Revision exercise A1

1 Find an approximate answer to each of these calculations by rounding each number to 1 significant figure.
   a) 580×83
   b) $\dfrac{63 \cdot 2}{3 \cdot 8}$
   c) $\dfrac{28 \cdot 3^2}{0 \cdot 48}$

   Now use a calculator to see how close your approximations are to the correct answers.

2 Simplify each of these, writing the answers in standard form.
   a) $5 \cdot 4 \times 10^6 \times 2 \cdot 7 \times 10^{-3}$
   b) $9 \cdot 54 \times 10^3 \div 4 \times 10^4$

3 Multiply out these brackets.
   a) $(x + 7)(x + 1)$
   b) $(a - 3)(a + 5)$
   c) $(2y - 4)(y + 1)$
   d) $(x - 5)(2x + 1)$
   e) $(4a + b)(a - b)$

4 Simplify these expressions.
   a) $2a^2 \times a^3$
   b) $10a^2 \div 2a$
   c) $(a^3)^2 \times a^3 \div a^4$
   d) $12a^2b \times 2a^2b^3$
   e) $6x^2y^2z^2 \div 2xy^2z$
   f) $8a^2b \times \dfrac{3abc}{6ab^2}$

5 Factorise each of these fully.
   a) $3a + 6b - 12c$
   b) $2a + 3ab$
   c) $a^2b - 3ab^2$
   d) $2x^2y - 6xy$
   e) $7abc + 14a^2b$
   f) $9a^2 + 3b^2 - 6c^2$
   g) $5pq - 10$
   h) $2a - 4a^2 + 6a^3$
   i) $100abc - 50ac$

6 Factorise these, where possible.
   a) $x^2 - 49$
   b) $x^2 - 1$
   c) $x^2 + 16$
   d) $x^2 - y^2$
   e) $121 - b^2$

7 Factorise these expressions.
   a) $x^2 - 16x + 63$
   b) $2x^2 - 8x - 42$
   c) $3x^2 - 8x + 4$
   d) $2x^2 + x - 15$
   e) $3x^2 - 48$
   f) $2x^2 - 11x - 21$
   g) $6x^2 - 27x - 15$
   h) $5x^2 - 21x + 18$
   i) $8x^2 - 6x - 5$
   j) $6x^2 - 11x - 10$

8 Simplify these fractions.
   a) $\dfrac{x^2 + 2x - 3}{4x - 4}$
   b) $\dfrac{x^2 + 2x + 1}{x^2 - x - 2}$
   c) $\dfrac{2x^2 + 2x}{x^2 + 4x + 3}$
   d) $\dfrac{x^2 - 7x + 10}{2x^2 - 11x + 5}$
   e) $\dfrac{x^2 + 8x + 15}{x^2 - 25}$
   f) $\dfrac{3x^2 + 15x + 12}{2x^2 + 7x - 4}$

9 Solve each of these equations.
   a) $3x^2 - 5x - 12 = 0$
   b) $2x^2 + x - 28 = 0$
   c) $3x^2 - 192 = 0$

10 The attractive force between two objects is inversely proportional to the square of their distance apart. The force is $0 \cdot 24$ units when the distance is 15 units.
   a) What will be the force when the distance is 30 units?
   b) Find a formula for the force, F, in terms of the distance apart, d.

11 Copy and complete the table to find the corresponding values of y.

	x	y
$y \propto x$	2	5
$y \propto x^2$	10	
$y \propto x^3$	10	
$y \propto \dfrac{1}{x}$	10	
$y \propto \dfrac{1}{x^2}$	10	

12 For each of these relationships
(i) find the variation, using $\propto$
(ii) find the formula.

a)

x	5	10
y	10	5

b)

x	5	50
y	10	100

c)

x	2	10
y	0·1	0·02

d)

x	2	10
y	0·1	2·5

Revision exercise B1

1 Write these in index form as simply as possible.
   a) The reciprocal of n
   b) The cube root of m
   c) The square root of $\frac{1}{n}$

