Arrange the digits

Mixed methods

Prerequisite knowledge

- Understanding of place value

Why do this problem?

It offers opportunities for being systematic by using trial and improvement. It can also be used as a stepping stone to discuss digital roots of 9 as on the resource sheet on the CD-ROM.

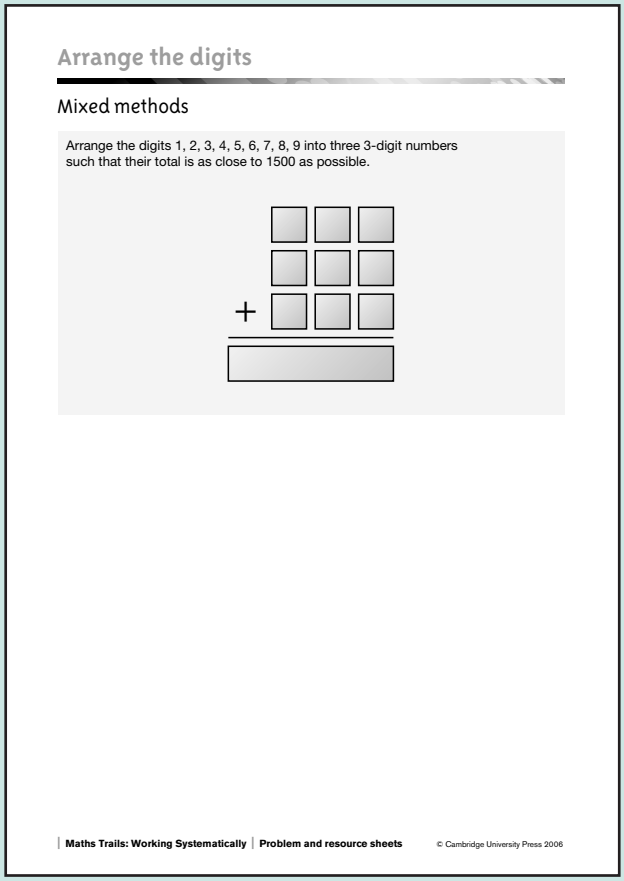
Time

One lesson

Resources

CD-ROM: problem sheet, resource sheet on the digital roots of 9

NRICH website (optional): www.nrich.maths.org, October 1997, 'Arrange the digits'


Arrange the digits

Mixed methods

Arrange the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 into three 3-digit numbers such that their total is as close to 1500 as possible.

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Introducing the problem

Present the problem and invite students to come to the board/OHP and place the nine digits in any order in the sum. Ask pupils to calculate the total of the digits arranged in this way. Ask the class how they might rearrange the digits to make a total closer to 1500.

Repeat this process, discussing strategies the pupils are using to improve the total.

Main part of the lesson

Encourage the class to work on the problem in small groups, concentrating on drawing out ideas and strategies for making the route to the solution more efficient. Say that you will stop them after ten minutes to share their thoughts.

Use this opportunity to move away from trial and improvement to some more strategic approaches:

- What could the sum of the numbers in the left-hand column be?
- How many different ways can you make the left-hand column sum to 13, 14, 15?
- What is the largest and what is the smallest possible value of the sum of the three sets of three digits with each of these left-hand column possibilities? If, for example, you're using 9, 1 and 3 to make a total of 13 in the left-hand column, then the largest total is found by using 6, 7 and 8 in the middle column. The smallest total is found by using 2, 4 and 5 in the middle column.

Pupils can now use trial and improvement to find totals closer to 1500. Allow them time to investigate hundred totals of 13, 14 and 15 in this way.

Plenary

- Why can't you make 1500?

A systematic approach to the problem can identify that the nearest value is 1503.

At this point you might like to discuss digital roots of 9. An example of what happens with

the digit 2 as it is moved between columns is given on the resource sheet. The questions below the table could be used for homework and followed up in the next lesson. Alternatively, this could be included in the main part of the lesson and a problem involving the same principles (such as the one given below) can be used in the plenary.

A follow-up session might explore an alternative problem that involves digital roots of 9. For example, think of a 3-digit number, reverse the digits, take the smaller from the larger; you will end up with a number that is divisible by 9.

- Is this always the case?
- Can you explain why?

Solution notes

Definition

The *digital root* of a number is the sum of all of its digits, computed recursively until only one digit remains. For example, the digital root of 457 is 7 (4 + 5 + 7 = 16; 1 + 6 = 7).

The solution cannot be 1500 because its digital root is not nine. The nearest solution is 1503. A more complete explanation of the concept of digital roots of nine can be found in the notes for this question on the NRICH website.