Core 1 January 2009

1 $\sqrt{45} + \frac{20}{\sqrt{5}} = \sqrt{9}\sqrt{5} + \frac{20\sqrt{5}}{\sqrt{5}\sqrt{5}}$

$= 3\sqrt{5} + \frac{20\sqrt{5}}{5} = 3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$

2 i) $(x^{1/3})^6 = x^{6/3} = x^2$

ii) $\frac{3y^4 \times 10^3 y^3}{2y^5} = \frac{3000y^7}{2y^5}$

$= 1500y^2$

3 $3x^{2/3} + x^{1/3} - 2 = 0$

Let $t = x^{1/3}$

$3t^2 + t - 2 = 0$

$(3t-2)(t+1) = 0$

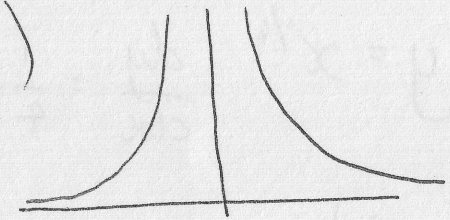
$t = 2/3$ or $t = -1$

$x^{1/3} = \frac{2}{3}$ or $x^{1/3} = -1$

$x = \left(\frac{2}{3}\right)^3$ or $x = (-1)^3$

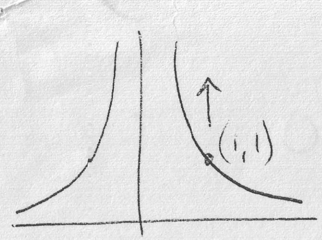
$x = \frac{8}{27}$ or $x = -1$

4 i)



ii) to move by -3 in x you must replace x with $x - -3 = x+3$

$y = \frac{1}{(x+3)^2}$

iii) 

will go to $\underline{(1,4)}$

or replace y with $y/4$

$\frac{y}{4} = \frac{1}{x^2} \quad y = \frac{4}{x^2}$ at $x = 1$ $y = 4$

5 i) $y = 10x^{-5} \quad \frac{dy}{dx} = -50x^{-6}$

ii) $y = \sqrt[4]{x} \quad y = x^{1/4} \quad \frac{dy}{dx} = \frac{1}{4}x^{-3/4}$

iii) $y = x(x+3)(1-5x)$

$y = x(x - 5x^2 + 3 - 15x) = x(-5x^2 - 14x + 3)$

$y = -5x^3 - 14x^2 + 3x$

$\frac{dy}{dx} = -15x^2 - 28x + 3$

6 i) $5(x^2 + 4x) - 8$

$= 5[(x+2)^2 - 4] - 8$

$= 5(x+2)^2 - 20 - 8$

$= \underline{5(x+2)^2 - 28}$

ii) $\underline{x = -2}$

iii) $b^2 - 4ac$

$400 - 4 \times 5 \times -8$

$= 400 + 160 = \underline{560}$

iv) 2 real roots (because the discriminant is greater than 0).

7 i) If $x = 10$

$30 + 4y - 10 = 0$

$4y = {}^{-}20 \quad \therefore \quad y = {}^{-}5$

$k = {}^{-}5$

ii) $\sqrt{(10-2)^2 + (1 - {}^{-}5)^2} = \sqrt{64+36} = \sqrt{100}$

$= 10$

iii) $(6, {}^{-}2)$ radius of 5

iv) The midpoint of AB is

$\left(\frac{10+2}{2}, \frac{1 \; {}^{-}5}{2}\right) = (6, {}^{-}2)$

So the midpoint of AB is then centre of the circle and the length of AB is double the radius.

8 i) Use quadratic formula - be careful!!

$5 - 8x - x^2 = 0$

$a = {}^{-}1 \quad b = {}^{-}8 \quad c = 5$

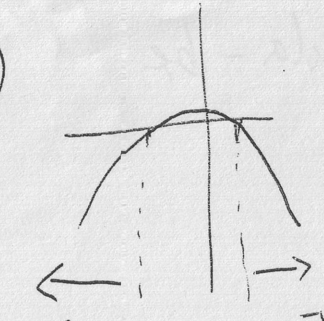
$$x = \frac{- {}^{-}8 \pm \sqrt{({}^{-}8)^2 - (4 \times {}^{-}1 \times 5)}}{-2}$$

$$x = \frac{8 \pm \sqrt{64+20}}{-2} = \frac{8 \pm \sqrt{84}}{-2}$$

$$\frac{8 \pm \sqrt{4}\sqrt{21}}{-2} = \frac{8 + 2\sqrt{21}}{-2} \text{ or } \frac{8 - 2\sqrt{21}}{-2}$$

$$= -4 - \sqrt{21} \text{ or } -4 + \sqrt{21}$$

ii)



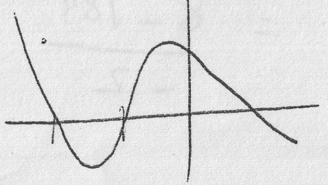
$5 - 8x - x^2 \le 0$

$x \le -4 - \sqrt{21}$

and

$x \geq -4 + \sqrt{21}$

iii) This has one more root than ↑ at $x = -4$ and is a -ve cubic



$(-4-\sqrt{21}, 0)$

$(-4, 0)$

$(-4+\sqrt{21}, 0)$

and $(0, 20)$

(just put 0 into $(5-8x-x^2)(x+4)$
5×4

9) If $y = x^3 + px^2 + 2$

Stationary point at $x = 4$ means $\frac{dy}{dx} = 0$ at $x = 4$

$\frac{dy}{dx} = 3x^2 + 2px = 0$

at $x = 4$ $3 \times 16 + 8p = 0$

$8p = -48$

$p = -6$

$\frac{dy}{dx} = 3x^2 - 12x$

$\frac{d^2y}{dx^2} = 6x - 12$ at $x = 4$

$\frac{d^2y}{dx^2} = 24 - 12 = 12$

Positive so it is a minimum point.

10 i) $y = x^2 + x$

$\frac{dy}{dx} = 2x + 1$ at $x = 2$

$\frac{dy}{dx} = 5$

ii) Gradient of normal is $\frac{-1}{5}$

at $x = 2$ $y = 2^2 + 2 = 6$

$\frac{y-6}{x-2} = \frac{-1}{5} \Rightarrow y - 6 = \frac{-1}{5}(x-2)$

$5y - 30 = -x + 2$

$5y + x - 32 = 0$

iii) If $y = kx - 4$ is a tangent then it meets the curve in 1 place

ie <u>one</u> solution to the simultaneous equation. $y = x^2 + x$ and $y = kx - 4$

(although there will probably be 2 values of k where this happens).

$x^2 + x = kx - 4$

$x^2 + x - kx + 4 = 0 \Rightarrow x^2 + (1-k)x + 4 = 0$

If there is one solution the discriminant $= 0$

$(1-k)^2 - 4 \times 1 \times 4 = 0$

$(1-k)^2 - 16 = 0$

You could expand then factorise

$(1-k)^2 = 16$

$1 - k = \pm 4$

$1 - k = 4$ $\quad$ $1 - k = -4$

$\underline{\underline{k = -3}}$ $\quad$ $\underline{\underline{k = 5}}$