



1	(i)	10 4 2 3 5 13 7 2 2 4 5 8 5 3 10 5 5 3	M1 M1 A1	First bundle starting with 10 4 2 and has at least one more bag in it Second bundle correct All bundles correct	[3]
	(ii)	Decreasing order: 13 10 10 8 7 5 5 5 5 5 4 4 3 3 3 2 2 2 13 10 2 10 8 7 5 5 5 5 4 4 3 3 3 2 2	M1 M1 A1	A value missing from written out list may be treated as a misread and lose the A mark only Sorting into decreasing order (may be implied from first bundle starting with 13) If each row sorted, award first M1 only Second and third bundles correct All bundles correct	[3]
	(iii)	Each person has roughly the same number of bags <u>or</u> the total weights are more evenly spread	B1	Saying that (i) gives a more even/equal allocation Five bundles in either part \oplus B0	[1]
	Total = 7				
2	(i)	a = number of apple cakes b = number of banana cakes c = number of cherry cakes	B1 B1	Identifying variables as 'number of cakes' Indicating a as apple, b as banana and c as cherry.	[2]
	(ii)	$4 \times 30 = 3 \times 40 = 4 \times 30 = 120$ $\frac{a}{30} + \frac{b}{40} + \frac{c}{30} = 30 \times 40 \times 30$ $4a + 3b + 4c = 120$ or $X = 4, Y = 3, Z = 4$	M1 A1	Any reasonable attempt 4, 3 and 4	[2]
	(iii)	$a + b + c \geq 30$ (or $a + b + c = 30$) $0 \leq a \leq 20, 0 \leq b \leq 25, 0 \leq c \leq 10$ (no need to say 'all integer')	B1 M1 A1	Constraint from total number of cakes correct All three upper constraints correct All three lower constraints correct also	[3]
	(iv)	$4a + 3b + 2c$	B1	Any multiple of this expression	[1]
Total = 8					
3	(i) a	$9 \times 2 = 18$	B1	18	[1]
	b	Since the graph is simple, the two nodes of order 5 are each connected to every other node and hence every node has order at least 2 (exactly 2)	B1 B1	Explicitly using the fact that the graph is simple Deducing that each node has order at least 2 or that all other nodes have order 2	[2]
				A diagram on its own is not enough	
	c	$3 \times 5 = 15$ and $18 - 15 = 3$ but the orders of the other nodes must sum to at least $3 \times 3 = 9$ (must sum to more than 3)	B1 B1	Or, the nodes of order 5 contribute $5 + 5 = 10$ arcs But there are only 9 arcs available	[2]
				A simply connected graph with 6 nodes and 9 arcs, with at least one odd node	
	(ii)	 or equivalent	M1 A1	For such a graph with node orders 1, 3, 3, 3, 3, 5	[2]
	(iii)	 or equivalent	M1 A1	A simply connected graph with 6 nodes and 9 arcs, with at least one even node For such a graph with node orders 2, 2, 2, 4, 4, 4	[2]
Total = 9					

4	(i)	<table><tr><td></td><td>1</td><td>4</td><td>5</td><td>3</td><td>2</td><td>7</td><td>6</td></tr><tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td></tr><tr><td>A</td><td>0</td><td>4</td><td>5</td><td>3</td><td>2</td><td>5</td><td>6</td></tr><tr><td>B</td><td>4</td><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>6</td></tr><tr><td>C</td><td>5</td><td>1</td><td>0</td><td>3</td><td>4</td><td>6</td><td>7</td></tr><tr><td>D</td><td>3</td><td>2</td><td>3</td><td>0</td><td>2</td><td>6</td><td>4</td></tr><tr><td>E</td><td>2</td><td>4</td><td>4</td><td>2</td><td>0</td><td>6</td><td>6</td></tr><tr><td>F</td><td>5</td><td>7</td><td>6</td><td>6</td><td>6</td><td>0</td><td>10</td></tr><tr><td>G</td><td>6</td><td>6</td><td>7</td><td>4</td><td>6</td><td>10</td><td>0</td></tr></table> <p>Order: A E D B C G F</p> <p>Minimum spanning tree:</p> <p>Total weight: 16 (or 1600 m)</p>		1	4	5	3	2	7	6		A	B	C	D	E	F	G	A	0	4	5	3	2	5	6	B	4	0	1	2	4	7	6	C	5	1	0	3	4	6	7	D	3	2	3	0	2	6	4	E	2	4	4	2	0	6	6	F	5	7	6	6	6	0	10	G	6	6	7	4	6	10	0	M1 M1 dep A1 B1 B1 B1	FIRST THREE MARKS ARE FOR WORK ON THE TABLE ONLY (Starting by) choosing row E in column A Choosing more than one entry from column A Correct entries chosen (or all transposed) Correct order, listed or marked on arrows or table, or arcs listed AE ED DB BC DG AF Tree (correct or follow through from table, provided solution forms a spanning tree) 16 or 1600m (or follow through from table or diagram, provided solution forms a spanning tree)
	1	4	5	3	2	7	6																																																																					
	A	B	C	D	E	F	G																																																																					
A	0	4	5	3	2	5	6																																																																					
