C1 sample exam paper

- The paper is 1 hour 30 minutes long.

- No calculators may be used.

1. Express $\dfrac{1 + \sqrt{2}}{3 - \sqrt{2}}$ in the form $a + b\sqrt{2}$.

(4)

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2. (a) Express 16^x in the form 2^{ax} where a is an integer to be found.

(2)

(b) Hence solve the equation
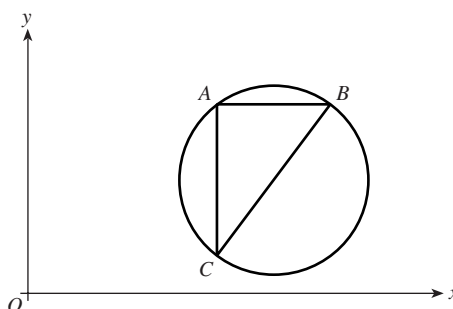
$$2^{x^2} = 16^x.$$

(3)

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3. The curve C has equation $y = f(x)$ and the point $Q(1, 7)$ lies on C.
Given that $f'(x) = (x^{\frac{3}{2}} - 2x^{-\frac{3}{2}})^2 + 2$, find an equation of the normal to C at Q.

(4)

4.

In the figure the coordinates of points A, B and C are $(5, 5)$, $(8, 5)$ and $(5, 1)$ respectively.

(a) Show that $AB^2 + AC^2 = BC^2$.

(2)

A circle is drawn which passes through the points A, B and C.

(b) Find the coordinates of the centre of the circle and the radius of the circle.

(2)

(c) Write down an equation of the circle.

(2)

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5. The curve C has equation $y = 2x^3 - x^2$. The points A and B both lie on the curve C having coordinates (1, 1) and (2, 12) respectively.

   (a) Show that the gradient of C at B is 5 times the gradient of C at A.

   (5)

   (b) Find an equation for the tangent to C at B.

   (3)

6. (a) Find, in terms of k, the roots of the equation

   $$x^2 + 2kx - 5 = 0.$$

   (3)

   (b) Prove that, for all real values of k, the roots of $x^2 + 2kx - 5 = 0$ are real and different.

   (2)

   (c) Given that $k = \sqrt{3}$, find the roots of the equation in the form $p\sqrt{3} + q\sqrt{2}$ where p and q are integers.

   (3)

7. The points $A(-4, -2)$, $B(6, 4)$ and $C(4, -4)$ are the vertices of triangle ABC.

   (a) Find an equation for the line passing through A and B in the form $ax + by + c = 0$, where a, b and c are integers.

   (4)

   Point D is the mid-point of AB.

   (b) Show that CD is perpendicular to AB.

   (4)

8. The curve C has equation

   $$y = 2x + \frac{10}{x}, \quad x > 0.$$

   (a) Find $\dfrac{dy}{dx}$.

   (2)

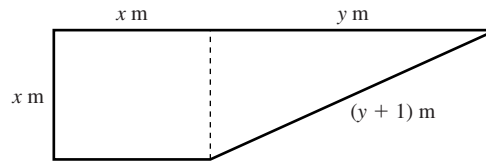   The coordinates of the stationary point on C are $(\sqrt{a}, b\sqrt{a})$, where a and b are integers.

   (b) Find the value of a and the value of b.

   (4)

   (c) Determine the nature of the stationary point.

   (2)

9.

The diagram represents the floor plan of an attic room which consists of a square of side x m and a triangle of sides x m, y m and $(y + 1)$ m.

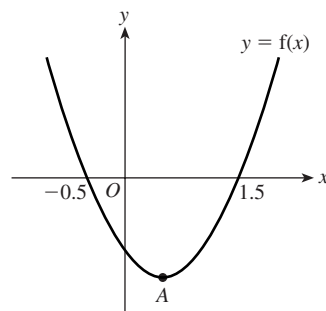The perimeter of the room is 10 m.
(a) Show that $3x + 2y = 9$.

(2)

The area of the floor is $5.5 \, \text{m}^2$.
(b) By forming another equation in x and y, find the value of x and the value of y.

(9)

10.

The figure shows a sketch of the curve with equation $f(x) = 4x^2 - 4x - 3$.
Given that $4x^2 - 4x - 3 = a(x + b)^2 + c$, for all x, where a, b and c are constants,

(a) calculate the value of a, b and c.

(4)

(b) Hence, or otherwise, find the coordinates of the minimum point A.

(2)

On separate diagrams sketch the curve with equation

(c) $y = f(x + 3)$,

(3)

(d) $y = 3f(x)$.

(2)

On each diagram, show clearly the coordinates of the minimum point, and of each point at which the curve crosses the x-axis.

END