8 (i) On a single diagram, sketch the curves with the following equations. In each case state the coordinates of any points of intersection with the axes.

(a) $y = a^x$, where a is a constant such that $a > 1$. [2]

(b) $y = 2b^x$, where b is a constant such that $0 < b < 1$. [2]

(ii) The curves in part (i) intersect at the point P. Prove that the x-coordinate of P is

$$\frac{1}{\log_2 a - \log_2 b}.$$ [5]

9 A geometric progression has first term a, where $a \neq 0$, and common ratio r, where $r \neq 1$. The difference between the fourth term and the first term is equal to four times the difference between the third term and the second term.

(i) Show that $r^3 - 4r^2 + 4r - 1 = 0$. [2]

(ii) Show that $r - 1$ is a factor of $r^3 - 4r^2 + 4r - 1$. Hence factorise $r^3 - 4r^2 + 4r - 1$. [3]

(iii) Hence find the two possible values for the ratio of the geometric progression. Give your answers in an exact form. [2]

(iv) For the value of r for which the progression is convergent, prove that the sum to infinity is $\frac{1}{2}a(1 + \sqrt{5})$. [4]

OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4722

Core Mathematics 2

Monday **10 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

1 Simplify $(3+2x)^3 - (3-2x)^3$. [5]

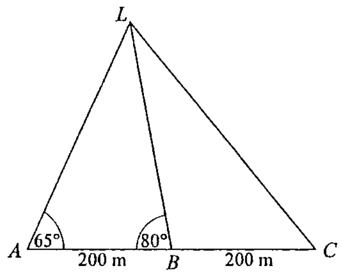
2 A sequence $u_1, u_2, u_3, \dots$ is defined by

$$u_1 = 2 \quad \text{and} \quad u_{n+1} = \frac{1}{1-u_n} \text{ for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3, u_4 and u_5. [3]

(ii) Deduce the value of u_{200}, showing your reasoning. [4]

3

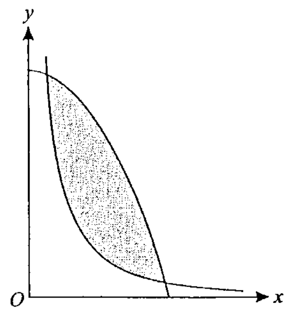


A landmark L is observed by a surveyor from three points A, B and C on a straight horizontal road, where $AB = BC = 200$ m. Angles LAB and LBA are $65°$ and $80°$ respectively (see diagram). Calculate

(i) the shortest distance from L to the road, [4]

(ii) the distance LC. [3]

4



The diagram shows a sketch of parts of the curves $y = \frac{16}{x^2}$ and $y = 17 - x^2$.

(i) Verify that these curves intersect at the points (1, 16) and (4, 1). [1]

(ii) Calculate the exact area of the shaded region between the curves. [7]

5 (i) Prove that the equation

$$\sin\theta \tan\theta = \cos\theta + 1$$

can be expressed in the form

$$2\cos^2\theta + \cos\theta - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\sin\theta\tan\theta = \cos\theta + 1,$$

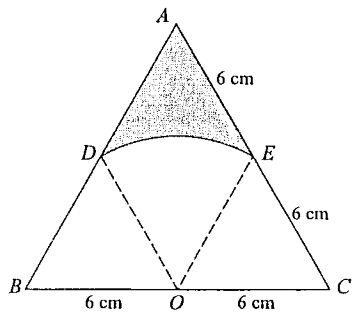
giving all values of θ between $0°$ and $360°$. [5]

6 (a) Find $\int x(x^2+2)\,dx$. [3]

(b) (i) Find $\int \frac{1}{\sqrt{x}}\,dx$. [3]

(ii) The gradient of a curve is given by $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$. Find the equation of the curve, given that it passes through the point (4, 0). [3]

7



The diagram shows an equilateral triangle ABC with sides of length 12 cm. The mid-point of BC is O, and a circular arc with centre O joins D and E, the mid-points of AB and AC.

(i) Find the length of the arc DE, and show that the area of the sector ODE is 6π cm^2. [4]

(ii) Find the exact area of the shaded region. [4]

[Questions 8 and 9 are printed overleaf.]

[Turn over