More ideas

Imagine a triangle/square/pentagon

You will need pens, paper or individual whiteboards and some prepared scripts.

This visualisation will help to develop language associated with properties of shapes.

Imagine a triangle and draw it on a piece of paper or individual whiteboard but do not show it to the class.

Describe the triangle, asking pupils to reproduce what you describe on their pieces of paper or whiteboards. Pupils then reveal their triangles and check.

You could describe the shape in a deliberately vague way to initiate a discussion about the need for more precise language.

Alternatively, you could describe the shape and its orientation in sufficient detail for pupils all to have the same drawing. This would lead to a discussion concerning how the differing interpretations of pupils relate to what you said. (You might like to write a couple of scripts to facilitate this and allow you to refer back.)

Next, invite pupils to work in pairs to draw a triangle and write a description that they can read out to enable other pupils (either the whole class or in small groups) to reproduce the original shape.

Pull together ideas about useful descriptors such as equality of sides and sizes of angles, a side of a triangle being parallel to an edge of the whiteboard, number of sides, position of vertices in relation to the page, etc.

This can be repeated for different shapes.

Describe and create

Pupils will need access to a range of 2-D or 3-D shapes. For example, a pair of pupils may have four congruent squares and two equilateral triangles; another pair may have a square, a hexagon and three circles.

This activity can be used simply to encourage language of position – 'to the left of', 'on top', 'behind', ... – but also more precise language associated with the properties of shapes such as equal angles and side length.

Pupils work in pairs with a small number of shapes at their disposal. One pupil imagines an arrangement of some or all of the shapes, and then describes that arrangement to their partner. The partner's task is to reproduce the arrangement being described.

There are a number of ways this can be done. For example:

- Pupils can sit with their backs to each other so that the pupil describing cannot see the progress of their partner.
- Alternatively, and possibly a useful way to introduce the idea, the pupil describing the arrangement can watch and therefore obtain feedback as their partner responds (you might like to suggest sitting on hands when describing).
- Another intermediate strategy is for the describer to have the arrangement made in front of them (but hidden from their partner who has a matching set of objects) so they can see how what they are describing has been interpreted.

Feely bags

You will need some cloth drawstring bags ('feely bags') and a range of 2-D or 3-D shapes or a set of shapes that are closely related, such as a range of different triangles or quadrilaterals.

This activity focuses on the language of properties of shapes. It is good to have a selection of shapes relating to the current topic.

Start with a range of shapes visible to the class. Selecting each shape in turn, state:

This is a ___. How do you know?

For example, pupils might reply: 'I know it is an equilateral triangle because it has three sides of equal length.'

Explain to the class that they will work in pairs. For each round of the activity one pupil will need to state the shape *and* how they know. The second pupil will be a 'critical friend' and comment on the accuracy of their reasoning.

Here are a couple of examples of how this activity can be done in pairs:

- All the shapes are placed in the bag and pupils take turns to put their hand in the bag, select a shape and say what the shape is, and why, before removing it.
- One pupil puts a shape into the bag without their partner seeing. The partner feels the shape, stating what it is and how they know.

Floor robot

This activity can be carried out with a floor robot or as 'people maths', but if you use a pupil to be the robot, it's very difficult for them not to do what they *think* they're being told to do. It might be better for you or a teaching assistant to be the robot.

The aim of this activity is for pupils to develop estimation skills, precise directional language and to consolidate their knowledge of properties of shapes.

This is just one example of a large number of possible activities involving the use of a floor robot. These floor robots allow pupils to do some basic programming in the form of being able to enter a series of commands which:

- move the robot forward one length or back one length;
- turn the robot right (clockwise) or left (anti-clockwise) through one right angle.

A series of instructions (normally achieved by pressing buttons in sequence) can be programmed into the robot. For example:

FFFFRFFFFRFFFFRFFFFR followed by pressing 'go'

will result in the floor robot drawing a square.

To start with, you could model a couple of examples with you as the robot and the whole class giving you instructions.

Pupils can then work in groups to program the robot to draw a particular shape, or to make a repeating pattern, or to traverse a series of obstacles returning to the starting point. As pupils work, they will discuss their visualisations of 'What will happen if ... ?'.

Five-pointed stars

You will need a picture of a five-pointed star ready on the board. Pupils will need pencil and paper.

This activity is in two parts, the first emphasising the need for precision and the second focusing on visualising and describing a visualisation.

Ask the class if they have ever tried to draw a five-pointed star without taking their pen off the paper or going over the same line twice. Some will say ‘yes’ and some ‘no’. Show the class the star on the board and ask them to:

- try to draw the star in one go;
- (when they have been successful) imagine describing to someone on the telephone how to draw the star.

The class can be encouraged to experiment using pencil and paper and to make some notes about their descriptions. They may like to test their ideas with the person sitting next to them.

