Birmingham City Council

Birmingham
Resources for
Understanding
Mathematics

C if U can

B even better!

Birmingham City Council

Solve these simultaneous equations

$3x + 5y = 4$
$6x + y = 26$

You **must** show your working
Do **not** use trial and improvement

Solve the simultaneous equations

$x + 2y = 4$

$3x - 4y = 7$

This time, start by multiplying the first equation by 3

How will this booklet help you to get a better grade in maths?

- This booklet is helpful to students entering GCSE at higher tier
- The booklet focuses on B grade questions that extend what you already know at C grade
- Even if you can't do some of the C grade work, you can often do some of the easier B grade questions and this will boost your overall mark
- The booklet focuses on the topics that get lots of marks
- Each topic has hints and tips to help you through the questions

Each topic starts with a reminder of facts you need to know, followed by a grade C question to get your brain up to speed. There are then three B grade questions, two of which have hints. You can then try a B grade question completely on your own.

if U can do even

etter!!!

Trigonometry

B-asics

B-efore ….. you do the trigonometry questions, remind yourself of these facts and have a go at this Pythagoras question

For trigonometry, you need to remember ……

SOH $\sin = \frac{\text{opp}}{\text{hyp}}$

CAH $\cos = \frac{\text{adj}}{\text{hyp}}$

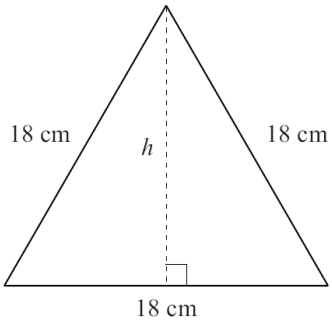
TOA $\tan = \frac{\text{opp}}{\text{adj}}$

So how can you change these around to find any side or any angle

Pythagoras Theorem

The diagram shows an equilateral triangle of side length 18cm

Calculate the height of the triangle (marked h in the diagram)



Not drawn accurately

C

Now **B** brave and try these

SUPERGROW GARDEN CENTRE

ROSES	SHRUBS
£x each	£y each

(a) Megan buys 4 roses and 3 shrubs.
She pays £33
Use this information to write down an equation in r and s

(b) Josh buys 6 roses and 6 shrubs
He pays £57
Use this information to write down another equation in r and s

(c) Solve your equations simultaneously to find the values of r and s
You **must** show your working.
Do **not** use trial and improvement

When you have your two equations,

- **double all the values in the one from part (a)**
- **eliminate the s by subtracting (b) from (a).**
- **work out the value of r**
- **substitute in that value to find s**

B

Simultaneous equations

B-asics

B-efore ….. you solve the simultaneous equations using elimination, (removing one of the variables)
try the C grade question which involves solution by graph

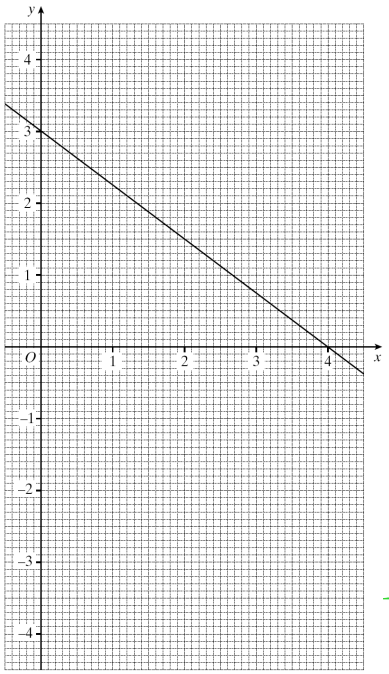
Remember…..

When you solve these equations, you can multiply one of them so that one of the variables is the same, then try eliminating
(like the first B grade question!)

You can eliminate one of the variables by either adding the two equations or subtracting them.