2 Write each of these as a whole number or a fraction.
   a) 4^{-1} **b)** 5^0
   c) $25^{\frac{1}{2}}$ **d)** 2^4
   e) $8^{\frac{2}{3}}$ **f)** $125^{-\frac{2}{3}}$
   g) $\left(\frac{1}{8}\right)^{-\frac{1}{3}}$ **h)** $64^{\frac{5}{6}}$
   i) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$ **j)** $\left(\frac{1}{12}\right)^{-2}$

3 Write each of these as a whole number or a fraction.
   a) $8^0 \times 25^2$ **b)** $4^2 \times 25^{\frac{1}{2}}$
   c) $12^2 \times 4^{-2}$ **d)** $6^3 \div 9^{\frac{3}{2}}$
   e) $5^2 - 4^3 + 3^4$ **f)** $25^{\frac{3}{2}} \times 64^{\frac{1}{3}}$
   g) $14^2 \times 49^{-1}$ **h)** $\left(\frac{4}{5}\right)^{-2} \times \left(\frac{16}{9}\right)^{\frac{1}{2}}$

4 Work out these. Give the answers either exactly or to 5 significant figures.
   a) 1.43^3 **b)** 0.87^5
   c) 2^{12} **d)** 7.9^{-4}

5 Work out these. Give the answers either exactly or to 5 significant figures.
   a) $59049^{\frac{1}{5}}$ **b)** $7.9^{\frac{1}{4}}$
   c) $4000^{\frac{1}{6}}$ **d)** $32768^{\frac{3}{5}}$

6 Write each of these as a power of a prime number, as simply as possible.
   a) 128 **b)** 27^2
   c) $49^{\frac{1}{3}}$ **d)** $9^2 \div 81^{-\frac{1}{2}}$
   e) $2^n \times 32^{n+1}$

7 Write each of these as a product of the prime factors. Use indices where possible.
   a) 40 **b)** 90
   c) 136 **d)** 588

8 Write each of these as a product of powers of prime numbers, as simply as possible.
   a) 15^2 **b)** $40^{\frac{1}{3}}$
   c) $14^3 \times 56^{\frac{1}{3}}$ **d)** $72^{\frac{3}{2}} \times 24^{-\frac{1}{2}}$

9 Rearrange each of these formulae to make a the subject.
   a) $3a + 5c = 2b - 5a$
   b) $3(a + 2b) = 2b + 5a$

10 Rearrange each of these formulae to make p the subject.
   a) $p + q = 2(q - 3p)$
   b) $t = \dfrac{2(p - 1)}{p}$
   c) $\dfrac{1}{p} = \dfrac{1}{q} + \dfrac{1}{s}$
   d) $T = p + \dfrac{2p}{q}$
   e) $\dfrac{1}{2p - 1} = \dfrac{2a}{p + 1}$
   f) $p^2 + 4a = 2b$

11 A sector has an angle of $75°$ and a radius of $6.5\,$cm. Calculate
   a) the arc length.
   b) the sector area.

12 Calculate the sector angle of each of these sectors.
   a)

   7.2 cm

   8.3 cm

   b)

   Area = 29 cm^2

   4.8 cm
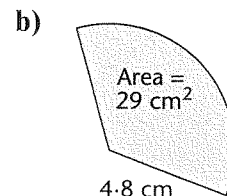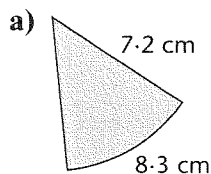
13 Calculate the radius of the circle with each of these sectors.

a) 5·2 cm

48°

b)

245°

Area = 50 cm²
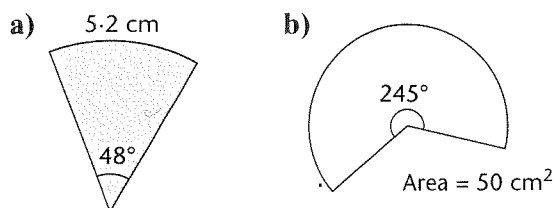
14 Jo blows up a spherical balloon until it has a radius of 12 cm.
Find the volume of air she has blown into the balloon.

15 A concrete water tower has its internal volume in the shape of an inverted cone. The radius of the top is 3·6 m. The depth of the cone is 10 m.

