

Algorithms Exercise #3

a. Environment

- I. OS: Windows 11
- II. Compiler: GNU GCC Compiler
- III. IDE: Code::Blocks 20.03

b. Results

I. Method:

i. Main function:

- (1) The main function handles input and output.
- (2) Using vector to store the elements of profit table.
- (3) Using while loop to process each testcase and query.

```

20 int main(){
21     int t; //testcase
22     cin>>t;
23     while(t--){
24         int r,c;
25         cin>>r>>c; //row(day), column(course)
26         vector<vector<int>> table(r, vector<int>(c));
27         for(int i=0;i<r;i++){
28             for(int j=0;j<c;j++){
29                 cin>>table[i][j];
30             }
31         }
32         int q; //query
33         cin>>q;
34         while(q--){
35             int d;
36             cin>>d;
37             int ans=maxprofit(r,c,table,d);
38             cout<<ans<<"\n";
39         }
40     }
41     return 0;
42 }

```

ii. Dynamic programming:

- (1) The 'maxprofit' function employs a bottom-up dynamic programming method to solve resource allocation problem.
- (2) It uses 2D vector 'dp' to store intermediate results, the value of $dp[i][j]$ represents the maximum profit of assigning days to i courses.
- (3) Because each course should study at least one day, the size of dp column is $d-c+1$.
- (4) Recursively define the value of an optimal solution by iterative approach. Initialize with the value of the previous row

$$dp[i][j] = dp[i-1][j]$$
- (5) Consider all possible k and update the value using the optimal substructure: $dp[i][j] = \max(dp[i][j], dp[i-1][j-k+1] + table[k-1][i-1])$.
- (6) Termination condition: The result is stored in $dp[c][d-c+1]$

- (7) The reason of using ' $\text{int } a = \min(r, d - c + 1);$ ' and ' $\text{for(int } k = 1; k \leq \min(j, a); k++)$ ' is to deal with the condition that if $r < d - c + 1$ or $d > r * c$, ensuring the proper allocation of the dynamic programming table.

```

6 int maxprofit(int r, int c, vector<vector<int>>&table, int d){
7     int a = min(r, d - c + 1);
8     vector<vector<int>> dp(c + 1, vector<int>(d - c + 2, 0));
9     //dp: row=c, column=d-c+1
10    for(int i = 1; i <= c; i++){
11        for(int j = 1; j <= d - c + 1; j++){
12            dp[i][j] = dp[i - 1][j];
13            for(int k = 1; k <= min(j, a); k++){
14                dp[i][j] = max(dp[i][j], dp[i - 1][j - k + 1] + table[k - 1][i - 1]);
15            }
16        }
17    }
18    return dp[c][d - c + 1];
19 }

```