Simultaneous equations and graphs

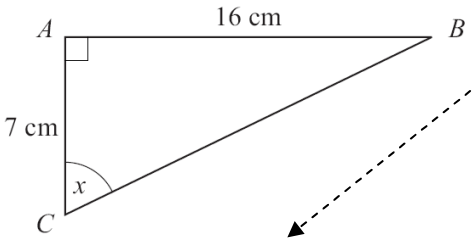
The graph of 4y + 3x = 12 has been drawn on the grid.
Draw another line on the grid to solve the simultaneous equations
4y + 3x = 12
y = 2x - 4



C

Now **B** brave and try these

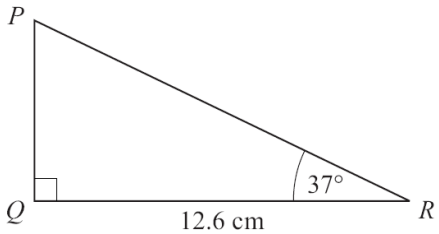
(a) In triangle ABC, angle A = 90°, AB = 16cm and AC = 7cm



You could start by labelling the sides of the triangle – which one is the hypotenuse, which is opposite and which is adjacent to angle x?

Calculate the value of *x*

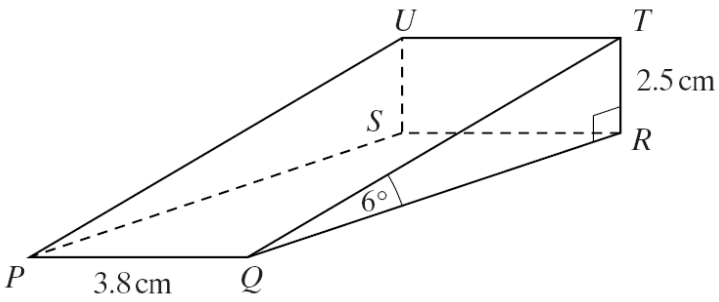
(b) In triangle PQR, angle Q = 90°, angle R = 37° and QR = 12·6cm



Calculate the length of PR

B

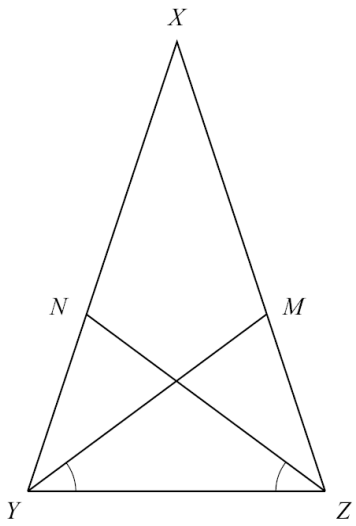
The diagram shows a door wedge with a rectangular horizontal base PQRS
The sloping face PQTU is also rectangular.
PQ = 3·8cm and angle TQR = 6°
The height TR is 2·5cm



This question has a number of steps to go through before you can find the answer. A good start might be to draw in the line that you are asked to find, then work out which triangles you are going to be working with. Don't forget what you know about Pythagoras!

Calculate the length of the diagonal PT

XYZ is an isosceles triangle in which XZ = XY
M and N are points on XZ and XY such that angle MYZ = angle NZY



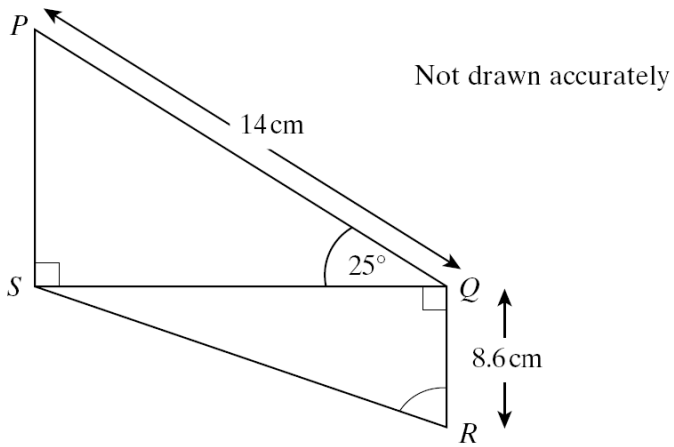
Prove that triangles YMZ and ZNY are congruent

ABC and XYZ are similar triangles with right angles at B and Y
AC = 13cm, BC = 5cm and YZ = 12·5cm

What scale factor increases 5 to 12·5?
(In other words 5 x ? = 12·5)
Whatever the scale factor is, you must multiply the 13 by this

Work out the length of XY

In the diagram, PQ = 14cm and QR = 8·6cm
Angle PSQ = angle SQR = 90°
Angle PQS = 25°



Calculate angle R

Graphs

B-asics

B-efore ….. you draw the quadratic and cubic graphs, remind yourself of how to draw a straight line graph

Remember ….