3·6 m

10 m
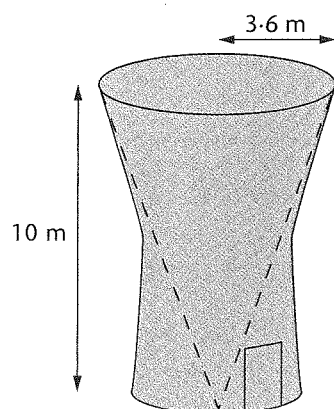
Calculate the volume of water which can be stored in the tower.

16 Paul measures out 250 g flour, 150 g butter and 120 g sugar, all to the nearest 10 g.
Calculate the upper and lower bounds of the total mass of flour, butter and sugar.

17 Two pieces of string measure 19·7 cm and 11·4 cm, to the nearest millimetre.
Calculate
a) the upper bound of the total length of the two pieces placed end to end.
b) the lower bound of the difference between the lengths of the two pieces.

18 The space for some kitchen base units is measured as 1000 mm to the nearest millimetre. Two base units are 500 mm each, to the nearest millimetre.
a) Explain why the two units will not necessarily fit into the space.
b) Calculate the upper bound of the gap remaining if the two units do fit in.

19 The length of a side of a cube is given as 4·6 cm to the nearest millimetre.
Calculate the upper and lower bounds of the volume of the cube, giving your answers to 3 significant figures.

20 A 100 m race was won in a time of 13·62 seconds, correct to the nearest hundredth of a second.
Calculate, to 3 significant figures, the upper bound of the average speed
a) if the distance is 100 m to 3 significant figures.
b) if the distance is 100·0 m to 4 significant figures.

21 Jane walks on an exercise machine for 7·2 minutes at a speed of 130 metres per minute, both measurements to 2 significant figures.
Calculate the upper bound of the distance she walks.

22 A town has a population of 94 300 to the nearest 100. Its area is 156 km², to the nearest square kilometre.
Calculate the lower bound of its population density, giving your answer to 3 significant figures.

Revision exercise C1

1 **a)** Two triangles are similar. One has a base 6 cm long and an area of 20 cm². The other has a base 18 cm long. What is its area?

   b) A cone has a height of 12 cm, a surface area of 380 cm² and a volume of 480 cm³. A similar cone has a volume of 60 cm³. What are its height and surface area?

   c) Three similar cylindrical cans have diameters of 5 cm, 8 cm and 10 cm. The smallest can holds 150 ml. What is the capacity of each of the other two cans?

2 Two bottles are similar. Their heights are in the ratio 1 : 1·5. The larger one holds 2700 ml. What does the smaller one hold?

3 The areas of two similar pieces of paper are in the ratio 1 : 8. The larger piece of paper is 21·0 cm wide. What is the width of the smaller piece?

4 Draw a set of axes with the x-axis from 0 to 16 and the y-axis from 0 to 8. Plot the points A(4, 5), B(9, 5), C(9, 2) and D(4, 2) and join them to form a rectangle. Enlarge the rectangle by a scale factor of ⁻1·5 using the point (8, 4) as the centre of enlargement. Write down the coordinates of the enlarged rectangle.

5 Copy the diagram and find
   a) the centre of enlargement.
   b) the scale factor.

6 The probability that Zoe is late for school is 0·2. The probability that she is on time for school is 0·65. What is the probability that Zoe is either on time or late for school?

7 Assuming that the events in question **6** are independent, find the probability that Zoe is late for school on Monday and on time for school on Tuesday.

8 The probability that it is sunny on any day in January is 0·3. Find the probability that for two days in January
   a) both are sunny.
   b) one of the days is sunny.

9 The probability that the school netball team will win any match is 0·4. The probability that they draw any match is 0·1. The team play two matches. Find the probability that the team
   a) loses both matches.
   b) does not lose both matches.
   c) wins one of the two matches and draws the other.

10 The probability that it rains on 15 July is 0·1. The probability that it rains on 16 July is also 0·1. Find the probability that it
   a) rains on both days.
   b) rains on one of the two days.

