Solutions June 2008

1 i) $\frac{1}{16}$ $\frac{1}{4^2}$ $= 4^{-2}$

ii) $4^3 = 64$.

iii) $\sqrt{4} = 2$ $2^3 = 8$ $\therefore 4^{3/2}$

2 i) $y = (x-2)^2$

ii) $y = -(x^3 - 4)$

3. i) $\sqrt{200} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

ii) $\frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

iii) $5\sqrt{8} - 3\sqrt{2}$
$5\sqrt{4}\sqrt{2} - 3\sqrt{2}$
$10\sqrt{2} - 3\sqrt{2}$
$= 7\sqrt{2}$

4) $2x - 7x^{1/2} + 3 = 0$
let $y = x^{1/2}$ so $y^2 = x$

$\therefore 2y^2 - 7y + 3 = 0$

$(2y-1)(y-3) = 0$

$2y - 1 = 0$ or $y - 3 = 0$
$y = 1/2$ $y = 3$

$y^2 = x$

$(1/2)^2 = x$ or $3^2 = x$
$1/4 = x$ $9 = x$

5) $y = 8\sqrt{x} + x$

$y = 8x^{1/2} + x$

$\frac{dy}{dx} = 4x^{-1/2} + 1 \quad = 4\left(\frac{1}{\sqrt{4}}\right) + 1$

$= \frac{7}{3}$

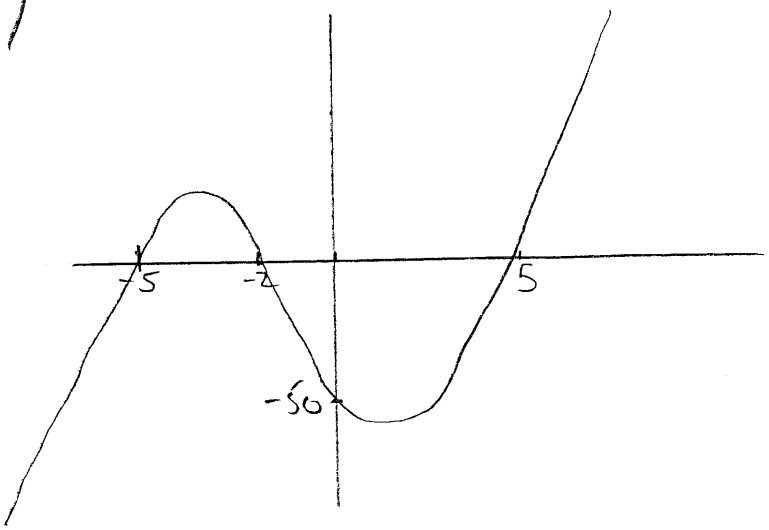
6) $(x-5)(x+2)(x+5)$

$(x-5)(x^2+7x+10)$

$x^3 + 7x^2 + 10x - 5x^2 - 35x - 50$

$x^3 + 2x^2 - 25x - 50$

ii)



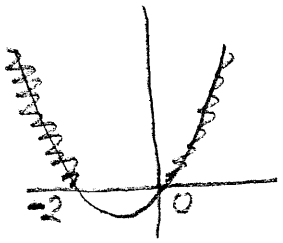
7 i) $8 < 3x - 2 < 11$ (+2)

$10 < 3x < 13$

$\frac{10}{3} < x < \frac{13}{3}$

ii) $y^2 + 2y \geq 0$

$y(y+2) \geq 0$



$y \leq -2$ or $y \geq 0$

8) $y = x^3 - kx^2 + x - 3$

i) $\frac{dy}{dx} = 3x^2 - 2kx + 1$

ii) $3(1)^2 - 2k(1) + 1$

$3 - 2k + 1 = 0$

$4 - 2k = 0$

$k = 2$

iii) $\frac{d^2y}{dx^2} = 6x - 2k$ $= 6x - 4$ a

sub $k=2$

$= 6 - 4 = 2$

$\frac{d^2y}{dx^2} > 0$ ∴ minimum point

8.iv) $\frac{dy}{dx} = 3x^2 - 4x + 1$ (as $k=2$)

$3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1)$

$3x = 1$ or $x = 1$

$x = 1/3$

9) i) $(x-2)^2 + (y-1)^2 = 10^2$ *

$x^2 - 4x + 4 + y^2 - 2y + 1 = 100$

$x^2 - 4x + y^2 - 2y + 5 = 100$

$x^2 - 4x + y^2 - 2y = 95$

$x^2 - 4x + y^2 - 2y - 95 = 0$

ii) $(5, k)$ $x > 0$

$5^2 - 4(5) + y^2 - 2y - 95 = 0$

$25 - 20 + y^2 - 2y - 95 = 0$

$y^2 - 2y - 90 = 0$

$(y \quad)$

$\frac{+2 \pm \sqrt{4 + 4 \times 90}}{2} = \frac{+2 \pm \sqrt{364}}{2}$

$= \frac{2 \pm \sqrt{4} \times \sqrt{91}}{2}$

$= \frac{2 \pm 2\sqrt{91}}{2}$

$= 1 \pm \sqrt{91}$

would have been easier to use * equation

9. iii) centre (2,1) point (-3,9)

Distance between points $= \sqrt{5^2 + 8^2}$

$= \sqrt{25 + 64}$

$= \sqrt{89}$

$\therefore \sqrt{89} < 10$ so inside

iv) Tangent at (8,9)

Gradient of normal through (2,1) (8,9)

$= \frac{\Delta y}{\Delta x} = \frac{8}{6} = \frac{4}{3}$

$\therefore$ tangent grad $= -\frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{3}{4}(x - 8)$

$y = -\frac{3}{4}x + 6 + 9$

$y = -\frac{3}{4}x + 15$

10) i) $2x^2 - 6x + 11$

$2(x^2 - 3x) + 11$

$2\left(\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right) + 11$

$2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$

ii) $y = 2x^2 - 6x + 11$ from $2(x - 3/2)^2 + \frac{13}{2}$

vertex is $(3/2, 13/2)$

iii) Discriminant $36 - 4 \times 2 \times 11$

$36 - 88$

$= -52$

iv) Discriminant < 0 ∴ no real roots

v) $y = 2x^2 - 6x + 11$ $\quad 7x + y = 14$, $y = 14 - 7x$

$14 - 7x = 2x^2 - 6x + 11$

$= 2x^2 + x - 3$

$0 = (2x + 3)(x - 1)$

$0 = 2x + 3$ or $0 = x - 1$

$2x = -3$ $\quad x = 1$

$x = -3/2$

∴ $y = 14 + 7 \times 3/2$ or $y = 14 - 7$

$y = 14 + \frac{21}{2}$ $\quad y = 7$

$y = \frac{49}{2}$ $\quad (1, 7)$

$\left(-3/2, \frac{49}{2}\right)$