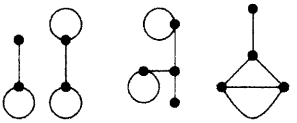
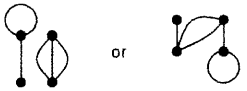
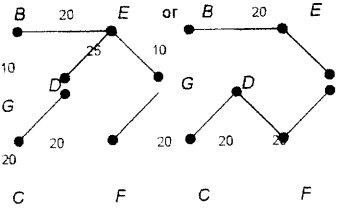
1	(i)	2 4 3 3 2 5 4 Box 1 **2 4 2** Box 2 **3 3** Box 3 **5** Box 4 **4**	M1 A1 [2]	For packing these seven weights into boxes with no more than 8 kg total in each box For this packing
	(ii)	5 4 4 3 3 2 2 Box 1 **5 3** Box 2 **4 4** Box 3 **3 2 2**	B1 M1 A1 [3]	For putting the weights into decreasing order (may be implied from packing) For packing the seven weights into three boxes with no more than 8 kg total in each box For this packing
	(iii)	15×2^2 = **60** seconds	M1 A1 [2]	For a correct calculation For 60 or 60 seconds or 1 minute 7
2	(i)	 graph *A* graph *B* graph *C* other solutions: 	M1 A1 [2] --- M1 A1 [2] --- M1 A1 [2]	Graphs may be in any order For a reasonable attempt For a graph that is topologically equivalent to one of these graphs For a different reasonable attempt For a graph that is topologically equivalent to one of these graphs For another different reasonable attempt For a graph that is topologically equivalent to one of these graphs
	(ii)	The graphs each have four **odd** nodes, but Eulerian graphs have no odd nodes.	B1 [1]	For any recognition that the nodes are not all even 7

3	(i)	**Travelling salesperson**	B1 [1]	Identifying TSP by name
	(ii)	***A – B – E – G – F – D – C – A*** 130 (minutes) Shortest possible time ≤ **130 minutes**	M1 A1 B1 B1 [4]	For starting with *A – B – E – G - …* For this closed tour For 130 For less than or equal to their time, **with units**
	(iii)	Order of connecting: ***B, E, G, F, D, C***  Lower bound = 10 + 15 + 95 = **120 minutes**	B1 M1 A1 M1 M1 A1 [6]	For a valid vertex order (or arc order) for their starting point For a diagram or listing showing a tree connecting the vertices *B, C, D, E, F* and *G* but not *A* For a diagram showing one of these trees (vertices must be labelled but arc weights are not needed) For stating or using the total weight of their tree For stating or using *AB* and *AD* or 10 + 15 For 120 or calculating 25 + their 95. **with units**
	(iv)	***A – B – E – G – F – C – D – A*** or this in reverse	M1 A1 [2]	For a reasonable attempt For a valid tour of weight 125
				13

4	(i)	$x \leq 2$ $y \geq 1$ $y \leq 2x$ $x + y \leq 4$	B1 B1 B1 B1 [4]	Strict inequalities used, penalise first time only All inequalities reversed, penalise first time only
	(ii)	(2, 1), (2, 2) (½, 1) (1⅓, 2⅔)	B1 B1 B1 [3]	Both of these This vertex in any exact form This vertex in any exact form or correct to 3 sf
	(iii)	x y $P = x + 2y$ 2 1 4 2 2 6 ½ 1 2½ 1⅓ 2⅔ 6⅔ $x = 1\frac{1}{3}$, $y = 2\frac{2}{3}$ (may be given in coordinate form) $P = 6\frac{2}{3}$	 M1 A1 A1 [3]	 Evidence of checking value at any vertex or using a sliding profit line Their x and y values at maximum in any exact form or correct to 3 sf Their maximum P value in any exact form or correct to 3 sf
	(iv)	x y $Q = 2x - y$ 2 1 3 2 2 2 ½ 1 0 1⅓ 2⅔ 0 $Q = 0$ (x, y) can be any point on the **line segment joining (½, 1) and (1⅓, 2⅔)**	 M1 A1 A1 [3]	 Evidence of checking value at any vertex or using a sliding profit line 0 (cao) The edge of the feasible region where $y = 2x$ No follow through
	(v)	$P = Q \Rightarrow 2x - y = x + 2y$ $\Rightarrow x = 3y$ $y = \frac{1}{3}x$ lies entirely in the shaded region	M1 A1 A1 [3]	For considering $P = Q$, or equivalent For this line, or any equivalent reasoning For explanation of why there are no solutions
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5 (i) $2x - 5y + 2z + s = 10$
$2x + 3z + t = 30$ — B1 [1] Slack variables used correctly