The type of equation tells you what shape the graph will be

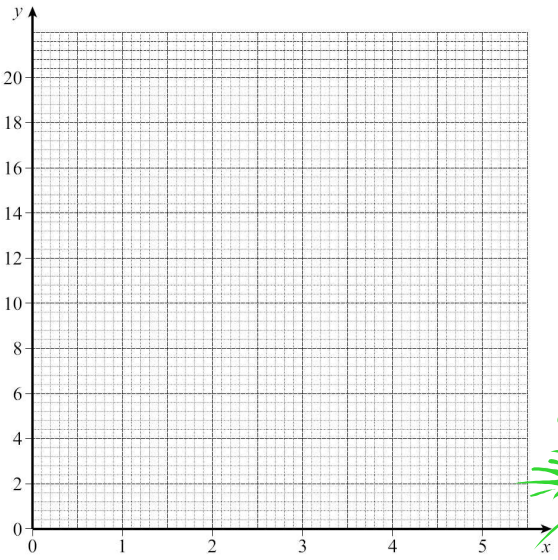
Look for a pattern in the table of values so you know you are on the right track

Graphs of linear equations

(a) Complete the table of values for
y = 3x + 4

x	0	1	2	3	4	5
y	4		10		16	19

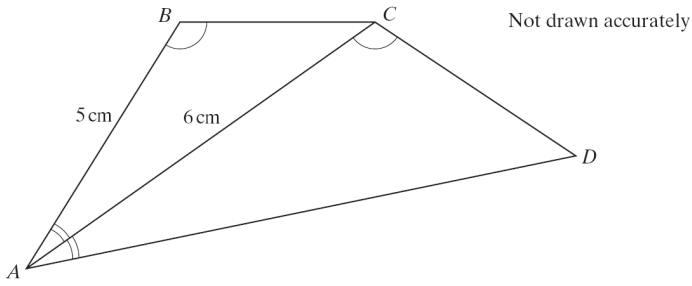
(b) On the grid draw the graph of y = 3x + 4 for values of x from 0 to 5



C

Now **B** brave and try these

Triangles ABC and ADC are similar.
AB = 5cm and AC = 6cm



Calculate the length of AD

AB is enlarged to AC
By what scale factor?
So AC will be enlarged to AD by the same scale factor

B

Similarity and congruence

B-asics

B-efore ….. you do the B grade questions, try this C grade enlargement question

Remember….

Similar shapes have the same sized angles but the lengths of the sides are enlarged by the same scale factor

Congruent shapes are identical to each other

You can prove that shapes are congruent if

(a) all the sides are the same or

(b) two pairs of sides are equal and the angle between them is equal or

(c) two pairs of angles are equal and on pair of corresponding sides is equal or

(d) In the case of two triangles, both triangles have a right angle, the hypotenuses are equal and one pair of corresponding sides is equal

Enlargement

The big trapezium is an enlargement of the small trapezium with a scale factor of 4

