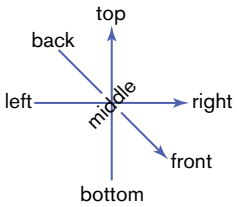
Noughts and crosses

Problem sheet

Imagine a $3 \times 3 \times 3$ cube, made up from 27 unit cubes, all of which are made from clear plastic that can be filled with ease.

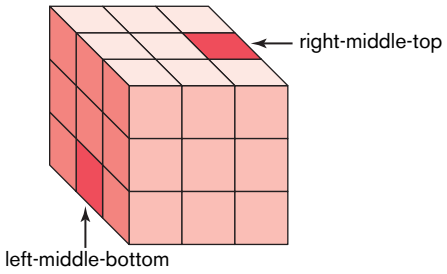
A unit cube is located according to its position with respect to the three axes or directions, given in order:

- left, middle, right;
- front, middle, back;
- top, middle, bottom.



A marble is placed in the unit cube at left-middle-bottom.

Another is placed at right-middle-top.



Where should the third marble be placed to make a winning line of three marbles?

How many winning lines go through middle-middle-middle?

How many different types of winning lines are there?

How many winning lines are there altogether?

Extension activities

How many winning lines of four are there altogether in a $4 \times 4 \times 4$ cube?

How many winning lines of n are there altogether in an $n \times n \times n$ cube?

How many winning lines of three are there altogether in a $4 \times 4 \times 4$ cube?