

Symmetry challenge

Deduction

Prerequisite knowledge

- Different forms of symmetry (rotation and reflection)

Why do this problem?

The problem invites pupils to work systematically in a spatial environment. It is a problem that is accessible to most pupils even if they need support in organising and presenting their ideas and ensuring the completeness of their solution.

Time

One lesson

Resources

CD-ROM: problem sheet, resource sheets of blank grids and examples

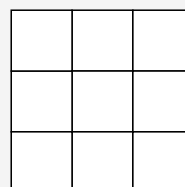
NRICH website (optional):
www.nrich.maths.org, October 2003, 'Symmetry challenge'

You might also need squared paper.

Symmetry challenge

Deduction

Systematically explore the range of symmetric designs that can be created by shading parts of the motif below. Use ordinary squared paper to record your results.



Keep a note of any observations and insights you gain.
What can be achieved? What cannot?
Explain all conclusions reached.

Maths Trails: Working Systematically | Problem and resource sheets

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Introducing the problem

Using the first two 3×3 shaded grids provided on the first resource sheet, discuss reflection symmetry. The two additional blank grids could be used to consolidate pupils' understanding.

Invite each pupil to produce a symmetrical shading on their problem sheet.

Ask the pupils to put themselves into groups according to the number of squares they have shaded on their grid and answer the following questions:

- Are all the patterns you have symmetrical?
- Which are the same and which are different?
- Are there any more with the same number of squares shaded?

These questions will lead to some discussion within the groups.

Main part of the activity

Bring the group discussions back to the whole class ready to discuss some of the resulting issues and introduce the problem. Issues might include:

- What counts as being the same?
- How did the groups check they had all the possibilities?

Introduce the main problem. Ask the groups for suggestions on how they might organise themselves, remembering that they will need to convince the other pupils that they have all the solutions.

As you work with the class the following questions might be useful prompts:

- Try shading 1 square only. What symmetries are possible?
- Now try shading 2, 3, 4, ... squares only. What symmetries are now possible?

- The 3×3 grid has 4 lines of symmetry and rotation symmetry of order 4. How might this help?
- How will you record your findings?

Plenary

The plenary should focus on persuasion that the approach adopted by particular groups will yield all solutions. It is important not to get too bogged down with listing all the possibilities.

Pupils could share out all the possibilities between the group and produce one or more image each, which could then be displayed in an organised fashion to emphasise a system that has been adopted.

The problem can be extended to discuss larger square grids, e.g. 4×4 , and whether there are any differences between even and odd lengths of side. The activity 'Isometrically' is also an extension possibility.

Solution notes

These solution notes refer to reflection symmetry only and give the number of patterns which are symmetrical about the vertical line through the middle of the square.

1 with no squares shaded

3 with one square shaded

6 with two

10 with three

12 with four

12 with five (complements four)

10 with six (complements three)

6 with seven (complements two)

3 with eight (complements one)

1 with all nine shaded

Total 64 symmetrical shadings