# IDEA 24

## FRACTIONAL FROGS

This problem is about a confused, love-struck frog who cannot make up her mind but happens to be expert at measuring fractional distances. The situation is as follows:

Felicity Frog is sitting on a stone right in the middle of a river. On one bank are all her mates who have been out on a tadpole night prior to Felicity's marriage to Herbert. On the other bank sits Herbert awaiting his true love to join him. The problem is that Felicity just cannot decide whether to swim to her pals, who are shouting to her: 'You'll be tied to the kitchen sink . . . ' and 'Don't do it . . . ', or to the other bank where Herbert is dolefully croaking his love-song to her.

Felicity sets off and swims towards Herbert, but when she is exactly halfway between the middle of the river and Herbert she has a change of mind and swims back towards her pals. After swimming exactly half the distance she originally swam towards Herbert, Felicity has another change of mind, at which point she swims half the previous distance back towards Herbert. Assuming Felicity continue in this state of indecision, where will she end up?

The problem can be developed by changing the fractional distance, say to $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ or any non-vulgar fraction . . . where does that description come from? The idea is to see for each fractional distance where the final point in the river is and how this compares to the fraction under exploration.

Who said mathematics isn't sexy?



# IDEA 25

## TWO MORE FRACTION PROBLEMS

The ideas below are intended to help students use and apply their knowledge of fractions and, as such, they will need to have some knowledge of subtracting and adding fractions.

**PROBLEM 1**

Differencing and differencing

Write a list of fractions which have denominators in the sequence 2, 3, 4, 5, 6, 7 . . . i.e. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, etc.

Find the differences between adjacent pairs of fractions.

| *Adjacent fraction* | | *Difference* |
|---|---|---|
| $\frac{1}{2} - \frac{1}{3}$ | = | $\frac{1}{6}$ |
| $\frac{1}{3} - \frac{1}{4}$ | = | $\frac{1}{12}$ |
| $\frac{1}{4} - \frac{1}{5}$ | = | . . . |

What do you notice about these answers? What happens if you now take the difference of the differences?

**PROBLEM 2**

Cumulative calculations

Write a list of fractions which have denominators of powers of 2, i.e. 1, 2, 4, 8, 16 . . .

$\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc.

What happens when the following calculations are made?

$\frac{1}{1}$

$\frac{1}{1} + \frac{1}{2}$

$\frac{1}{1} + \frac{1}{2} + \frac{1}{4}$

Write a list of fractions which have denominators in the triangular number sequence, i.e.

$\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{15}$, etc.

What happens when the following calculations are made?

| | | |
|---|---|---|
| $\frac{1}{1}$ | = | $\frac{1}{1}$ |
| $\frac{1}{1} + \frac{1}{3}$ | = | $\frac{4}{3}$ |
| $\frac{1}{1} + \frac{1}{3} + \frac{1}{6}$ | = | $\frac{9}{6}$ |
| $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10}$ | = | . . . |

Explore the pattern in the answers.

What happens if the sequence starts:

$\frac{1}{3}$

$\frac{1}{3} + \frac{1}{6}$

$\frac{1}{3} + \frac{1}{6} + \frac{1}{10}$.