Core1 May 2005

1. $x^2-6x-40 \geq 0$

$(x+4)(x-10) \geq 0$

$\therefore$ Critical values -4 and 10

if $x < -4$ then $(x+4)<0$ and $(x-10)<0$

so $(x+4)(x-10)>0$

if $x > 10$ then $(x+4)>0$ and $(x-10)>0$

so $(x+4)(x-10)>0$

So if $x \leq -4$ or $x \geq 10$ then $(x+4)(x-10) \geq 0$

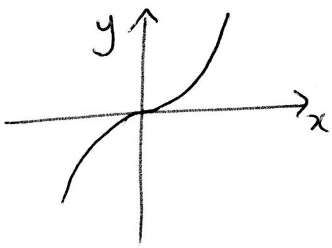
2. i) $3(x^2+4x)+7 = 3(x+2)^2-12+7$

$= 3(x+2)^2-5$

$\therefore$ $a=2$ $b=-5$

ii) $x=-2$

3. i)



ii) Either a reflection in y axis or a reflection in the x axis (as $-x^3 = (-x)^3$)

iii) $y=(x-p)^3$

4. This is a quadratic in x^3. So let $k = x^3$

$\therefore \quad x^6 + 26x^3 - 27 = 0$

$= \quad k^2 + 26k - 27 = 0$

$(k+27)(k-1) = 0 \qquad \therefore \quad k = -27, 1$

$\therefore \quad x^3 = -27 \qquad x^3 = 1$

So $x = -3$ or $x = 1$

5. a) $2x^{2/3} \times 3x^{-1} = 6x^{(2/3 + -1)} = 6x^{-1/3}$

b) $2^{40} \times 4^{30} = 2^{40} \times (2^2)^{30} = 2^{40} \times 2^{60} = 2^{100}$

c) $\dfrac{26}{4-\sqrt{3}} \times \dfrac{4+\sqrt{3}}{4+\sqrt{3}} = \dfrac{26(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})} = \dfrac{26(4+\sqrt{3})}{16-3}$

$= \dfrac{26(4+\sqrt{3})}{13} = 2(4+\sqrt{3}) = \underline{\underline{8+2\sqrt{3}}}$

6. i) $(x^2+2x+1)(3x-4)$

$3x^3 + 6x^2 + 3x - 4x^2 - 8x - 4$

$= 3x^3 + 2x^2 - 5x - 4$

ii) $f'(x) = 9x^2 + 4x - 5$

iii) $f''(x) = 18x + 4$

7. i) a) $b^2 - 4ac \qquad 6^2 - 4 \times 1 \times 9 = 0$

b) $(-10)^2 - 4 \times 1 \times 12 = 100 - 48 = 52$

c) $(-2)^2 - 4 \times 1 \times 5 = 4 - 20 = -16$

ii) a) Fig 3 - 1 root at $x = -3$

b) Fig 2 - 2 roots both +ve.

c) Fig 5 - no real roots ∴ does not meet x-axis.

8. i) Circle with centre (0,0) radius 5.

ii) $x^2 + y^2 = 25 \quad 2x + y - 5 = 0 \Rightarrow y = 5 - 2x$

$$x^2 + (5-2x)^2 = 25$$
$$x^2 + 25 + 4x^2 - 20x = 25$$
$$5x^2 - 20x = 0$$
$$5(x^2 - 4x) = 0 \qquad 5x(x-4) = 0$$

∴ $x = 0$ or $x = 4$

$(0,5) \quad (4,-3)$.

9. i) $4x - 3y + 5 = 0 \Rightarrow 4x + 5 = 3y \Rightarrow y = \frac{4}{3}x + \frac{5}{3}$

∴ Gradient is $\frac{4}{3}$

ii) l_2 has gradient $\frac{-3}{4}$ and passes through (1,2)

So $\frac{y-2}{x-1} = \frac{-3}{4} \qquad y - 2 = \frac{-3}{4}(x-1) \Rightarrow 4y - 8 = -3x + 3$

$\Rightarrow 3x + 4y - 11 = 0$

iii) at x-axis $y = 0$ so for l_1 $4x - 3y + 5 = 0$

$$4x + 5 = 0$$
$$x = -\frac{5}{4}$$

at y-axis $x = 0$

So for l_2 $3x + 4y - 11 = 0$

$$4y = 11 \qquad y = \frac{11}{4}$$

$\therefore\ P = \left(\frac{-5}{4}, 0\right) \quad Q = \left(0, \frac{11}{4}\right)$

So midpoint is $\left(\frac{0 + {}^{-5}/_4}{2}, \frac{0 + \frac{11}{4}}{2}\right) = \left(\frac{-5}{8}, \frac{11}{8}\right)$

iv) $\sqrt{\left(0 - \frac{-5}{4}\right)^2 + \left(\frac{11}{4} - 0\right)^2} = \sqrt{\frac{25}{16} + \frac{121}{16}} = \sqrt{\frac{146}{16}} = \frac{\sqrt{146}}{4}$

10. i) $y = \frac{1}{3}x^3 - 9x \quad \frac{dy}{dx} = \frac{3}{3}x^2 - 9 = x^2 - 9$

ii) $x^2 - 9 = 0 \quad x = 3$ or $-3 \quad \therefore\ y = \frac{1}{3}3^3 - 9 \times 3 = {}^{-}18$

or $y = \frac{1}{3}(-3)^3 + 9 \times 3 = 18$

So $(3, {}^{-}18)$ $(-3, 18)$

iii) $\frac{d^2y}{dx^2} = 2x$ if $x = 3$ $\frac{d^2y}{dx^2} = 6 \therefore (3, {}^{-}18)$ is a minimum.

if $x = {}^{-}3$ $\frac{d^2y}{dx^2} = {}^{-}6 \therefore (-3, 18)$ is a maximum.

iv) Gradient of $24x + 3y + 2 = 0$ is -8

as $3y = -24x - 2 \Rightarrow y = {}^{-}8x - {}^{2}/_{3}$

$\therefore$ Equate gradient of curve to -8

$x^2 - 9 = {}^{-}8 \quad x^2 = 1 \quad \therefore\ x = 1$ or -1

on curve if $x = 1$ $y = \frac{1}{3} - 9 = -8\,{}^{2}/_{3}$

if $x = {}^{-}1$ $y = {}^{-}\frac{1}{3} + 9 = 8\,{}^{2}/_{3}$

on line if $x = 1$ $y = -8\,{}^{2}/_{3}$

if $x = {}^{-}1$ $y = 8 - {}^{2}/_{3} = 7\,{}^{1}/_{3}$

These points are on line and curve

So $p = 1$ $q = {}^{-}8\,{}^{2}/_{3}$.