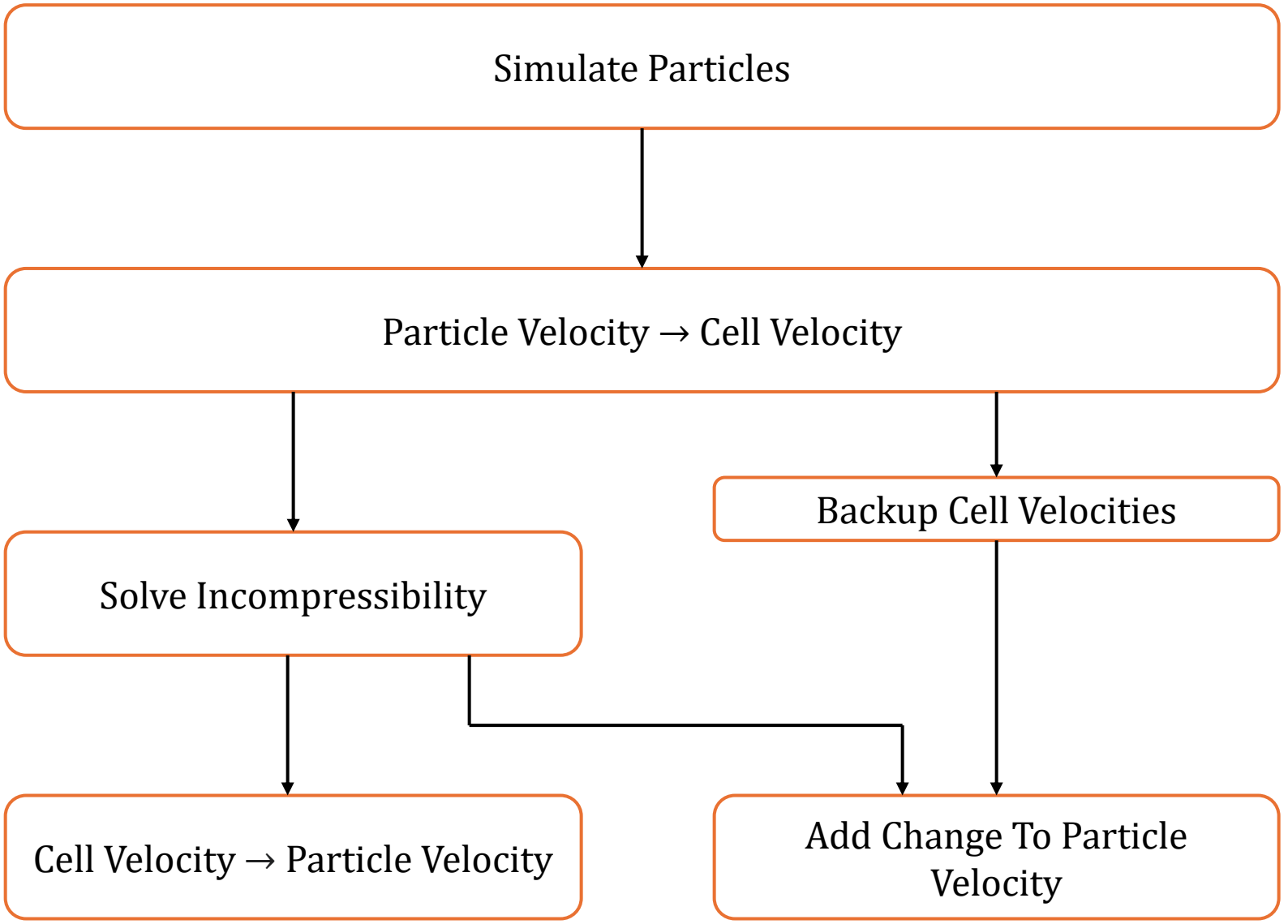
HW3
2D Fluid Simulation

Demo

PIC/FLIP Pipeline


Simulate Particles
Particle Velocity → Cell Velocity
Backup Cell Velocities
Solve Incompressibility
Cell Velocity → Particle Velocity
Add Change To Particle Velocity
PIC
FLIP

