# Sequences and series

## Generalising and creating formulae

### Prerequisite knowledge

- Triangle numbers
- Consecutive numbers

### Why do this problem?

It is a good context in which to explore triangle numbers in a way that identifies them as more than just a sequence but also numbers that have some pleasing properties when added.

### Time

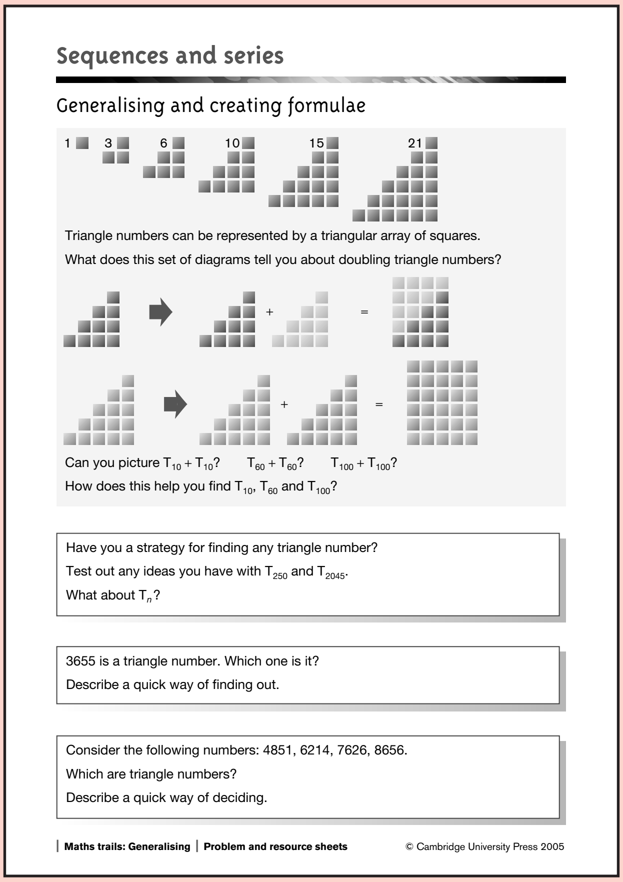
One lesson

### Resources

CD-ROM: pupil worksheet; two sets of triangular arrays cut out from OHTs

NRICH website (optional): www.nrich.maths.org, May 2004, ‘Sequences and series’

Using interlocking cubes will help pupils visualise what is happening. For demonstration on an OHP, pairs of triangular arrays with the same triangle number represented as blocks can be turned and overlaid to give an animated effect. An interactive tool is available on the NRICH site which allows a choice of triangle number.



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Triangle numbers can be represented by a triangular array of squares.

What does this set of diagrams tell you about doubling triangle numbers?

Can you picture $T_{10} + T_{10}$? $T_{60} + T_{60}$? $T_{100} + T_{100}$?

How does this help you find $T_{10}$, $T_{60}$ and $T_{100}$?

Have you a strategy for finding any triangle number?

Test out any ideas you have with $T_{250}$ and $T_{2045}$.

What about $T_n$?

3655 is a triangle number. Which one is it?

Describe a quick way of finding out.

Consider the following numbers: 4851, 6214, 7626, 8656.

Which are triangle numbers?

Describe a quick way of deciding.

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### Introducing the problem

Ask the class:

- Why is a triangle number so called?
- Can you make a triangle number with your cubes?
- What triangle numbers can they suggest?
- What is the smallest triangle number? What is the largest? (!)
- What can you say about the product of any two consecutive numbers? (always even)

### Main part of the lesson

Introduce the idea of doubling triangle numbers using appropriate resources to demonstrate joining two identical triangle numbers together.

- How did they fit together?

Repeat this process several times. Now introduce pupils to the subscript notation used in the problem.

- Can you imagine $T_{10} + T_{10}$? …

The aim is to draw out the relationship between the adjacent sides of the rectangle.

Ask the class to work in small groups to find a strategy for working backwards. Some questions to start them off are:

- Given a rectangle, can you find the triangle number that has been doubled?
- Does this work for any rectangle?
- How can you use this to quickly calculate the value of any triangle number?
- Given any number, how could you work out

if it is a triangle number? (it has to be half the product of two consecutive numbers)

At one or two points during the main activity, draw out some of the observations pupils are making in order to:

- share ideas;
- clarify ideas;
- encourage explanation;
- refocus activities.

## Plenary

Assess and reflect on pupils' understanding by asking them whether several numbers you suggest are triangle numbers and how they know. See the problem sheet for suggestions.

Some pupils may say that they can picture them as a staircase; this is a valid response at their level of understanding.

## Solution notes

$T_n = \frac{1}{2}n(n+1)$, which gives:

$T_{10} = 55$
$T_{60} = 1830$
$T_{100} = 5050$
$T_{250} = 31\,375$
$T_{2045} = 2\,092\,035$

Consider 3655. If 3655 is a triangle number, 7310 can be expressed as $n(n+1)$.

4851 and 7626 are triangle numbers. 6214 and 8656 are not.