Great dodecahedron

Modelling a situation

Prerequisite knowledge

- Experience of drawing circles using a pair of compasses
- Ability to recognise and describe equilateral and isosceles triangles, and simple symmetry

Why do this problem?

This problem requires pupils to construct an icosahedron and to compare its structure with that of a great dodecahedron, which is quite an unusual task. An introductory discussion based around an image of a dodecahedron offers pupils opportunities to be creative and share their ideas with no holds barred!

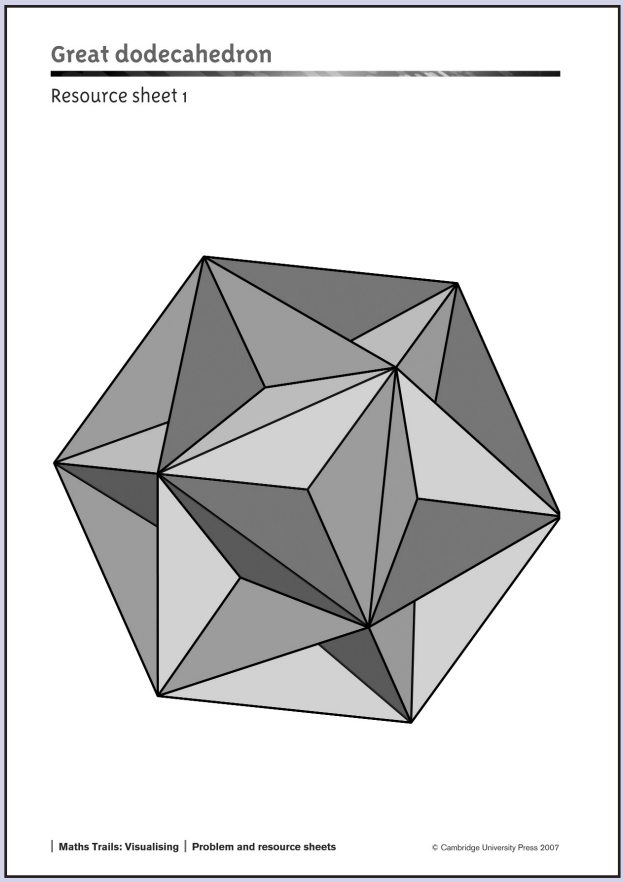
The problem also gives opportunities to pay particular attention to the comprehension phase of the problem-solving model.

Time

One lesson

Resources

Materials and tools for pupils to make icosahedra: card, compasses, scissors, glue or elastic bands
Solid icosahedron



CD-ROM: resource sheets 1 and 2; animation
NRICH website (optional): www.nrich.maths.org, September 2003, 'Dodecawhat'

Introducing the problem

Display the image of the great dodecahedron (resource sheet 1). Invite the class to look carefully at the diagram. The idea of this introduction is to encourage the class to describe what they see in the picture. At first, different pupils will identify different shapes and have different perspectives. If they describe carefully, others will be able to spot what they have noticed. The following prompts may be useful:

- What shapes can you see?
- Do certain parts of the diagram stick out or stick in? (We could call them peaks and valleys.)

From these initial questions, pupils often identify equilateral triangles, isosceles triangles, stars, different colours, repeating patterns and the outlines of hexagons, pentagons, tetrahedra and even a cube.

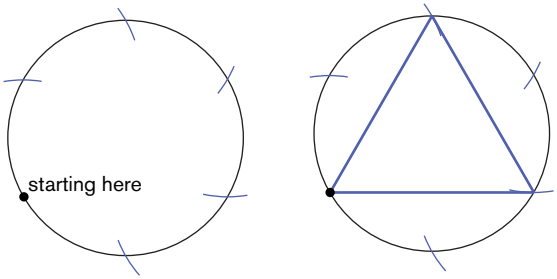
Main part of the lesson

Show the class the image of the icosahedron (resource sheet 2).

- What do we need to make this shape? [20 equilateral triangles]

Introduce pupils to the idea that they are going to make their own model and then compare the icosahedron with the great dodecahedron they looked at earlier.

Ask the class to work in pairs and draw 20 circles on their sheets of card. Six circles per card, each with a radius of 4 cm, is a good size if working on A4 (this will leave four spare circles just in case). Divide the circumference of each circle into three by stepping round six times with compasses set at the radius length. It might help to draw in the sides of the equilateral triangle.



Ask the class to lightly score the sides of the triangles and then cut out the circles. They should fold the card towards themselves along the scored lines and use the folded-back segments as tabs to glue the triangles together.

At this point you may like to show the class a partly completed version of the icosahedron so that they can see how the triangles are fitted together. (It is possible to leave these segments on the outside of the model and use elastic bands to join pieces together, which makes a nice lampshade!)

Plenary

Put up the image of the great dodecahedron again and ask the class what similarities and differences they can see between this and their icosahedron. They might notice that the great dodecahedron is an icosahedron with the faces 'pushed inwards' to form an 'inside-out' tetrahedron. Therefore the great dodecahedron has $3 \times 20 = 60$ faces. To illustrate this, show the class the animation of the icosahedron moving over the great dodecahedron. You could ask how they would go about making a great dodecahedron.

Solution notes

Not applicable.