A square deal

Interpretation

Prerequisite knowledge

- Familiarity with the terms square number, cube number, triangular number and prime number
- Knowledge of powers of 2
- Mental methods of addition, subtraction, multiplication and division

Why do this problem?

Having met the idea of interpreting information in a systematic way in ‘A mixed-up clock’, this problem lends itself to a similar approach but in a slightly less structured environment so that pupils’ confidence is increased.

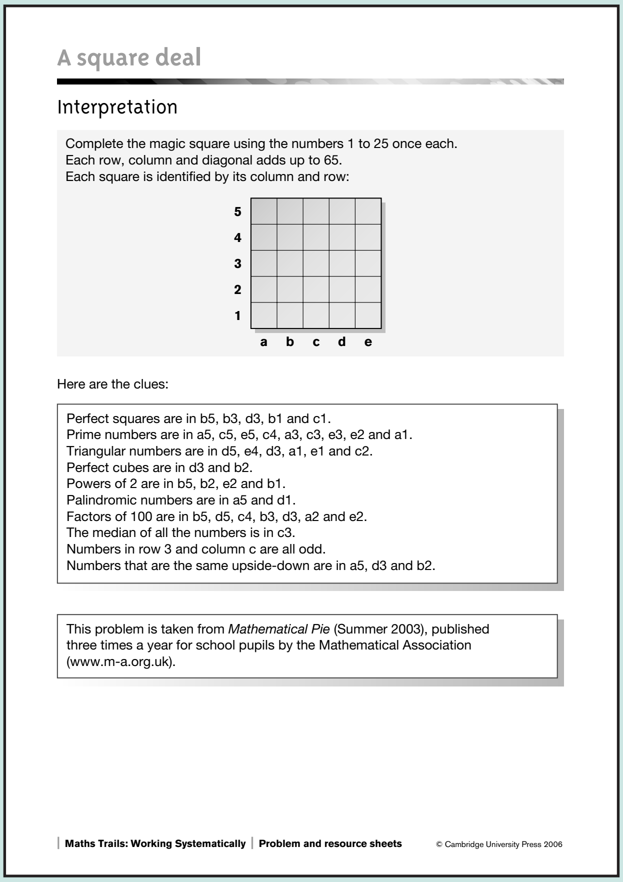
Time

One lesson

Resources

CD-ROM: problem sheet, resource sheet with blank squares

NRICH website (optional): www.nrich.maths.org, October 2003, ‘A square deal’; November 1999, ‘Magic squares 3 × 3’ and ‘Magic squares 4 × 4’


A square deal

Interpretation

Complete the magic square using the numbers 1 to 25 once each.
Each row, column and diagonal adds up to 65.
Each square is identified by its column and row:

Here are the clues:

Perfect squares are in b5, b3, d3, b1 and c1.
Prime numbers are in a5, c5, e5, c4, a3, c3, e3, e2 and a1.
Triangular numbers are in d5, e4, d3, a1, e1 and c2.
Perfect cubes are in d3 and b2.
Powers of 2 are in b5, b2, e2 and b1.
Palindromic numbers are in a5 and d1.
Factors of 100 are in b5, d5, c4, b3, d3, a2 and e2.
The median of all the numbers is in c3.
Numbers in row 3 and column c are all odd.
Numbers that are the same upside-down are in a5, d3 and b2.

This problem is taken from *Mathematical Pie* (Summer 2003), published three times a year for school pupils by the Mathematical Association (www.m-a.org.uk).

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Introducing the problem

Introduce the idea of a magic square to the class by explaining that in a magic square all the rows, columns and diagonals add to the same total and no number appears more than once. Put up two examples of partly completed magic squares (resource sheet provided):

		6
		1
4	3	8

4	9	6	15
			1
11			
5	16	3	10

Ask pupils what the magic square total is in each case and invite them to suggest how they might complete each square.

Main part of the lesson

Show the class the empty 5 × 5 grid and explain that this magic square has a magic total of 65 and uses the numbers 1 to 25 once each. The rows and columns have been labelled so that they can be referred to easily. In this case, none of the numbers are given and, although it is possible to complete a magic square without any further information, there are clues to help complete it.

In a similar way to the problem ‘A mixed-up clock’, give the pupils time to read the clues and ask them to talk to a partner about:

- where they might start;
- what it might be useful to record. This might be writing possible numbers in pencil in the grid itself and/or next to each clue. Using pencil will give the flexibility to rub out options that are ruled out later. For example, the perfect cubes are 1 and 8 and these numbers could be written in both d3 and b2. The first clue tells us that a perfect square must be in d3 so you could cross or rub out 8.

Invite pairs to share their suggestions with the whole class – this might also include discussion about terms such as 'powers of 2' and 'palindromic numbers'. The following prompts and questions may help to lead them into the problem:

- How can we find the median?
- Are we given several clues about any of the squares? If so, can we use this information straight away?
- How will we keep track of the numbers we have used?

Pupils should be encouraged to begin solving the problem in pairs. They will quickly discover that the clues can be used in different orders and that there is no single route to a solution. It may be appropriate to bring everyone together at various stages to talk about particular difficulties or findings.

Plenary

The nature of the problem means that it is self-checking so there is little point in discussing the answer as such. What might be more interesting is to trace back the possible stages in its solution, particularly for pupils who may have made slight errors along the way.

You may like to leave pupils to think about whether the 5 × 5 magic square can have more than one arrangement of its digits. Starting with a simpler case, how many different solutions are there for a 3 × 3 magic square?

Solution notes

The completed square looks like this:

5	11	4	17	10	23
4	24	12	5	18	6
3	7	25	13	1	19
2	20	8	21	14	2
1	3	16	9	22	15
	a	**b**	**c**	**d**	**e**