CHAPTER 1

Number review

Starter: Little and large

A speed-up sheet is available for this starter.

The starter is designed to stimulate a discussion about the meaning and importance of place value. Note that such questions can have more than one solution, for example: $95 + 74 = 94 + 75$.

If your students have a sense of humour, you could show them 9752^4, 975^{42}. Beware – this opens a whole can of worms!

Exercise 1.1

Special care needs to be taken when one (or more) of the intermediate digits is a zero, like four thousand and twenty-six. Not all English-speaking mathematicians use the same spoken system: the Americans would say 'one hundred forty' rather than 'one hundred and forty'.

Worksheet 1.1

This wordsearch will give students extra practice in changing numbers in figures to words. As an extension to Exercise 1.1, it can be used after questions 1 and 2.

Exercise 1.2

The introduction to this exercise shows two methods for addition: Method 1 is better for pencil and paper methods, while Method 2 is better for mental arithmetic. Note that you can't have 'multiplication sums' – a sum is an addition problem. You could use this exercise to check that the students know the names for the other processes: difference (subtraction), product (multiplication) and quotient (division).

Worksheet 1.2/1.3

This worksheet can be used after the content of Sections 1.2 and 1.3 have been covered. Since the addition and subtraction are straightforward, it can be done before, or as an extension to, question 1 of both Exercises 1.2 and 1.3.

Exercise 1.3

Once again there are two methods: Method 1 is better for subtraction by pencil and paper methods, while Method 2 is better for mental arithmetic.

Exercise 1.4

A speed-up sheet is available for question 1 of this exercise.

This exercise enables students to practise and learn multiplication facts for products up to 10 by 10. Although some students will work out, for example, 6 times 7 by repeated addition, it is better to learn as many of these results as possible, since they are used in harder work to come (e.g. long multiplication, factorising etc.).

Worksheet 1.4

This can be used after question 1 of Exercise 1.4, since completing the times table grid in question 1 will give students an advantage when they start working on these puzzles.

Exercise 1.5

To multiply by 10, do you really move the decimal one place to the right? Or does it stay where it is, and the digits all move one place to the left? Your students may have differing opinions, but both are valid points of view. This should be a straightforward exercise, but do urge your class to be careful, as mistakes are inevitably *big* ones, namely 10 times too big or 10 times too small: imagine taking a dose of medicine that was 10 times too small, or spending 10 times as much money as you meant to.

Worksheet 1.5

Section A can be done before starting Exercise 1.5, since all the numbers are whole. Section B can be done after questions 1 to 3 of Exercise 1.5, as it reinforces the same type of multiplication questions. Section C is similar to Exercise 1.5 question 4, but it gives a choice of answers, so students may find it easier to do the worksheet first.

Exercise 1.6

As with multiplication, dividing by powers of 10 should be straightforward. Encourage disciplined counting of place moves. A common slip is to subconsciously 'guess' that 34 divided by 1000 must be 0.34 because it looks about right, whereas careful counting reveals a ten-fold error.

Worksheet 1.6

Since all Section A numbers are whole, it can be used as a start, before Exercise 1.6. Section A can be followed by Exercise 1.6 questions 1 to 3, or students can move straight on to Section B1, as the questions are similar. Section B2 mixes examples of 'multiply' and 'divide' if more are needed. Since the answers are given to choose from, use Section C before question 3 in Exercise 1.6.

Exercise 1.7

The introduction shows three methods for long multiplication, and students may well have seen all three of these before. Encourage them to use whichever method they feel most confident with. Method 2 does carry some risks – it can be cumbersome when the numbers are large, since some of the boxes contain long strings of zeros, and a zero can easily become lost in the working, with disastrous results.

Worksheet 1.7

Introduce the grid method of multiplication with this worksheet before students start Exercise 1.7.

Exercise 1.8

All three methods for division shown in the introduction are good ones to use. When using Method 1, to divide by 18, for example, some students find it helpful to write out a list of multiples of 18 first – 18, 36, 54, 72, 90, 108, 126, 144, 162, 180 – so that they can readily see at a glance how many 18's go into 80.

Exercise 1.9

This exercise gives practice in recognising when to use the 'processes' vocabulary associated with $+$, $-$, $\times$, $\div$ introduced in the previous sections. There are some clues in the vocabulary: 'share' ($\div$) in question 1, 'total' ($+$) and 'change' ($-$) in question 2, for example.

Worksheet 1.9

Students repeat the use of $\times$, $+$ and $-$ in these problem solving exercises, so they are appropriate to use before Exercise 1.9 which involves a mixture of functions.

Exercise 1.10

Students may like to use 'two minuses make a plus', but this is only true of two minus signs **next** to each other, e.g. $- - 6$ is $+ 6$; contrast this with $- 6 + - 1 = - 7$, where two minuses make an even greater minus!

When ordering a mixed list of positive and negative numbers, it can be helpful to remember that 'smallest' is the same as 'most negative', and largest is 'most positive'.

When students have finished using their calculators for questions 6 and 7, you could encourage them to check their earlier answers by calculator, too.

Worksheet 1.10/1.11

It incorporates $+$, $-$ and $\times$ with negative numbers, so it needs to be used after the content of Sections 1.10 and 1.11 have been covered. The number line helps to make 'moving up/down' visual. Calculations similar to questions 1 to 3 of Exercises 1.10 and 1.11 are involved.

Exercise 1.11

In effect, the students can initially set the $+$ and $-$ signs to one side in this exercise. Perform the multiplication or division, then restore the signs, making the result – if there was originally one minus sign, + if there were two.

Exercise 1.12

Rounding should be quite straightforward, though one mistake that does sometimes crop up is to omit the place-holder zeros, e.g. the answer to 'round 352 to 2 significant figures' is given as 35 rather than 350.

Worksheet 1.12

Since the worksheet includes rounding to d.p. as well as to the nearest whole number/10/s.f., it can be used for extension after Exercise 1.12.

Exercise 1.13

Do emphasise that 1 significant figure is sufficient when estimating. Some students are reluctant to work this approximately, and will round 129×11 to 130×11 rather than 100×10. In the GCSE exam, students should use 1 s.f. approximations unless a question specifically tells them differently.

CHAPTER 2
Ratio and proportion

Starter: Squares

A speed-up sheet is available for this starter.

There are very many possible combinations of correct shading. Encourage students to work in a systematic way rather than haphazardly. For example, when shading 1 red for every 2 blues, they should try to break the shape up into natural blocks of 3 squares as much as possible.

Exercise 2.1

In this context, ratios may be thought of as another way of writing fractions; so, in the same way as two fractions can be equivalent, so can two ratios. This is a non-calculator exercise, but students could check their answers with a calculator, using the fraction key to cancel down the ratios automatically.

Worksheet 2.1

This gives students practice in simplifying ratios. It provides a different format for them to consolidate the process, and can be used before or after students try Exercise 2.1.

Exercise 2.2

The key to success with these problems is to add up the whole-number parts of the ratios to get the total share, and scale the total from there. Often the ratio contains *three* numbers, so the ratios can no longer be treated in the same way as fractions.

Exercise 2.3

The GCSE examination does not insist that students use the unitary method, but it is a powerful way of seeing whether a multiplication or division should be carried out.

Exercise 2.4

When converting from km to cm, it is easy to miscount by one power of 10. Although the metric system naturally scales in multiples of 1000 (e.g. m, then km), the two units with which students are most familiar are m and cm, where 100 cm = 1 m. It is worth emphasising that 1 km = 100 000 cm.

CHAPTER 3
Decimals

Starter: Getting the point

Much of this chapter concerns place value of decimals. The starter, on ordering decimals, reinforces this essential concept.

Worksheets 3S.1 and 3S.2

The starter assumes that students will have confidence in place value and writing decimals. These two worksheets support students who will need practice at this first.

Exercise 3.1

Rounding of decimals is a necessary consequence of some types of calculator work (e.g. finding square roots) and is also used in estimation. This exercise provides routine practice in standard rounding situations. Examination questions will always state the degree of rounding to be applied.

Worksheet 3.1

The worksheet gives more interesting practice for the easier examples.

Exercise 3.2

Some students find significant figures a more difficult concept than decimal places. A moderately common error, when first meeting the topic, is to think that 3814 to 2 s.f. is 38; consider discussing this with the students and asking what '3', or its equivalent in other numbers, represents.

This can lead on to a discussion about examples like 0.000 302 in which the first four zeros are not significant figures, but the zero between the 3 and the 2 is! The initial zeros are merely acting as place holders (they tell you where the decimal point is, in effect), whereas the zero between the 3 and 2 is indicating the number of hundred-thousandths in this decimal.

Rounding to 1 significant figure is required on the specification, but it is useful for students to be able to round to 3 significant figures for questions which require rounding 'to an appropriate level of accuracy'.

