OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4724**

Core Mathematics 4

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

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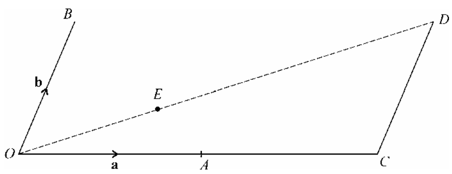
1 $\dfrac{x^4+1}{x^2+1}=x^2-1+\dfrac{2}{x^2+1}$	B1	For correct leading term x^2 in quotient
	M1	For evidence of correct division process
	A1	For correct quotient x^2-1
	A1 **4**	For correct remainder 2
	$\boxed{4}$	
2 **(i)** $(1-2x)^{-\frac{1}{2}}=1+(-\frac{1}{2})(-2x)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-2x)^2+$		
$+\dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-2x)^3+\dots$	M1	For 2nd, 3rd or 4th term OK (unsimplified)
$=1+x+\frac{3}{2}x^2+\frac{5}{2}x^3$	A1	For $1+x$ correct
	A1	For $+\frac{3}{2}x^2$ correct
	A1 **4**	For $+\frac{5}{2}x^3$ correct
(ii) Valid for $\lvert x\rvert<\frac{1}{2}$	B1 **1**	For any correct expression(s)
	$\boxed{5}$	
3 $\int_0^1 x\mathrm{e}^{-2x}\,\mathrm{d}x=\left[-\frac{1}{2}x\mathrm{e}^{-2x}\right]_0^1-\int_0^1-\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$	M1	For attempt at 'parts' going the correct way
	A1	For correct terms $-\frac{1}{2}x\mathrm{e}^{-2x}-\int-\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$
$=\left[-\frac{1}{2}x\mathrm{e}^{-2x}-\frac{1}{4}\mathrm{e}^{-2x}\right]_0^1$	M1	For consistent attempt at second integration
	M1	For correct use of limits throughout
$=\frac{1}{4}-\frac{3}{4}\mathrm{e}^{-2}$	A1 **5**	For correct (exact) answer in any form
	$\boxed{5}$	
4 **(i)** 	B1	For C correctly located on sketch
	B1	For D correctly located on sketch
	B1√ **3**	For E correctly located wrt O and D
(ii) $\overrightarrow{AE}=\frac{1}{3}(2\mathbf{a}+\mathbf{b})-\mathbf{a}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$	M1	For relevant subtraction involving $\overrightarrow{OE}$
	A1	For correct expression for $(\pm)\overrightarrow{AE}$ or $\overrightarrow{EB}$
Hence AE is parallel to AB	A1	For correct recognition of parallel property
i.e. E lies on the line joining A to B	A1 **4**	For complete proof of required result
	$\boxed{7}$	
5 **(i)** $4x+x\dfrac{\mathrm{d}y}{\mathrm{d}x}+y+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$	B1	For correct terms $x\dfrac{\mathrm{d}y}{\mathrm{d}x}+y$
	B1	For correct term $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$
Hence $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{4x+y}{x+2y}$	M1	For solving for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
	A1 **4**	For any correct form of expression
(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow y=-4x$	M1	For stating or using their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$
Hence $2x^2+(-4x^2)+(-4x)^2=14$	M1	For solving simultaneously with curve equn
i.e. $x^2=1$	A1	For correct value of x^2 (or y^2)
So the two points are $(1,-4)$ and $(-1,4)$	A1 **4**	For both correct points identified
	$\boxed{8}$	

