Square coordinates

Stepping into a problem

Prerequisite knowledge

- Familiarity with coordinates (only needs to be the first quadrant but could use all four)
- Knowledge of properties of a square

Why do this problem?

This problem helps to reinforce coordinate notation and consolidates a more general understanding of what defines a square.

The problem also gives opportunities to pay particular attention to the comprehension phase of the problem-solving model.

Time

One lesson

Resources

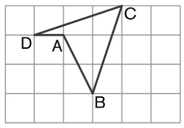
Squared paper; individual whiteboards and pens (optional)
CD-ROM: problem sheet; interactivity
NRICH website (optional):
www.nrich.maths.org, February 2005, 'Square coordinates'

Square coordinates

Problem sheet

We can describe the journey from A to B to C to D and back to A in the diagram below as follows:

A→ ↓↓B→ ↑↑↑C← ← ← ↓D→A
or A 1→2↓ B 1→3↑ C 3←1↓ D 1→A



On squared paper, identify a starting point, A. Draw the line AB described as:

A→ → → ↓↓B or A 3→2↓ B

Draw at least one square ABCD for which AB is a side.

Draw squares ABCD for which one side is given below:

- A 1→ 1↑ B
- A 2→ 1↑ B
- A 3→ 1↑ B
- A 2→ 2↑ B
- A 3→ 2↑ B

Complete the arrow notation for each square above.

What do you notice?

Formulate and describe a general instruction for the construction of a square when you are given one of its sides.

Using the rule you developed, decide whether any of the collections of points below form a square. If so, which ones?

1 (8, 3), (7, 8), (2, 7), (3, 2)
2 (3, 3), (7, 4), (8, 8), (4, 7)
3 (16, 19), (18, 22), (21, 20), (19, 17)
4 (4, 20), (21, 19), (20, 2), (3, 3)

Explain how you decided.

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Introducing the problem

Ask the class to imagine some squared paper upon which they are going to construct some shapes. Explain that you will ask them to imagine a starting point and give them some instructions about where to move. When you say 'click', they should mark another point and join it to the first point with a straight line. Say that you will go on in this way – giving instructions and saying 'click' – until you have brought them back to where they started. You will then ask them what shape they've drawn.

(You may like to ask pupils to draw the shapes on a piece of squared paper or on individual whiteboards.)

Here are some examples you could try:

1 Pick a starting point.
Move right four squares. Click.
Move up four squares. Click.
Move left four squares. Click.
Move down four squares. Click.
What shape have you drawn?
[a square]

2 Pick a starting point.
Move left three squares. Click.
Move up two squares. Click.
Move right six squares. Click.
Move down two squares. Click.
Move left three squares. Click.
What shape have you drawn?
[a rectangle]

3 Pick a starting point.
Move up four squares. Click.
Move up three squares and left two squares. Click.
Move down three squares and left two squares. Click.
Move down four squares. Click.
Move right four squares. Click.
What shape have you drawn?
[a house-shaped pentagon!]

Further examples of shapes might include an L-shape, various triangles, T-shape and so on. Diagonal lines make the visualisation more challenging.

Main part of the lesson

Introduce the class to the notation in this problem. Do this by showing them:

A → ↓ ↓ B → ↑ ↑ ↑ C ← ← ← ↓ D → A

and asking them what they think it means. [From starting point A, move right one and down two, and click for point B. Then move right one and up three, and click for point C. And so on.]

Pupils need to grasp two issues about the notation: first, the letters stand for vertices; second, an arrow represents a move of magnitude one unit in the direction indicated. An alternative version of this notation is:

A 1→2 ↓ B 1→3 ↑ C 3←1 ↓ D 1→ A

Once the notation is understood, ask pupils to visualise the instructions and invite them to draw the resulting shape on their pieces of paper or whiteboards. [it is a quadrilateral, an arrowhead]

A *tilted square* is a square with no horizontal sides. Use the interactivity to give some examples, such as a square with vertices at (10, 10), (17, 13), (14, 20) and (7, 17). Ask pupils to describe instructions to draw each square using the notation already introduced.

Pupils should now be able to tackle the problem sheet.

As you walk around the class the following prompts might be useful:

- Have you actually drawn the squares?
- Have you written down the notation for the whole square?
- Can you see any patterns in the notations?
- Can you explain the patterns?

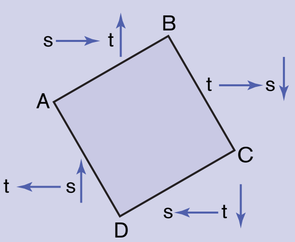
Plenary

The plenary can focus on the equivalence of different patterns. Pupils can work on convincing each other that two patterns which at first look different do produce the same square.

Solution notes

All the shapes 1 – 4 on the problem sheet are squares except the second.

On every side, there is a pattern in the difference between *x*- and *y*-coordinates as shown in the diagram:



Pattern of direction and distance

(starting at A): $\xrightarrow{s}\ \overset{t}{\uparrow}\ \xrightarrow{t}\ \overset{s}{\downarrow}\ \xleftarrow{s}\ \overset{t}{\downarrow}\ \xleftarrow{t}\ \overset{s}{\uparrow}$

This could be written:

$\xrightarrow{s}\ \overset{t}{\uparrow}\ \overset{s}{\downarrow}\ \xrightarrow{t}\ \xleftarrow{s}\ \overset{t}{\downarrow}\ \overset{s}{\uparrow}\ \xleftarrow{t}$

Note both examples contain

$\overset{t}{\uparrow}\ \xrightarrow{s}\ \overset{t}{\downarrow}\ \xleftarrow{s}$ and

$\overset{s}{\uparrow}\ \xrightarrow{t}\ \overset{s}{\downarrow}\ \xleftarrow{t}$

The opposing pairs of direction and distance (in any order) reflect the need to get back to the starting point. This can be a useful checking mechanism.