

FURTHER SURDS PROBLEMS

EUDOXUS' LADDER

STUDENT RESOURCE

One way of calculating approximations to $\sqrt{2}$ which was used by the ancient Greeks is called Eudoxus' ladder.

1	1	1/1
3	2	3/2
7	5	7/5
17	12	17/12
41	29	41/29
99	70	99/70

- Work out how each row of numbers in the first two columns is obtained from the row above.
- Check that the fractions in the third column give better and better approximations to $\sqrt{2}$.
- Simplify $(1 - \sqrt{2})^2$, $(1 - \sqrt{2})^3$, $(1 - \sqrt{2})^4$, $(1 - \sqrt{2})^5$, $(1 - \sqrt{2})^6$.
- What do you notice? To help explain what you find, work out $(a - b\sqrt{2})(1 - \sqrt{2})$.
- What happens to $(1 - \sqrt{2})^n$ when n become larger and larger? Use this to explain why the Eudoxus' ladder gives fractions which tend to $\sqrt{2}$.
- Explain why the fractions in the third column are alternately bigger and smaller than $\sqrt{2}$.
- Now invent a ladder for $\sqrt{3}$ by considering powers of $(1 - \sqrt{3})$.