After a short time, ask for volunteers to give you instructions as if you were on the other end of the phone. As they speak, draw exactly what they tell you to. (It is good to try to go wrong if you can!) Several pupils might like to try this.

Pupils will then need to spend some time thinking about how to improve the precision of their descriptions. This will involve using some form of reference system. Lead this discussion towards using a circle.

Now ask pupils to imagine a circle with six points equally spaced around the circumference.

- Where did you put the first point? How about the third? [nearly halfway round from the first point]

Say that you are going to ask the class to imagine drawing lines joining points and you will then ask them to describe what shape they finish with. There are a range of possibilities here, for example:

- join each point to the next point round [hexagon];
- join each point to the next-but-one point [equilateral triangle with every other point missed out; you might like to ask how to describe drawing the other equilateral triangle];
- join every third point [a line].

Continue in the same way with more points as the class becomes more confident.

Stand up, sit down

For this activity, you will need a row of chairs at the front of the class for everyone to see.

Pupils will be using visualisations to plan ahead in this activity.

Explain to the class that they are going to work in groups on a practical task. Invite four pupils to come and stand in a line in front of the chairs. The rules are:

- A move means you either stand up or sit down.
- You are only allowed to move if the person immediately to your right is seated and everyone else to your right is standing, unless you are the person standing at the extreme right.

- The person on the extreme right can stand or sit no matter what everyone else is doing.

In order for the pupils to get a feel for the rules, you could try some examples. Looking at four standing pupils:

- ask the third person from the right to sit and ask who can move next [the fourth person];
- ask who can move first [only the person on the extreme right].

The aim is for pupils to find the minimum number of moves that will result in all four people being seated. Ask for a volunteer to keep count of the number of moves. Divide the rest of the class in half and ask one half to 'instruct' the four pupils so that they all end up sitting. Challenge the other half of the class to try to improve on the number of moves.

Ask the class if they know whether they have definitely managed to do it in the smallest number of moves. The class can now move off in groups to work on the problem, trying to identify the minimum number of moves and being able to justify what they think.

If you wish to take this idea further, see the activity 'Fleas' in *Points of Departure 3*, published by the Association of Teachers of Mathematics in 1989 (ISBN 0 900095 80 6).

Flippin'

You will need four or more coins or discs with differently coloured faces.

This activity uses visualising to plan ahead and to identify patterns. It can be extended further – see 'Seven flipped' on the NRICH website (www.nrich.maths.org, May 2006).

Ask the class to imagine four coins lying on a table with the heads facing up. Tell the class that they are only allowed to turn two coins over at a time. The aim is to have all the coins with their tails facing up. Without writing anything down, ask them to spend one minute visualising how they would do this. Then tell them to turn to a partner in order to convince themselves that they can describe the moves to the rest of the class.

Give the class some opportunity to share their ideas, particularly noting any symmetry in their answers. For example, they might turn over the coins two on the left, then two on the right (or vice versa), or they may turn the two outside coins over first, then the two inside (or vice versa). It might be worth discussing why any of these are equivalent. [The position of the coins doesn't matter; what does matter is that you can do it by turning every coin once and you need a system for doing that.] If necessary, use coins to demonstrate this.

Challenge the class to describe how to flip 100 coins over to someone on the other end of a telephone line. They are most likely to give an answer which assumes the coins are in a line to start with so you could draw their attention to this fact, emphasising that you didn't say that at the beginning!

Now ask pupils what happens with three coins when you can only flip two at a time [it's not possible].

Then change the rules. This time flip three coins at a time and start with four coins with their heads face up. Can pupils describe some moves they would make in order for the coins to be tails up? [For example, HHHH, TTTH, THHT, HHTH, TTTT.] This will be challenging to visualise.

Musical maths

You might want to use a drum stick and some percussion instruments or objects that will sound when you hit them. Alternatively, you could simply clap and tap various parts of your body (head, shoulders, etc.).

This activity centres on factors and multiples.

Begin a rhythm: clap, clap, click (your fingers), and continue until the repetition is obvious. Ask pupils what you did first, second, etc. Follow on with questions such as:

- What would I be doing on the 20th beat? [clap]

There are lots of variations on this with more than two different sounds or movements and a repeat pattern involving more than three beats.

Invite two pupils to come to the front. They will both clap (C) and tap their head (T) in a repeating rhythm of three beats. Ask one to demonstrate CCT, CCT, CCT, ... Then ask the other to do TCC, TCC, TCC, ... Questions you might ask the class include:

- If both pupils start at the same time, when will they both tap their head at the same time? [never] Why? [one is tapping on multiples of 3, the other on one more than a multiple of 3]
- Are there other ways the pupils could have clapped and tapped for this to be the case? [e.g. CCT and CTC]
- What might you do to ensure that they do tap at the same time? How could you predict when this was? [either change so that the repeat length is different or alter the number of claps and taps]

Ask pupils to investigate and prepare one example to bring back as a challenge for the rest of the class, for example CCCT, CCCT, CCCT, ... and CCT, CCT, CCT, ... [Ts coincide on all multiples of 12].

