Geoboard

Solution notes

The tables below show the numbers of different triangles you can draw for different point circles. The highlighted row is explained in more detail below.

Looking at just odd numbers of points:

Points on circle	Distance between the base vertices of the family of triangles					
	1	**2**	**3**	**4**	**5**	**6**
7	3	1				
9	4	2	1			
11	5	3	2			
13	6	4	3	1		
15	7	5	4	2	1	
17	8	6	5	3	2	
19	9	7	6	4	3	1

Looking at just even numbers of points:

Points on circle	Distance between the base vertices of the family of triangles					
	1	**2**	**3**	**4**	**5**	**6**
6	2	1				
8	3	2				
10	4	3	1			
12	5	4	2	1		
14	6	5	3	2		
16	7	6	5	3	2	
18	8	7	6	4	3	1

Considering the highlighted row:

13	6	4	3	1		

Starting with a 13-point circle, the first family of triangles is produced by joining two adjacent points, leaving 11 free points that can be the third vertex of a triangle.

The symmetry of the circle means that of the 11 points remaining, 6 (= {11 + 1}/2) can be used to form unique triangles.

The next family of triangles is formed by joining two points that have one point between them (distance 2 units apart). This leaves 10 free points to form the third vertex of the triangle. In this case the symmetry means that there are 5 triangles that can be formed from the remaining points. However, one of them has been included in the first family because one of the new sides is formed by joining adjacent points; this reduces the total to 4.

The third family can be created in a similar way starting with a side formed by joining two points distance 3 units apart and leaving 9 free points. Because of the symmetry it is interesting to note that it is possible to draw 5 unique triangles in this family (the same as for the previous family). However, two of these triangles can be found in the previous two families, leaving 3 new triangles that are completely unique.

The pattern continues.

When dropping from an odd to an even number of free points the number of triangles reduces by 2, but from even to odd they reduce by 1.