Vp250 Problem Set 11

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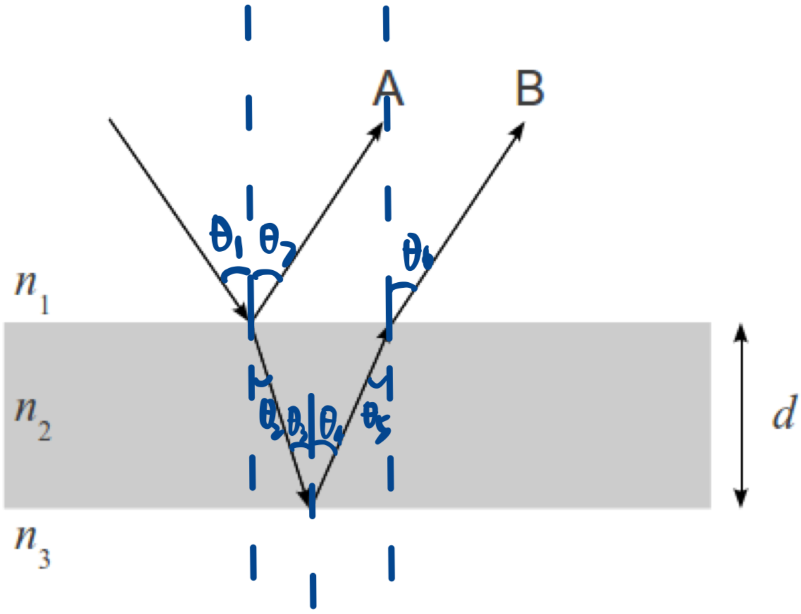
Problem 1

Label all the angle in the graph



We want to prove $\theta_7 = \theta_6$.
By reflection, $\theta_1 = \theta_2$ and $\theta_3 = \theta_4$.
By the theorem of parallel lines, $\theta_2 = \theta_3$ and $\theta_4 = \theta_5$. So we can see that $\theta_2 = \theta_5$.
By law of refraction, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{\sin\theta_6}{\sin\theta_5}$.
So we can conclude that $\theta_1 = \theta_6$, which implies $\theta_6 = \theta_7$. So A and B are parallel.

Problem 2

(a) At least two.

(b) Using n sheets, the maximum light intensity can only bu obtained when the angles between the polarization axis of the sheet and the polarized axis of light are equal. So

$$\cos^{2n}\frac{90^\circ}{n} \geqslant 0.6 \Rightarrow n \geqslant 5$$

Hence, it is at least 5 sheets.

Problem 3

(a) Suppose there are two waves E_1 and E_2 and the phase lag of wave E_2 is ϕ. So

$$E_1 = E\cos(kx - \omega t) \qquad E_2 = 2E\cos(kx - \omega t - \phi)$$

The combined light is

$$\begin{aligned} E = E_1 + E_2 &= E\cos(kx-\omega t) + 2E(\cos(kx-\omega t)\cos\phi + \sin(kx-\omega t)\sin\phi) \\ &= (E + 2E\cos\phi)\cos(kx-\omega t) + 2E\sin\phi\sin(kx-\omega t) \\ &= \sqrt{(E+2E\cos\phi)^2 + (2E\sin\phi)^2}\cos(kx-\omega t+\varphi) \end{aligned}$$

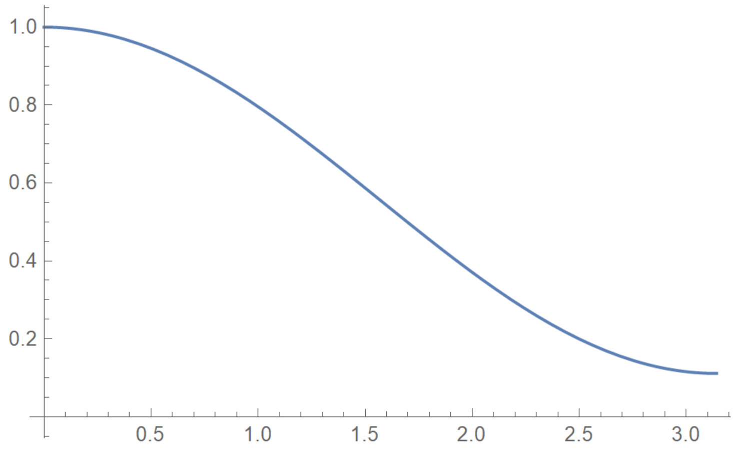
Then the intensity can be represented as

