OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4724**

Core Mathematics 4

Monday **12 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$. [4]

2 **(i)** Expand $(1 - 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2. [3]

(ii) Find the coefficient of x^2 in the expansion of $\dfrac{(1 + 2x)^2}{(1 - 3x)^2}$ in ascending powers of x. [4]

3 **(i)** Express $\dfrac{3 - 2x}{x(3 - x)}$ in partial fractions. [3]

(ii) Show that $\displaystyle\int_1^2 \frac{3 - 2x}{x(3 - x)}\,dx = 0$. [4]

(iii) What does the result of part **(ii)** indicate about the graph of $y = \dfrac{3 - 2x}{x(3 - x)}$ between $x = 1$ and $x = 2$? [1]

4 The position vectors of three points A, B and C relative to an origin O are given respectively by

$$\overrightarrow{OA} = 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k},$$
$$\overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$$
and $$\overrightarrow{OC} = 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.$$

(i) Find the angle between AB and AC. [6]

(ii) Find the area of triangle ABC. [2]

5 A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2.

(i) Write down a differential equation which models this situation. [2]

(ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]

6 **(i)** Show that the substitution $u = e^x + 1$ transforms $\displaystyle\int \frac{e^{2x}}{e^x + 1}\,dx$ to $\displaystyle\int \frac{u - 1}{u}\,du$. [3]

(ii) Hence show that $\displaystyle\int_0^1 \frac{e^{2x}}{e^x + 1}\,dx = e - 1 - \ln\left(\frac{e + 1}{2}\right)$. [5]

7 Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k}),$$

where a is a constant.

(i) Given that the lines are skew, find the value that a cannot take. [6]

(ii) Given instead that the lines intersect, find the point of intersection. [2]

8 **(i)** Show that $\int \cos^2 6x \, dx = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$. [3]

(ii) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} x\cos^2 6x \, dx$. [6]

9 A curve is given parametrically by the equations

$$x = 4\cos t, \qquad y = 3\sin t,$$

where $0 \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Find $\frac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent at the point P, where $t = p$, is

$$3x\cos p + 4y\sin p = 12.$$ [3]

(iii) The tangent at P meets the x-axis at R and the y-axis at S. O is the origin. Show that the area of triangle ORS is $\frac{12}{\sin 2p}$. [3]

(iv) Write down the least possible value of the area of triangle ORS, and give the corresponding value of p. [3]