

## SOLUTIONS

4736

D1

June 2007

FINAL

1	(i) Example: $V \rightarrow P \rightarrow Q \rightarrow X \rightarrow R \rightarrow V$ or $P \rightarrow Q \rightarrow X \rightarrow R \rightarrow V$ (ii) It passes through J twice (iii) 5 (iv) $t$ neither $B$ semi-Eulerian (v) $d = 2$ $h = 1$ (vi) There are 4 odd nodes ( $V, P, X$ and $Z$ ). To connect these we must add 2 arcs	B1 B1 1 B1 1 B1 1 B1 2 B1 B1 2 M1 2 A1 9	Any valid cycle (closed and does not repeat vertices, need not be a Hamiltonian cycle) Or it includes a cycle (accept 'loop') If graphs are not specified, assume $d$ is first If graphs are not specified, assume $d$ is first $t, B, h \rightarrow B'$ only Seen or implied For 2																																																															
2	(i) $d + t + g = 120$ (ii) "Area of grass is not more than 4 times (area of) decking" (iii) $d = t$ (iv) $g = 40$ $\min d = 10$ $\min t = 20$ (v) $5g + 10d = 20t$ or $g + 2d = 4t$ (vi) Minimise $g + 2d + 4t$ Subject to $d + t + g = 120$ $g + 4d = 0$ $d + t = 0$ $g = 40$ and $d = 10, t = 20, g = 0, d = 0$	B1 1 B1 B1 1 B1 B1 B1 3 B1 M1 B1 M1 3 10	For this equality. Condone an inequality Identifying the constraint in words (not just 'grass is less than or equal to 4 times decking' though) Do not accept $d = t$ Do not accept $g = 40$ $d = 10$ $t = 20$ Or any positive multiple of this For a reasonable attempt at setting up the minimisation problem using their expressions For dealing with this slack variable correctly (variables on LHS and constant on RHS) For a complete (or most) formulation (accept $d$ and $t = 0$ , or their min values for $d, t$ )																																																															
3	(i) <table><tr><td></td><td>8</td><td>6</td><td>9</td><td>7</td><td>5</td><td>Comps</td><td>Swaps</td></tr><tr><td>After 1st pass</td><td>6</td><td>8</td><td>9</td><td>7</td><td>5</td><td>1</td><td>1</td></tr><tr><td>After 2nd pass</td><td>6</td><td>8</td><td>9</td><td>7</td><td>5</td><td>1</td><td>0</td></tr><tr><td>After 3rd pass</td><td>6</td><td>7</td><td>8</td><td>9</td><td>5</td><td>3</td><td>2</td></tr><tr><td>After 4th pass</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>4</td><td>4</td></tr></table> <p>Comparisons must be 1, 2, 3 or 4 with total = 10 Swaps must be 0, 1, 2, 3 or 4 and no more than corresponding number of comparisons</p>		8	6	9	7	5	Comps	Swaps	After 1st pass	6	8	9	7	5	1	1	After 2nd pass	6	8	9	7	5	1	0	After 3rd pass	6	7	8	9	5	3	2	After 4th pass	5	6	7	8	9	4	4	M1 M1 M1 M1 M1 B1 B1 6	Bubble sort or decreasing order loses first 4 marks 1st pass correct 2nd pass correct, follow through from 1st pass 3rd pass correct, follow through from 2nd pass 4th pass correct Counting comparisons for at least three passes Counting swaps for at least three passes																							
	8	6	9	7	5	Comps	Swaps																																																											
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	(ii) <table><tr><td>Step 1</td><td>A</td><td>8</td><td>6</td><td>9</td><td>7</td><td>5</td></tr><tr><td>Step 2</td><td>A</td><td>6</td><td>9</td><td>7</td><td>5</td><td>X 8</td></tr><tr><td>Step 3</td><td>A</td><td>9</td><td>7</td><td>5</td><td>B</td><td>6</td></tr><tr><td>Step 4</td><td>A</td><td>7</td><td>5</td><td>C</td><td>9</td><td></td></tr><tr><td>Step 4</td><td>A</td><td>5</td><td></td><td>B</td><td>6</td><td>7</td></tr><tr><td>Step 4</td><td>A is empty</td><td></td><td>B</td><td>6</td><td>7</td><td>5</td></tr><tr><td>Step 6</td><td>N</td><td>3</td><td></td><td></td><td></td><td></td></tr><tr><td>Step 7</td><td>A</td><td>6</td><td>7</td><td>5</td><td>8</td><td>9</td></tr><tr><td>Step 8</td><td>Display</td><td>6</td><td>7</td><td>5</td><td>8</td><td>9</td></tr></table>	Step 1	A	8	6	9	7	5	Step 2	A	6	9	7	5	X 8	Step 3	A	9	7	5	B	6	Step 4	A	7	5	C	9		Step 4	A	5		B	6	7	Step 4	A is empty		B	6	7	5	Step 6	N	3					Step 7	A	6	7	5	8	9	Step 8	Display	6	7	5	8	9	M1 M1 M1 M1 M1 M1 A1 5 11	For identifying that 6 > B or the sub set [9] For identifying that 9 > C or the sub set [7] For identifying that 7 > B For identifying that 5 > B For the final A list or the display correct
Step 1	A	8	6	9	7	5																																																												
Step 2	A	6	9	7	5	X 8																																																												
Step 3	A	9	7	5	B	6																																																												
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Step 7	A	6	7	5	8	9																																																												
Step 8	Display	6	7	5	8	9																																																												

