# Number pyramids

## Generalising from number

### Prerequisite knowledge

- Basic number bonds
- Trying all possibilities systematically
- The idea of maximum and minimum values

### Why do this problem?

The problem focuses on identifying a pattern, and from this making reasoned judgements about how to solve a problem – for example, recognising that to obtain a maximum value it is necessary to put the largest numbers in particular positions.

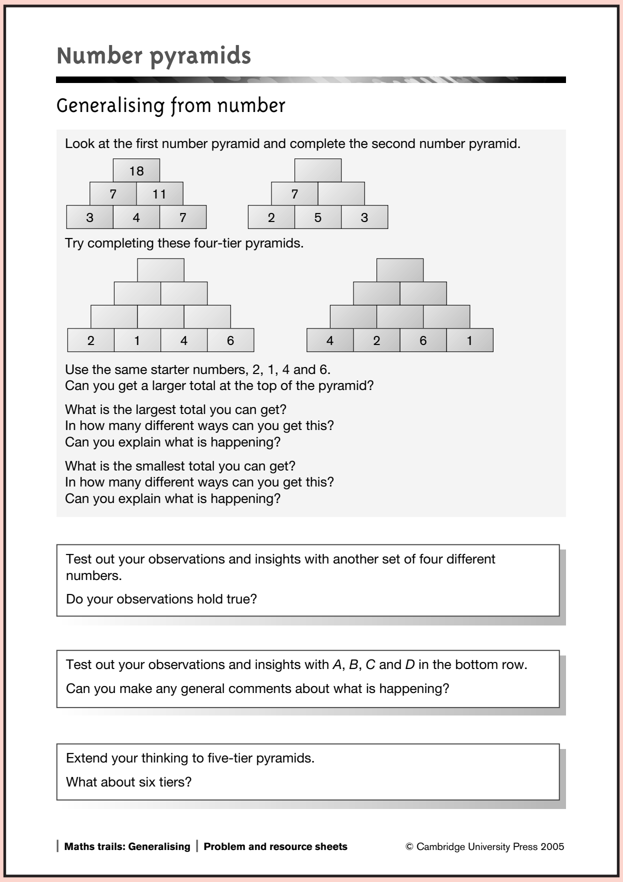
### Time

One lesson

### Resources

CD-ROM: pupil worksheet; resource sheets of blank four-, five- and six-tier pyramids

NRICH website (optional): www.nrich.maths.org, May 2004, 'Number pyramids' (includes an interactive tool that enables pupils to explore three- and four-tier pyramids)


Number pyramids

Generalising from number

Look at the first number pyramid and complete the second number pyramid.

Try completing these four-tier pyramids.

Use the same starter numbers, 2, 1, 4 and 6.
Can you get a larger total at the top of the pyramid?

What is the largest total you can get?
In how many different ways can you get this?
Can you explain what is happening?

What is the smallest total you can get?
In how many different ways can you get this?
Can you explain what is happening?

Test out your observations and insights with another set of four different numbers.
Do your observations hold true?

Test out your observations and insights with *A*, *B*, *C* and *D* in the bottom row.
Can you make any general comments about what is happening?

Extend your thinking to five-tier pyramids.
What about six tiers?

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### Introducing the problem

Draw a three-tier pyramid on the board and model selecting each pair of numbers, putting their sum in the block above until you reach the vertex. No dialogue is needed at this stage, but use actions in a way that emphasises the pyramid's structure (point at two numbers, then at the 'result' block and put in the sum). This can be done once or twice. Then, selecting another set of three 'base' numbers, invite individual pupils to 'fill in the blanks'.

Working with you and using a four-tier pyramid with base numbers 2, 1, 4 and 6, ask the class to find the top number. Then ask them, on their own or in pairs, to investigate the pyramid by tackling some of the questions on the problem sheet.

### Main part of the lesson

Although it is possible to extend this problem to five or more tiers there is a great deal of pattern and structure that can be discovered and shared with the class by concentrating on four-tier pyramids.

Stop after about 15 minutes to discuss some of the observations pupils have made. During the discussion, list questions pupils are posing or develop a question from their observations (such as the following). Share the list with the class to stimulate further work where pupils test and/or explain some of the ideas that have emerged.

- Why do the largest numbers need to be on the inside of the base for the maximum top number and on the outside for the minimum?

- Given base numbers of 1, 2, 4 and 6, what other top numbers are possible?
- What do you notice about the top numbers? (they are all odd)
- The given base numbers were one odd and three even. What would happen if the base numbers were all odd, all even, or two of each?
- How many of each base number are used to make the top number?

## Plenary

Pull together some of the answers to the above questions by testing ideas such as the following.

- Using a new set of base numbers, what would you do to make the maximum or minimum top number?
- Can you select four base numbers that are not all even and make an even top number?

It is possible to extend the task for homework to investigate similar questions with pyramids of more layers.

## Solution notes

You can get the largest top number by putting the two biggest base numbers in the middle of the pyramid base.

19 is the smallest possible top number with base numbers of 1, 2, 4 and 6.

To get the smallest total, you need to put the two smallest base numbers in the middle of the pyramid base.

Labelling the four base numbers as $A$, $B$, $C$ and $D$, the top number is always $A + 3B + 3C + D$.

Pascal's triangle gives the number of times each of the base numbers is used to obtain the vertex number:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

and so on.