Big powers

Deduction

Prerequisite knowledge

- Knowledge of positive indices but not the rules of multiplication and division etc.

Why do this problem?

The problem draws on patterns in the powers of numbers and uses this as a basis for solution. The idea that using an organised approach enables you to look at the sums of the two powers is a powerful application of being systematic and organised in the presentation of results. It is not enough to spot the pattern of the sequence in the powers; more important is to justify that pattern.

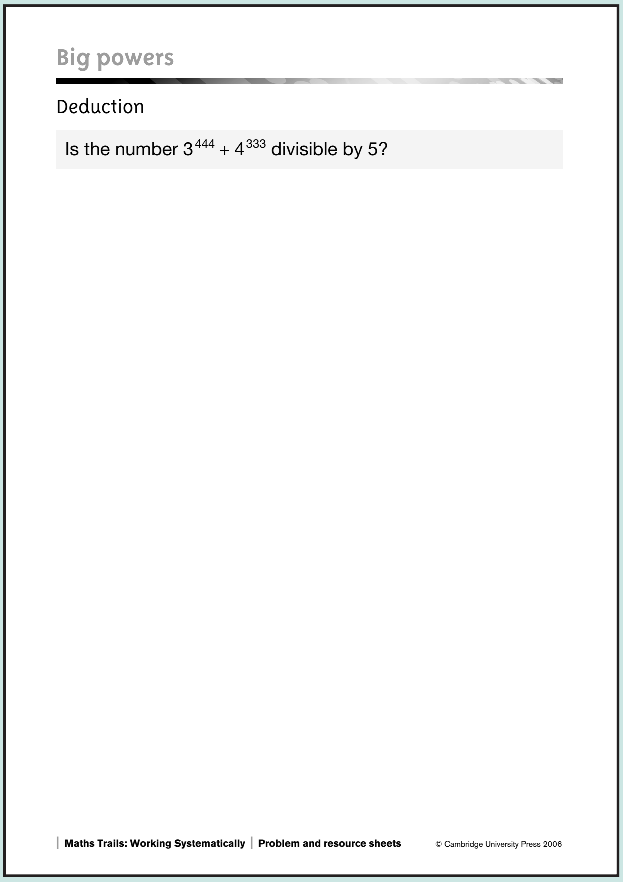
A number of similar problems are on the NRICH website (a list of three of them are given in the resources section) which utilise the same reasoning. These could either be used as a follow up to this problem or a precursor to the problem to help offer a bridge into it.

Time

Less than one lesson

Resources

CD-ROM: problem sheet (not strictly necessary)


Big powers

Deduction

Is the number $3^{444} + 4^{333}$ divisible by 5?

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NRICH website (optional): www.nrich.maths.org: January 1998, 'Big powers'; June 2002, 'Power crazy'; November 2003, 'What an odd factor'; October 2002, 'Remainder'

Spreadsheet software (optional)

Introducing the problem

Start by showing the problem that the pupils will be tackling in the main part of the lesson. Explain that the aim is to answer, and to justify the answer, to this question.

The main focus of the introduction is to recapitulate divisibility rules, which in turn may give a strong hint about how to tackle the problem itself. Useful rules might be those for 2, 3, 4, 5, 6 and 8. Ask pupils questions such as:

- How can you tell if 103 is divisible by 2, 3, 4 or 5?
- Can a number be divisible by 2 and 5? How about 3 and 5?
- How would you know that a number is divisible by 6?
- How can you tell if a number is divisible by 8?

Main activity

Continuing from the introduction, the focus turns to patterns in powers of 2 rather than multiples of 2. With the class, generate the first few powers of 2:

2, 4, 8, 16, 32, 64, 128, ...

- What do you notice about the units digit?
- Extend the sequence to see if the cyclic pattern continues.

- Can you justify any hypotheses concerning the continuation of the sequence? (Pupils could justify this continuation in terms of getting to a units digit of 8 then, multiplying by 2 results in a 6 in the units column.)
- How large is this cycle? (4)
- What would the units digit of 2^{20} be? (6)
- What about 2^{2002}? (4)

This relates to modulo 4 arithmetic.

Restate the main problem and ask the pupils to think about this for five minutes on their own. If they have any ideas, or strategies for the solution, they should make some notes to share with the group a little later.

After five minutes stop the class and ask pupils to share their ideas.

The longer term aim is for the group to identify patterns in the units digit of the powers of 3 and of 4 and that it might be possible to utilise these patterns. Groups can produce posters of their findings for display.

The importance is the justification, possibly using the idea of cycles, that the pattern continues forever.

Plenary

Discuss the posters and findings of the group and identify well-explained solutions that show how the solvers have been systematic in their analysis of the problem.

If not used in the main part of the lesson, it may also be appropriate to offer one of the follow-up problems from the NRICH website.

Solution notes

The sum is divisible by 5.