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Core 4 Jan 06

① $\dfrac{x^3-3x^2}{x^2-9} = \dfrac{x^2(x-3)}{(x-3)(x+3)} = \dfrac{x^2}{x+3}$

② $\sin y = xy + x^2$

$\frac{dy}{dx}\cos y = y + x\frac{dy}{dx} + 2x$

$\frac{dy}{dx}\cos y - x\frac{dy}{dx} = y+2x$

$\frac{dy}{dx}(\cos y - x) = y+2x$

$\frac{dy}{dx} = \dfrac{y+2x}{\cos y - x}$

③ i) $3x^3-2x^2+x+7 \equiv (x^2-2x+5)(Ax+B) + Rx+S$

$3x^3-2x^2+x+7 \equiv Ax^3-2Ax^2+5Ax+Bx^2-2Bx+5B+Rx+S$

$\equiv Ax^3+(B-2A)x^2+(5A-2B+R)x+(5B+S)$

$3=A$ $\qquad A=3$

$-2=B-2A$ $\qquad B=4$

$1=5A-2B+R$ $\qquad R=-6$

$7=5B+S$ $\qquad S=-13$

$\therefore$ Quotient is $3x+4$

Remainder is $-6x-13$

ii) if Remainder is $0x+0$ then

R S

$3=A$ (x^3)

$-2=B-2A$ (x^2)

$a=5A-2B$ (x)

$b=5B$ (unit)

$A=3$ $B=4$

$a=15-8=7$

$b=5\times 4=20$

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④ i) $\int x\sec^2 .dx$

u $\frac{dv}{dx}$

looks like parts. Learn!

$u=x$ $\qquad \frac{dv}{dx}=\sec^2 x$

$\frac{du}{dx}=1$ $\qquad v=\tan x$

$uv-\int v\frac{du}{dx}.dx$ $\qquad x\tan x-\int \tan x.dx$

formula book!

$\int x\sec^2x.dx = x\tan x - \ln|\sec x| + k$

ii) $\tan^2 x = \sec^2 x - 1$

$\int x\tan^2 x\,.dx = \int x(\sec^2 x-1)\,.dx$

$= \int x\sec^2 x - x\,.dx$

$= x\tan x - \ln|\sec x| - \frac{x^2}{2} + k$

⑤ i) $x=t^2$ $\quad y=2t$ $\qquad \frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$

$\frac{dy}{dt}=2$ $\quad \frac{dx}{dt}=2t$ $\qquad \frac{dy}{dx}=\frac{2}{2t}=\frac{1}{t}$

ii) at $(p^2, 2p)$ $\quad t=p$

So equation is $\dfrac{y-2p}{x-p^2}=\dfrac{1}{p}$

$y-2p=\frac{1}{p}(x-p^2)$

$py-2p^2=x-p^2$

$py = x+p^2$

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iii) at $(9,6)$ $p = 3$

$\therefore$ Equation is $3y = x + 9$ — ①

at $(25, -10)$ $p = -5$

$\therefore$ Equation is $-5y = x + 25$ — ②

$5 \times ① + 3 \times ②$

$$15y = 5x + 45$$
$$-15y = 3x + 75$$
$$= \quad 0 = 8x + 120$$
$$-120 = 8x$$
$$x = -15$$

if $x = -15$ $\underline{\underline{y = -2}}$ $\quad (3y = -15 + 9)$

⑥ $x = \sin^2\theta$ $\quad I = \int \sqrt{\frac{x}{1-x}} \, .dx = \int \sqrt{\frac{\sin^2\theta}{1-\sin^2\theta}}$

$\frac{dI}{dx} = \sqrt{\frac{\sin^2\theta}{1-\sin^2\theta}}$ $\qquad \frac{dx}{d\theta} = 2\sin\theta\cos\theta$

use chain rule:

let $u = \sin\theta$

$x = u^2$ $\qquad \frac{du}{d\theta} = \cos\theta$

$\frac{dx}{du} = 2u$

$\frac{dx}{d\theta} = \frac{dx}{du} \times \frac{du}{d\theta} = 2\sin\theta\cos\theta$

$\frac{dI}{d\theta} = \sqrt{\frac{\sin^2\theta}{\cos^2\theta}} \; 2\sin\theta\cos\theta$.

$\frac{dI}{d\theta} = \frac{\sin\theta}{\cos\theta} 2\sin\theta\cos\theta$

$\frac{dI}{d\theta} = 2\sin^2\theta$

$I = \int 2\sin^2\theta \, .d\theta$

4 ii) if $x = 1$ $\quad \sin^2\theta = 1$ $\quad \therefore \theta = \frac{\pi}{2}$

if $x = 0$ $\quad \sin^2\theta = 0$ $\quad \therefore \theta = 0$

$$\int_0^1 \sqrt{\frac{x}{1-x}} \, .dx = \int_0^{\pi/2} 2\sin^2\theta \, .d\theta$$

$$= \int_0^{\pi/2} 1 - \cos 2\theta \, .d\theta$$

$$= \left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$$

$$= \left(\frac{\pi}{2} - \frac{1}{2}\sin\pi\right) - (0 - 0) = \underline{\underline{\pi/2}}.$$

⑦ $\frac{11 + 8x}{(2-x)(1+x)^2} = \frac{A}{2-x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$