Method Details

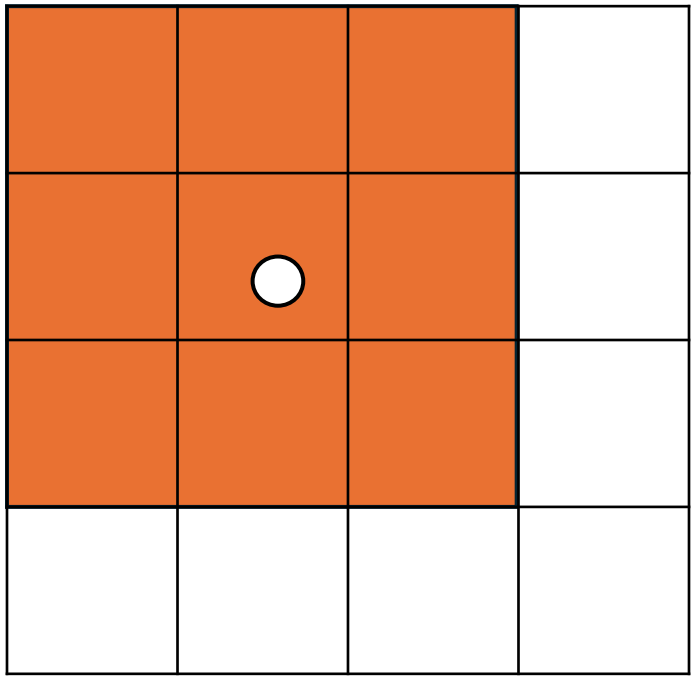
Simulate Particles

Integration

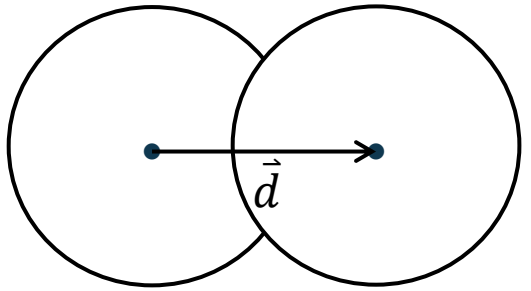
Boundary Collision

Relaxation

Relaxation



3x3 Local Lookup



$$\overrightarrow{p_{corr}} = \frac{1}{2} * \frac{2r - |\vec{d}|}{|\vec{d}|} * \vec{d}$$

