Tilted squares

Generalising and creating formulae

Prerequisite knowledge

- Areas of rectangles, squares and triangles

Why do this problem?

This problem not only looks at areas and the invariance of a square under rotation but can also lead to pupils establishing Pythagoras' theorem through structured exposure to 'tilted squares'. One of the most important concepts used in this problem is 'what makes a square a square'.

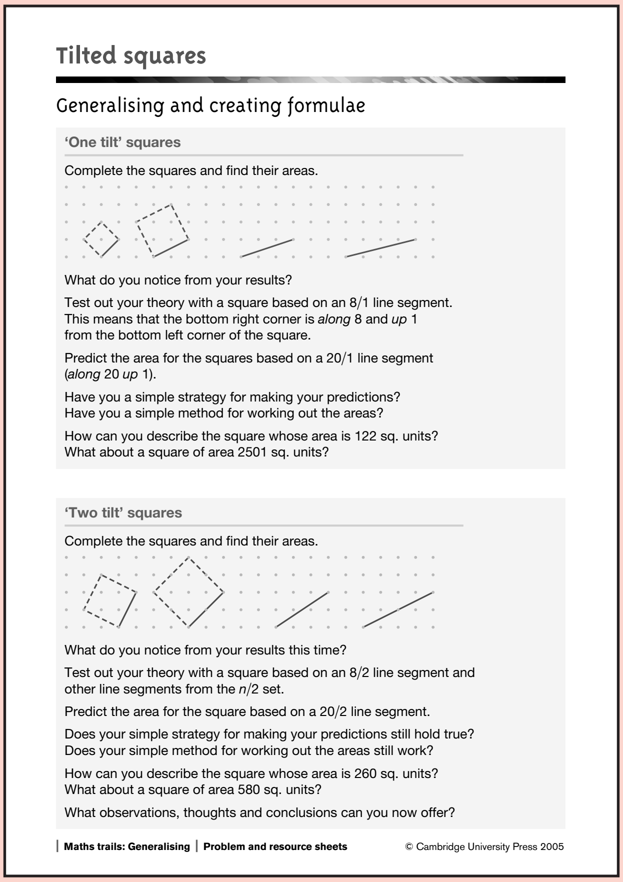
Time

Two lessons

Resources

CD-ROM: pupil worksheet; OHTs of dotty square grid and cut-out coloured squares

NRICH website (optional): www.nrich.maths.org, September 2004, 'Tilted squares' (three interactive tools can support the exploration and discussion throughout the problem-solving process); www.nrich.maths.org, October 2004, 'Square it' (an interactive game that uses tilted squares).



Tilted squares

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'One tilt' squares

Complete the squares and find their areas.

What do you notice from your results?

Test out your theory with a square based on an 8/1 line segment. This means that the bottom right corner is *along* 8 and *up* 1 from the bottom left corner of the square.

Predict the area for the squares based on a 20/1 line segment (*along* 20 *up* 1).

Have you a simple strategy for making your predictions? Have you a simple method for working out the areas?

How can you describe the square whose area is 122 sq. units? What about a square of area 2501 sq. units?

'Two tilt' squares

Complete the squares and find their areas.

What do you notice from your results this time?

Test out your theory with a square based on an 8/2 line segment and other line segments from the *n*/2 set.

Predict the area for the square based on a 20/2 line segment.

Does your simple strategy for making your predictions still hold true? Does your simple method for working out the areas still work?

How can you describe the square whose area is 260 sq. units? What about a square of area 580 sq. units?

What observations, thoughts and conclusions can you now offer?

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Introducing the problem

The main aim of the introduction is to revisit the properties of a square and consolidate the idea that squares do not have to have vertical and horizontal sides but can be 'tilted'.

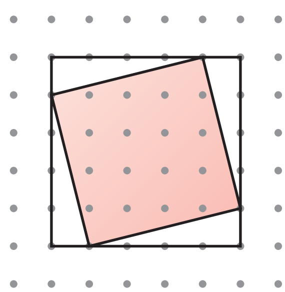
Display on an overhead projector a cut-out square from the resource sheet and ask pupils to say what shape it is and why they know it is a square.

Tilt the square and ask them again, confirming that the shape is still a square.

Now place the same square on the dotty square grid so that its base is horizontal and ask again if it is a square. Tilt the square on the grid and discuss the invariance of the properties of the square.

Choose another pre-prepared square that will 'fit'. Place the square on the grid and ask for ideas about how pupils might calculate its area. There may be a number of ideas, including estimation. At some point in this process turn the square so that it 'fits' the grid and lead a discussion about calculating its area:

Area of surrounding square minus area of 4 triangles	=	area of surrounding square minus area of 2 rectangles



Note: the shaded square would be described as a 4/1 square because the bottom right corner is *along* 4 and *up* 1 from the bottom left corner.

Main part of the lesson

Set pupils the task of trying to find a relationship between the areas of 'one tilt' squares and the amount of tilt (across and up).

After a few minutes stop the class and discuss findings. The area of a 'one tilt' square is $n^2 + 1$, where n is the 'along' measurement.

Ask pupils to suggest a hypothesis for the area of 'two tilt' squares – perhaps $n^2 + 2$. Now ask them to test this hypothesis and establish whether it is true or, if not, what they believe to be true.

Using the problem sheet as a structure, pupils can work towards generalising their findings for squares with any tilt – possibly leading to Pythagoras' theorem.

Plenary

This plenary will need to reflect the variety of levels of outcome that might be expected from pupils of varying ability and experience with problem solving. It is important at this stage to be clear about your core objectives. For example:

- If pupils are to have a secure concept of the 'squareness' of a square then some examples that test this out with a range of tilted squares, rhombi or other quadrilaterals will be sufficient.
- If you hope to reinforce pupils' skills with finding areas of triangles and quadrilaterals such as 'tilted squares' using the concept of complementary areas, then looking at extensions to grid shapes whose area can be found by using a surrounding rectangle would also be a fruitful focus for the plenary.
- The plenary may involve pulling some of the results together, working towards a generalisation for any degree of tilt.

With a very able class some discussion may have taken place during the session as they make progress towards the generalisation, in which case establishing a general rule (Pythagoras' theorem) can then be followed up with applications of the theorem in the subsequent sessions.

Solution notes

Using n for the *along* measurement and m for the *up* measurement:

The area of **'one tilt' squares** is $n^2 + 1$.

The square with area 122 sq. units is a 11/1 square, and that of area 2501 sq. units is a 50/1 square.

The area of **'two tilt' squares** is $n^2 + 4$.

The square with area 260 sq. units is a 16/2 square, and that of area 580 sq. units is a 24/2 square.

The area of **'three tilt' squares** is $n^2 + 9$.

The square with area 153 sq. units is a 12/3 square, and that of area 3145 sq. units is a 56/3 square.

The area of **'*m* tilt' squares** is $n^2 + m^2$.