Coordinate patterns

Generalising from patterns

Prerequisite knowledge

- Knowledge of 2-D coordinates in all four quadrants
- Properties of squares
- Properties of isosceles triangles

Why do this problem?

This problem is based on ideas found in the *Coordinate patterns* booklet in the SMP 11–16 series published by Cambridge University Press. It is a wonderful context for using coordinates and appreciating underlying mathematical structures.

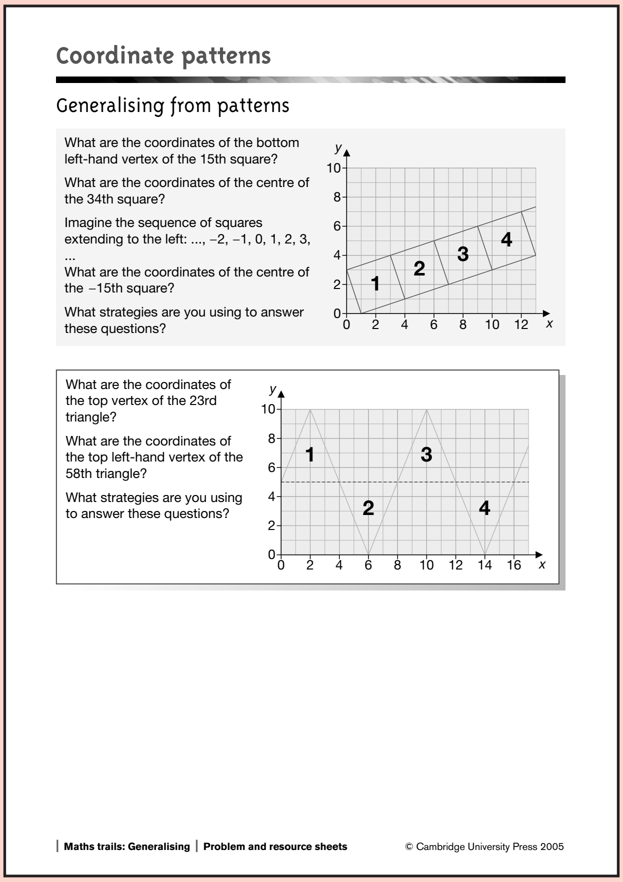
The problem encourages pupils to communicate their image of the patterns and how they are generated, to describe what they are doing and the assumptions they are making. The three examples are based on similar but broadening contexts.

Time

One or two lessons

Resources

CD-ROM: pupil worksheet


Coordinate patterns

Generalising from patterns

What are the coordinates of the bottom left-hand vertex of the 15th square?

What are the coordinates of the centre of the 34th square?

Imagine the sequence of squares extending to the left: ..., −2, −1, 0, 1, 2, 3, ...

What are the coordinates of the centre of the −15th square?

What strategies are you using to answer these questions?

What are the coordinates of the top vertex of the 23rd triangle?

What are the coordinates of the top left-hand vertex of the 58th triangle?

What strategies are you using to answer these questions?

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NRICH website (optional): www.nrich.maths.org, September 2004, ‘Coordinate patterns’

Introducing the problem

Show the first diagram to the group. Ask them to describe what they see on the picture and what is happening off the picture.

- Can you see any patterns?
- How far do these patterns extend?
- What do you notice about the x-coordinate of the top right-hand vertex of every square? (bottom left, etc.)
- What about the y-coordinates?
- Can you explain the patterns and predict where the next square will be, and the next, and the next, ... ?

Now extend to the notion of coordinate pairs (x, y).

It is important to draw out pupils’ descriptions and explanations of patterns as this will support their work in the main part of the lesson.

Main part of the lesson

Pupils address the questions in the problem, discussing strategies in pairs to feed back to the rest of the group part-way through the lesson. Emphasise that the strategy is often more interesting than the answer! So, for example, one group might notice that the bottom left-hand corner of the first square is at (1, 0) and that each subsequent left-hand corner is ‘along 3 and up 1’. From there, they find the 15th left-hand corner and hence the centre of this square. Another group might find the centre of the first square and notice that every subsequent centre can be found by a similar ‘stepping’ method to the one previously described. (There may only be time for the ‘squares’ pattern in the first lesson.)

When discussing strategies ask pupils:

- What is a good strategy?
- Did different groups identify different strategies?
- Are different strategies equally valid?
- Are some strategies more ‘elegant’?
- Can some pupils visualise particular strategies more easily than others?

Plenary

The key questions to address in the plenary are:

- What strategies are you using to answer the questions?
- Are all strategies equally efficient?
- If you did the problem again, which strategy would you use and why?

Solution notes

Squares

The coordinates of bottom left-hand corner of the 15th square are (43, 14).

The coordinates of centre of the 34th square are (101, 35).

More generally (pupils are not expected to use this notation):

If C_n is the centre of square n, the coordinates of C_n satisfy the equations $x_n = 3n - 1$ and $y_n = n + 1$.

If L_n is the bottom left-hand vertex of square n, the coordinates of L_n satisfy the equations $x_n = 3n - 2$ and $y_n = n - 1$.

Triangles

The coordinates of the vertex of the 23rd triangle are (90, 10).

The coordinates of the top left-hand vertex of the 58th triangle are (228, 5).

If T_n is the top/bottom vertex of triangle n, the coordinates of T_n satisfy the equations $x_n = 4n - 2$, and $y_n = 10$ when n is odd and $y_n = 0$ when n is even.

If L_n is the leftmost vertex of triangle n, the coordinates of L_n satisfy the equations $x_n = 4n - 4$ and $y_n = 5$.

And more ...

The vertices of triangle 0 are (–6, 1), (2, 1) and (2, 7). All even-numbered triangles are transposed by a multiple of 8 horizontally and 6 vertically.

The vertices of triangle 1 are (2, 1), (2, 7) and (10, 7). All odd-numbered triangles are transposed by a multiple of 8 horizontally and 6 vertically.

The 20th triangle is even and transposed 10 times to the right. Therefore its coordinates are $(-6 + 10 \times 8,\ 1 + 8 \times 6) = (74, 49)$, (82, 61) and (82, 67).

By the same method the coordinates of triangle –35 can be found by a transposition 18 times to the left, giving (–142, –107), (–142, –101) and (–134, –101).