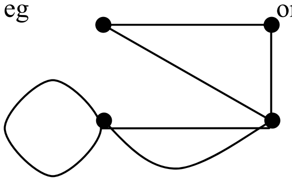
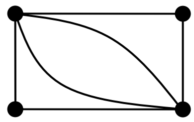
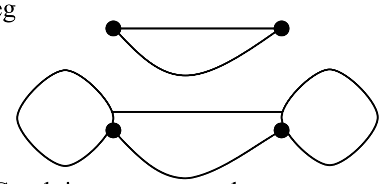
4736 Decision Mathematics 1

1	**(i)**	Biggest/largest/last number (only) (Not showing effect on a specific list)	B1	Accept bubbling to left unless inconsistent with part (ii): Smallest/first number	[1]
	(ii)	2 1 3 4 5 horizontally or vertically (may see individual comparisons/swaps) [For reference: original list was 3 2 1 5 4] 4 comparisons and 3 swaps (both correct)	M1 A1	Or bubbling to left: 1 3 2 4 5 Watch out for shuttle sort used If not stated, assume that comparisons come first	[2]
	(iii)	1 2 3 4 5 One (more pass after this)	M1 A1	FT from their first pass with their bubbling if possible Watch out for 'One swap (in 2nd pass)'	[2]
	(iv)	$(3000 \div 500)^2 \times 0.2$ = 7.2 seconds	M1 A1	$6^2 \times 0.2$ or $8{\times}10^{-7} \times 9{\times}10^6$ or any equivalent calculation cao UNITS	[2]
				Total =	**7**

2	**(i)**	eg  or  - Graph is not simple - Two of the vertices are joined by two arcs (if appropriate) - It has a 'loop' (if appropriate) - For a simple graph each vertex must have order 3 or less	M1 A1 B1	A graph with four vertices of orders 2, 2, 4, 4 (ignore any vertex labels) A connected graph Recognition that their graph is not simple (although it is connected). Need not use the word 'simple'.	[3]
	(ii)	eg  Graph is not connected	M1 A1 B1	Any graph with four vertices of orders 2, 2, 4, 4 (that is topologically different from that in part (i)) A graph that is not connected Recognition in words that their graph is not connected	[3]
				Total =	**6**

3	(i)	$y \le x + 2$ $x + 2y \ge 6$ $(y \ge -\frac{1}{2}x + 3)$ $2x + y \le 12$ $(y \le -2x + 12)$	M1 M1 M1 A1	Line $y = x + 2$ in any form Line $x + 2y = 6$ in any form Line $2x + y = 12$ in any form All inequalities correct	[4]
	(ii)	$x + 2y = 6$ and $y = x + 2 \Rightarrow (\frac{2}{3}, 2\frac{2}{3})$ $y + 2x = 12$ and $y = x + 2 \Rightarrow (3\frac{1}{3}, 5\frac{1}{3})$ $y + 2x = 12$ and $x + 2y = 6 \Rightarrow (6, 0)$	M1 A1 A1 B1	Follow through if possible Calculating from their lines or implied from either A mark $(\frac{2}{3}, \frac{8}{3})$ (art (0.7, 2.7)) $(\frac{10}{3}, \frac{16}{3})$ (art (3.3, 5.3)) (6, 0) cao	[4]
	(iii)	$(\frac{2}{3}, 2\frac{2}{3}) \Rightarrow 11\frac{1}{3}$ $(3\frac{1}{3}, 5\frac{1}{3}) \Rightarrow 32\frac{2}{3}$ $(6, 0) \Rightarrow 30$ At optimum, $x = 3\frac{1}{3}$ and $y = 5\frac{1}{3}$ Maximum value = $32\frac{2}{3}$	M1 A1 A1	Follow through if possible Testing vertices or using a line of constant profit (may be implied) Accept $(3\frac{1}{3}, 5\frac{1}{3})$ identified (ft) $32\frac{2}{3}$ (air 32.6 to 32.7) (ft)	[3]
	(iv)	$5\times3\frac{1}{3} + k\times5\frac{1}{3} \ge 5\times6 + k\times0$ $\Rightarrow k \ge 2.5$	M1 M1 A1	$5\times3\frac{1}{3} + k\times5\frac{1}{3}$ (ft) or implied $5\times6 + k\times0$ or 30 or implied Greater than or equal to 2.5 (cao)	[3]
				Total =	**14**

4	**(i)**	*A*: 1 | 0 *B*: 4 | 5; 6 5 *C*: 5 | 6; 6 *F*: (9) | (16); 16 *H*: 7 | 12; 12 *D*: 3 | 3; 4 3 *E*: 2 | 2; 2 *G*: 6 | 10; 10 *J*: (10) | (16); 16 *K*: 8 | 14; 14 Route = $A - E - B - G - H - K$ Length = 14 metres	M1 M1 A1 B1 B1 B1 B1	Both 6 and 5 shown at *B* All temporary labels correct including *F* and *J* No extra temporary labels All permanent labels correct (may omit *F* and/or *J*) cao Order of labelling correct (may omit *F* and/or *J*, may reverse *F* and *J*) cao $A - E - B - G - H - K$ cao 14 cao	**[7]**
	(ii)	Without using *CJ*: Route = $A - E - B - G - F - J$ Length = 21 metres	 B1 B1	Follow through their **(i)** $A - E - B - G - F - J$ 21	**[2]**
	(iii)	More than 2 metres (Answer of 'more than 7 metres' or '7 metres' $\Rightarrow$ M1, A0)	M1 A1	2 (cao) More than, or equivalent (Answer of 3 or $\geq 3 \Rightarrow$ SC1)	**[2]**
				Total =	**11**