11 If it is fine when he gets up there is a probability of 0·7 that Richard will cycle to school. If it is raining when he gets up there is a probability of 0·05 that he cycles. The weatherman estimates that there is a 20% chance that it will rain tomorrow morning. Using the weatherman's estimate, find the probability that Richard will cycle to school tomorrow.
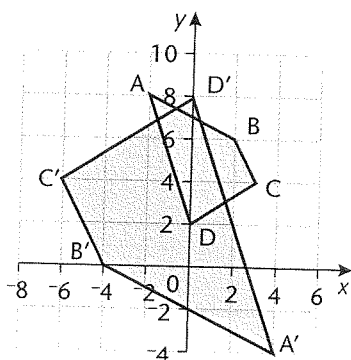
**12** In a game I toss a coin and spin one of these fair spinners.
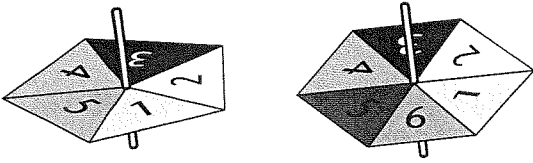
If I toss a head I spin the five-sided spinner.
If I toss a tail I spin the six-sided spinner.
I need a 5 to win the game.
What is the probability that I win the game?

13 Find the length of the line joining each of these pairs of points.
Give your answers correct to 2 decimal places.
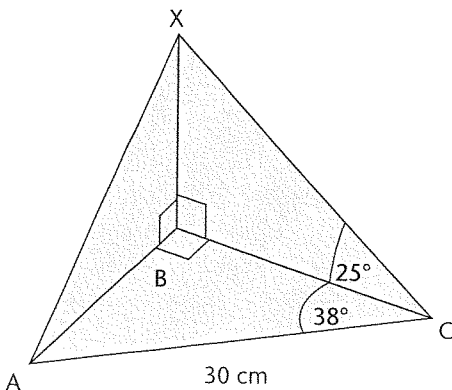**a)** A(2, 2) and B(4, 7)
b) C(-2, 9) and D(5, 3)

14 A street light suspended 7·5 m above the ground illuminates a circle with circumference 30·8 m.
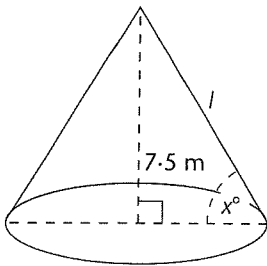
Calculate
a) the angle marked $x°$.
b) the length l.

15 In the diagram, triangle ABC is horizontal with angle ABC = 90°.
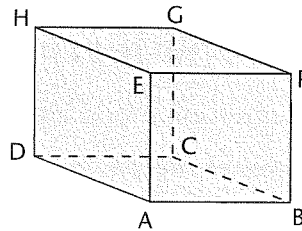XB is vertical.

Find the length of XB.

16 Calculate the distance between each of these pairs of points.
a) (0, 1, 4) and (2, 6, 5)
b) (3, 2, -4) and (-1, 6, 7)
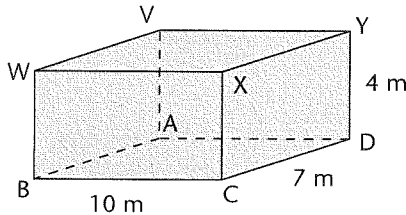**c)** (-1, -2, 1) and (3, 10, 4)

17 ABCDEFGH is a cuboid.

Specify the letters which define the angle between
a) the line BH and the plane DCGH.
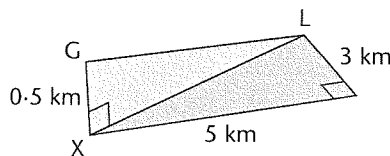**b)** the line AG and the plane BCGF.

18 A classroom measures 10 m by 7 m by 4 m.

Calculate
a) the angle between BY and ABWV.
b) the angle between AC and ADYV.
c) the length BY.

19 A glider, G, is 3 km east and 5 km south of its landing strip, L.
It is at an altitude of $\frac{1}{2}$ km above a point X on the ground.