Worksheet 3.2

Students may need some more reinforcement of basic rules and large numbers, before they deal with numbers smaller than 1.

Exercise 3.3

Addition and subtraction of decimals need not be any harder than addition and subtraction of whole numbers. Make sure the numbers are lined up at the decimal point, and use extra zeros (as shown on page 48 in the textbook) to square off the numbers at the right-hand edge.

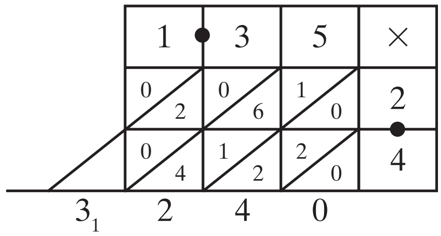
Worksheet 3.3–3.5

This worksheet can be used with Exercises 3.3 to 3.5. Students may be helped with adding and taking away decimals (Exercise 3.3) and multiplying and dividing decimals (Exercises 3.4 and 3.5) by doing questions that are set in a context.

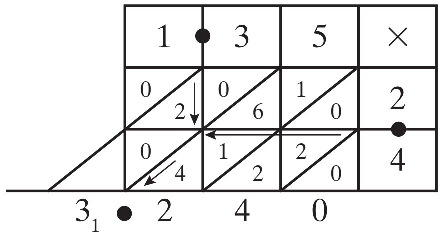
Exercise 3.4

Multiplication with decimals is best done by temporarily removing the decimal point. The decimal may be restored at the end by counting (as shown on page 50 in the text book), or in some cases by estimation, though this requires confident numerical intuition.

Another method is available to those who use the Gelosia method (Method 3 on page 12). This time you write the decimals into the numbers at the start, putting the point symbols on the lines, so for 1.35 × 2.4:



Next, look for the place where the decimal point lines intersect…



…and then run this down the diagonal into the final number. Voilá!

Exercise 3.5

In dividing by decimal numbers, the key idea is to first multiply both numbers by 10, or 100, until they are integers, then carry out a regular division. Occasionally you might see students dividing the final answer by 10 or 100 again: be on the lookout for this error by checking some answers early on in the exercise.

Exercise 3.6

A speed-up sheet is available for questions 3 and 4 of this exercise.

Reading scales is a straightforward topic that should cause little difficulty. Encourage students to work out the value of one step – say, 0.1, 0.2,... 1, 2,... etc. – and check it before interpreting the arrowed reading.

Exercise 3.7

Money problems are often used as a vehicle for testing the understanding of decimals. But money should be rounded to 2 decimals, so discourage £3.6 when it should be £3.60. Also, watch out for £3.60p which could lose marks in the GCSE examination.

There is a useful shortcut method for problems like question 4, which asks for 6 items at £5.99. Make the items £6 instead, to get £6 × 6 = £36, then remove the extra £0.01 × 6 = £0.06, to get £35.94.

Worksheet 3R

This is a puzzle that gives students practice in all aspects of the chapter.

CHAPTER 4

Fractions

Starter: Fractions of shapes

A speed-up sheet is available for question 2 of this starter.

The starter reminds students of the precise meaning of fractions, but the diagrams do have other uses, too: weaker candidates often find such diagrams a useful way of carrying out addition of fractions. Recent Edexcel GCSE exam questions have drawn on this idea, using, for example, a three by five grid to help students add one-third and two-fifths.

Worksheets 4S.1 and 4S.2

Worksheet 4S.1 can be used after students have completed Starter question 1. Worksheet 4S.2 gives practice in shading of fractions and extends to comparing fractions on the same grid size.

Exercise 4.1

The first example shows how to convert a fraction into a decimal by dividing the top by the bottom. Though not an Edexcel GCSE requirement, you may wish to show your students how to do this using the calculator's fraction key too – enter the fraction, press = (or EXE), then just touch the fraction key again, to toggle between the equivalent fractional and decimal forms.

Worksheet 4.1A

After Exercise 4.1, question 1, students can work on worksheet question 1 to carry out more conversions to decimals that are not recurring. Question 2 can be used before attempting Exercise 4.1, question 2.

Worksheet 4.1B

This can be used before Exercise 4.2 since it includes visual representation of equivalent fractions and simplification, and 'lead-through' examples.

Exercise 4.2

This is a straightforward exercise on simplifying fractions by cancelling to their lowest terms. A calculator's fraction key will simplify fractions automatically, but students need only know how to do this by calculation.

Exercise 4.3

This non-calculator exercise should be straightforward. Students could perhaps use a calculator afterwards as a check: divide the top of the answer by the bottom to recover the original decimal.

Exercise 4.4

Adding and subtracting fractions is normally taught as an arithmetic algorithm, but some candidates like to use diagrams as well. GCSE examiners report that candidates who draw diagrams consistently score high marks in these questions.

Worksheet 4.4A

This provides extra work after questions 1 and 2 of Exercise 4.4. Problem questions involve adding and subtracting fractions with the same denominator, or where one is a factor of the other.

Worksheet 4.4B

The worksheet combines diagrams with mixed numbers and improper fractions in the context of problem questions. Students can carry it out before working on Exercise 4.5.

Exercise 4.5

Calculators can handle mixed fractions, and usually have a key to toggle between mixed and top-heavy forms, so can be used for checking if required.

Exercise 4.6

In many cases it is actually easier to multiply or divide with fractions than to add or subtract them! Some students don't realise this, and so they (unnecessarily) arrange both fractions to have a common denominator first. With division, check that students have converted into the corresponding multiplication problem first, before cancelling any common factors.

Exercise 4.7

Question 1 asks for unit fractions only; when students progress to question 2 they should work out, for example, $\frac{1}{5}$ of 100 first, before attempting $\frac{3}{5}$; by breaking the problem into two stages like this, slips are far less likely to occur.

Worksheet 4.7

Questions on finding fractions of a quantity can be used after work on Exercise 4.7, question 1.

Review exercise 4

A speed-up sheet is available for questions 11, 12 and 13 of this exercise.

Internet Challenge 4

This is quite a tricky challenge, and question 7 is not immediately obvious. The point is that $\frac{5}{8}$ of an amount is hard to judge, but $\frac{5}{8}$ is $\frac{1}{2}$ plus $\frac{1}{8}$. So you could divide one sack of corn into eight equal looking piles, and the other four sacks into equal looking halves, giving you eight piles each worth $\frac{1}{2}$ a sack and eight piles each worth $\frac{1}{8}$ of a sack. Then give each chicken coop one of each type of pile.

CHAPTER 5

Percentages

Starter: How many per cent?

A speed-up sheet is available for this starter.

There are six ways of finishing with an N before the final T. Each of these can be reached from one of two E's, which in turn can be reached from either of two C's. Each C can be prefaced with eight different PER combinations. So the total number of ways is $8 \times 2 \times 2 \times 6 = 192$.

Exercise 5.1

A speed-up sheet is available for question 6 of this exercise.

The exercise carries two key principles. One is that to multiply/divide by 100 one moves the decimal point two places right/left; students should find this straightforward. The other is the way to work with equivalent fractions, so it is important that the concepts of Chapter 4 (and particularly Section 4.1) are secure.

Exercise 5.2

Errors in percentage change problems often arise when students guess what to do with numbers – typically, they divide by 100 instead of multiplying. The risk of this error may be avoided by writing the quantity as a fraction first, then multiplying by 100.

Worksheet 5.2/5.3

This worksheet supports Exercises 5.2 and 5.3.

Hopefully, most students will have a feel for either working out or simply 'seeing' 10% of a quantity. If not, since the quantities end with a zero, perhaps first finding 'one-tenth' offers an alternative starting point. Failing that, using the fact that 10% is equivalent to '0.1' may give rise to some early calculator operations.

The worksheet is intended to create a sense of achievement through thinking of the connections between percentage amounts. For example, 10% leads to 5% or 20% by halving or doubling both the percentage and the amount respectively.

Exercise 5.3

Again, the scope for errors can be greatly reduced by encouraging students first to convert the given percentage to a decimal (or a fraction of 100), and then to conduct the multiplication. Although students will probably write this down as a single calculation, it should be thought of as a two-stage process.

Exercise 5.4

Success here depends on being able to spot 10% of an amount by inspection. Weaker students might benefit from a warm-up – 'What is 10% of 60, 10% of 130, 10% of 1600?' – before studying the example on VAT. Since the rate of VAT is currently 17.5% $(10 + 5 + 2.5)$, this example lends itself very nicely to mental methods, and problems like this are quite common in non-calculator exams.

Questions 6 and 7 of this exercise are aimed at more able students and show the multiplicative nature of percentages as operators. So finding 20% of £60 and then a further 50% is $0.2 \times 0.5 \times 50 = 0.1 \times 60$, i.e. 10% of £60, rather than 70% of £60.

