

4724 Core Mathematics 4

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| <p>1 Method for finding magnitude of any vector Method for finding scalar prod of any 2 vectors Using $\cos \theta = \frac{\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} \cdot 2\mathbf{i} + \mathbf{j} + \mathbf{k}}{ \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} 2\mathbf{i} + \mathbf{j} + \mathbf{k} }$ 70.9 (70.89, 70.893) WWW; 1.24 (1.237)</p> | <p>M1 M1 M1 A1</p> | <p>Expect $\sqrt{14}$ and $\sqrt{6}$ Expect $1.2 + (-2)1 + 3.1 = 3$ Correct vectors only. Expect $\cos \theta = \frac{3}{\sqrt{14}\sqrt{6}}$ 4 Condone answer to nearest degree (71)</p> |
| <p>2 (i) Correct format $\frac{A}{x+1} + \frac{B}{x+2}$ $-\frac{1}{x+1}$ or $A = -1$ $+\frac{2}{x+2}$ or $B = 2$</p> <hr/> <p>(ii) $\int \frac{1}{x+1} dx = \ln(x+1)$ or $\ln x+1$ or $\int \frac{1}{x+2} dx = \ln(x+2)$ or $\ln x+2$ $A \ln x+1 + B \ln x+2 + c$ ISW</p> | <p>M1 A1 A1 B1 $\sqrt{A1}$</p> | <p>stated or implied by answer 3 Expect $-\ln x+1 + 2 \ln x+2 + c$</p> |
| <p>3 <u>Method 1 (Long division)</u> Clear correct division method at beginning Correct method up to & including x term in quot <u>Method 2 (Identity)</u> Writing $(x^2 + 2x - 1)(x^2 + bx + 2) + cx + 7$ Attempt to compare cfs of x^3 or x^2 or x or const Then: $b = -4$ $c = -1$ $a = 5$</p> | <p>M1 M1 M1 M1 A1 A1 A1</p> | <p>x^2 in quot, mult back & attempt subtraction [At subtraction stage, cf $(x^4) = 0$] [At subtraction stage, cf $(x^3) = 0$] Probably equated to $x^4 - 2x^3 - 7x^2 + 7x + a$ 5</p> |
| <p>4 $\frac{d}{dx}(x^2 y) = x^2 \frac{dy}{dx} + 2xy$ $\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$ Substitute $(x,y) = (1,1)$ and solve for $\frac{dy}{dx}$ $\frac{dy}{dx} = -\frac{11}{7}$ WWW Gradient normal = $-\frac{1}{\frac{dy}{dx}}$ $7x - 11y + 4 = 0$ AEF</p> | <p>B1 B1 M1 M1 A1 M1 A1</p> | <p>s.o.i.; or v.v. Solve now or at normal stage. [This dep on either/both B1 earned] Implied if grad normal = $\frac{7}{11}$ Numerical or general, awarded at any stage 6 No fractions in final answer.</p> |

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| <p>5 (i) Use $3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} - 5\mathbf{k}$ only</p> <p>Use correct method for scalar prod of <u>any</u> 2 vectors</p> <p>Obtain $6 + 4 - 10$, state = 0 & deduce perp AG</p> | <p>M1</p> <p>M1</p> <p>A1 3</p> | <p>(indep) May be as part of $\cos \theta = \frac{a \cdot b}{ a b }$</p> |
