Think of two numbers

Generalising from number

Prerequisite knowledge

- It is useful to have done 'Number tricks' before tackling this problem
- The ability to use some symbolic representation is necessary to take the solution to its full extent

Why do this problem?

This builds on 'Number tricks' because it is about similar structures, but it has an interesting twist at the end (related to the 9 times table).

The mental arithmetic at the start of the session sets the scene without giving the game away.

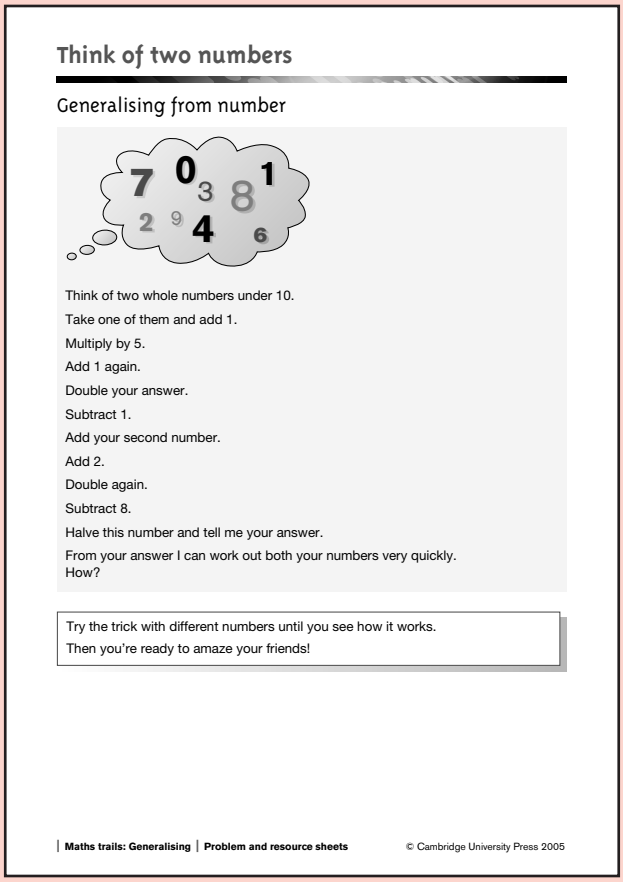
Time

Up to one lesson

Resources

CD-ROM: pupil worksheet

NRICH website (optional): www.nrich.maths.org, May 2003, 'Think of two numbers'
Pupils need mini-whiteboards for the main part of the lesson.



Think of two numbers

Generalising from number

Think of two whole numbers under 10.
Take one of them and add 1.
Multiply by 5.
Add 1 again.
Double your answer.
Subtract 1.
Add your second number.
Add 2.
Double again.
Subtract 8.
Halve this number and tell me your answer.
From your answer I can work out both your numbers very quickly.
How?

Try the trick with different numbers until you see how it works.
Then you're ready to amaze your friends!

Introducing the problem

Begin the lesson with some quick-fire mental arithmetic which concentrates on the 9 times table and adding 9 to numbers.

Main part of the lesson

Pupils work in pairs and start by recording their two chosen numbers on one of their whiteboards. Together, they follow your instructions as you go through the operations on the problem sheet and record their final answer on the second whiteboard.

Ask all pairs to reveal their final answer by holding up the appropriate whiteboard.

Impress the class by selecting several pairs and predicting their starting numbers. Pairs can show their starting numbers to the class to verify your prediction. (If arithmetic lets them down, there is a good chance (1 in 4) another pair will have the same final answer and may have got their arithmetic correct!)

Try to elicit from the class the importance of looking at several sets of numbers in order to identify patterns and relationships.

Ask pupils, in groups, to unpick what is happening: suggest that they pool sets of solutions (starting numbers and finishing number), recording sets of three numbers on a large sheet of paper in the middle of the table. It might be useful for more than one pair to try each of the starting numbers as a self-check. While pupils are working, go round the groups and encourage them to write results in a way that will help reveal patterns, e.g. putting the

final number underneath the two starting numbers.

There may be opportunities to stop the class to share ideas. At the least, expect pupils to have noticed the relationship between the digits of the final number and the starting numbers (+1/−1). More able pupils may work to a more algebraic solution.

Plenary

Very quickly, pupils should be able to establish the +1/−1 relationship, but why does it occur?

A discussion along the lines that the first number is multiplied by 5 and then by 2, so that it ends up in the tens column, and the second number is multiplied by 2 and then divided by 2, meaning it stays in the units column, will help pupils at least begin to get a feel for what is going on.

Of course, this is not the full algebraic solution, but it may be appropriate to leave it here.

Solution notes

Algebraic manipulation reveals that the final number is $10x + y + 9$, where x and y are the starting numbers.

To find the two digits, you therefore subtract 9 from the answer, and the tens unit is then x and the units digit is y. This is why doing the 9 times table at the beginning of the lesson may be useful, in helping pupils spot that you need to subtract 9 rather than using the +1/−1 method.