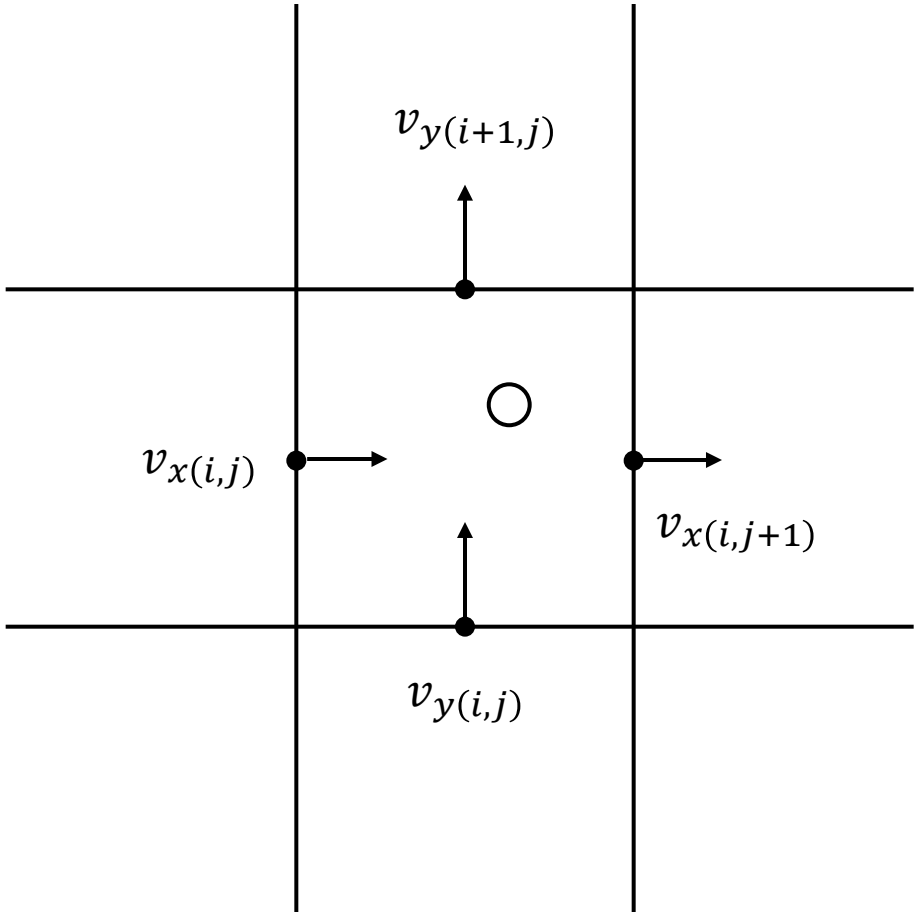
Transferring Velocities

MAC Grid

To Cells

From Cells

MAC Grid

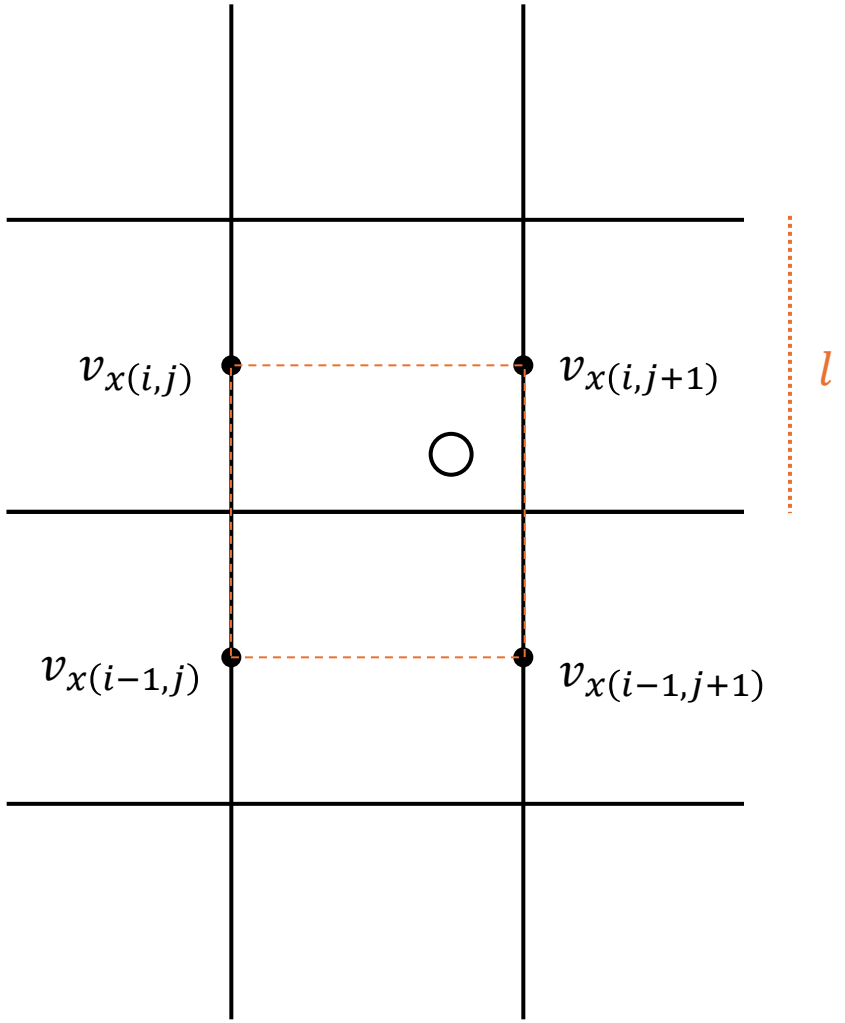


Fluid Cells

Air Cells

Solid Cells

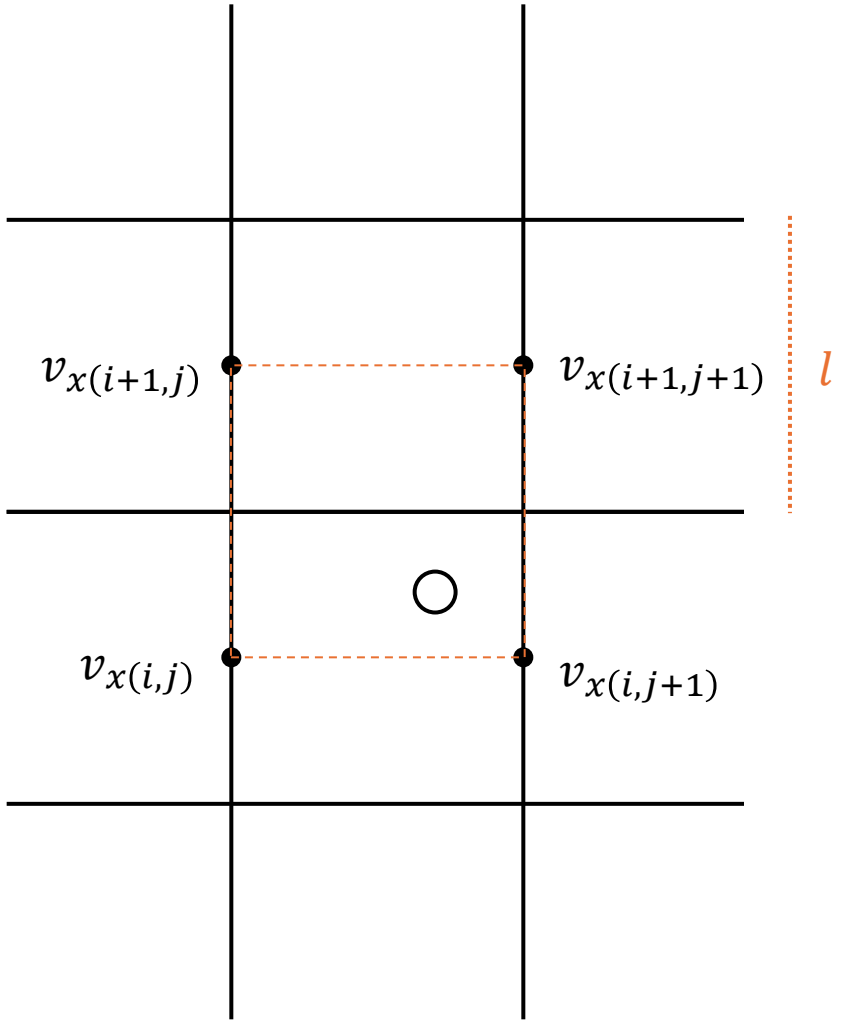
Bilinear Interpolation on MAC Grid



$$(i,j) = (\lfloor \frac{y - \frac{l}{2}}{l} \rfloor, \lfloor \frac{x}{l} \rfloor)$$