| <p>(ii) Produce 3 equations in s and t</p> <p>Solve 2 of the equations for s and t</p> <p>Obtain $(s, t) = \left(\frac{3}{5}, \frac{12}{5}\right)$ or $\left(\frac{9}{22}, \frac{18}{11}\right)$ or $\left(\frac{3}{19}, \frac{33}{19}\right)$</p> <p>Substitute their values in 3rd equation</p> <p>State/show inconsistency & <u>state non-parallel</u> \therefore skew</p> | <p>*M1</p> <p>dep*M1</p> <p>A1</p> <p>dep*M1</p> <p>A1 5</p> | <p>of the type $5 + 3s = 2 + 2t$, $-2 - 4s = -2 - t$ and $-2 + 2s = 7 - 5t$</p> <p><u>Or</u> Eliminate s (or t) from 2 pairs dep*M1</p> <p>($5t=12, 11t=18, 19t=33$) or ($5s=3, 22s=9, 19s=3$) A1, A1</p> <p>State/show inconsistency & <u>state non-parallel</u> \therefore skew WWW A1</p> |
| <p>6 (i) $1 - 4ax + \dots$</p> <p>$\frac{-4. - 5}{1.2}(ax)^2$ or $\frac{-4. - 5}{1.2}a^2x^2$ or $\frac{-4. - 5}{1.2}ax^2$</p> <p>$\dots + 10a^2x^2$</p> <p>(ii) f.t. (their cf x) + b(their const cf) = 1</p> <p>f.t. (their cf x^2) + b(their cf x) = -2</p> <p>Attempt to eliminate 'b' and produce equation in 'a'</p> <p>Produce $6a^2 + 4a = 2$ AEF</p> <p>$a = \frac{1}{3}$ and $b = \frac{7}{3}$ only</p> | <p>B1</p> <p>M1</p> <p>A1 3</p> <p>$\sqrt{B1}$</p> <p>$\sqrt{B1}$</p> <p>M1</p> <p>A1</p> <p>A1 5</p> | <p>Do not accept $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ unless 10 also appears</p> <p>Expect $b - 4a = 1$</p> <p>Expect $10a^2 - 4ab = -2$</p> <p>Or eliminate 'a' and produce equation in 'b'</p> <p>Or $6b^2 + 4b = 42$ AEF</p> <p>Made clear to be only (final) answer</p> |
| <p>7 (i) Perform an operation to produce an equation connecting A and B (or possibly in A or in B)</p> <p>$A = 2$</p> <p>$B = -2$</p> <p>(ii) Write $4 \sin \theta$ as $A(\sin \theta + \cos \theta) + B(\cos \theta - \sin \theta)$</p> <p>and re-write integrand as $A + \frac{B(\cos \theta - \sin \theta)}{\sin \theta + \cos \theta}$</p> <p>$\int A d\theta = A\theta$</p> <p>$\int \frac{B(\cos \theta - \sin \theta)}{\sin \theta + \cos \theta} d\theta = B \ln(\sin \theta + \cos \theta)$</p> <p>Produce $\frac{1}{4}A\pi + B \ln \sqrt{2}$ f.t. with their A, B</p> | <p>M1</p> <p>A1</p> <p>A1 3</p> <p>M1</p> <p>$\sqrt{B1}$</p> <p>$\sqrt{A2}$</p> <p>$\sqrt{A1}$ 5</p> | <p>Probably substituting value of θ, or comparing coefficients of $\sin x$, and/or $\cos x$</p> <p>WW scores 3</p> <p>A and B need not be numerical – but, if they are, they should be the values found in (i).</p> <p>general or numerical</p> <p>general or numerical</p> <p>Expect $\frac{1}{2}\pi - \ln 2$ (Numerical answer only)</p> |
| <p>8 (i) $\frac{dx}{dt}$ or $-kx^{\frac{1}{2}}$ or $kx^{\frac{1}{2}}$ seen</p> <p>$\frac{dx}{dt} = -kx^{\frac{1}{2}}$ or $\frac{dx}{dt} = kx^{\frac{1}{2}}$</p> <p>(ii) Separate variables or invert, + attempt to integrate</p> <p>Correct result for their equation after integration</p> <p>Subst $(t, x) = (0, 2)$ into eqn containing k &/or c dep*M1</p> <p>Subst $(t, x) = (5, 1)$ into eqn containing k & c dep*M1</p> <p>Subst $x = 0.5$ into