

**ADVANCED GCE UNIT
MATHEMATICS**

Core Mathematics 3

THURSDAY 18 JANUARY 2007

4723/01

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

- 1 Find the equation of the tangent to the curve $y = \frac{2x+1}{3x-1}$ at the point $(1, \frac{3}{2})$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. [5]

- 2 It is given that θ is the acute angle such that $\sin \theta = \frac{12}{13}$. Find the exact value of

(i) $\cot \theta$, [2]

(ii) $\cos 2\theta$. [3]

- 3 (a) It is given that a and b are positive constants. By sketching graphs of

$$y = x^5 \quad \text{and} \quad y = a - bx$$

on the same diagram, show that the equation

$$x^5 + bx - a = 0$$

has exactly one real root.

[3]

- (b) Use the iterative formula $x_{n+1} = \sqrt[5]{53 - 2x_n}$, with a suitable starting value, to find the real root of the equation $x^5 + 2x - 53 = 0$. Show the result of each iteration, and give the root correct to 3 decimal places. [4]

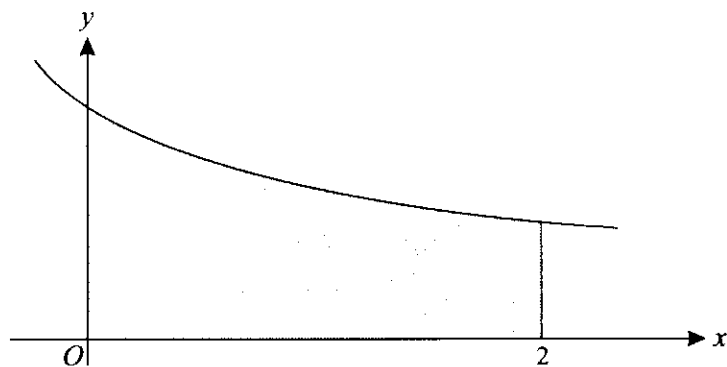
- 4 (i) Given that $x = (4t + 9)^{\frac{1}{2}}$ and $y = 6e^{\frac{1}{2}x+1}$, find expressions for $\frac{dx}{dt}$ and $\frac{dy}{dx}$. [4]

- (ii) Hence find the value of $\frac{dy}{dt}$ when $t = 4$, giving your answer correct to 3 significant figures. [3]

- 5 (i) Express $4 \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

- (ii) Hence solve the equation $4 \cos \theta - \sin \theta = 2$, giving all solutions for which $-180^\circ < \theta < 180^\circ$. [5]

6

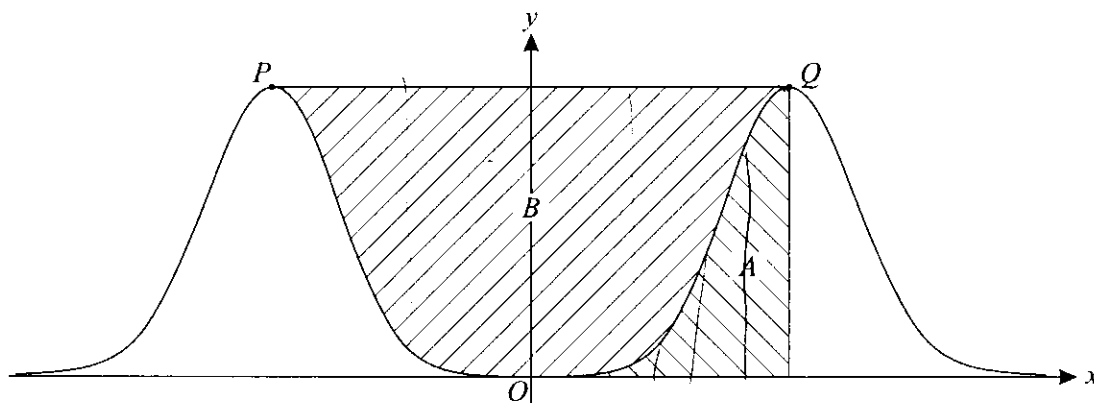


The diagram shows the curve with equation $y = \frac{1}{\sqrt{3x+2}}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$.

- (i) Find the exact area of the shaded region. [4]
 - (ii) The shaded region is rotated completely about the x -axis. Find the exact volume of the solid formed, simplifying your answer. [5]
- 7 The curve $y = \ln x$ is transformed to the curve $y = \ln\left(\frac{1}{2}x - a\right)$ by means of a translation followed by a stretch. It is given that a is a positive constant.
- (i) Give full details of the translation and stretch involved. [2]
 - (ii) Sketch the graph of $y = \ln\left(\frac{1}{2}x - a\right)$. [2]
 - (iii) Sketch, on another diagram, the graph of $y = \left|\ln\left(\frac{1}{2}x - a\right)\right|$. [2]
 - (iv) State, in terms of a , the set of values of x for which $\left|\ln\left(\frac{1}{2}x - a\right)\right| = -\ln\left(\frac{1}{2}x - a\right)$. [2]

[Questions 8 and 9 are printed overleaf.]

8



The diagram shows the curve with equation $y = x^8 e^{-x^2}$. The curve has maximum points at P and Q . The shaded region A is bounded by the curve, the line $y = 0$ and the line through Q parallel to the y -axis. The shaded region B is bounded by the curve and the line PQ .

- (i) Show by differentiation that the x -coordinate of Q is 2. [5]
- (ii) Use Simpson's rule with 4 strips to find an approximation to the area of region A . Give your answer correct to 3 decimal places. [4]
- (iii) Deduce an approximation to the area of region B . [2]

9 Functions f and g are defined by

$$\begin{aligned} f(x) &= 2 \sin x & \text{for } -\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi, \\ g(x) &= 4 - 2x^2 & \text{for } x \in \mathbb{R}. \end{aligned}$$

- (i) State the range of f and the range of g . [2]
- (ii) Show that $gf(0.5) = 2.16$, correct to 3 significant figures, and explain why $fg(0.5)$ is not defined. [4]
- (iii) Find the set of values of x for which $f^{-1}g(x)$ is not defined. [6]

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