

# 4723 Core Mathematics 3

<p>1 (i) Show correct process for composition of functions</p> <p>Obtain <math>(-3 \text{ and hence}) -23</math></p>	<p><b>M1</b> numerical or algebraic; the right way round</p> <p><b>A1 2</b></p>
<p>(ii) <u>Either</u>: State or imply <math>x^3 + 4 = 12</math></p> <p>Attempt solution of equation involving <math>x^3</math></p> <p>Obtain 2</p> <p><u>Or</u>: Attempt expression for <math>f^{-1}</math></p> <p>Obtain <math>\sqrt[3]{x-4}</math> or <math>\sqrt[3]{y-4}</math></p> <p>Obtain 2</p>	<p><b>B1</b></p> <p><b>M1</b> as far as <math>x = \dots</math></p> <p><b>A1 3</b> and no other value</p> <p><b>M1</b> involving <math>x</math> or <math>y</math>; involving cube root</p> <p><b>A1</b></p> <p><b>A1 (3)</b> and no other value</p>
<p>2 (i) Obtain correct first iterate 2.864</p> <p>Carry out correct iteration process</p> <p>Obtain 2.877</p> <p><math>[3 \rightarrow 2.864327 \rightarrow 2.878042 \rightarrow 2.876661 \rightarrow 2.876800]</math></p> <p>(ii) State or imply <math>x = \sqrt[3]{31 - \frac{5}{2}x}</math></p> <p>Attempt rearrangement of equation in <math>x</math></p> <p>Obtain equation <math>2x^3 + 5x - 62 = 0</math></p>	<p><b>B1</b> or greater accuracy 2.864327...; condone 2 dp here and in working to find at least 3 iterates in all after at least 4 steps; answer required to exactly 3 dp</p> <p><b>M1</b></p> <p><b>A1 3</b></p> <p><b>B1</b></p> <p><b>M1</b> involving cubing and grouping non-zero terms on LHS</p> <p><b>A1 3</b> or equiv with integers</p>
<p>3 (a) State correct equation involving <math>\cos \frac{1}{2}\alpha</math></p> <p>Attempt to find value of <math>\alpha</math></p> <p>Obtain 151</p> <p>(b) State or imply <math>\cot \beta = \frac{1}{\tan \beta}</math></p> <p>Rearrange to the form <math>\tan \beta = k</math></p> <p>Obtain 69.3</p> <p>Obtain 111</p>	<p><b>B1</b> such as <math>\cos \frac{1}{2}\alpha = \frac{1}{4}</math> or <math>\frac{1}{\cos \frac{1}{2}\alpha} = 4</math></p> <p>or ...</p> <p><b>M1</b> using correct order for the steps</p> <p><b>A1 3</b> or greater accuracy; and no other values between 0 and 180</p> <p><b>B1</b></p> <p><b>M1</b> or equiv involving <math>\sin \beta</math> only or <math>\cos \beta</math> only; allow missing <math>\pm</math></p> <p><b>A1</b></p> <p><b>A1 4</b> or greater accuracy; and no others between 0 and 180</p>
<p>4 (i) Obtain derivative of form <math>kh^5(h^6 + 16)^n</math></p> <p>Obtain correct <math>3h^5(h^6 + 16)^{-\frac{1}{2}}</math></p> <p>Substitute to obtain 10.7</p> <p>(ii) Attempt multn or divn using 8 and answer from (i)</p> <p>Attempt 8 divided by answer from (i)</p> <p>Obtain 0.75</p>	<p><b>M1</b> any constant <math>k</math>; any <math>n &lt; \frac{1}{2}</math>; allow if <math>-4</math> term retained</p> <p><b>A1</b> or (unsimplified) equiv; no <math>-4</math> now</p> <p><b>A1 3</b> or greater accuracy or exact equiv</p> <p><b>M1</b></p> <p><b>A1</b> <math>\sqrt{3}</math> or greater accuracy; allow <math>0.75 \pm 0.01</math>; following their answer from (i)</p>