Worksheet 5.4

This supplements the work on VAT in Section 5.4.

Calculating VAT at 17.5% has appeared on KS3 SATs papers and also on GCSE examination papers. It often causes students difficulty, perhaps by the inclusion of the 'half' in the percentage.

Hopefully, students will have completed Worksheet 5.2/5.3 before tackling this one. The method used is identical in that it breaks down the percentage into manageable chunks which can then be processed by halving amounts.

Initially the student has to work out 10% of the quantity which is then halved to give 5% and again to give 2.5%. The same is done with the amount, and finally the sums are combined to produce the 17.5%.

As students gain confidence, they may develop their own 'style' of presenting the solution, but otherwise the template shown will suffice.

Exercise 5.5

A common error with percentage increase/decrease is that students work out the size of the change but forget to add/subtract this to the starting amount. Watch for this at the beginning of the exercise – for instance, 50 students are absent, but the question asks for the number present, 1200.

Worksheet 5.5

This supports the Exercise 5.5 questions on two types of interest.

With limited information provided, students are asked to consider which bank or building society offers the better deal. Initially this is done by comparing rates of interest, then developed through calculator work to making actual monetary comparisons using equal amounts of investment.

This is extended by looking at the difference when a combined lump sum is invested in the institution offering the highest rate of interest against one paying the lowest rate.

Finally, students search for adverts from newspapers and magazines with details about interest rates, and illustrate their findings in tabular and graphical form.

Exercise 5.6

More able students might enjoy solving compound interest problems by repeated application of a multiplying factor, for example, £8000 after 3 years at 5% can be worked out as $£8000 \times 1.05 \times 1.05 \times 1.05$. This is very efficient, but is conceptually more difficult, so it is not to everyone's liking!

Worksheet 5R

This worksheet revises words used in calculations about money.

Review Exercise 5

A speed-up sheet is available for question 9 of this exercise.

Internet Challenge 5

The inflation challenge makes the point that we are all affected by percentages. Keener students might want to find out how high the rate of inflation has been in the past, in countries experiencing hyperinflation.

CHAPTER 6
Powers and roots

Starter: Off and on

▶▶ Two speed-up sheets are available for this starter, one for lower ability students which has only 25 bulbs and a line for each switch, and another for more able students, which has 100 bulbs and 10 lines for switching.

Students may quickly realise that lights corresponding to prime numbers will be switched off: for example, 5 has only two factors, 1 and 5, so it gets switched on and then off again. Most non-prime numbers will behave in a similar way – 6 has four factors, 1, 2, 3, 6, so is switched on, off, on, off. The only lights that will remain on are those with an **odd** number of factors, i.e. the square numbers 1, 4, 9, …

Exercise 6.1

▶▶ A speed-up sheet is available for question 4 of this exercise.

When checking to see whether a number is prime, you need not check past the square root of the given number. So for 61, since it not divisible by any of 2, 3, 4, 5, 6, 7, then it will be prime. (In fact, you need only to test for prime candidates up to the square root, i.e. 2, 3, 5, 7, but many Foundation students would find this extra concept confusing.)

Exercise 6.2

Every positive number has both a positive and a negative square root, but only question 9 of this exercise requires both to be found. Weaker candidates may benefit from being given a list of the first 15 square numbers (1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225).

Exercise 6.3

Square roots found using a calculator usually fill the screen with digits, so some rounding is required. GCSE exam questions will always state how much rounding is wanted.

Exercise 6.4

This might be a good place to point out an awkward feature of many calculators: ask your students to work out on a calculator the square and the cube of -2. The correct answers are 4 and -8, but many will get -4 for the square of -2. This is because the key strokes $\boxed{-}$ $\boxed{2}$ $\boxed{x^2}$ are often interpreted by calculators to mean the negative of $\boxed{2^2}$, and not the square of negative 2. Play safe and put brackets in: $\boxed{(}$ $\boxed{-}$ $\boxed{2}$ $\boxed{)}$ $\boxed{x^2}$, and all will be well.

Exercise 6.5

Most of the questions provide routine practice with powers and roots, but question 4 may lead students to discover the laws of indices for themselves in anticipation of the next section. With that in mind, a class discussion about the answers to question 4 may be very beneficial.

Exercise 6.6

Although the laws of indices are easy to use, watch for slips in certain questions. For example, 7^{200} divided by 7^{100} invites the mistaken answer of 7^2 in question 1k).

In question 4b), a common misunderstanding is to think that n^0 must be 0, when it is in fact 1. More able students might enjoy a proof of this, along the lines of x^3 divided by $x^3 = x^0$ (laws of indices), but also x^3 divided by $x^3 = 1$ (dividing a quantity by itself). Weaker students might prefer simply to accept the result as a calculator fact. Be prepared to explain why 0^0 gives an error!

Exercise 6.7

Prime factor trees are good fun, and students will enjoy getting them right. Care must be taken when multiplying back up to get the HCF or LCM, since slips can easily be made at this stage. A useful check is that HCF $\times$ LCM = product of original pair of numbers.

Exercise 6.8

This very simple topic often causes far more confusion than it should. Weaker students might benefit from doing many more practice questions.

Exercise 6.9

Over the years this has been taught as BODMAS, BEDMAS, BMDAS and so on, but the current fashion for BIDMAS is the best of all, since it maintains the correct priority of 'Indices' without using awkward terms like order or exponent.

The textbook makes the important point that DM are equally important; after them, AS are also equally important. So $7 - 3 + 2$ is *not* $7 - 5$ ('doing A first'); you simply do the A and S in the order you find them, to get 6.

Emphasise the importance of the implied brackets in a division problem, shown in Example part b) on page 117. Some students may prefer to work out the top and the bottom of the fraction separately, and then execute the division. This is inefficient, but examiners quite like it, since the method is clear to see.

Exercise 6.10

Only a light touch on standard index form is required at Foundation level. Students need to be able to interpret a calculator display and enter calculations involving standard form. Many students will find it easier to use the power button on their calculator to enter standard form rather than the 'EXP' button as this avoids the common mistake of wrongly pressing the '×' button before the 'EXP' button.

There are some good books and videos available on the theme of powers of 10, in which each image shows an object 10 times bigger (or smaller) than the one before; these are a nice visual counterpart to the exercise. For details of suppliers, you could do an internet search – perhaps using a search engine that draws its name from a high power of 10!

W *Worksheet 6R*

There is one worksheet for this chapter. It incorporates all the functions from the chapter and practice in using a calculator efficiently: a calculator is essential to complete the worksheet.

Internet Challenge 6

A speed-up sheet is available for this challenge.

This challenge reminds us all that some quantities are simply too big to record using ordinary (fixed point) notation. Standard form is then very useful.

CHAPTER 7

Working with algebra

Starter: Right or wrong?

The starter connects with the theme of BIDMAS from Chapter 6. It is designed to highlight any remaining misconceptions about the order in which arithmetic processes are carried out, before formulae and algebraic descriptions are developed.

Exercise 7.1

Substituting numerical values into simple formulae looks quite basic, but students need to keep an eye on the units. Special care is needed with compound measures (e.g. speed), so that in 2d) the speed is km/hour but the time is in minutes. In such questions it is always wise to convert units first, before applying the formula.

Exercise 7.2

Watch the details here: many students will instinctively write $n - 4$ for 1c), when it actually wants $4 - n$.

Exercise 7.3

There is plenty of BIDMAS practice here. In particular, watch for expressions like $2x^2$, where the square must be done before the ×2. Question 2 parts a) and b) help reinforce this point.

W *Worksheet 7.3A*

This introduces Exercise 7.3 by building on previous arithmetical processes. The numbers are linked to letters of the alphabet which in turn are related to an arithmetical expression which the student has to work out.

The exercise has the interest of involving coded messages and leads into the evaluation of some simple algebraic expressions on Worksheet 7.3B.

W *Worksheet 7.3B*

This introduces simple algebraic expressions after the arithmetical ones of Worksheet 7.3A, and fits in with work in Exercise 7.3, questions 1 to 5.

The student puts the value of the letter into an algebraic expression to calculate its value. This value is then represented by a letter which leads to a message being constructed. This exercise is a useful way for lower ability students to explore the processes of algebra.

Exercise 7.4

When all the terms are positive, collecting like terms is comparatively easy. If there are some minus signs as well, then students should remember that each sign belongs to the piece of algebra that comes immediately after it, so $3h^2 - 2h^2 - 2h^2 + 4h^2$ may be re-ordered as $3h^2 + 4h^2 - 2h^2 - 2h^2$, for example. Some students find it helpful to be shown the terms, with their signs, written on separate cards – so that as you re-order the terms, the signs move with them.

W *Worksheet 7.4*

This worksheet forms a structured beginning to the work on simplifying expressions in Exercise 7.4 with questions in the style of those in KS3 SATs tests and at GCSE.

The idea developed in the work is that like terms can be combined, whereas unlike terms cannot. The initial work to understand the idea is through arithmetical problems which are relatively straightforward: students build up blocks by adding together algebraic terms.

Exercise 7.5

This exercise builds on the ideas of Chapter 6, Exercise 6.6. Watch for the answer to 2i) which many students may give as x^1; while this is correct, you may want to remind them that the index 1 does not need to be written.

Exercise 7.6

Watch carefully to ensure that students are able to answer the later parts of questions 1 and 2 correctly (where the bracket is multiplied by a negative number). This is an essential skill students need to tackle the harder parts of question 4 successfully.

Exercise 7.7

The grid method works quite nicely for those who feel uncomfortable with FOIL; for either method, the signs must be watched carefully. In question 5 you may need to remind the class that $(a + 3)^2$ is not $a^2 + 9$ (though question 4 aims to make this point).

W *Worksheet 7.7*

Though challenging, this exercise enables students to ease into dealing with both algebra and the idea of using brackets by making use of a diagrammatic approach, often helpful for students who find these areas difficult.

This preliminary work may give uncertain students access to a range of questions in Exercise 7.7. Relating the algebraic expressions to the length and width of a rectangle gives them something concrete to think about whilst processing the terms. Rectangles made sufficiently large will allow working to be contained within them.

The end questions enable some simple quadratic expressions to be introduced in a similar way.

Exercise 7.8

At Higher as well as Foundation tier, many students have a poor understanding of the distinction between equations and identities. This exercise emphasises the difference between them, and also provides further practice in multiplying out a perfect square bracket.

Exercise 7.9

The instruction 'factorise' can mean several different things; in this exercise it means 'take out a common factor'. Make sure that your students do this fully. For example, in question 2a) an answer of $2(4a - 6)$ would be considered only partially correct.

Exercise 7.10

Students find this topic surprisingly difficult. Some find it helpful to replace the symbols with simple numerical values and solve the problem by arithmetic, then to look at the steps: Did they add, multiply etc? Then they can go back and apply the same steps to the symbols as they did to the numerical values.

Exercise 7.11

An alternative method is the reverse flow diagram, which some students like and others hate! For question 2a), for example, it would work like this:

What do I do to turn x into y?
Start with x
Multiply by 2
Add 1
End up with y

Now write the inverse instructions alongside these:

Start with x	End with x
Multiply by 2	Divide by 2
Add 1	Subtract 1
End up with y	Start with y

Finally, read the new instructions in reverse order (bottom to top) to get:

$$x = \frac{y - 1}{2}$$

Of course, this method only works provided the new subject, x, appears only once on the right-hand side.

Internet Challenge 7

A speed-up sheet is available for this challenge.

Algebra is the language of mathematics, so it is a good idea to check the meaning of certain key words here.

You may want to emphasise the meaning of 'expand' (= get rid of the brackets) and 'factorise' (which can mean many things, but usually in algebra = put brackets in, i.e. reverse of expand). Sometimes students will meet two of these words together, e.g. 'expand and simplify' (= get rid of brackets and collect up like terms).

CHAPTER 8
Equations

Starter: Triangular arithmagons

A speed-up sheet is available for this starter.

The last arithmagon is the hardest of them all. Students will probably find a solution by trial and improvement (i.e. experimentation), but you could show them how the problem can be built up by writing x in one of the circles, to generate a simple linear equation. Alternatively, come back and show them this when they have covered more of the material in the chapter.

Exercise 8.1

This introductory exercise revises vocabulary from the previous chapter, but also trains the eye to look for the key characteristics of an equation in preparation for the following work.

Exercise 8.2

Despite careful demonstrations of the balance method, some students will simply write down the answer by inspection, with no working. Explain to them that this is discouraged because they need to develop and practise formal methods in order to solve harder problems where the answer is not obvious by sight. Furthermore, a wrong answer with no working will score no marks at GCSE, whereas a wrong answer with sensible working may score most of the marks.

Exercise 8.3

Here are those harder problems! In each question the trick is to solve, for example, for $3a$ first, then divide by 3 to get the value of a. As a general rule, the questions in this exercise require additions or subtractions first (to collect like terms), then multiplications or divisions at the end (to find the value of the letter). Some students will instinctively try to do the multiplication or division first – perhaps because they have been overly successful in learning BIDMAS!

Exercise 8.4

All these problems could be done by dividing away the common factor as the first step. This is a good method for teachers' own use, but is likely to confuse the students, who are far more likely to feel comfortable with the multiplying out approach shown in the example.

Exercise 8.5

No new skills here, but these problems are much longer than the simpler ones that the students have solved earlier in the chapter. If they find it heavy going, you might want to shorten the exercise and come back to it again later in the course.

Exercise 8.6

The quadratic equations in this section can all be reduced to the form $x^2 =$ a number, so they are quite easy to solve. All the answers are exact, but do encourage the students to give both positive and negative options.

Exercise 8.7

'Mystery number' problems are great fun, and are used here to practise the techniques from earlier in the chapter. Students who finish ahead of the class could make up their own 'mystery number' problems and try them out on a friend. Don't let them make the problems too hard; the idea is that someone else should be able to solve them!

Exercise 8.8

▶▶ A speed-up sheet is available for this exercise.

If you have computers available, consider doing this exercise with a spreadsheet such as Excel. Of course, the students need to know how to do the questions using a calculator too, so you could get them to go on and solve one or two of these to 2 decimal places as a calculator homework.

Exercise 8.9

▶▶ A speed-up sheet is available for question 2 of this exercise.

Some students find the $>$ and $<$ signs intimidating, but the 'thermometer diagrams' for inequalities go a long way towards making the whole topic more visual and user-friendly. Solving inequalities should be no more difficult than solving the corresponding equations, which in this exercise are not too daunting at all.

Internet Challenge 8

Gauss worked in a huge number of fields, including algebra; much of his work is central to topics studied by sixth-form mathematicians nowadays. He was also a fine geometer. This challenge aims to give a flavour of his versatility, as well as his phenomenal output of original work.

CHAPTER 9
Number sequences

Starter: Circles, lines and regions

▶▶ A speed-up sheet is available for this starter.

This starter generates three interesting sequences – the integers 1, 2, 3, 4, …, the triangular numbers (0), 1, 3, 6, … and a third sequence that begins 1, 2, 4, 8, 16. Since the doubling pattern seems so strong, students may be surprised to find that the next term is not 32 (it is actually 31, then 57). Beware of jumping to conclusions too quickly!

Exercise 9.1

▶▶ A speed-up sheet is available for questions 1 to 5 of this exercise.

The GCSE may ask students to extend a given pattern of numbers by looking at diagrams. Notice that these are all linear – the terms increase by a constant amount each time – so the pattern is quite easy to spot and extend.

W *Worksheet 9.1*

This worksheet can be used as an introduction to number sequences, possibly prior to Exercise 9.1.

Students start by creating a number pattern using the familiar 4 times and 5 times tables: they generate the actual numbers and can also appreciate the nature of the sequence in a visual form.

They then build another number sequence from the original one, going (in algebraic terms) from $4n$ to $4n + 1$ and from $5n$ to $5n + 1$. At this stage, however, they are unlikely to recognise the algebraic results.

Elicit observations based on looking either at the number sequence or at the pattern formed in the coloured circles. This should prompt discussion about the findings, which are then developed further through Exercise 9.1.

The work continues with a different starting point, where students are asked to investigate along (by now) more familiar lines. Thus, $6n$ and $6n + 1$, and $3n$ and $3n + 1$, are introduced.

The worksheet ends with an open question inviting further study.

Exercise 9.2

This exercise contains a mixture of linear and non-linear sequences. You could ask students to go back over the exercise at the end, and identify the sequences which go up (or down) in equal steps, so that they learn to recognise linear sequences at sight.

Exercise 9.3

Again, this is a mixture of linear and non-linear sequences. At the end you could pick out the characteristics of a formula that matches a linear sequence, $an + b$, in preparation for Section 9.4.

W *Worksheet 9.3/9.4*

This worksheet can be used to introduce Exercises 9.3 and 9.4.

The familiar 2 times and 3 times table are developed using a rule to produce a sequence of numbers.

The arrangement of the 2 times table should allow students to see a pattern progressing in 2×1, 2×2, 2×3 etc., and to appreciate that the '2' remains constant, whereas the second number is a variable. By replacing this second number with the letter n (for number) pupils may arrive at '$2n$'. Further, the rule extends this $2n$ to $2n + 1$, and subsequently to the formula: Value = $2n + 1$.

Students then create a 'vertical-line' graph that shows them that the values increase consistently, and also that, when the highest points (the crosses) are joined up, a straight line is produced.

This approach can be used to make a formula for some of the sequences in Exercise 9.4, by starting with a simple multiplication table and then trying to adjust it by addition or subtraction to create the given sequence.

Exercise 9.4

Finding an expression for the nth term works well if students build up their formula in a logical and disciplined way, as shown in the student textook. Watch for the very common error of thinking that 3, 5, 7, 9, 11… is somehow described by '$n + 2$' when this sequence really means '$2n$ plus something'.

Exercise 9.5

A speed-up sheet is available for questions 2 and 3 of this exercise.

Although many examination questions are about linear sequences, students will meet others, too. This exercise runs through the most common non-linear sequences, including square numbers, triangular numbers and even numbers.

Worksheet 9.5

This worksheet is an easy way into the work of Exercise 9.5, introducing the theme of number sequences through gradual stages. Students generate a sequence of shapes from a single square box, before working on a sequence of double, then triple, rows of boxes.

For each sequence, students record the number of squares and the number of outside edges of each shape, to identify a number sequence or pattern. From this information, they describe the sequences in words. The objective of question 10 is to discover whether there can be the same number of squares as outside edges: this works for triple-box Shape 6.

As an extension, students should be encouraged to make a generalisation for each table of values, and possibly come up with an algebraic relationship.

Review Exercise 9

A speed-up sheet is available for questions 8, 11, 12 and 13 of this exercise.

Worksheet 9R

This can be used as part of the Review Exercise, to help students become more familiar with the terms related to number sequences by using them in the context of sentences. It will greatly help students to unscramble the terms in this exercise if, during progress through the chapter, the terms have been drawn attention to and discussed, and if students have been asked to make a list of the words and possibly their page references.

Internet Challenge 9

You might wish to extend this challenge into a set of lessons on Fibonacci numbers; there is a huge amount of varied material on the internet. Students could conduct their own research and present their findings to the class, perhaps in a short talk or a poster.

CHAPTER 10

Coordinates and graphs

Starter: Matchstick puzzles

The matchstick puzzles are here to stimulate spatial thinking prior to the chapter on coordinates; they also provide some light relief from the algebra of the preceding chapters. The first three puzzles should be quite straightforward to solve; the fourth is devious, requiring that you move the right-hand match a tiny amount to the right, so forming a small square gap between all four matches at the centre of the cross.

Exercise 10.1

A speed-up sheet is available for questions 4 to 6 of this exercise.

Much of the exercise will be revision, though the idea of working out the midpoint of a line segment will be new. If any student has difficulty remembering which is x and which is y, you could offer one of several memory aids, such as:

- Go along the street then up the building.
- Here comes y; reach for the sky.

Exercise 10.2

A speed-up sheet is available for questions 3 to 6 of this exercise.

The exercise gives further practice of the previous ideas, but on a grid in which x and y may now be negative. When both coordinates are negative, students should have no problem, but when one is positive and one negative they need to concentrate on plotting the point in the correct quadrant.

Exercise 10.3

Coordinates in 3-D are a natural extension of 2-D, and should not pose any new conceptual challenges. Do encourage your students to read the axes carefully, however, to make sure that they give their x, y and z coordinates in the right order. (Some books and teachers use x and y as ground-level coordinates, and z for height above or below the x–y plane.)

Exercise 10.4

▶▶ A speed-up sheet is available for questions 2 to 13 of this exercise.

Plotting of linear graphs follows on nicely from the work on linear number patterns covered in Chapter 9. Although teachers find this topic easy, many students will need a lot of practice.

Exercise 10.5

This exercise is on finding the gradient of a line by drawing a triangle underneath the line. Make sure students don't get into the bad habit of simply counting squares – they must look at the scales on both axes.

Exercise 10.6

Students also need to be able to write down the gradient of the line from its equation. The exercise gives some practice on this, and also introduces the idea of finding the equation of a line from its gradient and y intercept.

Exercise 10.7

▶▶ A speed-up sheet is available for this exercise.

Students only need to be able to solve simultaneous equations graphically. Remind students not using the speed-up sheets that they may need to extend their lines in order to find out where the lines cross.

Exercise 10.8

▶▶ A speed-up sheet is available for questions 1 to 6 of this exercise.

The textbook warns students not to join the points with a ruler; a freehand curve is required. It can be very helpful to turn your exercise book (or question paper) so that your hand is inside the curvature of the points; the wrist then acts as a natural compass and a smooth curve is quite easily drawn.

When questions contain terms like $2x^2$, do check that students are doing the squaring first, then the doubling.

Review Exercise 10

▶▶ A speed-up sheet is available for questions 2, 3, 6, 7, 8, 10 and 12 to 15 of this exercise.

Internet Challenge 10

We use the word 'parallel' in all sorts of everyday ways, as well as in mathematics. As an addition to question 5, you can ask: What is the 38th parallel?

CHAPTER 11
Measurements

Starter: Scales

Reading scales is still an important skill, despite the digital age in which we now live. This starter gives some warm-up practice.

W *Worksheets 11S.1 and 11S.2*

Worksheet 11S.1 is a light-hearted exercise in measuring lines, and Worksheet 11S.2 puts the reading of scales in a practical context.

Exercise 11.1

It has been said that because many commuters do not understand how to read a transport timetable, they fail to notice when trains are late! The key here is to realise that each vertical column of the table describes one train (or bus).

Make sure that students are writing am or pm when converting 24-hour clock times into 12-hour clock times.

W *Worksheet 11.1*

Students can do this worksheet before trying Exercise 11.1 questions 1 and 2. Times are given, rather than being read from a clock, and questions provide easy practice in various time formats.

Exercise 11.2

The metric system has been designed so that sub-units work in multiples of 1000: 1000 grams = 1 kilogram, 1000 mm = 1 m and so on. Of course, the conversion that students will have had greatest familiarity with is 100 cm = 1 m, and this can easily trigger slips with the other units. Be watchful.

Problems involving conversion of square units require the conversion factor to be applied repeatedly – so there are $100 \times 100 = 10\,000\ \text{cm}^2$ in $1\ \text{m}^2$. Emphasise this point as much as you can because this topic is usually answered very poorly in the GCSE examination.

W *Worksheet 11.2*

This can be worked on after question 1 of Exercise 11.2 is covered.

Exercise 11.3

The conversions here are quite simple to carry out, but students will be expected to learn the approximate equivalents given in the textbook for pounds, inches, feet, miles, pints and gallons.

W *Worksheet 11.3*

Use this before Exercise 11.3 question 1 to introduce a mixture of metric and imperial units and their abbreviations.

Exercise 11.4

The cover-up triangle is a great tool for teaching any compound measure; it effectively allows students to learn three formulae for the price of one. Point out that students must check compatibility of units first, though – it is no use multiplying a speed in km/h by a time in minutes.

W *Worksheet 11.4/11.5*

This takes students through the requirements of questions in Exercise 11.4, and question 6 of the worksheet is a lead-in for Exercise 11.5 questions.

Exercise 11.5

Again the cover-up triangle is a very useful device. Watch the units in question 3, where the students must use the fact that 1 $m^3 = 100 \times 100 \times 100 = 1\,000\,000$ cm^3.

Exercise 11.6

For an amount like 63 kg to the nearest kg, the bounds are always symmetric: $62.5 \leqslant$ value < 63.5. Strictly speaking it is meaningless to ask for the 'greatest value' – there isn't one. But some exam questions might ask this anyway. If so, you are simply expected to state the upper bound, namely 63.5. (Some students are unhappy with this, and try to write 63.499, but any number below 63.5 is always going to be too small.)

W *Worksheet 11.6*

As an alternative to Exercise 11.6, this worksheet reinforces rounding to the nearest whole number.

Review Exercise 11

▶▶ A speed-up sheet is available for question 3 of this exercise.

Internet Challenge 11

Students will enjoy comparing the speeds of real objects to see if their judgement was correct. It is a good idea to choose a system of units beforehand – miles per hour or kilometres per hour – and try to track down all the answers in that same system of units.

CHAPTER 12
Interpreting graphs

Starter: Lower and lower

▶▶ A speed-up sheet is available for this starter.

The idea of cross-section is related to the rate of change on the corresponding graph. Ask your students how they think a container with two different cross-sections might differ in behaviour from a container with only one.

Exercise 12.1

▶▶ A speed-up sheet is available for questions 1 to 5 of this exercise.

Most of the graphs in this exercise are linear; you might want to emphasise to your students that when things change at a constant rate, the corresponding graph will always be a straight line. As an example, there was a time when travel agents would charge a standing fee plus a percentage commission for money exchanges – resulting in a straight-line conversion graph that did not pass through the origin. Nowadays it is usual to charge commission only, so now the graph would pass through the origin.