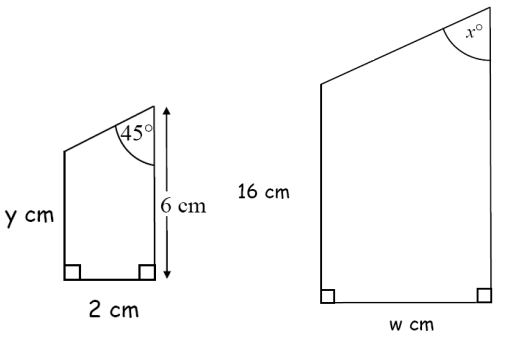
Find the value of

(a) w

(b) y

(c) x

(d) Work out the area of the big trapezium



C

Now **B** brave and try these

(a) Complete the table of values for $y = 2x^2 - 4x - 1$

x	–2	–1	0	1	2	3
y	15		–1		–1	5

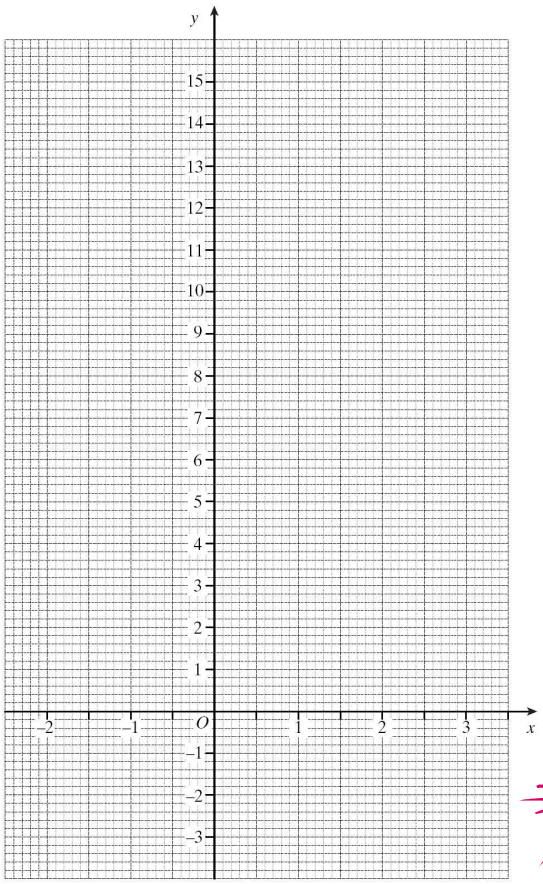
(b) On the grid opposite, draw the graph of $y = 2x^2 - 4x - 1$ for values of x from -2 to +3

(c) An approximate solution of the equation $y = 2x^2 - 4x - 1$ is $x = 2{\cdot}2$

(i) Explain how you can find this from the graph.

(ii) Use your graph to write down another solution of this equation

Look on your graph where x = 2·2. What do you notice?



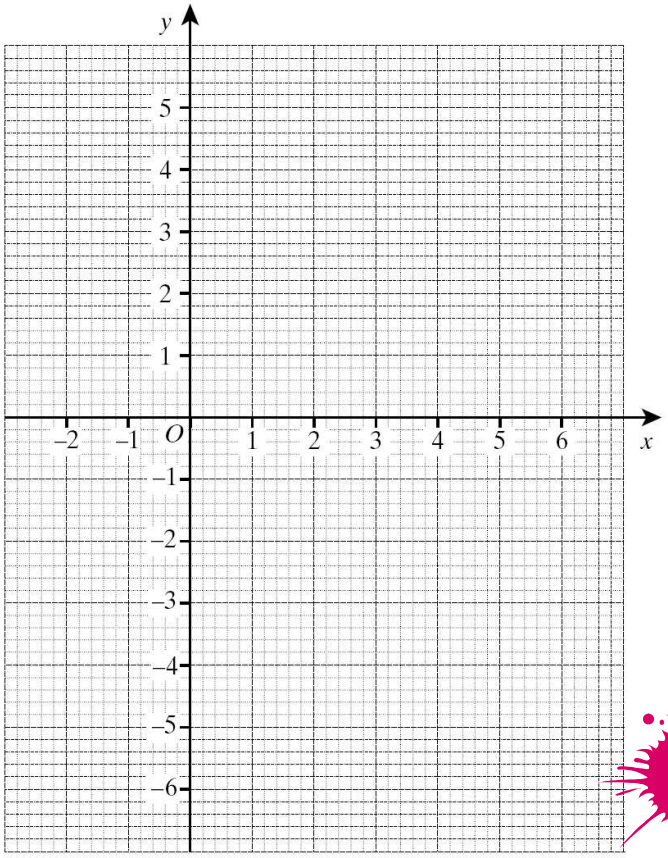
B

(a) Complete the table of values for $y = x^2 - 4x - 1$

x	–1	0	1	2	3	4	5
y		–1	–4		–4	–1	4

(b) On the grid, draw the graph of
$y = x^2 - 4x - 1$ for values of x from – 1 to + 5

(c) Use your graph to solve the equation
$x^2 - 4x - 1 = 0$



Again, you are looking at where the graph crosses the x axis (two points)

(a) Factorise $r^6 - 3r^4$

(b) Factorise $x^2 + 5x - 14$

(c) Hence solve the equation $x^2 + 5x - 14 = 0$

(a) Expand and simplify (p + 7)(p + 2)

