Make 100

Modelling and optimisation

Prerequisite knowledge

- Formulae in a spreadsheet

Why do this unit?

The design and creation of the spreadsheet involves pupils in having a sound understanding of the mathematical context. Pupils will need to problem solve in order to produce a spreadsheet which will support them in their search for a solution.

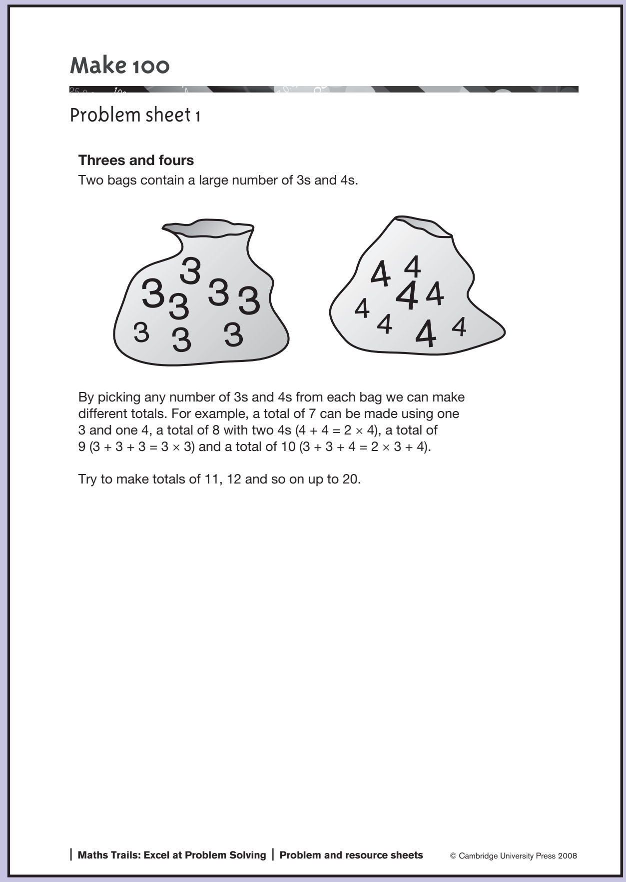
Time

Two lessons

Resources

CD-ROM: spreadsheet, problem sheets 1 and 2

NRICH website (optional): www.nrich.maths.org, October 2005, 'Data chunks'; September 1999, 'Euclid's algorithm I'



Make 100

Problem sheet 1

Threes and fours

Two bags contain a large number of 3s and 4s.

By picking any number of 3s and 4s from each bag we can make different totals. For example, a total of 7 can be made using one 3 and one 4, a total of 8 with two 4s ($4 + 4 = 2 \times 4$), a total of 9 ($3 + 3 + 3 = 3 \times 3$) and a total of 10 ($3 + 3 + 4 = 2 \times 3 + 4$).

Try to make totals of 11, 12 and so on up to 20.

| Maths Trails: Excel at Problem Solving | Problem and resource sheets © Cambridge University Press 2008

Introducing the unit

Introduce the problem 'Threes and fours' on problem sheet 1.

Collect and display successful combinations of 3s and 4s.

- What can you say about the totals? [We can make them all, and some can be made in more than one way.]
- Which ones can you do in more than one way? [12, 15, 16, 18, 19, 20]
- How do you know you have all the possible ways of making them? [We need to be systematic and try all the arrangements or find a rule.]

The next stage is to work together to develop a systematic approach, sharing findings. You will need to discuss with the group their ideas, giving them time to talk in pairs before bringing ideas together. Your aim could be to end up with a table showing all possible combinations.

This could look like:

	3	**6**	**9**	**12**	**15**	**18**
4	7	10	13	16	19	22
8	11	14	17	20	23	26
12	15	18	21	24	27	30
16	19	22	25	28	31	34
20	23	26	29	32	35	38

Working with tables on paper will make the structuring of the spreadsheet easier to grasp later in the activity. Note that the table above has the multiples of 4s and 3s as row and column headers so that the principal operation within the cells of the table is addition. Check that pupils understand that the table will contain every possible successful combination.

Main part of the unit

Introduce the problem 'Make 100' on problem sheet 2.

- How is this problem the same as and different from the 'Threes and fours' problem? [The total is still made from threes and fours but one specific result is required, 100, and we want to know exactly how many combinations will produce that result.]

Ask the group to spend a few minutes thinking on their own about the problem and how they might solve it, before moving them into pairs to compare ideas. When ready ask the whole group to share their approaches.

Draw pupils' attention to the use of a table in the 'Threes and fours' problem and discuss how a spreadsheet might be able to help, by systematically producing all possible combinations in a two-way table.

Open the sheet '3 & 4'

- What formula would you expect to be in cell C4? [Pupils may say =C3+B4.]

Show the formula in C4. It is formed from C3 and B4 but has $ signs in front of the 3 and the B. Look at some other cells to confirm similar use of the $ sign. Explain the purpose of the dollar sign (see the sheet 'Help - Table of 2 variables'). Use sheet '3 & 4 blank' to reinforce pupils' understanding of absolute references:

- Create the formula =C$3+$B4 in cell C4.
- Select C4 and copy.
- Select the range C4:I9 and paste.

Start the pupils working on the problem and constructing their own spreadsheets. They will need to:

- plan on paper;
- implement and test their spreadsheet;
- identify solutions and notice any patterns [solutions lie along a diagonal line].

As a whole group, discuss key points including a comparison of efficiency between different designs. The sheet '3 & 4' on the spreadsheet may be helpful.

Plenary

The plenary can focus on explaining why the solutions lie on a line and how this can be used to generate all solutions.

- How are the solutions related? [Adjacent solutions are always four along and three up.]
- What does that really mean? [Moving four along is a change of four 3s (12) and moving three up is a change of three 4s (12). Replacing four 3s by three 4s, or vice versa, produces the next solution along the line.]
- Can we now use this idea to generate all the solutions without a spreadsheet? [Yes, find one solution and from that replace groups of three 4s with four 3s, and then, starting again from the original solution, replace groups of four 3s with three 4s. This is the same as moving up and down the line of solutions in the table.]

Solution notes

Make 100

There are nine combinations which make 100.

$25 \times 4 + 0 \times 3$
$22 \times 4 + 4 \times 3$
$19 \times 4 + 8 \times 3$
$16 \times 4 + 12 \times 3$
$13 \times 4 + 16 \times 3$
$10 \times 4 + 20 \times 3$
$7 \times 4 + 24 \times 3$
$4 \times 4 + 28 \times 3$
$1 \times 4 + 32 \times 3$