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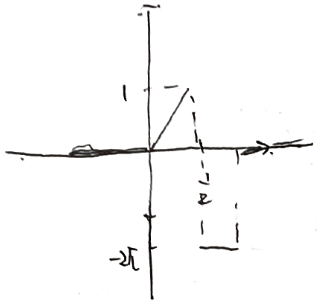
Problem 1.

(1) $f(t) = H(t) \cdot g(t)$ $\qquad (g(t) = 2 + t - t^2 + t^3 + e^t \sin t$

$$L[H(t) \cdot g(t) : t \to s] = e^{-0 \cdot s} \cdot \bar{g}(s) = \frac{2}{s} + \frac{1}{s^2} - \frac{2}{s^3} + \frac{6}{s^4} + \frac{1}{(s-1)^2+1}$$

(2)



$\Rightarrow f(t) = 2t \cdot H(t) + (-2\pi - 2t) \cdot H(t - \frac{1}{2}) + (2\pi) H[t-1]$

$$L[f(t) : t \to s] = e^{-0 \cdot s} \cdot 2 \cdot \frac{1}{s^2} + e^{-\frac{1}{2} \cdot s} \left(-2 \frac{1}{s^2} - (2\pi + 1)\frac{1}{s}\right) + e^{-1 \cdot s} \cdot 2\pi \cdot \frac{1}{s}$$

$$= \frac{2}{s^2} - \frac{2e^{-\frac{s}{2}}}{s^2} - \frac{(2\pi+1) \cdot e^{-\frac{s}{2}}}{s} + \frac{2\pi \cdot e^{-s}}{s}$$

Problem 2

$$f(s) = \frac{\cancel{3s+3}\,2}{s-1} + \frac{-2s-3}{s^2+2s+5} = \frac{2}{s-1} - 2 \cdot \frac{x+1}{(x+1)^2+4} - 1 \cdot \frac{1}{(x+1)^2+4}$$

$$\Rightarrow L^{-1}\left(\frac{2}{x-1}\right) - 2L^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\} \cdot L^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$$

$$= 2e^t - e^{-t} \cdot L^{-1}\left\{\frac{s}{s^2+4}\right\} \cdot L^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$$

$$= 2e^t - 2e^{-t} \cos(2t) - \frac{1}{2} e^{-t} \sin(2t).$$

Problem 3.

(a) $s^2 \bar{y}(s) - \cancel{5y\,5s}\; s f(0) - \dot{f}(0) - 5s\bar{y}(s) + 5f(0) + 6\bar{y}(s)$.

$$\Rightarrow (\cancel{s^2 - 5s + 6})\, \bar{y}(s) = \frac{2s-9}{s^2-5s+6} = \frac{2s-9}{(s-2)(s-3)} = \frac{5}{s-2} - \frac{3}{s-3} = 5e^{2t} - 3e^{3t}$$

(b). $s^2 \bar{y}(s) - s f(0) - \dot{f}(0) - 1\bar{y}(s) = \frac{1}{(s-2)^2}$ $\Rightarrow \bar{y}s = \dfrac{\frac{1}{(s-2)^2} + \frac{3}{2}}{(s^2-1)} = \dfrac{3s^2 - 12s + 14}{2s^4 - 8s^3 + 6s^2 + 8s - 8}$

$$\Rightarrow L(s) = -\frac{29}{36(s+1)} + \frac{5}{4(s-1)} - \frac{4}{9(s-2)} + \frac{1}{3(s-2)^2}$$

$$= -\frac{29}{36} e^{-t} + \frac{5}{4} \cdot e^{t} - \frac{4}{9} \cdot e^{2t} + \frac{e^{2t}}{3}$$

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(c).

$-\ddot{x}+x=0. \quad \ddot{x}-x=0. \quad \Rightarrow y_0=-1 \quad x'(0)=4. \quad (x_0)=1.$

$\ddot{y}-y=0.$

$\Rightarrow [s^2 X(s) - s(x(0)) - \dot{x}(0)] - X(s)=0. \qquad (s^2-1)[X(s)] = s \Rightarrow [X(s)] = \frac{s+1}{s^2-1} = \frac{1}{s-1}$

$\Rightarrow x=e^{t} \qquad y(t) = -\dot{x} = -e^{t}$

(d). $4[s^2 y(s) - s[y(0)] - y'(0)] + 4(s y(s) - y(0)) + 5y(s) = \mathcal{L}\{4H(t) - 4H(t-\pi)\}$