Calculate
a) the bearing of the point X from L.
b) the distance the glider has to fly to reach the landing strip.
c) the glider's inclination as measured from L.

Revision exercise D1

1   Draw a histogram to show the following
    distribution of the weights, to the nearest
    kilogram, of 50 Year 9 students.

Weight (W kg)	Frequency (f)
$32 \leqslant W < 34$	1
$34 \leqslant W < 39$	5
$39 \leqslant W < 43$	7
$43 \leqslant W < 47$	8
$47 \leqslant W < 51$	14
$51 \leqslant W < 59$	9
$59 \leqslant W < 70$	6

2   This histogram shows the masses,
    in grams, of plums picked in
    an orchard.
    How many plums were picked?
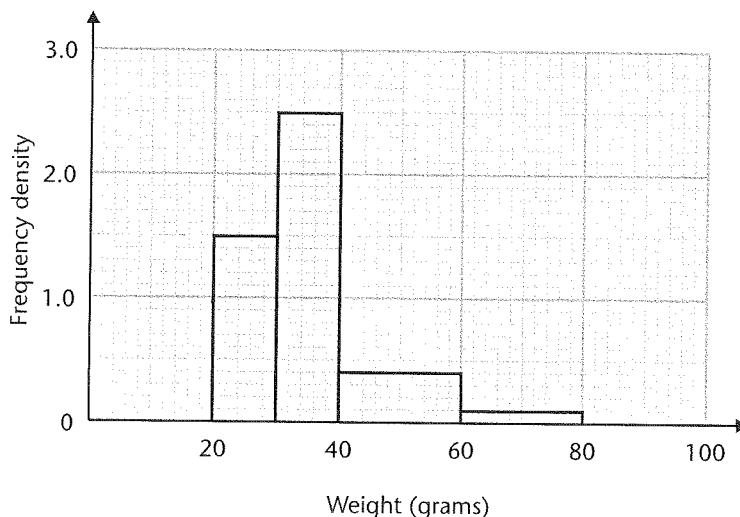
3   The heights of students in two classes are measured.
    The results are given in the tables.

Class 11A	
Height (H cm)	Frequency
$130 \leqslant H < 140$	1
$140 \leqslant H < 150$	4
$150 \leqslant H < 160$	9
$160 \leqslant H < 170$	8
$170 \leqslant H < 180$	2
$180 \leqslant H < 190$	2

Class 11B	
Height (H cm)	Frequency
$120 \leqslant H < 130$	4
$130 \leqslant H < 140$	5
$140 \leqslant H < 150$	8
$150 \leqslant H < 160$	3
$160 \leqslant H < 170$	3
$170 \leqslant H < 180$	1

Show the data on two histograms.

4   In the following questions O is the centre of each circle.
In each question calculate the size of each angle or length marked with a letter.

a)

b)

c)

d)

AD = DB

e)

AB = AT

f)

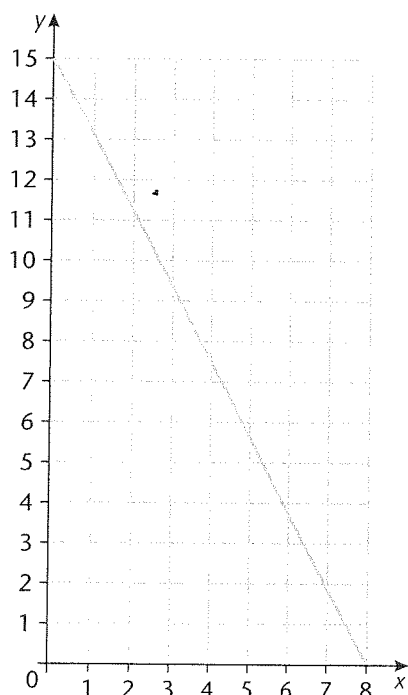**g)**

h)

i)

j)

