Consecutive sums

Problem sheet 1

Adding consecutive numbers

First do it the long way. Add up 1 + 2 + 3 + 4 + 5. What's the answer?

Now see the pattern another way.

The sequence can be written a second time in reverse order.

1 + 2 + 3 + 4 + 5
5 + 4 + 3 + 2 + 1

Notice that each number in the lower line makes a sum of 6 with the number directly above it.

Why do all these individual pairs have the same total?

In all, there are 5 of these pairs, all of which make 6, so the total of all the numbers in both lines must be 5 lots of 6 (that's 30).

Both lines have the same total so they must be 15 each (30 divided by 2).
So now we know that the numbers 1 to 5 have a sum of 15.

What about the sum of the numbers 1 to 10?
Write them down in line and again in a second line but backwards.
What will each number in the lower line make with the number directly above it this time?
How many pairs will there be?
So what is the total of all the numbers together in both lines?
And then you'll know the total in each line.
You have found the answer to 1 + 2 + … + 10 but without having to add them all!

Try finding the sum of 1 to 20 using this method.

What is the sum of the numbers 1 to 50? to 100? to 1000?

Could you write a formula explaining how to sum 1 to n?

Can you see how to use this method to add 10 + 11 + . . . + 20?

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