B	4	0	1	2	4	7	6																																																																					
C	5	1	0	3	4	6	7																																																																					
D	3	2	3	0	2	6	4																																																																					
E	2	4	4	2	0	6	6																																																																					
F	5	7	6	6	6	0	10																																																																					
G	6	6	7	4	6	10	0																																																																					
(ii)	Travelling salesperson (problem)	B1	Identifying TSP by name	[1]																																																																								
(iii)	Two shortest arcs from H: $12 + 13 = 25$ $25 + 16 = 41$ 4100 m	B1 M1 A1	$12 + 13$ or 25, or implied from final answer Adding their 25 to their 16 or for 41 (must be using two arcs from H) 4100 m or 4.1 km (correct and with units)	[3]																																																																								
(iv)	H A E D B C F G H $12 + 2 + 2 + 2 + 1 + 6 + 10 + 16 = 51$ 5100 m	M1 A1 M1 A1	(H) A E D B C ... Correct tour A substantially correct attempt at sum 5100m or 5.1 km (correct and with units)	[4]																																																																								
Total =				14																																																																								

5	<p>(i)</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> B <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>4</td><td>4</td></tr> <tr><td>4</td><td></td></tr> </table> </div> <div style="text-align: center;"> E <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>8/9</td><td>7</td></tr> <tr><td>7</td><td></td></tr> </table> </div> <div style="text-align: center;"> I <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>9/8</td><td>7</td></tr> <tr><td>7</td><td></td></tr> </table> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 10px;"> <div style="text-align: center;"> A <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>11</td><td>0</td></tr> <tr><td></td><td>2</td></tr> </table> </div> <div style="text-align: center;"> C <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>2</td><td>2</td></tr> <tr><td></td><td>2</td></tr> </table> </div> <div style="text-align: center;"> F <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>6/7</td><td>5</td></tr> <tr><td>6</td><td></td></tr> </table> </div> <div style="text-align: center;"> H <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>10/11</td><td>8</td></tr> <tr><td></td><td>8</td></tr> </table> </div> <div style="text-align: center;"> K <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>11/10</td><td>8</td></tr> <tr><td></td><td>9 8</td></tr> </table> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 10px;"> <div style="text-align: center;"> D <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>3</td><td>3</td></tr> <tr><td></td><td>3</td></tr> </table> </div> <div style="text-align: center;"> G <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>5</td><td>5</td></tr> <tr><td></td><td>6 5</td></tr> </table> </div> <div style="text-align: center;"> J <table border="1" style="border-collapse: collapse; margin: 0 auto;"> <tr><td>7/6</td><td>6</td></tr> <tr><td></td><td>7 6</td></tr> </table> </div> </div> <p>Note: H may have only a temporary label if left until last</p> <p>Route: $A D G J K$</p> <p>Number of speed cameras on route: 8</p>	4	4	4		8/9	7	7		9/8	7	7		11	0		2	2	2		2	6/7	5	6		10/11	8		8	11/10	8		9 8	3	3		3	5	5		6 5	7/6	6		7 6	<p>M1 Correct temporary labels at B to G, no extras</p> <p>M1 Correct temporary labels at H to J, no extras</p> <p>A1 All temporary labels correct</p> <p>B1 Order of becoming permanent correct (follow through their permanent labels)</p> <p>B1 All permanent labels correct</p> <p>B1 Correct route</p> <p>B1 8 (cao)</p>	[7]
4	4																																														
4																																															
8/9	7																																														
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9/8	7																																														
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7/6	6																																														
	7 6																																														
