IDEA 85

EXTENDING SEQUENCES BACKWARDS

There are many instances of exercises where students are either asked to fill in the missing numbers or to extend a sequence by a given number of terms. An extension of this is to extend sequences backwards as well as forwards.

Extending linear sequences backwards provides a context for students to work with negative numbers. For example, by extending the sequence 4, 7, 10, 13, 16 backwards by five terms we have 16, 13, 10, 7, 4, _, _, _, _, _, leading to the values, 16, 13, 10, 7, 4, 1 –2, –5, –8, –11.

By extending a quadratic sequence backwards, students can see the symmetry involved. For example, taking the quadratic sequence 1, 3, 6, 10, 15 backwards by five terms produces, 10, 6, 3, 1, 0, 1, 3, 6, 10, 15.

Extending the Fibonacci sequence backwards by several terms produces 13, –8, 5, –3, 2, 1, 0, 1, 1, 2, 3, 5, 8, 13. An interesting oscillation occurs.

What happens when we extend the numbers 1, 3, 4, 7, 11 from the Lucas sequence backwards?

What happens when we extend powers of two backwards?

Suppose we make up a sequence based upon two values of a and b, where to find the next value in the sequence we calculate $2a + b$. So a sequence starting with 2 and 3 creates the following: 2, 3, 7, 13, 27, 53 . . . What happens when we reverse this sequence? Suppose the rule was 2a – b?

IDEA 86

EXPLORING LINEAR SEQUENCES

This idea is based upon the knowledge that a linear sequence is 'nothing other' than a positive or a negative shift from an ordered list of multiples (commonly known as a multiplication 'table').

For example, the sequence 7, 11, 15, 19, 23 . . . is similar to the sequence 4, 8, 12, 16, 20 . . . The only difference is each term of the first sequence is three more than each number in the second sequence.

As the second sequence is multiples of four, the first sequence must be multiples of four plus three.

A further conceptual leap is for learners to appreciate that if the general term for the second sequence of numbers is $4n$, then the general term for the first sequence is $4n + 3$.

A task to help students practise and consolidate this knowledge is to set up a situation that works in reverse, where students set up questions and ask a partner to find solutions. The idea works as follows:

- Each student writes several lists of multiples, for example 5, 10, 15, 20, 25 . . .
- They also write the general term for each sequence, for example $5n$.
- For each sequence students add or subtract constant values, for example, by subtracting 2 from each number in the sequence we gain 3, 8, 13, 18, 23 . . . Again students write the general term for each sequence they create, for example, $5n - 2$.
- Students then swap their lists of sequences and have to work out the general terms for each sequence they are presented with.
- Students can self-check each other's answers.

This situation is ripe for students to draw graphs by making each sequence into a list of coordinate pairs and seeing for themselves the pairs of graphs create parallel lines.