The activity has been kept simple by only allowing one tap in a sequence and always at the end. This could be made more difficult by allowing the tap anywhere or having more than one tap.

People grid

You need a grid, which could be tiles on the floor, lines marked out in the playground, desks/chairs in the classroom or cones laid out in the hall. You will need to be able to identify the origin and your x- and y-axes.

This activity can be used for work on coordinates, but is really focusing on loci (particularly the early part).

To begin with, make sure pupils are familiar with the grid. Ask one pupil to stand on, for example (3, 4). Ask the class to imagine that the grid lines are like roads and that when you move from one point to another you must walk along the roads. Ask another pupil to stand on another grid point anywhere, for example, (6, 7). Demonstrate that this point is 6 'walking' units away from (3, 4), no matter what shortest route you take to get there.

Start again with just one person standing at a point of their choice. Ask the rest of the class to visualise where someone could stand who had to be exactly 2 units away. Ask them:

- Can you imagine all the places where that person could stand?

- Can you describe all the places?

Encourage pupils towards a description of the shape made by all the people as they stand at each of these positions [tilted square].

Now ask the class to imagine another point and the shape made by people who are standing five units away. (Make sure that this square overlaps the first one.)

- How many people would be needed to stand at all the possible positions? [in theory this is 20, but an overlap with the first square will mean it's only 18 (or 19 if it touches)]

The pupils will say 20, so discuss why this is not the case. Invite pupils to stand at the points mentioned so that they can check.

This activity can be repeated for different grid points and distances. If you would like pupils to apply this idea to a problem, then look at 'Lost' on the NRICH website (www.nrich.maths.org, February 2005).

Dissections

You will need paper squares and scissors.

This activity is a good opportunity for pupils to listen to, and use, geometric language such as 'vertex', 'right angle' etc., and to follow instructions.

Ask pupils to imagine a square sitting with its base horizontal and its sides vertical. Now ask them to 'cut' the square from the top right-hand corner to the bottom left-hand corner.

- What do you see? [they might say two triangles, right-angled, isosceles, one a mirror-image of the other along the cut line, etc.]

You might want to demonstrate this with the paper to convince pupils.

Now ask pupils to throw away the top right triangle and to imagine cutting the bottom left triangle from the vertex at the right angle to the middle of the opposite side.

- What do you see? [two right-angled triangles, similar to the first one but half its size and a quarter the area of the original square; they can be rearranged to form a square which is half the area of the original square]
- What would happen if you continued cutting in this way?

This can be demonstrated at various points if it helps.

This activity can be varied by cutting the original square in half horizontally/vertically or by using a different starting shape.

Fraction visualisations

You might need some paper squares, circles, triangles or rectangles.

This activity focuses on halving and halving again in a spatial context.

Ask pupils to imagine a square sitting with its base horizontal and its sides vertical. Invite them to draw an imaginary line through their square so that it is divided into two equal halves. Ask them to turn to a neighbour and describe where their line is. Share some of their ideas with the whole class. You may want to use paper to demonstrate some of the ideas to help the rest of the class.

- What if the line didn't have to be straight? What could it look like? [It might be curved, or be 'stepped'. If the two halves are identical, they will have rotation symmetry.]
- Where could you draw lines to divide the square into quarters?

Again, pupils could talk to each other and then share ideas. Encourage some creativity and ambition.

- Can you quarter the square with just one line? [no]
  How about two lines? How about three ... ?

All these ideas can be applied to different shapes and extended to three dimensions.

What do you see?

You will need some large cards prepared. One set, for this particular example, would be seven cards, four with £1 written on, two with £2 and one with £3. You will also need a pile of 8 books.

This activity encourages using visualisation as a mechanism for checking by thinking ahead about what you are expecting some statistical data to look like.

Invite seven pupils to stand holding the cards for everyone else to see. Explain that this represents the amount of pocket money received by these pupils. Having them stand in order (£1, £1, £1, £1, £2, £2, £3) will help the class.

Ask the class to visualise what a bar chart of this information would look like. Share their ideas with a partner and then with the whole class. They will need to clarify what their variable is [money, not the individual pupils] and that the bars will represent the frequency [one bar will be four high, one two high, etc.].

In the next example, invite three pupils to share eight books between them and ask the class to imagine a pie chart that represents how those books are shared. Draw out the important features of the pie chart: that it represents the whole (8 books) and so the basic division is one eighth of the circle. What fraction of the circle will represent the number of books each person has?

You can also try similar examples using different numbers and different contexts.

