Pair sums

Mixed methods

Prerequisite knowledge

- Mental methods of addition and subtraction
- Understanding of directed numbers

Why do this problem?

This task, which appears at first to be *too* open, is a good example of circumstances in which pupils can be encouraged to look carefully for information that they can be certain of and therefore find a way to a solution.

Time

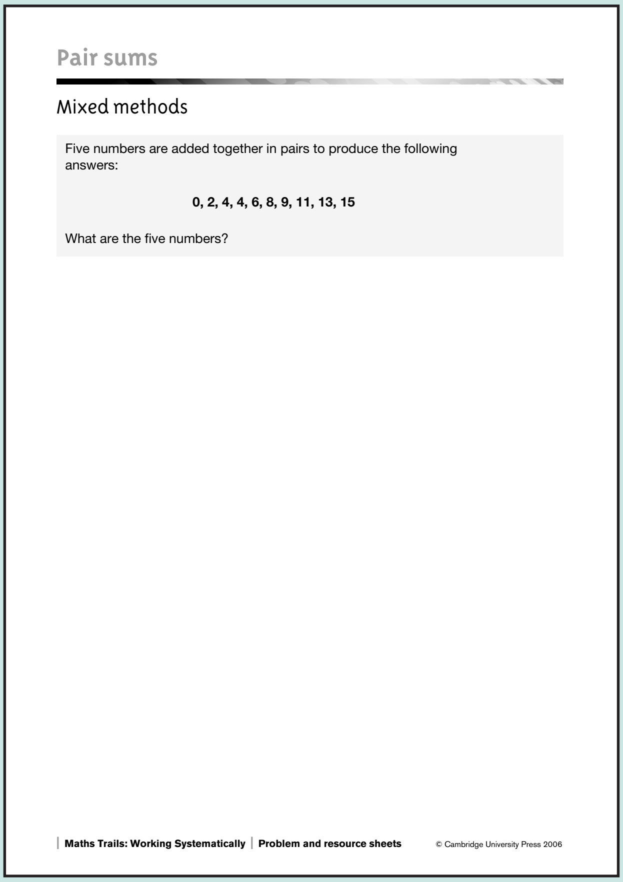
One lesson at most

Resources

CD-ROM: problem sheet (not strictly necessary)

NRICH website (optional): www.nrich.maths.org, January 1997, 'Pair sums'

A number line might be a useful aid for pupils who are less confident in working with negative numbers.



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Five numbers are added together in pairs to produce the following answers:

0, 2, 4, 4, 6, 8, 9, 11, 13, 15

What are the five numbers?

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Introducing the problem

An appropriate way to warm up would be to practise addition and subtraction of positive and negative numbers in a motivating context, for example by playing bingo.

Lead into the problem itself by writing up three numbers for the class to see, for example, 5, 2 and 10. Ask pupils to talk to their neighbours about the totals these numbers would produce if they were added together in pairs. Choose pupils to share their ideas with the class.

- How many pairs (and therefore how many totals) will these three numbers give altogether?
- How is the largest total produced?
- And the smallest total?
- If there were four numbers, how many totals would there be?

Try another set of numbers, this time including both positive and negative integers. After these initial discussions, the pupils will be familiar with the idea of the problem itself.

Main part of the lesson

Move on to the main problem by saying to the class that they are now going to try a similar problem, but the other way round (i.e. being given the totals and finding the original numbers). Have the totals on the board ready and draw the pupils' attention to them by asking:

- How many numbers are written?
- If these are totals produced by adding pairs of numbers in the same way as we have just done, how many numbers must we have started with?

Explain that the challenge this time is to find those five starting numbers. Having set them off on the task, perhaps in pairs, some questions and prompts will help to focus pupils' thinking if they are struggling:

- How can you make a total of zero?
- Why do you think the number 4 is written twice?
- What is the highest total and what does it tell you?
- What is the lowest total and what does it tell you?
- How might you go about beginning a solution?

After a few minutes it may be useful to draw the class back together to exchange findings.

Plenary

This problem is self-checking, as once the correct solution has been found, pupils will know it is right. This means that sharing the strategy or method for solving it is the most important aspect of the plenary and could be done by one or two pairs explaining their reasoning to everyone else.

An appropriate extension might be to ask:

- What is the smallest number of sums of pairs (i.e. *different* totals) you can make from six numbers? Is it possible to choose six numbers and put them into pairs all having the same total? For example, $3 + 7 = 10$, $2 + 8 = 10$, $1 + 9 = 10$, but $3 + 2 = 5$.

Pupils could be invited to explain how they know there is only one solution and perhaps to find a general rule.

Solution notes

The numbers were −1, 1, 3, 5, 10.

This is how the totals are produced:

$-1 + 1 = 0$	$3 + (-1) = 2$
$3 + 1 = 4$	$5 + (-1) = 4$
$5 + 1 = 6$	$5 + 3 = 8$
$10 + (-1) = 9$	$10 + 1 = 11$
$10 + 3 = 13$	$10 + 5 = 15$