Exercise 12.2

▶▶ A speed-up sheet is available for questions 1 to 9 of this exercise.

The simple scaling method for average speed shown in the example on paper 262–3 is often better than a formal division: if students try to tackle 6 km in 90 minutes by working out 6 divided by 90 on a calculator, then they are heading for difficulties with mismatched units.

Exam questions on this topic will always feature straight-line segments only, not curves.

Review Exercise 12

▶▶ A speed-up sheet is available for questions 4, 5 and 8 of this exercise.

Internet Challenge 12

Circles, ellipses and spirals occur in many real-life situations; this challenge reminds us that there are many other, less well-known curves too.

CHAPTER 13
Angles

Starter: Measuring angles

The starter provides some useful practice in measuring angles before students move on to a chapter about calculating them. Exercise 13.1 will include deciding whether each angle in the starter is acute, obtuse or reflex; you could bring that idea forward and discuss it as part of the starter instead.

W *Worksheet 13S*

Use this with the starter exercise. Questions 11 to 20 are quite a bit more challenging, involving two stages of calculation, so can be used or not depending on the level of your students.

Exercise 13.1

Make sure that students write down a reason for each answer in question 2. This is very important because GCSE questions will often be worth two marks per angle – one mark for the value, and the other mark for a brief reason, which should be stated precisely:

- Angles on a line add up to 180º
- Angles at a point add up to 360°
- Angles in a triangle add up to 180°
- Angles in a quadrilateral add up to 360°

W *Worksheet 13.1*

The crossnumber includes examples for all the facts covered in Section 13.1 and should reinforce them when used before or after Exercise 13.1.

Exercise 13.2

In this exercise, students need to use angles summing up to 180° (triangle) or 360° (quadrilateral), and then solve the corresponding equation. The individual steps are not too hard, but many students find this whole topic surprisingly difficult. Encourage them to break the solutions up into bite-size pieces, and to show their working at each stage.

Exercise 13.3

Careful application of the angle properties should lead to successful solutions here, but do watch for misidentifications of the complementary angles – e.g. writing 55° when 125° is meant. Such errors can easily be eliminated if students are urged to consider whether the final answer has to be acute or obtuse.

Once again, reasons may be asked for in the examination, and should be stated briefly but precisely:
- Vertically opposite angles are equal
- Alternate angles are equal
- Corresponding angles are equal
- Allied angles add up to 180°

Exercise 13.4

A speed-up sheet is available for questions 8 and 9 of this exercise.

The proof that angles in a quadrilateral add up to 360° is worth studying with your students; it emphasises the point that adding an extra side increases the angle sum by another 180°. This is an easy way of remembering the angle sums for pentagons, hexagons etc.

Questions about internal angles of regular polygons can be solved quite easily by working out the external angles first, using the principle that they always sum to 360°.

Exercise 13.5

A speed-up sheet is available for question 1 of this exercise.

Bearings are surprisingly difficult for many students, and you may need to give your class extra practice in drawing bearings before going on to measuring them. Errors of 180° are reasonably common, and may be avoided by estimating the value of a bearing prior to measuring it.

Review Exercise 13

A speed-up sheet is available for questions 9, 14 and 16 of this exercise.

Internet Challenge 13

A speed-up sheet is available for this challenge.

The four-colour theorem is well documented on the internet, so most answers should be quite easy to track down. In practice, a real mapmaker might want to colour all territories belonging to the same country in the same colour – so Alaska, which does not touch the rest of the USA, might still need to be the same colour. These kinds of considerations can prevent four colours from being sufficient.

CHAPTER 14
2-D and 3-D shapes

Starter: Inside and all round

A speed-up sheet is available for questions 3 and 4 of the starter.

The perimeters and areas in the starter can be obtained simply by counting methods. Note that question 2 deliberately avoids asking for the perimeter of the triangles since the diagonal lines are longer than 1 unit; you might wish to point this out and discuss with your class, either now or later in the chapter.

Exercise 14.1

The class might enjoy making a poster about properties of quadrilaterals as a way of summarising the work in this exercise. Note that it is possible for a shape to belong to more than on special category; for example, a square is also a special type of rectangle (and a parallelogram, and a rhombus….)

Worksheet 14.1

Polly continues work on naming shapes including triangle types and quadrilaterals. The worksheet also covers polygons, which were dealt with in Exercise 13.4.

Exercise 14.2

At this stage, students usually deal well with perimeters and areas of shapes made from rectangles. The formula for the area of a triangle can cause some confusion – is it half the base or half the height, or both? You might want to show your students that $\frac{1}{2} \times (16 \times 6)$, $(\frac{1}{2} \times 16) \times 6$ and $16 \times (\frac{1}{2} \times 6)$ all give the same answer, but $(\frac{1}{2} \times 16) \times (\frac{1}{2} \times 6)$ does not.

Worksheet 14.2/14.5

The worksheet covers the calculation of areas and perimeter of rectangles (Section 14.2) and of compound shapes (Section 14.5): the shapes are easier than those in Exercise 14.5.

Exercise 14.3

This exercise combines two topics (perimeter of a shape and simple equations) into one. GCSE candidates find this combination surprisingly hard, and you may well need to provide your students with some extra practice questions. (They are, in effect, using algebra to model a real-world situation, and have not had much previous experience of doing this.)

Exercise 14.4

The formula for the area of a trapezium is given in the examination, but some students find it awkward to use at first. You could, if necessary, introduce it in a verbal way first – 'Find the average of the two parallel sides, then multiply this by the width between them' – and move on to the algebraic form once students are able to do the calculations confidently.

Exercise 14.5
Compound shapes are great fun! While many of these problems will be done by adding up the areas of two or three smaller shapes, some, like 2f), are better done by starting with an oversize rectangle and subtracting the four corner parts.

Exercise 14.6
The formula for the volume of a prism will be given in the examination. Some students like to think of cubes and cuboids as being special types of prism, so you need only one formula to cover all these shapes.

Review Exercise 14
▶▶ A speed-up sheet available for question 8 of this exercise.

Internet Challenge 14
This challenge finishes with two puzzles about counting triangles. Students may find it helpful to number (or label) the different regions within each diagram, to help keep track of which ones are being used.

CHAPTER 15
Circles and cylinders

Starter: Three and a bit…
This starter is intended to get the students used to working with approximations to π; it also highlights some interesting aspects of how to use brackets, roots and power keys on a calculator.

Exercise 15.1
These questions are all quite short; they should not cause conceptual difficulties, but wrong answers may arise from two common slips:

- Selecting the wrong formula (πr^2 instead of $2\pi r$ or vice versa)
- Selecting the diameter instead of the radius (or vice versa)

Stronger students will enjoy working with both area and circumference problems as presented in the textbook. Weaker students might benefit from solving an extra exercise just on circumference, then another just on area, before having to deal with both together.

Exercise 15.2
Many of these questions pick up on the idea of compound areas from the previous chapter, but use circular sections instead of (or as well as) rectangles or triangles. Students should decide carefully at the start whether they are going to add up or subtract various parts to get the required answer.

Exercise 15.3
Although the term 'sector' is used here, GCSE questions at Foundation Tier will be based on semicircles and quadrants; students need to know the definition of a sector, but not its general formula.

Exercise 15.4
If you have an interactive whiteboard you could set up an Excel spreadsheet to work out the circumference of a circle, then use Excel's 'Goal Seek' to work backwards, a nice way of checking students' answers.

Exercise 15.5
Although the cylinder formulae are not given in the GCSE examination, a cylinder is a special type of prism – and the prism formula is given. You may need to remind students that the area formula is only for the curved surface area; to find the total surface area of a cylinder, two circular ends must also be added.

Exercise 15.6
Watch to make sure that students simplify their answers fully: you may see $\pi 5^2$ rather than 25π, for example.

Internet Challenge 15
The Earth is approximately (but not exactly) spherical, so questions 3 and 4 allow the use of the formula for the circumference of a circle to be used to find the distance all the way round the world.

We perhaps imagine that people who travel 'round the world' do so by following a longest possible route, i.e. a Great Circle, but actually only astronauts can do this! The route followed by a UK round-the-world sailor, for example, actually comprises a long journey south, a short (but difficult) circuit around the southern ocean, and then a long northerly return home.

CHAPTER 16
Pythagoras' theorem

Starter: Finding squares and square roots on your calculator
Some Pythagoras questions will be rigged to make the answers come out exactly, while others require the use of a calculator and subsequent rounding. The starter prepares the ground for this by looking at what happens when you try to find the square roots of different numbers.

Exercise 16.1
This chapter takes an unusual slant on introducing Pythagoras' Theorem as a way of checking whether or not a given triangle is right angled. This means that the

calculations are very simple, as no square rooting is necessary. By the time students also have to do this (in the next exercise), they should be well versed in quoting and applying the theorem.

