# Core 2 January 2005

1. $(3+2x)^3 - (3-2x)^3$

$\binom{3}{0}3^3 + \binom{3}{1}3^2(2x) + \binom{3}{2}3(2x)^2 + \binom{3}{3}(2x)^3$

$-\left(\binom{3}{0}3^3 + \binom{3}{1}3^2(-2x) + \binom{3}{2}3(-2x)^2 + \binom{3}{3}(-2x)^3\right)$

These will cancel because they're the same.

You're subtracting 2 negative terms $(-2x)$, $(-2x)^3$ so these will be positive, to make

$2\times\binom{3}{1}3^2(2x) + 2\times\binom{3}{3}(2x)^3$

$= 2\times3\times3^2\times2x + 2\times1\times2^3x^3$

$= 108x + 16x^3$

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2. $U_1 = 2 \qquad U_{n+1} = \dfrac{1}{1-U_n}$

$U_2 = \dfrac{1}{1-2} = -1$

$U_3 = \dfrac{1}{1-^-1} = \dfrac{1}{2}$

$U_4 = \dfrac{1}{1-\frac{1}{2}} = \dfrac{1}{\frac{1}{2}} = 2$

$U_5 = \dfrac{1}{1-2} = -1$

It will keep repeating!

$\therefore U_6 = \frac{1}{2}$

Every 3rd term (multiple of 3) will be ½ (ie $U_3, U_6, U_9, U_{12}$ etc)

So $U_{198} = \frac{1}{2}$

$U_{199} = 2$

$U_{200} = -1$

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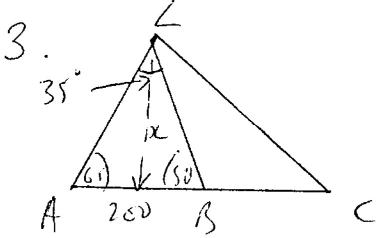
3.



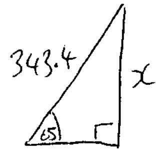

i) The shortest distance from L to the road is perpendicular to the road as shown ↑

We need to find LA first using the sine rule.

$\dfrac{200}{\sin 35} = \dfrac{LA}{\sin 80} \Rightarrow LA = \dfrac{200\sin 80}{\sin 35}$

$LA = 343.4°$

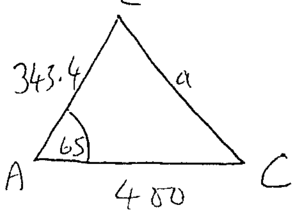
So




$\sin 65 = \dfrac{x}{343.4}$

$x = 343.4\sin 65 = \underline{311.2m}$

ii) Use the cosine rule to find LC




$a^2 = b^2 + c^2 - 2bc\cos 65$

$a^2 = 400^2 + 343.4^2 - 2\times400\times343.4\times\cos 65$

$a^2 = 161821.87$

$a = 402.3\ m$

4\.

i) $y = \frac{16}{x^2}$ $\quad y = 17 - x^2$

if $x = 1$ $\quad y = 16$ on both $\quad y = \frac{16}{1}$ & $y = 17 - 1^2$

if $x = 4$ $\quad y = 1$ on both $\quad y = \frac{16}{16}$ $\quad y = 17 - 4^2$

) $\int_1^4 17 - x^2 \, dx - \int_1^4 16x^{-2} \, dx$

$\left[17x - \frac{x^3}{3}\right]_1^4 - \left[-16x^{-1}\right]_1^4$

$= \left(\left(17 \times 4 - \frac{64}{3}\right) - \left(17 - \frac{1}{3}\right)\right) - \left(\frac{-16}{4} - \frac{-16}{1}\right)$

$= 68 - \frac{64}{3} - 16\frac{2}{3} - (12)$

$= 68 - 21\frac{1}{3} - 16\frac{2}{3} - 12 = \underline{\underline{18}}$

5\.

$\sin\theta \tan\theta = \sin\theta \frac{\sin\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$

So $\frac{\sin^2\theta}{\cos\theta} = \cos\theta + 1$

$\sin^2\theta = \cos^2\theta + \cos\theta$

$1 - \cos^2\theta = \cos^2\theta + \cos\theta$

$0 = 2\cos^2\theta + \cos\theta - 1$

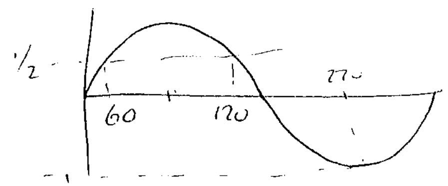
ii) if $\sin\theta \tan\theta = \cos\theta + 1$

Then $2\cos^2\theta + \cos\theta - 1 = 0$

Factorise $(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = -1$ or $\cos\theta = \frac{1}{2}$

Between $0 \leq \theta \leq 360$




$\theta = 270°$

or $\theta = 60°$ or $\theta = 120°$

6\. a) $\int x(x^2 + 2) \, dx = \int x^3 + 2x \, dx = \frac{x^4}{4} + x^2 + k$

b) i) $\int \frac{1}{\sqrt{x}} \, dx = \int x^{-1/2} \, dx = 2x^{1/2} + k$

ii) if $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$ then $y = 2x^{1/2} + k$

if it passes through $(4, 0)$ $\quad 0 = 2 \times 4^{1/2} + k$

$0 = 2 \times 2 + k$

$k = -4$

$\underline{\underline{y = 2x^{1/2} - 4}}$

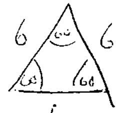
7. i) DO and DE are 6.

