# Core 2 May 2005

1. i) $u_1 = 3\times1-1 \quad u_2 = 3\times2-1 \quad u_3 = 3\times3-1$

$2, 5, 8$

The sequence is an arithmetic progression.

ii) $\sum_{n=1}^{100} u_n = \frac{1}{2}\times100\times(2\times2+99\times3) = \underline{15050}$

2. i) $r\theta = 12$ ii) $\frac{1}{2}r^2\theta = 36$

ii) $\frac{1}{2}r(r\theta) = 36 \quad \frac{1}{2}r\times12 = 36 \quad 6r = 36 \quad \underline{r=6} \quad \therefore \underline{\theta = 2}$

iii) Area of segment is area of sector − area of triangle

$36 - \frac{1}{2}\times6^2\times\sin 2 = \underline{19.6\,\text{cm}^2}$

3. i) $\int(2x+1)(x+3)\,dx \quad \int 2x^2+7x+3\,dx = \frac{2}{3}x^3+\frac{7}{2}x^2+3x+k$

ii) $\int_0^9 x^{-1/2}\,dx = \left[2x^{1/2}\right]_0^9 = 2\times\sqrt{9}-2\sqrt{0} = \underline{6}$

4. i) $c^2 = a^2+b^2-2ab\cos C \quad \therefore \quad \cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{5^2+6^2-9^2}{2\times5\times6}$

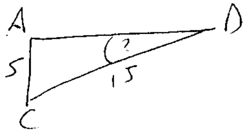
$= \frac{25+36-81}{60} = \frac{-20}{60} = \frac{-1}{3}$

$\cos^2C+\sin^2C = 1 \quad \left(\frac{-1}{3}\right)^2+\sin^2C = 1 \quad \sin^2C = \frac{8}{9}$

$\sin C = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$.

ii) Angle DAC = Angle ACB (alternate)

So $\sin DAC = \frac{2\sqrt{2}}{3}$




Using Sine rule

$\frac{\sin ADC}{5} = \frac{\sin DAC}{15} \Rightarrow \sin ADC = \frac{5\sin DAC}{15}$

$= \frac{5}{15}\times\frac{2\sqrt{2}}{3} = \frac{2\sqrt{2}}{9} \quad \sin^{-1}\left(\frac{2\sqrt{2}}{9}\right) \quad \therefore DAC = 18.3°$

5. i) If $x+1$ is a factor $f(-1)=0 \quad -1-a+b=0$

$b-a=1 \Rightarrow b = a+1$

If $x-3$ has remainder 16

$f(3) = 16 \quad 27+3a+b = 16$

$3a+b = -11$

$3a+(a+1) = -11 \quad 4a+1 = -11 \quad 4a = -12 \quad \underline{a=-3}$

$\therefore b = a+1 \quad b = -3+1 \quad \underline{b=-2}$

ii) $f(x) = x^3-3x-2$

$f(2) = 2^3-3\times2-2 = 8-6-2 = \underline{0}$

$\therefore (x+1)(x-2) = x^2-x-2$

$x^3-3x-2 = (x^2-x-2)(x+d)$

$x^3-3x-2 = x^3-x^2-2x+dx^2-dx-2d$

$= x^3+(d-1)x^2+(-d-2)x-2d$

$\therefore \underline{d=1}$

So $f(x) = (x+1)^2(x-2)$

6. i) $(x^2)^3 + 3(x^2)^2\left(\frac{1}{x}\right) + 3(x^2)\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$

$= x^6+3x^3+3+\frac{1}{x^3}$

ii) $\int\left(x^2+\frac{1}{x}\right)^3dx = \int x^6+3x^3+3+\frac{1}{x^3}\,dx$

$x^{-3}$

$= \frac{x^7}{7}+\frac{3}{4}x^4+3x-\frac{1}{2}x^{-2}+k$

7. i) $\log_5 15+\log_5 20-\log_5 12$ ← this term may cause a problem so.

$\log_5\left(\frac{15\times20}{12}\right) = \log_5 25 = \underline{\underline{2}}$

ii) $y = 3\times 10^{2x}$

$\log_{10} y = \log_{10}(3\times 10^{2x})$

$\log_{10} y = \log_{10} 3 + 2x\log_{10}10$

$2x = \log_{10} y - \log_{10} 3$

$2x = \log_{10}(\frac{1}{3}y)$

$x = \frac{1}{2}\log_{10}(\frac{1}{3}y)$ $\therefore$ $a = \frac{1}{2}$ $b = \frac{1}{3}$

8. i) $100000\times 0.9^3 = \underline{\underline{72900}}$.

ii) $100\,000\times 0.9^x < 5000$

$0.9^x < 0.05$

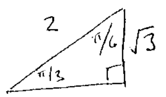
$\log 0.9^x < \log 0.05$

$x\log 0.9 < \log 0.05$

$x > \frac{\log 0.05}{\log 0.9} = 28.4$

So 28.4 years after 2001 so it will be in $\underline{2030}$.

iii) $\frac{100000(1-0.9^{30})}{1-0.9} = \underline{\underline{957609}}$.

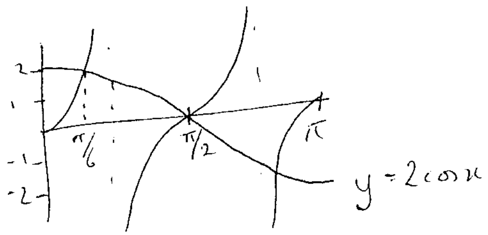
9. a) i) 


$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\tan\frac{\pi}{3} = \sqrt{3}$

if $x = \frac{1}{6}\pi$ then $2\cos x = \frac{2\times\sqrt{3}}{2}$

$\tan 2x = \tan\frac{\pi}{3} = \sqrt{3}$

$\underline{\frac{2\times\sqrt{3}}{2} = \sqrt{3}}$

ii) 


other roots are

$\underline{\underline{\pi/2}}$ and $\pi - \pi/6$

$= \underline{\underline{\frac{5\pi}{6}}}$

b) i) $\frac{1}{2}\times 0.1\times(0.1003 + 2(0.2027 + 0.3093) + 0.4225)$

$= 0.0774$

ii) 

Overestimate.

The tops of the trapezia are above the curve.