Make 37

Framing

Prerequisite knowledge

- Familiarity with even and odd numbers
- Mental methods of addition and subtraction

Why do this problem?

This problem, which on first glance appears very simple, has little structure; encouraging pupils to explore in order to 'get into' the mathematics. The fact that an answer is impossible may be a revelation to students, but the mathematical reason behind this requires relatively low level knowledge – it is how to explain this reason clearly that makes the problem more challenging and pushes students' understanding. It might be necessary to simplify the problem (for example, 'pick any six numbers from the bags above so that their total is 21') so that addition errors are minimised.

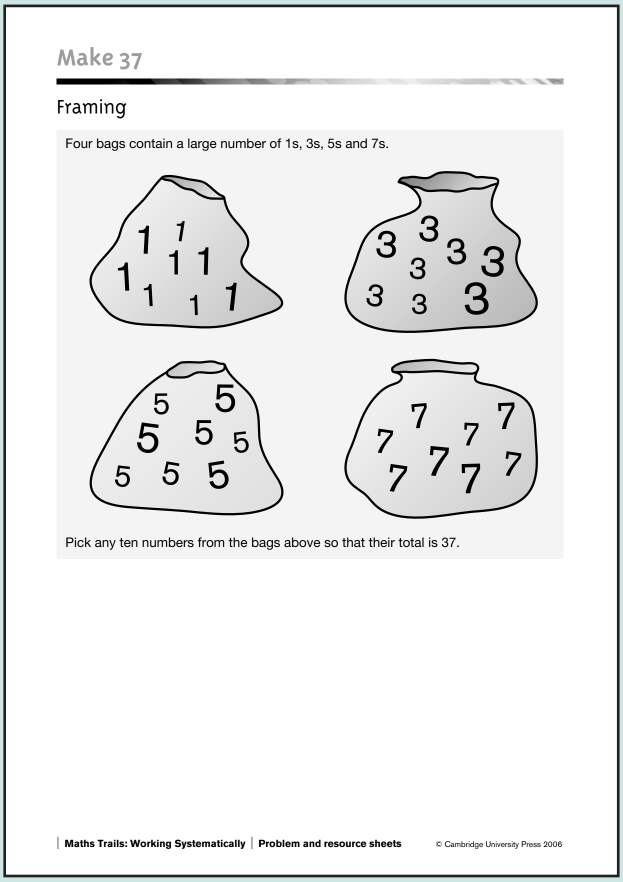
Time

Half a lesson or more

Resources

CD-ROM: problem sheet.

NRICH website (optional): www.nrich.maths.org, October 2003, 'Make 37'


Make 37

Framing

Four bags contain a large number of 1s, 3s, 5s and 7s.

Pick any ten numbers from the bags above so that their total is 37.

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A simple spreadsheet programmed to calculate the sum of the chosen numbers might be a useful tool.

Introducing the problem

A good way to start is by simply stating what the problem is – get the group to choose any ten numbers and find the total. Then give the pupils time to play, trying out different numbers for themselves. It might be a good idea to encourage them to check their results with a neighbour.

Main part of the lesson

After the initial investigation period, ask pupils what numbers they *have* been able to make. Ask questions such as:

- What numbers can you make?
- Have you kept a record of the totals?
- What do you notice about your totals?
- What can you say about these types of numbers?

Turning pupils' attention to the properties of the numbers that *are* possible will encourage them to make conjectures. Have they noticed that all the totals are even?

This may alert them to the fact that making 37 is much harder than they thought or does not seem to be possible. The crux of the question is to explain *why* it is not possible to make 37 and to generalise the circumstances. At this point, encourage pupils to talk to a partner about why a solution is impossible. Invite them to share their thoughts using the following prompts:

- Can you make 37 with a different number of numbers? How many?
- Can you explain why?
- How do you know two odd numbers added together make an even? It is not enough to give a few examples or say 'I just do'.
- Can you convince us all of your conclusion (prove it)?
- What have you learnt today that you did not know before?

Drawing diagrams of odd and even numbers (see the solution notes) may help pupils to visualise the mathematics.

Plenary

A follow-up question along the lines of 'How could you change the problem so that we *could* make 37 from ten numbers?' provides a useful assessment opportunity during the plenary. It can sometimes be the case that, as teachers, we believe pupils have fully understood a concept. In fact, providing a slightly different context or altering the parameters a little sometimes reveals that pupil's understanding is not as deep as we might at first have thought.

Solution notes

The sum of an even number of odd numbers cannot make an odd number. You can make 36 and 38 using 10 numbers but not 37. You can make 37, but by using 9 numbers. Here are some examples:

36 (10 numbers): $5 + 5 + 5 + 5 + 5 + 3 + 3 + 3 + 1 + 1$

38 (10 numbers): $1 + 1 + 1 + 3 + 3 + 5 + 5 + 5 + 7 + 7$

37 (9 numbers): $5 + 5 + 5 + 5 + 5 + 5 + 5 + 1 + 1$

One way of thinking of this is if you have two odd numbers and add them together, the one that is left over from each odd number makes a pair and can be shared. So two odd numbers make an even.

Another approach – every odd number ends in 1, 3, 5, 7 or 9. It is not necessary to look at all possible combinations of all pairs of odd numbers, as just the digits can be considered. An addition table can be used:

	1	3	5	7	9
1					
3					
5					
7					
9					