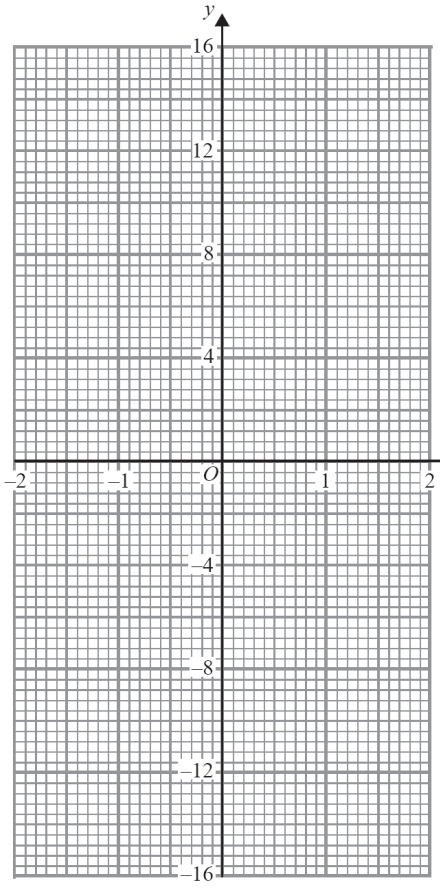
(b) Factorise $2x^2 + 3x - 5$

Use the same method as for the previous question but watch out for the negative sign.
What will be the 2 factors that multiply to give $2x^2$?
Which of the numbers in the factor pair for 5 should be multiplied by 2x to end up with + 3x
(This is quite tough!!)

(a) Complete the table of values for $y = x^3 + 3x$

x	–2	–1	0	1	2
y	–14		0		

(b) On the grid, draw the graph of $y = x^3 + 3x$



Compound interest

B-asics

B-efore ….. you solve the compound interest problems
Remind yourself about using percentages by answering the C grade percentage change question

Remember, compound interest is used to add an increasing amount each year.

There is a formula for working out compound interest
But if you don't know it, you can solve the problem by working out the first year, getting the new total, then finding the interest on that amount, adding it on ……. and so on

Percentage change

Hajra's weekly pay this year is £240
This is 20% more than her weekly pay last year

Bill says 'This means Hajra's weekly pay last year was £192'

Bill is wrong

(a) Explain why

(b) Work out Hajra's weekly pay last year

Now **B** brave and try these

(a) Expand and simplify $(x + 3)(x - 4)$

(b) Expand and simplify $(2x + 5)(3x - 4)$

(c) Factorise $x^2 + 7x + 10$

Put this expression into two pairs of brackets as in part (b)
Start with the x^2
The look at the factor pairs of 10 and find a pair that will add to 7

Quadratics and factorising

B-asics

B-efore ….. you do the quadratics questions, remind yourself of what you know about factorising in general and try this C grade question on expanding brackets

Remember….

Expand means the opposite of factorising – another way of saying 'expand' is 'multiply out'

When you are asked to factorise, take out the largest factor possible

Simplify means collect like terms

Expanding brackets

(a) Multiply out 6(3p + q)

(b) Multiply out -2(2p + 3q)

(c) Multiply out and simplify 6(3p + q) – 2(2p + 3q)

C

Now **B** brave and try these

(a) Rob invested £2500 in a building society account at 2% per year compound interest.
Calculate the total amount in the account after three years

Find 2% of £2500, add it on (year one), then find 2% of the new amount (year two), add it on again – and keep going!

(b) Jane invested some money in a building society account.
Interest of 3% was added to the account after one year.
The total in the account after the interest was added was £221·45.
How much money did Jane invest?

Here you need to find the amount that when multiplied by 1·03 = £221·45.

(Why do you multiply by 1·03?)

B

(a) George invests £400 for three years at 4% compound interest.
Show that, at the end of the three years, he has a total investment of £449·95 to the nearest penny

Use the same method as for the last question

(b) Calculate the percentage interest from £400 to £449·95

What do you multiply £400 by to get £449·95?

