Vp250 Problem Set 6

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Problem 1

$$\bar{v}(t) = (v_x(t), v_y)(t), v_z(t))$$
$$\bar{F}(t) = (-qE_0, -qv_z(t)B_0, qv_y(t)B_0)$$
$$\bar{a}(t) = \frac{\bar{F}}{m} = (-\frac{qE_0}{m}, -\frac{qv_z(t)B_0}{m}, \frac{qv_y(t)B_0}{m})$$
$$\dot{v}_y = \frac{qv_z(t)B_0}{m}$$
$$\dot{v}_z = -\frac{qv_y(t)B_0}{m}$$

Solving the differential equation we can obtain:

$$\bar{v}(t) = (v_{0x} - \frac{qE_0}{m}t, v_{0y}\cos\frac{qB_0}{m}t, -v_{0y}\sin\frac{qB_0}{m}t)$$

And so the position:

$$\bar{r}(t) = \int \bar{v}(t) = (v_{0x}t - \frac{qE_0}{2m}t^2, v_{0y}\frac{m}{qB_0}\sin\frac{qB_0}{m}t, v_{0y}\frac{m}{qB_0}\cos\frac{qB_0}{m}t - v_{0y}\frac{m}{qB_0})$$

Problem 2

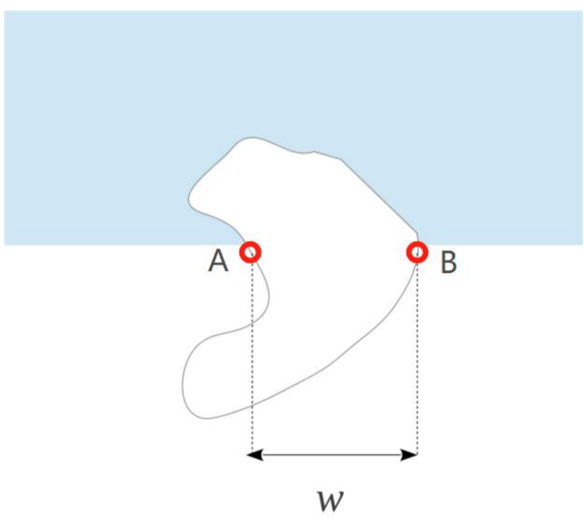
(a) The force acting on the plane through ab is $F_{ab} = Il_{ab}B = Jwhl_{ab}B$.($l_{ab}$ is the distance from one end to plane ab)
The force acting on the plane through cd is $F_{cd} = Il_{cd}B = Jwhl_{cd}B$.($l_{cd}$ is the distance from one end to plane cd)
So the preassure difference is

$$\Delta p = \frac{F_{cd}}{wh} - \frac{F_{ab}}{wh} = J(l_{ab} - l_{cd})B = JlB$$

(b)
$$\Delta p = JlB \Rightarrow 1.01 \times 10^5 = J \times 35 \times 10^{-3} \times 2.2 \Rightarrow J = 1.3 \times 10^6 A/m^2$$

Problem 3

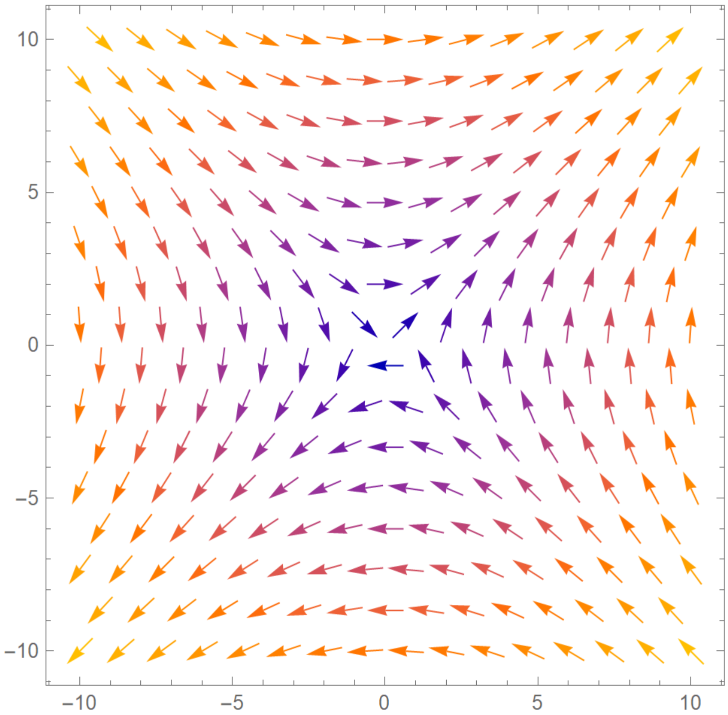
The graph with highlighted boundary point is shown below:



$$\mathrm{d}\bar{F} = I\mathrm{d}\bar{l} \times \bar{B}$$
$$|\bar{F}| = \int_{curve} I|\mathrm{d}\bar{l} \times \bar{B}| = I|\int_{curve} \mathrm{d}\bar{l} \times \bar{B}| = I|\int_A^B \mathrm{d}\bar{l} \times \bar{B}| = I|\overrightarrow{(AB)} \times \bar{B}| = IBw$$

