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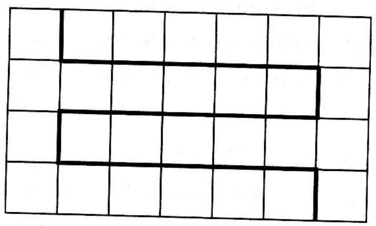
LONGEST PATH THROUGH A RECTANGLE

This idea is one I have used to develop students' understanding of quadratic sequences and requires students to have access to 1 cm-square grid paper and works as follows:

- Draw a rectangle on square grid paper, for example 7 by 4.
- Choose a grid point on one edge of the rectangle but not at a corner.
- Draw horizontal and vertical lines, which do not meet or cross each other to create a path to the opposite side (again not a corner).

The task is to find the longest possible path.

Using a 7-by-4 rectangle the longest path is 19 as in the diagram below:



Students can attempt to generalize for rectangles measuring 7 by n.

What happens with 6-by-n rectangles?

What is the general result for any rectangle m by n?

What happens if the problem is based upon an equilateral triangle drawn on an isometric-dot grid? For this problem the idea is to move from a point on one side (similarly not at a corner) to either of the other two sides and again taking the longest route possible. Perhaps, unsurprisingly, the result is connected to, though not the same as, the triangular number sequence.

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NUMBER ROUTE PROBLEMS

The phrase 'Students being responsible for their learning' is one that springs to my mind when I use an idea such as 'Number route problems'. Here students set up and solve each other's problems and I am interested in how to create a culture of problem-posing and problem-solving so students are encouraged to engage with such ways of working. One aspect of the teacher's role is to model a situation so students know what they need to do to develop skills and concepts.

This idea is to set up situations such as:

- I start with a number (s), multiply it by six to gain an answer (a).
- I start with the same number (s), add 24 and multiply this total by two to gain the same answer (a).
- Using these statements, what was my starting number?

Clearly guesswork and trial and improvement will eventually provide the correct values for s and a and, in the first instance, this is fine. However, this task also has rich potential for students to practise and consolidate solving equations with an unknown on both sides of the equation.

Thus, in the example above we have the following:

$6s = a$

$(s + 24) \times 2 = a$.

Which can be combined to form the equality: $6s = 2s + 48$. Solving this we have $s = 12$ and $a = 72$.

By choosing their own starting values and answers, students can easily create such number route problems.

The idea can be developed as follows:

- Start with a number (s), square it and add five to get an answer (a).
- Start with the same number, multiply it by seven and subtract one to get the same answer. Because the situation creates a quadratic equation, there will be two solutions, i.e. $s = 6$ and $s = 1$.

Algebraically the equation $s^2 + 5 = 7s - 1$ can be constructed leading to $s^2 - 7s + 6 = 0$. Factorizing this we gain $(s - 6)(s - 1) = 0$.