A gardener grows two batches of sunflower plants, batch A and batch B.
The diagram shows the cumulative frequency of the heights of the plants in batch A

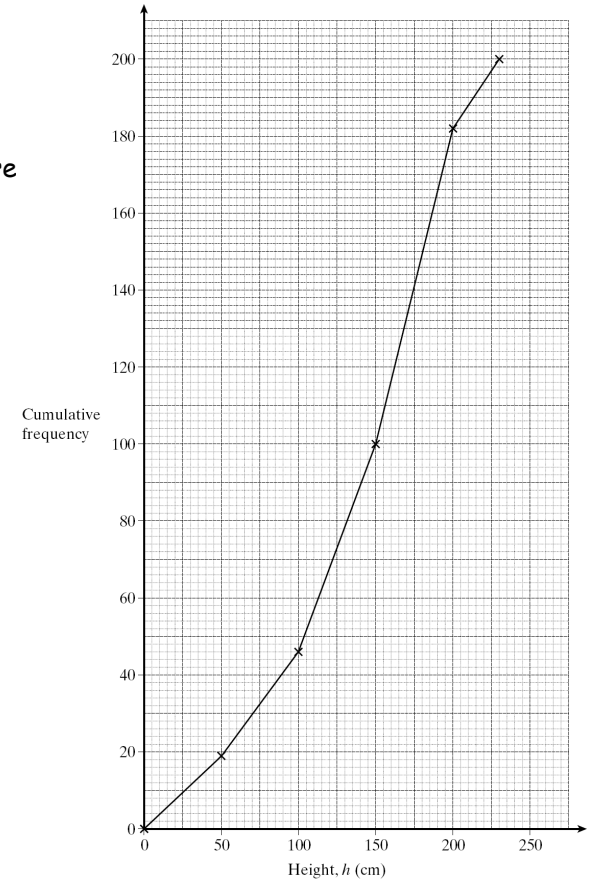
(a) Estimate the number of plants in batch A that are under 100cm tall

(b) The plants in batch B are fed with a special fertiliser.
The heights of these plants are shown in the cumulative frequency table.

Height, h (cm)	Cumulative frequency
$h \leqslant 50$	3
$h \leqslant 100$	17
$h \leqslant 150$	68
$h \leqslant 200$	146
$h \leqslant 250$	200

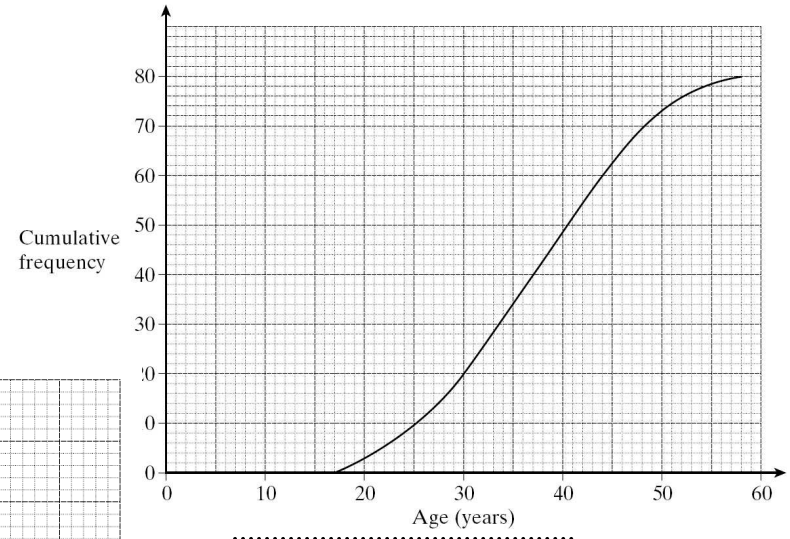
(c) On the same axes, draw a cumulative frequency diagram to represent batch B

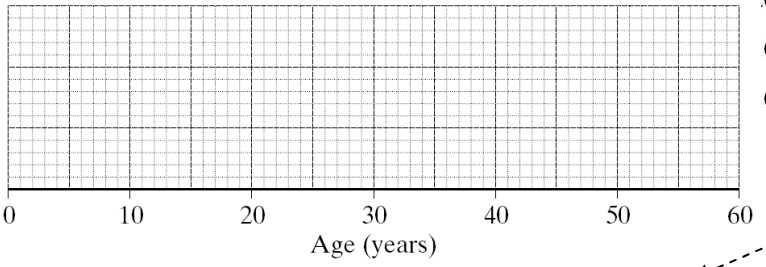
What is the difference between the median heights



