Changing places

Generalising from games and investigations

Prerequisite knowledge

- Positional language
- The ability to approach the problem in a systematic way

Why do this problem?

In a similar way to 'Colour wheels', this problem demonstrates the power of mathematical reasoning to predict and explain in a seemingly non-mathematical situation.

Time

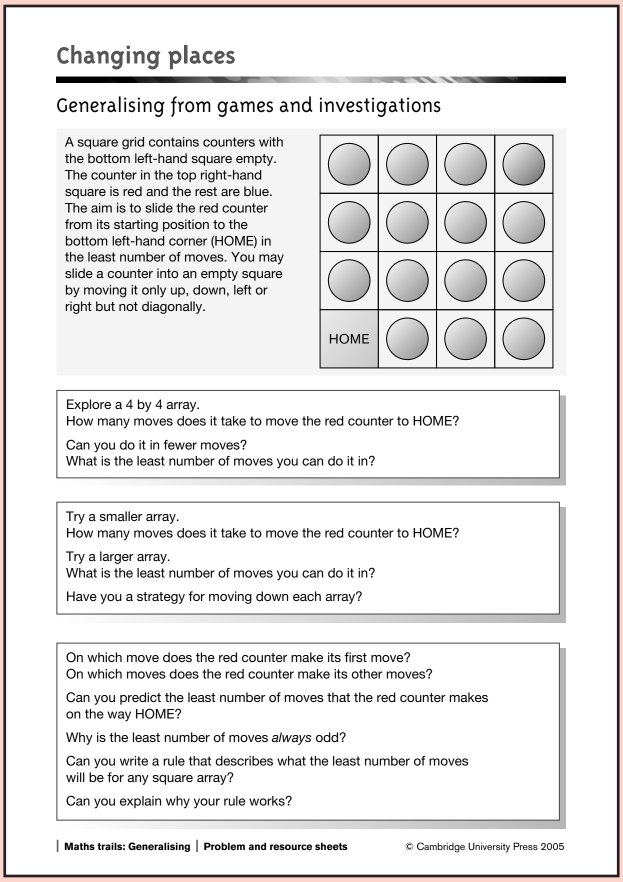
One lesson

Resources

It might be useful to have a selection of grid sizes on OHTs and OHT counters for demonstration and discussion purposes.

CD-ROM: pupil worksheet; OHTs of blank grids

NRICH website (optional): www.nrich.maths.org, September 2004, 'Changing places' (includes an interactive version of this problem, including the ability to change the grid size)



Changing places

Generalising from games and investigations

A square grid contains counters with the bottom left-hand square empty. The counter in the top right-hand square is red and the rest are blue. The aim is to slide the red counter from its starting position to the bottom left-hand corner (HOME) in the least number of moves. You may slide a counter into an empty square by moving it only up, down, left or right but not diagonally.

HOME

Explore a 4 by 4 array.
How many moves does it take to move the red counter to HOME?
Can you do it in fewer moves?
What is the least number of moves you can do it in?

Try a smaller array.
How many moves does it take to move the red counter to HOME?
Try a larger array.
What is the least number of moves you can do it in?
Have you a strategy for moving down each array?

On which move does the red counter make its first move?
On which moves does the red counter make its other moves?
Can you predict the least number of moves that the red counter makes on the way HOME?
Why is the least number of moves *always* odd?
Can you write a rule that describes what the least number of moves will be for any square array?
Can you explain why your rule works?

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Introducing the problem

Using a 4 by 4 OHT grid and counters, ask the class to suggest moves.

- What are the possible first moves?
- How many other moves are possible after the first move?

The second question helps the class to see that, although the situation appears very open, it is in fact quite constrained.

Main part of the lesson

Set the class the task of using the grid (in pairs) so that after a short period of time they will be able to suggest the smallest number of moves necessary to complete the activity. Ask the class how they will record their moves in order to be able to feed back.

Some questions you might wish to consider:

- How many moves did you take to move the red counter to HOME?
  What is the least number of moves you can do it in?
- How many moves did you have to make before you could move the red counter for the first time?

The class could then spend time considering different sized arrays – looking for a generalisation that links moves to grid size, first for square grids and/or generally for rectangular grids.

Plenary

The aim of the plenary should be to pull findings together and find ways of representing generalisations and explaining any patterns pupils have discovered. A number of useful questions are on the problem sheet.

Solution notes

For a 4 by 4 array, it is possible to complete the change in 21 moves.

On grids from 2 by 2 to 5 by 5, the results are as follows:

Grid size	Moves
2 by 2	5
3 by 3	13
4 by 4	21
5 by 5	29

This makes it clear that the difference in the number of moves is always 8 – but why?

Imagine a square grid of size n. For the first move of the red counter, it is necessary to slide the counters along to make a space. This requires $(n-1)+(n-2)=2n-3$ moves of the blue counters.

After this there is a series of repeated moves. The red counter moves every third go, travelling down the diagonal of the array. The red counter moves $n-2$ squares down and $n-1$ squares across, making a total of $2n-3$ moves.

Each time the red moves, two blue counters fill the space, creating a new space for the red. Therefore the total number of moves for the red and blue is $3(2n-3)$.

Hence, the total number of moves to be made
$= (2n-3)+1+3(2n-3)$
$= 2n-3+1+6n-9$
$= 8n-11$.