Lower Cell Coordinate

Bilinear Interpolation on MAC Grid



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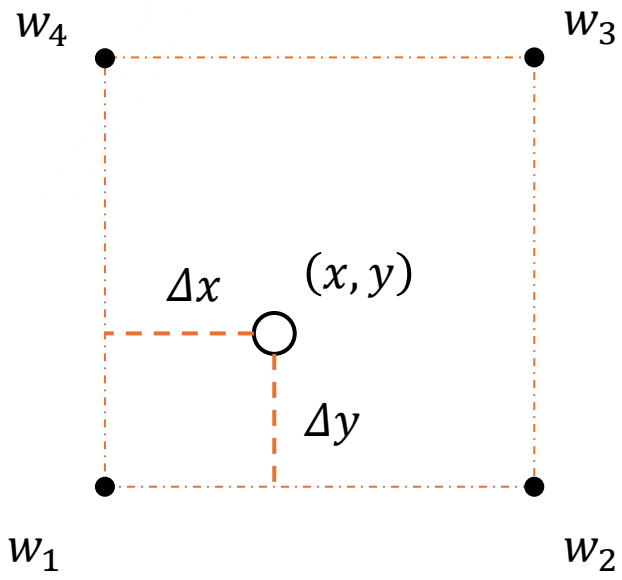
Lower Cell Coordinate

Bilinear Interpolation on MAC Grid



$$\Delta x = x - \lfloor \frac{x_{sample}}{l} \rfloor \qquad \Delta y = y - \lfloor \frac{y_{sample}}{l} \rfloor$$

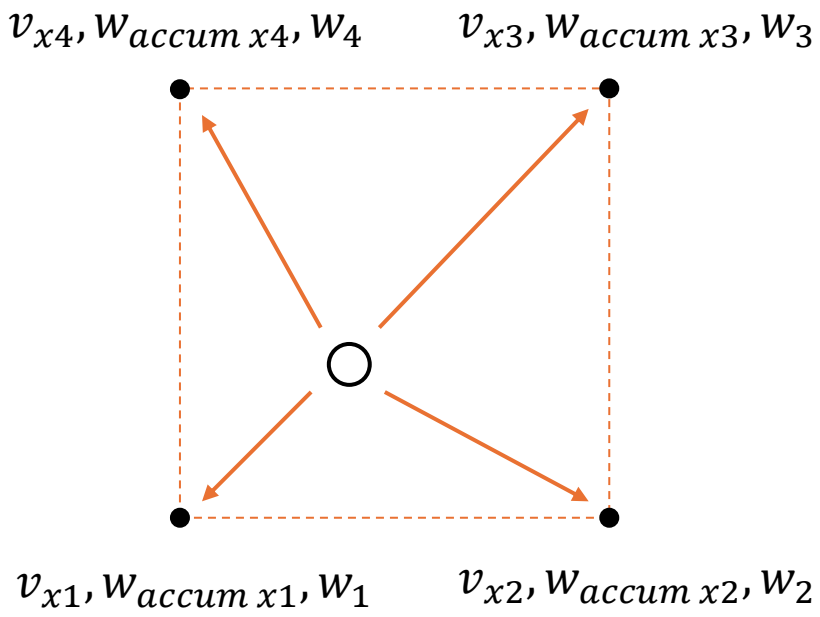
$$w_1 = \left(1 - \frac{\Delta x}{l}\right)\left(1 - \frac{\Delta y}{l}\right)$$

