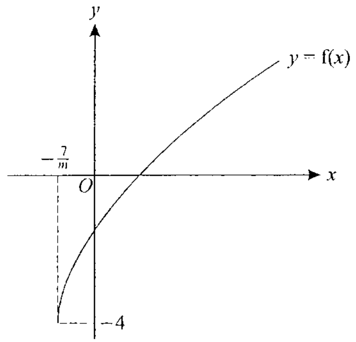
9



The function f is defined by $f(x) = \sqrt{(mx + 7)} - 4$, where $x \geqslant -\frac{7}{m}$ and m is a positive constant. The diagram shows the curve $y = f(x)$.

(i) A sequence of transformations maps the curve $y = \sqrt{x}$ to the curve $y = f(x)$. Give details of these transformations. [4]

(ii) Explain how you can tell that f is a one-one function and find an expression for $f^{-1}(x)$. [4]

(iii) It is given that the curves $y = f(x)$ and $y = f^{-1}(x)$ do not meet. Explain how it can be deduced that neither curve meets the line $y = x$, and hence determine the set of possible values of m. [5]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4723

Core Mathematics 3

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

 [Turn over

1 The function f is defined for all real values of x by

$$f(x) = 10 - (x + 3)^2.$$

(i) State the range of f. [1]

(ii) Find the value of $\mathrm{ff}(-1)$. [3]

2 Find the exact solutions of the equation $|6x - 1| = |x - 1|$. [4]

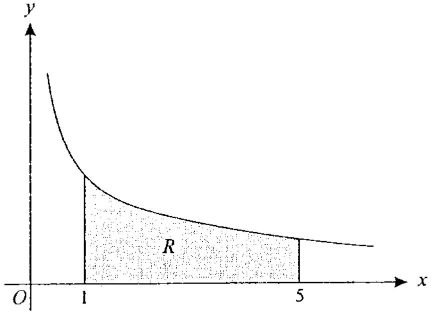
3 The mass, m grams, of a substance at time t years is given by the formula

$$m = 180e^{-0.017t}.$$

(i) Find the value of t for which the mass is 25 grams. [3]

(ii) Find the rate at which the mass is decreasing when $t = 55$. [3]

4 (a)



The diagram shows the curve $y = \dfrac{2}{\sqrt{x}}$. The region R, shaded in the diagram, is bounded by the curve and by the lines $x = 1$, $x = 5$ and $y = 0$. The region R is rotated completely about the x-axis. Find the exact volume of the solid formed. [4]

(b) Use Simpson's rule, with 4 strips, to find an approximate value for

$$\int_1^5 \sqrt{(x^2 + 1)}\,dx,$$

giving your answer correct to 3 decimal places. [4]

5 (i) Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence solve the equation $3\sin\theta + 2\cos\theta = \frac{7}{2}$, giving all solutions for which $0° < \theta < 360°$. [5]

6 (a) Find the exact value of the x-coordinate of the stationary point of the curve $y = x\ln x$. [4]

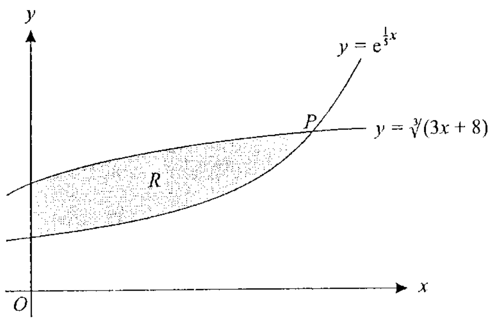
(b) The equation of a curve is $y = \dfrac{4x + c}{4x - c}$, where c is a non-zero constant. Show by differentiation that this curve has no stationary points. [3]

7 (i) Write down the formula for $\cos 2x$ in terms of $\cos x$. [1]

(ii) Prove the identity $\dfrac{4\cos 2x}{1 + \cos 2x} \equiv 4 - 2\sec^2 x$. [3]

(iii) Solve, for $0 < x < 2\pi$, the equation $\dfrac{4\cos 2x}{1 + \cos 2x} = 3\tan x - 7$. [5]

8



The diagram shows part of each of the curves $y = e^{\frac{1}{5}x}$ and $y = \sqrt[3]{(3x + 8)}$. The curves meet, as shown in the diagram, at the point P. The region R, shaded in the diagram, is bounded by the two curves and by the y-axis.

(i) Show by calculation that the x-coordinate of P lies between 5.2 and 5.3. [3]

(ii) Show that the x-coordinate of P satisfies the equation $x = \frac{5}{3}\ln(3x + 8)$. [2]

(iii) Use an iterative formula, based on the equation in part **(ii)**, to find the x-coordinate of P correct to 2 decimal places. [3]

(iv) Use integration, and your answer to part **(iii)**, to find an approximate value of the area of the region R. [5]

[Question 9 is printed overleaf.]

[Turn over