C2 sample exam paper

- The paper is 1 hour 30 minutes long.
- Calculators may be used.

1. Find $\int (4x + \sqrt[3]{x})\,dx$

(4)

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2. The sum S of an arithmetic series is given by

$$S = \sum_{r=1}^{20} (5r + 3).$$

(a) Write down the first three terms of the series.

(2)

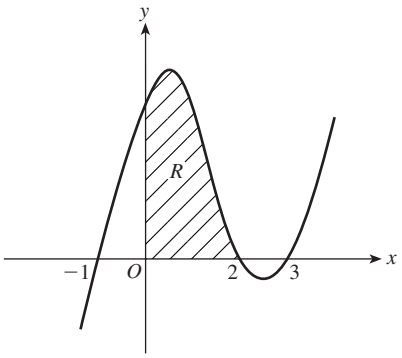
(b) Find the common difference of the series.

(1)

(c) Calculate the value of S.

(2)

3.



The figure shows the curve C with equation $y = f(x)$.

$$f(x) = x^3 - 4x^2 + x + 6.$$

The curve C crosses the x-axis at $(-1\ 0)$, $(2,\ 0)$ and $(3,\ 0)$.

The shaded region R is bounded by C, the positive x-axis and the positive y-axis.

Find the area of R.

(5)

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4. The speed, $v\,\text{m s}^{-1}$, of a car at time t seconds is modelled by

$$v = 0.006(400t - 4.9t^2), \qquad 0 \leqslant t \leqslant 20.$$

(a) Copy and complete the following table showing the speed of the car at 5 second intervals. Give the values of v to 2 decimal places.

t	0	5	10	15	20
v	0		21.06		36.24

(2)

The distance, s metres, travelled by a car in the first 20 seconds is given by the area under the graph of v against t that lies between the curve, the line $t = 20$ and the t-axis.

(b) Use the trapezium rule, with all the values from your table, to estimate s.

(4)

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5. $x_{n+1} = \dfrac{a - 3x_n}{x_n}, \qquad n > 0, \quad a > 3, \quad x_1 = 1.$

(a) Find, in terms of a, expressions for

(i) x_2, (ii) x_3

(3)

Given that $x_3 > 2$,

(b) find the range of values of a.

(4)

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6. The first 3 terms, in ascending powers of x, of the binomial expansion $(1 + bx)^n$ are $1 + 28x + 336x^2$.
Given that n is a positive integer, find the value of n and the value of b.

(8)

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7. A geometric series has third term 18 and sixth term 486.

(a) Show that the common ratio is 3.

(4)

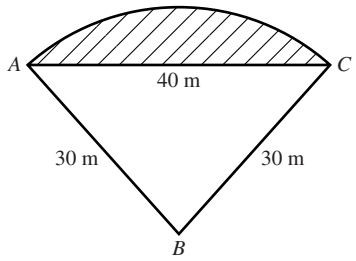
(b) Find the first term of the series.

(2)

(c) Hence calculate the sum of the first 15 terms.

(2)

8.



The figure shows a plan of a garden. The triangle ABC is a lawn and the shaded area is a flower-bed.

There is a straight line of length 40 m from the point A to the point C. The points are joined by an arc of the circle whose centre is B and whose radius is 30 m as shown in the figure.

(a) Show that the size of the angle ABC is 1.46 radians to 2 decimal places.

(3)

Calculate, giving your answers to 3 significant figures,

(b) the area of the entire garden,

(2)

(c) the area of the flower-bed.

(4)

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9. Given that $2 \log_3 y = c - \log_3 x$, $x > 0$, $y > 0$,

where c is the constant, and $y = 2$ when $x = 4$,

(a) show that

$$y = 4x^{-\frac{1}{2}}$$

(4)

Given that $\log_3 y = 1 - 2\log_3 x$, $x > 0$, $y > 0$,

(b) show that $yx^a = b$, stating the value of a and the value of b.

(3)

(c) Hence solve the simultaneous equations

$$2 \log_3 y = c - \log_3 x, \quad \log_3 y = 1 - 2 \log_3 x, \; x > 0, \; y > 0$$

giving your answers to 3 significant figures.

(3)

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10. Find all the values of θ, correct to 1 decimal place, in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$ for which

(a) $\tan(2\theta) = 0.4$ **(3)**

(b) $\sin\theta + \cos^2\theta = 0.2$ **(7)**

END