On the edge

Multipurpose

Prerequisite knowledge

- Properties of squares

Why do this problem?

This problem is intriguing in that basic geometrical reasoning leads to the conjecture that being able to rearrange the squares ought to be possible. The practical nature of the problem makes it appealing and it can lead to interesting suggestions for generalisations.

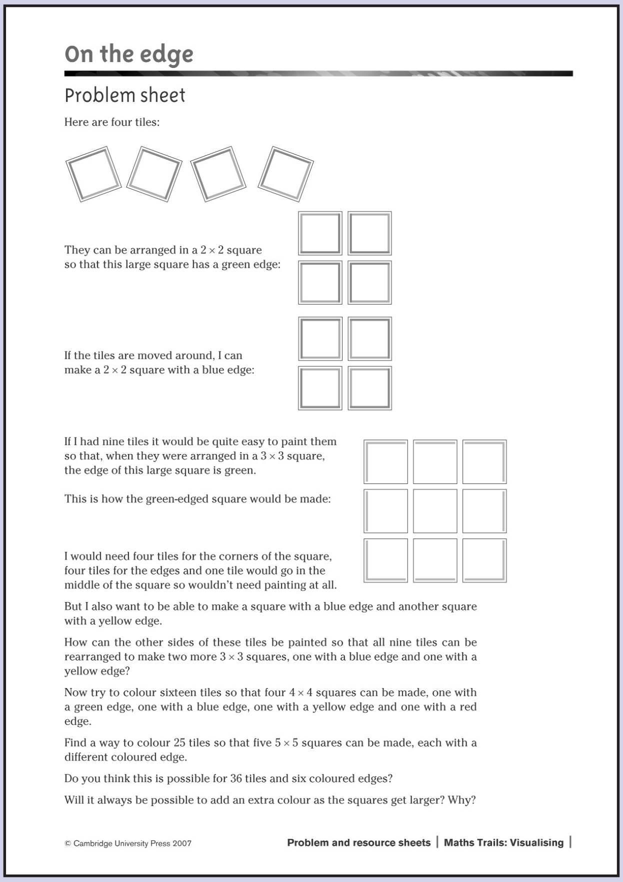
The problem also gives opportunities to pay particular attention to the comprehension phase of the problem-solving model.

Time

One lesson

Resources

Coloured pencils or pens; plenty of blank paper (already cut up into small squares)
CD-ROM: problem sheet
NRICH website (optional): www.nrich.maths.org, September 2004, 'On the edge'


On the edge

Problem sheet

Here are four tiles:

They can be arranged in a 2×2 square so that this large square has a green edge:

If the tiles are moved around, I can make a 2×2 square with a blue edge:

If I had nine tiles it would be quite easy to paint them so that, when they were arranged in a 3×3 square, the edge of this large square is green.

This is how the green-edged square would be made:

I would need four tiles for the corners of the square, four tiles for the edges and one tile would go in the middle of the square so wouldn't need painting at all.

But I also want to be able to make a square with a blue edge and another square with a yellow edge.

How can the other sides of these tiles be painted so that all nine tiles can be rearranged to make two more 3×3 squares, one with a blue edge and one with a yellow edge?

Now try to colour sixteen tiles so that four 4×4 squares can be made, one with a green edge, one with a blue edge, one with a yellow edge and one with a red edge.

Find a way to colour 25 tiles so that five 5×5 squares can be made, each with a different coloured edge.

Do you think this is possible for 36 tiles and six coloured edges?

Will it always be possible to add an extra colour as the squares get larger? Why?

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Introducing the problem

Ask the class to imagine a square.

- How many edges does it have? [4]

Now ask them to imagine another square and join it side-to-side to the first square.

- What shape do you see? [rectangle]
- Is it standing on a narrow edge or its long edge?

Suggest that pupils turn their shape so everyone is imagining the same thing: the rectangle 'lying down', so that the horizontal side is longer than the vertical side.

- How many edges does this rectangle have? [4]
- How many square edges are there? [$2 \times 4 = 8$]
- How many edges of the two small squares are on the outside of this rectangle? [6: 2 of each square on the long edges and 1 of each square on the short]
- How many edges of the original squares are hidden? [2]
- How many edges are there altogether? [8, which equals the number of square edges]
- Add another square on one side of the rectangle, so that it is even longer. Does it matter which end we add the square to? [no]
- How many edges of the squares are on the 'outside' of the new rectangle and how many on the 'inside'? [8 and 4 respectively, which matches the total number of square edges: $3 \times 4 = 8 + 4$]

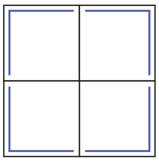
Write the results on the board.

Continue adding squares. Ask if the class can see a pattern in the results.

- Can you explain why?
- Will the number of edges of squares on the perimeter ever equal the number on the inside? [no] Why?

Main part of the lesson

Draw four squares on the board, this time placed together to form a larger square. Draw a coloured line just inside the outside edge of the large square.



- How many small square edges are there altogether? [$4 \times 4 = 16$]
- How many square edges are on the outside and so have a line drawn along them? [8]
- How many square edges are on the inside (with no line drawn along them)? [8]
- How can we check whether that is right? [$8 + 8 = 16 = 4 \times 4$ edges]

Suggest that if there are 8 edges on the inside perhaps the four squares can be rearranged so that the 8 square edges with no lines are on the outside. Invite someone to try.

Now ask the class to imagine they have a 3×3 square with a line around the edge.

- How many small square edges are there all together? [$9 \times 4 = 36$]
- How many coloured square edges are around the edge of the large square? [$4 \times 3 = 12$]

So it sounds as if we might be able to move the squares around twice more and colour with two different colour lines as we have 24 edges left. In theory this seems possible.

This leads into the task on the problem sheet, so ask pupils to test the theory. They could then try 4×4 squares, 5×5 squares, and so on. Make it clear that at the end of the session they will be asked:

- how they did it;
- what strategies they used.

Discuss the need to record in order to be able to feed back.

Groups who complete the task should be challenged to offer an explanation of how to do it. They could produce a poster describing the steps they took and strategies they used. What facts did they make particular use of?

Plenary

Pupils can feed back using the posters they have produced. Use the opportunity to pull out some important points. For example, in a 3×3 square, the small squares will be marked on 0, 1 or 2 edges depending on their positions. The square in the middle has to go in two corners for the second two arrangements. Similarly a corner square will:

- either have to go to the middle and then to a corner;
- or be in the middle of an edge on the next two arrangements.

A middle edge square cannot stay in the same position for the second two arrangements as only 3 sides will be coloured: it will have to go to a corner and an edge.

These strategies need discussing in the plenary along with effective recording methods – mathematics is not just about tables of results and numbers!

You may like to leave pupils with the challenge of squares of other sizes. Their aim should be to record their results in such a way that you can check they are right!

Solution notes

It is possible to colour the edges of the squares so that they can be rearranged.