5 (a)	Obtain integral of form $k(3x + 7)^{10}$ Obtain (unsimplified) $\frac{1}{10} \times \frac{1}{3} (3x + 7)^{10}$ Obtain (simplified) $\frac{1}{30} (3x + 7)^{10} + c$	<b>M1</b> any constant $k$ <b>A1</b> or equiv <b>A1 3</b>
(b)	State $\int \pi \left(\frac{1}{2\sqrt{x}}\right)^2 dx$ Integrate to obtain $k \ln x$  Obtain $\frac{1}{4} \pi \ln x$ or $\frac{1}{4} \ln x$ or $\frac{1}{4} \pi \ln 4x$ or $\frac{1}{4} \ln 4x$ <b>A1</b> Show use of the log $a - \log b$ property Obtain $\frac{1}{4} \pi \ln 2$	<b>B1</b> or equiv involving $x$ ; condone no $dx$ <b>M1</b> any constant $k$ involving $\pi$ or not; or equiv such as $k \ln 4x$ or $k \ln 2x$  <b>M1</b> not dependent on earlier marks <b>A1 5</b> or similarly simplified equiv
6 (i)	<u>Either:</u> Refer to translation and reflection State translation by 1 in negative $x$ -direction  State reflection in $x$ -axis <u>Or:</u> Refer to translation and reflection State reflection in $y$ -axis State translation by 1 in positive $x$ -direction	<b>B1</b> in either order; allow clear equivs <b>B1</b> or equiv but now using correct terminology <b>B1 3</b> using correct terminology <b>B1</b> in either order; allow clear equivs <b>B1 (3)</b> with order reflection then translation clearly intended
(ii)	Show sketch with attempt at reflection of 'negative' part in $x$ -axis Show (more or less) correct sketch	<b>M1</b> and curve for $0 < x < 1$ unchanged <b>A1 2</b> with correct curvature
(iii)	Attempt correct process for finding at least one value  Obtain $1 - \frac{1}{2}\sqrt{3}$ Obtain $1 + \frac{1}{2}\sqrt{3}$	<b>M1</b> as far as $x = \dots$ ; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$ <b>A1</b> or exact equiv <b>A1 3</b> or exact equiv; give <b>A1A0</b> if extra incorrect solution(s) provided
7 (i)	Attempt use of product rule for $xe^{2x}$ Obtain $e^{2x} + 2xe^{2x}$ Attempt use of quotient rule Obtain unsimplified $\frac{(x+k)(e^{2x} + 2xe^{2x}) - xe^{2x}}{(x+k)^2}$ Obtain $\frac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$	<b>M1</b> obtaining $\dots + \dots$ <b>A1</b> or equiv; maybe within QR attempt <b>M1</b> with or without product rule <b>A1</b> <b>A1 5 AG</b> ; necessary detail required
(ii)	Attempt use of discriminant Obtain $4k^2 - 8k = 0$ or equiv and hence $k = 2$ Attempt solution of $2x^2 + 2kx + k = 0$  Obtain $x = -1$ Obtain $-e^{-2}$	<b>M1</b> or equiv <b>A1</b> <b>M1</b> using their numerical value of $k$ or solving in terms of $k$ using correct formula <b>A1</b> <b>A1 5</b> or exact equiv

8 (i)	State or imply $h = 1$ Attempt calculation involving attempts at $y$ values  Obtain $a(1 + 4 \times 2 + 2 \times 4 + 4 \times 8 + 2 \times 16 + 4 \times 32 + 64)$ Obtain 91	<b>B1</b> <b>M1</b> addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 $y$ values any constant $a$  <b>A1 4</b>
(ii)	State $e^{x \ln 2}$ or $k = \ln 2$ Integrate $e^{kx}$ to obtain $\frac{1}{k}e^{kx}$ Obtain $\frac{1}{\ln 2}(e^{6 \ln 2} - e^0)$ Simplify to obtain $\frac{63}{\ln 2}$	<b>B1</b> allow decimal equiv such as $e^{0.69x}$ <b>M1</b> any constant $k$ or in terms of general $k$ <b>A1</b> or exact equiv <b>A1 4</b> allow if simplification in part (iii)
(iii)	Equate answers to (i) and (ii)  Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$	<b>M1</b> provided $\ln 2$ involved other than in power of $e$ <b>A1 2</b> <b>AG</b> ; necessary correct detail required

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9 (i)	State at least one of $\cos \theta \cos 60 - \sin \theta \sin 60$ and $\cos \theta \cos 30 - \sin \theta \sin 30$ Attempt complete multiplication of identities of form $\pm \cos \cos \pm \sin \sin$ Use $\cos^2 \theta + \sin^2 \theta = 1$ and $2 \sin \theta \cos \theta = \sin 2\theta$ Obtain $\sqrt{3} - 2 \sin 2\theta$	<b>B1</b>  <b>M1</b> with values $\frac{1}{2}\sqrt{3}$ , $\frac{1}{2}$ involved <b>M1</b> <b>A1 4</b> <b>AG</b> ; necessary detail required
(ii)	Attempt use of 22.5 in right-hand side Obtain $\sqrt{3} - \sqrt{2}$	<b>M1</b> <b>A1 2</b> or exact equiv
(iii)	Obtain 10.7 Attempt correct process to find two angles Obtain 79.3	<b>B1</b> or greater accuracy; allow $\pm 0.1$ <b>M1</b> from values of $2\theta$ between 0 and 180 <b>A1 3</b> or greater accuracy and no others between 0 and 90; allow $\pm 0.1$
(iv)	Indicate or imply that critical values of $\sin 2\theta$ are $-1$ and $1$ Obtain both of $k > \sqrt{3} + 2$ , $k < \sqrt{3} - 2$ Obtain complete correct solution	<b>M1</b> <b>A1</b> condoning decimal equivs, $\leq \geq$ signs <b>A1 3</b> now with exact values and unambiguously stated