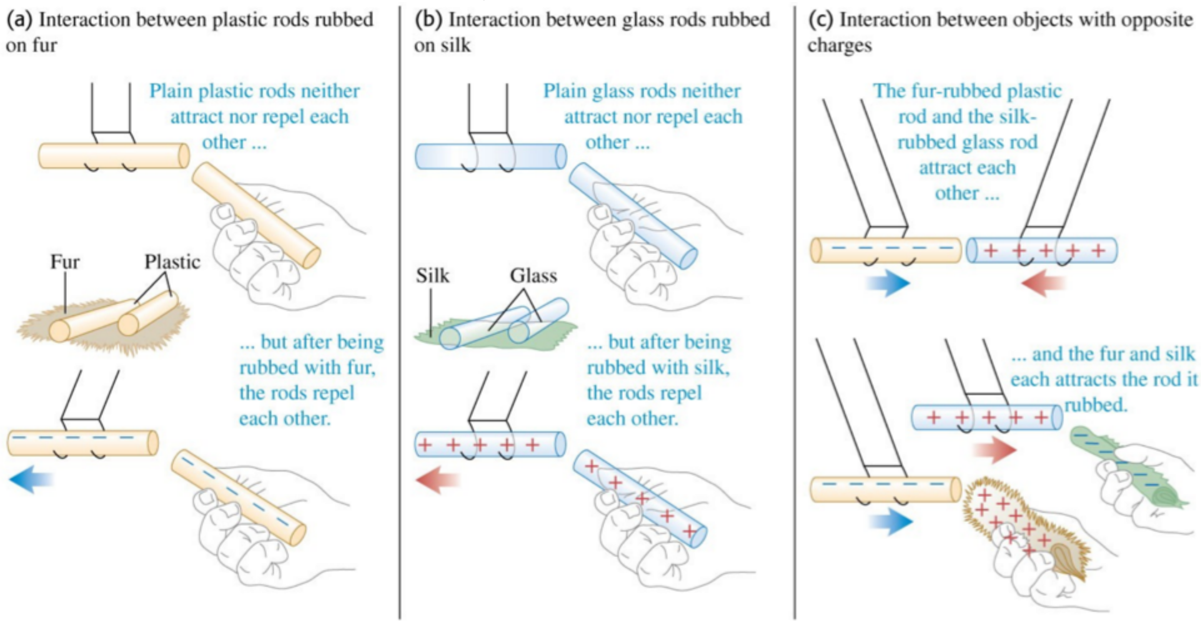
VP250 S1 RC1 Tong Jin

I.



CONSERVATION OF CHARGE

The algebraic sum of all the electric charges in any closed system is zero.

QUANTIZATION OF CHARGE

Any electric charge is a multiple of the electron's (or proton's) charge

$$Q = n \cdot (\pm e), \qquad \textit{where } n = 0, 1, 2, \ldots$$

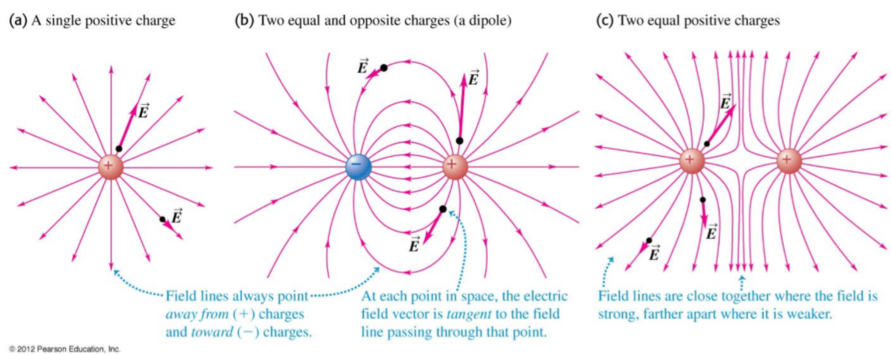
Coulomb's Law: ☆ direction (omitted here)