<table>
<tr><td>5</td><td>(i)</td><td>
<table>
<tr><td></td><td>E</td><td>W</td></tr>
<tr><td>A</td><td>x</td><td>$3 - x$</td></tr>
<tr><td>B</td><td>y</td><td>$3 - y$</td></tr>
<tr><td>C</td><td>$4 - x - y$</td><td>$x + y - 1$</td></tr>
</table>
Total cost = £$(250x + 250(3-x)$
$+ 200y + 140(3-y)$
$+ 300(4-x-y) + 280(x+y-1))$
$=$ £$(2090 - 20x + 40y)$ (AG)
</td><td>B1
B1
B1

M1

A1</td><td>$AW = 3 - x$
$BW = 3 - y$
$CE = 4 - x - y$, in any form

An appropriate calculation for their table

Leading to given result</td><td>[5]</td></tr>
<tr><td></td><td>(ii)</td><td>$2090 - 20x + 40y \le 2150$
$\Rightarrow -20x + 40y \le 60$
$\Rightarrow -x + 2y \le 3$ (AG)</td><td>B1</td><td>Showing where the given inequality comes from</td><td>[1]</td></tr>
<tr><td></td><td>(iii)</td><td>$50(3-x) + 40(3-y) + 60(x+y-1)$
$= 210 + 10x + 20y$
So need to maximise $x + 2y$ (AG)</td><td>M1
A1</td><td>Follow through their table
Correct expression
$210 + 10x + 20y$</td><td>[2]</td></tr>
<tr><td></td><td>(iv)</td><td>
<table>
<tr><td>P</td><td>x</td><td>y</td><td>s</td><td>t</td><td>-</td></tr>
<tr><td>1</td><td>-1</td><td>-2</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>-1</td><td>**2**</td><td>1</td><td>0</td><td>3</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>3</td></tr>
</table>
</td><td>

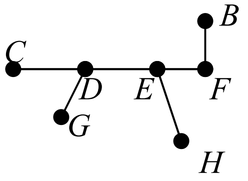
B1
B1</td><td>Rows and columns may be in any order
-1 -2 in objective row
Constraint rows correct</td><td>[2]</td></tr>
<tr><td></td><td>(v)</td><td>Pivot on the 2 in the y column
<table>
<tr><td>1</td><td>-2</td><td>0</td><td>1</td><td>0</td><td>3</td></tr>
<tr><td>0</td><td>-0.5</td><td>1</td><td>0.5</td><td>0</td><td>1.5</td></tr>
<tr><td>0</td><td>1.5</td><td>0</td><td>-0.5</td><td>1</td><td>1.5</td></tr>
</table>
Pivot on 1.5 in the x column
<table>
<tr><td>1</td><td>0</td><td>0</td><td>$\frac{1}{3}$</td><td>$1\frac{1}{3}$</td><td>5</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>$\frac{1}{3}$</td><td>$\frac{1}{3}$</td><td>2</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>$-\frac{1}{3}$</td><td>$\frac{2}{3}$</td><td>1</td></tr>
</table>
$x = 1, y = 2$
</td><td>B1

M1
A1

M1

A1
B1</td><td>Correct choice of pivot from y column
Follow through their tableau and valid pivot if possible
Pivot row correct
Other rows correct

Correct choice of pivot
Follow through their tableau and valid pivot if possible
Correct tableau
Correct answer only</td><td>[6]</td></tr>
<tr><td colspan="5">**Total =**</td><td>**16**</td></tr>
</table>

6	**(a)(i)**	Route Inspection (problem)	B1	Or Chinese postman (problem)	[1]
	(ii)	Odd nodes are A, B, C and D $AB = 250$ $\quad AC = 100$ $\quad AD = 200$ $CD = \underline{200}$ $\quad BD = \underline{250}$ $\quad BC = \underline{350}$ $\quad 450 \quad\quad 350 \quad\quad 550$ Repeat AC and $BFED = 350$ Length of shortest route = 3350 metres	B1 M1 A1 B1	Identifying odd nodes (may be implied from working) Pairing odd nodes (all three pairings considered) M mark may not be implied 350 as minimum 3350 m or 3.35 km UNITS	[4]
	(iii)	C is an odd node, so we can end at another odd node. $AB = 250$ $\quad AD = 200$ $\quad BD = 250$ Repeat $AD = 200$ Length of route = 3200 metres Route ends at B	 M1 A1 B1	Working need not be seen May be implied from answer 3200 B	[3]
	(b)(i)	$D - G - C - A - E - F - B - H - D$ 1580 metres $A - C - D$ - G then method stalls	M1 A1 B1	Correct cycle If drawn then arcs must be directed 1580 Identifying the stall	[3]
	(ii)	$BF = 100$ $FE = 50$ $ED = 100$ $DG = 80$ $EH = 110$ $DC = 200$  Order of adding nodes: $B\ F\ E\ D\ G\ H\ C$ Total weight of tree = 640 metres	M1 A1 B1 A1 B1	Use of Prim's algorithm to build tree (e.g. an attempt at list of arcs or order of adding vertices). NOT Kruskal Correct arcs chosen (listed or seen on tree) A correct tree with vertices labelled Order stated or clearly implied 640	[5]
	(iii)	Lower bound = 640 + 100 + 200 = 940 940 metres $\leq$ shortest tour $\leq$ 1580 metres	M1 A1	300 + weight of their tree their 940 $\leq$ length $\leq$ their 1580 (condone use of < here)	[2]
				Total =	**18**

For reference: