HW3

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Nr.1

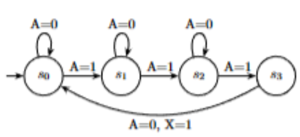
State encoding:
FSM for X:
s_i: state in which A has been 1 for i cycles, if the next states make $X=1$ (4 `1` in total occurs, reset to s0)
(Y =0 if not be shown on the arc)

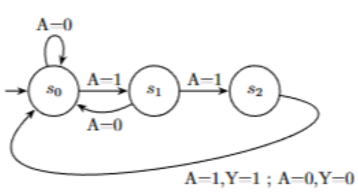
	s0	s1	s2	s3
S0	0	0	0	0
S1	0	0	1	1
S2	0	1	0	1

FSM for Y:
s_i: state in which A has been 1 consectively for i cycles. (3 `1` consectively occurs, reset to s0)

	s0	s1	s2
S0	0	0	0
S1	0	0	1
S2	0	1	0

State transition diagram:
X, Y=0 if it's omitted on the arc.





State transiion table:
FSM for X:

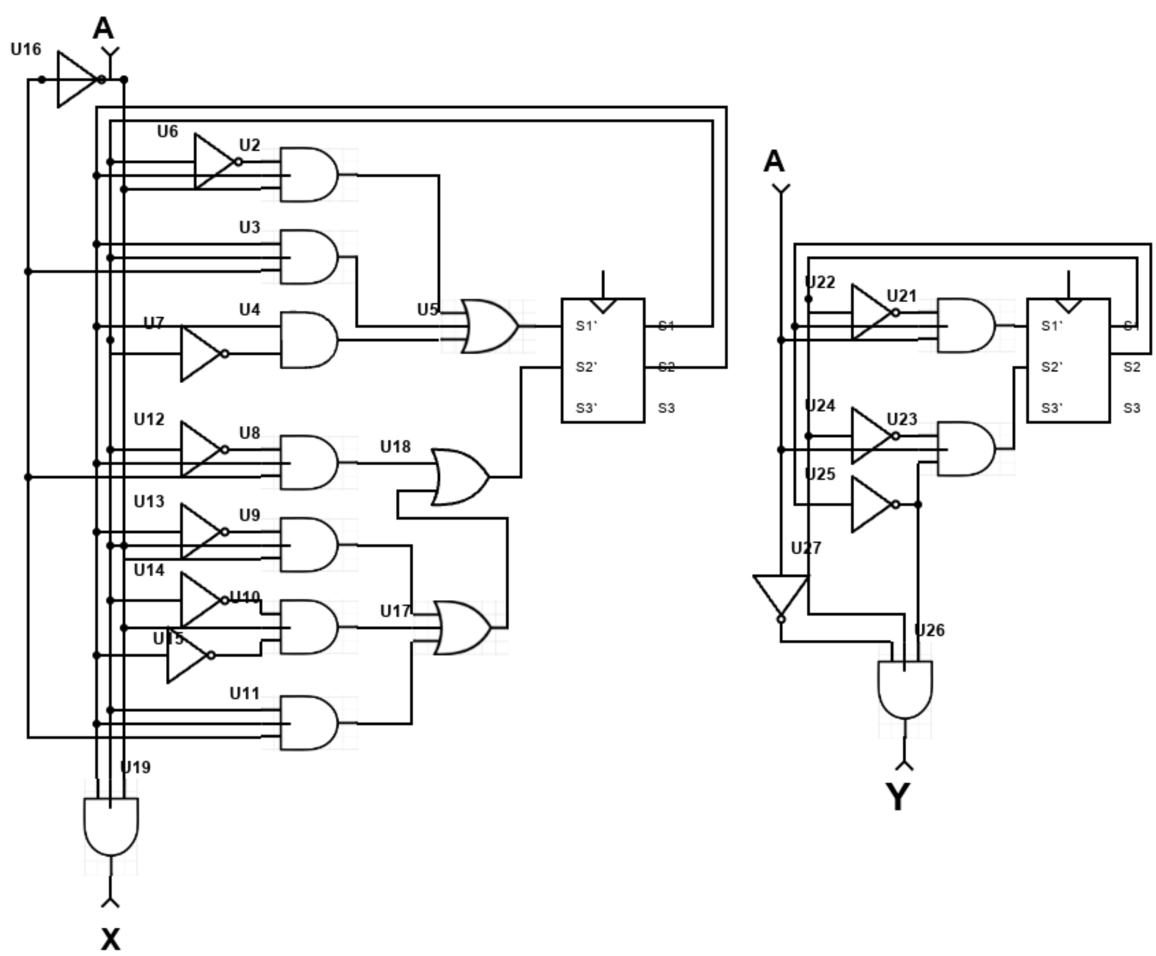
X	S[0:1]	A	s`	S`[0:1]
0 0	00	0 1	s0 s1	00 01
0 0	01	0 1	s1 s2	01 10
0 0	10	0 1	s2 s3	10 11
0 1	11	0 1	s3 s0	11 00

$S_0' = \bar{S}_0 S_1 A + S_0 \bar{S}_1 + S_0 S_1 \bar{A}$
$S_1' = \overline{S_0 S_1} A + \bar{S}_0 S_1 \bar{A} + S_0 \bar{S}_1 A + S_0 S_1 \bar{A}$

$X = S_0 S_1 A$
FSM for Y:

Y	S[0:1]	A	s`	S`[0:1]
0 0	00	0 1	s0 s1	00 01
0 0	01	0 1	s0 s2	00 10
1 0	10	0 1	s0 s0	00 00

$S_0' = \bar{S}_0 S_1 A$
$S_1' = \overline{S_0 S_1} A$
$Y = S_0 \bar{S}_1 \bar{A}$
schematic



