Core 1 Jan 05

1. i) $\frac{1}{11^2} = \frac{1}{121}$ ii) $100^{3/2} = (100^{1/2})^3 = 10^3 = \underline{1000}$

iii) $\sqrt{50} + \frac{6}{\sqrt{3}} = 5\sqrt{2} + \frac{2\sqrt{3}\sqrt{3}}{\sqrt{3}} = 5\sqrt{2} + 2\sqrt{3}$

2. $2x^2 - 12x + p = q(x-r)^2 + 10$

$2x^2 - 12x + p = 2(x^2 - 6x) + p$ $\quad (x-3)^2 = x^2 - 6x + 9$

$2[(x-3)^2 - 9] + p$

$= 2(x-3)^2 - 18 + p = 2(x-3)^2 + 10$

Then $10 = p - 18 \Rightarrow p = 28$

$\therefore 2(x-3)^2 - 18 + 28 = 2(x-3)^2 + 10$

$p = 28 \quad q = 2 \quad r = 3$

3. i) $y = 5\sqrt{2x}$ ii) A translation of 3 units in the ~~positive~~ negative y direction.

4. $x^2 - 3y + 11 = 0 \quad 2x - y + 1 = 0$

$y = 2x + 1$

$x^2 - 3(2x+1) + 11 = 0$

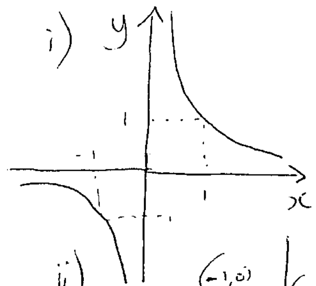
$x^2 - 6x - 3 + 11 = 0 \Rightarrow x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

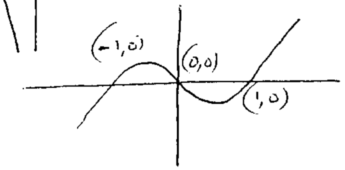
$x = 2$ or $x = 4$

if $x = 2$ $y = 5$ if $x = 4$ $y = 9$

Q5. i)



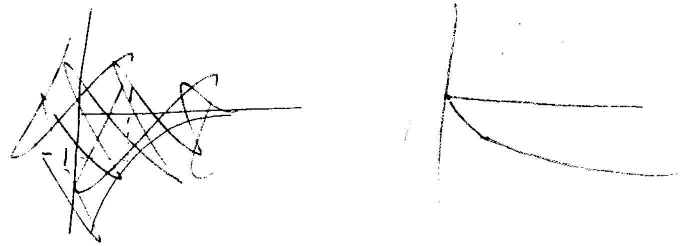
ii)



$y = x(x^2 - 1)$

$x = 0, -1,$ or 1 to make $y = 0$

iii)



Q6. i) $-2x^2 + 7x + 3$

$b^2 - 4ac$ is $7^2 - 4(-2)3$

$= 49 + 24 = 73$

The discriminant is positive so $-2x^2 + 7x + 3 = 0$ has 2 real roots (solutions)

ii) For equal or repeated roots the discriminant must be 0

$(p+1)^2 - 4 \times (2) \times (8) = 0$

$p^2 + 2p + 1 - 64 = 0$

$p^2 + 2p - 63 = 0$

$(p+9)(p-7) = 0 \quad \therefore \quad \underline{p = -9 \text{ or } 7}$

Q7. i) $y = \frac{1}{2}x^4 - 3x$ so $\frac{dy}{dx} = \frac{4}{2}x^3 - 3 = \underline{2x^3 - 3}$

ii) $(2x^2+3)(x+1)$

$= 2x^3 + 2x^2 + 3x + 3$ so $\frac{dy}{dx} = \underline{6x^2 + 4x + 3}$

iii) $y = \sqrt[5]{x} = x^{1/5}$ so $\frac{dy}{dx} = \underline{\frac{1}{5}x^{-4/5}}$