The ages of 80 workers in a factory are represented by the cumulative frequency diagram.
The youngest worker is 17 and the oldest if 57

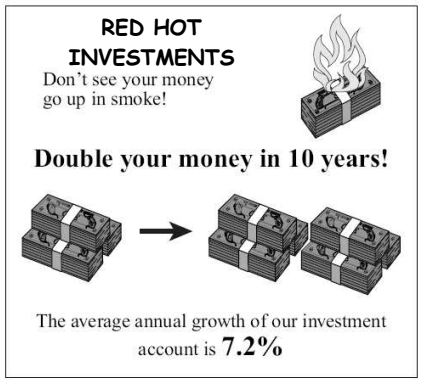
(a) Those workers who were aged 50 or over were offered early retirement.
Use the cumulative frequency diagram to estimate how many workers were offered early retirement.





Find the median, range and quartiles for the box plot

(b) Use the information in the diagram to draw a box plot on the grid above



Sarah has £2000 to invest.

Will she double her money in 10 years with Red Hot Investments?
Show all your working

Circle theorems

B-asics

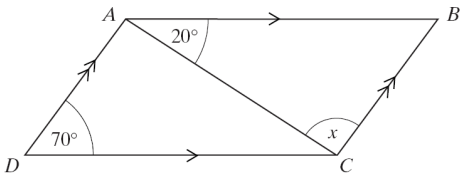
B-efore ….. you do the circle theorems questions, remind yourself of what you know about angle facts

think about…

angles on a straight line
angles in a triangle and a quadrilateral
angles around a point
vertically opposite angles
alternate angles
corresponding angles
angles in polygons (exterior and interior)

Angle facts

The diagram shows a parallelogram ABCD.
Angle BAC = 20°, angle ADC = 70°
Show that angle x is a right angle



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C

Now **B** brave and try these

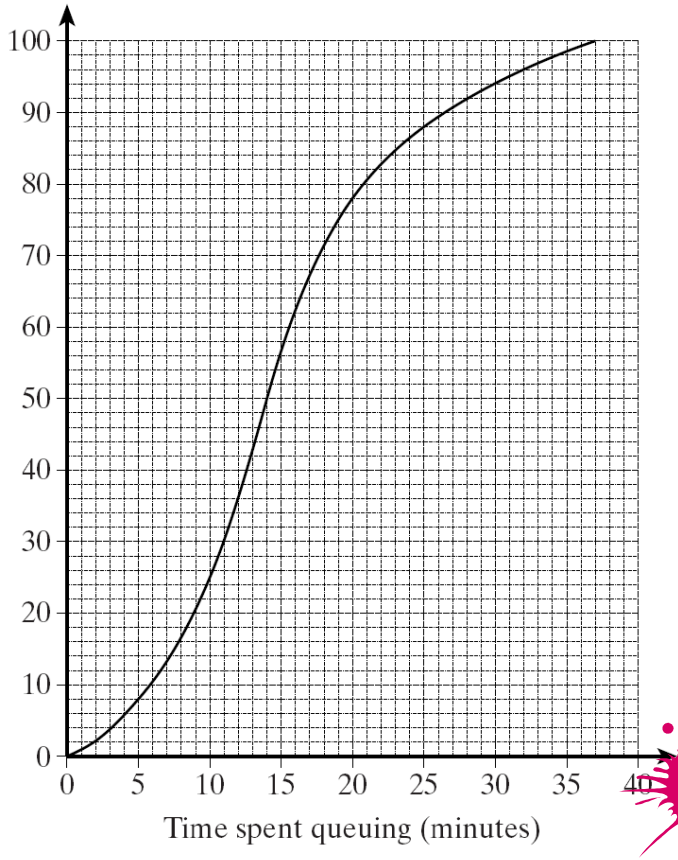
The time, in minutes, spent queuing in a post office by each of 100 customers is summarised by the cumulative frequency diagram.

Use the cumulative frequency diagram to estimate

(a) how many customers queued for more than 25 minutes

(b) the median queuing time

(c) the interquartile range of the queuing times



Find the upper and lower quartiles and then find the difference between them

B

Cumulative frequency

B-asics

B-efore ….. you do the circle theorems questions, remind yourself of what you know about averages and try this question on the finding the median

Remember…..

