27

MAGNETIC FIELD AND MAGNETIC FORCES

27.1. **IDENTIFY** and **SET UP:** Apply $\vec{F} = q\vec{v} \times \vec{B}$ to calculate $\vec{F}$. Use the cross products of unit vectors from Chapter 1. $\vec{v} = (+4.19\times10^4 \text{ m/s})\hat{i} + (-3.85\times10^4 \text{ m/s})\hat{j}$.

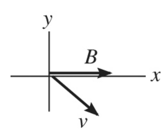
(a) EXECUTE: $\vec{B} = (1.40 \text{ T})\hat{i}$.

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})[(4.19\times10^4 \text{ m/s})\hat{i}\times\hat{i} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{i}]$.

$\hat{i}\times\hat{i} = 0$, $\hat{j}\times\hat{i} = -\hat{k}$.

$\vec{F} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})(-3.85\times10^4 \text{ m/s})(-\hat{k}) = (-6.68\times10^{-4} \text{ N})\hat{k}$.

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.



The right-hand rule gives that $\vec{v}\times\vec{B}$ is directed out of the paper ($+z$-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$.

Figure 27.1a

$\vec{F}$ is in the $-z$-direction. This agrees with the direction calculated with unit vectors.

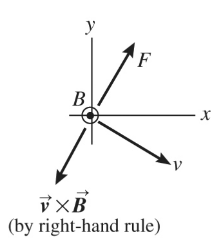
(b) EXECUTE: $\vec{B} = (1.40 \text{ T})\hat{k}$.

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})[(+4.19\times10^4 \text{ m/s})\hat{i}\times\hat{k} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{k}]$.

$\hat{i}\times\hat{k} = -\hat{j}$, $\hat{j}\times\hat{k} = \hat{i}$.

$\vec{F} = (-7.27\times10^{-4} \text{ N})(-\hat{j}) + (6.68\times10^{-4} \text{ N})\hat{i} = [(6.68\times10^{-4} \text{ N})\hat{i} + (7.27\times10^{-4} \text{ N})\hat{j}]$.

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.



The direction of $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

Figure 27.1b

27.2. **IDENTIFY:** The net force must be zero, so the magnetic and gravity forces must be equal in magnitude and opposite in direction.

SET UP: The gravity force is downward so the force from the magnetic field must be upward. The charge's velocity and the forces are shown in Figure 27.2. Since the charge is negative, the magnetic force is opposite to the right-hand rule direction. The minimum magnetic field is when the field is perpendicular to $\vec{v}$. The force is also perpendicular to $\vec{B}$, so $\vec{B}$ is either eastward or westward.

EXECUTE: If $\vec{B}$ is eastward, the right-hand rule direction is into the page and $\vec{F}_B$ is out of the page, as required. Therefore, $\vec{B}$ is eastward. $mg = |q|vB\sin\phi$. $\phi = 90°$ and

$$B = \frac{mg}{v|q|} = \frac{(0.195\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{(4.00\times10^{4}\text{ m/s})(2.50\times10^{-8}\text{ C})} = 1.91\text{ T}.$$

EVALUATE: The magnetic field could also have a component along the north-south direction, that would not contribute to the force, but then the field wouldn't have minimum magnitude.

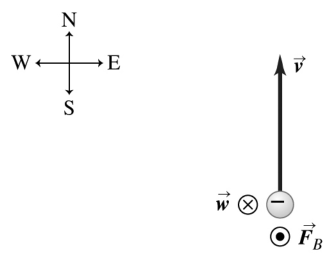


Figure 27.2

27.3. **IDENTIFY:** The force $\vec{F}$ on the particle is in the direction of the deflection of the particle. Apply the right-hand rule to the directions of $\vec{v}$ and $\vec{B}$. See if your thumb is in the direction of $\vec{F}$, or opposite to that direction. Use $F = |q|vB\sin\phi$ with $\phi = 90°$ to calculate F.

SET UP: The directions of $\vec{v}$, $\vec{B}$, and $\vec{F}$ are shown in Figure 27.3.

EXECUTE: **(a)** When you apply the right-hand rule to $\vec{v}$ and $\vec{B}$, your thumb points east. $\vec{F}$ is in this direction, so the charge is positive.

(b) $F = |q|vB\sin\phi = (8.50\times10^{-6}\text{ C})(4.75\times10^{3}\text{ m/s})(1.25\text{ T})\sin 90° = 0.0505\text{ N}$

EVALUATE: If the particle had negative charge and $\vec{v}$ and $\vec{B}$ are unchanged, the particle would be deflected toward the west.

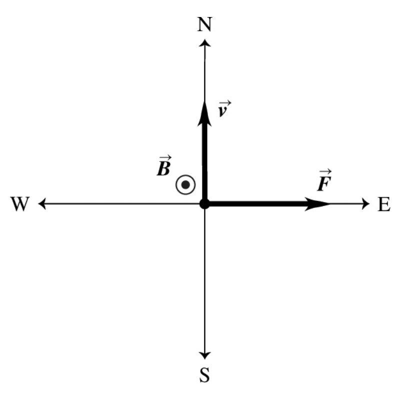


Figure 27.3

27.4. **IDENTIFY:** Apply Newton's second law, with the force being the magnetic force.

SET UP: $\hat{j}\times\hat{i} = -\hat{k}$.

EXECUTE: $\vec{F} = m\vec{a} = q\vec{v} \times \vec{B}$ gives $\vec{a} = \dfrac{q\vec{v} \times \vec{B}}{m}$ and

$$\vec{a} = \frac{(1.22\times10^{-8}\ \text{C})(3.0\times10^{4}\ \text{m/s})(1.63\ \text{T})(\hat{j}\times\hat{i})}{1.81\times10^{-3}\ \text{kg}} = -(0.330\ \text{m/s}^2)\hat{k}.$$

EVALUATE: The acceleration is in the $-z$-direction and is perpendicular to both $\vec{v}$ and $\vec{B}$.

27.5. IDENTIFY: Apply $F = |q|vB\sin\phi$ and solve for v.

SET UP: An electron has $q = -1.60\times10^{-19}$ C.

EXECUTE: $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-15}\ \text{N}}{(1.6\times10^{-19}\ \text{C})(3.5\times10^{-3}\ \text{T})\sin 60°} = 9.49\times10^{6}\,\text{m/s}.$

EVALUATE: Only the component $B\sin\phi$ of the magnetic field perpendicular to the velocity contributes to the force.

27.6. IDENTIFY: Apply Newton's second law and $F = |q|vB\sin\phi$.

SET UP: ϕ is the angle between the direction of $\vec{v}$ and the direction of $\vec{B}$.

EXECUTE: **(a)** The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{|q|vB}{m} = \frac{(1.6\times10^{-19}\ \text{C})(1.40\times10^{6}\ \text{m/s})(7.4\times10^{-2}\ \text{T})}{(9.11\times10^{-31}\ \text{kg})} = 1.82\times10^{16}\ \text{m/s}^2.$$

(b) If $a = \frac{1}{4}(1.82\times10^{16}\ \text{m/s}^2) = \dfrac{|q|vB\sin\phi}{m}$, then $\sin\phi = 0.25$ and $\phi = 14.5°$.

EVALUATE: The force and acceleration decrease as the angle ϕ approaches zero.

27.7. IDENTIFY: Apply $\vec{F} = q\vec{v} \times \vec{B}$.

SET UP: $\vec{v} = v_y\hat{j}$, with $v_y = -3.80\times10^{3}$ m/s. $F_x = +7.60\times10^{-3}$ N, $F_y = 0$, and $F_z = -5.20\times10^{-3}$ N.

EXECUTE: **(a)** $F_x = q(v_yB_z - v_zB_y) = qv_yB_z$.

$B_z = F_x/qv_y = (7.60\times10^{-3}\ \text{N})/[(7.80\times10^{-6}\ \text{C})(-3.80\times10^{3}\ \text{m/s})] = -0.256\ \text{T}.$

$F_y = q(v_zB_x - v_xB_z) = 0$, which is consistent with $\vec{F}$ as given in the problem. There is no force component along the direction of the velocity.

$F_z = q(v_xB_y - v_yB_x) = -qv_yB_x$. $B_x = -F_z/qv_y = -0.175\ \text{T}.$

(b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y.

(c) $\vec{B}\cdot\vec{F} = B_xF_x + B_yF_y + B_zF_z = (-0.175\ \text{T})(+7.60\times10^{-3}\ \text{N}) + (-0.256\ \text{T})(-5.20\times10^{-3}\ \text{N})$

$\vec{B}\cdot\vec{F} = 0$. $\vec{B}$ and $\vec{F}$ are perpendicular (angle is 90°).

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{v}\cdot\vec{F}$ is also zero.

27.8. IDENTIFY and **SET UP:** $\vec{F} = q\vec{v}\times\vec{B} = qB_z[v_x(\hat{i}\times\hat{k}) + v_y(\hat{j}\times\hat{k}) + v_z(\hat{k}\times\hat{k})] = qB_z[v_x(-\hat{j}) + v_y(\hat{i})].$

EXECUTE: **(a)** Set the expression for $\vec{F}$ equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40\times10^{-7}\ \text{N})}{-(-5.60\times10^{-9}\ \text{C})(-1.25\ \text{T})} = -106\ \text{m/s}.$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40\times10^{-7}\ \text{N})}{(-5.60\times10^{-9}\ \text{C})(-1.25\ \text{T})} = -48.6\ \text{m/s}.$$

(b) v_z does not contribute to the force, so is not determined by a measurement of $\vec{F}$.

(c) $\vec{v}\cdot\vec{F} = v_xF_x + v_yF_y + v_zF_z = \dfrac{F_y}{-qB_z}F_x + \dfrac{F_x}{qB_z}F_y = 0;\ \theta = 90°.$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{B}\cdot\vec{F}$ is also zero.

27.9. IDENTIFY: Apply $\vec{F} = q\vec{v}\times\vec{B}$ to the force on the proton and to the force on the electron. Solve for the components of $\vec{B}$ and use them to find its magnitude and direction.

SET UP: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force on the proton is in the $+y$-direction, $B_y = 0$ and $\vec{B} = B_x\hat{i} + B_z\hat{k}$. For the proton, $\vec{v}_p = (1.50\text{ km/s})\hat{i} = v_p\hat{i}$ and $\vec{F}_p = (2.25\times10^{-16}\text{ N})\hat{j} = F_p\hat{j}$. For the electron, $\vec{v}_e = -(4.75\text{ km/s})\hat{k} = -v_e\hat{k}$ and $\vec{F}_e = (8.50\times10^{-16}\text{ N})\hat{j} = F_e\hat{j}$. The magnetic force is $\vec{F} = q\vec{v}\times\vec{B}$.

EXECUTE: **(a)** For the proton, $\vec{F}_p = q\vec{v}_p\times\vec{B}$ gives $F_p\hat{j} = ev_p\hat{i}\times(B_x\hat{i} + B_z\hat{k}) = -ev_pB_z\hat{j}$. Solving for B_z gives $B_z = -\dfrac{F_p}{ev_p} = -\dfrac{2.25\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(1500\text{ m/s})} = -0.9375\text{ T}$. For the electron, $\vec{F}_e = -e\vec{v}_e\times\vec{B}$, which gives $F_e\hat{j} = (-e)(-v_e\hat{k})\times(B_x\hat{i} + B_z\hat{k}) = ev_eB_x\hat{j}$. Solving for B_x gives

$B_x = \dfrac{F_e}{ev_e} = \dfrac{8.50\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(4750\text{ m/s})} = 1.118\text{ T}$. Therefore $\vec{B} = 1.118\text{ T}\hat{i} - 0.9375\text{ T}\hat{k}$. The magnitude of the field is $B = \sqrt{B_x^2 + B_z^2} = \sqrt{(1.118\text{ T})^2 + (-0.9375\text{ T})^2} = 1.46\text{ T}$. Calling θ the angle that the magnetic field makes with the $+x$-axis, we have $\tan\theta = \dfrac{B_z}{B_x} = \dfrac{-0.9375\text{ T}}{1.118\text{ T}} = -0.8386$, so $\theta = -40.0°$. Therefore the magnetic field is in the xz-plane directed at 40.0° from the $+x$-axis toward the $-z$-axis, having a magnitude of 1.46 T.

(b) $\vec{B} = B_x\hat{i} + B_z\hat{k}$ and $\vec{v} = (3.2\text{ km/s})(-\hat{j})$.

$\vec{F} = q\vec{v}\times\vec{B} = (-e)(3.2\text{ km/s})(-\hat{j})\times(B_x\hat{i} + B_z\hat{k}) = e(3.2\times10^3\text{ m/s})[B_x(-\hat{k}) + B_z\hat{i}].$

$\vec{F} = e(3.2\times10^3\text{ m/s})(-1.118\text{ T}\hat{k} - 0.9375\text{ T}\hat{i}) = -4.80\times10^{-16}\text{ N}\hat{i} - 5.724\times10^{-16}\text{ N}\hat{k}.$

$F = \sqrt{F_x^2 + F_z^2} = 7.47\times10^{-16}\text{ N}$. Calling θ the angle that the force makes with the $-x$-axis, we have $\tan\theta = \dfrac{F_z}{F_x} = \dfrac{-5.724\times10^{-16}\text{ N}}{-4.800\times10^{-16}\text{ N}}$, which gives $\theta = 50.0°$. The force is in the xz-plane and is directed at 50.0° from the $-x$-axis toward either the $-z$-axis.

EVALUATE: The force on the electrons in parts (a) and (b) are comparable in magnitude because the electron speeds are comparable in both cases.

27.10. IDENTIFY: Knowing the area of a surface and the magnetic field it is in, we want to calculate the flux through it.

SET UP: $d\vec{A} = dA\hat{k}$, so $d\Phi_B = \vec{B}\cdot d\vec{A} = B_z dA.$

EXECUTE: $\Phi_B = B_zA = (-0.500\text{ T})(0.0340\text{ m})^2 = -5.78\times10^{-4}\text{ T}\cdot\text{m}^2$. $|\Phi_B| = 5.78\times10^{-4}\text{ Wb}$.

EVALUATE: Since the field is uniform over the surface, it is not necessary to integrate to find the flux.

27.11. IDENTIFY and SET UP: $\Phi_B = \int\vec{B}\cdot d\vec{A}$.

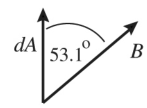
Circular area in the xy-plane, so $A = \pi r^2 = \pi(0.0650\text{ m})^2 = 0.01327\text{ m}^2$ and $d\vec{A}$ is in the z-direction. Use Eq. (1.18) to calculate the scalar product.

EXECUTE: **(a)** $\vec{B} = (0.230\text{ T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel $(\phi = 0°)$ so $\vec{B}\cdot d\vec{A} = B\,dA$.

