Han Yi bei 515370910123

1\. $\cancel{m a =}$ $m\ddot{x} + b\dot{x} + kx = f(t)$. $\frac{k}{m} = \omega_0^2$ $\frac{b}{m} = 2\varepsilon\omega_0$ $\Rightarrow 2\varepsilon\omega_0 = 5$.

$\Rightarrow \ddot{x} + 2\varepsilon\omega_0\dot{x} + \omega_0^2 x = \frac{F(t)}{m}$ $\Rightarrow$ $\cancel{x(t) =}$ $\ddot{x} + 5\dot{x} + 4x = 0$.

if $0 \le \varepsilon < 1$ $x(t) = e^{-\varepsilon\omega_0 t}\left(x_0 \cdot \cos\omega_d t + \frac{v_0 + \varepsilon\omega_0 x_0}{\omega_d}\sin\omega_d t\right)$

$\varepsilon = 1$ $x(t) = e^{-\omega_0 t}\left(x_0 + (v_0 + \omega_0 x_0)t\right)$

$\varepsilon > 1$ $x(t) = \frac{1}{2\omega_0\sqrt{\varepsilon^2-1}}\left[(v_0 + (\varepsilon + \sqrt{\varepsilon^2-1})\omega_0 x_0) \cdot e^{-\omega_0(\varepsilon - \sqrt{\varepsilon^2-1})t} - (v_0 + (\varepsilon - \sqrt{\varepsilon^2-1})\omega_0 x_0)e^{-\omega_0(\varepsilon+\sqrt{\varepsilon^2-1})t}\right]$.

$\Rightarrow \lambda^2 + 5\lambda + 4 = 0$ $\Rightarrow$ $\lambda = -4 / -1$. $\Rightarrow x(t) = C_1 e^{-4t} + C_2 e^{-t}$

① $x(0) = 1$ $\dot{x}(0) = 0$. $\begin{cases} C_1 + C_2 = 1 \\ -4C_1 - C_2 = 0. \end{cases}$ $C_1 = -\frac{1}{3}$ $C_2 = \frac{4}{3}$ $\Rightarrow x(t) = -\frac{1}{3} \cdot e^{-4t} + \frac{4}{3} \cdot e^{-t}$

② $x(0) = 1$ $\dot{x}(0) = 5$ $\begin{cases} C_1 + C_2 = 1 \\ -4C_1 - C_2 = -5 \end{cases}$ $\begin{cases} C_1 = \frac{4}{3} \\ C_2 = -\frac{1}{3} \end{cases}$ $\Rightarrow x(t) = \frac{4}{3} \cdot e^{-4t} - \frac{1}{3}e^{-t}$

2\. $\lambda^2 + 3\lambda + 2 = 0$. $\Rightarrow \lambda = -2 / -1$ $\Rightarrow$ $y_c = C_1 e^{-t} + C_2 \cdot e^{-2t}$

$\begin{cases} C_1' e^{-t} + C_2' e^{-2t} = 0 \\ -C_1' e^{-t} - 2C_2' e^{-2t} = \frac{1}{1+e^{-t}} \end{cases}$ $\Rightarrow \begin{cases} C_1' = \frac{e^{t}}{1+e^{-t}} \\ C_2' = -\frac{e^{2t}}{1+e^{-t}} \end{cases}$

$\Rightarrow C_1 = e^{t} - \ln(1+e^{t}) + \hat{C}_1$

$C_2 = e^{t} - \frac{e^{2t}}{2} - \ln[1+e^{t}] + \hat{C}_2$

$\Rightarrow y = 1 - e^{-t} \cdot \ln(1+e^{t}) + e^{-t} - \frac{1}{2} - e^{-2t} \cdot \ln[1+e^{t}] + C$.

$x(0) = 1$ $\Rightarrow \frac{3}{2} - 2\ln 2 + C = 1$ $\Rightarrow$ $C = 2\ln 2 - \frac{1}{2}$

$(x(t))' = C_1' \cdot e^{-t} - C_1 \cdot e^{t} + C_2' \cdot e^{-2t} - 2C_2 \cdot e^{-2t}$.

Han Yibo. 5093259(v012)

3: $2x^2+bx+3=0$. $\Rightarrow y_c$. $y_p = A\sin\frac{t}{2} + B\cos\frac{t}{2}$

$$-\frac{A}{4}\sin\frac{t}{2} - \frac{B}{4}\cos\frac{t}{2} + \frac{Ab}{2}\cos\frac{t}{2} - \frac{B\cdot b}{2}\sin\frac{t}{2} + A\sin\frac{t}{2} + B\cos\frac{t}{2} = 4\sin\frac{t}{2}$$

$$\begin{cases} -\frac{1}{2}\cdot A - \frac{b}{2}\cdot B + A = 4 \\ -\frac{1}{2}\cdot B + \frac{b}{2}\cdot A + B = 0 \end{cases} \Rightarrow \begin{cases} \frac{1}{2}A - \frac{b}{2}B = 4 \\ \frac{1}{2}B + \frac{b}{2}A = 0 \end{cases}$$

$w = \frac{1}{2}$ $F = -4$, $m = 2$ $k = 3$

$\Rightarrow R = \sqrt{(3-2\times\frac{1}{4})^2 + (b\times\frac{1}{2})^2} = \sqrt{\frac{25}{4} + \frac{1}{4}b^2} < 0.2.$

$\Rightarrow A = \frac{F}{R} = \frac{4}{\sqrt{\frac{25}{4}+\frac{1}{4}b^2}} < 0.2.$

since $b>0$.

$\sqrt{\frac{25}{4}+\frac{1}{4}b^2} > 20.$ $\frac{25}{4}+\frac{1}{4}b^2 > 400$. $\Rightarrow b > 15\sqrt{7}$.

4. $x_p(t) = \frac{F}{\sqrt{(k-mw^2)^2+(bw)^2}} \cdot \sin(wt+\varphi)$. $G = \frac{1}{\sqrt{(k-mw^2)^2+(bw)^2}} \Rightarrow$

$\cancel{R}=$. $G = \frac{1}{\sqrt{(40-100\cdot w^2)^2+(50w)^2}}$ $A = G$.

~~Gmax~~ $G = \cancel{\frac{1}{\quad}}$ $R = 10\cdot\sqrt{100w^4 - 80w^2 + 16 + 25w^2}$

$\Rightarrow w^2 = \frac{11}{40}$, we get maximum value: $R = 10\sqrt{\frac{135}{16}} = \frac{15}{2}\sqrt{15}$

$\Rightarrow w = \pm\frac{\sqrt{110}}{20}$