IDEA

67

SQUARE 9-PIN GEOBOARD 3

Returning to a square 9-pin geoboard, just three non-congruent squares can be formed; the areas of the squares are 1, 2 and 4 (square units). By comparison to a 2-by-2 4-dot grid, two of the squares (having areas 2 and 4) could be considered 'new' squares.

How many 'new' squares, each with different areas, can be made on a 16-dot grid? What is the total number of squares that can be formed on a 16-dot grid?

By increasing the size of the geoboard what new squares and total number of squares can be formed?

Is it possible to predict the new and the total number of squares for successively larger square grids?

By comparing the areas of the squares with the problem about vectors in Idea 61, opportunities exist for students to engage in pre-Pythagoras concept formation.

So far all the problems described here and in Ideas 43 and 44 have been formulated in the 2D plane. Imagine a 3D geoboard . . . make up some problems of your own.

IDEA

68

AREA OF 20CM^2

This idea provides students with opportunities to develop their understanding of the concept of the area of a rectangle beyond the simplistic, formulaic notion of 'length multiplied by width'.

The problem might also be used as a pre-Pythagoras task, as the solutions to the problem can later be related to Pythagoras' theorem.

The task is to find all the rectangles, on a square grid, with an area of 20cm^2 such that the corners of the rectangle always lie on a grid point.

There are three 'easy' solutions, i.e. a 1-by-20, a 2-by-10 and a 4-by-5. There are four more solutions, none of which are congruent rectangles to the three listed. One of the solutions is arrived at by the following calculation: $(1^2 + 3^2) \times 2$ or the sum of two square numbers multiplied by two . . . but what does the picture look like?

This idea could be used as a systematic attempt to sum square numbers.

The following areas will provide students much to think about: 50 cm^2, 72 cm^2, 90 cm^2 and 100 cm^2, each of which has a minimum of nine solutions.

A whole-class task could be to find all the possible solutions from 1 up to 100. To do this small groups of students could be given ten or a dozen different numbers between 1 and 100 to find all possible solutions; there could be some interesting display work to emerge.