eqn with their k & c subst dep*M1</p> <p>$t = 8.5$ (8.5355339)</p> | <p>M1</p> <p>A1 2</p> <p>*M1</p> <p>A1</p> <p>dep*M1</p> <p>dep*M1</p> <p>dep*M1</p> <p>A1 6</p> | <p>k non-numerical; i.e. 1 side correct</p> <p>i.e. both sides correct</p> <p>Based <u>only</u> on above eqns or $\frac{dx}{dt} = x^{\frac{1}{2}}$, $-x^{\frac{1}{2}}$</p> <p>Other than omission of 'c'</p> <p>or substitute (5, 1)</p> <p>or substitute (0, 2)</p> <p>[1 d.p. requested in question]</p> |

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| 9 | <p>(i) Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{\frac{dy}{dp}}{\frac{dx}{dp}}$</p> <p>$= \frac{2t}{3t^2}$ or $\frac{2p}{3p^2}$</p> <p>Find eqn tgt thro (p^3, p^2) or (t^3, t^2), their gradient</p> <p>$3py - 2x = p^3$ AG</p> | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> | <p>Or conv to cartes form & att to find $\frac{dy}{dx}$ at P</p> <p>Using $y - y_1 = m(x - x_1)$ or $y = mx + c$</p> <p>4 Do not accept t here</p> |
| | <p>(ii) Substitute $(-10, 7)$ into given equation</p> <p>Satis attempt to find at least 1 root/factor</p> <p>Any one root</p> <p>All 3 roots</p> <p>$(-1, 1)$, $(-64, 16)$ and $(125, 25)$</p> | <p>*M1</p> <p>dep*M1</p> <p>A1</p> <p>A1</p> <p>A1</p> | <p>to produce a cubic equation in p</p> <p>Inspection/factor theorem/rem theorem/t&i</p> <p>-1 or -4 or 5</p> <p>$-1, -4$ and 5</p> <p>5 All 3 sets; no f.t.</p> |
| 10 | <p>(i) $(1 - x^2)^{\frac{3}{2}} \rightarrow \cos^3 \theta$</p> <p>$dx \rightarrow \cos \theta d\theta$</p> <p>$\frac{1}{(1 - x^2)^{\frac{3}{2}}} dx \rightarrow \sec^2 \theta (d\theta)$ or $\frac{1}{\cos^2 \theta} (d\theta)$</p> <p>$\int \sec^2 \theta (d\theta) = \tan \theta$</p> <p>Attempt change of limits (expect 0 & $\frac{1}{6}\pi / 30$)</p> <p>$\frac{1}{\sqrt{3}}$ AEF</p> | <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> | <p>May be implied by $\int \sec^2 \theta d\theta$</p> <p>Use with $f(\theta)$; or re-subst & use 0 & $\frac{1}{2}$</p> <p>6 Obtained with no mention of 30 anywhere</p> |
| | <p>(ii) Use parts with $u = \ln x$, $\frac{dv}{dx} = \frac{1}{x^2}$</p> <p>$-\frac{1}{x} \ln x + \int \frac{1}{x^2} (dx)$ AEF</p> <p>$-\frac{1}{x} \ln x - \frac{1}{x}$</p> <p>Limits used correctly</p> <p>$\frac{2}{3} - \frac{1}{3} \ln 3$</p> <p><u>If substitution attempted in part (ii)</u></p> <p>$\ln x = t$</p> <p>Reduces to $\int t e^{-t} dt$</p> <p>Parts with $u = t$, $dv = e^{-t}$</p> <p>$-te^{-t} - e^{-t}$</p> <p>$\frac{2}{3} - \frac{1}{3} \ln 3$</p> | <p>*M1</p> <p>A1</p> <p>A1</p> <p>dep*M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> | <p>obtaining a result $f(x) + / - \int g(x)(dx)$</p> <p>Correct first stage result</p> <p>Correct overall result</p> <p>5</p> |