Under the ribbon

Modelling and optimisation

Prerequisite knowledge

- Pythagoras' theorem

Why do this unit?

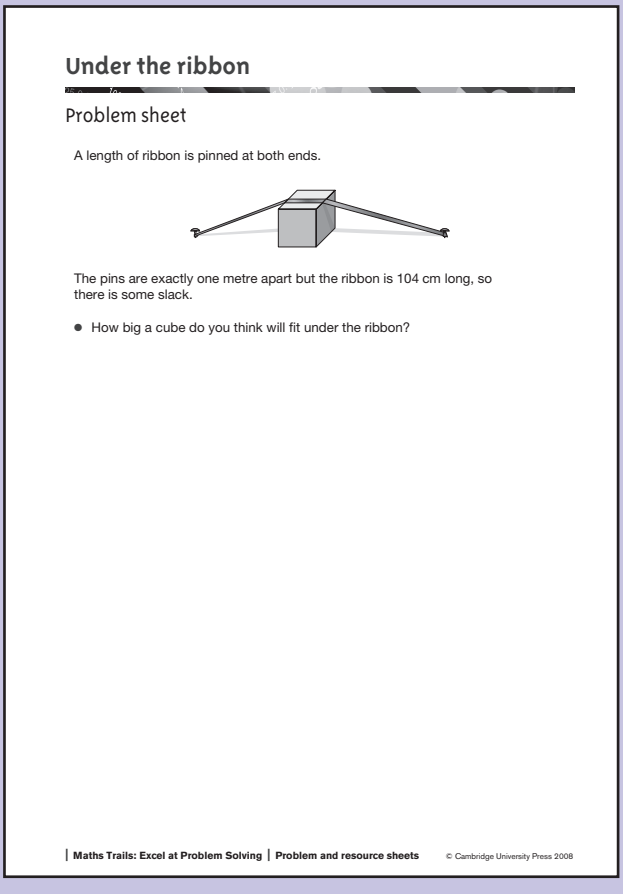
Here the spreadsheet gives access to a problem where the algebraic approach could prove difficult. The answer to the problem is surprising.

Time

One lesson

Resources

Ribbon, metre rulers, drawing pins and plastic construction cubes
CD-ROM: spreadsheet, resource sheet, problem sheet
NRICH website (optional): www.nrich.maths.org, January 2005, 'Slippage'; November 2007, 'Under the ribbon'



Under the ribbon

Problem sheet

A length of ribbon is pinned at both ends.

The pins are exactly one metre apart but the ribbon is 104 cm long, so there is some slack.

- How big a cube do you think will fit under the ribbon?

Maths Trails: Excel at Problem Solving | Problem and resource sheets

Introducing the unit

Introduce the group to the problem 'Under the ribbon' on the problem sheet.

- How big a cube do you think will fit under the ribbon?

Record pupils' estimations for later.

Direct the group to experiment with real ribbon and cubes of different sizes to arrive at a confirmation of their estimate or to settle on a revised value.

- Where is it best to place the cube? [centre]
- What is your estimate now? [record new estimates]
- What could we change about the experiment to get a more accurate answer to the main question? [Use smaller cubes, use more accurate measuring equipment.]

This last question leads towards finding an answer by calculation.

Main part of the unit

Explain to pupils that to clarify their thinking they will use their calculators to work through a single calculation based on their estimates.

Show diagram A from the resource sheet to help pupils visualise the situation.

- What do we know? [100 cm between the pins and ribbon length of 104 cm]
- What do we want to find? [the cube size that fits under a ribbon length of 104 cm]

Discuss what is known in relation to what is wanted and draw pupils towards seeing the possible value of using Pythagoras' theorem.

However, we do not appear to have sufficient information as the 'sloping length' of the ribbon depends on the height of the cube. So we need to estimate a height for the cube and use that to work out a length for the ribbon.

Discuss the merits of a spreadsheet when

approaching a problem which requires trial and improvement in the search for a solution.

In diagrams B and C on the resource sheet the estimate for the sake of discussion has been taken as 6 cm.

Starting with our estimate for the length of the side of the cube we can find the shortest length of ribbon that this cube could fit under.

- If we calculate the ribbon length with our estimate for the cube size, would it be 104 cm? [Probably not. If an estimate is too low the calculated length of the ribbon will be less than 104 cm and this tells us we need to try a larger cube. If our calculation gives a ribbon length greater than 104 cm we know our estimate for the cube is too high.]
- What is the calculation we need to do? [Using diagram C and working with a cube side of 6 cm we can calculate the hypotenuse and use this to find the length of the ribbon.]

Ask pupils to make the calculation using their own estimate. Ask them to suggest an improved second value for the cube side.

Explain that a row in a spreadsheet can work like the calculation they have just done.

Pupils need to discuss and plan their own spreadsheet. There may come a point in this process where you may find it helpful to show pupils the sheet 'Finding the cube side' on the spreadsheet. This can be deconstructed and an explanation given of each formula that has been used.

In this sheet the first column shows cube sides being tested. These advance in equal steps. The final column shows the full ribbon length passing over that cube. This needs to be 104 cm. Cell B5 can be changed to give an improved starting value, and the formula in cell B6 can be adjusted to give a small step size, finer increments giving an increasingly more accurate value for the length of the side of the cube.

Pupils need to implement their own designs, or replicate, or use the sheet 'Finding the cube side'.

When the groups have made progress, return to whole-group discussion. Compare solutions and the effectiveness of different methods.

Plenary

The calculation so far has been based on a cube placed at the centre but this is an assumption that it might be useful to revisit.

- For a specific cube size is the length of ribbon shortest when the cube is centred on 50 cm? [Yes. See the sheet 'Testing the centre position'.]

In 'Testing the centre position' on the spreadsheet you can change the cube side length in cell B4. The left section length increases from 0 to 100 in steps of 1 cm. We need to scan down column I to see where the shortest length of ribbon appears [when the cube is centred at 50 cm, that is, 47 cm from one end for a 6 cm cube].

Solution notes

The side of the cube is 13.3 cm (to 3 s.f.). Surprised?