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SYMMETRY IN 3D

This problem is one I have used dozens of times and recently began talking about during a meal with some mathematics advisors . . . a typical meal-time conversation when mathematicians get together! I was intrigued to see just how challenging the problem became and how keen my colleagues were to ascertain the 'right' answer.

The question is again simple to pose yet holds hidden complexities. I might wish to use the problem as a mind-imagery task or I might want students to make the solids under consideration and use the models to work on the task. I might do both of these, starting with an imagery problem and later allowing students to use solids. My initial question is:

'How many planes of symmetry does a cube (or a hexahedron) have?'

I want students to explain where planes of symmetry slice through the cube and describe the cross-sectional shapes formed by different planes of symmetry.

Further questions can be:

- What planes of symmetry does an octahedron have?
- What connections exist between the planes of symmetry of a cube and the planes of symmetry of an octahedron?
- Where are the axes of rotational symmetry of a cube and what is the order of rotational symmetry for each axis?
- Where are the axes of rotational symmetry of an octahedron and what is the order of rotational symmetry for each axis?

Making the solids and showing what cross-sectional shapes are formed, and using straws to show axes of rotational symmetry, can provide students with interesting challenges.

IDEA

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SHAPES FROM FOUR LINKING CUBES

Drawing 3D shapes on isometric paper is a skill some students can do intuitively while others require careful instructions to help them become confident. We might make good use of the teaching strategy of students who 'can' helping those who 'cannot', however, being able to represent 3D shapes in a 2D picture is only a means to an end. Providing students with problems where drawing diagrams may help in the solution is more important.

One is to give students just four linking cubes and ask them to find all the different shapes by joining them together and to draw the different solutions they find. Discounting rotations there are less than a dozen possible solutions.

This simple-to-pose problem has several aspects; further questions can be raised such as:

- What does 'different' mean in relation to the shapes made?
- What symmetries does each shape possess?
- What different surface areas are there for the shapes?
- Can students prove they have found a complete set of shapes?

If a whole-class approach to the last question is felt to be desirable, this problem might be a useful context for engaging all the students in discussions about the notion of proof. One approach might be to invite individuals to draw their solutions on the screen/board and use this collection of results to discuss how a proof about completeness might be developed.