Angle DOE is $60^\circ$ or $\frac{\pi}{3}$!

Arc length $= r\theta = \frac{\pi}{3} \times 6 = \underline{\underline{2\pi}}$.

Area $= \frac{1}{2} r^2 \theta = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = 18\frac{\pi}{3} = \underline{\underline{6\pi}}$.

ii) The parallelogram ADOE has area of 2 triangles



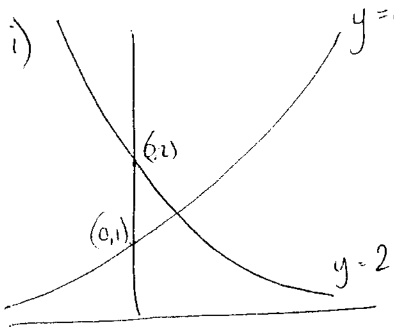

$\frac{1}{2} bc \sin A = \frac{1}{2} \times 6 \times 6 \times \sin 60$

$= 15.588$

Double to get parallelogram $= 31.1769$

Subtract area of sector $(6\pi)$ $= \underline{\underline{12.3 \text{ cm}^2}}$

8. i)




ii) At point of intersection $a^x = 2b^x$

(take log to base 2)

$\log_2(a^x) = \log_2(2b^x)$

$x \log_2 a = \log_2 2 + x \log_2 b$

$x \log_2 a = 1 + x \log_2 b$

$x \log_2 a - x \log_2 b = 1$

$x = \frac{1}{\log_2 a - \log_2 b}$ <u>A very easy 5 marks!</u>

9.

| ① | ② | ③ | ④ |
|---|---|---|---|
| $a$ | $ar$ | $ar^2$ | $ar^3$ |

i) The <u>difference</u> between ④ and ① is 4 × the <u>difference</u> between ③ and ②.

$(ar^3 - a) = 4(ar^2 - ar)$

$ar^3 - a = 4ar^2 - 4ar$

$(\div a)$ $r^3 - 1 = 4r^2 - 4r$

$r^3 - 4r^2 + 4r - 1 = 0$

ii) if $r=1$ is a factor

then putting $r=1$ into $r^3 - 4r^2 + 4r - 1$ should be 0

$1^3 - 4 \times 1^2 + 4 \times 1 - 1 = 0$ so $r=1$ is a factor

$(r-1)(Ar^2 + Br + C) = r^3 - 4r^2 + 4r - 1$

$Ar^3 + Br^2 + Cr - Ar^2 - Br - C = r^3 - 4r^2 + 4r - 1$

$A = 1$

$B - A = -4 \rightarrow B = -3$

$C - B = 4 \rightarrow C = 1$

$-C = -1$ ✓

So $(r-1)(r^2 - 4r + 1)$

$\cancel{(r-1)(r-2)(r-2)}$

$\cancel{r^3 - 4r^2 + 4r}$

$A = 1 \quad B = -3 \quad C = 1$

$(r-1)(r^2-3r+1)$

↳ use quadratic formula

iii) $r = \dfrac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2} = \dfrac{3 \pm \sqrt{5}}{2}$

$r = \dfrac{3+\sqrt{5}}{2}$ or $\dfrac{3-\sqrt{5}}{2}$

if $r=1$ then it's not a geometric progression so 2 solutions not 3

iv) Progression is convergent if $-1 < r < 1$

$r = \dfrac{3-\sqrt{5}}{2}$

$S_\infty = \dfrac{a}{1-r} = \dfrac{a}{1-\left(\frac{3-\sqrt{5}}{2}\right)} = \dfrac{a}{\frac{2-3+\sqrt{5}}{2}}$

$= \dfrac{2a}{-1+\sqrt{5}} = \dfrac{2a}{\sqrt{5}-1}$ rationalise denominator

$\dfrac{2a}{\sqrt{5}-1} \times \dfrac{\sqrt{5}+1}{\sqrt{5}+1} = \dfrac{2a(\sqrt{5}+1)}{(\sqrt{5})^2 - 1^2}$

$= \dfrac{2a(\sqrt{5}+1)}{4} = \dfrac{1}{2}a(\sqrt{5}+1)$

Finished! Tricky!

I've done this really quickly but I think it's right some of the numerical answers may not be correct so, //.