# The fire-fighter

## Modelling and optimisation

**Prerequisite knowledge**

- Pythagoras' theorem

**Why do this unit?**

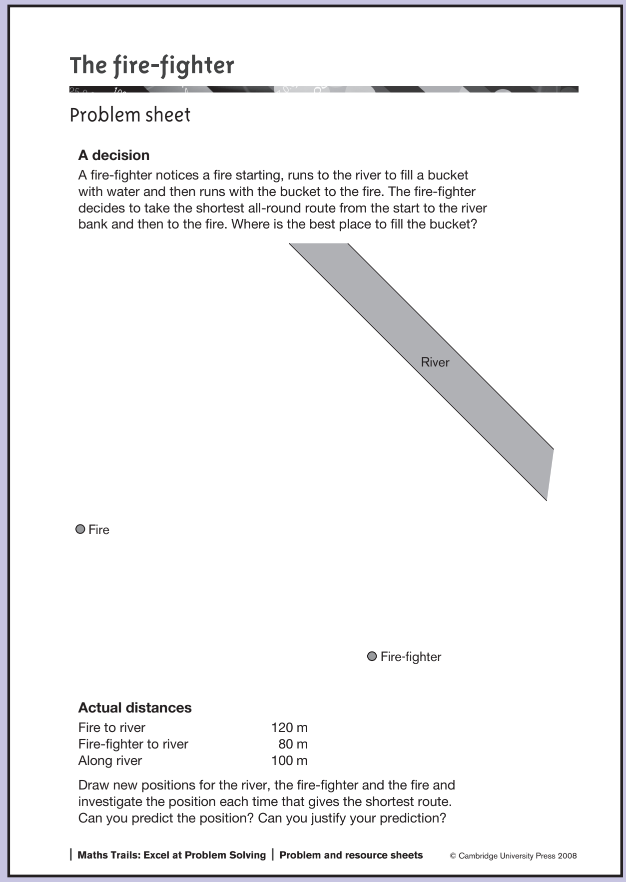
This activity will reinforce the use of a spreadsheet for optimisation problems. Pupils are required to use what they have learned in the first part of the problem to help them solve the second part.

**Time**

Two lessons

**Resources**

A length of string, a couple of drawing pins and some keys on a key ring
CD-ROM: spreadsheet, problem sheet, resource sheets 1 and 2
NRICH website (optional): www.nrich.maths.org, January 2005, 'Where to land'; November 2007, 'The fire-fighter's car keys'



The fire-fighter

Problem sheet

**A decision**

A fire-fighter notices a fire starting, runs to the river to fill a bucket with water and then runs with the bucket to the fire. The fire-fighter decides to take the shortest all-round route from the start to the river bank and then to the fire. Where is the best place to fill the bucket?



**Actual distances**

| | |
|---|---|
| Fire to river | 120 m |
| Fire-fighter to river | 80 m |
| Along river | 100 m |

Draw new positions for the river, the fire-fighter and the fire and investigate the position each time that gives the shortest route. Can you predict the position? Can you justify your prediction?

### Introducing the unit

Show the group the problem on the problem sheet. This requires pupils to find the shortest distance to a fire via a river, to collect water. Pupils will need their own copy of the problem sheet for individual work.

- What does 'shortest all-round route' mean? [There are two parts to the route: towards the river, then away from the river to the fire. The combined distance needs to be a minimum.]

Discuss with pupils how they might find the position along the river bank which will give the minimum distance by using a ruler and trial and improvement.

- How can we define this point? [Suggest that the distance can be measured from the nearest point on the river to the fire-fighter – point A on resource sheet 1.]

The diagram may distort slightly in the process of printing or photocopying but for the first part of the activity this is not a problem.

Give pupils a few minutes to find the 'optimal' position through measurement and trial and improvement, and share findings.

- How could we improve accuracy? [use Pythagoras' theorem]

### Main part of the unit

Show the group the sheet 'River start' on the spreadsheet and spend some time making sense of the way the sheet is constructed. [The sheet calculates the total distance from the fire-fighter to the fire via the river based on the diagrams on the problem and resource sheets with $d = 100$ mm, $a = 80$ mm and $b = 120$ mm (a nominal scale of 1 mm to 1 m).]

Check that the use of Pythagoras' theorem and the **SQRT** function in Excel is clear. For example, the Excel formula =SQRT(49) would return the value 7.

Take pupils through the process of scanning down the 'Combined distance' column to locate the minimum value and the position between A and B where this occurs. If some pupils suggest that the ratio split of AB matches the ratio of $a$:$b$ ask the group if that can be justified as a general result. If not, suggest that more examples could be useful and move to the second activity where pupils create, measure and record their own results.

Encourage pupils to try different positions for the river, the fire and the fire-fighter, keeping the distance along the river a constant (100 mm in their scale say, as this will make comparisons of results easier).

- What might you record? [The distance of the best position along the bank from a fixed point (for example the nearest point on the river to the fire-fighter), and distances from the fire and from the fire-fighter directly to the river – see lengths $a$ and $b$ on resource sheet 1.]

Record the findings for different lengths and distances and discuss any conjectures pupils might have about the best place on the river to run to. [For example, the further the fire-fighter is from the river, the further up the river towards the fire he will need to aim for. The distance along the river seems to be linked closely to the distances the fire-fighter and fire are from the river.]

By changing the values for the distances $a$ and $b$ in cells A2 and A3 in 'River' on the spreadsheet, results can be checked. The minimum is automatically picked out. Note in particular the use of absolute references (e.g. $A$1) in this sheet.

If time allows, pupils can make their own versions of the sheets 'River start' and 'River', but if the sheets are used as ready-made utilities pupils still need to visualise the route. Return to the diagram to confirm results as necessary.

Pupils may begin to see possible relationships and start to justify what they see without too much prompting but there is likely to be a need to focus discussion.

- Is there any connection between the $a$ and $b$ values and the position on AB which produces the shortest route?
- What might be useful $a$, $b$ values to try? [For example one approach could be to explore $a = 2b$ for different values of $a$, then restart using other multiples of $b$.]
- Can you justify any of your conjectures? [The diagram in resource sheet 2 shows an equivalent problem with a much simpler solution – if the fire-fighter ran the same distance to the river edge but from the other side of the river (assuming the river has no width), the best position to fill the bucket would be on that line so that the overall route was a straight line. This means that the two triangles are similar and so the ratio of corresponding sides confirms the fact that the best point along the river divides $d$ in the ratio $a:b$.]

## Plenary

On a pin board mark a horizontal line and then, some way above the left-hand side of the line, push in one pin. Push in the other pin at a different height above the line towards the right-hand side. Put a length of string through a key ring. Place the ends of the string one over each pin so that the keys hang above the line. Explain that you will slowly release the string so that the keys cross the line.

- Can you predict the position where the keys will cross the line?

Invite pupils to put a pin at the predicted point. Discuss the general relationship between the position of the pins and the point where the keys cross.

- How is this problem related to the fire-fighter activity? [It reduces to a shortest route problem.]

Use the sheet 'Car keys' on the spreadsheet to confirm hypotheses.

## Solution notes

The required position is located as follows: if the point on the river's edge nearest the fire-fighter (A) is a distance $a$ from the fire-fighter, and the point on the river's edge nearest the fire (B) is a distance $b$ from the fire, then the shortest route uses a point that splits AB in the proportion $a:b$.

The keys will slip to hang as low as possible for any given length of string. They will touch the line where the route from pin to line to other pin is a minimum. They touch the line at this point rather than any other point because the increased string length available for any other position would have allowed the keys to hang somewhere lower than the line and they would slip to do so.