Fibonacci

Developing construction skills

Prerequisite knowledge

- Understanding the idea of a sequence, and that a sequence can be generated using different methods. For example, 2, 4, 6, 8, … can be created by starting with 2 and forming each new term by adding 2 to the previous term, or by multiplying the term number n by 2 ($2n$)
- Create formulae in a spreadsheet

Why do this unit?

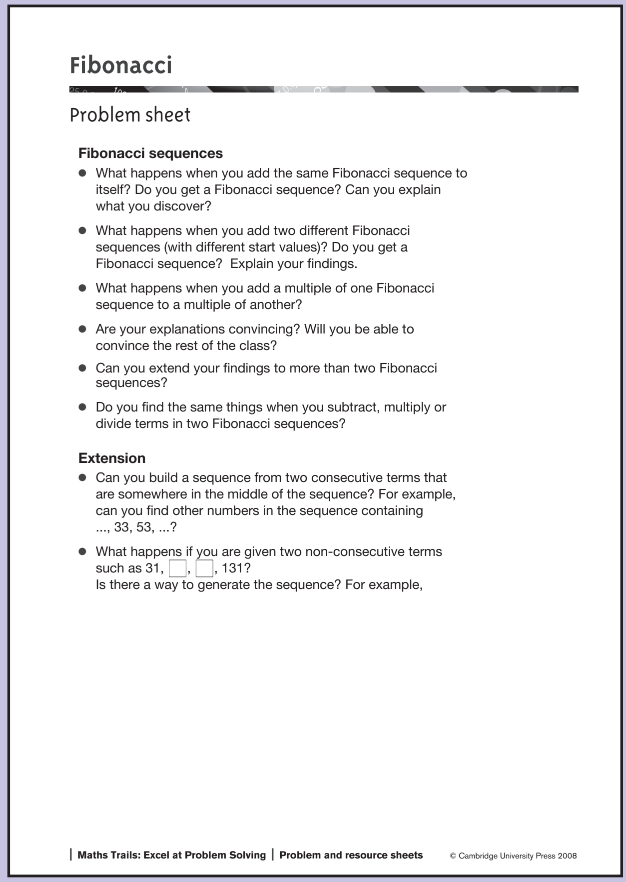
This activity encourages pupils to become familiar with a well-known sequence and explore its properties. Pupils make hypotheses and then either justify or disprove them. This activity is also a useful context in which to reinforce the understanding involved in key technical skills: editing a formula, **Copy**, **Paste** and **Fill**.

Time

One or two lessons

Resources

A3 sheets of paper and pens
CD-ROM: spreadsheet, problem sheet
NRICH website (optional): www.nrich.maths.org, October 2000, '1 step 2 step'; May 2005, 'Sheep talk'. These problems result in the Fibonacci sequence and make useful complements to this activity.


Fibonacci

Problem sheet

Fibonacci sequences

- What happens when you add the same Fibonacci sequence to itself? Do you get a Fibonacci sequence? Can you explain what you discover?
- What happens when you add two different Fibonacci sequences (with different start values)? Do you get a Fibonacci sequence? Explain your findings.
- What happens when you add a multiple of one Fibonacci sequence to a multiple of another?
- Are your explanations convincing? Will you be able to convince the rest of the class?
- Can you extend your findings to more than two Fibonacci sequences?
- Do you find the same things when you subtract, multiply or divide terms in two Fibonacci sequences?

Extension

- Can you build a sequence from two consecutive terms that are somewhere in the middle of the sequence? For example, can you find other numbers in the sequence containing …, 33, 53, …?
- What happens if you are given two non-consecutive terms such as 31, ☐, ☐, 131?
Is there a way to generate the sequence? For example,

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Introducing the unit

The Fibonacci number sequence (named by the French mathematician Edouard Lucas after the 13th century Italian mathematician) starts: 1, 1, 2, 3, … Similar sequences that have a different initial pair (1, 3 or 2, 5 for example) are called Lucas numbers. Other sequences of numbers formed using slightly different rules have particular names (e.g. Pell numbers). In this activity, the process of adding terms to produce a next term we will call the 'Fibonacci process' and the resulting sequences 'Fibonacci-like' sequences.

Ask pupils to imagine the numbers 2, 4, 6, 8, …

- What is the next number in the sequence? [10] And the next? [12] And the next? [14]
- How about the 10th term? [20] How do you know? [There are several valid answers including 2×10.]
- Now imagine another sequence 1, 2, 4, 7, … Can you describe what is happening here? [The step size is increasing by one.]
- Can you tell me the next three terms in this sequence? [11, 16 and 22]

Main part of the unit

Ask the group to think about 0, 1, 1, 2, 3, 5, …

- Can you describe how each term is formed and give the next three terms? [Add two consecutive terms to obtain the next term; 8, 13, 21]

Explain that this is called the Fibonacci sequence – it is a sequence where each term is formed by adding the two previous terms.

Demonstrate how to create a Fibonacci sequence. Start from an empty spreadsheet or use the blue tabbed sheets on the spreadsheet provided. Inspect cells and ask pupils what each formula calculates until the sheet's structure is understood.

Draw pupils' attention to how the first two values control all that follows. Change the values to demonstrate this – here the 'Fibonacci' sheet could be used. Discuss the structure through questions such as:

- What initial values make the 5th term 13? [2 and 3]

Once a general feel for a Fibonacci sequence has been established ask:

- What happens if we add two of these sequences together? (see 'Adding-T' on the spreadsheet)

Encourage pupils to suggest hypotheses which they can begin to test for themselves.

- How can you check the sum is a Fibonacci sequence? [The sum itself could be generated by creating the sequence from its first two terms.]

Ask pupils to investigate other properties of Fibonacci-like sequences, using the problem sheet for guidance.

Encourage pupils to write their conjectures on A3 conjecture sheets and post them up on the wall to write on as they test, explain and extend their ideas. These will form the focus of the plenary.

As the pupils work on the tasks encourage them to record their findings and explanations on their conjecture sheets, emphasising the importance of being able to produce a convincing argument. Encourage them to ask questions of their own if they notice something of interest.

Pupils who are struggling to produce the spreadsheet for themselves might benefit from being given a pre-prepared spreadsheet (see the blue tab sheets on the spreadsheet).

Note: You might wish to introduce the notation $T_n = T_{n-2} + T_{n-1}$ to those pupils who have a good grasp of algebraic notation.

Extension

- Given two consecutive terms, e.g. 33, 53, subtracting 33 from 53 gives the preceding term (20, 33, 53). Repeat this process until sufficient terms are found.
- Given 31, x, y, 131, then $31 + x = y$ and $x + y = 131$. Solve these equations simultaneously.

Plenary

Encourage pupils to share the range of findings they have made.

- Were any of your findings unexpected?
- Which arguments were most convincing?

You can use the pupils' conjecture sheets to support this part of the lesson.

Solution notes

Adding or subtracting two Fibonacci-like sequences or multiplying by a constant results in a Fibonacci-like sequence. Multiplying or dividing sequences does not.

Adding

a	b	$a+b$	$a+2b$	$2a+3b$	...
+	+	+	+	+	+
c	d	$c+d$	$c+2d$	$2c+3d$	...
↓	↓	↓	↓	↓	↓
$a+c$	$b+d$	$a+c+b+d$	$a+c+2b+2d$	...	...

Subtraction and multiplication by a constant work similarly.