Tree tops

Modelling and optimisation

Prerequisite knowledge

- Experience of producing a model using a spreadsheet
- Formulae in a spreadsheet

Why do this unit?

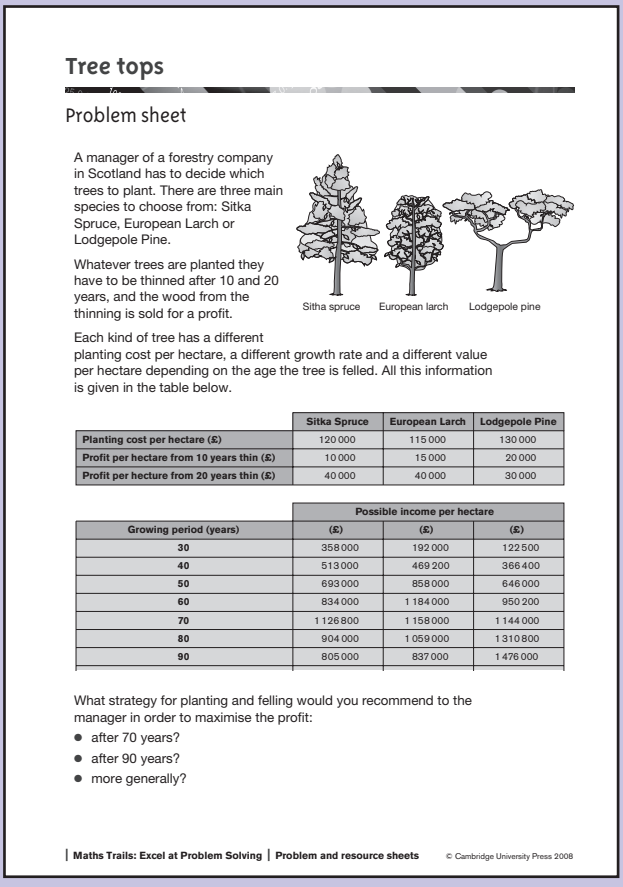
Interpreting the information given on the problem sheet is a significant part of this activity. The problem encourages pupils to experiment with different models. Very basic ideas can lead quickly to useful outcomes and encourage pupils to extend their models. The use of the spreadsheet can encourage a systematic approach to variation and reduce calculations.

Time

Two lessons

Resources

CD-ROM: spreadsheet, problem sheet
NRICH website (optional):
www.nrich.maths.org, June 2003, 'Tree tops'



Tree tops

Problem sheet

A manager of a forestry company in Scotland has to decide which trees to plant. There are three main species to choose from: Sitka Spruce, European Larch or Lodgepole Pine.

Whatever trees are planted they have to be thinned after 10 and 20 years, and the wood from the thinning is sold for a profit.

Sitha spruce European larch Lodgepole pine

Each kind of tree has a different planting cost per hectare, a different growth rate and a different value per hectare depending on the age the tree is felled. All this information is given in the table below.

	Sitka Spruce	European Larch	Lodgepole Pine
Planting cost per hectare (£)	120 000	115 000	130 000
Profit per hectare from 10 years thin (£)	10 000	15 000	20 000
Profit per hecture from 20 years thin (£)	40 000	40 000	30 000

	Possible income per hectare		
Growing period (years)	(£)	(£)	(£)
30	358 000	192 000	122 500
40	513 000	469 200	366 400
50	693 000	858 000	646 000
60	834 000	1 184 000	950 200
70	1 126 800	1 158 000	1 144 000
80	904 000	1 059 000	1 310 800
90	805 000	837 000	1 476 000

What strategy for planting and felling would you recommend to the manager in order to maximise the profit:

- after 70 years?
- after 90 years?
- more generally?

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Introducing the unit

Present the problem given on the problem sheet.

Ask the pupils to spend a few minutes on their own making sense of the problem, writing notes if it helps them clarify their ideas. After two or three minutes, ask pupils to talk in pairs about what they have noticed and what they might try first, telling them that you will want to share their ideas in five minutes…
Think – Pair – Share

The aim is for pupils to notice issues such as a planting of 20 years or less is not profitable and the minimum time for trees to grow and make a profit needs to be 30 years.

Gather ideas and make a list (including the 30-year minimum time for a profit) of possible ways forward on the board. For example:

- Find the profits for each type of tree after 30, 40, 50 years.
- Find the profits for two plantings of a given species over 70, 80, 90, 100 years. (Note that there are only a limited number of possibilities here: 70 = 30 + 40; 80 = 30 + 50 or 40 + 40; 90 = 30 + 60 or 40 + 50; 100 = 30 + 70 or 40 + 60 or 50 + 50 – see 'One species, two plantings' on the spreadsheet).
- Find the profits for two plantings of two different species.

Main part of the unit

Introduce the sheet 'Data' on the spreadsheet. Using the idea of the potential profits from single plantings, invite pupils, in pairs, to discuss, plan and implement a spreadsheet to calculate profits for different time spans. Remind them that they need to produce evidence to support the advice they would give to the manager.

- What will you need to include on your spreadsheet? [how much it costs to plant, the profits from each thinning and the felling, totals for each of the trees for 30, 40, …, 90 years]

Remind pupils that their aim is to present a spreadsheet clearly so that it explains why they are making their recommendation to the manager. As they finish each model they should write their recommendation at the bottom of the sheet, add their names and print it for discussion at the end of the lesson.

As pairs finish the single planting model and are able to make a recommendation, ask them to consider a more complex model of mixed planting over different periods. Again have pupils print their recommendations for discussion later.

During this part of the activity identify models that can inform the discussion in the plenary. For example, models that:

- are elegantly represented using a spreadsheet;
- show a systematic approach;
- are giving high profits based on multiple plantings of one tree species or mixed planting for discussion in the plenary.

Pupils who are struggling with producing models of their own may find it helpful to use the models started in the sheets 'One species, one planting' and 'One species, two plantings' on the spreadsheet (green tabs). If using the prepared spreadsheets, pupils will need to spend time making sense of the model.

Prior to the plenary ask pairs to choose one of their models to share.

Plenary

Have pairs of pupils swap models and talk about their findings. Ask pairs if they were given models and recommendations they were particularly impressed with (or use one or two models you have identified). Discuss what makes these good examples and how they might be improved further, for example by planting more species. Other ideas might include considering mixed plantings so that you could get profits in the shorter and longer term and reduce risk of problems (e.g. disease) that might arise from planting just one species.

Solution notes

It you were to plant a single species, the most profitable tree after 50 years is the Sitka Spruce, but after 70 years it is the European Larch and, for 80 years or more, the Lodgepole Pine.

After 70 years some trees are losing value and after 90 years all trees are losing value.

A better strategy might be to plant the same species successively for less time. For example, two 50-year plantings of Sitka Spruce gives a higher return than any single planting over 100 years. Alternatively, mixing the species and planting for different periods may be more profitable – for example, over 90 years, 30 years of Sitka Spruce and 60 years of European Larch (see 'One species models' on the spreadsheet).