IDEA

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DOUBLING AND DOUBLING AND . . .

There are several problems based upon the idea of the speed at which growth occurs as a result of continually doubling. A classic problem is the grains of rice and the chessboard problem. Here a story about placing one grain of rice on the first square, two grains of rice on the second square, four grains of rice on the third square . . . leads to a question about how many grains of rice there will be if this process continues for all 64 squares.

The important concept is for students to see how quickly the values increase; such problems also cause students to work on the arithmetic of doubling and, as in the problem above, addition. Here are two more problems based upon the same concept.

1 Take a sheet of newspaper and tear it in half. Place the two halves together. This is Stage 1. Tear Stage 1 pile in half and place these two halves together. This is Stage 2. Repeat this process a lot of times.

  The problem is as follows: If the paper was, say, 0.01 cm thick what will the height of the pile be after 10 stages, 20 stages, 50 stages? What is the smallest number of stages you will need to get higher than Mt Everest?

2 Starting with 1p, how many times must this be doubled to get to £1 million?

IDEA

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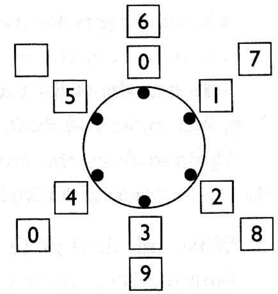
DOUBLING IN MODULAR ARITHMETIC

I first met this idea in the Association of Teachers of Mathematics (ATM) journal *Mathematics Teaching* 84; the purpose is for students to consider properties of numbers and explore a different branch of mathematics, i.e. modular, or 'clock' arithmetic.

To illustrate how modular arithmetic works an example of mod 6 is given below.



In mod 6 all numbers are represented by one of the following values: 0, 1, 2, 3, 4, 5.

So 6 is represented by zero, 7 is represented by 1, 8 is represented by 2, and so on.

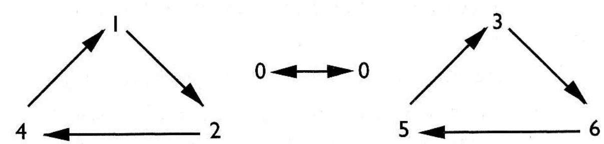
This idea is to consider what happens when numbers are successively doubled in different mods. In mod 6, starting with 1, a doubling chain forms as follows:

$1 \to 2 \to 4 \to 8$. However, 8 in mod 6 is 2, so we gain the following: $1 \to 2 \leftrightarrow 4$.

Considering chains for numbers that have not so far appeared i.e. 0, 3 and 5, we gain the following in formation: $0 \to 0 \quad 3 \to 0 \quad 5 \to 4 \leftrightarrow 2$.

Collecting all this information together we have: $1 \to 2 \leftrightarrow 4 \to 5$ and $3 \to 0$.

Exploring doubling chains in mod 7, the following three diagrams are formed:



The task is for students, working in groups, to produce a lot of information for doubling in different mods, say up to mod 50, with different mods allocated to different groups. Using sugar paper and large marker pens students can produce a display for analysis.