MATHEMATICS   **4723**

Core Mathematics 3

Wednesday   **18 JANUARY 2006**   Afternoon   1 hour 30 minutes

Additional materials:
    8 page answer booklet
    Graph paper
    List of Formulae (MF1)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

*   Write your name, centre number and candidate number in the spaces provided on the answer booklet.
*   Answer **all** the questions.
*   Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
*   You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

*   The number of marks is given in brackets [ ] at the end of each question or part question.
*   The total number of marks for this paper is 72.
*   Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
*   **You are reminded of the need for clear presentation in your answers.**
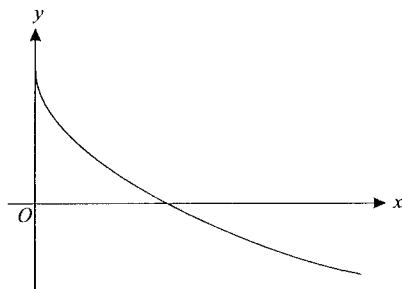

This question paper consists of 4 printed pages.

1   Show that $\displaystyle\int_{2}^{8} \frac{3}{x}\,dx = \ln 64$. [4]

2   Solve, for $0° < \theta < 360°$, the equation $\sec^2\theta = 4\tan\theta - 2$. [5]

3   (a)   Differentiate $x^2(x+1)^6$ with respect to x. [3]

    (b)   Find the gradient of the curve $y = \dfrac{x^2 + 3}{x^2 - 3}$ at the point where $x = 1$. [3]
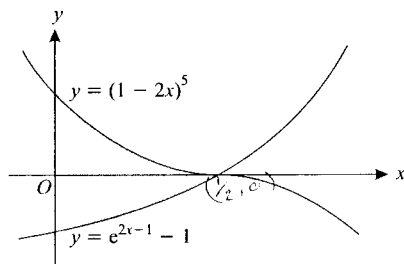
4

The function f is defined by $f(x) = 2 - \sqrt{x}$ for $x \geqslant 0$. The graph of $y = f(x)$ is shown above.

   (i)   State the range of f. [1]

  (ii)   Find the value of ff(4). [2]

 (iii)   Given that the equation $|f(x)| = k$ has two distinct roots, determine the possible values of the constant k. [2]

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The diagram shows the curves $y = (1 - 2x)^5$ and $y = e^{2x-1} - 1$. The curves meet at the point $\left(\frac{1}{2}, 0\right)$. Find the exact area of the region (shaded in the diagram) bounded by the y-axis and by part of each curve. [8]

6 **(a)**

| t	0	10	20
X	275	440	

The quantity X is increasing exponentially with respect to time t. The table above shows values of X for different values of t. Find the value of X when $t = 20$. [3]

(b) The quantity Y is decreasing exponentially with respect to time t where

$$Y = 80e^{-0.02t}.$$

(i) Find the value of t for which $Y = 20$, giving your answer correct to 2 significant figures. [3]

(ii) Find by differentiation the rate at which Y is decreasing when $t = 30$, giving your answer correct to 2 significant figures. [3]

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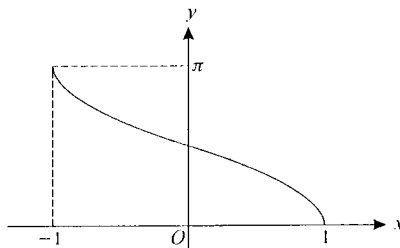


The diagram shows the curve with equation $y = \cos^{-1} x$.

(i) Sketch the curve with equation $y = 3\cos^{-1}(x - 1)$, showing the coordinates of the points where the curve meets the axes. [3]

(ii) By drawing an appropriate straight line on your sketch in part **(i)**, show that the equation $3\cos^{-1}(x - 1) = x$ has exactly one root. [1]

(iii) Show by calculation that the root of the equation $3\cos^{-1}(x - 1) = x$ lies between 1.8 and 1.9. [2]

(iv) The sequence defined by

$$x_1 = 2, \qquad x_{n+1} = 1 + \cos\left(\tfrac{1}{3}x_n\right)$$

converges to a number α. Find the value of α correct to 2 decimal places and explain why α is the root of the equation $3\cos^{-1}(x - 1) = x$. [5]

[Questions 8 and 9 are printed overleaf.]

[Turn over

8
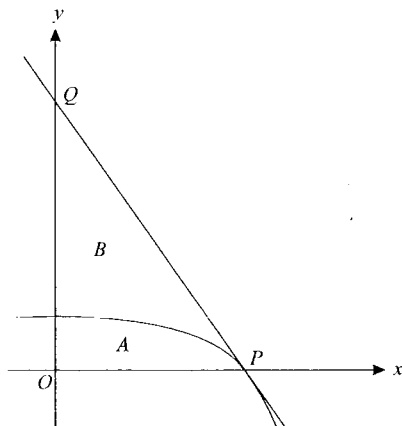
The diagram shows part of the curve $y = \ln(5 - x^2)$ which meets the x-axis at the point P with coordinates $(2, 0)$. The tangent to the curve at P meets the y-axis at the point Q. The region A is bounded by the curve and the lines $x = 0$ and $y = 0$. The region B is bounded by the curve and the lines PQ and $x = 0$.

(i) Find the equation of the tangent to the curve at P. [5]

(ii) Use Simpson's Rule with four strips to find an approximation to the area of the region A, giving your answer correct to 3 significant figures. [4]

(iii) Deduce an approximation to the area of the region B. [2]

9 (i) By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$ [4]

(ii) Determine the greatest possible value of

$$9 \sin\left(\tfrac{10}{3}\alpha\right) - 12 \sin^3\left(\tfrac{10}{3}\alpha\right),$$

and find the smallest positive value of α (in degrees) for which that greatest value occurs. [3]

(iii) Solve, for $0^\circ < \beta < 90^\circ$, the equation $3 \sin 6\beta \operatorname{cosec} 2\beta = 4$. [6]