How to Use a Cross Table to Solve A Interview Question

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Question

- Suppose there are 5 coins. Four of the coins are fair with P(heads)=0.5, and one of the coins is biased with P(heads)=0.75. Suppose a coin is randomly selected and flipped 4 times resulting in 4 heads.

- What is the probability that the selected coin was biased? That is, what is P(biased coin selected | HHHH)?

Note: The following calculation only retains four decimal places for the convenience of calculation

- Baye's formula:

$$P(A|B) = \frac{P(AB)}{P(B)}$$

- Assume:

  A= biased coin selected

  B= HHHH

  👉$P(biased\ coin\ selected \mid HHHH) = P(A|B) = \frac{P(AB)}{P(B)}$

Note: HHHH means flipped 4 times resulting in 4 heads)

Cross Table

5 coins
One biased with P(heads)=0.75
Four fair with P(heads)=0.5

		A=biased with coin	$\bar{A}$=fair with coin	Row Total
B=HHHH	cell prob row prob col prob	P(AB)	P($\bar{A}$B)	
$\bar{B}$=non-HHHH	cell prob row prob col prob	$P(A\bar{B})$	$P(A\bar{B})$	
Column Total		P(biased)=P(A)	P(fair)=P($\bar{A}$)	1

Cross Table

$$P(B) = P(AB)+P(\bar{A}B)$$
$$P(\bar{B})=P(A\bar{B}) + P(\bar{A}\bar{B})$$

		A=biased with coin	$\bar{A}$=fair with coin	Row Total
B=HHHH	cell prob row prob col prob	$0.2 * (0.75^4) = 0.0633$	$0.8 * (0.5^4) = 0.05$	P(HHHH) $=P(B) = P(AB)+P(\bar{A}B)$
$\bar{B}$=non-HHHH	cell prob row prob col prob	$0.2 * (1 - 0.75^4) = 0.1367$	$0.8 * (1 - 0.5^4) = 0.75$	P(non-HHHH) $=P(\bar{B})=P(A\bar{B}) + P(\bar{A}\bar{B})$
Column Total		P(biased)=P(A)	P(fair)=P($\bar{A}$)	1

Cross Table

$$row\ prob = \frac{cell\ prob}{corresponding\ row\ prob}$$

		A=biased with coin	$\bar{A}$=fair with coin	Row Total
B=HHHH	cell prob	$0.2 * (0.75^4) = 0.0633$	$0.8 * (0.5^4) = 0.05$	P(HHHH)= $0.0633 + 0.05 = 0.1133$
	row prob			
	col prob			
non-HHHH	cell prob	$0.2 * (1 - 0.75^4) = 0.1367$	$0.8 * (1 - 0.5^4) = 0.75$	P(non-HHHH)= $0.1367 + 0.75 = 0.8867$
	row prob			
	col prob			
Column Total		P(biased)= $\frac{1}{5} = 0.2$	P(fair)=$\frac{4}{5} = 0.8$	1

Cross Table

$$col\ prob = \frac{cell\ prob}{corresponding\ col\ prob}$$

		A=biased with coin	$\bar{A}$=fair with coin	Row Total
B=HHHH	cell prob	$0.2 * (0.75^4) = 0.0633$	$0.8 * (0.5^4) = 0.05$	P(HHHH)= $0.0633 + 0.05 = 0.1133$
	row prob	$\frac{0.0633}{0.1133} = 0.5587$	$\frac{0.05}{0.1133} = 0.4413$	
	col prob			
non-HHHH	cell prob	$0.2 * (1 - 0.75^4) = 0.1367$	$0.8 * (1 - 0.5^4) = 0.75$	P(non-HHHH)= $0.1367 + 0.75 = 0.8867$
	row prob	$\frac{0.1367}{0.8867} = 0.1542$	$\frac{0.75}{0.8867} = 0.8458$	
	col prob			
Column Total		P(biased)= $\frac{1}{5} = 0.2$	P(fair)=$\frac{4}{5} = 0.8$	1

		A=biased with coin	Fair with coin	Row Total
B=HHHH	cell prob	$0.2 * (0.75^4) = 0.0633$	$0.8 * (0.5^4) = 0.05$	P(HHHH)= 0.0633 + 0.05 = 0.1133
	row prob	$\frac{0.0633}{0.1133} = 0.5587$	$\frac{0.05}{0.1133} = 0.4413$	
	col prob	$\frac{0.0633}{0.2} = 0.3165$	$\frac{0.05}{0.8} = 0.0625$	
non-HHHH	cell prob	$0.2 * (1 - 0.75^4) = 0.1367$	$0.8 * (1 - 0.5^4) = 0.75$	P(non-HHHH)= 0.1367 + 0.75 = 0.8867
	row prob	$\frac{0.1367}{0.8867} = 0.1542$	$\frac{0.75}{0.8867} = 0.8458$	
	col prob	$\frac{0.1367}{0.2} = 0.6835$	$\frac{0.75}{0.8} = 0.9375$	
Column Total		P(biased)= $\frac{1}{5} = 0.2$	P(fair)=$\frac{4}{5} = 0.8$	1

$$P(\textit{biased coin selected} \mid HHHH) = P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.0633}{0.1133} = 0.5587$$