Exercise 16.2

All the problems in this exercise require squaring and adding, since the hypotenuse is being found in each case. You may wish to remind students at this stage that the longest side in a right angled triangle is always opposite to the right angle.

Exercise 16.3

All the problems in this exercise require squaring and subtracting, since the hypotenuse is given in each case. Students should check that their answer for the missing short side is actually shorter than the given hypotenuse.

Review Exercise 16

In the examination, candidates will need to decide whether to square and add or square and subtract. This exercise provides practice at recognising the difference between these two types of Pythagoras problems.

Internet Challenge 16

This challenge aims to draw out some of the richness of Pythagorean triples. It is to be hoped that students will at least come away having learned that 3, 4, 5 and 5, 12, 13 are two common triples that, together with their simple multiples, often feature in examination questions.

CHAPTER 17

Transformations

Starter: Monkey business

Seven of the monkeys are congruent; the exceptions are E (too fat) and I (too thin). Note that H is congruent to six other monkeys, though it is a reversed (mirror image) version of the other six.

Exercise 17.1

▶▶ Speed-up sheets are available for questions 1 to 4 of this exercise.

A line (or plane) of symmetry divides an object into two congruent parts. You might want to emphasise the difference between two important mathematical words at this stage:

Congruent = same shape and same size
Similar = same shape, but one may be larger or smaller than the other

Exercise 17.2

▶▶ Speed-up sheets are available for questions 1 to 12 of this exercise.

In this exercise on constructing reflected images, the mirror lines may be vertical, horizontal or diagonal. It is always a good idea to rotate the book, exam paper or whatever so that the mirror line is vertical; mistakes are then far less likely.

Exercise 17.3

▶▶ Speed-up sheets are available for questions 1 to 6 of this exercise.

Although this is a straightforward exercise on translation, some students find this topic rather abstract, and are happier using tracing paper. This is allowed in the examination, and should be encouraged in this chapter whenever necessary.

Exercise 17.4

▶▶ A speed-up sheet is available for questions 1 to 3 of this exercise.

Most students will find little difficulty with this topic. Do watch out for shapes with no symmetry at all; they are not order 0, as some students might think, but order 1.

Exercise 17.5

▶▶ Speed-up sheets are available for questions 1 to 6 of this exercise.

Rotations about points other than the origin are quite difficult to visualise. Use of tracing paper is to be encouraged, and the students can then stab the centre of rotation with a pencil or compass prior to rotating the tracing.

Exercise 17.6

▶▶ Speed-up sheets are available for questions 1 to 6 of this exercise.

These questions carry a good number of marks in the examination, and should prove quite accessible to most students. Errors are usually caused by their misreading instructions, or trying to do too much at once. With care, there are easy marks to be had here.

Exercise 17.7

▶▶ Speed-up sheets are available for questions 1 to 4 of this exercise.

There are two types of enlargement problems at GCSE (see also Exercise 17.8). Sometimes, as here, the candidates merely have to make the shape bigger on a grid, without worrying about specifying a centre of enlargement. Students normally find this type of enlargement quite simple.

Questions 12 and 13 of this exercise illustrate the relationship between enlargement and perimeter, area and volume. Students are not expected to know that the area increases by the square of the scale factor or the volume by the cube of the scale factor. However, they need to understand the terms similar and congruent.

Exercise 17.8

▶▶ Speed-up sheets are available for questions 1 to 4 of this exercise.

Here is the harder type of enlargement, when a centre is specified. Strictly speaking, such problems should be done by a 'ray' method as shown in the textbook, though this can be fiddly in practice. Stronger students might like to count the number of squares across and up from the centre, then multiply this by the enlargement factor, a kind of vector method (although they won't call it that) to determine the position of the image; rays can then be traced back as a check.

Review Exercise 17

Speed-up sheets are available for questions 2, 5 to 13 and 15 of this exercise.

Internet Challenge 17

A speed-up sheet is available for this challenge.

The chapter contains a lot of specialist geometrical vocabulary; the Internet Challenge builds on this. Some of these definitions are central to the Foundation GCSE, while others are provided as enrichment.

CHAPTER 18
Constructions

Starter: Round and round in circles

The circles starter is here to give some practice in compass technique. Some students will hold the compass by the pencil-carrying arm rather than the grip provided at the top; this will tend to throw the radius out of alignment as the circle is drawn. The starter requires that all seven circles are drawn with the same radius – a good check on the students' motor skills!

As an aside, the two coloured examples in the textbook show exactly the same mathematical diagram, but our visualisation of it changes dramatically when the colour palette is changed.

Exercise 18.1

Speed-up sheets are available for questions 1 to 5 of this exercise.

Practice in these various constructions is necessary for two reasons – to recognise the different types, and to develop accuracy. GCSE examiners use an overlay and will allow a tolerance of something like ± 2 mm in the accuracy of the drawing.

You may prefer either to use a speed-up sheet here, or to get students to draw the triangles on blank paper. Exam questions normally give one printed line segment to start the solution off.

Exercise 18.2

Speed-up sheets are available for questions 1 to 9 of this exercise.

At Foundation level, students need to know how to construct:

1. an angle bisector
2. the perpendicular bisector of a line
3. a perpendicular from a point on a line
4. a perpendicular from a point to a line.

The exam would normally provide the printed angle or line segment, so a speed-up sheet is a good idea.

As an extension activity for quicker students, you could get them to draw a triangle of their choice, then construct the three angle bisectors; these should all meet at a single point (the incentre). Likewise they could construct the perpendicular bisectors of the three sides; these also meet at a single point (the circumcentre).

Exercise 18.3

Speed-up sheets are available for questions 1 to 14 of this exercise.

This exercise requires a combination of mathematical understanding and skilful drawing. Make sure that pencils are sharp, and discourage students from using ink, since corrections are then almost impossible to make.

A speed-up sheet should be used: in the examination, locus problems will usually ask students to draw a required locus over the top of a given diagram.

Exercise 18.4

A speed-up sheet is available for question 11 of this exercise.

Some students find this topic very hard. You could use centimetre cubes and get them to build simple rectangles, L-shapes etc., then draw the result and show the drawing to a friend who has to describe the original object.

Students who study Graphic Design will be at a significant advantage here, since isometric paper is regularly used in that subject. Maybe they can help their less confident friends!

Review Exercise 18

Speed-up sheets are available for questions 1, 4 to 12 and 14 to 22 of this exercise.

Internet Challenge 18

The Platonic solids provide a nice way of rounding off this topic; you could develop this challenge into a classroom activity. If you have a classroom kit of interlocking plastic polygons (Jovo or Polydron, for example) then your students could make nets of all the Platonic solids quite quickly, and fold them up to check that they are right.

CHAPTER 19
Collecting data

Starter: Tally ho!

A speed-up sheet is available for this starter.

A tally chart is very simple indeed – but there is an important point to be made here: tallying does not have much value if the groups are too numerous (Myra) or too few (Enrico). You need a manageable number of groups, and they should be wide enough to score several hits in most of them (Sunita).

Exercise 19.1

This exercise looks at ways in which data might have been collected in an unfair way.

Sometimes the questions in a survey might, themselves, be unfair: 'Don't you agree that…' is a bad way to start a question, since it implies a preferred response. This is called a leading question.

Even good questions might produce an unfair result if they are asked to a sample that does not reflect the population in a fair way; for example, the first 50 children to arrive at school are probably not typical of the school as a whole. This is called a biased sample.

Exercise 19.2

The way in which data are processed will depend on whether they are categorical, discrete or continuous, so it is important that students can recognise the difference – which is precisely the goal of this exercise.

Exercise 19.3

▶▶ A speed-up sheet is available for questions 2 to 5 of this exercise.

Tally charts are a good way of building up frequency tables. If your students find this very easy, then you could show them how to calculate the mean from a frequency table; this is developed in the next chapter of the book (Exercise 20.4).

Exercise 19.4

▶▶ A speed-up sheet is available for questions 1 to 3 of this exercise.

Two-way tables are a useful way of organising certain types of data. Usually totals are given at the end of each row and each column; your students should check that both sets of totals add up to the same overall total number of items.

Review Exercise 19

▶▶ A speed-up sheet is available for questions 1, 2, 5 and 11 of this exercise.

Internet Challenge 19

Whether we like it or not, we are all statistics – there is a substantial penalty (question 3) for failing to submit a census return.

CHAPTER 20

Working with statistics

Starter: Sharing the cards

The ideas that lie behind mean, median and mode are introduced informally, in the guise of a number card puzzle. In the work that follows, all three of these will be used to describe the average value of a data set.

Exercise 20.1

The exercise provides plenty of practice at calculating mean, median and mode – these are all different ways of describing a 'typical' member of the data set.

