

Generalising from games and investigations

Prerequisite knowledge

- Simple mental addition and subtraction
- Knowledge of factors and multiples

Why do this problem?

This problem provides a motivating context which encourages pupils to engage in reasoning about numbers. 'Got it now' demonstrates how knowledge of factors and multiples can lead to powerful generalisations and 'winning the game'.

The game of 'Got it' can be easily adapted so that the arithmetic does not get in the way of the generalising. It is a good idea to familiarise yourself with the game beforehand so that you can play strategically.

Time

One or two lessons

Resources

CD-ROM: pupil worksheet

NRICH website (optional):
www.nrich.maths.org, February 2002, 'Got it now' (the interactive version of the game

Got it now

Generalising from games and investigations

The game 'Got it' is a version of a well-known game called 'Nim'.

It is an adding game for two. You play against the computer or against a friend.

Start with a target of 23. Set the range of available numbers from 1 to 4.

Players take turns to add a whole number from 1 to 4 to the running total.

The player who hits the target of 23 wins the game.



Play the game several times. Can you always win?

Can you find a winning strategy?

Does your strategy depend on whether or not you go first?

Change the game by choosing a new target.

Test the strategy you found earlier. Does it need adapting?

Can you work out a winning strategy for any target?

Is it best to start the game? Always?

Change the game again, returning to a target of 23 but using a different range of numbers this time.

Test the strategies you found earlier. Do they need adapting?

Can you work out a winning strategy for any range of numbers?

Is it best to start the game? Always?

Can you work out a winning strategy for any target and any range of numbers?

Maths trails: Generalising | Problem and resource sheets

© Cambridge University Press 2005

makes an engaging starting point); a challenging extension or follow-up activity is the game of 'Last biscuit' (October 1998)

Introducing the problem

Explain how to play 'Got it' and play against the class using 23 as the target number and selecting numbers between 1 and 4 inclusive. Invite pupils to decide whether they go first or second. (It might be useful to play strategically so that you win, thereby giving the class an incentive to investigate the mathematics!)

Play a second time against pupils, asking them to try to identify anything you do that they think is relevant.

After the second game, invite pupils to share their thoughts.

- When did it become obvious that you couldn't win?
- Why was it impossible to win at that stage?

- Did you notice any patterns that you think might be significant?

Main part of the lesson

Challenge pupils to play in pairs against each other, recording the game as they see best. Their aim is to find a winning strategy which they will test by playing against you.

After a suitable length of time (e.g. 10 minutes) bring the class together briefly and invite pairs to share anything interesting they have found which might help them to win the game. The following questions may be useful:

- What total do you have to make to guarantee that you will land on the target number on your next move and win?
- Why?

It is worth commenting at this point:

- So, is this *not* a game about 23, but a game about 18? Or is it a game about something else?

The discussion at this stage will help to focus pupils and will give those who are struggling some extra insights. Following this sharing of ideas, give pupils more time to play the game together and develop their strategies further, stopping to ask:

- How can you make sure you can reach that penultimate total? And the one before that? ...
- What number might you want to start with?

When appropriate, ask one pair to play against you so that they can prove they are able to win. Invite them to explain how they knew they would beat you. Ask other pairs to give details of any different strategies.

Finally, get pupils to work out a winning strategy for any target and any range of numbers. You might like to give some examples, including a target which is a multiple of one more than the highest number available to pick.

Plenary

Play against the pupils (or two pairs against each other) with a target number and range of numbers of someone else's choice.

Use this game to highlight the strategies being used, including the decision about who starts.

A challenging extension or follow-up activity is the game of 'Last biscuit' which can also be found on the NRICH site.

Solution notes

To be sure that you hit the target of 23, you must make sure you get to 18.

18 is 5 away from 23, and as your opponent can only choose a number between 1 and 4 inclusive, whatever they pick, you will always be able to win.

Working backwards using the same argument, you must ensure you hit targets of 13, 8 and 3 so that you can guarantee making it to 23. As

the first target is 3 and this is one of the numbers you can choose, you want to go first.

The series of targets can be thought of as stepping stones which must be trodden on to get to the final target number. If the range of numbers to choose from consists of consecutive numbers starting at 1 and going up to n , then the stepping stones will be $n + 1$ apart. If $n + 1$ is a multiple of the target number, then you need to go second to win. If not, you should go first.