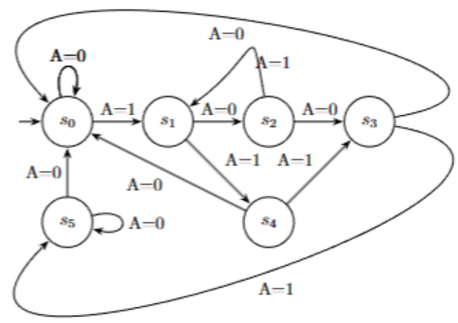
Nr.2

States encoding:
s_0: pattern `0` occurs
s_1: pattern `01` occurs
s_2: pattern `010` occurs
s_3: pattern `0100` or `0111` occurs
s_4: pattern `011` occurs
s_5: pattern `1` occurs
A: input
X: output
We consider the FST for pattern recognition task is cylic(for asylic situation just ending in the s_3).

	s0	s1	s2	s3	s4	s5
S0	0	0	0	0	1	1
S1	0	0	1	1	0	0
S2	0	1	0	1	0	1

States transition diagram:

> Initial state can be s_0 or s_5
> Only output 1 when s_3 is reached, then reset.(for acyclic siutation, end here)



States transition table:

X	s	S[2:0]	S`[2:0]	s`	A
0 0	s0	000	001 000	s1 s0	0 1
0 0	s1	001	010 100	s2 s4	0 1
1 0	s2	010	011 001	s3 s1	0 1
0 0	s3	011	000 101	s0 s5	0 1
0 1	s4	100	000 011	s0 s3	0 1
0 0	s5	101	000 101	s0 s5	0 1

Next state:
$S_0 = \overline{S_0 S_1 S_2} A + \bar{S}_0 S_1 \bar{S}_2 + S_0 S_1 \bar{S}2A + \bar{S}1S2A$
$S_1 = S_0 \overline{S_1 S_2 A} + \bar{S}_0 S_1 \bar{S}_2 \bar{A}$
$S_2 = S_0 \overline{S_1 S_2} A + S_0 S_1 \bar{S}_2 A + S_0 \bar{S}_1 S_2 A$

Output:
$X = \bar{S}_0 \bar{S}_1 \bar{S}_2 A + \bar{S}_0 \bar{S}_1 S_2 A$

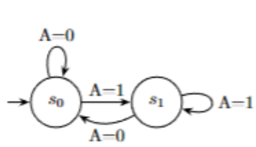
Nr.3

This is usually considered as a mealy machine, in which the output depend both on inputs and current states. But taking input as part of states makes it a moore machine.

States encoding:
$s_0 : A_{n-1} = 0$
$s_1 : A_{n-1} = 1$
$s_0 : S = 0, s_1 : S = 1$
States transition diagram:

> Notice that A_{n-1} is equvalent to S

> Ouput $Z = S\bar{B} + SAB$, which is not shown in graph because it's just $combinatoric\_func(S, A, B)$



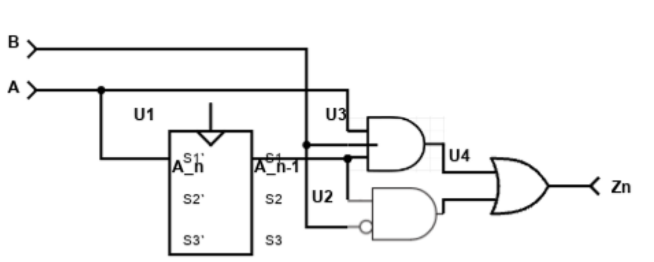
States transition table:

S	Bn	An(S`)	Zn
0	0	0 1	0 0
0	1	0 1	0 0
1	0	0 1	1 1
1	1	0 1	0 1

Next states and output:

$S' = A_n$
$Z_n = A_{n-1}\bar{B}_n + A_{n-1}A_nB_n$

Schematic:



Nr.4

Prove that any Moore machine can be transformed into an equivalent Mealy machine:
For Moore machine, $\exists f : s_i \rightarrow Y$, we can easily associate a boolean function and input x such that $bool(s_i, x) = f(s_i)$, the new relation of output with x and s_i implies it's a mealy machine now.

Prove that any Mealy machine can be transformed into an equivalent Moore machine:
We again use a construction to prove any mealy machine can be transformed.
$\forall s_i, x_i, f(x_i, s_i)$ where x_i is input ,f()is output, we define new states:
$s_i' = s_i x_i$, $s_i'' = s_i \bar{x}_i$.
Let $f'(s_i') = f(s_i, x_i), f''(s_i') = f(s_i, \bar{x}_i)$

Now we get a moore machine with the same evaluation of the mealy machine.

Program explaination:

> see the `main.py` which implement the functionality specified in the assignment.

Notice:
The count of minterms in s_i is count of 1 in it. Because the min-terms are those $bool_{s_k}$ such that $S_0(i, j) = 1$

Usage:

```
# change the fsm matrix in main.py
fsm = [ [1,1,1],
        [0,0,0],
        [2,2,2]]

python main.py
```

Test:

```
python main.py

The input matrix is:
[2, 3, 1, 0]
[1, 3, 2, 1]
[2, 3, 3, 1]
[3, 3, 3, 3]


The s0 matrix is:
[0, 1, 1, 0]
[1, 1, 0, 1]
[0, 1, 1, 1]
[1, 1, 1, 1]
The count of min-terms in s0 is 12

The s1 matrix is:
[1, 1, 0, 0]
[0, 1, 1, 0]
[1, 1, 1, 0]
[1, 1, 1, 1]
The count of min-terms in s1 is 11
```