$$w_2 = \left(\frac{\Delta x}{l}\right)\left(1 - \frac{\Delta y}{l}\right)$$

$$w_3 = \left(\frac{\Delta x}{l}\right)\left(\frac{\Delta y}{l}\right)$$

$$w_4 = \left(1 - \frac{\Delta x}{l}\right)\left(\frac{\Delta y}{l}\right)$$

Particle Velocities to Cells



For each particle

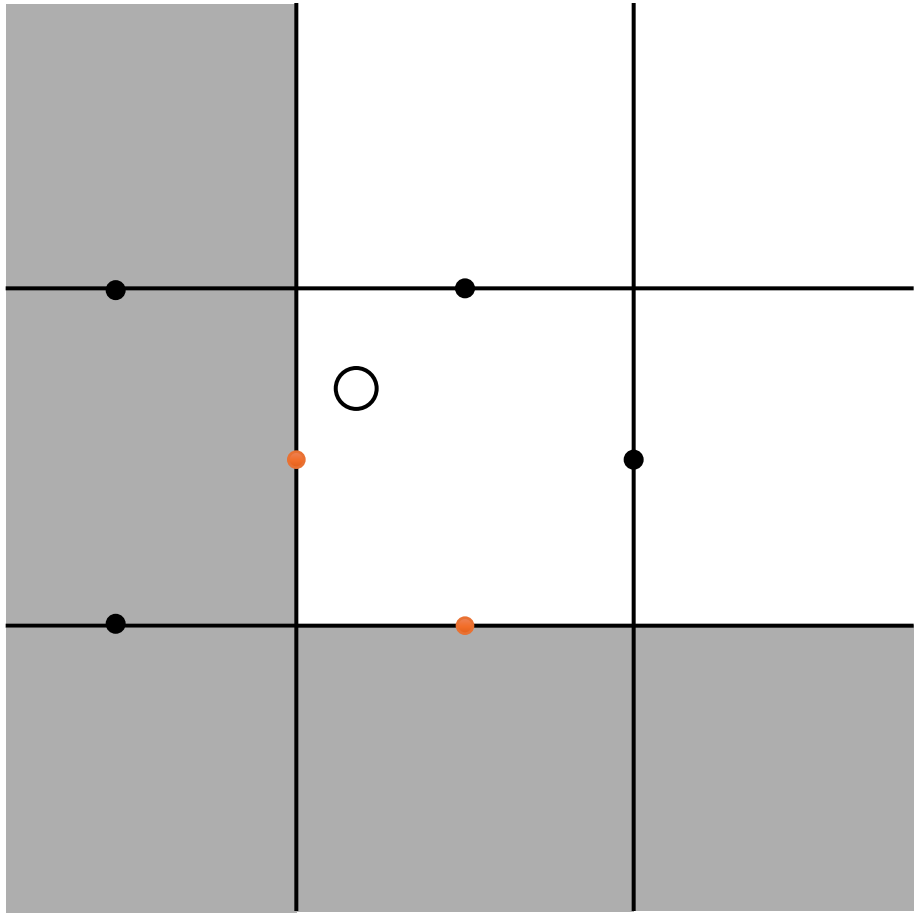
$$v_{xn} \mathrel{+}= v_{px} * w_n$$

$$w_{accum\ n} \mathrel{+}= w_n$$

For each cell

$$v_{xn} = \frac{v_{xn}}{w_{accum\ xn}}$$

Particle Velocities to Cells

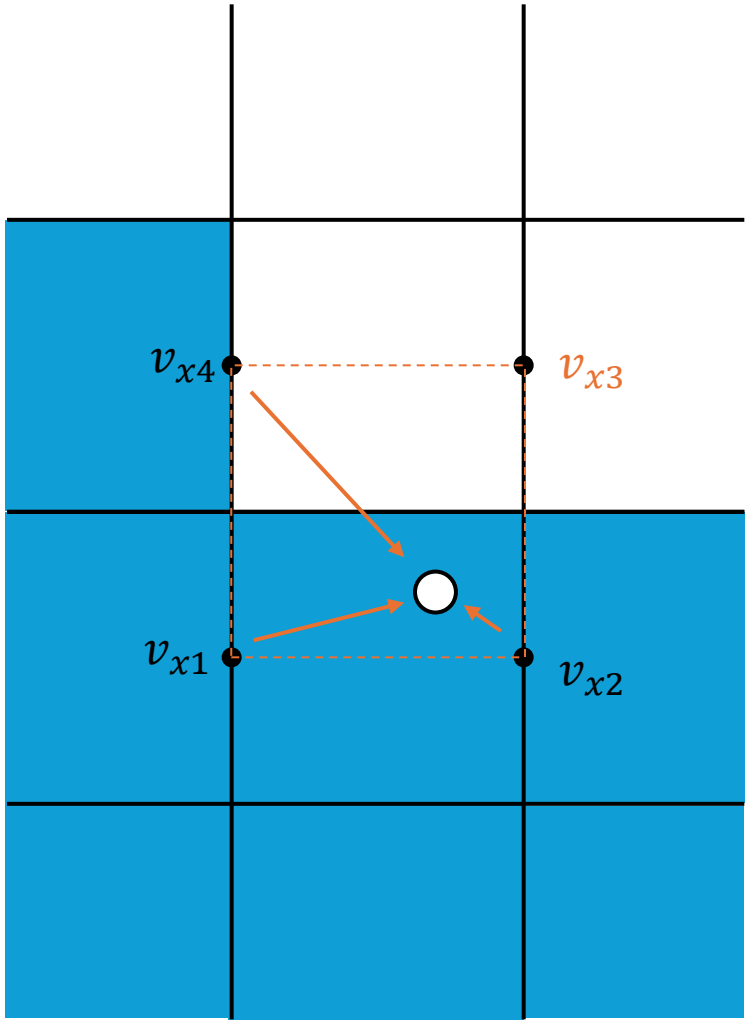


If we set the cell velocity component to zero
→ Particle sticks to ceiling

Remove solid cells on the ceiling or we can keep its velocity but treat it as zero when solving incompressibility

