IDEA 87

GRAPHING FACTOR PAIRS

Finding opportunities for students to see concepts emerge in different contexts, and see how concepts are connected together in different ways, is necessary if mathematics is to be perceived as a coherent discipline.

This idea is initially aimed at the upper KS2–lower KS3 age range and will require students to have access to a simple (non-scientific) calculator and connects together concepts of factors and drawing graphs.

Choose a number that has several pairs of factors, for example, 24.

Write these pairs of factors as coordinate pairs, i.e. (1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1).

By plotting these points on a graph and joining them together with a smooth curve, students will effectively drawing the graph of $y = 24/x$.

An extension is to consider values that are not divisors of 24, for example 5, 7, 9. By examining the graph to see what the missing ordinates are in each coordinate pair (5, ?), (7, ?) etc, students can see how accurate, or otherwise, their readings are. Once the missing ordinates have been determined (within degrees of accuracy provided by the graph), students can check how close they are by multiplication [$5 \times 4.8 = 24$] and/or by division [$24 \div 5 = 4.8$].

Students will have departed from working with whole number solutions at this point. However, the key issue is that all the points between whole number coordinate pairs continue to produce points on the graph.

When are the two dimensions equal? This question could be used as a task in its own right and is aimed at students making sense of the concept of a square root, particularly if different degrees of accuracy are searched for prior to directing any of the students to the existence of the $\sqrt{}$ key.

What does the graph look like for a starting value other than 24?

What happens if graphs for different starting values are drawn on the same pair of axes?

IDEA 88

EXPANDING AND FACTORIZING QUADRATICS

This is a 'practise and consolidation' idea and requires students to learn the 'mechanics' of multiplying out expressions in pairs of brackets. Teachers will have different ways of explaining this, although my preferred approach is to use a two-way grid method as for multiplying numbers greater than ten together.

Inviting students to work in pairs may prove useful.

Provide students with sheets of A3 or sugar paper.

Ask them to draw a line down the middle of their sheet (portrait orientation) and write their name(s) at the top of each half.

On the left-hand side of the line they write a number of pairs of factorized expressions, for example $(x + 2)(x + 5)$.

Sarah and Tom	Sarah and Tom
$(x + 2)\ (x + 5)$	$x^2 + 7x + 10$

Using whatever method they know/have been taught, they work out the expansion of their expression, for example, $x^2 + 7x + 10$ and write this on the other side of the line.

Students could do a lot of these. I suggest they start with easy expressions and gradually write harder ones, i.e. starting with factors involving (+ and +), then (+ and –) or (– and +), then (– and –) with the x terms having a coefficient of 1.

Each person now has a number of factorized expressions on the left-hand side of their paper and the equivalent expansions on the right-hand side. Next they cut their sheet down the dividing line and swap their expanded expressions with another person. Pairs now try to work out what the factorizations are for each expression. Students subsequently check their answers with one another.