

# IDEA 29

## FIVE NUMBERS IN A RING 1

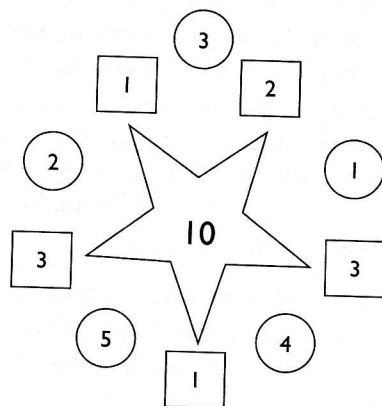
This idea begins with writing the numbers 1, 2, 3, 4 and 5 in any order in a (pentagonal) ring.

Next, calculate the positive difference between adjacent pairs of numbers and write these answers in between (perhaps in another colour to differentiate them from the five starting numbers).

Now add together all the differences and write the total in the middle of the ring.

In the example below the result can be written as follows:

$(3 - 1) + (4 - 1) + (5 - 4) + (5 - 2) + (3 - 2)$  which is equal to 10.



Questions can now be worked on such as:

- What other totals can be achieved using the same five starting numbers written in a different order?
- Why can only a certain set of totals be made?
- What happens if we start with 6, 7, 8, etc. consecutive numbers? How many different answers are there for each situation?
- Generalize the minimum total for  $n$  consecutive numbers starting at 1?
- Generalize the maximum total for  $n$  consecutive numbers starting at 1?

# IDEA 30

## FIVE NUMBERS IN A RING 2

The following ideas are developments from Idea 29.

- 1 What happens if, instead of finding the positive difference, we calculate the product between adjacent pairs and total up these products? What answers are possible now?
- 2 What happens if we calculate quotients and total these results? Because division is not commutative we will need to determine a direction of travel around the ring of numbers. However, this can generate two further problems:
  - What is the total if the direction of travel around the ring is reversed?
  - What is the total if the values created from adjacent pairs are always written as fractions?
- 3 What happens if we turn adjacent pairs of numbers into coordinate pairs and these are plotted as points? Students can explore shapes that are produced. Again we will need to be consistent in determining the direction of travel around the ring, although by plotting points using both directions of travel around the ring, the property of reflection in the line  $y = x$  can emerge.
  - What are the names, symmetries, areas and perimeters of the shapes so formed?