

## Generalising from patterns

- Simple factors
- Multiples

It demonstrates the power of using mathematical knowledge to predict and explain patterns in a visual context that is not immediately obvious as mathematical.

## One lesson

CD-ROM: pupil worksheet

NRICH website (optional):  
www.nrich.maths.org, April 2004, 'Colour wheels' (includes simple animations that can help pupils to visualise the sequences)

## Generalising from patterns

As the wheel goes round, a trail is painted on the ground:  
BRGBRGBRGBRGBRGBRGB...



Make sure that you can predict where the blues will appear.  
Can you predict where the red and green marks will appear?

Can you predict the colour of the 18th mark?  
The 19th? The 31st? The 59th? The 299th?  
The 3311th? The 96312th?

How did you work it out?

Now consider wheels that produce:

- BBYGBBYGBBYG...
- BYBRBYBRBYBRBYR...
- RRRBYYRRRBYYRRRBYYRRRBYYRRRBYY...
- RRRRRRRRRRRRRRRRRRRRRRRRRRR...

What will the 24th colour be in each case? The 49th colour?  
The 100th?

How did you work it out?

What if ... ?

- Suppose a wheel has 6 markings. Where must red be painted on it so that the 100th mark made is red?
- Consider other wheels (with more or fewer markings) that would give you a red in the 100th position.
- The 3rd mark on a wheel is the only red mark and the 100th mark made by the wheel is also red. How many marks are there on the wheel altogether? How many marks could there be on the wheel if more than one of them is red?

Maths trails: Generalising | Problem and resource sheets

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Modelling the creation of colours can be achieved by using a circular disc with colours marked on its circumference. Ask pupils to close their eyes and imagine a wheel which produces a similar pattern and then describe or draw on whiteboards what they see.

After doing the first sequence as a group encourage pupils to work in pairs or small groups on the rest of the problem.

Encourage pupils to:

- build up ideas by describing what they see – ‘Can you continue the pattern of colours?’
- model the wheel either physically or by visualising, using a number line to see where each of the colours appears – ‘What colour is the number 4? What other numbers are red?’

- look for patterns and note where the repeats occur – ‘How often is there a red mark?’
- build upon their notion of multiples and divisibility – ‘Can you describe anything that is the same about the “blue” numbers?’

The ‘What if ... ?’ questions provide opportunities to extend this problem further.

Discuss methods and findings from different groups regarding divisibility and remainders. Did different groups tackle the problems in different ways? Invite pupils to suggest other methods they could have used and/or comment on a method you might have chosen as a teacher problem solver.

## Solution notes

**For the first wheel**, BRGBRGBRG..., the repeating unit is BRG.

The 18th colour will be G, the 19th B, the 31st B, the 59th R, the 299th R, the 3311th R, and the 96312th G.

Multiples of 3 will be G; one less than (or two more than) multiples of 3 will be R; one more than (or two less than) multiples of 3 will be B.

**For the second wheel**, BBYGBBYGBBYG..., the repeating unit is BBYG.

The 24th colour will be G, the 49th B, and the 100th G.

Multiples of 4 will be G; one or two more than multiples of 4 will be B; three more than multiples of 4 will be Y.

**For the third wheel**, BYBRBYBRBYBRBYBR..., the repeating unit is BYBR.

The 24th colour will be R, the 49th B, and the 100th R.

Multiples of 4 will be R; one or three more than multiples of 4 will be B; two more than multiples of 4 will be Y.

**For the fourth wheel**, RRRBBYRRRBBY..., the repeating unit is RRRBBY.

The 24th colour will be Y, the 49th R, and the 100th B.

Multiples of 6 will be Y; one, two or three more than multiples of 6 will be R; four or five more than multiples of 6 will be B.

**For the fifth wheel**, RRBRRRRBRRRRBRR..., the repeating unit is RRBRR.

The 24th colour will be R, the 49th R, and the 100th R.

Two less than multiples of 5 are B; all the others are R.

### What if ... ?

The 4th mark must be red but of course any of the other marks could be red too.

To get a red in the 100th position, find the remainder when 100 is divided by the number of markings and make that position red.

If there is only one red mark on the wheel and the 100th mark made by the wheel is red then the wheel must have 97 markings.