B is constant over the circular area so

$\Phi_B = \int\vec{B}\cdot d\vec{A} = \int B\,dA = B\int dA = BA = (0.230\text{ T})(0.01327\text{ m}^2) = 3.05\times10^{-3}\text{ Wb}.$

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11a.



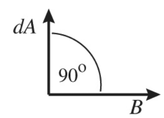
$\vec{B}\cdot d\vec{A} = B\cos\phi dA$
with $\phi = 53.1°$.

Figure 27.11a

B and ϕ are constant over the circular area so $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\cos\phi dA = B\cos\phi\int dA = B\cos\phi A$

$\Phi_B = (0.230\text{ T})\cos 53.1°(0.01327\text{ m}^2) = 1.83\times 10^{-3}\text{ Wb}.$

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11b.



$\vec{B}\cdot d\vec{A} = 0$ since $d\vec{A}$ and $\vec{B}$ are perpendicular ($\phi = 90°$).
$\Phi_B = \int \vec{B}\cdot d\vec{A} = 0.$

Figure 27.11b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.12. **IDENTIFY:** Knowing the area of a surface and the magnetic flux through it, we want to find the magnetic field needed to produce this flux.

SET UP: $\Phi_B = BA\cos\phi$ where $\phi = 60.0°$.

EXECUTE: Solving $\Phi_B = BA\cos\phi$ for B gives $B = \dfrac{\Phi_B}{A\cos\phi} = \dfrac{3.10\times 10^{-4}\text{ Wb}}{(0.0280\text{ m})(0.0320\text{ m})\cos 60.0°} = 0.692\text{ T}.$

EVALUATE: This is a fairly strong magnetic field, but not impossible to achieve in modern laboratories.

27.13. **IDENTIFY:** The total flux through the bottle is zero because it is a closed surface.

SET UP: The total flux through the bottle is the flux through the plastic plus the flux through the open cap, so the sum of these must be zero. $\Phi_{\text{plastic}} + \Phi_{\text{cap}} = 0$.

$\Phi_{\text{plastic}} = -\Phi_{\text{cap}} = -BA\cos\phi = -B(\pi r^2)\cos\phi.$

EXECUTE: Substituting the numbers gives $\Phi_{\text{plastic}} = -(1.75\text{ T})\pi(0.0125\text{ m})^2\cos 25° = -7.8\times 10^{-4}\text{ Wb}.$

EVALUATE: It would be very difficult to calculate the flux through the plastic directly because of the complicated shape of the bottle, but with a little thought we can find this flux through a simple calculation.

27.14. **IDENTIFY:** When $\vec{B}$ is uniform across the surface, $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\phi.$

SET UP: $\vec{A}$ is normal to the surface and is directed outward from the enclosed volume. For surface *abcd*, $\vec{A} = -A\hat{i}$. For surface *befc*, $\vec{A} = -A\hat{k}$. For surface *aefd*, $\cos\phi = 3/5$ and the flux is positive.

EXECUTE: **(a)** $\Phi_B(abcd) = \vec{B}\cdot\vec{A} = 0.$

(b) $\Phi_B(befc) = \vec{B}\cdot\vec{A} = -(0.128\text{ T})(0.300\text{ m})(0.300\text{ m}) = -0.0115\text{ Wb}.$

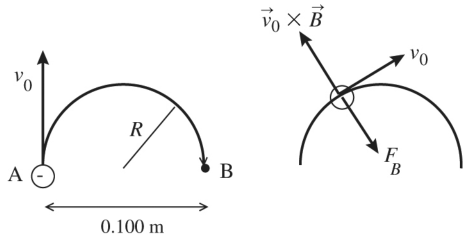
(c) $\Phi_B(aefd) = \vec{B}\cdot\vec{A} = BA\cos\phi = \frac{3}{5}(0.128\text{ T})(0.500\text{ m})(0.300\text{ m}) = +0.0115\text{ Wb}.$

(d) The net flux through the rest of the surfaces is zero since they are parallel to the *x*-axis. The total flux is the sum of all parts above, which is zero.

EVALUATE: The total flux through any closed surface, that encloses a volume, is zero.

27.15. **(a) IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$ to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v}\times\vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0 \times \vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{rad}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0 \times \vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's second law to calculate the magnitude of $\vec{B}$:

$\sum \vec{F} = m\vec{a}$ gives $\sum F_{rad} = ma \quad F_B = m(v^2/R)$.

$F_B = |q| vB\sin\phi = |q| vB$, so $|q| vB = m(v^2/R)$.

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}.$$

(b) IDENTIFY and **SET UP:** The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is πR, so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^{6}\text{ m/s}} = 1.11\times10^{-7}\text{ s}.$

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.16. IDENTIFY: Newton's second law gives $|q|vB = mv^2/R$. The speed v is constant and equals v_0. The direction of the magnetic force must be in the direction of the acceleration and is toward the center of the semicircular path.

SET UP: A proton has $q = +1.60\times10^{-19}$ C and $m = 1.67\times10^{-27}$ kg. The direction of the magnetic force is given by the right-hand rule.

EXECUTE: (a) $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0500\text{ m})} = 0.294\text{ T}.$

The direction of the magnetic field is out of the page (the charge is positive), in order for $\vec{F}$ to be directed to the right at point A.

(b) The time to complete half a circle is $t = \pi R/v_0 = 1.11\times10^{-7}$ s.

EVALUATE: The magnetic field required to produce this path for a proton has a different magnitude (because of the different mass) and opposite direction (because of opposite sign of the charge) than the field required to produce the path for an electron.

27.17. IDENTIFY and **SET UP:** Use conservation of energy to find the speed of the ball when it reaches the bottom of the shaft. The right-hand rule gives the direction of $\vec{F}$ and $F = |q|v\,B\sin\phi$ gives its magnitude. The number of excess electrons determines the charge of the ball.

EXECUTE: $q = (4.00\times10^{8})(-1.602\times10^{-19}\text{ C}) = -6.408\times10^{-11}\text{ C}.$

speed at bottom of shaft: $\frac{1}{2}mv^2 = mgy$; $v = \sqrt{2gy} = 49.5$ m/s.

$\vec{v}$ is downward and $\vec{B}$ is west, so $\vec{v}\times\vec{B}$ is north. Since $q < 0$, $\vec{F}$ is south.

$F = |q| vB\sin\theta = (6.408\times10^{-11}\text{ C})(49.5\text{ m/s})(0.250\text{ T})\sin 90° = 7.93\times10^{-10}\text{ N}.$

EVALUATE: Both the charge and speed of the ball are relatively small so the magnetic force is small, much less than the gravity force of 1.5 N.

27.18. **IDENTIFY:** Since the particle moves perpendicular to the uniform magnetic field, the radius of its path is $R = \dfrac{mv}{|q|B}$. The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

SET UP: The alpha particle has charge $q = +2e = 3.20\times10^{-19}$ C.

EXECUTE: **(a)** $R = \dfrac{(6.64\times10^{-27}\text{ kg})(35.6\times10^{3}\text{ m/s})}{(3.20\times10^{-19}\text{ C})(1.80\text{ T})} = 4.104\times10^{-4}\text{ m} = 0.4104\text{ mm}$. The alpha particle moves in a circular arc of diameter $2R = 2(0.4104\text{ mm}) = 0.821\text{ mm}$.

(b) For a very short time interval the displacement of the particle is in the direction of the velocity. The magnetic force is always perpendicular to this direction so it does no work. The work-energy theorem therefore says that the kinetic energy of the particle, and hence its speed, is constant.

(c) The acceleration is

$a = \dfrac{F_B}{m} = \dfrac{|q|vB\sin\phi}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(35.6\times10^{3}\text{ m/s})(1.80\text{ T})\sin 90^\circ}{6.64\times10^{-27}\text{ kg}} = 3.09\times10^{12}\text{ m/s}^2$. We can also use $a = \dfrac{v^2}{R}$ and the result of part (a) to calculate $a = \dfrac{(35.6\times10^{3}\text{ m/s})^2}{4.104\times10^{-4}\text{ m}} = 3.09\times10^{12}\text{ m/s}^2$, the same result. The acceleration is perpendicular to $\vec{v}$ and $\vec{B}$ and so is horizontal, toward the center of curvature of the particle's path.

EVALUATE: **(d)** The unbalanced force $(\vec{F}_B)$ is perpendicular to $\vec{v}$, so it changes the direction of $\vec{v}$ but not its magnitude, which is the speed.

27.19. **IDENTIFY:** For motion in an arc of a circle, $a = \dfrac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.19. The mass of a proton is 1.67×10^{-27} kg.

EXECUTE: **(a)** $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90^\circ$ and $v = \dfrac{|q|BR}{m} = \dfrac{3(1.60\times10^{-19}\text{ C})(0.250\text{ T})(0.475\text{ m})}{12(1.67\times10^{-27}\text{ kg})} = 2.84\times10^{6}\text{ m/s}$.

(b) $F_B = |q|vB\sin\phi = 3(1.60\times10^{-19}\text{ C})(2.84\times10^{6}\text{ m/s})(0.250\text{ T})\sin 90^\circ = 3.41\times10^{-13}\text{ N}$.

$w = mg = 12(1.67\times10^{-27}\text{ kg})(9.80\text{ m/s}^2) = 1.96\times10^{-25}\text{ N}$. The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: **(c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

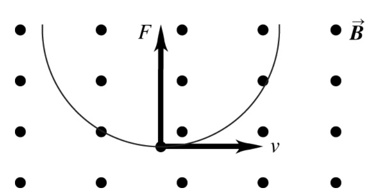


Figure 27.19

27.20. **IDENTIFY:** The magnetic field acts perpendicular to the velocity, causing the ion to move in a circular path but not changing its speed.

SET UP: $R = \dfrac{mv}{|q|B}$ and $K = \frac{1}{2}mv^2$. $K = 5.0\text{ MeV} = 8.0\times10^{-13}$ J.

EXECUTE: **(a)** Solving $K=\frac{1}{2}mv^2$ for v gives $v=\sqrt{2K/m}$.

$v=[2(8.0\times10^{-13}\text{ J})/(1.67\times10^{-27}\text{ kg})]^{1/2}=3.095\times10^7\text{ m/s}$, which rounds to 3.1×10^7 m/s.

(b) Using $R=\dfrac{mv}{|q|B}=(1.67\times10^{-27}\text{ kg})(3.095\times10^7\text{ m/s})/[(1.602\times10^{-19}\text{ C})(1.9\text{ T})]=0.17\text{ m}=17\text{ cm}.$

EVALUATE: If the hydride ions were accelerated to 20 MeV, which is 4 times the value used here, their speed would be twice as great, so the radius of their path would also be twice as great.

27.21. **(a) IDENTIFY** and **SET UP:** Apply Newton's second law, with $a=v^2/R$ since the path of the particle is circular.

EXECUTE: $\sum\vec{F}=m\vec{a}$ says $|q|vB=m(v^2/R)$.

$$v=\frac{|q|BR}{m}=\frac{(1.602\times10^{-19}\text{ C})(2.50\text{ T})(6.96\times10^{-3}\text{ m})}{3.34\times10^{-27}\text{ kg}}=8.35\times10^5\text{ m/s}.$$

(b) IDENTIFY and **SET UP:** The speed is constant so $t=\text{distance}/v$.

EXECUTE: $t=\dfrac{\pi R}{v}=\dfrac{\pi(6.96\times10^{-3}\text{ m})}{8.35\times10^5\text{ m/s}}=2.62\times10^{-8}\text{ s}.$

(c) IDENTIFY and **SET UP:** kinetic energy gained = electric potential energy lost.

EXECUTE: $\frac{1}{2}mv^2=|q|V$.

$$V=\frac{mv^2}{2|q|}=\frac{(3.34\times10^{-27}\text{ kg})(8.35\times10^5\text{ m/s})^2}{2(1.602\times10^{-19}\text{ C})}=7.27\times10^3\text{ V}=7.27\text{ kV}.$$

EVALUATE: The deutron has a much larger mass to charge ratio than an electron so a much larger B is required for the same v and R. The deutron has positive charge so gains kinetic energy when it goes from high potential to low potential.

27.22. **IDENTIFY:** An alpha particle has twice as much charge and about 4 times as much mass as a proton.

SET UP: $R=\dfrac{mv}{|q|B}$ and $K=\frac{1}{2}mv^2$. $K=(mv)^2/2m=p^2/2m$, so $mv=\sqrt{2mK}$.

EXECUTE: The kinetic energy is the same in both cases, so express the radius in terms of it.

$R=\dfrac{mv}{|q|B}=\dfrac{\sqrt{2mK}}{|q|B}$. Now take ratios of the radii for an alpha particle and a proton.

$$\frac{R_\alpha}{R_p}=\frac{\dfrac{\sqrt{2m_\alpha K}}{2eB}}{\dfrac{\sqrt{2m_p K}}{eB}}=\frac{1}{2}\sqrt{\frac{m_\alpha}{m_p}}=\frac{1}{2}\sqrt{\frac{6.64}{1.67}}=0.997,\text{ which gives}$$

$R_\alpha=0.997R_p=(0.997)(16.0\text{ cm})=16.0\text{ cm}$, which is the same as for the proton.

EVALUATE: The radius is proportional to $\dfrac{\sqrt{m}}{|q|}$. The alpha particle has twice the charge of the proton and about 4 times its mass, so the result is the same for both particles.

27.23. **IDENTIFY:** When a particle of charge $-e$ is accelerated through a potential difference of magnitude V, it gains kinetic energy eV. When it moves in a circular path of radius R, its acceleration is $\dfrac{v^2}{R}$.

SET UP: An electron has charge $q=-e=-1.60\times10^{-19}$ C and mass 9.11×10^{-31} kg.

EXECUTE: $\frac{1}{2}mv^2=eV$ and $v=\sqrt{\dfrac{2eV}{m}}=\sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(2.00\times10^3\text{ V})}{9.11\times10^{-31}\text{ kg}}}=2.65\times10^7\text{ m/s}$. $\vec{F}=m\vec{a}$ gives $|q|vB\sin\phi=m\dfrac{v^2}{R}$. $\phi=90^\circ$ and $B=\dfrac{mv}{|q|R}=\dfrac{(9.11\times10^{-31}\text{ kg})(2.65\times10^7\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.180\text{ m})}=8.38\times10^{-4}\text{ T}.$

EVALUATE: The smaller the radius of the circular path, the larger the magnitude of the magnetic field that is required.

27.24. **IDENTIFY:** The magnetic force on the beam bends it through a quarter circle.

SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18 \text{ cm}/(\pi/2) = 0.751 \text{ cm}$. Solving for the magnetic field: $B = mv/qR = (1.67\times10^{-27} \text{ kg})(1200 \text{ m/s})/[(1.60\times10^{-19} \text{ C})(0.00751 \text{ m})] = 1.67\times10^{-3} \text{ T}$.

EVALUATE: This field is about 10 times stronger than the earth's magnetic field, but much weaker than many laboratory fields.

27.25. **IDENTIFY** and **SET UP:** $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$ gives the total force on the proton. At $t = 0$,

$\vec{F} = q\vec{v}\times\vec{B} = q(v_x\hat{i} + v_z\hat{k})\times B_x\hat{i} = qv_zB_x\hat{j}$. **c)**

EXECUTE: (a) $\vec{F} = (1.60\times10^{-19} \text{ C})(2.00\times10^{5} \text{ m/s})(0.500 \text{ T})\hat{j} = (1.60\times10^{-14} \text{ N})\hat{j}$.

(b) Yes. The electric field exerts a force in the direction of the electric field, since the charge of the proton is positive, and there is a component of acceleration in this direction.

(c) In the plane perpendicular to $\vec{B}$ (the yz-plane) the motion is circular. But there is a velocity component in the direction of $\vec{B}$, so the motion is a helix. The electric field in the $+\hat{i}$-direction exerts a force in the $+\hat{i}$-direction. This force produces an acceleration in the $+\hat{i}$-direction and this causes the pitch of the helix to vary. The force does not affect the circular motion in the yz-plane, so the electric field does not affect the radius of the helix.

(d) IDENTIFY and **SET UP:** Use $\omega = |q|B/m$ and $T = 2\pi/\omega$ to calculate the period of the motion. Calculate a_x produced by the electric force and use a constant acceleration equation to calculate the displacement in the x-direction in time $T/2$.

EXECUTE: Calculate the period T: $\omega = |q|B/m$.

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2\pi(1.67\times10^{-27} \text{ kg})}{(1.60\times10^{-19} \text{ C})(0.500 \text{ T})} = 1.312\times10^{-7} \text{ s. Then } t = T/2 = 6.56\times10^{-8} \text{ s.}$$

$v_{0x} = 1.50\times10^{5} \text{ m/s}$.

$$a_x = \frac{F_x}{m} = \frac{(1.60\times10^{-19} \text{ C})(2.00\times10^{4} \text{ V/m})}{1.67\times10^{-27} \text{ kg}} = +1.916\times10^{12} \text{ m/s}^2.$$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_xt^2$.

$x - x_0 = (1.50\times10^{5} \text{ m/s})(6.56\times10^{-8} \text{ s}) + \tfrac{1}{2}(1.916\times10^{12} \text{ m/s}^2)(6.56\times10^{-8} \text{ s})^2 = 1.40 \text{ cm}$.

EVALUATE: The electric and magnetic fields are in the same direction but produce forces that are in perpendicular directions to each other.

27.26. **IDENTIFY:** After being accelerated through a potential difference V the ion has kinetic energy qV. The acceleration in the circular path is v^2/R.

SET UP: The ion has charge $q = +e$.

EXECUTE: $K = qV = +eV$. $\tfrac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19} \text{ C})(220 \text{ V})}{1.16\times10^{-26} \text{ kg}}} = 7.79\times10^{4} \text{ m/s}$.

$F_B = |q|vB\sin\phi$. $\phi = 90°$. $\vec{F} = m\vec{a}$ gives $|q|vB = m\dfrac{v^2}{R}$.

$$R = \frac{mv}{|q|B} = \frac{(1.16\times10^{-26} \text{ kg})(7.79\times10^{4} \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.874 \text{ T})} = 6.46\times10^{-3} \text{ m} = 6.46 \text{ mm}.$$

EVALUATE: The larger the accelerating voltage, the larger the speed of the particle and the larger the radius of its path in the magnetic field.

27.27. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85\times10^3 \text{ m/s})(1.35 \text{ T}) = 7898 \text{ N/C}$.

(a) If $q = 0.640\times10^{-9}$ C, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the opposite direction to the magnetic force.

(b) If $q = -0.320\times10^{-9}$ C, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).

EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.28. IDENTIFY: For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection. With only the magnetic force, $|q|vB = mv^2/R$.

EXECUTE: **(a)** $v = E/B = (1.56\times10^4 \text{ V/m})/(4.62\times10^{-3} \text{ T}) = 3.38\times10^6 \text{ m/s}$.

(b) The directions of the three vectors $\vec{\boldsymbol{v}}$, $\vec{\boldsymbol{E}}$, and $\vec{\boldsymbol{B}}$ are sketched in Figure 27.28.

(c) $R = \dfrac{mv}{|q|B} = \dfrac{(9.11\times10^{-31} \text{ kg})(3.38\times10^6 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(4.62\times10^{-3} \text{ T})} = 4.17\times10^{-3} \text{ m}$.

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi R}{v} = \frac{2\pi(4.17\times10^{-3} \text{ m})}{(3.38\times10^6 \text{ m/s})} = 7.74\times10^{-9} \text{ s}.$$

EVALUATE: For the field directions shown in Figure 27.28, the electric force is toward the top of the page and the magnetic force is toward the bottom of the page.

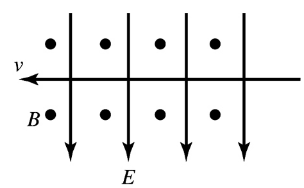


Figure 27.28

27.29. IDENTIFY: For the alpha particles to emerge from the plates undeflected, the magnetic force on them must exactly cancel the electric force. The battery produces an electric field between the plates, which acts on the alpha particles.

SET UP: First use energy conservation to find the speed of the alpha particles as they enter the region between the plates: $qV = 1/2\ mv^2$. The electric field between the plates due to the battery is $E = V_b/d$. For the alpha particles not to be deflected, the magnetic force must cancel the electric force, so $qvB = qE$, giving $B = E/v$.

EXECUTE: Solve for the speed of the alpha particles just as they enter the region between the plates. Their charge is $2e$.

$$v_\alpha = \sqrt{\frac{2(2e)V}{m}} = \sqrt{\frac{4(1.60\times10^{-19} \text{ C})(1750\text{V})}{6.64\times10^{-27} \text{ kg}}} = 4.11\times10^5 \text{ m/s}.$$

The electric field between the plates, produced by the battery, is

$$E = V_b/d = (150 \text{ V})/(0.00820 \text{ m}) = 18{,}300 \text{ V/m}.$$

The magnetic force must cancel the electric force:

$$B = E/v_\alpha = (18{,}300 \text{ V/m})/(4.11\times10^5 \text{ m/s}) = 0.0445 \text{ T}.$$

The magnetic field is perpendicular to the electric field. If the charges are moving to the right and the electric field points upward, the magnetic field is out of the page.

EVALUATE: The sign of the charge of the alpha particle does not enter the problem, so negative charges of the same magnitude would also not be deflected.

27.30. IDENTIFY: The velocity selector eliminates all ions not having the desired velocity. Then the magnetic field bends the ions in a circular arc.

SET UP: In a velocity selector, $E = vB$. For motion in a circular arc in a magnetic field of magnitude B', $R = \dfrac{mv}{|q|B'}$. The ion has charge $+e$.

EXECUTE: **(a)** $E = vB = (4.50\times10^3 \text{ m/s})(0.0250 \text{ T}) = 112 \text{ V/m}$.

(b) $B' = \dfrac{mv}{|q|R} = \dfrac{(6.64\times10^{-26} \text{ kg})(4.50\times10^3 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.125 \text{ m})} = 1.49\times10^{-2} \text{ T}$.

EVALUATE: By laboratory standards, both the electric field and the magnetic field are rather weak and should easily be achievable.

27.31. IDENTIFY: The velocity selector eliminates all ions not having the desired velocity. Then the magnetic field bends the ions in a circular arc.

SET UP: In a velocity selector, $E = vB$. For motion in a circular arc in a magnetic field of magnitude B, $R = \dfrac{mv}{|q|B}$. The ion has charge $+e$.

EXECUTE: **(a)** $v = \dfrac{E}{B} = \dfrac{155 \text{ V/m}}{0.0315 \text{ T}} = 4.92\times10^3 \text{ m/s}$.

(b) $m = \dfrac{R|q|B}{v} = \dfrac{(0.175 \text{ m})(1.60\times10^{-19} \text{ C})(0.0175 \text{ T})}{4.92\times10^3 \text{ m/s}} = 9.96\times10^{-26} \text{ kg}$.

EVALUATE: Ions with larger ratio $\dfrac{m}{|q|}$ will move in a path of larger radius.

27.32. IDENTIFY and **SET UP:** For a velocity selector, $E = vB$. For parallel plates with opposite charge, $V = Ed$.

EXECUTE: **(a)** $E = vB = (1.82\times10^6 \text{ m/s})(0.510 \text{ T}) = 9.28\times10^5 \text{ V/m}$.

(b) $V = Ed = (9.28\times10^5 \text{ V/m})(5.20\times10^{-3} \text{ m}) = 4.83 \text{ kV}$.

EVALUATE: Any charged particle with $v = 1.82\times10^6$ m/s will pass through undeflected, regardless of the sign and magnitude of its charge.

27.33. IDENTIFY: A mass spectrometer separates ions by mass. Since ^{14}N and ^{15}N have different masses they will be separated and the relative amounts of these isotopes can be determined.

SET UP: $R = \dfrac{mv}{|q|B}$. For $m = 1.99\times10^{-26}$ kg (^{12}C), $R_{12} = 12.5$ cm. The separation of the isotopes at the detector is $2(R_{15} - R_{14})$.

EXECUTE: Since $R = \dfrac{mv}{|q|B}$, $\dfrac{R}{m} = \dfrac{v}{|q|B} = \text{constant}$. Therefore $\dfrac{R_{14}}{m_{14}} = \dfrac{R_{12}}{m_{12}}$ which gives

$R_{14} = R_{12}\left(\dfrac{m_{14}}{m_{12}}\right) = (12.5 \text{ cm})\left(\dfrac{2.32\times10^{-26} \text{ kg}}{1.99\times10^{-26} \text{ kg}}\right) = 14.6 \text{ cm}$ and

$R_{15} = R_{12}\left(\dfrac{m_{15}}{m_{12}}\right) = (12.5 \text{ cm})\left(\dfrac{2.49\times10^{-26} \text{ kg}}{1.99\times10^{-26} \text{ kg}}\right) = 15.6 \text{ cm}$. The separation of the isotopes at the detector is $2(R_{15} - R_{14}) = 2(15.6 \text{ cm} - 14.6 \text{ cm}) = 2.0 \text{ cm}$.

EVALUATE: The separation is large enough to be easily detectable. Since the diameter of the ion path is large, about 30 cm, the uniform magnetic field within the instrument must extend over a large area.

27.34. IDENTIFY: The earth's magnetic field exerts a force on the moving charges in the wire.

SET UP: $F = IlB\sin\phi$. The direction of $\vec{F}$ is determined by applying the right-hand rule to the directions of I and $\vec{B}$. 1 gauss $= 10^{-4}$ T.

EXECUTE: (a) The directions of I and $\vec{B}$ are sketched in Figure 27.34a. $\phi = 90°$ so $F = (1.5\text{ A})(2.5\text{ m})(0.55\times10^{-4}\text{ T}) = 2.1\times10^{-4}\text{ N}$. The right-hand rule says that $\vec{F}$ is directed out of the page, so it is upward.

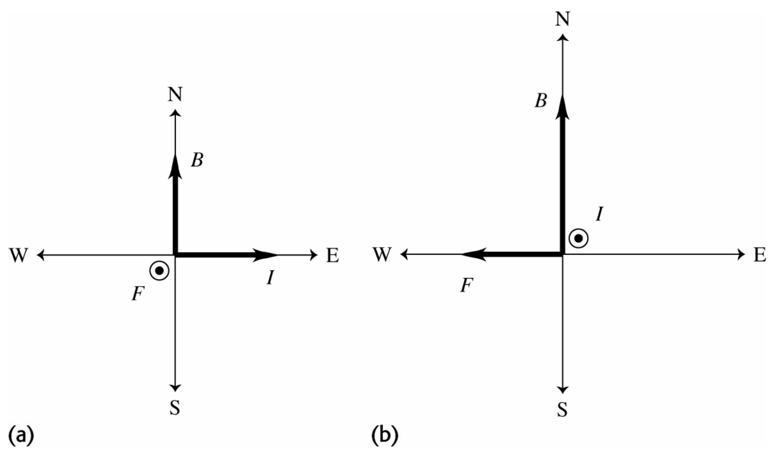


Figure 27.34

(b) The directions of I and $\vec{B}$ are sketched in Figure 27.34b. $\phi = 90°$ and $F = 2.1\times10^{-4}$ N. $\vec{F}$ is directed east to west.

(c) $\vec{B}$ and the direction of the current are antiparallel. $\phi = 180°$ so $F = 0$.

(d) The magnetic force of 2.1×10^{-4} N is not large enough to cause significant effects.

EVALUATE: The magnetic force is a maximum when the directions of I and $\vec{B}$ are perpendicular and it is zero when the current and magnetic field are either parallel or antiparallel.

27.35. IDENTIFY: Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\text{ m} - x$. Figure 27.35a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\text{ A})x(0.240\text{ T})$. $F_c = (4.50\text{ A})(0.600\text{ m} - x)(0.240\text{ T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50\text{ A})(0.600\text{ m})(0.240\text{ T}) = 0.648\text{ N}$ and is directed toward the bottom of the page in Figure 27.35a. $F_b = (4.50\text{ A})(0.300\text{ m})(0.240\text{ T}) = 0.324\text{ N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.35b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\text{ N})^2 + (0.324\text{ N})^2} = 0.724\text{ N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\text{ N}}{0.324\text{ N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.35b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

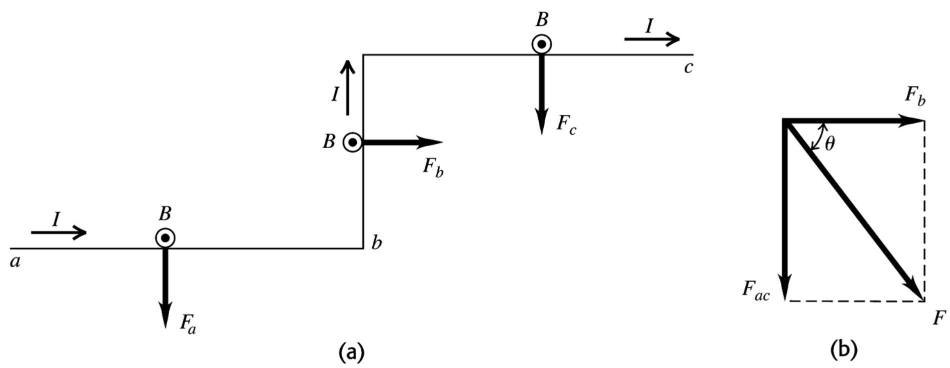


Figure 27.35

27.36. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: $l = 0.0500$ m is the length of wire in the magnetic field. Since the wire is perpendicular to $\vec{B}$, $\phi = 90°$.

