Cola can

Modelling and optimisation

Prerequisite knowledge

- Area of a rectangle and of a circle
- Volume of a cylinder

Why do this unit?

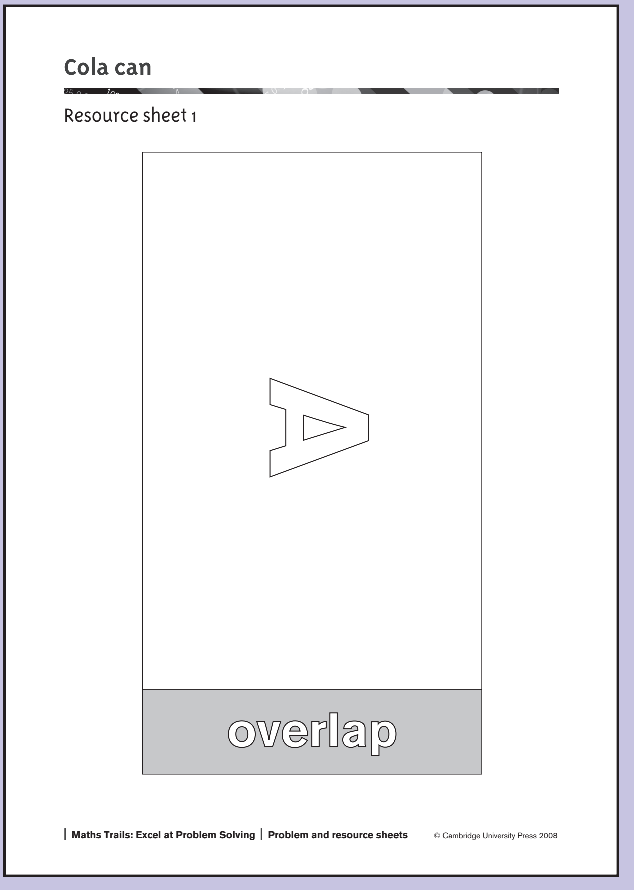
This problem utilises a spreadsheet to aid optimisation and demonstrates the value of a graph to support analysis.

Time

Two lessons

Resources

A sample of tins or cans of different sizes and proportions
CD-ROM: spreadsheet, resource sheets 1–8 (six rectangles can be made into cylinders using paper clips; solution cylinder)
NRICH website (optional): www.nrich.maths.org, July 2007, 'Gutter'; November 2007, 'Cola can'


Cola can

Resource sheet 1

A

overlap

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Introducing the unit

This unit is about creating a cylinder with a given volume that has the smallest possible surface area.

Show pupils the six cylinders made from the resource sheets.

- Which is the smallest? Which is the largest?

Include orderings based on the impression of volume and of surface area. First, working with surface area:

- How can we calculate the surface areas? [Find the area of the rectangle each cylinder was created from.]

Unwrap each cylinder to help illustrate the calculation and make measurements, or use the heights and widths given on the 'Dimension sheet' on resource sheet 7.

Compare the order based on calculations with the order based on impressions and discuss.

Now ask pupils to estimate the volumes of the cylinders A through to F from the resource sheets.

- How can we calculate those volumes? [Discussion might include recalling the volume formula, or deriving the formula by visualising a base area multiplied by height, as for a general prism.]
- What do we need to know? [radius]
- How can we find that? [Draw out the relationship between the width of the rectangle, the circumference and the radius of the cylinder.]

Calculate the radii and then the volumes of the cylinders.

Compare the new ordering based on calculation with the initial ordering made by impression.

Draw particular attention to the three cylinders that have similar volume but which were made from rectangles with different areas.

Main part of the unit

Show pupils a sample of commercially produced tins and cans. Discuss the value of minimising packaging materials.

A cola drink manufacturer has to keep costs low and might try to use as little aluminium as possible for the can.

- Can this be worked out? [If the volume is known (a cola can has a volume of 330 ml or 330 cm^3) a range of cylinders with different radii and heights can be made with that volume and it is then possible to calculate their surface areas.]

Ask pupils for a radius–height combination that they think might work for a volume 330 cm^3, or come close. Calculate the volume with the whole group. Invite suggestions for improved radius or height values. Ask pupils to make further calculations to improve their results. Draw out the advantages of increasing or decreasing just one of the variables and keeping the other fixed.

Take radius–height values from pupils which gave volume results close to 330 cm^3 and use these to calculate surface areas (including the bases and tops of the cans).

- Why is this approach to finding a minimum surface area inefficient? [Because even if the volume is close to 330 m^3, we have made many calculations without knowing if we have found the best (minimum) result for the surface area.]

If it does not arise naturally out of discussion, explain that it isn't necessary to guess both the radius and the height. If we know one of these we can work out the other using the volume of 330 cm^3.

- If we take a radius value of 10 cm, what will be the height? What is the complete surface area for this can? [0.525 cm, 661 cm^2 (both 3 s.f.)]
- If we take a height value of 10 cm, what will be the radius? What is the complete surface area? [3.24 cm, 270 cm^2 (both to 3 s.f.)]

Remind the group that the aim is to make the surface area as small as possible.

We could adjust the radius of 10 cm to get closer to the minimum surface area and continue until we are satisfied that we are close enough. However, these calculations can be done very efficiently on a spreadsheet.

Discuss with the group the possible structure of a suitable spreadsheet to do this before they attempt their own constructions, or use 'Changing the radius' on the spreadsheet as a prompt. Pupils will need to satisfy themselves that the formula in each cell performs the required calculation. In particular, look in column F (total surface area): the values in cells F6 to F8 suggest a revised start value of 3 and an increment of 0.1 (say). Further iterations of this process will continue to improve the accuracy of the result.

Instead of adjusting the radius it is possible to adjust the height – see the sheet 'Changing the height' on the spreadsheet.

Plenary

Graphs offer a valuable aid to visualisation. The sheet 'Changing the radius (graph)' on the spreadsheet contains a graph which shows the relationship between choice of radius and the resulting surface area. See also 'Changing the height (graph)'.

The sheet 'Graph help' explains how a graph can be created in Excel. Demonstrate the creation of one of the graphs.

- How does the graph help us answer the question 'Which can dimensions are best?'? [It identifies the minimum point and interprets the coordinates.]

Solution notes

The radius of the cylinder with minimum surface area is approximately 3.74 cm and its height is 7.50 cm (see resource sheet 8).