Interactive number patterns 2

Ready-made spreadsheets to explore mathematical ideas

Prerequisite knowledge

- Substitution into formulae
- Linear equations
- Quadratic expressions

Why do this unit?

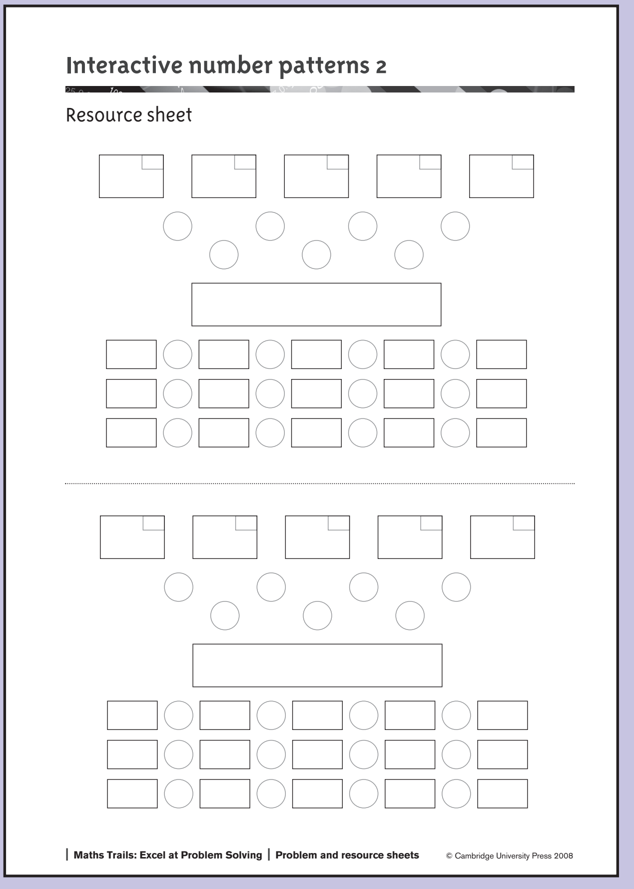
The interactive environment allows pupils to explore quadratic sequences and their algebraic representation. This unit is suitable for pupils for whom 'Interactive number patterns 1' is very straightforward.

Time

Two lessons

Resources

CD-ROM: spreadsheet, resource sheet
NRICH website (optional):
www.nrich.maths.org
September 2007, 'Interactive number patterns 2'


Interactive number patterns 2

Resource sheet

| Maths Trails: Excel at Problem Solving | Problem and resource sheets © Cambridge University Press 2008

Introducing the unit

Open 'Patterns (simple)' on the spreadsheet. Ensure that pupils understand:

- the slider at the top changes the red numbers;
- the spinner buttons on the bottom edge of the formula box change the numbers in the formula;
- the spinner buttons on the left of the screen hide or show parts of the display.

Set the formula to $n^2 + 7n + 4$ and the red numbers to read 1 to 5. Check with the group that substituting n as 1 into the expression gives 12, and continue with $n = 2$ through to 5 to produce 22, 34, 48 and 64.

Hide the blue numbers, ask for changes to the formula, and invite pupils to predict the hidden blue values. Include examples where the red n values range up to 20.

Continue this activity but hide the red values instead of the blue. Once the red values are hidden, set the formula back to $n^2 + 7n + 4$ and use the slider at the top to change the first blue number to 202.

- What red value produces the blue of 202? [11]
- How did you work that out? [Pupils may say 'trial and improvement'.]

Show the red values and reset the formula to $n^2 - 10n + 10$. Drag the slider to the extreme left to show blue values of 1, –6, –11, –14, –15.

- When n is 1 the formula gives 1. For what other n value will the formula also give 1? [9]
- Are there any other n values that produce 1? [No, after 9 the function continues ascending indefinitely.]
- Which n value produces 21? [11]
- Is there a second value which also produces 21? [Not within the limits of this spreadsheet, but $n = -1$ makes $n^2 - 10n + 10 = 21$.]

Make sure that pupils understand fully the process of substituting to find unknown or hidden values before moving to the main activity.

Main part of the unit

The aim of this part of the activity is for pupils to arrive at a method for determining the formula used to generate the terms within a sequence. It is common for pupils to use a method based on the difference between values, and then the difference between those difference values. It is less common for pupils to grasp the validity of this method. This activity is designed to improve pupil understanding of the 'difference method'.

Set the formula to n^2+3n+8 and move to 'Patterns with differences' on the spreadsheet. (Note that changing the quadratic expression and/or the red values on one sheet automatically updates the other.) Many values are displayed on this sheet and it is important that pupils are given time to understand each part.

There are three rows below the formula box. These display the values of the quadratic, linear and constant terms from the quadratic formula respectively. Change the terms in the formula box to help pupils grasp what each row shows. Verify that the three values added together produce the blue function values in the boxes at the top of the screen. Draw attention to the two rows of differences above the formula box and verify that the data showing is the correct difference for the blue function values. Change the red values using the slider and have pupils verify the new values displayed within the sheet.

- What changes when we alter the red values and what stays the same? [The first difference changes; the second difference remains unaltered.]
- Why? [The constant term contributes nothing to the first difference; the linear term contributes to the first difference but not the second; the coefficient (multiple) of the quadratic term is the only coefficient influencing the second difference.]

Return to 'Patterns (simple)'. Change the formula and ask pupils to write what the other sheet would show if they could see it. (There is a template for this on the resource sheet which enables pupils to record their solutions prior to discussion.)

Return to 'Patterns with differences' to compare pupil answers with the display and discuss.

Repeat this exercise as a group or ask pupils to work in pairs, with one of them altering the formula for the other to explain, until they are familiar with the role of differences.

Use 'Patterns with differences' and set the formula to $2n^2$ with no linear or constant term. Discuss the effect of the 2. Change the 2 to 3, 4 and then 5, discussing the effect until the group is confident that for a quadratic sequence the second difference will always be constant at double the coefficient of the quadratic term. Ask them to explain why this is true. Allow plenty of time for discussion as this is a central objective for this activity.

Ask pupils to work in groups to determine rules for finding a formula for any given sequence.

Plenary

Pupils discuss and evaluate some of their methods, leading to a whole-group challenge to determine hidden formulae for given sequences.