More number pyramids

Generalising from number

Prerequisite knowledge

- This problem builds on 'Number pyramids'

Why do this problem?

The main purpose of the problem is to work towards symbolic representation of rules.

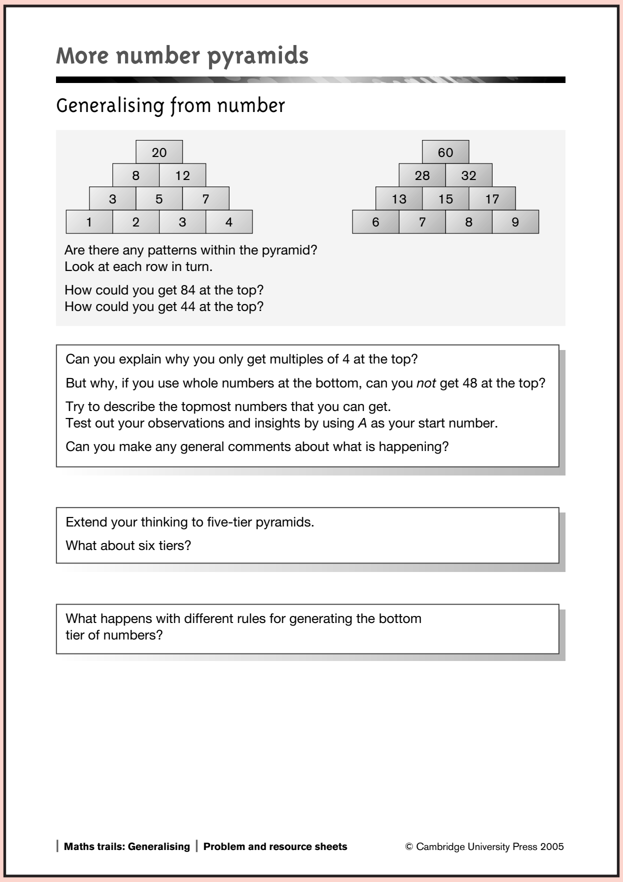
Time

One lesson

Resources

CD-ROM: pupil worksheet; resource sheets of blank four-, five- and six-tier pyramids

NRICH website (optional): www.nrich.maths.org, May 2005, 'More number pyramids' (includes an interactive tool that enables pupils to experiment with a four-tier pyramid)



More number pyramids

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Are there any patterns within the pyramid?
Look at each row in turn.

How could you get 84 at the top?
How could you get 44 at the top?

Can you explain why you only get multiples of 4 at the top?

But why, if you use whole numbers at the bottom, can you *not* get 48 at the top?

Try to describe the topmost numbers that you can get.
Test out your observations and insights by using *A* as your start number.

Can you make any general comments about what is happening?

Extend your thinking to five-tier pyramids.
What about six tiers?

What happens with different rules for generating the bottom tier of numbers?

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Introducing the problem

Ask pupils to give you a base number to put in the bottom left-hand corner of a four-tier pyramid. Fill in the rest of the bottom tier, adding one each time – do not explain what you are doing.

Model selecting each pair of numbers, putting their sum in the block above until you reach the vertex. Use actions in a way that emphasises the pyramid's structure (point at two numbers, then at the 'result' block and put in the sum). This can be done several times. It is useful to have a number less than 5 and one greater than 10 in two of the examples. Keep each completed pyramid to refer to later. Then draw out the rules involved in creating the pyramid by asking questions such as:

- How was the bottom line created?
- What patterns can you see in the pyramid?
- Will you always get odd numbers in the second row? Why?
- Can you explain why the third and fourth tiers only have even numbers?
- How would the pattern of odd/even numbers change if there were more tiers? Why?

Then, asking for another start number, ask pupils to complete the pyramid with you, explaining the rules they are using and the patterns they see.

Main part of the lesson

Look back at all the pyramids that the class have completed together.

- What numbers are at the top of each of the pyramids we have drawn?
- How can we get 84 at the top? (Referring to the two pyramids you have drawn, one with a number less than 5 and one with a number greater than 10 in the bottom left-hand corner, you could encourage a trial-and-improvement approach to achieving a top number of 84.)

Ask pupils to spend time looking for patterns which might enable them to work backwards from any top number, preferably without applying trial-and-improvement methods. The nice thing about this step is that pupils have the scope to tackle the problem in a way with which they feel most comfortable:

- some might continue with trial and improvement;
- some may notice that they can work backwards using the relationship 'halve +/– one';
- some may be able to achieve a solution using purely algebraic methods.

The problem sheet indicates other routes of enquiry, including considering pyramids with more than four tiers.

Plenary

Ask groups of four to six pupils to select one observation they have made and produce a poster for display to the rest of the class. Groups then go round the class in a circus fashion looking at each other's work.

Solution notes

For 84 the bottom left number is 9.

For 44 the bottom left number is 4.

The use of Pascal's triangle plus the fact that the base numbers are consecutive can yield an algebraic solution. If bottom left corner is A, the top number will be:

$$A + 3(A + 1) + 3(A + 2) + (A + 3) = 8A + 12$$

For a five-tier pyramid, if the bottom left corner is A, the top number will be $16A + 32$.

For a six-tier pyramid, if the bottom left corner is A, the top number will be $32A + 80$.

If you add a constant B instead of 1 to generate the bottom tier, the top number takes a similar form except that the second term will be a multiple of B. For example, a six-tier pyramid will have a top number of $32A + 80B$.