Problem 1.

y_c: $\lambda^2 - 2\lambda = 0$ $\lambda = 0/2$ $\Rightarrow y_c = A + B\cdot e^{2x}$.

y_p: Ce^x $(Ce^x)'' - 2(Ce^x)' = 2e^x \Rightarrow y_p = -2e^x$.

$y = A + Be^{2x} - 2e^x$ $\Rightarrow y(1) = -1$ $y'(1) = 0$

$\Rightarrow B = \frac{1}{e}$ $A = e - 1$

$\Rightarrow y = e - 1 + e^{2x-1} - 2e^x$

Question 2.

(1) $\lambda^2 + 2\lambda + 2 = 0 \Rightarrow \lambda = -1 \pm i$

$\Rightarrow y_c = e^{-x}(A\cos x + B\sin y)$.

y_p: e^x: Ce^x.

$x\cos x$: $D_1 x\cos x + D_2 x\sin x + D_3\cos x + D_4\sin x$...

$Ce^x + 2Ce^x + 2Ce^x = e^x \Rightarrow C = \frac{1}{5}$

$\Rightarrow y = e^{-x}(A\cos x + B\sin y) + Ce^x + (D_1 x\cos x + D_2 x\sin x + D_3\cos x + D_4\sin x)$

(2) $\lambda^4 + 5\lambda^2 + 4 = 0$. $\Rightarrow \lambda = \pm 2i, \pm i$

$\sin x\cos 2x$:

$y_p = B_1\sin x\cos 2x + B_2\sin 2x\cos x$

$+ B_3\cos x\cos 2x + B_4\sin x\sin 2x$.

$\Rightarrow y_c = A_1\cos 2x + A_2\sin 2x + A_3\cos x + A_4\sin x$

$\Rightarrow y = y_c + y_p = A_1\cos 2x + A_2\sin 2x + A_3\cos x + A_4\sin x + B_1\sin x\cos 2x + B_2\sin 2x\cos x$

$+ B_3\cos x\cos 2x + B_4\sin x\sin 2x$

(3) $\lambda^2 + 2\lambda + 1 = 0 \Rightarrow \lambda_1 = \lambda_2 = -1 \Rightarrow y_c = (A + Bx)\cdot e^{-x}$

y_p: xe^{-x}: $B_1\cdot x\cdot e^{-x} + B_2 e^{-x}$

$x\cos x$: $C_1 x\cos x + C_2 x\sin x + C_3\cos x + C_4\sin x$

$y = (A + Bx)\cdot e^{-x} + B_1 xe^{-x} + B_2 e^{-x} + C_1 x\cos x + C_2 x\sin x + C_3\cos x + C_4\sin x$

Question: $x = 4y - \dot{y}$

$\Rightarrow \ddot{y} - 4\dot{y} - 2\dot{y} + 8y + y = 0$.

$\Rightarrow y'' - 6y' + 9y = 0$.

$\Rightarrow \lambda^2 - 6\lambda + 9 = 0 \Rightarrow \lambda_1 = \lambda_2 = 3$.

$y = e^{3x}(A_1 + A_2 x)$

$y = \dot{x} - 2x$.

$\ddot{x} - 2\dot{x} - 4\dot{x} + 8x + x = 0$

$\ddot{x} - 6\dot{x} + 9x = 0$.

$x = e^{3x}(B_1 + B_2 y)$.