Inside out

Multipurpose

Prerequisite knowledge

- Properties of cubes

Why do this problem?

This problem is a three-dimensional version of ‘On the edge’ and is therefore placed in a ‘side loop’ of the trail. The reasoning links closely to ‘On the edge’ but pupils might find the 3-D visualising more challenging.

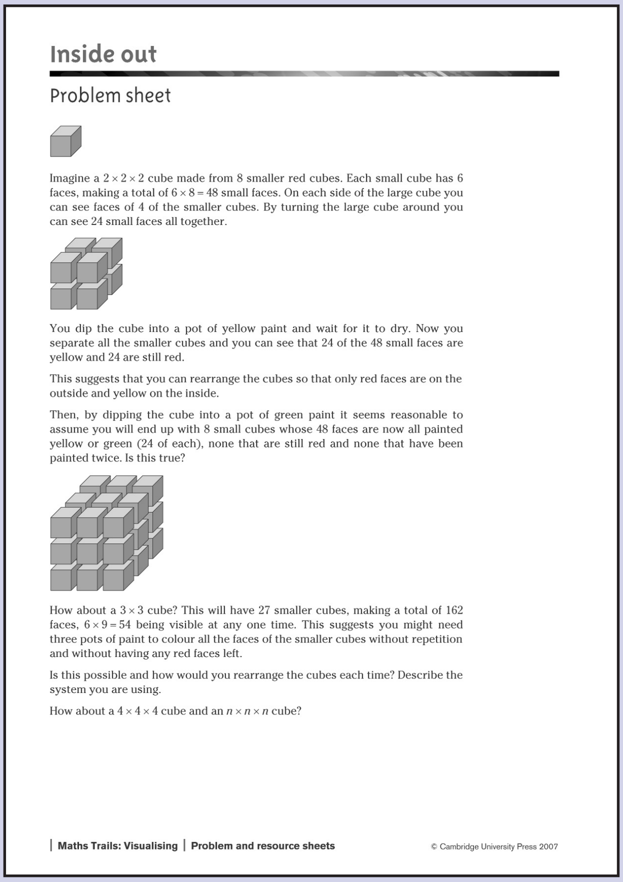
The problem also gives opportunities to pay particular attention to the comprehension and the planning, execution and interpretation phases of the problem-solving model.

Time

One lesson

Resources

A cube made from 8 smaller linking cubes for demonstration purposes; linking cubes
CD-ROM: problem sheet
NRICH website (optional): www.nrich.maths.org, September 2004, ‘Inside out’


Inside out

Problem sheet

Imagine a $2 \times 2 \times 2$ cube made from 8 smaller red cubes. Each small cube has 6 faces, making a total of $6 \times 8 = 48$ small faces. On each side of the large cube you can see faces of 4 of the smaller cubes. By turning the large cube around you can see 24 small faces all together.

You dip the cube into a pot of yellow paint and wait for it to dry. Now you separate all the smaller cubes and you can see that 24 of the 48 small faces are yellow and 24 are still red.

This suggests that you can rearrange the cubes so that only red faces are on the outside and yellow on the inside.

Then, by dipping the cube into a pot of green paint it seems reasonable to assume you will end up with 8 small cubes whose 48 faces are now all painted yellow or green (24 of each), none that are still red and none that have been painted twice. Is this true?

How about a 3×3 cube? This will have 27 smaller cubes, making a total of 162 faces, $6 \times 9 = 54$ being visible at any one time. This suggests you might need three pots of paint to colour all the faces of the smaller cubes without repetition and without having any red faces left.

Is this possible and how would you rearrange the cubes each time? Describe the system you are using.

How about a $4 \times 4 \times 4$ cube and an $n \times n \times n$ cube?

| Maths Trails: Visualising | Problem and resource sheets

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Introducing the problem

Ask the class to imagine a cube.

- How many faces, edges and vertices does it have? [6 faces, 12 edges, 8 vertices]
- What shape are the faces? [squares]

Now ask pupils to imagine taking a second, identical cube.

- How many faces, edges and vertices do the two cubes have altogether? [12, 24, 16]

Invite them to imagine sticking the two cubes together.

- How many uniquely different ways can you do this? [one, providing both cubes are identical and their faces have no distinguishing features]
- What shape does it make? [a cuboid two cubes long and one wide]
- How many faces does the cuboid have? [6]
- How many small faces can you see? [10]

Now ask pupils to imagine taking a third cube and adding it to the first two to create a cuboid 3 units long.

- How many small faces can you see? [$3 \times 4 + 2 = 14$]
- Can you see the same number of faces on each cube? [no; the end cubes have five faces showing, but the middle cube has only four]
- What happens if you add another cube, and another, and another, ... ?

This offers two main discussion points:

- the end cubes always have five faces showing but the inner ones have four;
- there is a pattern to the number of faces: keep adding 4.

Main part of the lesson

Now ask pupils to imagine that they have 8 cubes.

- How many faces are there altogether? [$6 \times 8 = 48$]
- If you place the cubes together to make a larger cube, what will this larger cube look like? [$2 \times 2 \times 2$]

Reveal a ready-made cube made from 8 smaller cubes.

- How many small faces are visible? [4 on each face of the large cube, so $6 \times 4 = 24$]

Now introduce the task on the problem sheet. Ask the class to imagine dipping the large cube into some yellow paint and pulling it out.

- How many small faces are painted? [24]
- So how many faces are left? [24]
- Can you rearrange the small cubes so that no yellow paint is visible?

When pupils have completed this, invite them to think about a $3 \times 3 \times 3$ cube.

- How many times could you dip the large cube into a paint pot in theory? How would you rearrange the cubes each time?

The following prompts for the $3 \times 3 \times 3$ cube may be useful as you walk around the class:

- Do all the small cubes have the same number of faces painted yellow?
- How many different colour paints do you think you might need? Why? [27×6 faces altogether, each dip $= 9 \times 6$ faces, therefore 3 dips and so 3 colours]
- Think about what would happen to each type of cube (i.e. cubes with different numbers of yellow faces) depending on where it was for the second dip. Where could it go?
- What about the other cubes?
- What about the cube hidden in the middle?
- How will you record your findings so that you can reproduce the solution and persuade someone else that you have done it correctly?

This could then be extended to a $4 \times 4 \times 4$ cube and even a $5 \times 5 \times 5$ cube.

Plenary

The plenary should include a discussion of the key findings, such as the middle cube having to go to the corners for the next two moves.

Pupils could talk about the potential of extending this problem to cuboids:

- What would need to be the case for this to work?
- Can anyone think of an example?

Solution notes

It is possible to paint the faces of unit cubes making up any size larger cube so that they can be rearranged to make differently coloured faces.