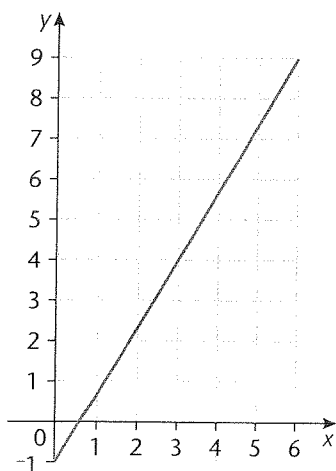**5** Find the equation of each of these lines.
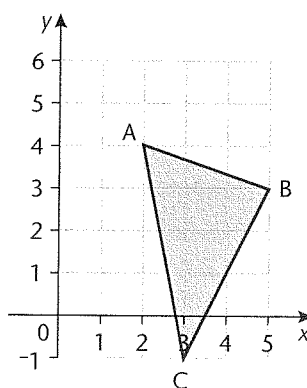
a)
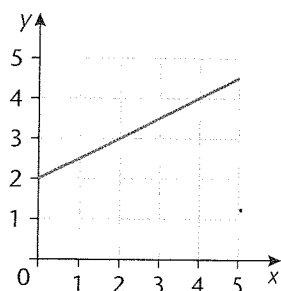
b)

c)

6 Find the equation of each of these lines.
a) A line with gradient 3, passing through $(0, 2)$
b) A line with gradient $^-2$, passing through $(1, 4)$
c) A line with gradient $\frac{1}{2}$, passing through $(2, 6)$

7 Find the equation of the line that joins each of these pairs of points.
a) $(4, 0)$ and $(6, 5)$
b) $(1, 2)$ and $(6, 4)$
c) $(2, 3)$ and $(5, ^-6)$

8 Find the equation of each of the three sides of the triangle ABC.

9 Find the equation of each of these lines.
a) A line parallel to $y = 7x + 4$, passing through $(5, 1)$
b) A line parallel to $2y = 5x + 4$, passing through $(4, 0)$
c) A line parallel to $5x + 4y = 3$, passing through $(2, ^-1)$

10 Find the equation of each of these lines.
a) A line perpendicular to $y = 2x + 4$, passing through $(2, 1)$
b) A line perpendicular to $3y = 2x + 7$, passing through $(0, 1)$
c) A line perpendicular to $2x + 5y = 4$, passing through $(1, ^-3)$

11 A mobile phone company calls 200 people, chosen at random, who subscribe to their company, to find out how satisfied they are with the service they receive.
Is this a satisfactory method of sampling? Give a reason for your answer.

STAGE
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12 Identify which type of sampling has been used in each of the following cases.

a) In order to determine whether the library facilities in a town are satisfactory, all the library cards are numbered and 100 questionnaires are sent out to the owners of cards selected using random numbers.

b) A factory employs 1500 people on machines, 400 on packing and distribution and 300 in the offices. A sample is taken containing 15 machine operators, 4 people from packing and distribution and 3 office workers.

13 Safia surveyed students in her school to find out their views about background music in shops.

The size of each year group in the school is shown in the table.

Year group	Boys	Girls
7	84	66
8	71	85
9	82	86
10	93	107
11	81	90
Total	411	434

Safia took a sample of 80 students.

a) Should she have sampled equal numbers of boys and girls in Year 7? Give a reason for your answer.

b) Calculate the number of students she should sample in Year 7.

14 A government officer is investigating whether there is a link between the income of families and the number of children they have.

The table shows the information in her database of families.

Number of children	Number of families
0	123
1	179
2	457
3	88
4 and over	45

She plans to interview a sample of 60 families.

How many should she select from each group to make the sample representative?

15 Sweet-tasting apples are used to make apple juice.

An apple grower needs to find out how 'sweet' a crop of apples from an orchard is.

He is advised to select a sample of 50 apples.

The orchard consists of 1500 trees, and each tree produces about 50 apples.

The grower decides to pick 50 apples from one tree that he selects at random.

Is this a satisfactory method?

Give reasons for your answer.

16 The manager of a newly-opened gym wants to find out how successful it is.

He decides to interview a sample of 50 of the members of the gym.

Describe how he might choose the 50 members using each of these methods.

a) Simple random sampling

b) Systematic random sampling

c) Stratified random sampling