EXECUTE: $F = IlB = (10.8\text{ A})(0.0500\text{ m})(0.550\text{ T}) = 0.297\text{ N}$.

EVALUATE: The force per unit length of wire is proportional to both B and I.

27.37. **IDENTIFY and SET UP:** The magnetic force is given by $F = IlB\sin\phi$. $F_I = mg$ when the bar is just ready to levitate. When I becomes larger, $F_I > mg$ and $F_I - mg$ is the net force that accelerates the bar upward. Use Newton's second law to find the acceleration.

EXECUTE: **(a)** $IlB = mg$, $I = \dfrac{mg}{lB} = \dfrac{(0.750\text{ kg})(9.80\text{ m/s}^2)}{(0.500\text{ m})(0.450\text{ T})} = 32.67\text{ A}$.

$V = IR = (32.67\text{ A})(25.0\ \Omega) = 817\text{ V}$.

(b) $R = 2.0\ \Omega$, $I = \varepsilon/R = (816.7\text{ V})/(2.0\ \Omega) = 408\text{ A}$.

$F_I = IlB = 92\text{ N}$.

$a = (F_I - mg)/m = 113\text{ m/s}^2$.

EVALUATE: I increases by over an order of magnitude when R changes to $F_I >> mg$ and a is an order of magnitude larger than g.

27.38. **IDENTIFY and SET UP:** $F = IlB\sin\phi$. The direction of $\vec{F}$ is given by applying the right-hand rule to the directions of I and $\vec{B}$.

EXECUTE: **(a)** The current and field directions are shown in Figure 27.38a (next page). The right-hand rule gives that $\vec{F}$ is directed to the south, as shown. $\phi = 90°$ and

$F = (2.60\text{ A})(1.00\times10^{-2}\text{ m})(0.588\text{ T}) = 0.0153\text{ N}$.

(b) The right-hand rule gives that $\vec{F}$ is directed to the west, as shown in Figure 27.38b. $\phi = 90°$ and $F = 0.0153\text{ N}$, the same as in part (a).

(c) The current and field directions are shown in Figure 27.38c. The right-hand rule gives that $\vec{F}$ is $60.0°$ north of west. $\phi = 90°$ so $F = 0.0153\text{ N}$, the same as in part (a).

EVALUATE: In each case the current direction is perpendicular to the magnetic field. The magnitude of the magnetic force is the same in each case but its direction depends on the direction of the magnetic field.

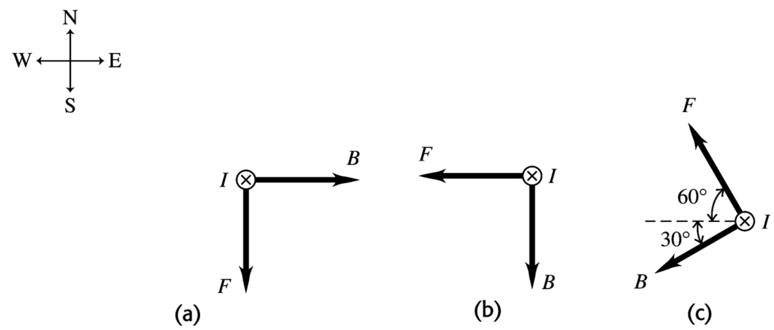


Figure 27.38

27.39. **IDENTIFY:** The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \frac{V}{R}$, where V is the battery voltage.

EXECUTE: **(a)** The forces are shown in Figure 27.39. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \frac{IlB}{g} = \frac{VlB}{Rg} = \frac{(175\text{ V})(0.600\text{ m})(1.50\text{ T})}{(5.00\,\Omega)(9.80\text{ m/s}^2)} = 3.21\text{ kg}.$

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

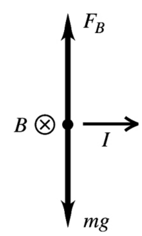


Figure 27.39

27.40. **IDENTIFY:** $\tau = IAB\sin\phi$. The magnetic moment of the loop is $\mu = IA$.

SET UP: Since the plane of the loop is parallel to the field, the field is perpendicular to the normal to the loop and $\phi = 90°$.

EXECUTE: **(a)** $\tau = IAB = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m})(0.19\text{ T}) = 4.7\times10^{-3}\text{ N}\cdot\text{m}.$

(b) $\mu = IA = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m}) = 0.025\text{ A}\cdot\text{m}^2.$

(c) Maximum area is when the loop is circular. $R = \frac{0.050\text{ m} + 0.080\text{ m}}{\pi} = 0.0414\text{ m}.$

$A = \pi R^2 = 5.38\times10^{-3}\text{ m}^2$ and $\tau = (6.2\text{ A})(5.38\times10^{-3}\text{ m}^2)(0.19\text{ T}) = 6.34\times10^{-3}\text{ N}\cdot\text{m}.$

EVALUATE: The torque is a maximum when the field is in the plane of the loop and $\phi = 90°$.

27.41. **IDENTIFY:** The wire segments carry a current in an external magnetic field. Only segments ab and cd will experience a magnetic force since the other two segments carry a current parallel (and antiparallel) to the magnetic field. Only the force on segment cd will produce a torque about the hinge.

SET UP: $F = IlB\sin\phi$. The direction of the magnetic force is given by the right-hand rule applied to the directions of I and $\vec{B}$. The torque due to a force equals the force times the moment arm, the perpendicular distance between the axis and the line of action of the force.

EXECUTE: **(a)** The direction of the magnetic force on each segment of the circuit is shown in Figure 27.41. For segments *bc* and *da* the current is parallel or antiparallel to the field and the force on these segments is zero.

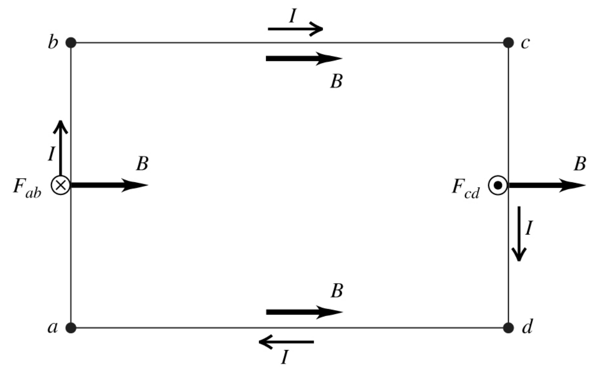


Figure 27.41

(b) $\vec{F}_{ab}$ acts at the hinge and therefore produces no torque. $\vec{F}_{cd}$ tends to rotate the loop about the hinge so it does produce a torque about this axis. $F_{cd} = IlB\sin\phi = (5.00\text{ A})(0.200\text{ m})(1.20\text{ T})\sin 90° = 1.20\text{ N}$

(c) $\tau = Fl = (1.20\text{ N})(0.350\text{ m}) = 0.420\text{ N}\cdot\text{m}$.

EVALUATE: The torque is directed so as to rotate side *cd* out of the plane of the page in Figure 27.41.

27.42. **IDENTIFY:** $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.42a for its original position and in Figure 27.42b after it has rotated $30.0°$.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.42a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0$. The net force on the coil is zero. $\phi = 0°$ and $\sin\phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0°$ and the net torque is $\tau = (1)(1.95\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0° = 0.113\text{ N}\cdot\text{m}$. The net torque is clockwise in Figure 27.42b and is directed so as to increase the angle ϕ.

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

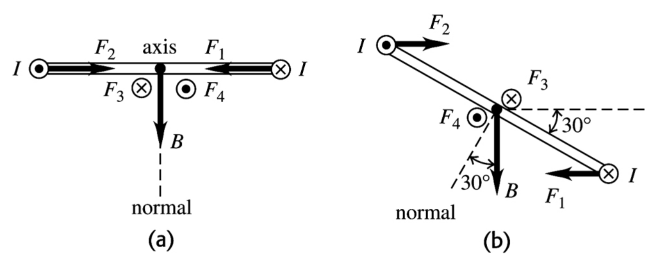


Figure 27.42

27.43. **IDENTIFY:** The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

SET UP: The magnetic torque is given by $\vec{\tau} = \vec{\mu}\times\vec{B}$, and the rotational form of Newton's second law is $\sum\tau = I\alpha$. The magnetic field is parallel to the plane of the loop.

EXECUTE: **(a)** The coil rotates about axis A_2 because the only torque is along top and bottom sides of the coil.

(b) To find the moment of inertia of the coil, treat the two 1.00-m segments as point-masses (since all the points in them are 0.250 m from the rotation axis) and the two 0.500-m segments as thin uniform bars rotated about their centers. Since the coil is uniform, the mass of each segment is proportional to its fraction of the total perimeter of the coil. Each 1.00-m segment is 1/3 of the total perimeter, so its mass is

(1/3)(210 g) = 70 g = 0.070 kg. The mass of each 0.500-m segment is half this amount, or 0.035 kg. The result is

$$I = 2(0.070\text{ kg})(0.250\text{ m})^2 + 2\tfrac{1}{12}(0.035\text{ kg})(0.500\text{ m})^2 = 0.0102\text{ kg}\cdot\text{m}^2.$$

The torque is

$$|\vec{\tau}| = |\vec{\mu}\times\vec{B}| = IAB\sin 90° = (2.00\text{A})(0.500\text{m})(1.00\text{m})(3.00\text{T}) = 3.00\text{ N}\cdot\text{m}.$$

Using the above values, the rotational form of Newton's second law gives

$$\alpha = \frac{\tau}{I} = 290\text{ rad/s}^2.$$

EVALUATE: This angular acceleration will not continue because the torque changes as the coil turns.

27.44. IDENTIFY and **SET UP:** Both coils A and B have the same area A and N turns, but they carry current in opposite directions in a magnetic field. The torque is $\vec{\tau} = \vec{\mu}\times\vec{B}$ and the potential energy is $U = -\mu B\cos\phi$. The magnetic moment is $\vec{\mu} = I\vec{A}$.

EXECUTE: **(a)** Using the right-hand rule for the magnetic moment, $\vec{\mu}$ points in the $-z$-direction (into the page) for coil A and in the $+z$-direction (out of the page) for coil B.

(b) The torque is $\vec{\tau} = \vec{\mu}\times\vec{B}$ which has magnitude $\tau = \mu B\sin\phi$. For coil A, ϕ = 180°, and for coil B, ϕ = 0°. In both cases, sin ϕ = 0, making the torque zero.

(c) For coil A: $U_A = -\mu B\cos\phi = -NIAB\cos 180° = NIAB.$

For coil B: $U_B = -\mu B\cos\phi = -NIAB\cos 0° = -NIAB.$

(d) If coil A is rotated slightly from its equilibrium position, the magnetic field will flip it 180°, so its equilibrium is unstable. But if the same thing it done to coil B, the magnetic field will return it to its original equilibrium position, which makes its equilibrium stable.

EVALUATE: For the stable equilibrium (coil B), its potential energy is a minimum, while for the unstable equilibrium (coil A), its potential energy is a maximum.

27.45. IDENTIFY: $\vec{\tau} = \vec{\mu}\times\vec{B}$ and $U = -\mu B\cos\phi$, where $\mu = NIA$. $\tau = \mu B\sin\phi$.

SET UP: ϕ is the angle between $\vec{B}$ and the normal to the plane of the loop.

EXECUTE: **(a)** $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $\hat{k}\times\hat{j} = -\hat{i}$. $U = -\mu B\cos\phi = 0$.

(b) $\phi = 0$. $\tau = NIAB\sin(0) = 0$, no direction. $U = -\mu B\cos\phi = -NIAB$.

(c) $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $-\hat{k}\times\hat{j} = \hat{i}$. $U = -\mu B\cos\phi = 0$.

(d) $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction, $U = -\mu B\cos(180°) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.46. IDENTIFY and **SET UP:** The potential energy is given by $U = -\vec{\mu}\cdot\vec{B}$. The scalar product depends on the angle between $\vec{\mu}$ and $\vec{B}$.

EXECUTE: For $\vec{\mu}$ and $\vec{B}$ parallel, $\phi = 0°$ and $\vec{\mu}\cdot\vec{B} = \mu B\cos\phi = \mu B$. For $\vec{\mu}$ and $\vec{B}$ antiparallel, $\phi = 180°$ and $\vec{\mu}\cdot\vec{B} = \mu B\cos\phi = -\mu B$.

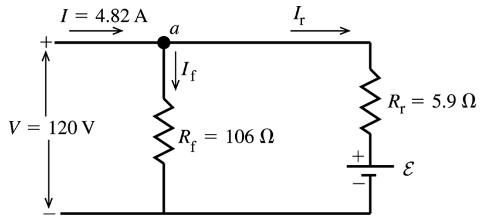
$U_1 = +\mu B$, $U_2 = -\mu B$.

$\Delta U = U_2 - U_1 = -2\mu B = -2(1.45\text{ A}\cdot\text{m}^2)(0.835\text{ T}) = -2.42\text{ J}.$

EVALUATE: U is maximum when $\vec{\mu}$ and $\vec{B}$ are antiparallel and minimum when they are parallel. When the coil is rotated as specified its magnetic potential energy decreases.

27.47. IDENTIFY: The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance R_r and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

SET UP: The circuit is sketched in Figure 27.47.



$\mathcal{E}$ is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

Figure 27.47

EXECUTE: **(a)** The field coils and the rotor are in parallel with the applied potential difference V, so $V = I_f R_f$. $I_f = \dfrac{V}{R_f} = \dfrac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$

(b) Applying the junction rule to point a in the circuit diagram gives $I - I_f - I_r = 0$.

$I_r = I - I_f = 4.82\text{ A} - 1.13\text{ A} = 3.69\text{ A}.$

(c) The potential drop across the rotor, $I_r R_r + \mathcal{E}$, must equal the applied potential difference V: $V = I_r R_r + \mathcal{E}$

$\mathcal{E} = V - I_r R_r = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$P_{in} = IV = (4.82\text{ A})(120\text{ V}) = 578\text{ W}.$

electrical power loss in the two resistances

$P_{loss} = I_f^2 R_f + I_r^2 R_r = (1.13\text{ A})^2(106\ \Omega) + (3.69\text{ A})^2(5.9\ \Omega) = 216\text{ W}.$

mechanical power output

$P_{out} = P_{in} - P_{loss} = 578\text{ W} - 216\text{ W} = 362\text{ W}.$

The mechanical power output is the power associated with the induced emf $\mathcal{E}$.

