4724 Core Mathematics 4

1 (a) $2x^2 - 7x - 4 = (2x+1)(x-4)$ or

$3x^2 + x - 2 = (3x-2)(x+1)$ **B1**

$\dfrac{2x+1}{3x-2}$ as final answer; this answer only   **B1**   Do not ISW

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(b) For correct leading term x in quotient   **B1**   <u>Identity method</u>

For evidence of correct division process   **M1**   M1: $x^3 + 2x^2 - 6x - 5 = Q(x^2 + 4x + 1) + R$

Quotient $= x - 2$ **A1**   M1: $Q = ax+b$ or $x+b, R = cx+d$ & ≥ 2 ops

   [N.B. If $Q = x+b$, this $\Rightarrow$ 1 of the 2 ops]

Remainder $= x - 3$ **A1**   A2: $a=1, b=-2, c=1, d=-3$ SR: <u>B</u>1 for two

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2 Parts with correct split of $u = \ln x, \dfrac{dv}{dx} = x^4$ ***M1**   obtaining result $f(x) +/- \int g(x)\, dx$

$\dfrac{x^5}{5}\ln x - \int \dfrac{x^5}{5}.\dfrac{1}{x}\,(dx)$ **A1**

$\dfrac{x^5}{5}\ln x - \dfrac{x^5}{25}$ **A1**

Correct method with the limits   dep***M1**   Decimals acceptable here

$\dfrac{4e^5}{25} + \dfrac{1}{25}$ ISW   (Not '+c')   **A1**   Accept equiv fracts; like terms amalgamated

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3 (i) $\dfrac{d}{dx}(x^2 y) = x^2\dfrac{dy}{dx} + 2xy$ or $\dfrac{d}{dx}(xy^2) = 2xy\dfrac{dy}{dx} + y^2$ ***B1**

Attempt to solve their differentiated equation for $\dfrac{dy}{dx}$ dep***M1**

$\dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - 2xy}$ only   **A1**   WWW **AG** Must have intermediate line &…

…could imply "=0" on 1st line

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(ii)(a) Attempt to solve **only** $y^2 - 2xy = 0$ & derive $y = 2x$ **B1**   **AG** Any effort at solving $x^2 - 2xy = 0 \rightarrow$ B0

Clear indication why $y = 0$ is not acceptable   **B1**   Substituting $y = 2x \rightarrow$ B0, B0

     2

(b) Attempt to solve $y = 2x$ simult with $x^2 y - xy^2 = 2$ **M1**

Produce $-2x^3 = 2$ or $y^3 = -8$ **A1**   AEF

$(-1, -2)$ or $x = -1, y = -2$ **only**   **A1**

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4	**(i)**	For (either point) + t(difference between vectors)	**M1**	't' can be 's', 'λ' etc.
		$\mathbf{r} = (3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})+ t(-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $2\mathbf{i} - \mathbf{j} - \mathbf{k})$	**A1**	'$\mathbf{r}$' must be '$\mathbf{r}$' but need not be bold
				Check other formats, e.g. $ta + (1-t)b$
			2	

	(ii)	State/imply that their $\mathbf{r}$		
		and their $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ are perpendicular	***M1**	N.B.This *M1 is dep on M1 being earned in (i)
		Consider scalar product = 0	dep*M1	
		Obtain $t = -\dfrac{1}{6}$ or $\dfrac{1}{6}$ or $-\dfrac{5}{6}$ or $\dfrac{5}{6}$	**A1**	
		Subst their t into their equation of AB	**M1**	
		Obtain $\dfrac{1}{6}(16\mathbf{i} + 13\mathbf{j} + 19\mathbf{k})$ AEF	**A1**	Accept decimals if clear
			5	

5	**(i)**	$(1-x)^{\frac{1}{2}} = 1 - \dfrac{1}{2}x - \dfrac{1}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \dfrac{1}{2}x + kx^2$, $k \neq -\dfrac{1}{8}$ or 0
		$(1+x)^{-\frac{1}{2}} = 1 - \dfrac{1}{2}x + \dfrac{3}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \dfrac{1}{2}x + kx^2$, $k \neq \dfrac{3}{8}$ or 0
		Product = $1 - x + \dfrac{1}{2}x^2$ ignoring x^3 etc	**B1**	**AG**; with (at least) 1 intermediate step (cf x^2)
			5	

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	(ii)	$\sqrt{\dfrac{5}{9}}$ or $\dfrac{\sqrt{5}}{3}$ seen	**B1**	
		$\dfrac{37}{49}$ or $1 - \dfrac{2}{7} + \dfrac{1}{2}\left(\dfrac{2}{7}\right)^2$ seen	**B1**	
		$\dfrac{\sqrt{5}}{3} \approx \dfrac{37}{49} \Rightarrow \sqrt{5} \approx \dfrac{111}{49}$	**B1**	**AG**
			3	

