Square it

Planning ahead

Prerequisite knowledge

- Knowledge of properties of a square

Why do this problem?

This is an intriguing and absorbing game that allows users to expand their knowledge of squares as seen on the coordinate plane. It is a quick and easy game to play that requires strategic and tactical thinking. When playing the game pupils need to visualise squares in planning ahead.

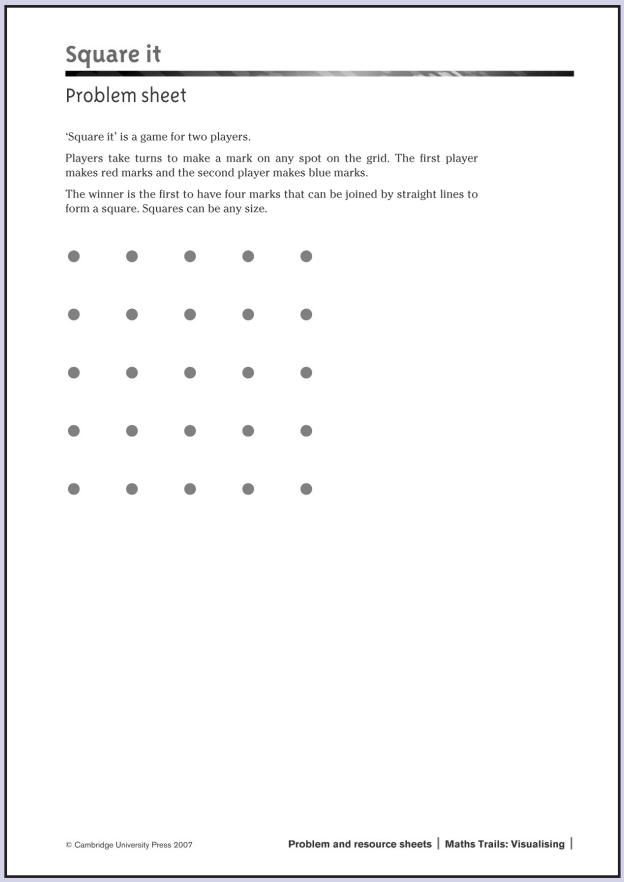
The problem also gives opportunities to pay particular attention to the analysis and synthesis, and planning, execution and interpretation phases of the problem-solving model.

Time

One lesson

Resources

A4 sheets, coloured pencils or pens
CD-ROM: problem sheet; resource sheets 1 and 2; interactivity
NRICH website (optional):
www.nrich.maths.org, October 2004, 'Square it'


Square it

Problem sheet

'Square it' is a game for two players.

Players take turns to make a mark on any spot on the grid. The first player makes red marks and the second player makes blue marks.

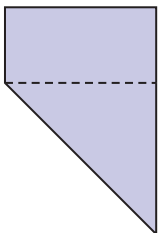
The winner is the first to have four marks that can be joined by straight lines to form a square. Squares can be any size.

Introducing the problem

Ask the class to imagine a square. Invite some to describe the square they have imagined, focusing on the language and properties associated with squares.

Ask everyone to make a square with A4 paper:

- Start with the piece of paper in portrait orientation. Take the bottom left-hand corner and fold it up to meet the right-hand vertical edge of the paper. Crease carefully. You will now have a shape like this (not to scale):



- Cut along the dashed line and discard the rectangle. If you unfold the paper, you now have a square.

Suggest that one pupil comes up and pins their square to the board. Ask another to pin up their square so that it looks different.

- Is it still a square? Is it the same square?

Main part of the lesson

Explain the rules of the game 'Square it'.

- The game is played on a dotty square grid by two players using different coloured pens.
- Players take turns to mark any spot with their coloured pen.

- The winner is the first to have four marks that can be joined by straight lines to form a square. Squares can be any size.

Using the interactivity, play the game as a class against the computer. Alternatively, the class could play against the teacher on a 5×5 dotty grid (see the problem sheet).

Encourage pupils to play against each other for about 5 minutes (see resource sheet 1).

Discuss strategies briefly using questions such as:

- Is one place better to start than another?
- How do you know?
- How many squares are possible in total?

Ask the class to pursue this last question: to find out the total number of squares and the number of squares possible from each point, using this to identify a strategy for trying to win the game. Remind them of the need to be systematic. It might be appropriate to use a 4×4 dotty grid at first (resource sheet 2).

If groups complete this they can start to play and unpick further strategies. For example, where should the second player go to optimise their chances of winning?

It is also possible to enlarge the grid on the interactivity for further challenges.

Plenary

Discuss the strategies pupils have found and then play against the computer, putting their strategies to the test.

Ask them which were good strategies and how they know. For example:

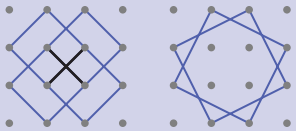
- A dot in the middle can be the vertex of more squares compared with a dot placed on the edge.
- Focusing on 'middle' points gives greater flexibility.

Solution notes

On a 4×4 dotty grid there are:

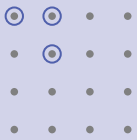
- nine 1×1 squares;
- four 2×2 squares;
- one 3×3 square;
- four small tilted squares;
- two larger tilted squares.

For example, for the small and large tilted squares:



So there is a total of 20 squares.

By symmetry, on a 4×4 dotty grid, there are only three possible positions for the points: at a vertex, on a side and in the middle.



The number of squares possible from each type of point are:

- three from a vertex point;
- five from a side point;
- seven from a middle point.