Another way of approaching this is to show pupils a table of data or a pie chart, bar chart, etc. Ask them to imagine the data represented in a different form and to describe what they see. It might be worth drawing out the equivalence of different orderings in a bar chart (where appropriate) or sectors in a pie chart.

Triangles, triangles, triangles, ...

You will need from the CD-ROM one of the two animations 'Sierpinski triangle' or 'Triangle quarters' and the storyboards from 'Cubes within cubes' for the class to complete.

Explain to the class that you will show them a short film which they will need to watch carefully in order to be able to describe at the end how to recreate what they have seen. You will ask them questions such as:

- How did it start?
- What happened first?
- Then what happened?
- What colours were used?
- When and where did the colours appear?

Play one of the animations through twice without pausing and then question the class in detail about what they saw. Give them feedback on the accuracy of their responses. If necessary, replay the animation, pausing to discuss and clarify points that have arisen in the discussion. Draw out the system to the developing pattern, the fractions shaded and so on.

Give pairs of pupils a storyboard to complete. You might like to use these for a display.

Travelling

You will need an image of a square with a locus around it along with a selection of rectangles, one of which made the locus. These images can be based on those from the interactivity 'Travelling' on the CD-ROM, which you will need towards the end of the activity.

This activity requires an understanding of loci and the ability to visualise at several levels: the basic shapes being used and the locus as that shape travels. Pupils are also encouraged to think about efficiency when problem solving.

Show the image of the square and locus to the class and explain that the locus was made by a point in the middle of a rectangle as the rectangle slid around the edge of the square. Explain to pupils that they have to decide which of the given rectangles made the locus.

- Can you rule out any of the rectangles immediately? Why?
- Which rectangles would make a locus with longer horizontal sides?
- Would the length of all the loci be the same?
- Can you order the rectangles according to the length of their loci?

Ask pupils to work in pairs to discuss their ideas; this should lead to a justification of their choice of rectangle. Pairs could join together in groups to see whether they agree and, if not, come to an agreement before sharing with the whole class.

Once pupils have convinced themselves of the correct answers, show them the interactivity.

Ask the class:

- If I chose a different locus, what criteria would you use to come to a decision more efficiently?

In other words, what have they learned?

The shepherd

You will need the interactivity 'Walk' from the CD-ROM.

This challenge is based on knowledge of distance–time graph construction; the visualisation helps in predicting what a graph will look like.

Show the interactivity and move the man around on the screen (using the arrow keys on the keyboard) to demonstrate the connection between the graph and the movement of the man.

Ask pupils to describe what is happening:

- How can you get a line to be drawn at the top/bottom of the graph? [the man needs to be as far away from the sheep as possible/standing by the sheep]

- Does the man have to be still to get a horizontal straight line? [no, just the same distance away from the sheep – a circle of points]
- How can you make the line steeper/shallower? [move faster/slower away from or towards the sheep]
- What would you have to do to make the line graph go up and then down? [move away from the sheep and then back]

There are some graphs on the interactivity which pupils can try to reproduce. Ask pupils to describe what they would do to create a particular-shaped graph and then test their ideas, discussing where things 'go wrong' – if they do.

There are two complementary approaches here:

- showing a graph and asking pupils how they would recreate it;
- describing a journey and asking the pupils what the graph would be like.

This visualisation is based on one on the NRICH website (www.nrich.maths.org, March/April 2006, 'Take your dog for a walk'), and similar problems can also be found there, such as 'You tell the story' (March/April 2006).

How far does it move?

You will need the interactivity 'How far' from the CD-ROM.

The problem provides a visual context in which to consider how distance–time graphs represent movement over time.

Show pupils the interactivity, varying the polygon and the positions of the point. Discuss the graphs that they produce. You might wish to ask questions such as:

- Why does the gradient of the graph change? [the point is travelling further for one unit of turn (around a vertex)]
- Why does the dot speed up and slow down at different stages of the 'journeys'? [similar argument to above related to the distance it has to travel per unit of turn]
- How can you make a horizontal line? [point in the centre of the shape or at a vertex]
- Why are there no horizontal lines if the point is in the centre of the side? [the point is never stationary (except momentarily)]

The pause button could be used to focus discussion on the different stages of the 'journeys' and to make conjectures about what will follow.

Having demonstrated the interactivity, possible approaches include:

- showing a graph and asking which polygon with the point in what position created the graph; give pupils plenty of time to discuss what they think – encourage convincing arguments;
- showing a polygon and the point in a particular position and asking pupils what graph will be produced; again, give plenty of time for pupils to visualise what is happening on their own, share ideas with a partner and then try to convince other members of the group.

Pupils' suggestions and arguments could be used for a display for all to see and discuss.

This visualisation is based on one on the NRICH website (www.nrich.maths.org, March/April 2006, 'How far does it move?'), and similar problems can also be found there, such as 'Speeding up, slowing down' (March/April 2006).