Painted cube

Generalising from games and investigations

Prerequisite knowledge

- Properties of a cube
- Knowledge of cube numbers and how they relate to volume

Why do this problem?

This offers a slight twist on a familiar problem through looking at what is *not* painted rather than what is.

The problem gives opportunities to apply knowledge of the properties of a cube, and to use 3-D visualisation and packing skills.

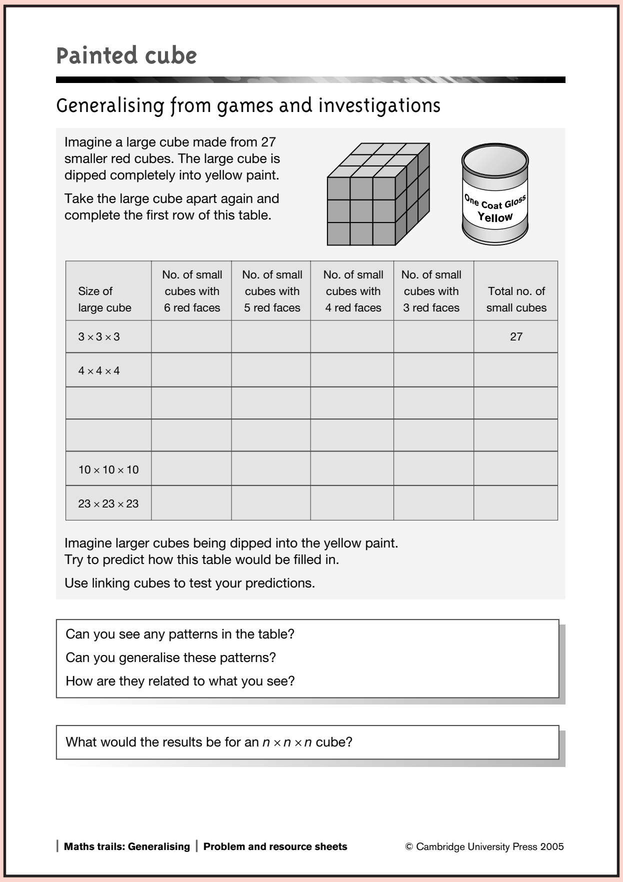
Time

One lesson

Resources

It is useful to have a set of cubes of different dimensions made from smaller interlocking cubes – perhaps two 3 × 3 × 3 cubes (see ‘Introducing the problem’). The effect of painting can be achieved by placing coloured stickers on each of the outside faces of the smaller cubes.

CD-ROM: pupil worksheet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Imagine a large cube made from 27 smaller red cubes. The large cube is dipped completely into yellow paint.

Take the large cube apart again and complete the first row of this table.

Size of large cube	No. of small cubes with 6 red faces	No. of small cubes with 5 red faces	No. of small cubes with 4 red faces	No. of small cubes with 3 red faces	Total no. of small cubes
3 × 3 × 3					27
4 × 4 × 4					
10 × 10 × 10					
23 × 23 × 23					

Imagine larger cubes being dipped into the yellow paint.
Try to predict how this table would be filled in.

Use linking cubes to test your predictions.

Can you see any patterns in the table?
Can you generalise these patterns?
How are they related to what you see?

What would the results be for an $n \times n \times n$ cube?

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NRICH website (optional): www.nrich.maths.org, June 2004, ‘Painted cube’ (includes an animation of dipping a 3 × 3 × 3 cube)

Introducing the problem

Show the pupils a 3 × 3 × 3 cube that has not been painted and discuss its properties. Include questions such as:

- How many small cubes is it made from?
- How many of the small cubes are visible?
- How many faces of how many small cubes are visible?

Ask what would happen if the cube was dipped into a pot of paint. Spend some time talking about the number of faces of the small cubes that will end up in the new colour or stay the same colour.

Show the ‘painted’ cube to confirm their conjectures or to help pupils in making conjectures of their own.

Main part of the lesson

Ask pupils how they might predict how many unpainted faces there would be in any size of cube. This may involve reminding them of the sorts of questions that you asked for a 3 × 3 × 3 cube and encouraging them towards investigating cubes of different sizes.

The class will then need to work individually or in small groups to collect data and try to establish some rules for very large cubes.

It may be worth pausing pupils’ work to emphasise that in the plenary it will not be enough just to have a rule but that pupils will need to be able to explain to the rest of the class why the rule works and how it relates to the cubes.

About 10 minutes before the end of the session ask pupils to spend some time in pairs producing convincing arguments for their findings to share with other members of the class during the plenary.

Plenary

This should focus on the *why*, not the *what*. Pupils may quickly offer solutions and formulae or verbal rules but the aim is to convince others that a rule is right by seeing how it relates to the context. It might be a good idea to choose two or three explanations that can form a classroom display.

An extension to cuboids is also worth considering if there is time.

Solution notes

Size of large cube:	$n \times n \times n$
No. of small cubes with 6 red faces:	$(n-2)^3$
No. of small cubes with 5 red faces:	$6(n-2)^2$
No. of small cubes with 4 red faces:	$12(n-2)$
No. of small cubes with 3 red faces:	8
Total no. of small cubes:	n^3