Left overs

Modelling and optimisation

Prerequisite knowledge

- Common factors and common multiples

Why do this unit?

The spreadsheet prompts the solver to see there is an underpinning structure to this problem context. It is up to the pupils to explain the structure and make use of it. For example, can the interval of 60 between solutions be explained and generalised?

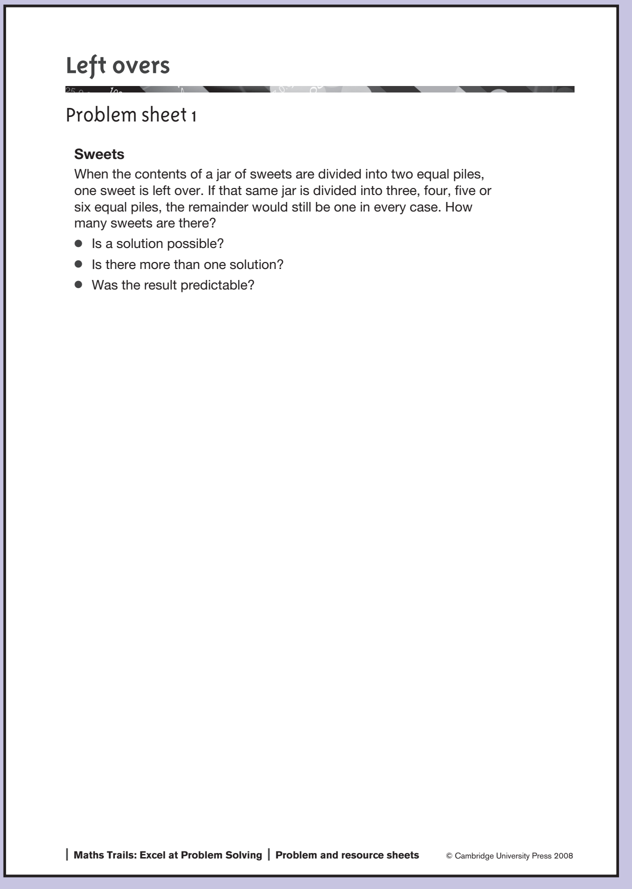
Time

Two lessons

Resources

CD-ROM: spreadsheet, problem sheets 1 and 2

NRICH website (optional): www.nrich.maths.org, November 1996, 'Remainders'; March 2007, 'The Chinese remainder theorem'


Left overs

Problem sheet 1

Sweets

When the contents of a jar of sweets are divided into two equal piles, one sweet is left over. If that same jar is divided into three, four, five or six equal piles, the remainder would still be one in every case. How many sweets are there?

- Is a solution possible?
- Is there more than one solution?
- Was the result predictable?

Maths Trails: Excel at Problem Solving | Problem and resource sheets © Cambridge University Press 2008

Introducing the unit

Show pupils 'Sweets' on problem sheet 1.

- Where would you start? What should we try? [Try some numbers.]
- Is a solution possible? Can you explain why this is so?

Have pupils work on their own for a few minutes, trying some numbers and recording any results and insights. After a short time stop the group. Share and draw out pointers to the structure of the problem. For example, the number of sweets must be odd (this leaves one when divided by 2) and must be one more than a multiple of 3, so already the possibilities are constrained (7, 13, 19, ...).

- Can we use this idea about multiples? [We can be more particular in the numbers we try. We also need the number of sweets to be one more than a multiple of 4, of 5 and of 6.]

Allow a short period in which pupils, working in pairs, continue to calculate and think. End the introduction ready to move into finding a solution or, if one has been found, pose the question:

- Could there be more than one solution?

Main part of the unit

Show pupils the sheet 'Plain sweets' on the spreadsheet.

Invite pupils to say what calculation is being done from looking at the numbers, and then click each cell to see the formula used and discuss the MOD function. [MOD gives the remainder when the first value is divided by the second. MOD(17, 3) would give 2 because when 17 is divided by 3 there is a remainder of 2.]

Take time throughout the following stages of the unit for mental arithmetic or calculator use if that is helpful.

- How would you recognise a solution in the spreadsheet? [a row of ones]

Scan down and ask pupils to look for a solution. Finding the row of ones is easy but we can make this even easier by using **conditional formatting**. Show the group the sheet 'Sweets' on the spreadsheet and briefly discuss what the conditional formatting is doing. [highlighting a cell which contains the value 1]

- The sheet starts at 80. Is 80 a good start value? [No, because there may be solutions less than 80.]

Discuss what would be better. Change the 80 to 1, drawing attention to the formula, for example =B3+1 in cell B4, which causes automatic recalculation. Scan down and record solutions as they occur.

- What do you notice? [Rows of ones appear at intervals of 60.]
- Was that interval predictable? [Solutions in intervals must be a multiple of 2, and a multiple of 3 and 4 and 5 and 6 – that is, the common multiple of 2, 3, 4, 5, and 6.]
- What is the first solution? [1] Why is that a solution? [Each pile receives no sweets with one left over.]

Encourage discussion, relating ways of expressing a general solution [any multiple of 60, plus 1] to the original problem.

Show the group the problem 'Remainders' on problem sheet 2.

Explain to pupils that they will be expected to produce their own spreadsheets and to justify their solutions to the rest of the group at the end of the session. Ask pupils to plan on paper before they move to the computer to use Excel.

The following prompts may be useful while the pupils are working:

- What formula did you need in each cell? [use MOD for remainders, a formula to create the first column values]
- What will you be looking for in the rows? [1, 2, …]
- Can you convince me that you have found the smallest number that works?

Plenary

Use examples from the pupil-produced spreadsheets to discuss results and insights including the value of using a spreadsheet and the efficiency of different layouts. The sheets 'Plain remainders' and 'Remainders' may be useful.

- Can you explain why the solutions appear at intervals of 60? [LCM]
- Can you explain why the smallest solution is 59? [60 would have a remainder of zero for all divisors. 59 is one less than a multiple of each of the divisors, so it will leave a remainder of 5 when divided by 6, a remainder of 4 when divided by 5, and so on.]
- If the remainder had to be two less than the number you divided by what would be the smallest solution? [58]

Solution notes

For 'Sweets', the first solution is 1 and then all the other solutions follow at intervals of 60 (there is an infinite number of solutions). For 'Remainders', the first positive solution is 59 with solutions continuing in steps of 60 (again, there is an infinite set of solutions).

The article 'The Chinese remainder theorem' on the NRICH website explores the mathematics of these problems in greater depth.