$P_{out} = P_{\mathcal{E}} = \mathcal{E} I_r = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W}$, which agrees with the above calculation.

EVALUATE: The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.11. In that example the rotor and field coils are connected in series and in this problem they are in parallel.

27.48. **IDENTIFY:** Apply $V_{ab} = \mathcal{E} + Ir$ in order to calculate I. The power drawn from the line is $P_{supplied} = IV_{ab}$.

The mechanical power is the power supplied minus the $I^2 r$ electrical power loss in the internal resistance of the motor.

SET UP: $V_{ab} = 120\text{ V}$, $\mathcal{E} = 105\text{ V}$, and $r = 3.2\ \Omega$.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir \Rightarrow I = \dfrac{V_{ab} - \mathcal{E}}{r} = \dfrac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}.$

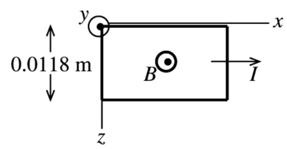
(b) $P_{supplied} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}.$

(c) $P_{mech} = IV_{ab} - I^2 r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}.$

EVALUATE: If the rotor isn't turning, when the motor is first turned on or if the rotor bearings fail, then $\mathcal{E} = 0$ and $I = \dfrac{120\text{V}}{3.2\ \Omega} = 37.5\text{ A}$. This large current causes large $I^2 r$ heating and can trip the circuit breaker.

27.49. **IDENTIFY:** The drift velocity is related to the current density by $J_x = n|q|v_d$. The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

SET UP and **EXECUTE:** **(a)** The section of the silver ribbon is sketched in Figure 27.49a.



$J_x = n|q|v_d$.

so $v_d = \dfrac{J_x}{n|q|}$.

Figure 27.49a

EXECUTE: $J_x = \dfrac{I}{A} = \dfrac{I}{y_1 z_1} = \dfrac{120\text{ A}}{(0.23\times 10^{-3}\text{ m})(0.0118\text{ m})} = 4.42\times 10^7\text{ A/m}^2.$

$v_d = \dfrac{J_x}{n|q|} = \dfrac{4.42\times 10^7\text{ A/m}^2}{(5.85\times 10^{28}/\text{m}^3)(1.602\times 10^{-19}\text{ C})} = 4.7\times 10^{-3}\text{ m/s} = 4.7\text{ mm/s}.$

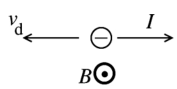
(b) magnitude of $\vec{E}$:

$|q|E_z = |q|v_d B_y$.

$E_z = v_d B_y = (4.7\times 10^{-3}\text{ m/s})(0.95\text{ T}) = 4.5\times 10^{-3}\text{ V/m}.$

direction of $\vec{E}$:

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.49b.

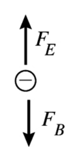


$\vec{v}\times\vec{B}\uparrow.$

$\vec{F}_B = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}\downarrow.$

Figure 27.49b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.49c.



$\vec{F}_E$ must oppose $\vec{F}_B$ so $\vec{F}_E$ is in the $-z$-direction.

Figure 27.49c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$-direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z = 0$ and $z = z_1$) that results from the electric field calculated in part (b). $\varepsilon_{\text{Hall}} = Ez_1 = (4.5\times 10^{-3}\text{ V/m})(0.0118\text{ m}) = 53\,\mu\text{V}.$

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.12. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

27.50. **IDENTIFY:** Apply $qn = \dfrac{-J_x B_y}{E_z}$.

SET UP: $A = y_1 z_1$. $E = \varepsilon/z_1$. $|q| = e$.

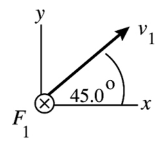
EXECUTE: $n = \dfrac{J_x B_y}{|q|E_z} = \dfrac{IB_y}{A|q|E_z} = \dfrac{IB_y z_1}{A|q|\varepsilon} = \dfrac{IB_y}{y_1|q|\varepsilon}.$

$$n = \frac{(78.0\text{ A})(2.29\text{ T})}{(2.3\times 10^{-4}\text{ m})(1.6\times 10^{-19}\text{ C})(1.31\times 10^{-4}\text{ V})} = 3.7\times 10^{28}\text{ electrons/m}^3.$$

EVALUATE: The value of n for this metal is about one-third the value of n calculated in Example 27.12 for copper.

27.51. **IDENTIFY:** Use $\vec{F} = q\vec{v}\times\vec{B}$ to relate $\vec{v}$, $\vec{B}$, and $\vec{F}$.

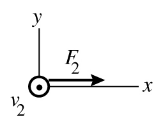
SET UP: The directions of $\vec{v}_1$ and $\vec{F}_1$ are shown in Figure 27.51a.



$\vec{F} = q\vec{v}\times\vec{B}$ says that $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$. The information given here means that $\vec{B}$ can have no z-component.

Figure 27.51a

The directions of $\vec{v}_2$ and $\vec{F}_2$ are shown in Figure 27.51b.



$\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$, so $\vec{B}$ can have no x-component.

Figure 27.51b

Both pieces of information taken together say that $\vec{B}$ is in the y-direction; $\vec{B} = B_y\hat{j}$.

EXECUTE: **(a)** Use the information given about $\vec{F}_2$ to calculate B_y: $\vec{F}_2 = F_2\hat{i}, \vec{v}_2 = v_2\hat{k}, \vec{B} = B_y\hat{j}$.

$\vec{F}_2 = q\vec{v}_2\times\vec{B}$ says $F_2\hat{i} = qv_2B_y\hat{k}\times\hat{j} = qv_2B_y(-\hat{i})$ and $F_2 = -qv_2B_y$.

$B_y = -F_2/(qv_2) = -F_2/(qv_1)$. $\vec{B}$ has the magnitude $F_2/(qv_1)$ and is in the $-y$-direction.

(b) $F_1 = qvB\sin\phi = qv_1\left|B_y\right|/\sqrt{2} = F_2/\sqrt{2}$.

EVALUATE: $v_1 = v_2$. $\vec{v}_2$ is perpendicular to $\vec{B}$ whereas only the component of $\vec{v}_1$ perpendicular to $\vec{B}$ contributes to the force, so it is expected that $F_2 > F_1$, as we found.

27.52. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $B_x = 0.650$ T. $B_y = 0$ and $B_z = 0$.

EXECUTE: $F_x = q(v_yB_z - v_zB_y) = 0$.

$F_y = q(v_zB_x - v_xB_z) = (7.26\times10^{-8}\text{ C})(5.85\times10^{4}\text{ m/s})(0.650\text{ T}) = 2.76\times10^{-3}$ N.

$F_z = q(v_xB_y - v_yB_x) = -(7.26\times10^{-8}\text{ C})(-3.11\times10^{4}\text{ m/s})(0.650\text{ T}) = 1.47\times10^{-3}$ N.

EVALUATE: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. We can verify that $\vec{F}\cdot\vec{v} = 0$. Since $\vec{B}$ is along the x-axis, v_x does not affect the force components.

27.53. **IDENTIFY:** In part (a), apply conservation of energy to the motion of the two nuclei. In part (b) apply $\left|q\right|vB = mv^2/R$.

SET UP: In part (a), let point 1 be when the two nuclei are far apart and let point 2 be when they are at their closest separation.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$. $U_1 = K_2 = 0$, so $K_1 = U_2$. There are two nuclei having equal kinetic energy, so $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = ke^2/r$. Solving for v gives

$$v = e\sqrt{\frac{k}{mr}} = (1.602\times10^{-19}\text{ C})\sqrt{\frac{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}{(3.34\times10^{-27}\text{ kg})(1.0\times10^{-15}\text{ m})}} = 8.3\times10^{6}\text{ m/s}.$$

(b) $\sum \vec{F} = m\vec{a}$ gives $qvB = mv^2/r$. $B = \dfrac{mv}{qr} = \dfrac{(3.34\times10^{-27}\text{ kg})(8.3\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(1.25\text{ m})} = 0.14\text{ T}.$

EVALUATE: The speed calculated in part (a) is large, nearly 3% of the speed of light.

27.54. IDENTIFY: The period is $T = 2\pi r/v$, the current is Q/t and the magnetic moment is $\mu = IA$.

SET UP: The electron has charge $-e$. The area enclosed by the orbit is πr^2.

EXECUTE: **(a)** $T = 2\pi r/v = 1.5\times10^{-16}$ s.

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1$ mA.

(c) $\mu = IA = I\pi r^2 = 9.3\times10^{-24}\ \text{A}\cdot\text{m}^2$.

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.55. IDENTIFY: The sum of the magnetic, electrical and gravitational forces must be zero to aim at and hit the target.

SET UP: The magnetic field must point to the left when viewed in the direction of the target for no net force. The net force is zero, so $\sum F = F_B - F_E - mg = 0$ and $qvB - qE - mg = 0$.

EXECUTE: Solving for B gives

$$B = \frac{qE + mg}{qv} = \frac{(2500\times10^{-6}\text{ C})(27.5\text{ N/C}) + (0.00425\text{ kg})(9.80\text{ m/s}^2)}{(2500\times10^{-6}\text{ C})(12.8\text{ m/s})} = 3.45\text{ T}.$$

The direction should be perpendicular to the initial velocity of the coin.

EVALUATE: This is a very strong magnetic field, but achievable in some labs.

27.56. IDENTIFY and SET UP: The maximum radius of the orbit determines the maximum speed v of the protons. Use Newton's second law and $a_{\text{rad}} = v^2/R$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$.

$v = \dfrac{|q|BR}{m} = \dfrac{(1.60\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{1.67\times10^{-27}\text{ kg}} = 3.257\times10^{7}$ m/s. The kinetic energy of a proton moving with this speed is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(3.257\times10^{7}\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$.

(b) The time for one revolution is the period $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(0.40\text{ m})}{3.257\times10^{7}\text{ m/s}} = 7.7\times10^{-8}$ s.

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2 = \frac{1}{2}\dfrac{|q|^2B^2R^2}{m}$. Or, $B = \dfrac{\sqrt{2Km}}{|q|R}$. B is proportional to $\sqrt{K}$, so if K is increased by a factor of 2 then B must be increased by a factor of $\sqrt{2}$. $B = \sqrt{2}(0.85\text{ T}) = 1.2\text{ T}$.

(d) $v = \dfrac{|q|BR}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{6.65\times10^{-27}\text{ kg}} = 1.636\times10^{7}$ m/s

$K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65\times10^{-27}\text{ kg})(1.636\times10^{7}\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$, the same as the maximum energy for protons.

EVALUATE: We can see that the maximum energy must be approximately the same as follows: From part (c), $K = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2$. For alpha particles $|q|$ is larger by a factor of 2 and m is larger by a factor of 4 (approximately). Thus $|q|^2/m$ is unchanged and K is the same.

27.57. IDENTIFY and SET UP: Use $\vec{F} = q\vec{v}\times\vec{B}$ to relate q, $\vec{v}$, $\vec{B}$ and $\vec{F}$. The force $\vec{F}$ and $\vec{a}$ are related by Newton's second law. $\vec{B} = -(0.120\text{ T})\hat{k}$, $\vec{v} = (1.05\times10^{6}\text{ m/s})(-3\hat{i} + 4\hat{j} + 12\hat{k})$, $F = 2.45$ N.

EXECUTE: **(a)** $\vec{F} = q\vec{v}\times\vec{B}$. $\vec{F} = q(-0.120\text{ T})(1.05\times10^{6}\text{ m/s})(-3\hat{i}\times\hat{k} + 4\hat{j}\times\hat{k} + 12\hat{k}\times\hat{k})$.

$\hat{\boldsymbol{i}}\times\hat{\boldsymbol{k}}=-\hat{\boldsymbol{j}},\ \hat{\boldsymbol{j}}\times\hat{\boldsymbol{k}}=\hat{\boldsymbol{i}},\ \hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}=0.$ $\vec{\boldsymbol{F}}=-q(1.26\times10^5\ \text{N/C})(+3\hat{\boldsymbol{j}}+4\hat{\boldsymbol{i}})=-q(1.26\times10^5\ \text{N/C})(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}}).$ The magnitude of the vector $+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}}$ is $\sqrt{3^2+4^2}=5.$ Thus $F=-q(1.26\times10^5\ \text{N/C})(5).$

$$q=-\frac{F}{5(1.26\times10^5\ \text{N/C})}=-\frac{2.45\ \text{N}}{5(1.26\times10^5\ \text{N/C})}=-3.89\times10^{-6}\ \text{C}.$$

(b) $\Sigma\vec{\boldsymbol{F}}=m\vec{\boldsymbol{a}}$ so $\vec{\boldsymbol{a}}=\vec{\boldsymbol{F}}/m.$

$\vec{\boldsymbol{F}}=-q(1.26\times10^5\ \text{N/C})(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=-(-3.89\times10^{-6}\ \text{C})(1.26\times10^5\ \text{N/C})(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=+0.490\ \text{N}(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}}).$

Then

$$\vec{\boldsymbol{a}}=\vec{\boldsymbol{F}}/m=\left(\frac{0.490\ \text{N}}{2.58\times10^{-15}\ \text{kg}}\right)(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=(1.90\times10^{14}\ \text{m/s}^2)(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=7.60\times10^{14}\ \text{m/s}^2\hat{\boldsymbol{i}}+5.70\times10^{14}\ \text{m/s}^2\hat{\boldsymbol{j}}.$$

(c) IDENTIFY and **SET UP:** $\vec{\boldsymbol{F}}$ is in the *xy*-plane, so in the *z*-direction the particle moves with constant speed 12.6×10^6 m/s. In the *xy*-plane the force $\vec{\boldsymbol{F}}$ causes the particle to move in a circle, with $\vec{\boldsymbol{F}}$ directed in toward the center of the circle.