$$I = \frac{(E + 2E\cos\phi)^2 + (2E\sin\phi)^2}{2\mu_0 c} = \frac{5E^2 + 4E^2\cos\phi}{2\mu_0 c} \Rightarrow I_{max} = \frac{9E^2}{2\mu_0 c}$$

So

$$I = I_{max}\left(\frac{5}{9} + \frac{4}{9}\cos\phi\right)$$

(b) The plot (take $I_{max} = 1$) is shown below



The minimum is $\frac{1}{9}I_{max} = \frac{E^2}{2\mu_0 c}$ when $\phi = \pi$.

Problem 4

(a)

$$\sin\theta_{rc} = \frac{1}{1.6} \Rightarrow \theta_{rc} = 38.68°$$

So the largest θ is $90° - \theta_{rc} = 51.32°$

(b)

$$\sin\theta'_{rc} = \frac{1.33}{1.6} \Rightarrow \theta'_{rc} = 56.23°$$

So the largest θ is $90° - \theta'_{rc} = 33.78°$

Problem 5

$$\sin 60° = \frac{\sqrt{3}}{2} = \frac{x}{1.62} \Rightarrow x = 1.40$$

Problem 6

(a) Since the index of reflection in the air is 1, which is less than that in the coat, there is a phase shift on the surface of the coat. But between the coat and the glass, $2.62 > 1.61$ such that there is no phase shift between them. Hence, only one has phase shift.
We want the light with $\lambda = 505 \times 10^{-9}[m]$ to be eliminated. Suppose the thickness of the film is t, So

$$2t = m\frac{\lambda}{2.62} \Rightarrow t = \frac{505 \times 10^{-9}m}{5.24} = 9.64 \times 10^{-8}m[m] \text{ where } m \in \mathbb{N}^*$$

So the minimum thickness is $9.64 \times 10^{-8}[m]$.

(b) $1.93 \times 10^{-7}[m]$, $2.89 \times 10^{-7}[m]$, $3.85 \times 10^{-7}[m]$.

Problem 7

Let $\theta_m = m\pi$ such that $I = I_{max}\cos^2(\frac{\pi dy}{\lambda l}) = I_{max}$.
When $I = I_{max}\cos^2(\frac{\pi dy}{\lambda l})$.

$$\cos(\frac{\pi dy}{\lambda l}) = \pm\frac{\sqrt{2}}{2} \Rightarrow \frac{dy}{\lambda l} = \frac{1}{4} + \frac{k}{2} \quad \text{for } k \in \mathbb{Z}\backslash\{0\}$$

So

$$y_m^+ = \frac{\lambda l}{d}(m + \frac{1}{4})$$
$$y_m^- = \frac{\lambda l}{d}(m - \frac{1}{4})$$

Since

$$y = l\tan\theta = l\theta$$
$$\theta_m^+ = \frac{\lambda}{d}(m + \frac{1}{4})$$
$$\theta_m^- = \frac{\lambda}{d}(m - \frac{1}{4})$$

So

$$\Delta\theta_m = \frac{\lambda}{2d}$$

It does not depend on m.

Problem 8

Suppose the pit depth is t. Since both have phase shift, we have

$$2t = (2m+1)\frac{\lambda}{2\times 1.8} \Rightarrow t = (2m+1)\frac{790\times 10^{-9}}{4\times 1.8} = 1.10\times 10^{-7}(2m+1)[m] \quad \text{where } m \in \mathbb{N}$$

So the minimum is $1.10\times 10^{-7}[m]$.