Median - the middle value when the values are placed in order, smallest to largest
If there is an even number of values, take the value half way between the middle pair.

Range - the difference between the smallest and the largest values.
Upper quartile - half way between the median and the largest value(a quarter of the way down through the ordered data)
Lower quartile - half way between the median and the lowest value (a quarter of the way up through the ordered data)

Interquartile range - the difference between the lower and upper quartile values (the middle half!)

Finding the median

Each day the number of pupils who were late for school was recorded.
The stem and leaf diagram shows the results for 15 days

Key 3 | 4 represents 34 pupils

0	2 3 6 8
1	2 2 3 4 6 7
2	1 3 6 9
3	4

(a) On how many days were more than 10 pupils late?

(b) Work out the median

(c) On the next day 11 pupils were late.
Tick the box to show the effect this value would have on the range

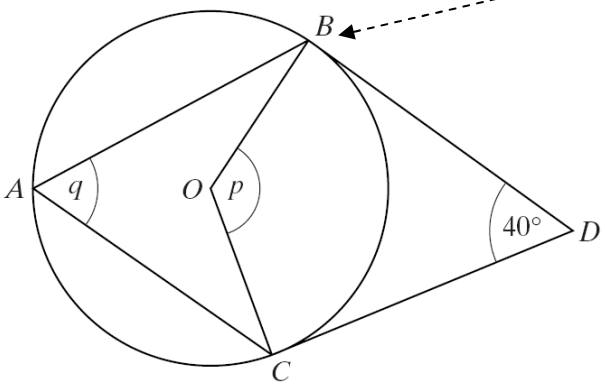
☐ Range would increase ☐ Range would stay the same ☐ Range would decrease

C

Now **B** brave and try these

A, B and C are points on the circumference of a circle with centre O
BD and CD are tangents
Angle BDC = 40°

One way you could start is by putting in the values of angles DCO and DBO – where a tangent meets a radius??



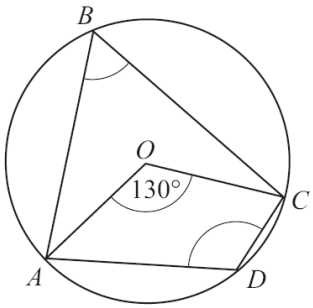
(a) Work out the value of p

(b) Hence write down the value of q

B

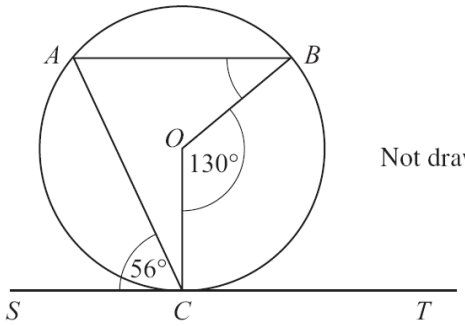
(a) A, B, C and D are points on the circumference of a circle centre O. Angle AOC = 130°.
Work out the size of angles ABC and ADC

You might start by thinking about the relationship between angle AOC and angle ABC



(b) ABC are three points on the circumference of a circle centre O. SCT is a tangent to the circle.
Angle SCA = 56° and angle COB = 130°.
Find the size of angle OBA

Which angles can you put in that you know straight away?



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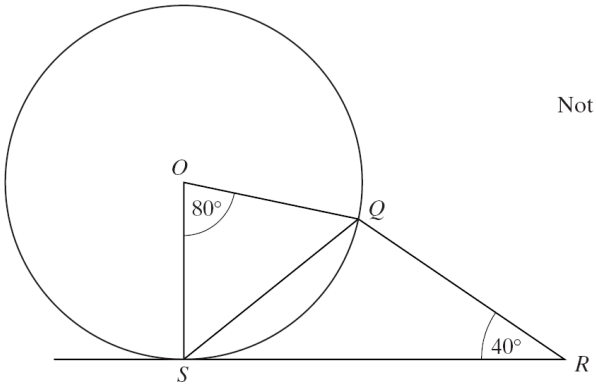
In the diagram below points Q and S lie in a circle centre O.
SR is a tangent to the circle at S
Angle QRS = 40° and angle SOQ = 80°



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Prove that triangle QSR is isosceles