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \iff E = \frac{1}{4\pi\epsilon_0} \frac{q_0}{r^2}$$

$q_0 \nearrow$ source charge

$*\ \epsilon_0 = 8.854 \times 10^{-2} \left[\frac{C^2}{N \cdot m^2}\right]$ *unit check

☆ Electric Field Lines:



① E tangential to the lines

② ≣ => E big / ≡ => E small

③ never across

Calculations:

dimension	object	charge density	infinitesimal charge
1D	line	λ [C/m]	$dq = \lambda \cdot dl$
2D	surface	σ [C/m^2]	$dq = \sigma\, dS$
3D	solid	ρ / ϱ [C/m^3]	$dq = \rho\, dV$

disk: $dS = 2\pi r \cdot dr$

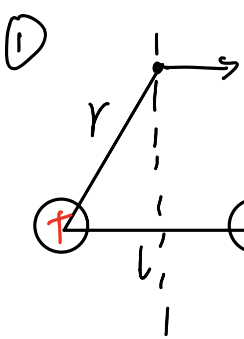
cylinder: $dV = \pi r^2 \cdot dh$

ball: $dV = 4\pi r^2 \cdot dr$

☆ personal favor:

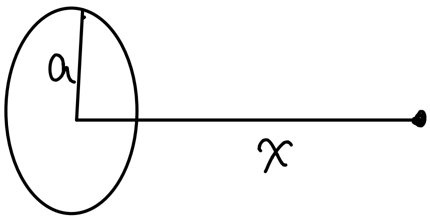
$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} \Rightarrow dq = \begin{cases} \lambda \cdot dl \\ \sigma \cdot ds \\ \rho \cdot dV \end{cases} \Rightarrow \text{related with } r$$

①



$$E = \frac{1}{4\pi\epsilon_0} \frac{q \cdot l}{r^3}$$

② cirde



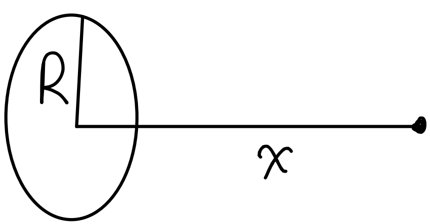
$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda \cdot dl}{x^2 + a^2}$$

$$dE_x = \frac{x\lambda \cdot dl}{(a^2+x^2)^{\frac{3}{2}}} \cdot \frac{1}{4\pi\epsilon_0}$$

$$\Rightarrow E_x = \frac{1}{4\pi\epsilon_0} \cdot \frac{x \cdot q}{(x^2+a^2)^{\frac{3}{2}}}$$ * $q = \lambda \cdot 2\pi a$

1. unit check
2. limit check: $x \to 0,\ E_x \to 0$; $x \to \infty,\ E_x \to 0$
   $a \to 0$, 点电荷 ($x \Rightarrow a$)

③ disk

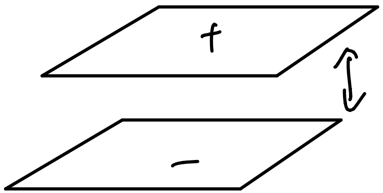


$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{R^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma \cdot 2\pi r \cdot dr}{x^2 + r^2}$$

$$\Rightarrow E_x = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{1+\frac{R^2}{x^2}}}\right)$$ * integral

limit check: $R \to 0,\ E_x \to 0$; $R \to \infty,\ E_x = \frac{\sigma}{2\epsilon_0}$

④



$E = 0$

$E = \frac{\sigma}{\epsilon_0}$

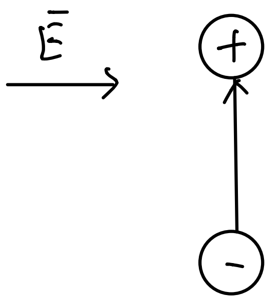
$E = 0$

★ regardless of distance

★ Dipoles:

