RLC Transfer Function

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Transfer function is the Laplace Transform of the State Space Equation.

From:
$\dot{x} = Ax + Bu$
$y = Cx + Du$;

$sX(s) = AX(s) + BU(s)$
$BU(s) = sX(s) - AX(s);\ \ BU(s) = X(s)[Is - A];$
$X(s) = [sI - A]^{-1}BU(s)$

$Y(s) = CX(s) + DU(s)$; Substitute $X(s)$ from above
$Y(s) = C[sI - A]^{-1}BU(s) + DU(s)$; Separate U(s)
$Y(s) = U(s)(C[sI - A]^{-1}B + D)$

$$\boldsymbol{\frac{Y(s)}{U(s)} = C[sI - A]^{-1}B + D}$$

Based on the state space we got from the previous page:

$$A = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}; B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M = [sI - A]^{-1}$$

$$M = \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}\right]^{-1} = \begin{bmatrix} S & -\frac{1}{C} \\ \frac{1}{L} & S + \frac{R}{L} \end{bmatrix}^{-1} = \frac{\begin{bmatrix} S + \frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & S \end{bmatrix}}{det \begin{bmatrix} S & -\frac{1}{C} \\ \frac{1}{L} & S + \frac{R}{L} \end{bmatrix}} = \frac{LC}{s^2LC + sRC + 1} \begin{bmatrix} S + \frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & S \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = CMB + D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{LC}{s^2LC + sRC + 1} \begin{bmatrix} S + \frac{R}{L} & \frac{1}{C} \\ -\frac{1}{L} & S \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} \frac{1}{(s^2LC + sRC + 1)} \\ \frac{SC}{(s^2LC + sRC + 1)} \end{bmatrix}$$