Nine colours

Mixed methods

Prerequisite knowledge

- Properties of a cube
- Ability to visualise and talk about 3-D shapes

Why do this problem?

There are a number of strategies that can be adopted to be successful in completing this problem but, as has been said several times in this book, it is the journey and the discussion of the route that different groups take rather than actually finding the solution which is important. It is a very effective context within which to discuss cubes and their properties, cube numbers and so on.

Time

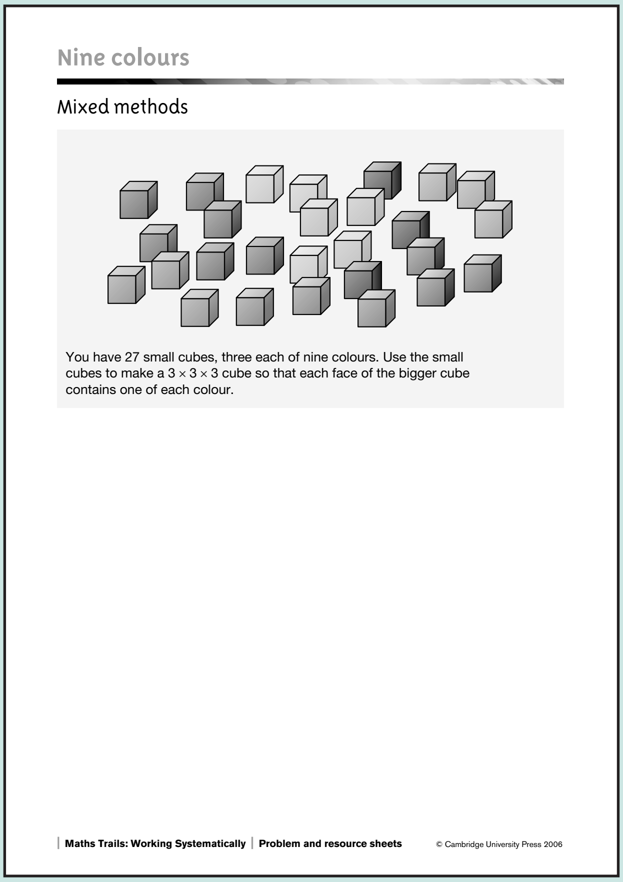
One lesson

Resources

CD-ROM: problem sheet

NRICH website (optional): www.nrich.maths.org, April 2001, 'Nine colours'

A3 sheets of paper for posters, coloured cubes


Nine colours

Mixed methods

You have 27 small cubes, three each of nine colours. Use the small cubes to make a $3 \times 3 \times 3$ cube so that each face of the bigger cube contains one of each colour.

| Maths Trails: Working Systematically | Problem and resource sheets © Cambridge University Press 2006

Introducing the problem

It is important that pupils have a good sense of the properties of a cube and a cube constructed of 27 smaller cubes ($3 \times 3 \times 3$) in order to be able to communicate ideas effectively with other pupils. An introduction which asks questions to refresh these ideas is therefore recommended:

- How many faces does the large (or any) cube have?
- How many edges?
- How many vertices?
- How many small cubes make the large cube?
- How many small faces altogether?
- How many small edges?
- How many small vertices?
- How many small faces are visible?

Main activity

Hand out the cubes, explain the task and give the pupils ten to fifteen minutes to experiment.

Stop to talk about strategies and what pupils may have noticed as useful in searching for a solution.

Useful questions to elicit possible strategies:

- How many small faces will be visible for each colour cube? ($6 = 54 \div 9$)
- How many cubes will have no faces visible, 1 face etc?
- Where are the cubes with 0, 1, ... faces visible?

From this, we know that the cube in the middle has no faces showing, so the other two cubes of that colour must be at two vertices of the large cube (so that 6 faces of this colour are visible).

- Can you have the same colour twice in each layer of the cube?
- Is it useful to consider taking three sets of three colours and ordering in all possible arrangements? For example,

  Y, G, R
  R, Y, G
  G, R, Y

Each of these methods is systematic.

If a group happens upon a solution the question would be whether they could do it again using their strategy – in fact this is worth a try!

If a group finishes, they should be encouraged to produce a poster that explains their method. The aim is that the method is sufficiently systematic to be reproducible. Any group that achieves an outcome without being systematic may be able to look at their solution to unravel a method, rather than just present an answer.

There is an interactive version of this problem on the NRICH website.

Plenary

Give pupils the opportunity to discuss the methods used and view each other's posters.

Ask the group to imagine a $2 \times 2 \times 2$ cube made with smaller cubes, two each of four colours. Can they visualise the solution?

For a finale ask the group to visualise a $4 \times 4 \times 4$ cube made of 16 colours (why 16?).

- Think about the number of cubes with different faces showing – how many are there of each?
- This time there are 8 cubes in the middle. What does this imply about the vertices of the larger cube?

Solution notes

One approach is to make a 3×3 square using all the nine colours for the bottom face.

Their position does not matter.

For each colour, move up one level, right one space (if the colour is on the far right already, then move to the left row), and backward one space (if the colour is on the back already, then move it to the front row).

Repeat the same procedure to fill the top level.

This ensures that there is one of each colour in each row and column, and on each level.

Because of this, each of the faces can only contain one of each colour. So, considering a whole face, all of the colours will just shift one place in each of the two other directions.