C1 Jan 06

1 i) $x^{1/3} = 2 \qquad x = 2^3 \qquad x = 8$

ii) $10^t = 1 \quad \therefore \quad t = 0$

iii) $(y^{-2})^2 = \frac{1}{81}$

$$y^{-2} = \frac{1}{9}$$

$$\frac{1}{y^2} = \frac{1}{9}$$

$$y^2 = 9$$
$$y = \pm 3$$

2) i) $(3x+1)^2 - 2(2x-3)^2$

$9x^2 + 3x + 3x + 1 \quad -2(4x^2 - 6x - 6x + 9)$

$9x^2 + 6x + 1 \quad -8x^2 + 24x - 18$

$x^2 + 30x - 17$

ii) coefficient of x^3

$2x^3 + 6x^3 + 4x^3$

$= 12x^3$

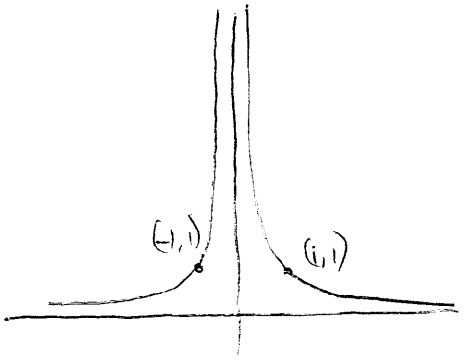
$= 12$

3. $y = 3x^5 - \sqrt{x} + 15$

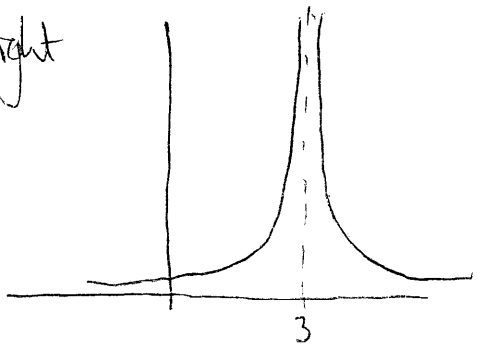
i) $\frac{dy}{dx} = 15x^4 - \frac{1}{2}x^{-1/2}$

$\frac{d^2y}{dx} = 60x^3 + \frac{1}{4}x^{-3/2}$

4) i $y = \frac{1}{x^2}$



ii) $y = \frac{1}{(x-3)^2}$ 3 → right



iii) $y = \frac{1}{x^2}$ to $y = \frac{2}{x^2}$ same as $\frac{y}{2}$ - stretch in parallel to y axes st 2

5) i) $x^2 + 3x$

$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$

ii) $y^2 - 4y - \frac{11}{4}$

$(y-2)^2 - 4 - \frac{11}{4}$

$(y-2)^2 - \frac{27}{4}$

iii) $x^2 + y^2 + 3x - 4y - \frac{11}{4} = 0$ 2 previous eqn used:

$\left(x + \frac{3}{2}\right)^2 + (y-2)^2 - \frac{27}{4} - \frac{9}{4} = 0$

$\therefore$ centre is $\left(-\frac{3}{2}, 2\right)$

$\left(x + \frac{3}{2}\right)^2 + (y-2)^2 = \frac{36}{4}$

iv) $\therefore r = \sqrt{\frac{36}{4}} = 3$

6) i) stationary point $y = x^3 - 3x^2 + 4$

$\frac{dy}{dx} = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$x = 0$ $x = 2$

sub into $y = x^3 - 3x^2 + 4$ $y = 4$ $y = 0$

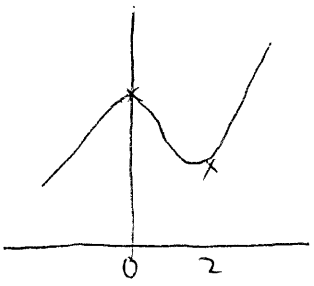
$(0, 4)$ $(2, 0)$

6 ii) $\frac{d^2y}{dx^2} = 6x - 6$

$x = 0$ $\qquad$ $x = 2$

$\frac{d^2y}{dx^2} = -6 \therefore$ -ve $\qquad$ $\frac{d^2y}{dx^2} = 6 \therefore$ +ve

iii)



As positive x^2

$\therefore x > 2$

$x < 0$

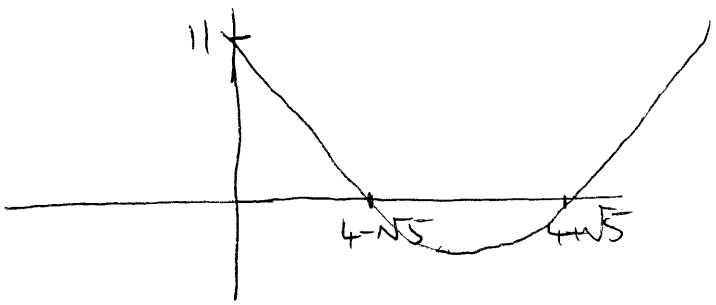
7 i) $x^2 - 8x + 11 = 0$

$+8 \pm \frac{\sqrt{64 - 44}}{2} = \frac{8 \pm \sqrt{20}}{2}$

$= \frac{8 \pm 2\sqrt{5}}{2}$

$= 4 \pm \sqrt{5}$

ii)



iii) $y - 8y^{1/2} + 11 = 0$ $\qquad$ let $x = y^{1/2} \therefore x^2 = y$.

so $x^2 - 8y + 11 = 0$ from previous

$x = 4 \pm \sqrt{5}$

$\therefore y = (4 \pm \sqrt{5})^2$

$y = 16 + 5 \pm 8\sqrt{5}$

$= 21 \pm 8\sqrt{5}$

8) i) $y = x^2 - 5x + 15$ $\quad 5x - y = 10$ $\quad y = 5x - 10$

$-10 = 5x = x^2 - 5x + 15$

$0 = x^2 - 10x + 25$

ii) $-10^2 - 4 \times 25 = 0$

iii) touches once $\therefore$ Tangent

iv) from previous $\quad x^2 - 10x + 25 = 0$

$(x - 5)(x - 5)$

$x = 5$

$\therefore y = 25 - 25 + 15 = \cancel{30}\ 15$

$(5, 15)$

v) gradient of tangent $= 5$ $\quad \therefore$ normal grad $= -\frac{1}{5}$

$y - y_1 = m(x - x_1)$

$y - 15 = -1/5 (x - 5)$

$y - 15 = -\frac{1}{5}x + 1$

$y = -\frac{1}{5}x + 16 \qquad (\times 5)$

$5y = -x + 80$

$x + 5y = 80$

9) Distance AC $= \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

Distance AB $= \sqrt{(p-5)^2 + 6^2}$

$= \sqrt{(p-5)^2 + 36}$

$\therefore \sqrt{(p-5)^2 + 36} = 2\sqrt{10}$

$(p-5)^2 + 36 = 40$

$p^2 - 10p + 25 + 36 = 40$

$p^2 - 10p + 21 = 0$

$(p-7)(p-3) = 0$

$p = 7$ or 3

ii) Point p $\quad y = 3x - 14$

$7 = 3x - 14$

$21 = 3x$

$x = 7.$

$\therefore$ B $(7,7)$ $\qquad$ A $(5,1)$

mid point $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\left(\frac{12}{2}, \frac{8}{2}\right)$

$(6,4)$