Seven squares

Generalising from patterns

Prerequisite knowledge

- Drawing systematically
- Making comparisons
- Basic properties of a square

Why do this problem?

Although the mathematical knowledge needed to tackle this problem is no more than 'Colour wheels' the context is not quite so easy to visualise and connections are a little less obvious but describing what is seen can help. The important idea here is that seeing the same pattern in different ways can generate the same mathematics. There is no one way that is 'right'.

The problem is a good opportunity to use algebraic symbols to explain the pattern but this is *not* essential. Lots of examples of other similar contexts are included.

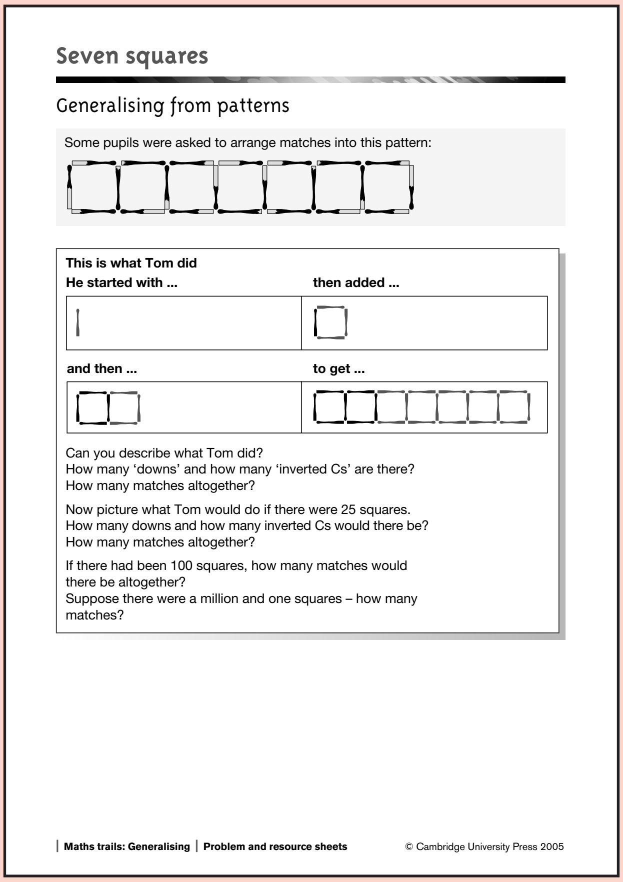
Time

One or two lessons

Resources

CD-ROM: pupil worksheet

NRICH website (optional): www.nrich.maths.org, September 2004, 'Seven squares' (animations on the NRICH site show interactive examples of how pupils might have created the first pattern)


Seven squares

Generalising from patterns

Some pupils were asked to arrange matches into this pattern:

This is what Tom did

He started with …	then added …
and then …	**to get …**

Can you describe what Tom did?
How many 'downs' and how many 'inverted Cs' are there?
How many matches altogether?

Now picture what Tom would do if there were 25 squares.
How many downs and how many inverted Cs would there be?
How many matches altogether?

If there had been 100 squares, how many matches would there be altogether?
Suppose there were a million and one squares – how many matches?

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Introducing the problem

Display a picture of the seven squares. It is not a good idea to draw the pattern in front of the class because this gives pupils a sense of 'the right way' (which of course there isn't).

Invite pupils to work in pairs, one pupil drawing or creating the pattern while the other pupil watches. The observer then describes in words the method used to their partner and the pair agree on a good way of explaining what they did to the rest of the class.

Ask two pairs, one after the other, to describe their different methods to you so that you can re-create them on the board. The aim is to use contrasting methods to identify a 'notation' for describing and then calculating the number of matches.

For example:

> The pattern could be made by first drawing 1 'down' match followed by 7 'back-to-front Cs'.
>
> Total number of matches:
> $1 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 1 + 7 \times 3 = 22$

Or

> The pattern can be made by first drawing 7 'across' matches then another 7 'across' matches below the first, and finally adding 8 'downs'.
>
> Total number of matches:
> $7 + 7 + 8 = 7 + 7 + (7 + 1) = 22$

Ask pupils appropriate questions in order to code what they have drawn. For example: 'What is the significance of the 7?'

Ask the class to code their methods. Those pairs that have created their patterns in the same way as those on the board could think of a different construction method.

Use this work to lead up to:

- Suppose there had been 100 squares – how many matches altogether?
- A million and one squares – how many matches?

Main part of the lesson

Select one or more of the suggested patterns in 'Follow-up activities' on the problem sheet. Ask pupils to work in small groups to create and code the patterns in as many different ways as they can. They should aim to generalise the patterns and give a convincing argument (prove) at the end of the session how they know the number of dots, lines or squares needed to create a pattern of any size.

Plenary

There are two main points to draw out of the plenary session. Firstly, share with pupils what you consider to be well-constructed arguments for the generalisation and why. Secondly, highlight the fact that pupils have produced the same generalisation by taking different routes.

Solution notes

Tom's method

For 7 squares, there are 1 down and 7 inverted Cs, so $1 + (7 \times 3) = 22$ matches.

For 25 squares, there would be 1 down and 25 inverted Cs, so $1 + (25 \times 3) = 76$ matches.

For 100 squares, there are 301 matches in total, i.e. $1 + (100 \times 3)$ matches.

For one million and one squares, there are 3 000 004 matches in total, i.e. $1 + (1\,000\,001 \times 3)$ matches.

When N is the number of squares, downs = 1, inverted Cs = N, so $3N + 1$ matches altogether.

Alan's method

For 7 squares, there are 14 matches along and 8 down, so $(7 \times 2) + (7 + 1) = 22$ matches.

For 25 squares, there would be 50 along and 26 down, so $(25 \times 2) + (25 + 1) = 76$ matches.

For 100 squares, there are 301 matches in total, i.e. $(100 \times 2) + (100 + 1)$ matches.

For one million and one squares, there are 3 000 004 matches in total, i.e. $(1\,000\,001 \times 2) + (1\,000\,001 + 1)$ matches.

When N is the number of squares, alongs = $N \times 2$, downs = $N + 1$, so $3N + 1$ matches altogether.

Ruth's method

For 7 squares, there are 1 square and 6 inverted Cs, so $4 + (6 \times 3) = 22$ matches.

For 25 squares, there would be 1 square and 24 inverted Cs, so $4 + (24 \times 3) = 76$ matches.

For 100 squares, there are 301 matches in total, i.e. $4 + (99 \times 3)$ matches.

For one million and one squares, there are 3 000 004 matches in total, i.e. $4 + (1\,000\,000 \times 3)$.

When N is the number of squares, squares = 1, inverted Cs = $N - 1$, so $3N + 1$ matches altogether.

Follow-up activities

Growing rectangles

Perimeter = $4 + 2N$, where N is the width.

Number of lines needed = $7 + 5(N - 1)$, where N is the width.

T-shapes

Number of small squares = $3N - 2$, where N is the height of the T.

Number of lines needed = $9N - 5$, where N is the height of the T.

L-shapes

Number of small squares = $2N - 1$, where N is the side of the large square.

Perimeter of L-shape = $4N$, where N is the side of the large square.

Squares in squares

Number of white squares = $(\sqrt{N} + 2)^2 - N$, where N is the number of grey squares.