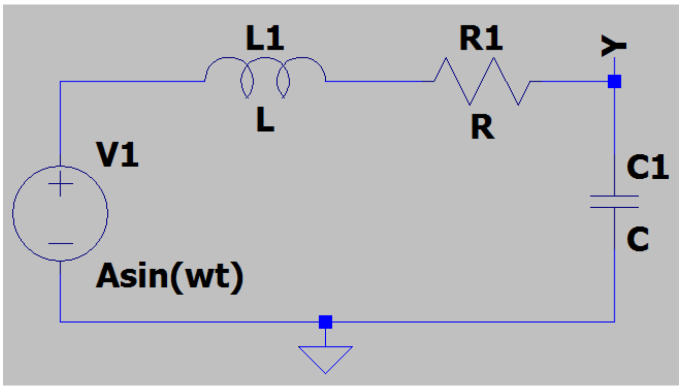
RLC Transfer Function

Monday, March 16, 2020 10:20 AM



Finding The State Space Formula from above circuit, with input is the V1 (sinusoidal) and the output we want to analyze are Y (C1 Voltage) and current in the circuit.

First we should understand the equation of state space which are:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Where, A, B, C, D is matrix. The u is the input, x is the state space variable, and y is the output. Since we want to know the Y output in the circuit, we can decide our state space variable is V_c and I. So :

$x_1 = V_c$; $x_2 = I$;

Finding $\dot{x_1}$:

$$I_c = C\frac{dV_c(t)}{dt}$$

$$\frac{dV_c(t)}{dt} = \frac{1}{C}I_c \ ; \ I_c = I = x_2 \ ; \quad \dot{\boldsymbol{x_1}} = \frac{\mathbf{1}}{\boldsymbol{c}}\boldsymbol{x_2};$$

Finding $\dot{x_2}$:

$$V = V_L + V_R + V_c$$

$$V = L\frac{di(t)}{dt} + IR + V_c$$

$$\frac{di(t)}{dt} = \left(\frac{V}{L} - \frac{IR}{L} - \frac{V_c}{L}\right) \ ; \ V = u \ ; I = x_2 \ ; \ V_c = x_1$$

$$\boldsymbol{\dot{x_2}} = -\frac{\mathbf{1}}{\boldsymbol{L}}\boldsymbol{x_1} - \frac{\boldsymbol{R}}{\boldsymbol{L}}\boldsymbol{x_2} + \frac{\mathbf{1}}{\boldsymbol{L}}\boldsymbol{u}$$

Finding y_1 and y_2 is simple since it is same with x_1 and x_2:

$y_1 = V_c$; $\boldsymbol{y_1} = \boldsymbol{x_1}$

$\boldsymbol{y_2} = \boldsymbol{x_2}$;

By put the $\dot{\boldsymbol{x_1}}$ and $\dot{\boldsymbol{x_2}}$, $\boldsymbol{y_1}$ and $\boldsymbol{y_2}$ to the state space equation formula we got:

$$A = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}; B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Bu; \ \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = C \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Du; \ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = C \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u;$$

Transfer function is the Laplace Transform of the State Space Equation.

From:
$\dot{x} = Ax + Bu$
$y = Cx + Du$;

$sX(s) = AX(s) + BU(s)$
$BU(s) = sX(s) - AX(s); \ \ BU(s) = X(s)[Is - A];$
$X(s) = \ [sI - A]^{-1}BU(s)$

$Y(s) = CX(s) + DU(s)$; Substitute $X(s)$ from above
$Y(s) = C[sI - A]^{-1}BU(s) + DU(s)$; Separate U(s)
$Y(s) = U(s)(C[sI - A]^{-1}B + D)$

$$\boldsymbol{\frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D}$$

Based on the state space we got from the previous page:

$$A = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}; B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$M = \ [sI - A]^{-1}$

$$M = \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \right]^{-1} = \begin{bmatrix} S & -\frac{1}{C} \\ \frac{1}{L} & S + \frac{R}{L} \end{bmatrix}^{-1} = \frac{\begin{bmatrix} S + \frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & S \end{bmatrix}}{det \begin{bmatrix} S & -\frac{1}{C} \\ \frac{1}{L} & S + \frac{R}{L} \end{bmatrix}} = \frac{LC}{s^2LC + sRC + 1} \begin{bmatrix} S + \frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & S \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = CMB + D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{LC}{s^2LC + sRC + 1} \begin{bmatrix} S + \frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & S \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} \frac{1}{(s^2LC + sRC + 1)} \\ \frac{SC}{(s^2LC + sRC + 1)} \end{bmatrix}$$