Core 2 - Specimen Paper

Q1. $(1-2x)^4$

$= \binom{4}{0}1^4 + \binom{4}{1}1^3(-2x) + \binom{4}{2}1^2(-2x)^2 + \binom{4}{3}1(-2x)^3 + \binom{4}{4}(-2x)^4$

$= 1 + -8x + 6\times1\times4x^2 + 4\times{-8x^3} + 16x^4$

$= 1 - 8x + 24x^2 - 32x^3 + 16^4$

Q2 i) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + k = -x^{-1}+k$

or $-\frac{1}{x}+k$

ii) if $\frac{dy}{dx} = \frac{1}{x^2}$ then $y = \frac{-1}{x}+k$

if this passes through $(1,3)$ then

$3 = \frac{-1}{1}+k \qquad 3 = -1+k$

$k = 4$

Therefore curve is $y = \frac{-1}{x}+4$

Q3 a) i) $\log_2 x^2 = 2\log_2 x$

ii) $\log_2(8x^2) = \log_2 8 + \log_2 x^2$

$= \log_2 2^3 + 2\log_2 x$

$= 3 + 2\log_2 x$

b) if $y^2 = 27$ then $\log_3 y^2 = \log_3 27$

$2\log_3 y = 3$

$\log_3 y = \frac{3}{2}$

Q4. i) $a = 3000$

$ar = 2400$

$r = \frac{ar}{a} = \frac{2400}{3000} = \frac{24}{30} = \frac{4}{5}$ or 0.8

if $n=20$ then no. sold is ar^{n-1}

$= 3000\times0.8^{19} = 43$

ii) Sum of first n terms if $n=20$

$S_{20} = \frac{a(1-r^n)}{1-r} = \frac{3000(1-0.8^{20})}{1-0.8} = 14827$

iii) Sum to infinity $S_\infty = \frac{a}{1-r} = \frac{3000}{1-0.8} = 15000$

Q5. i) $15\cos^2\theta = 13+\sin\theta$

Replace $\cos^2\theta$ with $1-\sin^2\theta$

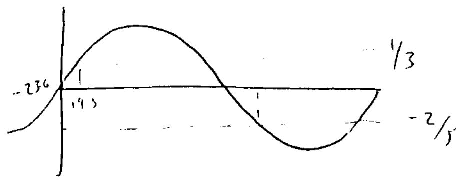
$15(1-\sin^2\theta) = 13+\sin\theta$

$15 - 15\sin^2\theta = 13+\sin\theta$

$15\sin^2\theta + \sin\theta - 2 = 0$

ii) Factorises to $(5\sin\theta+2)(3\sin\theta-1)$

$\therefore \sin\theta = -\frac{2}{5}$ or $\sin\theta = \frac{1}{3}$



$\sin^{-1}(\tfrac{1}{3}) = 19.5$ or 160.5

$\sin^{-1}(-\tfrac{2}{5}) = -23.6$ ← not in range

So $180+23.6 = 203.6$

or $360-23.6 = 336.4$

Q6. i) Using sine rule (Put calculator in Radians)

$$\frac{5}{\sin 2.1} = \frac{3}{\sin C} \Rightarrow \sin C = \frac{3\sin 2.1}{5}$$

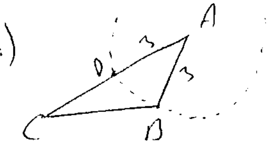
$C = \underline{0.544}$ (rads)

ii) Angles in a triangle add upto π rads

Angle at $A = \pi - 0.544 - 2.1 = 0.4972$

So area $= \frac{1}{2} \times 5 \times 3 \times \sin 0.4972 = \underline{\underline{3.58\,cm^2}}$

iii)



Perimeter $= 6 +$ arc 'BD' $= 6 + 3 \times 0.4972$
$= \underline{\underline{7.49\,cm}}$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 3^2 \times 0.4972$
$= \underline{\underline{2.24\,cm^2}}$