$= 4 - 4e^{-\pi s}$

$\Rightarrow (4s^2+4s+5)\, y(s) = \frac{4}{s} - \frac{4e^{-\pi s}}{s} \Rightarrow y(s) = \frac{4-4e^{-\pi s}}{s(4s^2+4s+5)}.$

$\Rightarrow y(x) = \frac{4}{s(4s^2+4s+5)} - 4\cdot\frac{e^{-\pi s}}{s(4s^2+4s+5)}. \qquad \frac{4}{5}H(t)+\frac{4}{5}e^{-\frac{t}{2}}\cos t - \frac{2}{5}e^{-\frac{t}{2}}\sin(t)$

$\frac{4}{s(4s^2+4s+5)} = \frac{4}{5s} - \frac{4}{5}\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+1} - \frac{2}{5}\cdot\frac{1}{(s+\frac{1}{2})^2+1} \qquad -4[H(t-\pi)]\cdot\left(\frac{4}{5}H(t-\pi) - \frac{4}{5}\cdot e^{-\frac{t-\pi}{2}}\cos(t-\pi) - \frac{2}{5}\cdot e^{-\frac{t-\pi}{2}}\sin(t-\pi)\right)$

(e). $[s^2 y(s) - s y(0) - y'(0)] + 2(s y(s) - y(0)) + 2y(s) = e^{-\pi t}.$

$y(s) = \frac{e^{-\pi t}}{s^2+2s+2} \Rightarrow y(s) = H(t-\pi)\cdot \mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+2}\right\} = H(t-\pi)\cdot\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$

$= H(t-\pi)\cdot e^{-t+\pi}\sin(t-\pi).$

Problem 4.

(a). $x = \frac{y-y'}{2} \qquad \frac{y'}{2} - \frac{y''}{2} = 2y - \frac{3y}{2} + \frac{3y'}{2} \qquad y''+2y'+y=0 \Rightarrow \lambda^2+2\lambda+1=0 \quad \lambda_1=\lambda_2=-1.$

$\Rightarrow y = (C_1+C_2 t)\cdot e^{-t}$

$x = \frac{1}{2}(C_1 e^{-t} + C_2 t\cdot e^{-t} + C_1 e^{-t} - C_2 e^{-t} + C_2\cdot t\cdot e^{-t})$

$= (C_1 - \frac{1}{2}C_2)\cdot e^{-t} + C_2\cdot t\cdot e^{-t}$

$\lambda_1=\lambda_2=-1$

(b). $A = \begin{vmatrix} 3 & -2 & -1 \\ 3 & -4 & -3 \\ 2 & -4 & 0 \end{vmatrix} \qquad \lambda=-5,\ \lambda_2=\lambda_3=2.$

$\Rightarrow \lambda=-5. \begin{pmatrix}1\\3\\2\end{pmatrix} \qquad \lambda_2=2. \begin{pmatrix}1\\0\\1\end{pmatrix} \begin{pmatrix}2\\1\\0\end{pmatrix}$

$\Rightarrow x = C_1\cdot e^{-5t} + C_2\cdot e^{2t} + 2C_3\cdot e^{-t}$

$y = 3C_1\cdot e^{-5t} + C_3\cdot t\cdot e$

$z = 2C_1 e^{-5t} + C_2 e^{2t}.$

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c). $x = y' - \cancel{5y} - 5e^{-t} \Rightarrow y'' - y' + 5e^{-t} = 5y' - 5y - 25e^{-t} - 3y + 2e^{3t}$

$y'' - 6y' + 8y = 2e^{3t} - 30e^{-t}$. $\lambda^2 - 6\lambda + 8\lambda = 0 \Rightarrow (\lambda - 4)(\lambda - 2) = 0 \Rightarrow \lambda_1 = 4, \lambda_2 = 2$. $y_c = C_1 e^{4t} + C_2 e^{2t}$

$y_p = A e^{3t} + B e^{-t}$ $\quad 9Ae^{3t} + Be^{-t} - 18Ae^{3t} + 6Be^{-t} + 8Ae^{3t} + 8Be^{-t} = 2e^{3t} - 30e^{-t}$

$\begin{cases} (9-18+8)A = 2 \\ (1+6+8)B = -30 \end{cases} \Rightarrow \begin{cases} A = -2 \\ B = -2 \end{cases} \Rightarrow y = C_1 e^{4t} + C_2 e^{2t} - 2e^{3t} - 2e^{-t}$

