One out, some under

Stepping into a problem

Prerequisite knowledge

- Knowledge of multiples and factors

Time

Two lessons

Why do this problem?

This problem requires pupils to visualise with numbers rather than in a geometric context.

The problem also gives opportunities to pay particular attention to the planning, execution and interpretation phase of the problem-solving model.

Resources

Pack of cards
NRICH website (optional): www.nrich.maths.org; for an extension activity look at 'Counting cards' (July 2003)

Introducing the problem

Prepare a stack of 10 cards in numerical order, from 1 on the top to 10 on the bottom. After pupils have had a chance to visualise each step (see below), use the cards to model what they have been visualising. This instant feedback will support pupils and keep them on task.

Ask the class to imagine a stack of 10 cards in order from 1 (ace) on the top to 10 on the bottom. Then ask the class:

- If I remove the top card and put it on the bottom, what is the card on the top now? [2]
- If I repeat this process, carrying on from where I finished, but this time move four cards, what is the card on the top this time? [6]
- If I move six cards this time, what is the number on the top now? [2]

Repeat with different numbers of cards until the class are confident.

Main part of the lesson

You may wish to do the following without a stack of cards. Alternatively, you might want to use a stack to start with to model the process and to check answers.

Ask the class to imagine:

- a stack of four cards;
- each card is labelled with a number 1, 2, 3 or 4;
- they are arranged in numerical order with the card labelled 1 on the top and the card labelled 4 at the bottom of the pack.

Say that the cards are dealt one out, one under, one out, one under, one out, one under, ...

- What is the last card left in your hand? [4]

Now ask the class to imagine a stack of 6 cards, dealing the cards in the same way.

- What is the last card left in your hand? [4]
- Are you surprised by the answer?

Now ask the group to consider a stack of 10 cards, dealing them in the same way.

- What would be the last card left in your hand? [4]
- How can you be sure and how can you check your theories?

Ask the group to investigate stacks of different numbers of cards and to justify any rules they discover.

Extension

- With a stack of 40 cards what is the last card left in your hand? [32]
- Consider a stack of 100 cards. What would be the last card left in your hand this time? [64]
- Can you always predict which will be the final card in your hand no matter how many there are in the pack?

Pupils could consider different numbers of cards out and look for further generalisations.

Plenary

To share their work, small groups can produce posters of their discoveries, theories and explanations.

Solution notes

If the number of cards is a power of 2, the last card in your hand is the last card in the pile, eg 16 cards, last card is 16.

For $2^n + 1$ cards, the last card is 2, e.g. 17 cards, last card is 2.

For $2^n - 1$ cards, the last card is the last but one card ($2^n - 2$), e.g. 15 cards, the last card is 14.

Number of cards	Last card
4	4
5	2
6	4
7	6
8	8
9	2
10	4
11	6
12	8
13	10
14	12
15	14
16	16
17	2
18	4
19	6
20	8

Explaining some of the results is demanding.