# Arithmagons

## Generalising from number

### Prerequisite knowledge

- Basic knowledge of directed numbers
- Addition and subtraction of fractions

### Why do this problem?

This family of problems involves the exploration of patterns and relationships among numbers forming arithmagons. The aim of the first session is for pupils to recognise that the sum of the two adjacent ‘vertex’ numbers of a triangular arithmagon equals the number in the middle of the side joining them. Pupils may be able to identify and use simple algebraic techniques to solve triangular arithmagons generally.

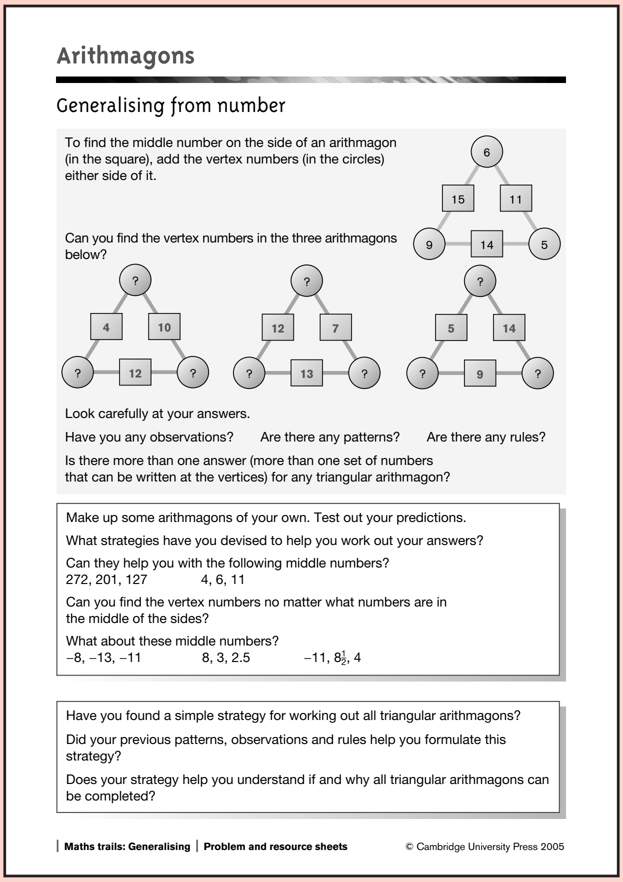
The second session focuses on square arithmagons and their particular properties, with the possibility of extension to *n*-sided arithmagons.

### Time

Two or more lessons – it may be worth revisiting this topic over a term or year with different (sided) arithmagons.

### Resources

CD-ROM: pupil worksheets ‘Arithmagons’ and ‘Square arithmagons’; OHTs/resource sheets with blank triangular and square arithmagons for teacher demonstration or pupil use



Arithmagons

Generalising from number

To find the middle number on the side of an arithmagon (in the square), add the vertex numbers (in the circles) either side of it.

Can you find the vertex numbers in the three arithmagons below?



Look carefully at your answers.

Have you any observations? Are there any patterns? Are there any rules?

Is there more than one answer (more than one set of numbers that can be written at the vertices) for any triangular arithmagon?

Make up some arithmagons of your own. Test out your predictions.

What strategies have you devised to help you work out your answers?

Can they help you with the following middle numbers?
272, 201, 127 4, 6, 11

Can you find the vertex numbers no matter what numbers are in the middle of the sides?

What about these middle numbers?
−8, −13, −11 8, 3, 2.5 −11, $8\frac{1}{2}$, 4

Have you found a simple strategy for working out all triangular arithmagons?

Did your previous patterns, observations and rules help you formulate this strategy?

Does your strategy help you understand if and why all triangular arithmagons can be completed?

Maths trails: Generalising | Problem and resource sheets © Cambridge University Press 2005

NRICH website (optional): www.nrich.maths.org, March 2005, ‘Arithmagons’ (includes an interactive tool that enables pupils to explore triangular arithmagons)

### Session 1: Introducing the problem

Start with a triangular arithmagon with the numbers in the vertices. Calculate the ‘middle’ numbers on each side in order to establish the rule in pupils’ minds:

> The middle number on each side of the arithmagon is equal to the sum of the two vertex numbers.

Then move to an arithmagon where pupils are given the middle numbers and have to work out the vertex numbers (not so easy). Use the suggested numbers on the problem sheet as the starting point. Pupils will need several minutes for this.

### Main part of the lesson

When pupils are beginning to obtain solutions to the three starters, invite them to share their findings.

- Have you all got the same solutions?
- Is there more than one answer?
- Do you notice any patterns?
- Are there any rules?

Ask pupils to make arithmagons of their own in order to test their predictions. The problem sheet suggests some follow-up questions and examples to push their thinking. When they have a theory and have convinced themselves it is always true, ask them to convince a neighbour. When they feel confident ask them to write their theory in a 'conjectures' area of the board or display area. The aim is for pupils to feel they can give a convincing argument to the rest of the class in the plenary session.

## Plenary

What rules have pupils found?

What different strategies did they use to find the rules? It is important to share the multiple approaches pupils have adopted.

Can they convince the class (and you) that their rule is always true? Key ideas could be kept on display for reference in the second session.

## Session 2: Introducing the problem

Start with a reminder of the findings from the session on triangular arithmagons, referring to the conjectures pupils 'proved'.

Introduce the idea of extending the principles to square arithmagons.

- Do you think square arithmagons will follow the same rules?
- Will they need adapting? If so, how?
- How might you go about testing whether any of the conjectures or versions of the conjectures still hold true?

This should lead to pupils testing their ideas with some square arithmagons. It might be useful to suggest some starting points – that is, a square arithmagon that works and one that does not. (See the problem sheet 'Square arithmagons' for suggestions.)

## Main part of the lesson

Pupils work towards refuting or supporting conjectures they made in the introductory part of the session and coming up with new conjectures which they can share in a similar way to before (convince yourself – convince a friend – prepare to convince the class).

## Plenary

The interesting thing here is that the mathematics feels counter-intuitive because what worked for triangular arithmagons does not work in the same way for square arithmagons.

- What is the same and what is different?

Applying the same rules leads to the fact that some square arithmagons have an infinite number of solutions and some have none.

- What are the conditions that make square arithmagons possible?

You might wish to end the session with the following question for pupils to consider:

- Can you extend this to other polygons? Do you think you can find similar rules? For example, will odd-sided arithmagons be like triangular arithmagons and even-sided like square ones?

## Solution notes

Some things to notice in triangular arithmagons:

> The sum of the middle numbers is twice the sum of the vertex numbers. One middle number plus the opposite vertex number is equal to the sum of the vertex numbers (which is equal to half the sum of the middle numbers).

While it is not intended that pupils at this stage use algebra to write the rule for triangular arithmagons, they should be able write the rule(s) in words.

For square arithmagons:

> There is an infinite number of solutions to square arithmagons provided the sums of opposite middle numbers are equal. Otherwise there is no solution.