(ii)

P	x	y	z	s	t	
1	-1	2	3	0	0	0
0	2	-5	2	1	0	10
0	2	0	3	0	1	30

M1 For overall structure correct, including two slack variable columns and column for RHS (condone omission of P column or labels)
A1 [2] For a completely correct initial tableau, with no extra constraints added (condone variations in order of rows or columns)

(iii) Pivot on x column since it is the only column with a **negative value in the objective row** — B1 For negative in objective row, top row, pay-off row, or equivalent
$10 \div 2 = 5$ $5 < 15$ so pivot on this row
$30 \div 2 = 15$ — B1 [2] For these two divisions shown

(iv) New row 2 = **row 2 ÷ 2** — B1 For dealing with the pivot row correctly
New row 1 = **row 1 + new row 2**
New row 3 = **row 3 – 2 × new row 2** — B1 [2] For dealing with the other rows correctly
May be coded by rows of table

1	0	-0.5	4	0.5	0	5
0	1	-2.5	1	0.5	0	5
0	0	5	1	-1	1	20

M1 For updating their pivot row correctly
M1 For a reasonable attempt at updating other rows
A1 [3] For correct values in tableau (condone consistent order of rows or columns). Do not follow through errors in initial tableau or pivot choice.

$x = 5$, $y = 0$, $z = 0$ — B1 For reading off x, y and z from their tableau
$P = 5$ — B1 For reading off P from their tableau
Not the maximum feasible value of P since there is still a **negative** value **in** the **objective row** — B1 [3] 'No' seen or implied and a correct reason

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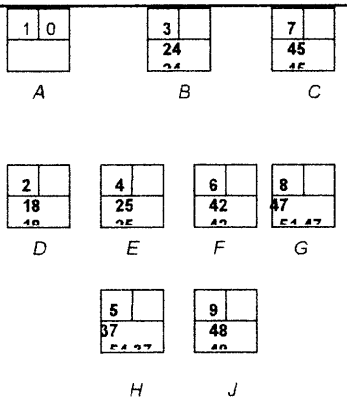
6	(a)			ANSWERED ON INSERT
			M1	Values correct at *B*, *D* and *E* (condone temporary labels implied from permanent labels)
			M1	Both 54 and 37 seen at *H* and both 51 and 47 seen at *G* (method)
			A1	All temporary labels correct and no extras
			B1	All permanent labels correct
			B1	Order of labelling correct (condone boxes consistently swapped over)
		A - *E* - *H* - *J* **48** metres	B1 B1 [7]	For this route, including end vertices (cao) For 48 (cao)
	(b)	*A* and *J* are the only odd nodes	B1	Identifying odd nodes (or by implication)
	(i)	48 + 300 = **348** metres	M1 A1 [3]	For their 48 + 300 (or their 300) 348 (cao)
	(ii)	Odd nodes *A*, *B*, *H*, *J* *AB* = **24** *AH* = **37** *AJ* = **48** *HJ* = **11** *BJ* = **38** *BH* = **34** Repeat *AB* and *HJ* = 35 300 - 30 = 270 metres Shortest distance = 270 + 35 = **305** metres	B1 B1 B1 M1 M1 A1 [6]	Identifying odd nodes (or by implication) For distances from *A* – or from their Dijkstra For distances *HJ*, *BJ*, *BH* correct Choosing their least pairing or by implication Or by implication 305 (cao)
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