OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS

4723

Core Mathematics 3

Thursday 8 JUNE 2006 Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

1 Find the equation of the tangent to the curve $y = \sqrt{4x+1}$ at the point (2, 3). [5]

2 Solve the inequality $|2x - 3| < |x + 1|$. [5]

3 The equation $2x^3 + 4x - 35 = 0$ has one real root.

(i) Show by calculation that this real root lies between 2 and 3. [3]

(ii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{17.5 - 2x_n},$$

with a suitable starting value, to find the real root of the equation $2x^3 + 4x - 35 = 0$ correct to 2 decimal places. You should show the result of each iteration. [3]

4 It is given that $y = 5^{x-1}$.

(i) Show that $x = 1 + \dfrac{\ln y}{\ln 5}$. [2]

(ii) Find an expression for $\dfrac{dx}{dy}$ in terms of y. [2]

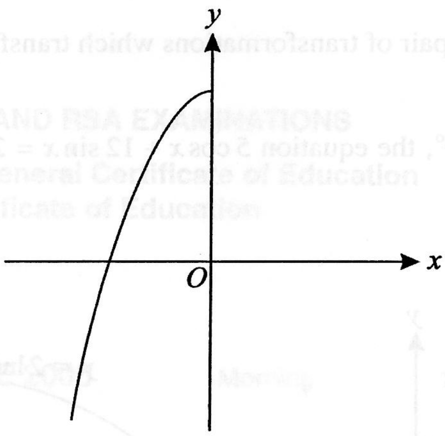
(iii) Hence find the exact value of the gradient of the curve $y = 5^{x-1}$ at the point (3, 25). [2]

5 **(i)** Write down the identity expressing $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. [1]

(ii) Given that $\sin\alpha = \frac{1}{4}$ and α is acute, show that $\sin 2\alpha = \frac{1}{8}\sqrt{15}$. [3]

(iii) Solve, for $0° < \beta < 90°$, the equation $5\sin 2\beta \sec\beta = 3$. [3]

6



The diagram shows the graph of $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2 - x^2, \qquad x \leqslant 0.$$

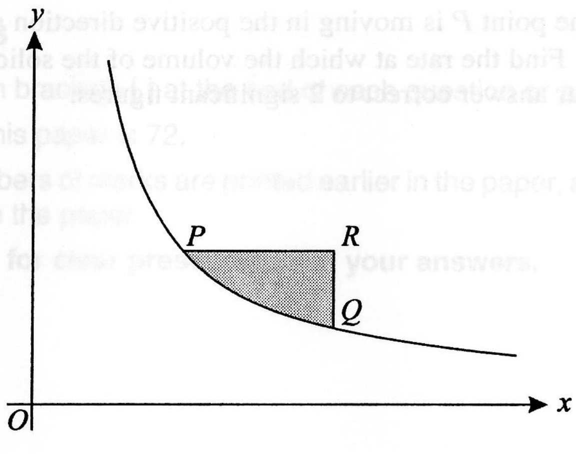
(i) Evaluate $\mathrm{ff}(-3)$. [3]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(iii) Sketch the graph of $y = \mathrm{f}^{-1}(x)$. Indicate the coordinates of the points where the graph meets the axes. [3]

7 (a) Find the exact value of $\displaystyle\int_1^2 \frac{2}{(4x-1)^2}\,\mathrm{d}x$. [4]

(b)



The diagram shows part of the curve $y = \dfrac{1}{x}$. The point P has coordinates $\left(a, \dfrac{1}{a}\right)$ and the point Q has coordinates $\left(2a, \dfrac{1}{2a}\right)$, where a is a positive constant. The point R is such that PR is parallel to the x-axis and QR is parallel to the y-axis. The region shaded in the diagram is bounded by the curve and by the lines PR and QR. Show that the area of this shaded region is $\ln\left(\frac{1}{2}\mathrm{e}\right)$. [6]

[Turn over

8 (i) Express $5\cos x + 12\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence give details of a pair of transformations which transforms the curve $y = \cos x$ to the curve $y = 5\cos x + 12\sin x$. [3]

(iii) Solve, for $0° < x < 360°$, the equation $5\cos x + 12\sin x = 2$, giving your answers correct to the nearest $0.1°$. [5]

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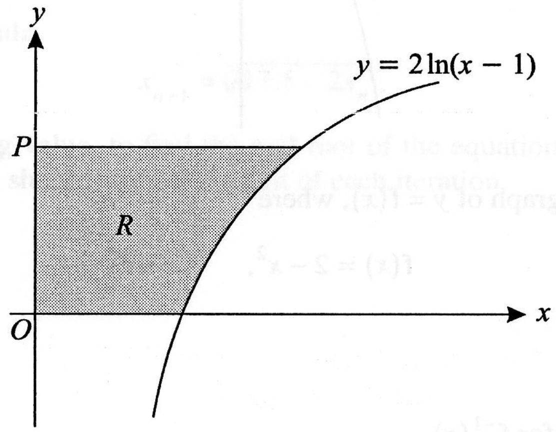

The diagram shows the curve with equation $y = 2\ln(x - 1)$. The point P has coordinates $(0, p)$. The region R, shaded in the diagram, is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The units on the axes are centimetres. The region R is rotated completely about the **y-axis** to form a solid.

(i) Show that the volume, V cm^3, of the solid is given by

$$V = \pi\left(e^p + 4e^{\frac{1}{2}p} + p - 5\right).$$ [8]

(ii) It is given that the point P is moving in the positive direction along the y-axis at a constant rate of 0.2 cm min^{-1}. Find the rate at which the volume of the solid is increasing at the instant when $p = 4$, giving your answer correct to 2 significant figures. [5]