Directly modify particle velocities instead when colliding with outer edges (already implemented for you)

Cell Velocities to Particles



v_{x3} is invalid

$$v_{pic\,x} = \frac{w_1 v_{x1} + w_2 v_{x2} + w_4 v_{x4}}{w_1 + w_2 + w_4}$$

$$v_{flip\,x} = v_{px} + \frac{w_1(v_{x1} - v_{xprev1}) + w_2(v_{x2} - v_{xprev4}) + w_4(v_{x4} - v_{xprev4})}{w_1 + w_2 + w_4}$$

$$v_{px} = \alpha_{flip} v_{flipx} + (1 - \alpha_{flip}) v_{picx}$$

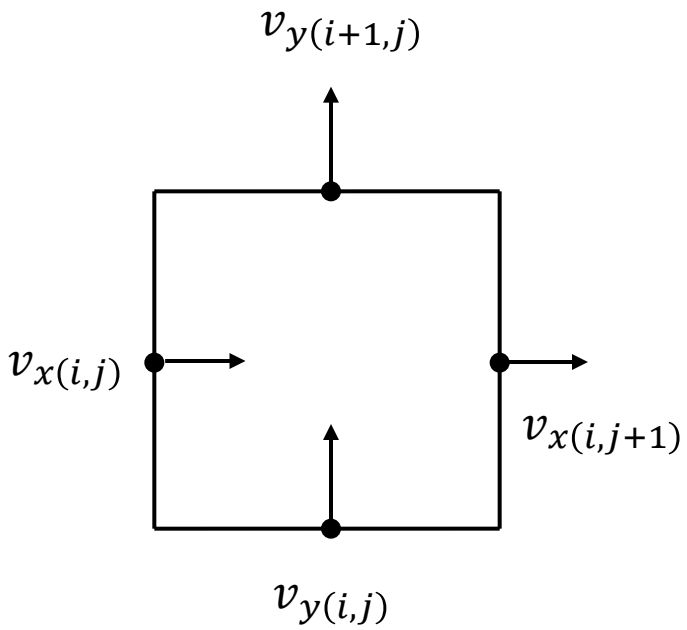
Incompressibility

Solving incompressibility

Density correction

- Cell densities
- Resting density

Solving Incompressibility

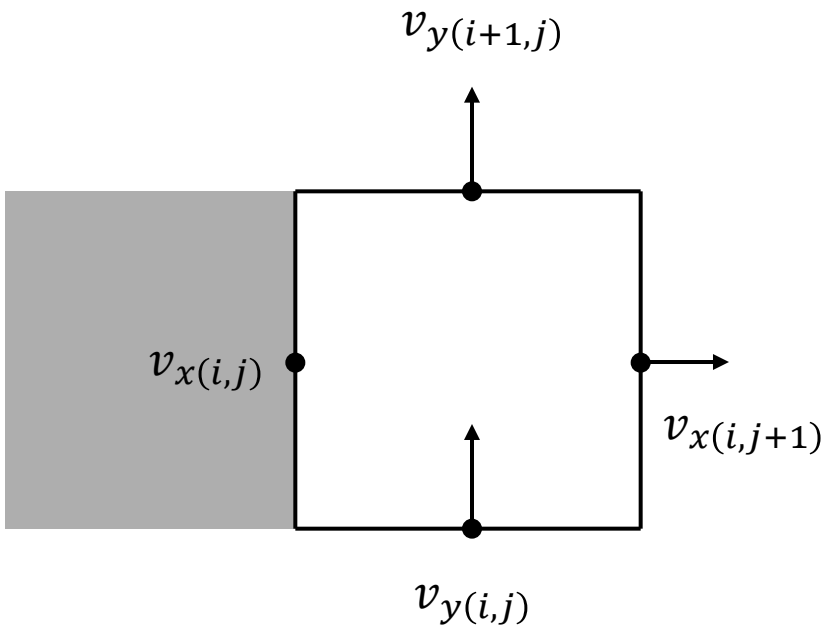


Total inflow should equal total outflow
→ Solve for divergence = 0

$$div = v_{top} + v_{right} - v_{left} - v_{bottom}$$

Bottom and left velocities are inverted when calculating divergence

Solving Incompressibility



Divergence correction should be distributed across available flow directions only