EXECUTE: $\sum\vec{\boldsymbol{F}}=m\vec{\boldsymbol{a}}$ gives $F=m(v^2/R)$ and $R=mv^2/F.$

$v^2=v_x^2+v_y^2=(-3.15\times10^6\ \text{m/s})^2+(+4.20\times10^6\ \text{m/s})^2=2.756\times10^{13}\ \text{m}^2/\text{s}^2.$

$F=\sqrt{F_x^2+F_y^2}=(0.490\ \text{N})\sqrt{4^2+3^2}=2.45\ \text{N}.$

$$R=\frac{mv^2}{F}=\frac{(2.58\times10^{-15}\ \text{kg})(2.756\times10^{13}\ \text{m}^2/\text{s}^2)}{2.45\ \text{N}}=0.0290\ \text{m}=2.90\ \text{cm}.$$

(d) IDENTIFY and **SET UP:** The cyclotron frequency is $f=\omega/2\pi=v/2\pi R.$

EXECUTE: The circular motion is in the *xy*-plane, so $v=\sqrt{v_x^2+v_y^2}=5.25\times10^6$ m/s.

$$f=\frac{v}{2\pi R}=\frac{5.25\times10^6\ \text{m/s}}{2\pi(0.0290\ \text{m})}=2.88\times10^7\ \text{Hz, and }\ \omega=2\pi f=1.81\times10^8\ \text{rad/s}.$$

(e) IDENTIFY and **SET UP:** Compare *t* to the period *T* of the circular motion in the *xy*-plane to find the *x*- and *y*-coordinates at this *t*. In the *z*-direction the particle moves with constant speed, so $z=z_0+v_zt.$

EXECUTE: The period of the motion in the *xy*-plane is given by $T=\dfrac{1}{f}=\dfrac{1}{2.88\times10^7\ \text{Hz}}=3.47\times10^{-8}$ s. In $t=2T$ the particle has returned to the same *x*- and *y*-coordinates. The *z*-component of the motion is motion with a constant velocity of $v_z=+12.6\times10^6$ m/s. Thus

$z=z_0+v_zt=0+(12.6\times10^6\ \text{m/s})(2)(3.47\times10^{-8}\ \text{s})=+0.874$ m. The coordinates at $t=2T$ are $x=R=0.0290$ m, $y=0,$ $z=+0.874$ m.

EVALUATE: The circular motion is in the plane perpendicular to $\vec{\boldsymbol{B}}$. The radius of this motion gets smaller when *B* increases and it gets larger when *v* increases. There is no magnetic force in the direction of $\vec{\boldsymbol{B}}$ so the particle moves with constant velocity in that direction. The superposition of circular motion in the *xy*-plane and constant speed motion in the *z*-direction is a helical path.

27.58. IDENTIFY: Apply $\vec{\boldsymbol{F}}=q\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}.$

SET UP: $\vec{\boldsymbol{v}}=v\hat{\boldsymbol{k}}.$

EXECUTE: **(a)** $\vec{\boldsymbol{F}}=-qvB_y\hat{\boldsymbol{i}}+qvB_x\hat{\boldsymbol{j}}.$ But $\vec{\boldsymbol{F}}=3F_0\hat{\boldsymbol{i}}+4F_0\hat{\boldsymbol{j}},$ so $3F_0=-qvB_y$ and $4F_0=qvB_x.$

Therefore, $B_y=-\dfrac{3F_0}{qv},$ $B_x=\dfrac{4F_0}{qv}$ and B_z is undetermined.

(b) $B=\dfrac{6F_0}{qv}=\sqrt{B_x^2+B_y^2+B_z^2}=\dfrac{F_0}{qv}\sqrt{9+16+\left(\dfrac{qv}{F_0}\right)^2B_z^2}=\dfrac{F_0}{qv}\sqrt{25+\left(\dfrac{qv}{F_0}\right)^2B_z^2},$ so $B_z=\pm\dfrac{\sqrt{11}F_0}{qv}.$

EVALUATE: The force doesn't depend on $B_z,$ since $\vec{\boldsymbol{v}}$ is along the *z*-direction.

27.59. IDENTIFY: For the velocity selector, $E = vB$. For circular motion in the field B', $R = \dfrac{mv}{|q|B'}$.

SET UP: $B = B' = 0.682$ T.

EXECUTE: $v = \dfrac{E}{B} = \dfrac{1.88\times10^4 \text{ N/C}}{0.682 \text{ T}} = 2.757\times10^4$ m/s. $R = \dfrac{mv}{qB'}$, so

$$R_{82} = \frac{82(1.66\times10^{-27} \text{ kg})(2.757\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.682 \text{ T})} = 0.0344 \text{ m} = 3.44 \text{ cm}.$$

$$R_{84} = \frac{84(1.66\times10^{-27} \text{ kg})(2.757\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.682 \text{ T})} = 0.0352 \text{ m} = 3.52 \text{ cm}.$$

$$R_{86} = \frac{86(1.66\times10^{-27} \text{ kg})(2.757\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.682 \text{ T})} = 0.0361 \text{ m} = 3.61 \text{ cm}.$$

The distance between two adjacent lines is $2\Delta R = 2(3.52 \text{ cm} - 3.44 \text{ cm}) = 0.16 \text{ cm} = 1.6 \text{ mm}$.

EVALUATE: The distance between the ^{82}Kr line and the ^{84}Kr line is 1.6 mm and the distance between the ^{84}Kr line and the ^{86}Kr line is 1.6 mm. Adjacent lines are equally spaced since the ^{82}Kr versus ^{84}Kr and ^{84}Kr versus ^{86}Kr mass differences are the same.

27.60. IDENTIFY: Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1 \text{ u} = 1.66\times10^{-27}$ kg.

EXECUTE: **(a)** During acceleration of the ions, $qV = \tfrac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2qV}{m}}$. In the magnetic field,

$$R = \frac{mv}{qB} = \frac{m\sqrt{2qV/m}}{qB} \text{ and } m = \frac{qB^2R^2}{2V}.$$

(b) $V = \dfrac{qB^2R^2}{2m} = \dfrac{(1.60\times10^{-19} \text{ C})(0.150 \text{ T})^2(0.500 \text{ m})^2}{2(12)(1.66\times10^{-27} \text{ kg})} = 2.26\times10^4$ V.

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\dfrac{2Vm}{qB^2}}$, so

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\Bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\Bigg|_{12} = 2\sqrt{\frac{2V(1 \text{ u})}{qB^2}}(\sqrt{14} - \sqrt{12}).$$

$\Delta D = 2\sqrt{\dfrac{2(2.26\times10^4 \text{ V})(1.66\times10^{-27} \text{ kg})}{(1.6\times10^{-19} \text{ C})(0.150 \text{ T})^2}}(\sqrt{14} - \sqrt{12}) = 8.01\times10^{-2}$ m. This is about 8 cm and is easily distinguishable.

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\dfrac{2(1.60\times10^{-19} \text{ C})(2.26\times10^4 \text{ V})}{12(1.66\times10^{-27} \text{ kg})}} = 6.0\times10^5$ m/s. This is very fast, but well below the speed of light, so relativistic mechanics is not needed.

27.61. IDENTIFY: The force exerted by the magnetic field is given by $F = IlB\sin\phi$. The net force on the wire must be zero.

SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in the figure with the problem in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.61a.

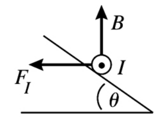
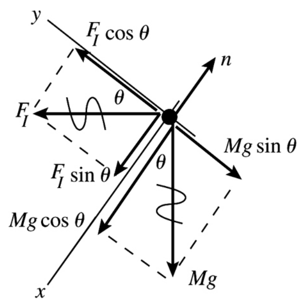


Figure 27.61a

The free-body diagram for the wire is given in Figure 27.61b.



EXECUTE: $\sum F_y = 0.$

$F_I \cos\theta - Mg\sin\theta = 0.$

$F_I = ILB\sin\phi.$

$\phi = 90°$ since $\vec{B}$ is perpendicular to the current direction.

Figure 27.61b

Thus $(ILB)\ \cos\theta - Mg\sin\theta = 0$ and $I = \dfrac{Mg\tan\theta}{LB}.$

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of *Mg* down the incline and the component of F_I up the incline is smaller; *I* must increase with θ to compensate. As $\theta \to 0$, $I \to 0$ and as $\theta \to 90°$, $I \to \infty$.

27.62. **IDENTIFY:** In the figure shown with the problem in the text, the current in the bar is toward the bottom of the page, so the magnetic force is toward the right. Newton's second law gives the acceleration. The bar is in parallel with the 10.0-Ω resistor, so we must use circuit analysis to find the initial current through the bar.

SET UP: First find the current. The equivalent resistance across the battery is $30.0\ \Omega$, so the total current is $4.00\,\text{A}$, half of which goes through the bar. Applying Newton's second law to the bar gives $\sum F = ma = F_B = ILB.$

EXECUTE: Equivalent resistance of the $10.0\text{-}\Omega$ resistor and the bar is $5.0\,\Omega$. Current through the $25.0\text{-}\Omega$ resistor is $I_{\text{tot}} = \dfrac{120.0\ \text{V}}{30.0\,\Omega} = 4.00\ \text{A}$. The current in the bar is 2.00 A, toward the bottom of the page. The force $\vec{F}_I$ that the magnetic field exerts on the bar has magnitude $F_I = IlB$ and is directed to the right. $a = \dfrac{F_I}{m} = \dfrac{IlB}{m} = \dfrac{(2.00\ \text{A})(0.850\ \text{m})(1.60\ \text{T})}{(2.60\ \text{N})/(9.80\ \text{m/s}^2)} = 10.3\ \text{m/s}^2$. $\vec{a}$ is directed to the right.

EVALUATE: Once the bar has acquired a non-zero speed there will be an induced emf (Chapter 29) and the current and acceleration will start to decrease.

27.63. **IDENTIFY:** $R = \dfrac{mv}{|q|B}.$

SET UP: After completing one semicircle the separation between the ions is the difference in the diameters of their paths, or $2(R_{13} - R_{12})$. A singly ionized ion has charge $+e$.

EXECUTE: **(a)** $B = \dfrac{mv}{|q|R} = \dfrac{(1.99\times10^{-26}\ \text{kg})(8.50\times10^{3}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.125\ \text{m})} = 8.46\times10^{-3}\ \text{T}.$

(b) The only difference between the two isotopes is their masses. $\dfrac{R}{m} = \dfrac{v}{|q|B} = \text{constant}$ and $\dfrac{R_{12}}{m_{12}} = \dfrac{R_{13}}{m_{13}}.$

$R_{13} = R_{12}\left(\dfrac{m_{13}}{m_{12}}\right) = (12.5\text{ cm})\left(\dfrac{2.16\times10^{-26}\text{ kg}}{1.99\times10^{-26}\text{ kg}}\right) = 13.6\text{ cm}.$ The diameter is 27.2 cm.

(c) The separation is $2(R_{13} - R_{12}) = 2(13.6\text{ cm} - 12.5\text{ cm}) = 2.2\text{ cm}.$ This distance can be easily observed.

EVALUATE: Decreasing the magnetic field increases the separation between the two isotopes at the detector.

27.64. IDENTIFY: Turning the charged loop creates a current, and the external magnetic field exerts a torque on that current.

SET UP: The current is $I = q/T = q/(1/f) = qf = q(\omega/2\pi) = q\omega/2\pi.$ The torque is $\tau = \mu B\sin\phi.$

EXECUTE: In this case, $\phi = 90°$ and $\mu = IA,$ giving $\tau = IAB.$ Combining the results for the torque and current and using $A = \pi r^2$ gives $\tau = \left(\dfrac{q\omega}{2\pi}\right)\pi r^2 B = \frac{1}{2}q\omega r^2 B.$

EVALUATE: Any moving charge is a current, so turning the loop creates a current causing a magnetic force.

27.65. IDENTIFY: The force exerted by the magnetic field is $F = ILB\sin\phi.$ $a = F/m$ and is constant. Apply a constant acceleration equation to relate v and d.

SET UP: $\phi = 90°.$ The direction of $\vec{F}$ is given by the right-hand rule.

EXECUTE: (a) $F = ILB,$ to the right.

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v^2 = 2ad$ and $d = \dfrac{v^2}{2a} = \dfrac{v^2 m}{2ILB}.$

(c) $d = \dfrac{(1.12\times10^4\text{ m/s})^2(25\text{ kg})}{2(2000\text{ A})(0.50\text{ m})(0.80\text{ T})} = 1.96\times10^6\text{ m} = 1960\text{ km}.$

EVALUATE: $a = \dfrac{ILB}{m} = \dfrac{(2.0\times10^3\text{ A})(0.50\text{ m})(0.80\text{ T})}{25\text{ kg}} = 32\text{ m/s}^2.$ The acceleration due to gravity is not negligible. Since the bar would have to travel nearly 2000 km, this would not be a very effective launch mechanism using the numbers given.

27.66. IDENTIFY: Apply $\vec{F} = I\vec{l}\times\vec{B}.$

SET UP: $\vec{l} = l\hat{k}.$

EXECUTE: (a) $\vec{F} = I(l\hat{k})\times\vec{B} = Il[(-B_y)\hat{i} + (B_x)\hat{j}].$ This gives

$F_x = -IlB_y = -(7.40\text{ A})(0.250\text{ m})(-0.985\text{ T}) = 1.82\text{ N}$ and

$F_y = IlB_x = (7.40\text{ A})(0.250\text{ m})(-0.242\text{ T}) = -0.448\text{ N}.$ $F_z = 0,$ since the wire is in the z-direction.

(b) $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.82\text{ N})^2 + (0.448\text{ N})^2} = 1.88\text{ N}.$

EVALUATE: $\vec{F}$ must be perpendicular to the current direction, so $\vec{F}$ has no z-component.

27.67. IDENTIFY: The magnetic field exerts a force on each of the three segments of the wire due to the current in them. The net force on the wire is the vector sum of these three forces.

SET UP: Label the three segments in the magnetic field 1, 2, and 3, as shown in Figure 27.67. The force on a current carrying conductor is $F = IlB\sin\phi,$ where ϕ is the angle between the direction of the current and the direction of the magnetic field. The direction of the force on each segment is given by the right-hand rule and is shown in the figure. The sum of $\vec{F}_1$ and $\vec{F}_3$ is the same as the force $\vec{F}_{13}$ on a wire 0.307 m long. Section 2 has length 0.800 m. The current in each segment is perpendicular to the magnetic field, so $\phi = 90°.$

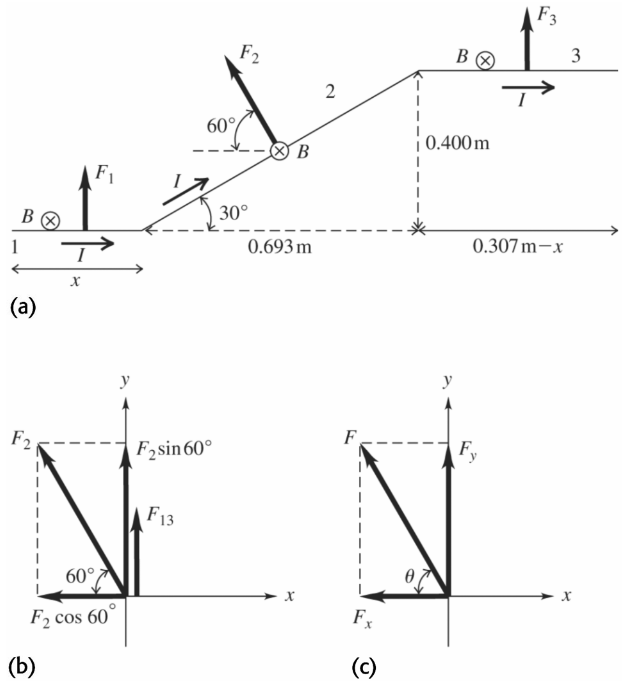


Figure 27.67

EXECUTE: $F_{13} = IlB\sin\phi = (6.50\text{ A})(0.307\text{ m})(0.280\text{ T})\sin 90° = 0.559\text{ N}.$
$F_2 = IlB\sin\phi = (6.50\text{ A})(0.800\text{ m})(0.280\text{ T})\sin 90° = 1.46\text{ N}.$ The forces and a coordinate system are shown in Figure 27.67b. $\vec{F}_2$ has been resolved into its x- and y-components.

$$F_x = F_{2x} + F_{13x} = -F_2\cos 60.0° = -(1.46\text{ N})(\cos 60.0°) = -0.730\text{ N}.$$

$$F_y = F_{2y} + F_{13y} = F_2\sin 60.0° + F_{13} = +(1.46\text{ N})(\sin 60.0°) + 0.559\text{ N} = +1.83\text{ N}.$$

F_x, F_y, and the resultant total force $\vec{F}$ are shown in Figure 27.67c. The resultant force has magnitude 1.97 N and is at $68.3°$ clockwise from the left-hand straight segment.

