1. Prove DeMorgan's Law. (5 points)

A	B	$\neg A \vee \neg B$	$\neg(A \wedge B)$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

$\Rightarrow \neg A \vee \neg B = \neg(A \wedge B)$

$\Rightarrow$ proved

2. Use algebraic manipulation to convert the following equation to sum-of-products form without any simplification: (5 points)

$$F = a (b + c'd) (c' + bd') + a'c (b' + bd)$$

$F = (ab + ac'd)(c' + bd) + a'cb' + a'cbd$

$= abc' + abd + ac'd + a'cb' + a'bcd$

3. Convert above equation to a sum-of-minterms form. (5 Points)

$F = abc'd + abc'd' + abcd + ab'c'd + a'b'cd + a'b'cd' + a'bcd$

4. Use DeMorgan's Law to find the inverse of the equation in Problem 2 until DeMorgan's Law cannot be further applied. (5 points)

$F' = [a(b + c'd)(c' + bd)' + a'c(b' + bd)]'$

$= [a(b + c'd)(c' + bd)]' [a'c(b' + bd)]'$

$= [a' + (b + c'd)' + (c' + bd)'] \quad [(a'c)' + (b' + bd)']$

$= [a' + b' \cdot (c'd)' + c \cdot (bd)'] \quad [a + c' + b(bd)']$

$= [a' + b'(c + d') + c(b' + d')] \quad [a + c' + b(b' + d')]$

$= a' + b'c + b'd' + cd'$

5. Problem 2.41 (5 points)

2.41 Convert the function F shown in the truth table in Table 2.11 to an equation. Don't minimize the equation.

TABLE 2.11 Truth table.

a	b	c	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$F = a'b'c + abc' + abc$

6. Problem 2.42 (5 points)

2.42 Use algebraic manipulation to minimize the equation obtained in Exercise 2.41.

$F = a'b'c + ab$

2.52 Determine whether the two circuits in Figure 2.81 are equivalent circuits, using (a) algebraic manipulation and (b) truth tables.

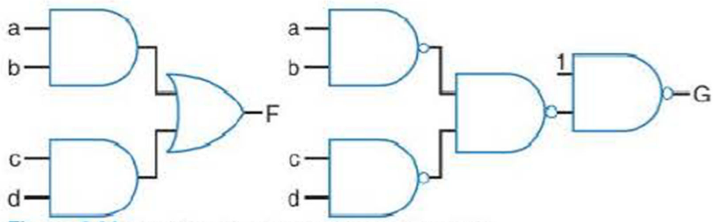


Figure 2.81 Combinational circuits for F and G.

left= $ab+cd$

right $= \neg(\neg(\neg(ab)\neg(cd)))$

(a) $\neg[\neg(ab)\neg(cd)) = ab+cd$ (de morgan law) $\Rightarrow$ left $=\neg$right

(b)

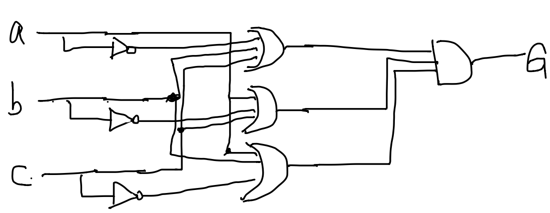
a	b	c	d	left	right
0	0	0	0	0	1
0	0	0	1	0	1
0	0	1	0	0	1
0	0	1	1	1	0
0	1	0	0	0	1
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	1	0

$\Rightarrow$ left= right $\Rightarrow$ proved

2.54 A museum has three rooms, each with a motion sensor (m0, m1, and m2) that outputs 1 when motion is detected. At night, the only person in the museum is one security guard who walks from room to room. Create a circuit that sounds an alarm (by setting an output A to 1) if motion is ever detected in more than one room at a time (i.e., in two or three rooms), meaning there must be one or more intruders in the museum. Start with a truth table.

2.55 Create a circuit for the museum of Exercise 2.54 that detects whether the guard is properly patrolling the museum, detected by *exactly* one motion sensor being 1. (If no motion sensor is 1, the guard may be sitting, sleeping, or absent.)

Output= $(a'bc)(ab'c)(abc')$



9. Problem 6.5. (10 points)

6.5 Perform two-level logic size optimization for `F(a,b,c,d) = a'bc' + abc'd' + abd` using a K-map.

cd \ ab	00	01	10	11
00	0	1	0	1
10	0	0	0	0
01	0	1	0	1
11	0	0	0	1

F = bc' + abd

10. Problem 6.7. (10 points)

6.7 Perform two-level logic size optimization for `F(a,b,c) = a'b'c + abc`, assuming input combinations `a'bc` and `ab'c` can never occur (those two minterms represent don't cares).

c \ ab	00	01	10	11
0	0	0	0	0
1	1	X	X	1

F = C

11. Problem 6.8. (10 points)

6.8 Perform two-level logic size optimization for `F(a,b,c,d) = a'bc'd + ab'cd'`, assuming that `a` and `b` can never both be `1` at the same time, and that `c` and `d` can never both be `1` at the same time (i.e., there are don't cares).

cd \ ab	00	01	10	11
00	0	0	0	X
10	0	0	1	X
01	0	1	0	X
11	X	X	X	X

F = ac + bd

12. Problem 6.9 (10 points)

6.9 Consider the function `F(a,b,c) = a'c + ac + a'b`. Using a K-map: (a) Determine which of the following terms are implicants (but not necessarily prime implicants) of the equation: `a'b'c', a'b', a'bc, a'c, c, bc, a'bc', a'b`. (b) Determine which of those terms are prime implicants of the function.

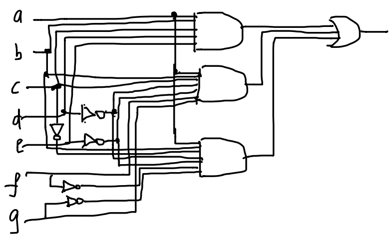
cd \ ab	00	01	10	11
00	0	1	0	0
10	1	1	1	1
01	0	1	0	0
11	1	1	1	1

(a) a'bc · a'c , c , bc, a'b'c', a'b

(b) a'bc, c

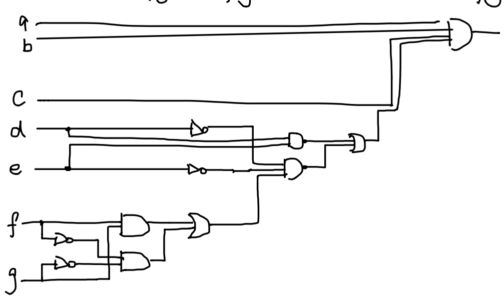
13. Problem 6.14 (10 points)

6.14 Using algebraic methods, reduce the number of gate inputs for the following equation by creating a multilevel circuit: `F(a,b,c,d,e,f,g) = abcde + abcd'e'fg + abcd'e'f'g'`. Assume only AND, OR, and NOT gates will be used. Draw the circuit for the original equation and for the multilevel circuit, and clearly list the delay and number of gate inputs for each circuit.



delay = 2
gate = 22 x 2 = 44

F = abc(de+d'e'fg+d'e'f'g') = abc(de+d'e'(fg+f'g'))



delay = 5

gate = 2 x 17 = 34