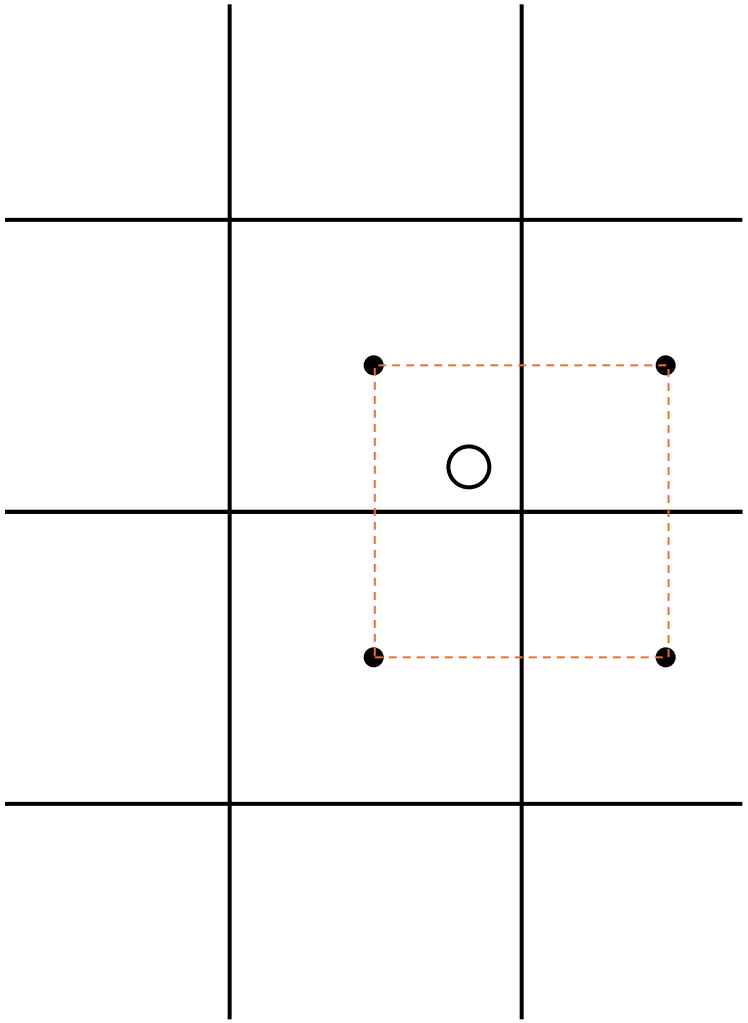
$$v_{corr} = o * \frac{div}{dirs}$$

o: Over relaxation constant to speed up convergence

$$1 < o < 2$$

Add correction to the left and bottom velocities
Deduct correction from the top and right velocities
(In this case, we shouldn't add anything to the left since it is a solid cell)