	<p>(ii)</p> <p>Odd nodes: $A I J K$</p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> $AI = 7$ $JK = 2$ 9 </div> <div style="text-align: center;"> $AJ = 6$ $IK = 4$ 10 </div> <div style="text-align: center;"> $AK = 8$ $IJ = 6$ 14 </div> </div> <p>Repeat AI and $JK \Rightarrow AB$ and JK</p> <p>Route (example): $K J D A B I K J G K H G F H E F C G D C A B C E B I E K$</p> <p>Number of speed cameras on route: 81</p>	<p>M1 Identifying or using $A I J K$</p> <p>A1 Weight of AI + weight of $JK = 9$</p> <p>A1 Weight of AJ + weight of $IK = 10$ (follow through weight of AI, AJ from (i) if necessary)</p> <p>M1 A list of 28 nodes that starts and ends with K</p> <p>A1 Such a list that includes each of AB, BI, JK (or reversed) twice</p> <p>B1 $72 + \text{weight of their least pairing}$</p>	[6]																																												
	<p>(iii)</p> <p>The only odd nodes are I and J so she only needs to repeat $IJ = 6$</p> <p>$72 + 6 = 78$</p>	<p>B1 Identifying I and J or IJ (not just implied from 6 or $72 + 6$ or 78)</p> <p>M1 Correct calculation (may be implied from 78)</p>	[3]																																												
Total = 16																																															

6	(i)	<table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th></tr><tr><td>1</td><td>-3</td><td>5</td><td>-4</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>2</td><td>-3</td><td>1</td><td>0</td></tr><tr><td>0</td><td>2</td><td>5</td><td>-8</td><td>0</td><td>1</td></tr></table>	P	x	y	z	s	t	1	-3	5	-4	0	0	0	1	2	-3	1	0	0	2	5	-8	0	1	B1	Correct use of two slack variable columns		
P	x	y	z	s	t																									
1	-3	5	-4	0	0																									
0	1	2	-3	1	0																									
0	2	5	-8	0	1																									
				B1	$\pm (-3 \ 5 \ -4)$ in objective row																									
				B1	1 2 -3 12 and 2 5 -8 40 in constraint rows	[3]																								
(ii)	The entries in rows 2 and 3 of the z column are negative Pivot on 1 in x column x and z columns have negative entries in obj. row but no value in z column is positive so choose x $12 \div 1 = 12$, $40 \div 2 = 20$ Least positive ratio is 12 so pivot on the 1			B1	Entries for potential pivots are not positive																									
				B1	Correct pivot choice (cao) (stated or entry ringed)																									
				B1	Follow through their table 'Negative in top row for x' and a correct explanation of choice of row 'least ratio $12 \div 1$ '	[3]																								
(iii)	<table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th></tr><tr><td>1</td><td>0</td><td>11</td><td>-13</td><td>3</td><td>0</td></tr><tr><td>0</td><td>1</td><td>2</td><td>-3</td><td>1</td><td>0</td></tr><tr><td>0</td><td>0</td><td>1</td><td>-2</td><td>-2</td><td>1</td></tr></table> $x = 12$, $y = 0$, $z = 0$	P	x	y	z	s	t	1	0	11	-13	3	0	0	1	2	-3	1	0	0	0	1	-2	-2	1			M1	Follow through their tableau if possible Correct method evident	
P	x	y	z	s	t																									
1	0	11	-13	3	0																									
0	1	2	-3	1	0																									
0	0	1	-2	-2	1																									
				A1	Correct tableau (if it is reasonable and possible, column representing RHS of equations must contain non-negative entries)																									
(iv)	z can increase without limit and increasing z will increase P			B1	Correct non-negative values for their tableau	[3]																								
				B1	Discussing the effect of increasing z Not just referring to pivoting in tableau	[1]																								
(v)	Initial tableau is unchanged except entry in z col of obj. row becomes +40 First iteration tableau is also unchanged except for this entry which becomes 31 36			B1	Describing change to obj. row of initial tableau or showing tableau that results Identifying 31 instead of -13 (cao) No other changes 36 stated (cao)	[4]																								
(vi)	Adding the constraints gives $3x - 5y + 7z \leq 52$ so $Q \leq 52$			B1	52	[1]																								
(vii)	$x - 3z = 12$ and $2x + 10z = 40$ (Accept \square) $\ominus 10z + 6z = 40 - 24$ $\ominus x = 15$ and $z = 1$			M1	Eliminating y terms (may be implied)																									
				M1	Trying to solve simultaneous equations																									
				A1	Correct values (may imply method marks)	[3]																								
Total =						18																								