$x = 3C_1 e^{4t} + C_2 e^{2t} - 8e^{3t} - 5e^{-t}$

D). $\begin{cases} x = -y'' - y \\ -y^{(4)} - y'' = -3y'' - 3y + 4y \end{cases}$ $\Rightarrow y^{(4)} - 2y'' + y = 0 \Rightarrow \lambda_1 = \lambda_2 = 1,\ \lambda_3 = \lambda_4 = -1$

$\Rightarrow y = (C_1 + C_2 t)\cdot e^{t} + C(C_3 + C_4 t)\cdot e^{-t}$

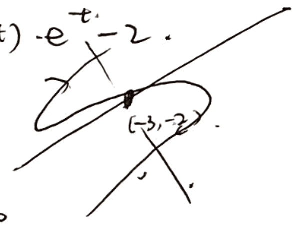
$\Rightarrow x = 2C_4 e^{-t} - 2C_2 e^{t} - 2(C_1 + C_2 t)\cdot e^{t} - 2C(C_3 + C_4 t)\cdot e^{-t}$

e). $x = \dfrac{3y - y'}{2}$ $\quad \dfrac{3y'}{2} - \dfrac{y''}{2} = 2y - \dfrac{3y}{2} + \dfrac{y'}{2} + 1 \Rightarrow y'' - 2y' + y = -2$

$\lambda_1 = \lambda_2 = 1 \Rightarrow y_c = (C_1 + C_2 t)\cdot e^{t} \Rightarrow y_p = -2 \Rightarrow y = (C_1 + C_2 t)\cdot e^{t} - 2$.

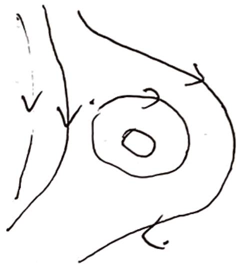
$x = C_1 e^{t} + C_2 - t e^{t} - 1 - \frac{C_2}{2} e^{t}$.

$\begin{cases} 2y - x + 1 = 0 \\ 3y - 2x = 0 \end{cases} \Rightarrow \begin{cases} x = -3 \\ y = -2 \end{cases}$



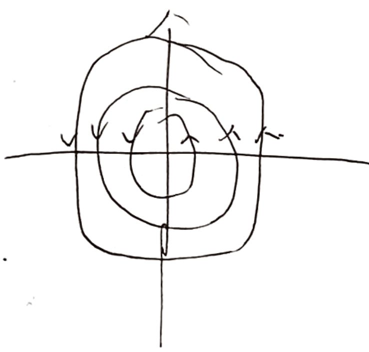
5. $\begin{cases} y = \dot{x} \\ \dot{y} - x + x^2 = 0 \end{cases} \Rightarrow \begin{pmatrix} 0 & 1 \\ 2x1 & 0 \end{pmatrix} \Rightarrow$ ~~$\lambda_1 = -1 \quad \lambda_2 = 1$~~. $\begin{cases} y = 0 \\ x = 0/1. \end{cases}$

1).



2) $\begin{cases} y = \dot{x} \\ \dot{y} = -2x^3 \end{cases} \Rightarrow \begin{pmatrix} 0 & 1 \\ -6x & 0 \end{pmatrix}$ $x = 0$.

$J = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$ $\lambda^2 = 0$, $\lambda_1 = \lambda_2 = 0$. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$



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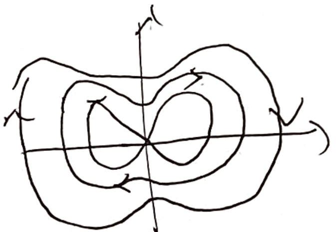

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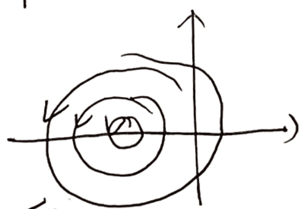

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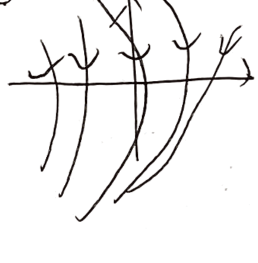

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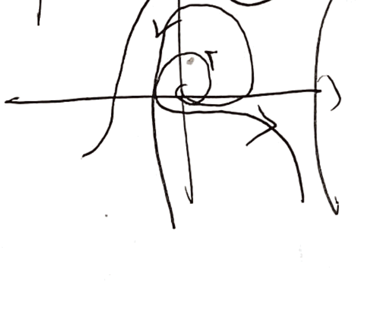

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