IDEA

38

DIAGONAL DIVERSIONS 1

The following problems are based upon lengths of lines and areas of shapes within regular polygons. The problems are written for students who have acquired knowledge of Pythagoras and trigonometry.

- In a regular pentagon all diagonals are the same length. What is the ratio of the length of a diagonal to the length of a side in a regular pentagon? Students can draw different sized pentagons and compare answers. The result is, of course, going to be 1.618 (to 3 decimal places), i.e. the Golden Ratio. If the inverse ratio is taken, between the length of a side and the length of a diagonal, then the answer will be 0.618, or 1.618 – 1. As with Idea 27, this task might also be used to create the quadratic $\Phi^2 - \Phi - 1 = 0$ arising from $1/\Phi = \Phi - 1$.
- Again for a regular pentagon, when a single diagonal is cut off, an isosceles triangle and an isosceles trapezium are formed. What is the ratio of the areas of these shapes?
- How many different length diagonals are there in different regular polygons?
- What different ratios of lengths of pairs of diagonals exist?
- What different ratios exist when comparing the lengths of different diagonals to the length of the side of the polygon?

IDEA

39

CARD TRICKERY 3

My thanks goes to Peter Hampson, the headteacher at my last school who showed me this trick which I have used many times in lots of lessons.

1 Using a full pack of cards turn over the top card and call out its face value (Jack, Queen and King cards count as face value of 10).
2 Count out extra cards so the face value of the first card and extra cards add up to a total of 12. For example, if the first card is a 9, count three more cards to make 12 and place these underneath the 9. This now makes the first pile (p).
3 Turn over the next card from the remaining pack and repeat as before. If this card is a 5 then count out a further seven cards (to make up to 12), place these under the 5, and make a second pile.
4 Keep repeating this until you reach a situation where there are not enough cards in the remaining pack to make a total of 12. Place these cards in a separate remainder pile (r).

Without looking at the cards on the top of each pile, it is possible to work out the total sum of the cards by knowing how many piles and how many remainder cards there are.

The following is a natural development and provides in-context opportunities for students to construct formulae to show how the total (T) can be calculated by knowing the number of piles (p) and the number of remaining cards (r).

Trying to explain *why* the formula connecting p, r and T works will provide a further, worthy challenge.

A development of this task is to change the variables within the problem. Suppose a pack of cards consisted of four suits with only ten cards per suit, or five suits and ten cards in each, what would the 'magic' totals be in each case?

How would the formula connecting p and r with T change?