Note that the mode identifies the most frequent item of data, but students must use the word 'most' with great care. Thus 'red is the most popular colour of car' is not the same as 'most people have red cars'; in the first statement, 'most' means 'more than any other colour' whereas in the second statement 'most' implies 'more than half'.

Exercise 20.2

Whereas the mean, median, and mode are all describing what a *typical* member of the data set might be like, the range is exploring how much *variety* there is amongst the data.

The range may not be particularly reliable, since its value is based on just two members of the data set, both of which are not typical, so an unusually high or low value can greatly affect the range. A better idea of the spread of the data set can be obtained by looking at a diagram of all the data points: this is done using the stem and leaf diagram in the next section.

Exercise 20.3

Stem and leaf diagrams are great fun – the students enjoy drawing them, and much useful statistics can be gleaned from them, too. You can immediately see whether the data are skewed or symmetric, for example, by turning the diagram on its side.

Some older textbooks might advise you to code 53 at 50 | 3 but this is now frowned on, since it could be interpreted as 503. Also, some books put commas between the leaves, but your students should not do this either. Finally, watch for vertical alignment of columns, which is very important: in the exam, it is a good idea for the students to draw thin vertical pencil lines as guides to help keep the diagram neat. They should always add a key.

Exercise 20.4

▶▶ A speed-up sheet is available for questions 1 to 3 of this exercise.

This exercise is all about finding the mean from a frequency table. Exam questions often provide a clue in the form of a spare column to the right of the given data. The method is standard; one reasonably common error is when students simply take the mean of the x values, ignoring the frequencies completely. Watch for this if anyone says they got an answer of 3.5 for question 1d).

Exercise 20.5

▶▶ A speed-up sheet is available for questions 1 to 4 of this exercise.

For grouped frequencies, the methods of the previous exercise are adapted so that students work with the midpoint of each interval; take care to check each midpoint value carefully. Exam questions often provide a clue in the form of two spare columns to the right of the given data.

Note that since the midpoint is being used instead of the original raw values, the figure obtained for the overall mean can be only an estimate of the true mean. In the GCSE exam, such questions usually contain the instruction, 'Calculate an estimate of the mean.'

The estimate could miss the true mean by a significant amount. For example, if you were told that 12 books cost between £0 and £10 each, and you used a midpoint of £5, there could be a large error, since many of the books might actually have cost £9.99.

Exercise 20.6

▶▶ A speed-up sheet is available for questions 2, 3, 4, 6 and 7 of this exercise.

Scatter diagrams are quite easy, and the examination will often ask students to add a few extra points to an existing graph. Care should be taken to read the scales carefully before plotting the points.

When drawing a line of best fit, a see-through ruler is a good idea; this makes it easier to judge when the points are balanced on either side of the line.

A nice extension, if there are not too many points, is to work out the mean of all the x values and the mean of all the y values, and use this to plot a 'mean point', which is marked on the graph with a distinctive symbol (an open circle, for example). You can then use this mean point as a pivot for the line of best fit. (The underlying mathematics, which the students do not need to know, is that the regression line of y on x always passes through the mean point.)

Review Exercise 20

▶▶ A speed-up sheet is available for questions 3, 4, 5 and 10 of this exercise.

Internet Challenge 20

▶▶ A speed-up sheet is available for this challenge.

It is convenient for us to look up the figures on the internet (secondary data), but how do countries establish their own population figures (how is the primary data collected)?

The 12 countries in the table are the twelve most populous, so they are not representative of the world as a whole; students would need to use a suitable method to ensure they were working with an unbiased sample. Stratified sampling (by continent) is one such method.

CHAPTER 21
Presenting data

Starter: Lies, damned lies and statistics

The starter is intended to show just how easy it is to present statistical diagrams in a misleading way. You might discuss what type of person would deliberately want to mislead in this manner – a politician's 'spin doctor' for example.

Exercise 21.1

▶▶ A speed-up sheet is available for questions 2, 5 and 6 of this exercise.

Pictograms and bar charts are good ways of displaying categorical data. (For numerical data there are more appropriate methods, as developed in Exercise 21.3 onwards.)

Exercise 21.2

▶▶ A speed-up sheet is available for question 7 of this exercise.

Pie charts are also used for categorical data. The essential teaching point is to compare the total number of items (e.g. 30) with the number of degrees in a full circle (360) and thus establish that each item is equivalent to a certain number of degrees (e.g. $360 \div 30 = 12$). Then multiply the frequencies by this number so that they now add up to 360, and thus can be used as the angles in degrees.

Pie chart questions carry quite a lot of marks in the exam. Students should make sure that each segment is clearly labelled, and also that they write the size of each angle in the centre of the circle.

Weaker students could be given some extra questions where they have to express percentages in pie chart form, and could use a percentage pie chart scale to do this (a percentage pie chart scale looks like a circular protractor but is labelled in percentages, so it has 100 ticks instead of 360) before returning to do more general pie chart questions.

Exercise 21.3

▶▶ A speed-up sheet is available for questions 1 to 4 of this exercise.

Vertical line graphs are used for discrete data, so students should not join the tops of the lines, as the intermediate values have no meaning.

Histograms are used for continuous data. At Foundation Tier GCSE all the class intervals will be of equal width, so students do not need to worry about working with frequency density: they can just plot frequency up the y

axis. Make sure they do not leave small gaps between adjacent columns!

Exercise 21.4

▶▶ A speed-up sheet is available for questions 1 to 3 of this exercise.

Students with a clear understanding of histograms should find frequency polygons quite straightforward: you can take a histogram and join the midpoints of the tops of the rectangles to obtain the corresponding frequency polygon.

'Why did we not join the tops of a vertical line graph, but now it's OK to join these points with a set of line segments?' Good question. The answer is that continuous data is now being displayed, so the intermediate values do have a meaning.

Exercise 21.5

▶▶ A speed-up sheet is available for questions 1 to 3 of this exercise.

Time series graphs are simply a plot of a variable quantity with time running along the *x* axis. Time series graphs can be misleading if the vertical scale does not extend all the way down to zero: look back to the Starter for an example of this.

Review Exercise 21

▶▶ A speed-up sheet is available for questions 1 to 3, 6 and 8 of this exercise.

Internet Challenge 21

Some of these quotes are reasonably well-known, while others are more obscure, but they all make interesting comments on how statistics may be used (or misused). Quotes 4 and 6 each refer to the 'average' – your students might want to discuss whether this is likely to refer to the mean, median or mode in each case.

CHAPTER 22
Probability

Starter: Fair game

If a ten-sided dice is not readily available, students could use a calculator's random number generator, which can automatically display a random decimal between 0 and 1 to three decimal places. Ignore the '0.' part and you immediately have a three-digit random number, though remember to count 0.62 as 620, for example. Just press the EXE (or =) key to obtain another number.

Exercise 22.1

▶▶ A speed-up sheet is available for questions 2 to 4 of this exercise.

Although percentages are used in common discussions about probability – 'There's a 10% chance of rain today' – you should encourage your students to express their probabilities as quantities between 0 and 1. Sometimes decimals are a natural way of doing this, while in other situations (dice throws, days of the week) fractions are more appropriate.

Exercise 22.2

▶▶ A speed-up sheet is available for questions 1, 2 and 4 of this exercise.

Some of the questions in this exercise are tricky, requiring clear and careful interpretation of the two-way table. For example, in question 3a) students need only to look down the 'Walk' column, and would give their probability out of 50, not 140. Actual exam questions at Foundation Tier are likely to be more straightforward calculations based on the overall total.

Exercise 22.3

▶▶ A speed-up sheet is available for questions 1, 3 and 5 of this exercise.

Some students may have met these diagrams at Key Stage 3, and perhaps know them as 'sample space diagrams'. This term is not needed at Foundation GCSE.

Exercise 22.4

A key principle here is that if you multiply a probability by the number of intended trials, then the result gives you a forecast of the likely frequency for that outcome (although the actual frequency will be a little different in most cases).

Many students do this intuitively, but you could give them a simple formula – n times p – if it helps.

Exercise 22.5

▶▶ A speed-up sheet is available for questions 7 and 12 of this exercise.

The vocabulary used in the introductory text – mutually exclusive, exhaustive – is used in the text to provide a framework for the students to understand why the numerical rules used in the examples actually work. Although these terms appear on the specification, they will not be used formally in the GCSE examination.

Review Exercise 22

▶▶ A speed-up sheet is available for questions 1, 3, 7, 8, 11, 14 and 15 of this exercise.

Internet Challenge 22

Although the examiner will not normally ask candidates to define the words used in probability theory, a sound knowledge of all the key vocabulary can help students to improve their understanding of this topic. The oft-quoted 'or means add' only applies when the outcomes are mutually exclusive; likewise, the rule 'and means multiply' only applies to independent events.