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FIBONACCI 1

The Fibonacci sequence appears in a variety of natural contexts and has many intriguing properties. There are also many ways the sequence can be generated. For example:

- The number of ways to ascend a set of stairs by going up one or two stairs at each step.
- The number of ways of making different amounts of money using £1 and £2 coins.
- The number of ways of filling a 2-by-n 'path' using paving slabs measuring 2-by-1 and 1-by-2. For example in a 2-by-4 path, there are five possible solutions.
- On a spreadsheet by typing:
  1 in cell A1,
  1 in cell A2,
  = A1 + A2 in cell A3 then filling down.

The following ideas are based upon tasks once the sequence has been generated

EXTENDING THE SEQUENCE.

Having generated the first few values of the sequence, work out all the values up to, say, the twentieth term. This can be a useful context for practising mental arithmetic.

How many terms does it take to get above 1 million?

GENERATING THE GOLDEN RATIO (1.618034 ...)

This is gained by dividing successive terms, i.e.
$2 \div 1, 3 \div 2, 5 \div 3 \ldots 10946 \div 6765$.

By reversing these calculations, i.e.
$1 \div 2, 2 \div 3, 3 \div 5 \ldots 6765 \div 10946$, a result of the previous limit less 1 is gained.

This task might be used as a context for setting up a quadratic equation, i.e. $1/\Phi = \Phi - 1$ leading to $\Phi^2 - \Phi = 1$.

GRAPHING THE ABOVE RESULTS

By graphing the two sets of results from the previous task on the same axes, students will have an opportunity to appreciate the notion of the values settling out to the Golden Ratio (Φ) and to the Golden Ratio minus 1 ($\Phi - 1$).

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FIBONACCI 2

These are further ideas that students can explore based upon the Fibonacci sequence and they draw upon a range of mathematical concepts and skills, for example, working with negative numbers, using and applying algebra and forming generalizations.

SUMMING A FIBONACCI SEQUENCE UP TO THE TENTH TERM

This sum can be calculated by multiplying the seventh term by 11. The question is, why? For example: 7, 4, 11, 15, 26, 41, 67, 108, 175, 283 sums to 737 which is 67×11.

SOLVING 'NUMBER CELL' PROBLEMS.

This idea is aimed at developing students' symbolic manipulating skills. Students can work in pairs, each producing a number of five-cell sequences, for example: 7, 3, 10, 13, 23 or 5, –2, 3, 1, 4, and so on.

Having done this they give their partner the first and the last value for each set of numbers, for example: 7 _ _ _ 23 or 5 _ _ _ 4. Students then have to try to calculate each other's missing values.

A development is for students to construct a formula to describe how f (the first number) and l (the last number) in each five-cell can be manipulated to find m (the middle number).

Using the symbols f, l and m, the equation $m = \frac{1}{3}(f + l)$ can be constructed.

Students can check this works for any values.

A further exploration aimed at producing formulae for finding the middle values when seven, nine and eleven-cell, etc. are considered (given the first and last values), reveals an interesting result. This will challenge students to construct formulae based upon the information gathered.