Hexominoes

Planning ahead

Prerequisite knowledge

- No specific knowledge required

Why do this problem?

This is an opportunity to demand detail from pupils. While it is not the only problem in the book where this is possible, it can be focused on in this activity.

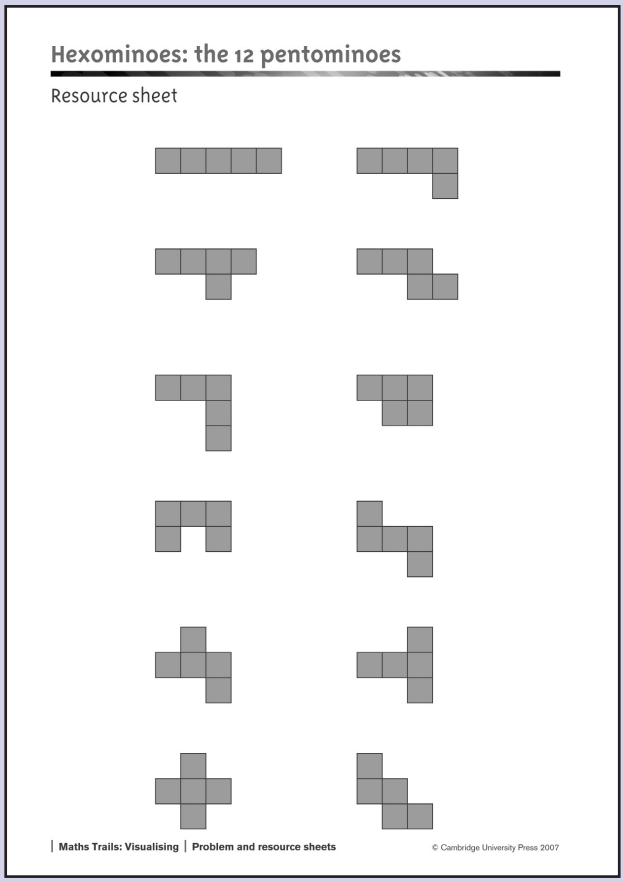
This problem also gives opportunities to pay particular attention to the analysis and synthesis, and planning, execution and interpretation phases of the problem-solving model.

Time

One or two lessons; you may wish to spend longer on the introductory part of the lesson by devoting a proportion of a precursor session or homework to it

Resources

Squares of paper (about 2.5 cm across); squared paper, coloured pens (optional)
CD-ROM: resource sheet; animation



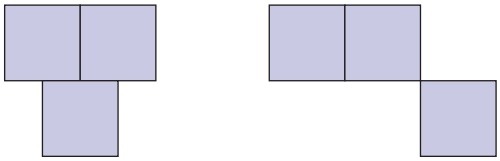
Introducing the problem

Begin with a demonstration and discussion using two of the small paper squares.

Ask the class to imagine two identical squares joined together so that they have a side in common.

- How many different shapes can you make like this? [only one, if you count 2-tall 1-wide as the same as 1-tall 2-wide]

Now introduce a third square. Say that squares have to join so that they have a whole side in common with no overlap, so the following are not allowed:



- How many unique shapes can you make now? [two, a long shape and an L-shape]

You may need to discuss what constitutes 'unique'. [new shapes cannot be created by rotating or flipping an existing shape]

Ask the class to draw on their squared paper all the possible pentominoes (shapes made from five squares joined edge to edge). Do not tell pupils how many there are. The aim is for them to think about how confident they are that they have found them all. The intention is not for them to complete the task at this stage.

Main part of the lesson

After a few minutes stop the class and ask how many pentominoes they have found. Walking round the class while they are working may reveal some points of interest; for example,

noticing duplicates of shapes they already have but in a slightly different orientation, or not adopting a systematic approach.

Indicate that you are going to stop them there and look instead at six square pieces (hexominoes) in a systematic way in the hope that this will help them to produce all the pentominoes confidently when they return to them later in the lesson.

Show the animation and ask the class to watch it very carefully: tell them that they will be asked to describe what happens in detail afterwards. (You might wish to show them the animation again after you hear the quality of the initial responses.) The aim is for pupils to be able to describe the animation in sufficient detail to someone who has not seen it before so that they can reproduce it themselves. When they have watched the complete animation, ask them about the colours of the squares, how they started and where they moved. Pupils should not write anything down; they should just focus on the animation.

Ask pupils to describe the animation using prompts such as:

- How did the animation start?
- What happened first?
- Then what happened?
- How did the square move?
- When did it change colour?
- What happened when the square returned to its starting point?
- Did other squares change colour?
- Where did the fifth square start?
- What happened when the fifth square had been around the other four squares once?

Show the animation again, telling pupils that they will be asked to reproduce the sequence of different hexominoes on squared paper when it is over. You will play the animation again towards the end of the lesson so they can check they have them all in the right order. They should focus on the animation as any attempt to just copy will mean they have not followed the organisation and so will not be able to reproduce the hexominoes.

Hand out enough paper squares so that each pair of pupils has six. Suggest that one of the pair moves the pieces while the other records on squared paper any new shapes as they appear. They might like to colour or number the squares to help them keep track of their actions.

Plenary

Show the hexominoes animation again, pausing it so that pupils can check each new hexomino and count to see how many there are. [35] More than 18 correct is good going.

Return to the pentomino problem and ask the group to produce the pentominoes in an order that reflects the same reasoning as for the hexominoes: this should now be ‘easy’ (you might like to set this for homework).

Show the group the twelve pentominoes in order so they can check they have all of them (see the resource sheet).

Solution notes

See the animation.