6 **(i)** $\theta = 0$ at the origin	B1	For the correct value
A is $(0, a\pi)$	B1	For the correct y-coordinate at A
B is $(a, 0)$	B1 **3**	For the correct x-coordinate at B
(ii) $\dfrac{dx}{d\theta} = a\cos\theta$	B1	For correct differentiation of x
$\dfrac{dy}{d\theta} = a(\cos\theta - \theta\sin\theta)$	M1	For differentiating y using product rule
Hence $\dfrac{dy}{dx} = \dfrac{\cos\theta - \theta\sin\theta}{\cos\theta} = 1 - \theta\tan\theta$	M1	For use of $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \Big/ \dfrac{dx}{d\theta}$
	A1	For given result correctly obtained
Gradient of tangent at the origin is 1	M1	For using $\theta = 0$
Hence equation is $y = x$	A1 **6**	For correct equation
	$\boxed{9}$	
6 **(i)** $L_1 : \mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$	M1	For correct RHS structure for either line
$L_2 : \mathbf{r} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1 **2**	For both lines correct
(ii) $3 + 2s = 3 + t,\ 6 + 3s = -1 - 2t,\ 1 - s = 4 + t$	M1	For at least 2 equations with two parameters
First pair of equations give $s = -1, t = -2$	M1	For solving any relevant pair of equations
	A1	For both parameters correct
Third equation checks: $1 + 1 = 4 - 2$	A1	For explicit check in unused equation
Point of intersection is $(1, 3, 2)$	A1 **5**	For correct coordinates
(iii) $2\times1 + 3\times(-2) + (-1)\times1 = (\sqrt{14})(\sqrt{6})\cos\theta$	B1	For scalar product of correct direction vectors
	B1	For correct magnitudes $\sqrt{14}$ and $\sqrt{6}$
	M1	For correct process for $\cos\theta$ with *any* pair of vectors relevant to these lines
Hence acute angle is $56.9°$	A1 **4**	For correct acute angle
	$\boxed{11}$	
8 **(i)** $I = \displaystyle\int \frac{1}{u^2(1+u)^2} \times 2u\,du = \int \frac{2}{u(1+u)^2}\,du$	M1	For any attempt to find $\dfrac{dx}{du}$ or $\dfrac{du}{dx}$
	A1	For '$dx = 2u\,du$' or equivalent correctly used
	A1 **3**	For showing the given result correctly
(ii) $2 \equiv A(1+u)^2 + Bu(1+u) + Cu$	M1	For correct identity stated
$A = 2$	B1	For correct value stated
$C = -2$	B1	For correct value stated
$0 = A + B$ (e.g.)	A1	For any correct equation involving B
$B = -2$	A1 **5**	For correct value
(iii) $2\ln u - 2\ln(1+u) + \dfrac{2}{1+u}$	B1$\sqrt{}$	For $A\ln u + B\ln(1+u)$ with their values
	B1$\sqrt{}$	For $-C(1+u)^{-1}$ with their value
Hence $I = \ln x - 2\ln(1+\sqrt{x}) + \dfrac{2}{1+\sqrt{x}} + c$	M1	For substituting back
	A1 **4**	For completely correct answer (excluding c)
	$\boxed{12}$	

9	$\frac{dx}{dt} = -k\sqrt{x}$	M1		For use of derivative for rate of change
		A1		For correct equation (neg sign optional here)
	$x = 100$ and $\frac{dx}{dt} = -1 \Rightarrow k = 0.1$	M1		For use of data and their DE to find k
	Hence equation is $\frac{dx}{dt} = -0.1\sqrt{x}$	A1		For any form of correct DE
	$\int x^{-\frac{1}{2}}\,dx = -0.1\int dt \Rightarrow 2x^{\frac{1}{2}} = -0.1t + c$	M1		For separation and integration of both sides
		A1		For $2x^{\frac{1}{2}}$ correct
		A1√		For $(\pm)kt$ correct (the numerical evaluation of k may be delayed until after the DE is solved)
		B1		For one arbitrary constant included (or equivalent statement of both pairs of limits)
	$x = 200, t = 0 \Rightarrow c = 2\sqrt{200}$	M1		For evaluation of c
	So when $x = 100$, $2\sqrt{100} = -0.1t + 2\sqrt{200}$	M1		For evaluation of t
	i.e. $t = 82.8$	A1	**11**	For correct value 82.8 (minutes)
			$\boxed{11}$	