

Soultion:

From 2.1 we already know that:

$$\vec{a}^{(1)} = W^{(1)}\vec{a}^{(0)} + \vec{b}^{(1)} \quad (1)$$

$$\vec{a}^{(2)} = W^{(2)}\vec{a}^{(1)} + \vec{b}^{(2)} \quad (2)$$

$$\vec{a}^{(3)} = W^{(3)}\vec{a}^{(2)} + \vec{b}^{(3)} \quad (3)$$

Therefore, we have (3):

$$\vec{a}^{(3)} = W^{(3)}\vec{a}^{(2)} + \vec{b}^{(3)} \quad (3)$$

Put (2) into (3), we have:

$$\vec{a}^{(3)} = W^{(3)}(W^{(2)}\vec{a}^{(1)} + \vec{b}^{(2)}) + \vec{b}^{(3)} \quad (4)$$

and we put (1) into (4), we have:

$$\begin{aligned} \vec{a}^{(3)} &= W^{(3)}(W^{(2)}(W^{(1)}\vec{a}^{(0)} + \vec{b}^{(1)}) + \vec{b}^{(2)}) + \vec{b}^{(3)} \\ &= W^{(3)}(W^{(2)}W^{(1)}\vec{a}^{(0)} + W^{(2)}\vec{b}^{(1)} + \vec{b}^{(2)}) + \vec{b}^{(3)} \\ &= W^{(3)}W^{(2)}W^{(1)}\vec{a}^{(0)} + W^{(3)}W^{(2)}\vec{b}^{(1)} + W^{(3)}\vec{b}^{(2)} + \vec{b}^{(3)} \end{aligned}$$

According to this question, we already know the weight and bias of network 1, which means we can know:

$$W^{(3)}, W^{(2)}, W^{(1)}, \vec{b}^{(1)}, \vec{b}^{(2)}, \vec{b}^{(3)}$$

So, according to the definition of this question "two networks are equivalent", we get the answer:

$$\tilde{W} = W^{(3)}W^{(2)}W^{(1)}$$

$$\tilde{b} = W^{(3)}W^{(2)}\vec{b}^{(1)} + W^{(3)}\vec{b}^{(2)} + \vec{b}^{(3)}$$