CORE 1 6 JUNE 2006

4) Points $A(1,3)$ $B(4,21)$ lie on curve $y = x^2 + x + 1$

i) grad AB $= \frac{21-3}{4-1} = \frac{18}{3} = \underline{6}$

ii) grad on curve when $x = 3$

$y = x^2 + x + 1$

$\frac{dy}{dx} = 2x + 1 \rightarrow$ when $x = 3$ $\quad 2x + 1 = \underline{7}$

2) i) evaluate $27^{-2/3} = \frac{1}{27^{2/3}} = \frac{1}{3^2} = \underline{\frac{1}{9}}$

ii) $5\sqrt{5} = 5^1 \times 5^{1/2} = \underline{5^{3/2}}$

iii) $\frac{1-\sqrt{5}}{3+\sqrt{5}} = \frac{(1-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \frac{3-4\sqrt{5}+5}{9-5}$

$= \frac{8-4\sqrt{5}}{4} = \underline{2-\sqrt{5}}$

3i) $2x^2 + 12x + 13$

$= 2(x^2 + 6x) + 13$

$= \underline{2(x+3)^2 - 5}$

ii) solve $2x^2 + 12x + 13 = 0$

$2(x+3)^2 - 5 = 0$

$2(x+3)^2 = 5$

$(x+3)^2 = 5/2$

$x + 3 = \pm\sqrt{5/2}$

4) i) $(x-4)(x-3)(x+1)$

$= (x-4)(x^2-2x-3)$

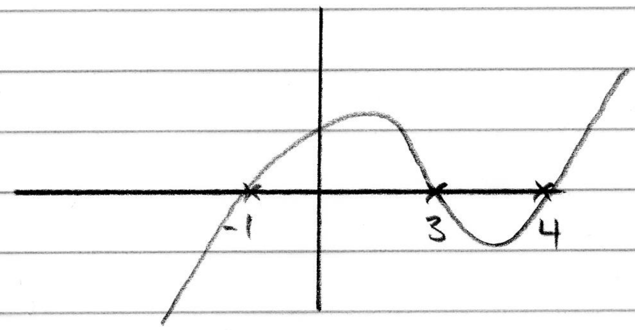
$= x^3-2x^2-3x-4x^2+8x+12$

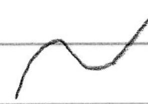
$= \underline{x^3-6x^2+5x+12}$

ii) sketch curve.

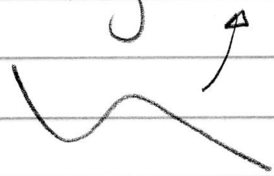
$(x-4)(x-3)(x+1)=0$

$x = 4$ or 3 or -1



- It's not asked for stationary points so don't differentiate
- positive cubic so  shape

iii) same points but negative cubic so



5) i) $1 < 4x - 9 < 5$

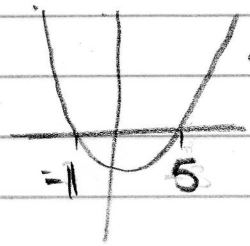
$10 < 4x < 14$

$\underline{\frac{5}{2} < x < \frac{7}{2}}$

ii) $y^2 \geq 4y+5$

$y^2-4y-5 \geq 0$

$(y+1)(y-5) \geq 0$



$y \geq 5$ or $y \leq -1$

6) i) solve $x^4 - 10x^2 + 25 = 0$

let $y = x^2$ $\quad y^2 - 10y + 25 = 0$

$(y-5)(y-5) = 0$

$y = 5$

if $y = x^2$ $\quad x = \pm\sqrt{5}$

ii) $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3$

find $\frac{dy}{dx} = 2x^4 - 20x^2 + 50$

$$\left.\begin{array}{l} 2x^4 - 20x^2 + 50 = 0 \\ x^4 - 10x^2 + 25 = 0 \end{array}\right\} \text{same as above.}$$

$\therefore$ 2 solutions $\sqrt{5}$ or $-\sqrt{5}$ so 2 stationary points

7) i) $y = x^2 - 5x + 4$ $\quad y = x - 1$ $\quad$ just make them equal

$x - 1 = x^2 - 5x + 4$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5$ or $x = 1$

ii) two points of intersect.

iii) ~~y=~~

7 iii) $y = x + c$ is tangent to $y = x^2 - 5x + 4$

$$\frac{dy}{dx} = 2x - 5$$

gradient of $y = x + c$ is 1
so

$$2x - 5 = 1$$
$$2x = 6$$
$$\underline{\underline{x = 3}}$$

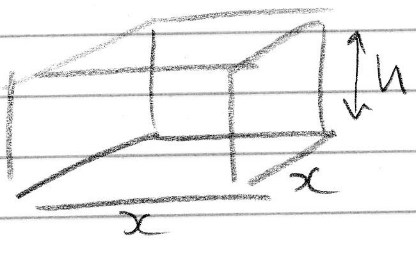
sub into curve to get y so $y = x^2 - 5x + 4$

$$y = 9 - 15 + 4$$
$$\underline{\underline{y = -2}}$$

sub into $y = x + c$

$$-2 = 3 + c$$
$$\underline{\underline{c = -5}}$$

8) Volume $= 8m^3$



A = surface area

$\underline{\text{volume}}$ $\quad 8 = hx^2$ ①

$\underline{\text{Suface area}}$ $\quad A = 2x^2 + 4xh$ ②

$h = \frac{8}{x^2}$ sub into ② $\quad A = 2x^2 + 4x\left(\frac{8}{x^2}\right)$

$$\underline{\underline{= 2x^2 + \frac{32}{x}}}$$

8ii) $A = 2x^2 + \frac{32}{x}$

$= 2x^2 + 32x^{-1}$

$\frac{dA}{dx} = \underline{\underline{4x - 32x^{-2}}}$

iii) $4x - 32x^{-2} = 0$

$4x - \frac{32}{x^2} = 0$

$4x^3 - 32 = 0$

$4x^3 = 32$

$x^3 = 8$

$x = 2$

$\frac{d^2A}{dx^2} = 4 + 32x^{-3}$

$= 4 + \frac{32}{x^3}$ at $x = 2$ $\frac{d^2A}{dx^2} = 8$

+ve so $\underline{\underline{\text{minimum}}}$

9) A(4,-2) B(10,6) C midpoint of AB

find i) coords of C $= \left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$

$= \underline{(7,2)}$

ii) length AC $= \sqrt{(7-4)^2 + (2--2)^2}$

$= \sqrt{9+16} = \sqrt{25} = \underline{\underline{5}}$

9 iii) equation of circle has AB as diameter.

$$(x-7)^2 + (y-2)^2 = 25$$

iv) tangent to circle at point A.

tangent at 90° to diameter

so grad AB $= \frac{6 - -2}{10-4} = \frac{8}{6} = \frac{4}{3}$

grad tangent $= \frac{-3}{4}$

equation of line

$$y - -2 = \frac{-3}{4}(x-4)$$

$$4(y+2) = -3(x-4)$$

$$4y + 8 = 12 - 3x$$

$$3x + 4y = 4$$