One step, two step

Deduction

Prerequisite knowledge

- It would be useful for pupils to be familiar with the idea of number sequences
- Experience of looking for, and explaining, number patterns

Why do this problem?

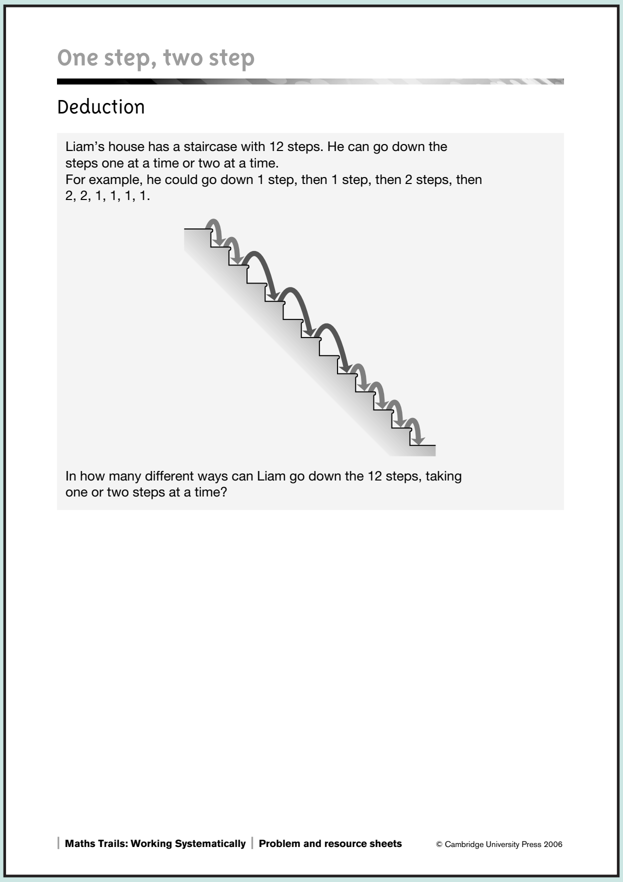
'One step, two step' is an excellent example of how simplifying a problem and working logically through a series of stages can shed light on the underlying mathematics. The problem also encourages pupils not just to spot patterns, but to try to understand why the particular sequence occurs. This problem gives considerable scope for levels of sophistication in the solutions pupils can offer. Whilst some pupils will do well to produce all possibilities for different staircases, the most able pupil will be challenged by the need to give a convincing argument that the sequence is Fibonacci.

Time

Two lessons

Resources

CD-ROM: problem sheet, resource sheets with 3, 4, 5, 6, 7 and 12 step staircases


One step, two step

Deduction

Liam's house has a staircase with 12 steps. He can go down the steps one at a time or two at a time.
For example, he could go down 1 step, then 1 step, then 2 steps, then 2, 2, 1, 1, 1, 1.

In how many different ways can Liam go down the 12 steps, taking one or two steps at a time?

Maths Trails: Working Systematically | Problem and resource sheets © Cambridge University Press 2006

NRICH website (optional): www.nrich.maths.org, October 2000, '1 step, 2 step'

A supply of centimetre squared paper might be useful once the pupils start to work on their own.

Introducing the problem

A quick mental and oral starter involving number sequences would be an appropriate way to begin this lesson. Ask pupils to give the next three numbers in these sequences, explaining how they worked them out:

- 21, 15, 10, ... (triangular numbers in descending order)
- 986, 991, 996, ... (adding 5)
- 4, 8, 16, ... (doubling)
- 82, 65, 50, 37, ... (square numbers plus one in descending order)
- 1.5, 1.5, 3, 4.5, 7.5, ... (each term is the sum of previous two terms)

Introduce the problem either by showing a set of 12 stairs on the board, which will help the class to visualise the problem and get them started. Check that pupils understand what the problem requires by asking some initial questions such as:

- Can you give me one way that Liam could walk down the 12 steps?
- How could we change this way to make it different? And again?
- Do you have any sense of approximately how many different ways there might be?

At this point, discuss the time it would take to find all the ways and the difficulty of making sure you have them all.

Main part of the lesson

Show a new staircase on the board with just 5 steps. Ask pupils to suggest a way of going down the stairs and draw it on the board using arrows. Invite the class to offer other ways, drawing each on a new staircase. Stop to discuss the need to work systematically to ensure that nothing is missed. Ask pupils to work in pairs to find all the ways for going down 5 steps and warn them that they must convince everyone that they have found them all. In the feedback, some pupils might have used a combinations approach, for example 1112, 1121, 1211, ... , and other pupils may have physically drawn out the steps, which is equally as valid. They should have found that there are 8 ways in total.

Having solved this simpler version of the problem, invite pupils to suggest how they might find the number of ways for going down 12 steps.

- Some may postulate that it will be double the number for 6 steps. This could be tested by one half of the class comparing, for example, 2 steps with 4 steps and the other half comparing 3 steps with 6 steps.
- Some may suggest starting more simply and looking for a pattern (which they should then try to justify). This is a very useful approach and at some point in the lesson will need to be pursued.

If the group has not considered organising by simplifying the problem inform them that this is a very useful method used by mathematicians for solving problems and looking for patterns. They should try to justify any patterns they find as they might just be coincidence and you could refer back to the doubling idea if it was developed earlier.

Remind the group that they have already found the number of ways for 5 steps. How many ways are there for walking down 4 steps? 3 steps? 2 steps? 6 steps? etc. Suggest that the class looks at these simpler cases – perhaps they could be divided if time is limited. Can they spot and justify any patterns?

Plenary

Bring the class together to look at the results they have collected. Ask pupils to help you complete a table up to, for example, 6 steps.

- What patterns do they notice in the numbers?
- Can they predict the number of ways for 12 steps using any patterns they identify?

You might like to mention that this particular sequence of numbers (given by the number of ways) is known as the Fibonacci sequence and appears in many instances in nature. More information about Fibonacci can be found on the NRICH website.

Solution notes

No. of steps	Ways	No. of ways
1	1	1
2	1,1 or 2	2
3	1,1,1 or 1,2 or 2,1	3 (= 2 + 1)
4	1,1,1,1 or 1,1,2 or 1,2,1 or 2,1,1 or 2,2	5 (= 3 + 2)
5	1,1,1,1,1 or 1,1,1,2 or 1,1,2,1 or 1,2,1,1 or 2,1,1,1 or 1,2,2 or 2,2,1 or 2,1,2	8 (= 5 + 3)
6	1,1,1,1,1,1 or 1,1,1,1,2 or 1,1,1,2,1 or 1,1,2,1,1 or 1,2,1,1,1 or ...	13 (= 8 + 5)

The number of ways is always the sum of the two numbers before it, i.e. the Fibonacci series.

12 steps: 233 ways