Noughts and crosses

Multipurpose

Prerequisite knowledge

- Knowledge of properties of cubes and experience of working with cubes made from smaller cubes

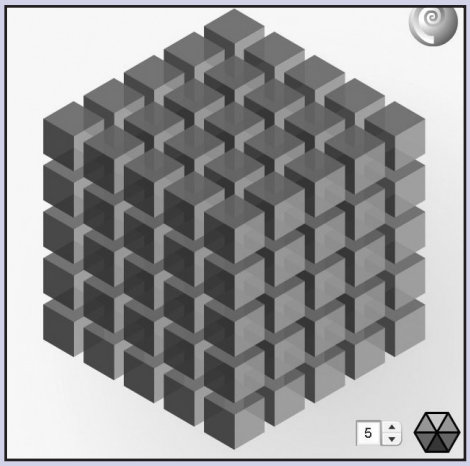
Why do this problem?

This is a good opportunity to explore aspects of generality in three dimensions and for pupils to discuss images and possible solutions, and find convincing arguments for the unique solution.

For some this may be a hard problem to visualise, which is not always made easier by diagrams.

Once pupils have met this activity it is a nice one to return to as a lesson starter, asking pupils to complete lines in their mind's eye.

This problem also gives opportunities to pay particular attention to the analysis and synthesis, and planning, execution and interpretation phases of the problem-solving model.



Time

One lesson

Resources

Individual whiteboards and pens; linking cubes (optional)
CD-ROM: problem sheet; resource sheet; interactivities 'Which lines' and 'Find the cube'
NRICH website (optional): www.nrich.maths.org, January 2007, 'Noughts and crosses'

Introducing the problem

Invite the class to imagine a 3×3 square grid.

- How many small squares are there? [9]
- How are they arranged? [3 rows of 3]

Ask the group to draw a 3×3 grid on their whiteboards (or on a piece of paper).

Now, ask them to imagine colouring one of their nine squares and to put up their hand if they can describe to the rest of the class where to colour in a square so that is the same as theirs. Explain that when the person describes their square you will ask everyone to fill in that square on their boards and you will do it too.

Repeat this activity several times with the aim of identifying some notation that fully describes the position of squares on the grid. Of course, you could use a numbered coordinate system but the rest of this lesson focuses on descriptive clues. The important point is, though, that you need two pieces of information: position across, and position up or down.

The resource sheet describes positions using a left–middle–right and top–middle–bottom notation.

Repeat the activity but this time describe lines of squares (see resource sheet for examples), like the winning lines in 'Noughts and crosses'. Emphasise not only the notation but that there are three types of lines: along the edge; horizontally or vertically through the middle; and diagonally.

- How many of each type of line are there? [4, 2, 2; therefore 8 possibilities in total]

Main part of the lesson

The aim of the starting point here is to agree a notation and encourage pupils to visualise what is happening. The problem sheet can act as a reminder.

Ask the group to imagine that they have a $3 \times 3 \times 3$ cube made up from 27 unit cubes.

- Can you devise a similar notation for describing positions of little cubes and lines of cubes within this larger cube (as if we were playing 3-D 'Noughts and crosses')?

You may wish to use the 'Find the cube' interactivity to help pupils become familiar with the notation.

A unit cube can be thought of as located according to the following positions with respect to the three axes:

- x-axis: left, middle, right;
- y-axis: front, middle, back;
- z-axis: top, middle, bottom.

Tell the group that a marble is placed in the unit cube found at middle-middle-top.

Another is placed at middle-middle-middle.

- Where should the third marble be placed to make a winning line of three marbles? [middle-middle-bottom]

Try some other examples with the group. Ask someone to come up with an example of their own. The interactivity 'Which lines' can be used to give feedback so that pupils are kept motivated. This interactivity allows you to locate lines of three. Clicking on a face gives a layer, then clicking on either a face or a corner star gives a winning line. For long diagonal winning lines, click directly on a corner star. (Hover on one of the vertices of the large cube to see a corner star.)

Hand out the problem sheet, which contains some extension activities.

Plenary

Use the plenary to revisit some oral visualisations similar to those in the introduction or in the main part of the lesson, for example:

- Imagine a 3×3 square with a cross in the left-top and one in the right-top. Where would a third cross have to go to win?
- How about trying to play 'Noughts and crosses' without paper?

Again, the interactivity can be used to check pupils' suggestions.

Solution notes

'Lines' of three cubes can be made from cubes joined face to face – for example, along an edge or across the middle of a side.

'Diagonals' are cubes joined edge to edge in a line – for example, the diagonal of a face (though there are hidden diagonals inside the cube).

'Long diagonals' are lines of three cubes joined vertex to vertex. These go through the middle of a $3 \times 3 \times 3$ cube from vertex to diagonally opposite vertex of the cube.

$3 \times 3 \times 3$ cube:

27 lines
18 diagonals
4 long diagonals
Total 49

$4 \times 4 \times 4$ cube:

48 lines
24 diagonals
4 long diagonals
Total 76

$n \times n \times n$ cube:

$3n^2$ lines
$6n$ diagonals
4 long diagonals