$\bar{p} = q \cdot \bar{l}$

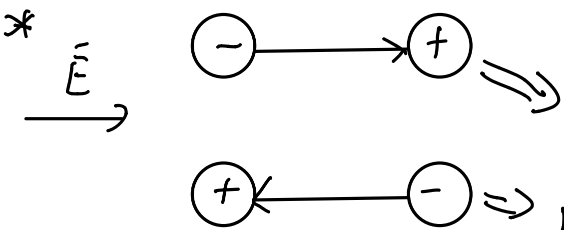
★ direction: negative to positive



⊙ $\bar{\tau} = \bar{p} \times \bar{E}$

$W = -\bar{p} \cdot \bar{E}$

*



all equilibrium

⇒ stable (final)

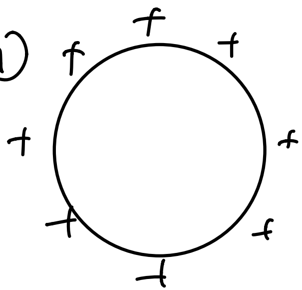
⇒ not stable

II. Electric Flux: $\Phi_E = \oint \bar{E} \cdot d\bar{A} = \oint \bar{E} \cdot \bar{n}\, dA$

(outwards)

Gauss's Law: $\oint \bar{E} \cdot d\bar{A} = \dfrac{q_{enclosed}}{\epsilon_0}$

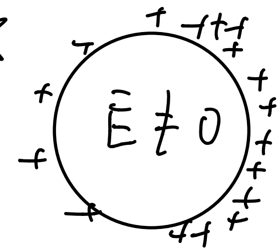
* powerful to calculate E (symmetry)

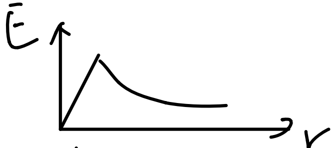
① 

outside: $E \cdot 4\pi r^2 = \dfrac{q}{\epsilon_0}$

inside: $\underline{E = 0}$

symmetry

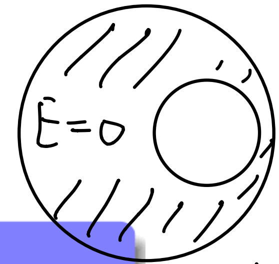


② charged ball: 

↓

$\bar{E} = \dfrac{1}{4\pi\epsilon_0} \cdot \dfrac{Q}{R^3}\, r$

General Fact. Under electrostatic conditions, the electric field at every point inside a conductor is zero.



$\vec{E}_{total} = \vec{E}_{ext} + \vec{E}_{int} = 0$

* hint: superposition

Property 1

Any excess charge, placed on a solid conductor, resides entirely on the conductor's surface.

Property 2a (hollow conductor with empty cavity)

For a hollow charged conductor with an empty cavity, there is no net charge on the cavity's surface.

Property 2b (cavity with a charge inside)

For a hollow charged (q_x) conductor with a cavity and a point charge q_c inside the cavity, the net charge on the cavity's surface is $-q$ and the charge on the conductor's surface is $q_x + q_c$.

conservation of charge

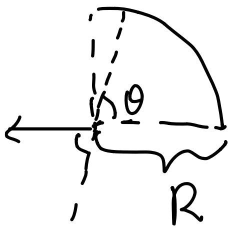
$\vec{E}$ normal to surface

$\vec{E}$ ~ surface charge density

Ex)

21.97 •• **CALC** Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius a that lies in the first quadrant, with the center of curvature at the origin. Find the x- and y-components of the net electric field at the origin.

Ans 1



$$\frac{\lambda \cdot \pi R}{2} = Q$$

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda \cdot R \cdot d\theta}{R^2}$$

$$dE_x = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda}{R} \cdot d\theta \cdot \cos\theta$$

$$E_x = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda}{R} \cdot \left[\sin\theta\right] \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{R} = \frac{1}{4\pi\epsilon_0 R} \cdot \frac{2Q}{\pi R} = \frac{Q}{2\pi^2 \epsilon_0 R^2}$$