The parcel

Modelling and optimisation

Prerequisite knowledge

- Nets and volumes of cuboids
- Formulae in a spreadsheet
- Construction of two-way tables
- Absolute referencing (optional)

Why do this unit?

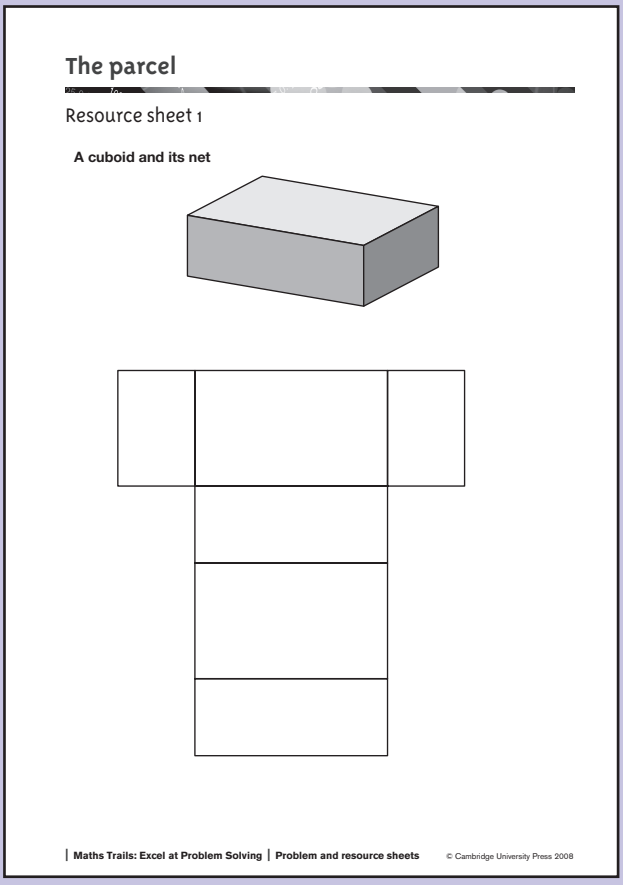
The aim is to model a situation to obtain a solution using trial and improvement. An initial task gives pupils time to become familiar with the setting for the second task by deconstructing a ready-made spreadsheet.

Time

One or two lessons

Resources

Individual whiteboards, a cuboid
CD-ROM: spreadsheet, problem sheets 1 and 2, resource sheets 1–3
NRICH website (optional): www.nrich.maths.org, January 2005, 'Sending a parcel'; July 2006, 'All wrapped up'; a number of optimisation problems were published in January 2005 including 'Fence it' (knowledge of area and perimeter of rectangles), 'Where to land' (requires Pythagoras' theorem) and 'Slippage' (requires some knowledge of trigonometry)


The parcel

Resource sheet 1

A cuboid and its net

Maths Trails: Excel at Problem Solving | Problem and resource sheets

Introducing the unit

The main objective of the introduction is to ensure pupils' confidence with the net(s) of a cuboid and the relationship between the dimensions of a cube and the measurements on a net – see resource sheet 1.

Show the class a cuboid.

- How many faces, edges and vertices does it have? [6, 12, 8]

Ask pupils to draw a net on their whiteboards or on paper. Discuss the equivalence of the different nets pupils have drawn. Ask pupils to select one edge and mark all the edges on their net that are the same length.

- How many different edge lengths are there? [3 – height, length and width]

Use the image on resource sheet 1 to establish how the width and the length of the net are made of three and four edge lengths respectively (see resource sheets 2 and 3 for examples).

Ensure pupils are comfortable with the relationship between the dimensions of a cuboid and the net before moving on to the main part of the unit.

Main part of the unit

This is in two parts. The first involves deconstructing the sheet 'Wrapping presents'

and the second part involves getting pupils to construct their own spreadsheet.

Show pupils the problem 'Wrapping presents' (problem sheet 1), which involves finding the largest cuboid that can be covered completely with a sheet of A3 paper. Ask them to spend a few minutes on their own trying to make sense of the problem, perhaps by drawing a diagram, and thinking about how they might go about finding a solution if they had a spreadsheet.

After a short time ask pupils to talk to a partner about their ideas and get ready to share thoughts with the rest of the class. It is worth spending some time while the pairs are working 'harvesting' some ideas to use in the next part of the lesson if suggestions are not forthcoming.

Share ideas with the whole class before looking at the spreadsheet 'Wrapping presents'. You might, however, prefer to construct the spreadsheet from scratch by talking through some of the ideas with the class, especially if pupils have come up with equally valid but quite different approaches. Examples of different approaches (including using spinners and two-way tables) can be found in the spreadsheets 'Parcel basic', 'Parcel with control for height', 'Parcel with spinner for height' and 'Parcel two-way table'.

- How was this sheet constructed? [Height is increasing by 1 unit, width and length are determined by the dimensions of the paper – see the diagrams on the spreadsheet.]
- What is the purpose of the spreadsheet? [It calculates lengths and widths from given heights and then calculates the volume.]
- Why are there two tables? [two orientations of the A3 sheet – see resource sheets 2 and 3]
- How does it help with finding a solution? [By scanning the Volume column it is easy to identify the maximum, 1512 cm^3, for both orientations (is that surprising?).]

Having spent time deconstructing the spreadsheet – building confidence both in its construction and how it relates to the original task and the net – pupils should be ready to tackle the task 'Sending a parcel' (problem sheet 2) in pairs. This problem also requires pupils to find a maximum volume but with different constraints. Remind pupils to plan what they will do with the spreadsheet before they start, checking their understanding of the problem after a few minutes.

Pupils who are finding the ideas difficult may wish to use the spreadsheet 'Parcel' to get started. Their aim will be to extend the table and create new tables with different heights, homing in on a solution.

Things to look out for:

- Some pupils may struggle with the need to deal with two variables – how could they do this systematically? [Keep one variable constant to start with and vary the other.]
- Some pupils may wish to use the idea of two-way tables and may need support with absolute referencing to do this.

Plenary

Discuss the different methods adopted by the group and their solutions. Key points to draw out include how their models enabled the groups to home in on solutions and why the use of a spreadsheet aided efficiency.

Solution notes

The maximum volume for part 2 of the problem is 74 052 cm^3 if the dimensions of the parcel are in cm (see the spreadsheet).