$\times (2-x)$ $\quad \frac{11+8x}{(1+x)^2} = A + \frac{B(2-x)}{1+x} + \frac{C(2-x)}{(1+x)^2}$

let $x = 2$ $\quad \frac{27}{9} = \underline{\underline{A = 3}}$

$\times (1+x)^2$ $\quad \frac{11+8x}{2-x} = \frac{A(1+x)^2}{2-x} + B(1+x) + C$

let $x = -1$ $\quad \frac{3}{3} = \underline{\underline{C = 1}}$

$$\frac{11+8x}{(2-x)(1+x)^2} = \frac{3}{2-x} + \frac{B}{1+x} + \frac{1}{(1+x)^2}$$

let $x=0$ (Just choose something that will make the calculation easier ie you would choose $x=17$!)

$$\frac{11}{2\times 1} = \frac{3}{2} + \frac{B}{1} + 1$$

$$5\tfrac{1}{2} = 2\tfrac{1}{2} + B$$

$\underline{B=3}$

$$\frac{11+8x}{(2-x)(1+x)^2} = \frac{3}{2-x} + \frac{3}{1+x} + \frac{1}{(1+x)^2}$$

ii) $3(2-x)^{-1} + 3(1+x)^{-1} + (1+x)^{-2}$

$$= 3\cdot 2^{-1}\left(1-\frac{x}{2}\right)^{-1} = \frac{3}{2}\left(1 + -1\times\left(\frac{-x}{2}\right) + \frac{-1\times -2}{2\times 1}\left(\frac{-x}{2}\right)^2\right)$$

$$+\ 3(1+x)^{-1} = 3\left(1 + -1(x) + \frac{-1\times -2}{2\times 1}(x)^2\right)$$

$$+\ (1+x)^{-2} = 1 + -2(x) + \frac{-2\times -3}{2\times 1}(x)^2$$

$$= \left(\frac{3}{2} + \frac{3x}{4} + \frac{3x^2}{8}\right) + \left(3 - 3x + 3x^2\right) + \left(1 - 2x + 3x^2\right)$$

$$= \frac{11}{2} - \frac{17x}{4} + \frac{51}{8}x^2$$

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8 i) $\dfrac{dy}{dx} = \dfrac{2-x}{y-3}$

$$(y-3)\frac{dy}{dx} = 2-x$$

$\int \cdot dx$

$$\frac{1}{2}y^2 - 3y = 2x - \frac{1}{2}x^2 + c$$

at $y=4$ $x=5$

$$\frac{1}{2}\times 4^2 - 3\times 4 = 2\times 5 - \frac{1}{2}5^2 + c$$

$$= 8 - 12 = 10 - \frac{25}{2} + c$$

$$-\frac{3}{2} = c$$

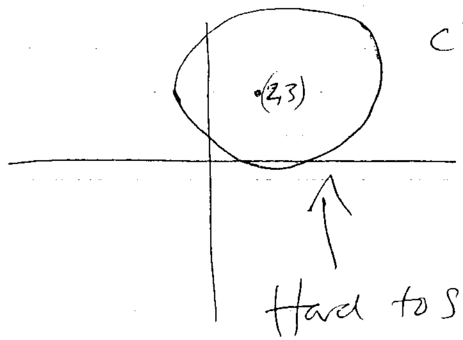
$$\underline{\underline{\frac{y^2}{2} - 3y = 2x - \frac{x^2}{2} - \frac{3}{2}}}$$

ii) $y^2 - 6y = 4x - x^2 - 3$

$$(x-2)^2 - 4 + (y-3)^2 - 9 = -3$$

$$(x-2)^2 + (y-3)^2 = 10$$

iii)



circle radius $\sqrt{10}$

centre $(2,3)$

Hard to sketch so write key info

7/

(9) i) To find the angle between the lines you only need to find the angle between the direction vectors

So $\begin{pmatrix}-8\\1\\-2\end{pmatrix} \cdot \begin{pmatrix}-9\\2\\-5\end{pmatrix}$

$= -8\times-9 + 1\times2 + -2\times-5$

$72 + 2 + 10 = 84$

$92 = \left|\begin{matrix}-8\\1\\-2\end{matrix}\right|\left|\begin{matrix}-9\\2\\-5\end{matrix}\right| \cos\theta$

$92 = \sqrt{64+1+4}\sqrt{81+4+25} \cos\theta$

$92 = \sqrt{69}\sqrt{110} \cos\theta$

$\cos\theta = \frac{84}{\sqrt{69}\sqrt{110}}$ $\quad \cos^{-1}\left(\frac{84}{\sqrt{69}\sqrt{110}}\right) = \theta = 15.4°$

ii) $4-8t = -2-9s$ — (1)

$2+t = a+2s$ — (2)

$-6-2t = -2-5s$ — (3)

Use (1) & (3) to find s & t

$4-8t = -2-9s$

$-6-2t = -2-5s$

$4-8t = -2-9s$

(−) $-24-8t = -8-20s$

8/ if $s=2$ $\quad 4-8t = -2-18$

$24-8t=0$

$t=3$

Now use (2) to find a

$2+t = a+2s$

$5 = a+4 \quad \therefore a=1$

$\begin{pmatrix}4\\2\\-6\end{pmatrix} + 3\begin{pmatrix}-8\\1\\-2\end{pmatrix} = \begin{pmatrix}4+-24\\2+3\\-6+-6\end{pmatrix}$

$= \begin{pmatrix}-20\\5\\-12\end{pmatrix}$

Easy Easy Easy Paper!!

You could all get an A on this!