Problem 4

(a) The graph:



(b) $(0,0) \to (0,L)$:

$$\bar{B}(\bar{r}) = (0, 0, \frac{B_0}{L} y)$$
$$\mathrm{d}\bar{l} = (0, \mathrm{d}y, 0)$$
$$\bar{F}_1 = \int I\mathrm{d}\bar{l} \times \bar{B} = I\int_0^L \frac{B_0}{L} y\mathrm{d}y\hat{n}_x = \frac{IB_0L}{2}\hat{n}_x$$

Magnitude: $\frac{IB_0y^2}{2L}$, direction: positive x axis.
$(0,L) \to (L,L)$:

$$\bar{B}(\bar{r}) = (0, 0, B_0)$$
$$\mathrm{d}\bar{l} = (\mathrm{d}x, 0, 0)$$
$$\bar{F}_2 = \int I\mathrm{d}\bar{l} \times \bar{B} = I\int_0^L -B_0\mathrm{d}x\hat{n}_y = -IB_0L\hat{n}_y$$

Magnitude: IB_0L, direction: negative y axis.
$(L,L) \to (L,0)$:

$$\bar{B}(\bar{r}) = (0, 0, \frac{B_0 y}{L})$$
$$\mathrm{d}\bar{l} = (0, \mathrm{d}y, 0)$$
$$\bar{F}_3 = \int I\mathrm{d}\bar{l} \times \bar{B} = I\int_L^0 \frac{B_0}{L} y\mathrm{d}y\hat{n}_x = -\frac{IB_0L}{2}\hat{n}_x$$

Magnitude: $\dfrac{IB_0y^2}{2L}$, direction: negative x axis.
$(L, 0) \to (0, 0)$:

$$\bar{B}(\bar{r}) = (0, 0, 0)$$
$$\mathrm{d}\bar{l} = (\mathrm{d}x, 0, 0)$$
$$\bar{F}_4 = \int I\mathrm{d}\bar{l} \times \bar{B} = 0$$

Magnitude: 0.

(c)

$$\bar{F}_{net} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 = -IB_0L\hat{n}_y$$

Magnitude: IB_0L, direction: negative y axis.

Problem 5

(a)

$$\mathrm{d}\bar{l} = \mathrm{d}x\hat{n}_x + \mathrm{d}y\hat{n}_y$$
$$\mathrm{d}\bar{F} = I\mathrm{d}\bar{l} \times \bar{B} = I(\mathrm{d}x\hat{n}_x + \mathrm{d}y\hat{n}_y) \times \bar{B} = IB\mathrm{d}y\hat{n}_y$$
$$\bar{F}_{net} = IB \oint \mathrm{d}y\hat{n}_y = 0$$

(b) Suppose θ is the angle between $(x, y, 0)$ and the positive x-axis.

$$\mathrm{d}\bar{\tau} = \bar{r} \times \mathrm{d}\bar{F} = (x, y, 0) \times IB\mathrm{d}y\hat{n}_y = IBx\mathrm{d}y\hat{n}_z = IBR\cos\theta R\cos\theta\mathrm{d}\theta\hat{n} = IBR^2\cos^2\theta\mathrm{d}\theta\hat{n}$$
$$\bar{\tau}_{net} = IBR^2 \int_0^{2\pi} \cos^2\theta\mathrm{d}\theta\hat{n}_z = \pi IBR^2\hat{n}_z$$

Problem 6

(a)

$$T = \frac{2\pi R}{v} = \frac{2\pi \cdot 5.3 \cdot 10^{-11}}{2.2 \cdot 10^6} = 1.51 \cdot 10^{-16}s$$

(b) Suppose that when the electron is orbiting, the charge is uniformly placed on the circle.

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}l}{\mathrm{d}t} \cdot \frac{\mathrm{d}Q}{\mathrm{d}l} = v\frac{\mathrm{d}Q}{\mathrm{d}l} = v\frac{e}{2\pi R} = \frac{2.2 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}}{2\pi \cdot 5.3 \cdot 10^{-11}} = 1.06 \times 10^{-3}A$$

(c)

$$\bar{\mu} = I \cdot \pi R^2\hat{n} = \frac{veR}{2}\hat{n} = \frac{2.2 \cdot 10^6 \cdot 1.6 \cdot 10^{-19} \cdot 5.3 \cdot 10^{-11}}{2}\hat{n} = 9.328 \times 10^{-24}\hat{n}A \cdot m^2$$