EVALUATE: Even though all three segments are perpendicular to the magnetic field, the direction of the force on the segments is not the same. Therefore we must use vector addition to find the force on the wire.

27.68. **IDENTIFY:** The torque exerted by the magnetic field is $\vec{\tau} = \vec{\mu} \times \vec{B}$. The torque required to hold the loop in place is $-\vec{\tau}$.

SET UP: $\mu = IA$. $\vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau = IAB\sin\phi$, where ϕ is the angle between the normal to the loop and the direction of $\vec{B}$.

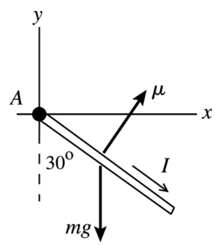
EXECUTE: **(a)** $\tau = IAB\sin 60° = (15.0\text{ A})(0.060\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 60° = 0.030\text{ N}\cdot\text{m}$, in the $-\hat{\boldsymbol{j}}$-direction. To keep the loop in place, you must provide a torque in the $+\hat{\boldsymbol{j}}$-direction.

(b) $\tau = IAB\sin 30° = (15.0\text{ A})(0.60\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 30° = 0.017\text{ N}\cdot\text{m}$, in the $+\hat{\boldsymbol{j}}$-direction. You must provide a torque in the $-\hat{\boldsymbol{j}}$-direction to keep the loop in place.

EVALUATE: **(c)** If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.69. **IDENTIFY:** For the loop to be in equilibrium the net torque on it must be zero. Use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to calculate the torque due to the magnetic field and $\tau_{mg} = mgr\sin\phi$ for the torque due to gravity.

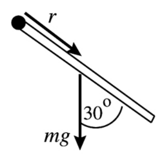
SET UP: See Figure 27.69a (next page).



Use $\sum \tau_A = 0$, where point A is at the origin.

Figure 27.69a

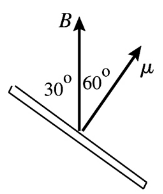
EXECUTE: See Figure 27.69b.



$\tau_{mg} = mgr\sin\phi = mg(0.400\text{ m})\sin 30.0°.$ The torque is clockwise; $\vec{\tau}_{mg}$ is directed into the paper.

Figure 27.69b

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\tau}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.69c.



$\vec{\tau}_B = \vec{\mu} \times \vec{B}$. For this torque to be counter-clockwise ($\vec{\tau}_B$ directed out of the paper), $\vec{B}$ must be in the $+y$-direction.

Figure 27.69c

$\tau_B = \mu B\sin\phi = IAB\sin 60.0°.$

$\tau_B = \tau_{mg}$ gives $IAB\sin 60.0° = mg(0.0400\text{ m})\sin 30.0°.$

$m = (0.15\text{ g/cm})2(8.00\text{ cm} + 6.00\text{ cm}) = 4.2\text{ g} = 4.2\times 10^{-3}\text{ kg}.$

$A = (0.0800\text{ m})(0.0600\text{ m}) = 4.80\times 10^{-3}\text{ m}^2.$

$$B = \frac{mg(0.0400\text{ m})(\sin 30.0°)}{IA\sin 60.0°}.$$

$$B = \frac{(4.2\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.0400\text{ m})\sin 30.0°}{(8.2\text{ A})(4.80\times 10^{-3}\text{ m}^2)\sin 60.0°} = 0.024\text{ T}.$$

EVALUATE: As the loop swings up the torque due to $\vec{B}$ decreases to zero and the torque due to mg increases from zero, so there must be an orientation of the loop where the net torque is zero.

27.70. **IDENTIFY** and **SET UP:** The force on a current-carrying bar of length l is $F = IlB$ if the field is perpendicular to the bar. The torque is $\tau_z = \mu B\sin\phi$.

EXECUTE: **(a)** The force on the infinitesimal segment is $dF = IBdl = IBdx$. The torque about point a is $d\tau_z = xdF\sin\phi = xIBdx$. In this case, $\sin\phi = 1$ because the force is perpendicular to the bar.

(b) We integrate to get the total torque: $\tau_z = \int_0^L xIBdx = \frac{1}{2}IBL^2$.

(c) For $F = ILB$ at the center of the bar, the torque is $\tau_z = F\left(\frac{L}{2}\right) = ILB\left(\frac{L}{2}\right) = \frac{1}{2}IBL^2$, which is the same result we got by integrating.

EVALUATE: We can think of the magnetic force as all acting at the center of the bar because the magnetic field is uniform. This is the same reason we can think of gravity acting at the center of a uniform bar.

27.71. IDENTIFY: Apply $\vec{F} = I\vec{l} \times \vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: **(a)** $F_{PQ} = (5.00\text{ A})(0.600\text{ m})(3.00\text{ T})\sin(0°) = 0\text{ N}.$

$F_{RP} = (5.00\text{ A})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 12.0\text{ N},$ into the page.

$F_{QR} = (5.00\text{ A})(1.00\text{ m})(3.00\text{ T})(0.800/1.00) = 12.0\text{ N},$ out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the *PR*-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the *x*-axis. $\tau = rF\sin\phi$ gives $\tau_{PQ} = r(0\text{ N}) = 0,$ $\tau_{RP} = (0\text{ m})F\sin\phi = 0$ and $\tau_{QR} = (0.300\text{ m})(12.0\text{ N})\sin(90°) = 3.60\text{ N}\cdot\text{m}.$ The net torque is $3.60\text{ N}\cdot\text{m}.$

(d) Using $\tau = NIAB\sin\phi$ gives

$\tau = NIAB\sin\phi = (1)(5.00\text{ A})\left(\frac{1}{2}\right)(0.600\text{ m})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 3.60\text{ N}\cdot\text{m},$ which agrees with our result in part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point *Q* will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB\sin\phi,$ ϕ is the angle between the plane of the loop and the direction of $\vec{B}$. In this problem, $\phi = 90°.$

27.72. IDENTIFY: For rotational equilibrium, the torques due to gravity and the magnetic field must balance around point *a*.

SET UP: From Problem 27.70 we have $\tau_z = \frac{1}{2}IBL^2.$

EXECUTE: **(a)** Balancing the two torques gives: $mg\frac{L}{2}\cos\theta = \frac{1}{2}IBL^2.$ Simplifying gives $ILB = mg\cos\theta.$

Putting in the numbers gives

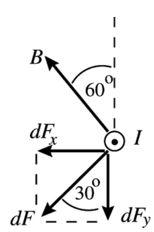
$I(0.150\text{ T})(0.300\text{ m}) = (0.0120\text{ kg})(9.80\text{ m/s}^2)\cos(30.0°)$, so $I = 2.26$ A.

(b) Gravity tends to rotate the bar clockwise about point *a*, so the magnetic force must be upward and to the left to tend to rotate the bar clockwise. Therefore the current must flow from *a* to *b*.

EVALUATE: If the current were from *b* to *a*, the bar could not balance.

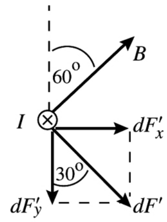
27.73. IDENTIFY: Use $dF = Idl\,B\sin\phi$ to calculate the force on a short segment of the coil and integrate over the entire coil to find the total force.

SET UP: See Figures 27.73a and 27.73b. The two sketches show that the *x*-components cancel and that the *y*-components add. This is true for all pairs of short segments on opposite sides of the coil. The net magnetic force on the coil is in the *y*-direction and its magnitude is given by $F = \int dF_y.$



Consider the force $d\vec{F}$ on a short segment *dl* at the left-hand side of the coil, as viewed in the figure with the problem in the textbook. The current at this point is directed out of the page. $d\vec{F}$ is perpendicular both to $\vec{B}$ and to the direction of *I*.

Figure 27.73a



Consider also the force $d\vec{F}'$ on a short segment on the opposite side of the coil, at the right-hand side of the coil in the figure with the problem in the textbook. The current at this point is directed into the page.

Figure 27.73b

EXECUTE: $dF = Idl\,B\sin\phi.$ But $\vec{B}$ is perpendicular to the current direction so $\phi = 90°.$

$dF_y = dF\cos 30.0 = IB\cos 30.0° dl.$

$F = \int dF_y = IB\cos 30.0° \int dl.$

But $\int dl = N(2\pi r),$ the total length of wire in the coil.

$F = IB\cos 30.0° N(2\pi r) = (0.950 \text{ A})(0.220 \text{ T})(\cos 30.0°)(50)2\pi(0.0078 \text{ m}) = 0.444 \text{ N}$ and $\vec{F} = -(0.444 \text{ N})\hat{j}$

EVALUATE: The magnetic field makes a constant angle with the plane of the coil but has a different direction at different points around the circumference of the coil so is not uniform. The net force is proportional to the magnitude of the current and reverses direction when the current reverses direction.

27.74. **IDENTIFY** and **SET UP:** The rod is in rotational equilibrium, so the torques must balance. Take torques about point P and use $\tau_z = \frac{1}{2}IBL^2$ from Problem 27.70.

EXECUTE: Balancing torques gives $mg\frac{L}{2}\cos\theta + \frac{1}{2}IBL^2 = T\sin\theta L,$ where L is the length of the bar and T is the tension in the string. Solving for T and putting in the numbers gives
$T = [(0.0840 \text{ kg})(9.80 \text{ m/s}^2)\cos(53.0°) + (12.0 \text{ A})(0.120 \text{ T})(0.180 \text{ m})]/[2\sin(53.0°)] = 0.472 \text{ N}.$

EVALUATE: If the current were reversed, the tension would be less than 0.472 N.

27.75. **IDENTIFY:** Apply $d\vec{F} = Id\vec{l} \times \vec{B}$ to each side of the loop.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I. Since the loop is in the xy-plane, $z = 0$ at the loop and $B_y = 0$ at the loop.

EXECUTE: **(a)** The magnetic field lines in the yz-plane are sketched in Figure 27.75.

(b) Side 1, that runs from (0,0) to (0,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y\,dy}{L}\hat{i} = \frac{1}{2}B_0 LI\hat{i}.$

Side 2, that runs from (0,L) to (L,L): $\vec{F} = \int_{0,y=L}^{L} Id\vec{l} \times \vec{B} = I\int_{0,y=L}^{L} \frac{B_0 y\,dx}{L}\hat{j} = -IB_0 L\hat{j}.$

Side 3, that runs from (L,L) to (L,0): $\vec{F} = \int_{L,x=L}^{0} Id\vec{l} \times \vec{B} = I\int_{L,x=L}^{0} \frac{B_0 y\,dy}{L}(-\hat{i}) = -\frac{1}{2}IB_0 L\hat{i}.$

Side 4, that runs from (L,0) to (0,0): $\vec{F} = \int_{L,y=0}^{0} Id\vec{l} \times \vec{B} = I\int_{L,y=0}^{0} \frac{B_0 y\,dx}{L}\hat{j} = 0.$

(c) The sum of all forces is $\vec{F}_{\text{total}} = -IB_0 L\hat{j}.$

EVALUATE: The net force on sides 1 and 3 is zero. The force on side 4 is zero, since $y = 0$ and $z = 0$ at that side and therefore $B = 0$ there. The net force on the loop equals the force on side 2.

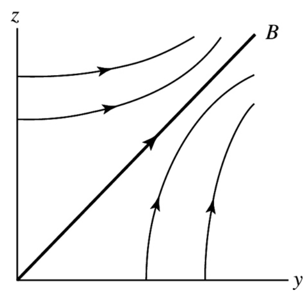


Figure 27.75

27.76. IDENTIFY: $I = \dfrac{\Delta q}{\Delta t}$ and $\mu = IA$.

SET UP: The direction of $\vec{\mu}$ is given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. I is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

EXECUTE: **(a)** $I_u = \dfrac{dq}{dt} = \dfrac{\Delta q}{\Delta t} = \dfrac{q_u v}{2\pi r} = \dfrac{ev}{3\pi r}$.

(b) $\mu_u = I_u A = \dfrac{ev}{3\pi r}\pi r^2 = \dfrac{evr}{3}$.

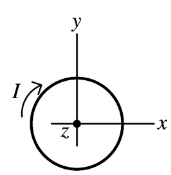
(c) Since there are two down quarks, each of half the charge of the up quark, $\mu_d = \mu_u = \dfrac{evr}{3}$. Therefore, $\mu_{\text{total}} = \dfrac{2evr}{3}$.

(d) $v = \dfrac{3\mu}{2er} = \dfrac{3(9.66\times10^{-27}\ \text{A}\cdot\text{m}^2)}{2(1.60\times10^{-19}\ \text{C})(1.20\times10^{-15}\text{m})} = 7.55\times10^7$ m/s.

EVALUATE: The speed calculated in part (d) is 25% of the speed of light.

27.77. IDENTIFY: Use $U = -\vec{\mu}\cdot\vec{B}$ to relate $U, \mu,$ and $\vec{B}$ and use $\vec{\tau} = \vec{\mu}\times\vec{B}$ to relate $\vec{\tau}, \vec{\mu},$ and $\vec{B}$. We also know that $B_0^2 = B_x^2 + B_y^2 + B_z^2$. This gives three equations for the three components of $\vec{B}$.

SET UP: The loop and current are shown in Figure 27.77.



$\vec{\mu}$ is into the plane of the paper, in the $-z$-direction.

Figure 27.77

EXECUTE: **(a)** $\vec{\mu} = -\mu\hat{k} = -IA\hat{k}$.

(b) $\vec{\tau} = D(+4\hat{i} - 3\hat{j})$, where $D > 0$.

