Han Yibei 5[illegible]

① $y=\sum_{i=0}^{\infty} a_n \cdot x^n$

$y''=\sum_{i=2}^{\infty} a_n \cdot n \cdot (n-1) \cdot x^{n-2}$ $\quad y'=\sum_{i=1}^{\infty} a_n \cdot n \cdot x^{n-1}$ $\quad -y' \cdot x = \sum_{i=1}^{\infty} \cdot n \cdot x^n$

$=\sum_{i=0}^{\infty} a_{n+2} \cdot (n+2)(n+1) \cdot x^n$ $\quad 4y = 4\sum_{i=0}^{\infty} a_n \cdot x^n$

$\Rightarrow \sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)\, x^n - \sum_{n=1}^{\infty} a_n \cdot n \cdot x^{n} + 4\sum_{n=0}^{\infty} a_n \cdot x^n$

x^0: $1\times2\, a_2 + 4a_0 = 0$. $\quad x^n$: $(n+1)(n+2)\, a_{n+2} + (4-n)\, a_n = 0$.

x^1: $2\times3\, a_3 - 1 \cdot a_1 + 4a_1 = 0$.

$\Rightarrow a_{n+2} = \frac{n-4}{(n+1)(n+2)} a_n$. $\Rightarrow a_n = \frac{n-6}{(n-1)\,n} a_{n-2}$.

$n=2k$: $\quad a_n = a_0 \cdot \prod_{i=1}^{k} \frac{2i-6}{(2i-1)\cdot 2i}$ $\quad \Rightarrow y_n = a_0 + a_1 x + \sum_{k=1}^{\infty} a_0 \cdot \prod_{i=1}^{k} \frac{2i-6}{(2i-1)\cdot 2i} \cdot x^{2k}$

$n=2k+1$ $\quad a_n = a_1 \cdot \prod_{i=1}^{k} \frac{(2i-5)}{2i\,(2i+1)}$. $\quad + \sum_{k=1}^{\infty} a_1 \cdot \prod_{i=1}^{k} \frac{2i-5}{2i(2i+1)} \cdot x^{2k+1}$

②. $\sin x = \cancel{(-1)} \sum_{x=0}^{\infty} (-1)^{n-1} \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$

$y'' = \sum_{i=0}^{\infty} a_{n+2}(n+2)(n+1) \cdot x^n$. $\quad -\sin x\, y = -\sin x \cdot \sum_{i=0}^{\infty} a_n \cdot x^n$

~~$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)\, x^n = \sum_{n=0}^{\infty} ($~~

$4\cdot3\cdot a_4 \cdot t^2 + 5\cdot4\cdot a_5 t^3 + \cdots = (t^1 - \frac{t^3}{6} + \frac{t^5}{120} + \cdots)(a_0 t + a_1 t + a_2 t^2 + \cdots)$

$\Rightarrow a_0 = 1$ $\quad a_1 = 0$ $\quad a_2 = 0$ $\quad a_3 = -\frac{1}{6}$ $\quad a_4 = 0$. $\quad a_5 = \frac{1}{120}$ $\quad a_6 = \frac{1}{180}$

$\Rightarrow$ $y = 1 - \frac{1}{6}(x-\pi)^3 + \frac{1}{120}(x-\pi)^5 + \frac{1}{180}(x-\pi)^6$

③. $(x-x^2)y'' = \sum_{n=2}^{\infty} a_n \cdot n(n-1) \cdot x^{n-1} - \sum_{n=2}^{\infty} a_n \cdot n \cdot (n-1) \cdot x^n$ $\quad -y = -\sum_{n=0}^{\infty} a_n \cdot x^n$

$(1-x)y' = \sum_{n=1}^{\infty} a_n \cdot n \cdot x^{n-1} - \sum_{n=1}^{\infty} a_n \cdot n \cdot x^n$.

$\Rightarrow \sum_{n=1}^{\infty} (n+1) \cdot n \cdot a_{n+1} \cdot x^n - \sum_{n=2}^{\infty} a_n \cdot n \cdot (n-1) \cdot x^n + \sum_{n=0}^{\infty} a_{n+1} \cdot n \cdot x^n - \sum_{n=1}^{\infty} a_n \cdot n \cdot x^n - \sum_{n=0}^{\infty} a_n \cdot x^n = 0$.

x^0: $a_1 = a_0$. $\quad x^1$: $2\times1 \cdot a_2 + 1\cdot a_2 - 1\cdot a_1 - a_1 = 0$ $\quad \Rightarrow 4a_2 - 2a_1 = 0$.

$\Rightarrow x^n\, (n+1) \cdot n \cdot a_{n+1} - n(n-1) \cdot a_n + n \cdot a_{n+1} - n \cdot a_n - a_n = 0$.

$(n+1)^2 \cdot a_{n+1} - (n-1)^2 a_n \Rightarrow$ $\quad a_{n+1} = \frac{(n-1)^2}{(n+1)^2} a_n$ $\quad \Rightarrow a_n = \frac{(n^2-2n)}{n^2} a_{n-1}$. $\quad -\sum_{n=1}^{\infty} n b_n x$

$y = a_0 + a_0 x + \frac{1}{2}a_0 x^2 + \sum_{n=3}^{\infty} (a_2 \cdot \prod_{i=3} \frac{(i^2-2i)}{i^2}) \cdot x^n$

$y_2 = y_1 \ln x + \sum_{n=0}^{\infty} b_n x^n \Rightarrow (x-x^2)y_2'' = (x-x^2)\ln x\, y_1'' + 2(1-x)y_1' - \frac{1-x}{x} y_1 + \sum_{n=2}^{\infty} n(n-1) b_n x^{n-1}$

b_{n-1} $\frac{(n-1)^2+1}{n^2}$ $b_{n-1} + \frac{4n^2 - b_n + 4}{k \quad n}$ a_n. $\quad \sum x^n$.