4

(i)

P	x	y	z	t	u	
1	1	1	1	0	0	0
0	1	1	1	1	0	12
0	1	1	0	0	1	10
0	1	10	0	0	1	45

B1 For correct use of three slack variable columns

B1 For +15/5 in objective row

B1 For 1/5, 12/5, 1/5, 10 and 3/10, 45 in constraint rows

3 For correct pivot choice (x)

B1 For negative in top row for x, or equivalent, and a correct explanation of choice of row (least ratio 10/1) (th their pivot column)

2

in their tableau, if possible for method marks

(ii)

P	x	y	z	t	u	
1	0	0	0	1	0	30
0	0	10	1	1	0	2
0	1	1	0	1	0	10
0	0	25	0	15	1	15

M1 For correct method evident for objective row

M1 For a correct method evident for pivot row

M1 For a correct method evident for other rows

A1 For correct tableau CAO

x = 10, y = 0

P = 30

B1 For correct values from their tableau

B1 6 For correct value from their tableau

(iv)

11 + 5(0.2) = 2 or x = 0

11 + 5(0.2) = 10 or x = 0

9(1) + 18(0.2) = 35 or x = 10

so all the constraints are satisfied

B1 For showing (not just stating) that constraints are satisfied

P = 3(1) + 5(0.2) = 32

which is bigger than 30 from (iii)

B1 2 For calculating 32, or equivalent eg 3x has increased by 3 but 5y has only decreased by 1

13

5 iii)

<i>t</i>	<i>h</i>
130	9 125
	125

<i>t</i>	<i>t</i>	<i>t</i>
8 100	4 90	7 85
100	90	95
<i>t</i>	<i>t</i>	<i>h</i>
4 70	2 125	3 65
90 70	25	65
		75

<i>t</i>
1 0

Shortest path from *J* to *B*: *J* → *C* → *H* → *B*

Length of path: 125 metres

iii) OMD nodes: *B*, *C*, *H*, *J*

<i>B</i> : 60	<i>B</i> : 35	<i>B</i> : 125
<i>J</i> : 90	<i>C</i> : 95	<i>C</i> : 70
150	150	195

Repeat *BH* and *CJ* for *BH*, *BH*, *BH*

150 + 765

Shortest route: 895 metres

iii)

Traveling salesperson problem

ANSWERED ON INSERT

M1 For correct initial temporary labels at *t* & *h*

M1 For correctly updating *t* and label at *h*

A1 For all temporary labels correct (including 11 allow extra 100 at *t*, 105 at *h*, 75 at *h* only)

B1 For order of becoming permanent correct

B1 For all permanent labels correct (if need not have a permanent label)

B1 For correct route (don't omit omission of *J* or *B*)

B1 For 125

B1 For identifying or using *B*; *t* & *h* implied

M1 For any three of these weights correct, or implied, or 11 from their (11)

A1 For identifying the pairing *BH*, *CJ* to repeat or 130 met (1)

M1 For 765 + their 150 (a valid pair, total)

A1 For 895 (only)

B1 For graph structure correct

M1 For a reasonable attempt at all weights (at least 9 correct, including the three given)

A1 For all 10 weights correct

4  
B1 16 For identifying TSP by name

6 (ii)

	1	5	2	4	3	6
1	-	8	7	10	1	-
5	6	-	5	6	-	14
2	3	5	-	8	4	10
4	-	6	8	-	3	8
3	-	-	4	3	-	-
6	-	14	10	8	-	-

Order: 1, 5, 2, 4, 3, 6

Minimum spanning tree



Total weight: 23 miles

- (ii) MST for reduced network = 8  
Two shortest arcs from B = 6 + 6 = 11  
Lower bound = 29 miles
- (iii) 1 - D - E - C - A - B - F  
 $8 + 3 + 4 + 3 + 6 = 24$   
 58 miles

## ANSWERS ON INSERT

- M2 For choosing row C in column 1
- M1 dep For choosing more than one entry from column 1
- A1 For correct entries chosen
- B1 For correct order, listed or marked on arrows or table, or arcs listed: B-C-E-F-D-A-B
- B1 For tree (correct or follow through from table provided solution forms a spanning tree)
- B1 For 24 or follow through from table or diagram, provided solution forms a spanning tree
- 6 For their 18 seen or implied
- M1 For 14 seen or implied
- A1 3 For 29 is not
- M1 For 1 - D - E - C - A - B
- A1 For correct total
- M1 4 For a substantially correct attempt at sum
- A1 13 For 58 is not