Automobile Engineering 3089620

1. Our limits of integration using $dz\,dr\,d\theta$ is;

$$r \leq z \leq 2-r^2$$
$$0 \leq r \leq 1$$
$$0 \leq \theta \leq 2\pi$$

$$\int_0^{2\pi}\int_0^1\int_r^{2-r^2} dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 [z]_r^{2-r^2}\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1 2-r^2-r\,dr\,d\theta$$

$$\int_0^{2\pi}\left[2r-\frac{r^3}{3}-\frac{r^2}{2}\right]_0^1 d\theta$$

$$\int_0^{2\pi}\left[2-\frac{1}{3}-\frac{1}{2}\right] d\theta$$

$$\int_0^{2\pi}\frac{7}{6}\,d\theta$$

$$\Rightarrow \frac{7}{6}(2\pi) = 7.330$$

Automobile Engineering 3089620

$\therefore$ The volume of the area in question is 7.330 units.

ii. Our limits of integration using $dr\,dz\,d\theta$ is;

$$\int_0^{2\pi}\int_1^2\int_0^{\sqrt{2-z}} dr\,dz\,d\theta + \int_0^{2\pi}\int_0^1\int_0^z dr\,dz\,d\theta$$

This is because taking the integration in the direction of the radius (r) splits the region into the paraboloid and the cone.

$$\int_0^{2\pi}\int_1^2 [r]_0^{\sqrt{2-z}}\,dr\,dz\,d\theta + \int_0^{2\pi}\int_0^1 [r]_0^z$$

$$\int_0^{2\pi}\int_1^2 \sqrt{2-z}\,dz\,d\theta + \int_0^{2\pi}\int_0^1 z\,dz\,d\theta$$

$$\int_0^{2\pi}\left[-\frac{2}{3}(2-z)^{3/2}\right]_1^2 d\theta + \int_0^{2\pi}\left[\frac{z^2}{2}\right]_0^1$$

$$\int_0^{2\pi}\left[-\frac{2}{3}(2-2)^{3/2}+\frac{2}{3}(2-1)^{3/2}\right]d\theta + \int_0^{2\pi}\frac{1}{2}\,d\theta$$

Automobile Engineering 3089620

$$\int_0^{2\pi} \frac{2}{3} d\theta + \int_0^{2\pi} \frac{1}{2} d\theta$$

$$\Rightarrow \left[\frac{2}{3}\theta\right]_0^{2\pi} + \left[\frac{1}{2}\theta\right]_0^{2\pi}$$

$$= \frac{4\pi}{3} + \frac{2\pi}{2}$$

$$= \frac{7}{3}\pi = 7.330$$