Roundabout

Stepping into a problem

Prerequisite knowledge

- It is not necessary to be able to calculate the circumference of a circle

Why do this problem?

This is a nice example of a problem that can be tackled practically with a shape and a coin. The interactivity allows opportunities for exploration beyond what is in the text.

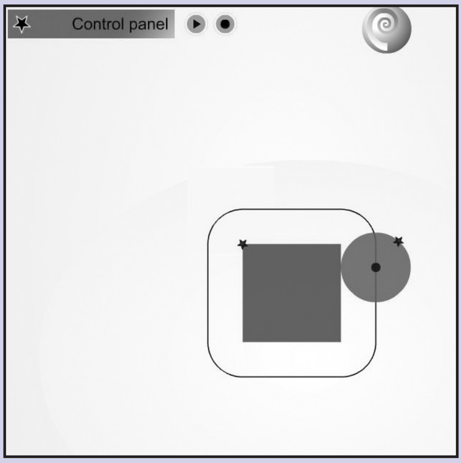
The problem also gives opportunities to pay particular attention to the analysis and synthesis phase of the problem-solving model.

Time

One lesson

Resources

Conjecture board to post conjectures and findings, divided into sections with one of the prompt cards in each section
CD-ROM: resource sheet; interactivity
NRICH website (optional): www.nrich.maths.org, February 2004, 'Roundabout'



Introducing the problem

Ask pupils to imagine a circle rolling along the top edge of a square. Find out which way their circle is rolling then suggest that you all agree left to right.

- What is the locus (path taken) by the centre of the circle as it rolls along the edge? [a straight line]
- What can you say about this straight line? [it is parallel to the edge of the square]
- What would make the distance between the locus of the centre and the edge of the square change? [changing the radius of the circle]

Use the interactivity to check pupils' visualisations.

Now ask pupils to imagine the circle continuing to roll when it comes to the corner of the square.

- What would happen at the corner? [the circle pivots on the point of the circumference touching the corner until it can roll along or down the next side]
- What is the locus of the centre as the circle pivots around a corner? [part of a circle] What part? [quarter]

Ask pupils to describe the locus of the centre of the circle (no hands!) as it rolls around the whole square [a square but with curved corners]. Give them time to think on their own first, then talk with a partner, then discuss in the whole group (*think – pair – share*).

Return to the interactivity to verify predictions.

Discuss the total effect of the locus of the centre at the four corners [it makes a full turn].

Discuss the effect of changing the dimensions of the square and the circle or of making the square into a rectangle.

- How could you easily double the length of the locus? [double the sides of the square and the diameter of the circle]
- Could you double the length of the locus by just changing the dimensions of the square? [yes, but it is not straightforward – you would need to introduce pi]

Main part of the lesson

During the main part of the lesson pupils can work in groups and start investigating the locus of the centre of the circle as it rolls around an equilateral triangle. Encourage them to make and test conjectures which might include:

- a conjecture concerning the total length of the curved portions of the locus;
- a conjecture concerning the length of the locus as the dimensions of the triangle and/or the circle are varied;
- conjectures about other non-equilateral triangles – what stays the same and what changes.

Ask each group to write their conjecture on a piece of A4 paper, show how they tested it and write their argument for rejecting or accepting their conjecture.

Once pupils have finished, ask groups to choose a prompt card (see the resource sheet) which interests them and which helps them to identify a conjecture or series of conjectures they can work on.

In each case, on an A4 sheet, pupils will identify the prompt card, write their conjectures and their arguments, and post them in appropriate sections of the conjecture board for the class to view and discuss at the end of the session.

Plenary

Give the class a few minutes to view other groups' findings.

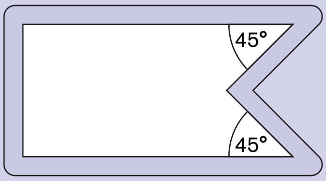
Identify two or three sets of findings of particular interest. This might be because of:

- the results;
- the approach the groups have adopted, for example in their choice of variables (number of zigzags or size or steepness);
- the way the results have been presented that makes the approach very clear;
- the aspects of problem-solving strategies emphasised.

Solution notes

Solutions will depend on the choice of conjectures pupils make but some things that might emerge are given below. The black line in the diagrams indicates the locus of the centre of a circle. Therefore the distance between the locus and the edge of the shape is always the radius of the circle. For the shaded shapes, the total length of the locus is the perimeter of the shape plus the circumference of the circle.

The unshaded pentagon is concave and so the length of the locus is the perimeter of the pentagon minus the diameter of the circle (as the circle 'sits' in the concave corner), plus the circumference of the circle, plus an extra quarter of the circumference.



In the case of the zigzag, which also has concave parts, the length of the locus is the length of the zigzag minus five diameters of the circle (where the circle 'sits' in the dips), plus two and a quarter circumferences of the circle. This can lead to some generalisations about zigzags with different numbers of 'zigs' and 'zags'.