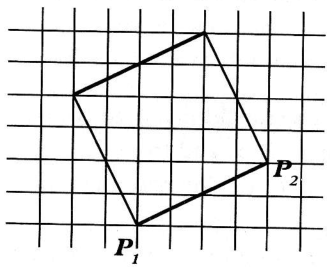
IDEA 61

EXPLORING COORDINATES AND VECTORS

These ideas build on Problem 2 described in Idea 60. The purpose is for students to see how the system of coordinates relates to the system of vectors.

PROBLEM 1

If P_1 and P_2 are adjacent corners of a square, find points P_3 and P_4 such that P_1, P_2, P_3 and P_4 form a square (there are two solutions to this problem).



Examine the four vectors formed between pairs of adjacent corners of the square.

Try to generalize.

PROBLEM 2

If P_1 and P_3 are opposite corners of a square P_1, P_2, P_3, P_4, find points P_2 and P_4. Examine the vectors formed between opposite corners of the square and relate these to the vectors formed between adjacent corners. Try to generalize.

PROBLEM 3

Work out the area of a number of squares and write one of the vectors formed between a pair of adjacent corners for each square.

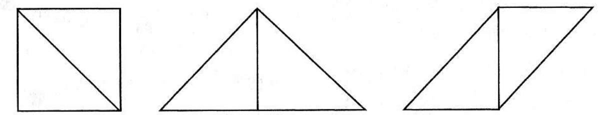
Look for a connection between the area and the chosen vector for each square. Try to generalize.

The general result is, of course, Pythagoras' theorem and though there is still much work to be done, i.e. to see the vector as the hypotenuse of a right-angled triangle, students will nevertheless be on the way towards making sense of the formula $a^2 + b^2 = c^2$.

IDEA 62

JOINING TRIANGLES

Shapes A, B and C are each made by joining two right-angled isosceles triangles together. The joining 'rule' is placing sides of equal length together.



SHAPE A SHAPE B SHAPE C

This idea might begin by starting with a square and slicing this into two triangles along a diagonal. The two triangles can be used to form Shape B; by carrying out a rotation of one triangle Shape C can be formed.

The following questions/tasks might then be considered:

- What is the angle sum for each shape?
- What are the symmetries of the shapes?
- What is the perimeter of each shape? (This can either be written algebraically or in surd form; the latter requiring knowledge of Pythagoras' theorem.)
- How many shapes can be made with three triangles? How many for four triangles?
- Questions about angle sums, symmetries and perimeter can be similarly considered.
- Students can be challenged to construct a system to show (prove) they have found all possible shapes using, say, four triangles.
- How many shapes can be formed if, say, four equilateral triangles are used instead of four right-angled isosceles triangles?
- What happens if 30°, 60° or 90° triangles are used?

BIG triangle templates may be useful for display purposes.