Q8. width $= x$ length $= x+10$

i) Perimeter $= 2(x+10) + 2x = 4x + 20$

which must be greater than 64

$4x + 20 > 64$

ii) Area $= x(x+10)$

$x(x+10) < 299$

$x^2 + 10x - 299 < 0$

$(x-13)(x+23) < 0$

From (ii) for $(x-13)(x+23)$ to be negative then exactly one of $x-13$ and $x+23$ must be negative

if $x < -23$ then $(x-13)$ and $(x+23)$ are both negative so their product is positive

if $x > 13$ then both are positive.

if $-23 < x < 13$ then $(x+23)$ is positive and $(x-13)$ is negative so this is the range we need

However, from (i) $4x + 20 > 64$ $4x > 44$

$x > 11$

combining the 2 $\underline{11 < x < 13}$

Q9 i) $y = 2x^2$ so $\frac{dy}{dx} = 4x$ if $x = 3$ $\frac{dy}{dx} = \underline{12}$

Gradient

ii) $y = 2x^2$

$\frac{dy}{dx} = 4x$ Gradient of tangent

Gradient of normal is $\frac{1}{8}$ $\therefore$ Gradient of tangent $= -8$

$\therefore \frac{dy}{dx} = -8$ so $4x = -8$

$x = -2$

if $x = -2$ $y = 2 \times (-2)^2 = \underline{8}$ $\underline{(-2, 8)}$

iii) That the gradient at P, is 6.

iv) $y = kx^2$ $\frac{dy}{dx} = 2kx$ if $\frac{dy}{dx} = 6$ at $x = 1$

$6 = 2 \times k \times 1 \Rightarrow 6 = 2k$

$\underline{k = 3}$

Q10. i) D is $(-2, 0)$ E is $(0, -1)$

Grad of DE $= \frac{-1-0}{0-2} = \frac{-1}{2}$

ii) Parallel to DE has same gradient $-\frac{1}{2}$

Goes through $(2, 3)$

$\frac{y-3}{x-2} = -\frac{1}{2}$

$y - 3 = \frac{-1}{2}(x-2) \Rightarrow 2y - 6 = -(x-2)$

$2y - 6 = 2 - x$

$2y + x - 8 = 0$

iii) Gradient of EF is $\dfrac{3 - -1}{2 - 0} = \dfrac{4}{2} = 2$

Grad of EF × Grad of DE

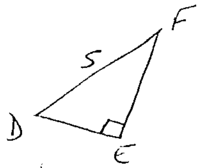
$= 2 \times -\frac{1}{2} = -1$

$\therefore$ EF and DE are perpendicular so DEF is rightangled.

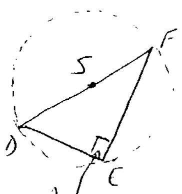
iv) D is $(-2, 0)$ F$(2, 3)$

Length DF $= \sqrt{(2 - -2)^2 + (3 - 0)^2} = \sqrt{16 + 9} = \underline{\underline{5}}$.

v) So far we know DEF looks like this



The circle that passes through them looks like this -



Angle on a diameter is 90°

The centre of the circle is the midpoint of DF eg $\left(\frac{-2+2}{2}, \frac{0+3}{2}\right) = \left(0, \frac{3}{2}\right)$

It has radius $\frac{5}{2}$

$\therefore (x - 0)^2 + \left(y + \frac{3}{2}\right)^2 = \left(\frac{5}{2}\right)^2$

$x^2 + y^2 - \frac{3}{2}y - \frac{3}{2}y + \frac{9}{4} = \frac{25}{4}$

$x^2 + y^2 - 3y + \frac{9}{4} - \frac{25}{4} = 0$

$x^2 + y^2 - 3y - \frac{16}{4} = 0$

$\underline{\underline{x^2 + y^2 - 3y - 4 = 0}}$