6	**(i)**	Produce at least 2 of the 3 relevant equations in t and s	**M1**	$1 + 2t = 12 + s$, $3t = -4s$, $-5 + 4t = 5 - 2s$
		Solve for t and s	**M1**	
		$(t, s) = (4, -3)$ AEF	***A1**	
		Subst $(4, -3)$ into suitable equation(s) & show consistency	dep*A1	Either into "3rd" eqn or into all 3 coordinates.
				N.B. Intersection coords not asked for
			4	

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	(ii)	Method for finding magnitude of any vector	***M1**	Expect $\sqrt{29}$ and $\sqrt{21}$
		Method for finding scalar product of any 2 vectors	***M1**	Expect -18
		Using $\cos\theta = \dfrac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ AEF for the correct 2 vectors	dep*M1	Should be $-\dfrac{18}{\sqrt{29}\sqrt{21}}$
		137 (136.8359) or 43.2(43.164…)	**A1**	2.39 (2.388236..) or 0.753(0.75335…) rads
			4	

7 (i) Correct (calc) method for dealing with $\dfrac{1}{\sin x}$ or $(\sin x)^{-1}$ **M1**

Obtain $-\dfrac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2}\cos x$ **A1**

Show manipulation to $-\text{cosec }x\cot x$ (or vice-versa) **A1** WWW **AG** with ≥ 1 line intermed working

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(ii) Separate variables, $\displaystyle\int(-)\dfrac{1}{\sin x\tan x}\,dx = \int\cot t\,dt$ **M1** or $\displaystyle\int\dfrac{1}{\sin x\tan x}\,dx = \int(-)\cot t\,dt$

<u>Style: For the M1 to be awarded, dx and dt must appear on correct sides or there must be</u> $\displaystyle\int$ <u>sign on both sides</u>

$\displaystyle\int-\text{cosec }x\cot x\,dx = \text{cosec }x$ (+c) **A1** or $\displaystyle\int\text{cosec }x\cot x\,dx = -\text{cosec }x$

$\displaystyle\int\cot t\,dt = \ln\sin t$ or $\ln|\sin t|$ (+c) **B1** or $\displaystyle\int-\cot t\,dt = -\ln\sin t$ or $-\ln|\sin t|$

Subst $(t,x) = \left(\dfrac{1}{2}\pi, \dfrac{1}{6}\pi\right)$ into their equation containing 'c' **M1** and attempt to find 'c'

$\text{cosec }x = \ln\sin t + 2$ or $\ln|\sin t| + 2$ **A1** WWW ISW; $\text{cosec }\dfrac{\pi}{6}$ to be changed to 2

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8 (i) $A(t+1) + B = 2t$ **M1** <u>Beware:</u> correct values for A and/or B can be ...
$A = 2$ **A1** … obtained from a wrong identity
$B = -2$ **A1** <u>Alt method:</u> subst suitable values into given…
…expressions

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(ii) Attempt to connect dx and dt **M1** But not just $dx = dt$. As **AG**, look carefully.
$dx = t\,dt$ s.o.i. AEF **A1**

$x + \sqrt{2x-1} \rightarrow \dfrac{t^2+1}{2} + t = \dfrac{(t+1)^2}{2}$ s.o.i. **B1** Any wrong working invalidates

$\displaystyle\int\dfrac{2t}{(t+1)^2}\,dt$ **A1** **AG** WWW The 'dt' must be present

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(iii) $\displaystyle\int\dfrac{1}{t+1}\,dt = \ln(t+1)$ **B1** Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t+1$

$\displaystyle\int\dfrac{1}{(t+1)^2}\,dt = -\dfrac{1}{t+1}$ **B1**

Attempt to change limits (expect 1 & 3) and use f(t) **M1** <u>or</u> re-substitute and use 1 and 5 on g(x)

$\ln 4 - \dfrac{1}{2}$ **A1** AEF (like terms amalgamated); if A0 A0 in (i),
then final A0

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9 **(i)** $A : \theta = \frac{1}{2}\pi$ (accept $90°$)   **B1**

         $B : \theta = 2\pi$ (accept $360°$)   **B2**   SR If B0 awarded for point B, allow B1 SR for

                                                     any angle s.t. $\sin \theta = 0$

                                                         3

     (ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}$ **M1**   or $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} \cdot \dfrac{\mathrm{d}\theta}{\mathrm{d}x}$ Must be used, not just quoted

         $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2 + 2 \cos 2\theta$ **B1**

         $2 + 2 \cos 2\theta = 4 \cos^2 \theta$ with ≥ 1 line intermed work   ***B1**

         $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4 \cos \theta}{2 + 2 \cos 2\theta}$ s.o.i.   **A1**   This & previous line are interchangeable

         $= \sec \theta$ dep***A1**   WWW **AG**

                                                          5

   (iii) Equating $\sec \theta$ to 2 and producing at least one value of θ **M1**   degrees or radians

         $(x =) -\dfrac{2}{3}\pi - \dfrac{\sqrt{3}}{2}$ **A1**   'Exact' form required

         $(y =) -2\sqrt{3}$ **A1**   'Exact' form required

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