$\vec{\mu} = -IA\hat{k}, \vec{B} = B_x\hat{i} + B_y\hat{j} + B_y\hat{k}$.

$\vec{\tau} = \vec{\mu}\times\vec{B} = (-IA)(B_x\hat{k}\times\hat{i} + B_y\hat{k}\times\hat{j} + B_z\hat{k}\times\hat{k}) = IAB_y\hat{i} - IAB_x\hat{j}$.

Compare this to the expression given for $\vec{\tau}$: $IAB_y = 4D$ so $B_y = 4D/IA$ and $-IAB_x = -3D$ so $B_x = 3D/IA$.

B_z doesn't contribute to the torque since $\vec{\mu}$ is along the z-direction. But $B = B_0$ and $B_x^2 + B_y^2 + B_z^2 = B_0^2$; with $B_0 = 13D/IA$. Thus $B_z = \pm\sqrt{B_0^2 - B_x^2 - B_y^2} = \pm(D/IA)\sqrt{169 - 9 - 16} = \pm12(D/IA)$.

That $U = -\vec{\mu}\,.\,\vec{B}$ is negative determines the sign of B_z: $U = -\vec{\mu}\cdot\vec{B} = -(-IA\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = +IAB_z$.

So U negative says that B_z is negative, and thus $B_z = -12D/IA$.

EVALUATE: $\vec{\mu}$ is along the z-axis so only B_x and B_y contribute to the torque. B_x produces a y-component of $\vec{\tau}$ and B_y produces an x-component of $\vec{\tau}$. Only B_z affects U, and U is negative when $\vec{\mu}$ and $\vec{B}_z$ are parallel.

27.78. **IDENTIFY:** The ions are accelerated from rest. When they enter the magnetic field, they are bent into a circular path. Newton's second law applies to the ions in the magnetic field.

SET UP: $K = \frac{1}{2}mv^2 = qV$. $R = \dfrac{mv}{qB}$, where q is the magnitude of the charge.

EXECUTE: **(a)** As the ions are accelerated through the potential difference V, we have $K = \frac{1}{2}mv^2 = qV$, which gives $v = \sqrt{\dfrac{2qV}{m}}$. In the magnetic field, $R = \dfrac{mv}{qB}$. Using the v we just found gives $R = \dfrac{mv}{qB} = \dfrac{m}{qB}\sqrt{\dfrac{2qV}{m}} = \sqrt{\dfrac{m}{q}}\dfrac{\sqrt{2V}}{B} = \dfrac{1}{B}\sqrt{\dfrac{2m}{q}}\sqrt{V}$. From this result we see that a graph of R versus $\sqrt{V}$ should be a straight line with a slope equal to $\dfrac{1}{B}\sqrt{\dfrac{2m}{q}}$.

(b) The graph of R versus $\sqrt{V}$ is shown in Figure 27.78. The slope of the best-fit line is $(6.355\text{ cm})/\sqrt{\text{kV}} = (0.06355\text{ m})/\sqrt{1000\text{ V}} = 0.00201\text{ m}\cdot\text{V}^{-1/2}$. We know that $\dfrac{1}{B}\sqrt{\dfrac{2m}{q}} = \text{slope}$, so

$$\frac{q}{m} = \frac{2}{[B(\text{slope})]^2} = \frac{2}{[(0.250\text{ T})(0.00201\text{ m}\cdot\text{V}^{-1/2}]^2} = 7.924\times10^{-6}\text{ C/kg},$$ which rounds to 7.92×10^{6} C/kg.

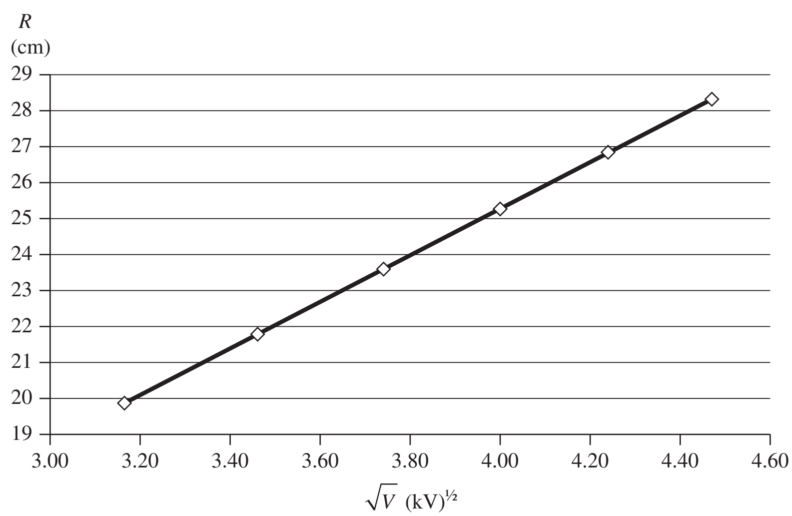


Figure 27.78

(c) Use our result for q/m: $v = \sqrt{\dfrac{2qV}{m}} = \sqrt{2(20.0\times10^{3}\text{ V})(7.924\times10^{6}\text{ C/kg})} = 5.63\times10^{5}\text{ m/s}$.

(d) Since $R = \dfrac{1}{B}\sqrt{\dfrac{2m}{q}}\sqrt{V}$, doubling q means that R is smaller by a factor of $\sqrt{2}$. Therefore $R = (21.1\text{ cm})/\sqrt{2} = 15.0\text{ cm}$.

EVALUATE: Besides the approach we have taken, the equation $R = \frac{1}{B}\sqrt{\frac{2m}{q}}\sqrt{V}$ can be graphed in other ways to obtain a straight line. For example, we could graph R^2 versus V, or even log$R$ versus log$V$. Ideally they should all give the same result for q/m. But differences can arise because we are dealing with less-than-ideal data points.

27.79. IDENTIFY and SET UP: The analysis in the text of the Thomson e/m experiment gives $\frac{e}{m} = \frac{E^2}{2VB^2}$. For a particle of charge e and mass m accelered through a potential V, $eV = \frac{1}{2}mv^2$.

EXECUTE: **(a)** Solving the equation $\frac{e}{m} = \frac{E^2}{2VB^2}$ for E^2 gives $E^2 = 2\left(\frac{e}{m}\right)B^2V$. Therefore a graph of E^2 versus V should be a straight line with slope equal to $2(e/m)B^2$.

(b) We can find the slope using two easily-read points on the graph. Using (100, 200) and (300, 600), we get $\frac{600\times10^8\ \text{V}^2/\text{m}^2 - 200\times10^8\ \text{V}^2/\text{m}^2}{300\ \text{V} - 100\ \text{V}} = 2.00\times10^8\ \text{V/m}^2$ for the slope. This gives

$e/m = (\text{slope})/2B^2 = (2.00\times10^8\ \text{V/m}^2)/[2(0.340\ \text{T})^2] = 8.65\times10^8\ \text{C/kg}$, which gives $m = 1.85\times10^{-28}$ kg.

(c) $V = Ed = (2.00\times10^5\ \text{V/m})(0.00600\ \text{m}) = 1.20$ kV.

(d) Using $eV = \frac{1}{2}mv^2$ to find the muon speed gives

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{2(8.65\times10^8\ \text{C/kg})(400\ \text{V})} = 8.32\times10^5\ \text{m/s}.$$

EVALUATE: Results may vary due to inaccuracies in determining the slope of the graph.

27.80. IDENTIFY and SET UP: If q is the magnitude of the charge, the cyclotron frequency is $\omega = \frac{qB}{m}$, where $\omega = 2\pi f$, and $R = mv/qB$.

EXECUTE: **(a)** Combining $\omega = \frac{qB}{m}$ and $\omega = 2\pi f$ gives $f = \left(\frac{1}{2\pi}\frac{q}{m}\right)B$. Therefore a graph of f versus B should be a straight line having slope equal to $q/2\pi m = (2e)/2\pi m = e/\pi m$. Solving for m gives $m = \frac{e}{\pi(\text{slope})}$. We use two points on the graph to calculate the slope, giving 7.667×10^6 Hz/T. Therefore

$$m = \frac{e}{\pi(\text{slope})} = e/[\pi(7.667\times10^6\ \text{Hz/T})] = 6.65\times10^{-27}\ \text{kg}.$$

(b) Apply $f = \left(\frac{1}{2\pi}\frac{q}{m}\right)B = qB/2\pi m$ to the electron and the proton.

Electron: $f_\text{e} = (1.602\times10^{-19}\ \text{C})(0.300\ \text{T})/[2\pi(9.11\times10^{-31}\ \text{kg})] = 8.40\times10^9\ \text{Hz} = 8.40\ \text{GHz}.$

Proton: $f_\text{p} = (1.602\times10^{-19}\ \text{C})(0.300\ \text{T})/[2\pi(1.67\times10^{-27}\ \text{kg})] = 4.58\times10^6\ \text{Hz} = 4.58\ \text{MHz}.$

For an alpha particle, $q = 2e$ and $m \approx 4m_\text{p}$, so q/m for an alpha particle is $(2e)/(4m_\text{p}) = \frac{1}{2}$ of what it is for a proton. Therefore $f_\alpha = \frac{1}{2}f_\text{p} = 2.3$ MHz.

For an alpha particle, $q = 2e$ and $m = 4(1836)m_\text{e}$, so q/m for an alpha particle is $2/[4(1836)] = 1/[2(1836)]$ what it is for an electron. Therefore $f_\alpha = \frac{1}{2(1836)}f_\text{e} = \frac{1}{3672}f_\text{e} = 2.3$ MHz.

(c) $R = mv/qB$ gives $v = RqB/m = (0.120\ \text{m})(3.2\times10^{-19}\ \text{C})(0.300\ \text{T})/(6.65\times10^{-27}\ \text{kg}) = 1.73\times10^6$ m/s.

$K = \frac{1}{2}mv^2 = (1/2)(6.65\times10^{-27}\ \text{kg})(1.73\times10^6\ \text{m/s})^2 = 1.0\times10^{-14}\ \text{J} = 6.25\times10^5\ \text{eV} = 625\ \text{keV} = 0.625\ \text{MeV}.$

EVALUATE: We could use $v = R\omega$ to find v in part (c), where $\omega = 2\pi f$.

27.81. **IDENTIFY** and **SET UP:** In the magnetic field, $R = \dfrac{mv}{qB}$. Once the particle exits the field it travels in a straight line. Throughout the motion the speed of the particle is constant.

EXECUTE: **(a)** $R = \dfrac{mv}{qB} = \dfrac{(3.20\times10^{-11}\text{ kg})(1.45\times10^{5}\text{ m/s})}{(2.15\times10^{-6}\text{ C})(0.420\text{ T})} = 5.14\text{ m}.$

(b) See Figure 27.81. The distance along the curve, d, is given by $d = R\theta$. $\sin\theta = \dfrac{0.25\text{ m}}{5.14\text{ m}}$, so $\theta = 2.79° = 0.0486\text{ rad}$. $d = R\theta = (5.14\text{ m})(0.0486\text{ rad}) = 0.25\text{ m}$. And

$t = \dfrac{d}{v} = \dfrac{0.25\text{ m}}{1.45\times10^{5}\text{ m/s}} = 1.72\times10^{-6}\text{s}.$

(c) $\Delta x_1 = d\tan(\theta/2) = (0.25\text{ m})\tan(2.79°/2) = 6.08\times10^{-3}\text{ m}.$

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where Δx_2 is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50\text{ m})\tan 2.79° = 0.0244\text{ m}$. Therefore,

$\Delta x = 6.08\times10^{-3}\text{ m} + 0.0244\text{ m} = 0.0305\text{ m}.$

EVALUATE: d is much less than R, so the horizontal deflection of the particle is much smaller than the distance it travels in the y-direction.

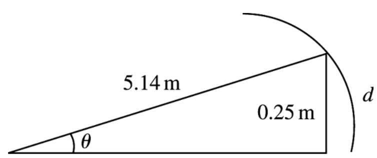


Figure 27.81

27.82. **IDENTIFY:** The electric and magnetic fields exert forces on the moving charge. The work done by the electric field equals the change in kinetic energy. At the top point, $a_y = \dfrac{v^2}{R}$ and this acceleration must correspond to the net force.

SET UP: The electric field is uniform so the work it does for a displacement y in the y-direction is $W = Fy = qEy$. At the top point, $\vec{F}_B$ is in the $-y$-direction and $\vec{F}_E$ is in the $+y$-direction.

EXECUTE: **(a)** The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W = qEy = \dfrac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2qEy}{m}}$.

(c) At the top, $F_y = qE - qvB = -\dfrac{mv^2}{R} = -\dfrac{m}{2y}\dfrac{2qEy}{m} = -qE$. $2qE = qvB$ and $v = \dfrac{2E}{B}$.

EVALUATE: The speed at the top depends on B because B determines the y-displacement and the work done by the electric force depends on the y-displacement.

27.83. **IDENTIFY** and **SET UP:** The torque on a magnetic moment is $\tau = \mu B\sin\phi$.

EXECUTE: $\tau = \mu B\sin\phi = (1.4\times10^{-26}\text{ J/T})(2\text{ T})(\sin 90°) = 2.8\times10^{-26}\text{ N}\cdot\text{m}$, which is choice (c).

EVALUATE: The value we have found is the maximum torque. It could be less, depending on the orientation of the proton relative to the magnetic field.

27.84. **IDENTIFY** and **SET UP:** For the nucleus to have a net magnetic moment, it must have an odd number of protons and neutrons.

EXECUTE: Only $^{31}P_{15}$ has an odd number of protons and neutrons, so choice (d) is correct.

EVALUATE: All the other choices have an even number of protons and an even number of neutrons.

27.85. **IDENTIFY** and **SET UP:** Model the nerve as a current-carrying bar in a magnetic field. The resistance of the nerve is $R = \frac{\rho L}{A}$, the current through it is $I = V/R$ (by Ohm's law), and the maximum magnetic force on it is $F = ILB$.

EXECUTE: The resistance is $R = \frac{\rho L}{A} = (0.6\ \Omega \cdot \text{m})(0.001\ \text{m})/[\pi(0.0015/2\ \text{m})^2] = 340\ \Omega.$

The current is $I = V/R = (0.1\ \text{V})/(340\ \Omega) = 2.9\times10^{-4}\ \text{A}$.

The maximum force is $F = ILB = (2.9\times10^{-4}\ \text{A})(0.001\ \text{m})(2\ \text{T}) = 5.9\times10^{-7}\ \text{N} \approx 6\times10^{-7}\ \text{N},$ which is choice (a).

EVALUATE: This is the force on a 1-mm segment of nerve. The force on the entire nerve would be somewhat larger, depending on the length of the nerve.