Core 1 - June 2007

① $(2x+5)^2-(x-3)^2$

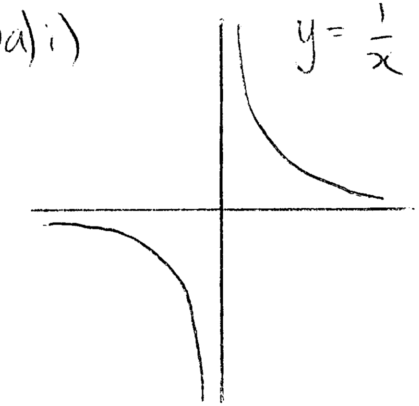
$= (2x+5)(2x+5) - (x-3)(x-3)$

$= (4x^2+10x+10x+25) - (x^2-6x+9)$

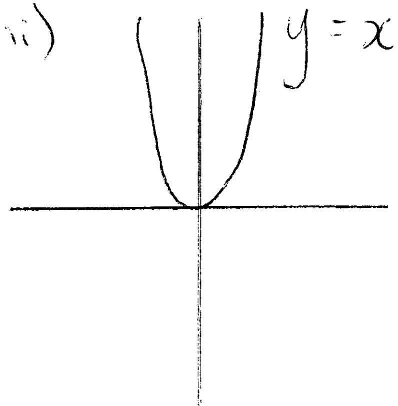
$= 4x^2+20x+25-x^2+6x-9$

$= \underline{\underline{3x^2+26x+16}}$

② a) i) $y=\frac{1}{x}$



ii) $y=x^4$



b) $y=x^3 \longrightarrow y=8x^3$

Either $\frac{y}{8}=x^3$

y replaced by $\frac{y}{8}$ so a stretch of 8 in the y direction

or $y=(2x)^3$

x replaced by $2x$ $\left(\frac{x}{1/2}\right)$

so a stretch of $\frac{1}{2}$ in the x direction

③ i) $3\sqrt{10}\times\sqrt{2}$

$=3\sqrt{2}\sqrt{5}\sqrt{2} = 3\times2\times\sqrt{5} = \underline{\underline{6\sqrt{5}}}$

ii) $\sqrt{500}+\sqrt{125}$

$=\sqrt{5}\sqrt{100}+\sqrt{5}\sqrt{25}$

$=10\sqrt{5}+5\sqrt{5} = \underline{\underline{15\sqrt{5}}}$

④ The discriminant is b^2-4ac

i) $\underset{a}{K}x^2 \underset{b}{-4}x + \underset{c}{K}$

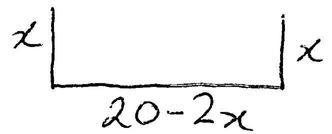
$(-4)^2 - 4\times K\times K$

$16-4K^2$

ii) For equal (or repeated) roots the discriminant is 0.

$$16 - 4k^2 = 0$$
$$16 = 4k^2$$
$$k^2 = 4$$
$$\underline{\underline{k = 2 \text{ or } -2}}$$

(5) i)



$$\text{Area} = x(20-2x)$$
$$A = 20x - 2x^2$$

ii) $$\frac{dA}{dx} = 20 - 4x$$

At max point $20 - 4x = 0$

$$20 = 4x$$
$$x = 5$$

$$\therefore \text{Area} = 5(20 - 2\times5)$$
$$\underline{\underline{= 50m^2}}$$

(6) $y = (x+2)^2$

$$(x+2)^4 + 5(x+2)^2 - 6 = 0$$
$$y^2 + 5y - 6 = 0$$
$$(y+6)(y-1) = 0$$

$$y = -6 \text{ or } y = 1$$

So $-6 = (x+2)^2$ ← Not possible! $(x+2)^2$ cannot be -6

$$1 = (x+2)^2$$

$$(x+2)(x+2) = 1$$
$$x^2 + 4x + 4 = 1$$
$$x^2 + 4x + 3 = 0$$
$$(x+3)(x+1) = 0$$

So $x = -3$
or $x = -1$

7) a) $f(x) = x + \frac{3}{x}$

$f(x) = x + 3x^{-1}$

$f'(x) = 1 - 3x^{-2}$

b) $y = x^{5/2}$

$\frac{dy}{dx} = \frac{5}{2} x^{3/2}$

at $x = 4$

$\frac{dy}{dx} = \frac{5}{2}(\sqrt{4})^3 = \frac{5}{2} 2^3$

$= \underline{\underline{20}}$

8) i) $x^2 + 8x + 15$

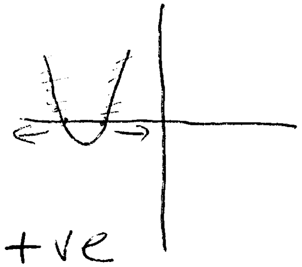
$= (x+4)^2 - 16 + 15$

$= (x+4)^2 - 1$

ii) Min value of y occurs at $x = -4$
this minimum value is -1
so vertex is $(-4, -1)$

