

Core 2 January 2008

① Area of sector = $\frac{1}{2} r^2 \theta = \frac{1}{2} \times 11^2 \times 0.7$

Area of triangle = $\frac{1}{2} ab \sin C = \frac{1}{2} \times 11 \times 11 \times \sin 0.7$

$$\frac{1}{2} \times 11^2 \times 0.7 - \frac{1}{2} \times 11^2 \times \sin 0.7 = 3.37 \text{ cm}^2$$

② $\int_1^7 \sqrt{x^2 + 3} \, dx$ 3 strips/intervals

$$x_0 = 1$$

$$y_0 = 2$$

$$x_1 = 3$$

$$y_1 = \sqrt{12}$$

$$x_2 = 5$$

$$y_2 = \sqrt{28}$$

$$x_3 = 7$$

$$y_3 = \sqrt{52}$$

$$\frac{1}{2} \times 2 \times (2 + \sqrt{52} + 2(\sqrt{12} + \sqrt{28})) = 20.92$$

③ i) $\log_a 2 + \log_a 3 = \log_a 6$

ii) $2 \log_{10} x - 3 \log_{10} y = \log_{10} \left(\frac{x^2}{y^3} \right)$

④ i) $\frac{\sin 62}{BD} = \frac{\sin 50}{16}$

$$BD = \frac{16 \sin 62}{\sin 50} = 18.44 \text{ cm}$$

ii) $a^2 = b^2 + c^2 - 2bc \cos A$

$$18.44^2 = 20^2 + 10^2 - 2 \times 20 \times 10 \cos A$$

$$\frac{18.44^2 - 20^2 - 10^2}{-2 \times 20 \times 10} = \cos A$$

$$\underline{\underline{A = 66.4^\circ}}$$

$$(5) \quad \frac{dy}{dx} = 12\sqrt{x} = 12x^{1/2}$$

$$y = \frac{12x^{3/2}}{3/2} + k = \underline{\underline{8x^{3/2} + k}}$$

Passes through (4, 50)

$$50 = 8 \times 4^{3/2} + k$$

$$50 = 8 \times 8 + k$$

$$k = -14$$

Equation is

$$y = 8x^{3/2} - 14$$

$$(6) \quad i) \quad 7, 9, 11$$

ii) Arithmetic

$$iii) \quad S_n = \frac{1}{2}n(2a + (n-1)d)$$

$$2200 = \frac{1}{2}n(2 \times 7 + (n-1)2)$$

$$2200 = \frac{1}{2}n(14 + 2n - 2)$$

$$2200 = \frac{1}{2}n(2n + 12)$$

$$4400 = 2n^2 + 12n$$

$$2200 = n^2 + 6n$$

$$n^2 + 6n - 2200 = 0$$

$$(n + 50)(n - 44) = 0 \quad n = -50 \text{ or } \underline{\underline{n = 44}}$$

⑦ i) The area under the x-axis is negative
the area above is positive

$$\begin{aligned}\text{ii) } \int_0^3 (x^2 - 3x) \cdot dx &= \left[\frac{x^3}{3} - \frac{3}{2}x^2 \right]_0^3 \\ &= \left(\frac{27}{3} - \frac{27}{2} \right) - (0) = \frac{-27}{6} = -\frac{9}{2}\end{aligned}$$

So the area is $\frac{9}{2}$

$$\int_3^5 x^2 - 3x \cdot dx = \left[\frac{x^3}{3} - \frac{3}{2}x^2 \right]_3^5$$

$$= \left(\frac{125}{3} - \frac{75}{2} \right) - \left(\frac{27}{3} - \frac{27}{2} \right)$$

$$= \left(\frac{250}{6} - \frac{225}{6} \right) - \left(\frac{-27}{6} \right)$$

$$= \frac{52}{6} = \frac{26}{3}$$

⑧ i) 4th term in $10 \times 0.8^3 = 5.12$

$$\begin{aligned}\text{ii) } S_n &= \frac{a(1-r^n)}{1-r} = \frac{10 \times (1-0.8^{20})}{1-0.8} \\ &= \underline{\underline{49.4}}\end{aligned}$$

$$iii) \frac{a}{1-r} - \frac{a(1-r^n)}{1-r} < 0.01$$

$$= \frac{10}{0.2} - \frac{10(1-0.8^n)}{0.2} < 0.01$$

$$10 - 10(1-0.8^n) < 0.002$$

$$10 - 10 + 10 \times 0.8^n < 0.002$$

$$10 \times 0.8^n < 0.002$$

$$0.8^n < 0.0002$$

$$\log 0.8^n < \log 0.0002$$

$$\begin{array}{l} | \\ | \\ | \\ \cdot \cdot \cdot \end{array} \quad n \log 0.8 < \log 0.0002$$

$$\quad \quad \quad n > \frac{\log 0.0002}{\log 0.8} = 38.2$$

$$\therefore \underline{\underline{n = 39}}$$

$$(9) i) \text{Min } (-90, -2) \quad \text{Max } (90, 2)$$

$$ii) a) 180 - \alpha$$

$$b) -\alpha$$

$$\hookrightarrow 2 \sin x = 2 - 3 \cos^2 x$$

$$2 \sin x = 2 - 3(1 - \sin^2 x)$$

$$2 \sin x = 2 - 3 + 3 \sin^2 x$$

$$3 \sin^2 x - 2 \sin x - 1 = 0$$

$$(3\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{3} \quad \text{or} \quad \sin x = 1$$

$$\hookrightarrow x = 90$$

$$\sin^{-1} = -19.5 \text{ or } -160.5$$

$$x = -19.5, 160.5, 90$$

$$\textcircled{10} \text{ i) } \binom{4}{0}(2x)^4 + \binom{4}{1}(2x)^3 5 + \binom{4}{2}(2x)^2 5^2 + \binom{4}{3}2x \cdot 5^3 + \binom{4}{4}5^4$$

$$= 16x^4 + 160x^3 + 600x^2 + 1000x + 625$$

$$\text{ii) } (16x^4 + 160x^3 + 600x^2 + 1000x + 625)$$

$$- (16x^4 - 160x^3 + 600x^2 - 1000x + 625)$$

$$= 320x^3 + 2000x$$

$$\text{iii) } 320x^3 + 2000x = 3680x - 800$$

$$320x^3 - 1680x + 800 = 0$$

$$4x^3 - 21x + 10 = 0$$

$$\textcircled{\div 80} \quad \text{if } x=2 \quad 4 \times 8 - 21 \times 2 + 10$$

$$= 32 - 42 + 10 = 0$$

$$4x^3 - 21x + 10 = (x-2)(Ax^2 + Bx + C)$$

$$= Ax^3 + Bx^2 + Cx - 2Ax^2 - 2Bx - 2C$$

$$= Ax^3 + (B-2A)x^2 + (C-2B)x - 2C$$

$$A = 4$$

$$B - 2A = 0$$

$$C - 2B = -21$$

$$-2C = 10$$

$$A = 4$$

$$B = 8$$

$$C = -5$$

$$4x^3 - 21x + 10 \equiv (x-2)(4x^2 + 8x - 5)$$

$$= (x-2)(2x+5)(2x-1)$$

$$x = 2 \text{ or } -\frac{5}{2} \text{ or } \frac{1}{2}$$
