Peaches

Modelling and optimisation

Prerequisite knowledge

- Simple formulae in Excel

Why do this unit?

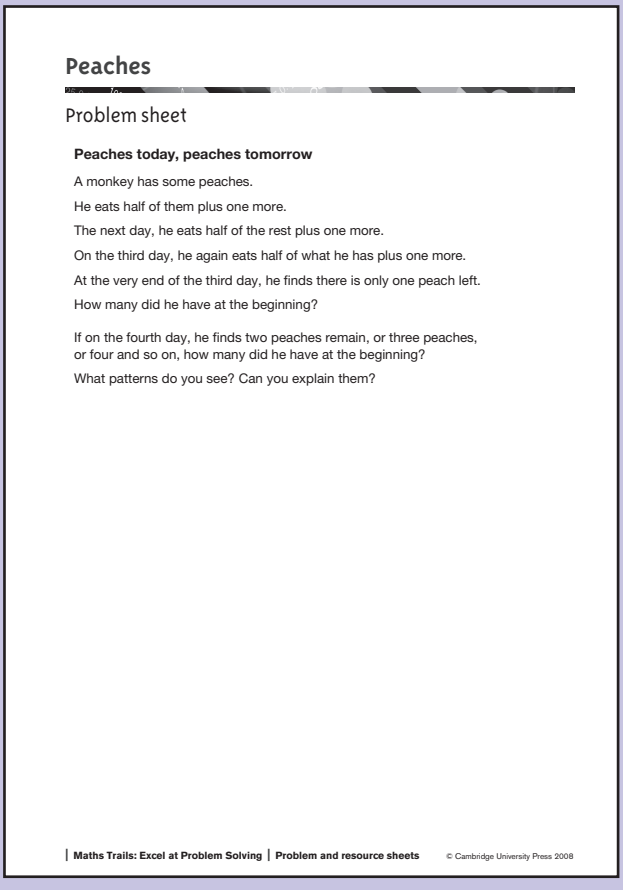
This activity shows how a spreadsheet can be used to provide results systematically so that patterns can be observed and relationships verified.

Time

One lesson

Resources

CD-ROM: spreadsheet, problem sheet
NRICH website (optional):
www.nrich.maths.org, November 2004, 'Peaches today, peaches tomorrow'



Peaches

Problem sheet

Peaches today, peaches tomorrow

A monkey has some peaches.

He eats half of them plus one more.

The next day, he eats half of the rest plus one more.

On the third day, he again eats half of what he has plus one more.

At the very end of the third day, he finds there is only one peach left.

How many did he have at the beginning?

If on the fourth day, he finds two peaches remain, or three peaches, or four and so on, how many did he have at the beginning?

What patterns do you see? Can you explain them?

Introducing the unit

Show pupils the problem sheet.

Allow a few moments for pupils to think about the problem and to share their ideas with a partner. Move into whole-group discussion and focus on trial and improvement. For example:

- How many peaches are left at the end of day 3 if there were 30 peaches to start with? [2]

Go through the calculation together in readiness for looking at the spreadsheet.

Main part of the unit

Go to 'Peaches 1' on the spreadsheet.

The sheet is set for 30 peaches. Discuss the contents of each cell and how that connects with the discussions so far.

Change the 30 in cell A3 to 20 and verify the values that appear. Point out that 20 could not be the solution because it does not result in a whole-number answer.

- Why do decimal values appear in the sheet? [Dividing numbers by 2 several times will frequently lead to a decimal answer.]

Ask the group to work in pairs using 'Peaches 1', and changing the value in A3, first to find the number of peaches that leave 1 at the end of day 3, and then to find the start numbers that leave 2, 3 or any whole number of peaches.

After a short time show the 'Peaches 2' sheet and ask the pupils why this sheet is an improvement. [We can see which numbers work, keep track of what we have tried, and notice patterns.]

Show the group how 'Peaches 2' has been created from 'Peaches 1' using **Fill down**.

Ask pupils to continue with the task. As they work, the following prompts may be useful:

- Why do negative values appear in the sheet? [Trying to take more than is there will generate a negative result.]

- Why is it good to start at 1? [We need to work systematically.]
- Which numbers help us answer the question 'How many peaches did the monkey start with?' [Whole-number values in the 'Peaches left after day 3' column.]
- Can you explain why whole numbers appear in steps of eight?

Extension

Working backwards can be a powerful problem-solving strategy. This problem offers an excellent opportunity to demonstrate that skill to some pupils using the 'Peaches backwards' sheet.

- Can you explain how this sheet solves the problem? [The 'half plus one' means that the other (uneaten) half lost one which needs adding back to get a true half. That is then doubled to return to the whole before halving. That process happens three times to get us back to the starting number of peaches.]

Plenary

Allow plenty of time for the main activity to be discussed and understood. Draw out key points that you have noticed while observing pupils working, including how a spreadsheet displays calculation results but does not explain any patterns or account for them.

Create a shared table of results, for example:

Number of peaches left	Peaches at start
1	22
2	30
3	38
4	46
5	...
...	...

- The pattern of start peaches goes up in steps of eight with 14 added in each case [$8n + 14$]. Why?

Solution notes

The 'Peaches backwards' sheet on the spreadsheet gives the first few solutions.

Whole numbers of peaches appear at regular intervals. A group of 8 peaches is halved three times until only one remains, so the remaining number at the end of the third day increases by one for every eight extra peaches at the start. For n peaches at the end of day 3 there must be $8n + 14$ peaches at the start. If the process took place over four days you would expect the step size to be 16 ($2^4 = 16$).

Explaining the 14 by working backwards.
Imagine you finish with n peaches. This means that on the penultimate day there would have been $2(n + 1) = 2n + 2$ peaches. And on the day before: $2(2n + 2 + 1) = 4n + 6$. And on the day before that: $2(4n + 6 + 1) = 8n + 14$.