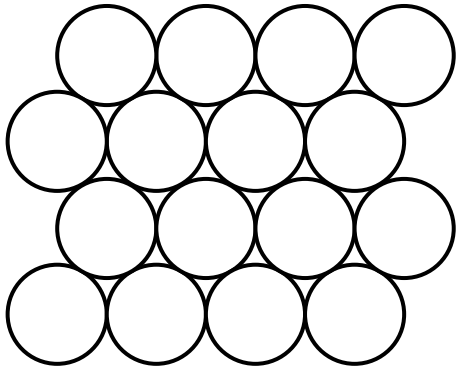
Density correction



Sample cell densities at the center of each cell

Add $1 * w_n$ to each sample corner

Density correction



Initial particle position

Calculate resting density of fluid cells

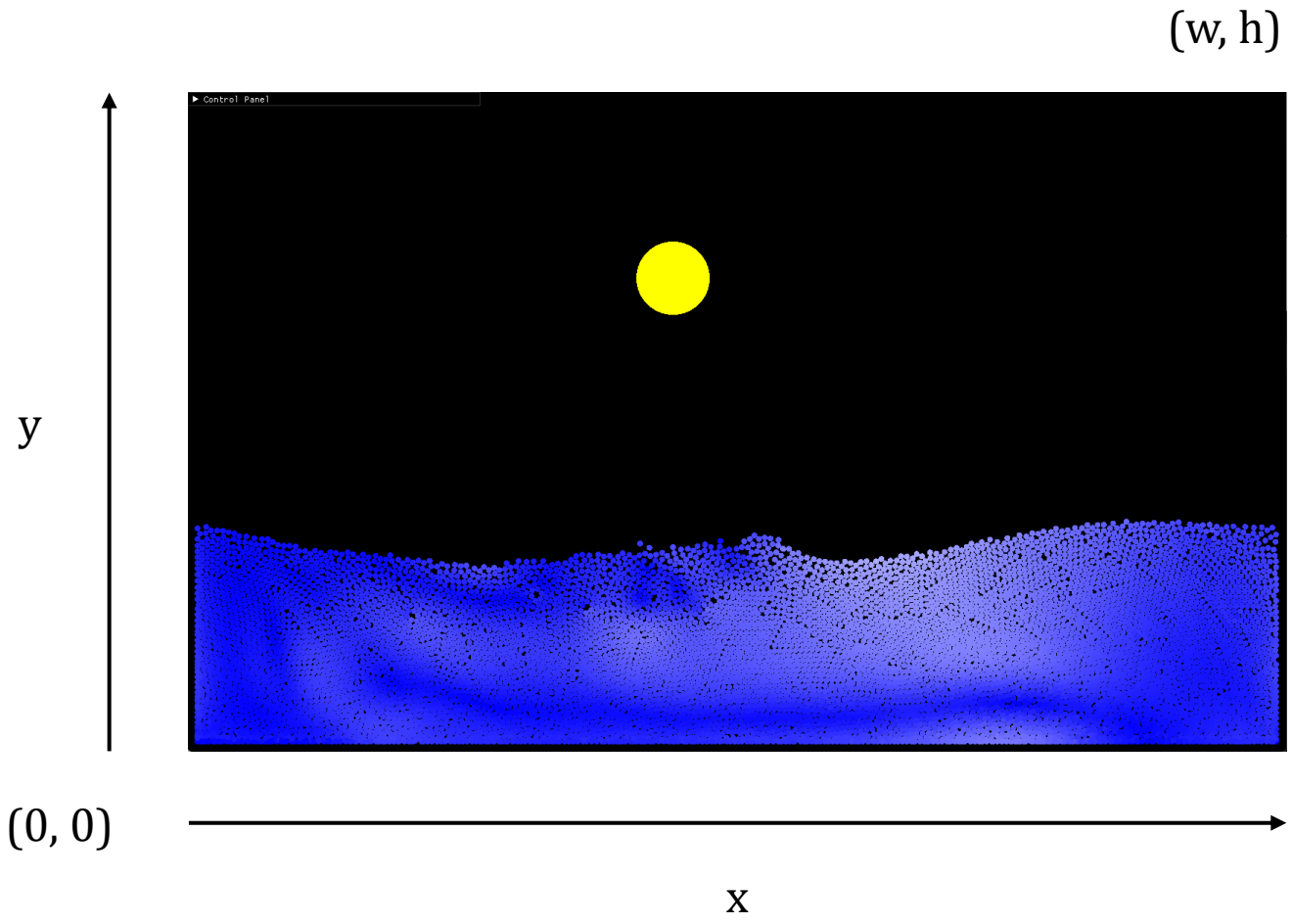
$$\rho_{rest} = \frac{Sum\ of\ fluid\ cell\ densities}{N_{fluid\ cells}}$$

Correct divergence when $\rho_{cell} > \rho_{rest}$

$$div\ -= k_{stiff}(\rho_{cell} - \rho_{rest})$$

Assignment Specifics

Scene coordinate setup



2d vectors are still stored in row major order.

Cell of particle at (x, y)
Cell[$\lfloor \frac{y}{l} \rfloor$][$\lfloor \frac{x}{l} \rfloor$]

To-dos

fluid.cpp

- particleRelaxation
- transferVelocities
- updateDensity
- solveIncompressibility

main.cpp

- Enter your student id in the window banner

```
181     // Create window
182     GLFWwindow* window = glfwCreateWindow(windowW, windowH, "Fluid Simulation | ID: 111111111", NULL, NULL);
```

Report

Implementation : Explain the reasoning behind your implementation

Results & Discussion : Experiment with different parameters

- Cell dim (main.cpp)
- Flip ratio
- Stiffness coefficient etc.

Bonus (Optional) : Any creativity

Miscellaneous (Optional) : Briefly go over changes that are not within the To-do sections.

- Variable changes
- Function rewrites etc.

Scoring

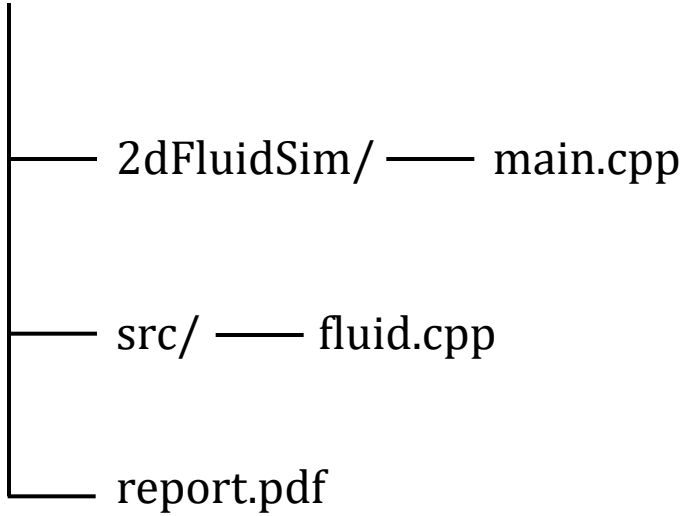
- particleRelaxation (15%)
- transferVelocities (25%)
  - Particle To Cell (10%)
  - Cell To Particle (15%)
- updateDensity & divergence correction (15%)
  - Cell densities (10%)
  - Resting density (2%)
  - Divergence correction (3%)
- solveIncompressibility (25%)
- Report (20%)
- Bonus (Up to 15%)

Submission

Include all files that you have modified.

Construct the submission folder following the project structure.

HW3_111111111_XXX.zip/



Questions

Thanks for listening