iii) $x^2 + 8x + 15 > 0$

$(x+3)(x+5) > 0$



if $x < -5$ $\quad -ve \times -ve = +ve$

if $-5 < x < -3$ $\quad -ve \times +ve = -ve$

if $x > -3$ $\quad +ve \times +ve = +ve$

$\therefore$ $(x+3)(x+5)$ is $+ve$ (>0)

if $x < -5$
or $x > -3$

9) i) $x^2+y^2-6x-k=0$

$(x-3)^2-9+y^2-k=0$

$(x-3)^2+y^2=k+9$

there are no y's so the y coordinate must be 0

If the radius is 4 then

$k+9=16$

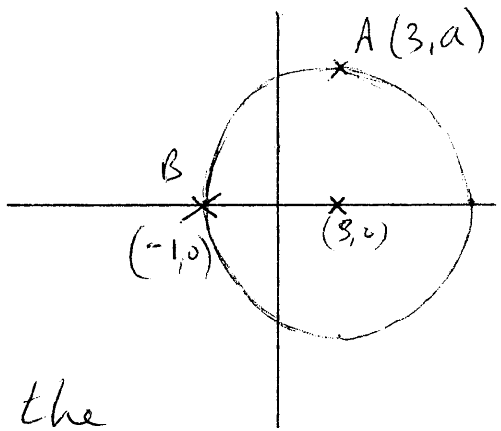
so $\underline{k=7}$

The centre of the circle is

$\underline{(3,0)}$

ii)

A sketch helps.



We know that the equation of the circle is

$(x-3)^2+y^2=16$ Find $\underset{\sim}{a}$ first

put in $x=3$

$(3-3)^2+y^2=16$

$y^2=16$

$y=4$ so $a=4$

So A is $(3,4)$ B is $(-1,0)$

Find the Length AB

$\sqrt{(3--1)^2+(4-0)^2}$

$=\sqrt{16+16}=\sqrt{32}=\sqrt{16}\sqrt{2}$

$=\underline{\underline{4\sqrt{2}}}$

iii) To find the equation of AB we first need to find the gradient

$\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{4-0}{3--1}=\dfrac{4}{4}=1$

Gradient $=1$

I'll use $(-1,0)$ but you can use $(3,4)$

$\dfrac{y-0}{x--1}=1$

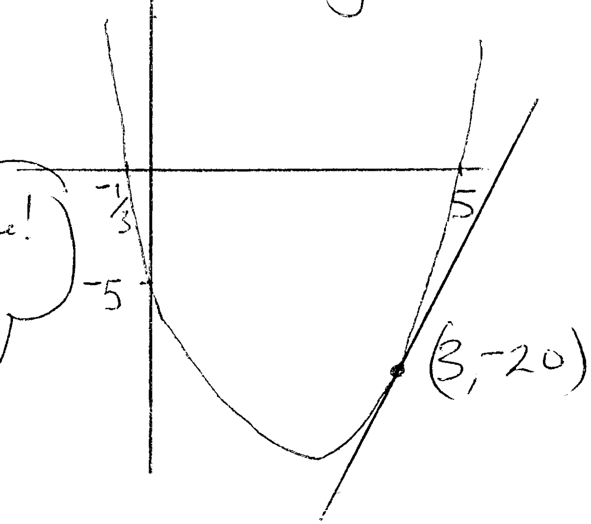
$\underline{\underline{y=x+1}}$

(10) i) $3x^2 - 14x - 5 = 0$

$(3x+1)(x-5) = 0$

So $x = \frac{-1}{3}$ $\quad x = 5$

ii) When $x = 0$ $\quad y = -5$



Not to scale! Not even close!!

iii) $y = 3x^2 - 14x - 5 = 0$

$\frac{dy}{dx} = 6x - 14$

The line $y = 4x + c$ has a gradient of 4

So $6x - 14 = 4$

$6x = 18$

$\underline{\underline{x = 3}}$

If $y = 3 \times 9 - 42 - 5 =$

$27 - 42 - 5 = -20$

The intercept will be:

$-20 = 4 \times 3 + c$

$-20 = 12 + c$

$c = -32$

$\underline{\underline{y = 4x - 32}}$

Because $(3, -20)$ has to 'work' in the equation as its on the line.