Q7. $y = -3x^2 - 9x + 30$ & $y = x^2 + 3x - 10$

$\therefore -3x^2 - 9x + 30 = x^2 + 3x - 10$

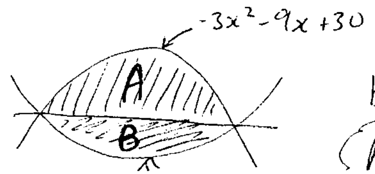
$0 = 4x^2 + 12x - 40$

$0 = 4(x^2 + 3x - 10)$

$= 4(x+5)(x-2)$

$\therefore x = -5$ or 2

if $x = -5$ $y = 0$ if $x = 2$ $y = 0$



You need to add these areas but the bottom is negative to make it positive you need $-B$ ∴

$(A) + (-B)$

So you need $\int_{-5}^{2} -3x^2 - 9x + 30 \, dx + -\int x^2 + 3x - 10 \, dx$

A B (negative)

Which can be written as

$$\int_{-5}^{2} (-3x^2 - 9x + 30) - (x^2 + 3x - 10) \, dx$$

$$= \int_{-5}^{2} (-4x^2 - 12x + 40) \, dx$$

iii)

$$= \left[-\frac{4}{3}x^3 - 6x^2 + 40x\right]_{-5}^{2}$$

$$= \left(-\frac{32}{3} - 24 + 80\right) - \left(\frac{500}{3} - 150 - 200\right)$$

$$= \left(56 - \frac{32}{3}\right) + 350 - \frac{500}{3}$$

$$406 - \frac{532}{3} = \frac{1218}{3} - \frac{532}{3}$$

$$= \frac{686}{3} = \underline{\underline{228\tfrac{2}{3}}}$$

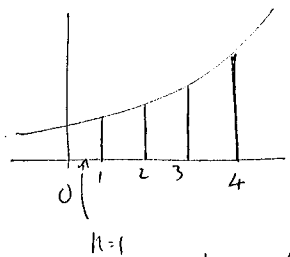
Q8. i) $y = 1.25^x$

$2 = 1.25^x$

$x \log 1.25 = \log 2$

$x = \frac{\log 2}{\log 1.25} = 3.11$

ii)



$\frac{1}{2} \times 1 \left(1.25^0 + 1.25^4 + 2\left(1.25^1 + 1.25^2 + 1.25^3\right)\right)$

iii) It is an overestimate as each trapezium extends above the curve because the curve bends upwards.

iv) Use more trapeziums with a smaller value for h.

Q9. i) $f(2) = 2^3 + a2^2 + b \times 2 - 6 = 8 + 4a + 2b - 6 = 2 + 4a + 2b$

$f(-2) = (-2)^3 + a(-2)^2 + b \times -2 - 6 = -8 + 4a - 2b - 6 = -14 + 4a - 2b$

$2 + 4a + 2b = 4a - 2b - 14$

$4b = -16$

$\underline{\underline{b = -4}}$

ii) $f(1) = 0$

$1^3 + a \times 1^2 + (-4) \times 1 - 6 = 0$

$1 + a - 4 - 6 = 0$

$\underline{\underline{a = 9}}$

iii) $f(x) = x^3 + 9x^2 - 4x - 6 = (x-1)(Ax^2 + Bx + C)$

$x^3 + 9x^2 - 4x - 6 = Ax^3 + Bx^2 + Cx - Ax^2 - Bx - C$

$= Ax^3 + (B-A)x^2 + (C-B)x - C$

x^3: $A = 1$

x^2: $9 = B - A \quad \therefore B = 10$

x: $-4 = C - 10 \quad \therefore C = 6$

$x^3 + 9x^2 - 4x - 6 = (x-1)(x^2 + 10x + 6)$

iv) $(x-1)(x^2 + 10x + 6) = 0$

Clearly $x = 1$ is a root

the discriminant of $x^2 + 10x + 6$ is $100 - 4 \times 1 \times 6$

$= 76$ (positive)

So 2 more roots.

$\therefore$ 3 roots in total