Power crazy

Modelling and optimisation

Prerequisite knowledge

- Powers higher than cube

Why do this unit?

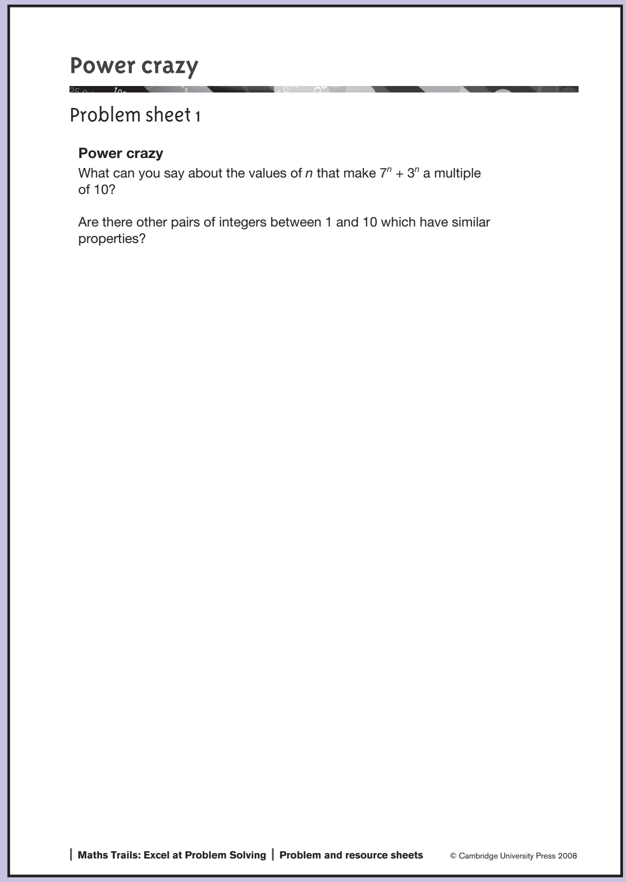
There is a surprising structure and pupils can move quickly into the problem-solving process of accounting for pattern. For example, can what is true for the 3 – 7 combination be explained and generalised?

Time

One lesson

Resources

Calculators
CD-ROM: spreadsheet, problem sheets 1 and 2
NRICH website (optional): www.nrich.maths.org, June 2002, 'Power crazy'; November 2003, 'What an odd fact(or)'



Power crazy

Problem sheet 1

Power crazy
What can you say about the values of n that make $7^n + 3^n$ a multiple of 10?

Are there other pairs of integers between 1 and 10 which have similar properties?

| Maths Trails: Excel at Problem Solving | Problem and resource sheets © Cambridge University Press 2008

Introducing the unit

Show the group the 'Power crazy' problem on problem sheet 1.

Ask the class how they would approach the first part of the problem.

- Where might you start? [Try some numbers, checking that 3 and 7 are raised to the same power.]

As pupils work individually, record their results in a table that they can all refer to. For example:

n	3^n	7^n	$3^n + 7^n$	Multiple of 10?
1	3	7	10	√
2	9	49	58	×
3	27	343	370	√
4	81	2401	2482	×
5	243	16807	17050	√
…	…	…	…	…
…	…	…	…	…

- Can we answer the question yet? [For the data available, it always appears to work for odd powers.]
- Do we know why? [The main part of the lesson explores this.]

Main part of the unit

Show the group the sheet 'Power crazy 1' on the spreadsheet. Briefly look at the contents of cells to see how the spreadsheet was created.

- What can you see? [patterns in the units digits; for example the units digit for powers of 3 follows a cycle 3, 9, 7, 1, 3, …]
- Can you say why you know these cycles continue? [Multiplying a number with a units digit of 3 by 3 will always give you a number with a units digit of 9 and then multiplying by 3 again will give a units digit of 7. One more time brings you back to a units digit of 1. A similar argument can be used with the pattern of the units digits in powers of 7.]

Draw out the connection between the values in the two columns and how you know that the cycle of multiples of 10 will continue. That is, in both patterns the units digit has a cycle of 4 with the 3s and 7s in the units column alternating but summing to make a multiple of 10.

- What other pairs of numbers do you think this might work with? [complements of 10: 1, 9; 2, 8; 4, 6]

Explain to pupils that they are to make the spreadsheet, first with 3 and 7 and then with other combinations. They may wish to copy tables so they have a record to refer to. Divide the class into three groups, each focusing on and justifying their results for one of the other three combinations [1, 9; 2, 8 and 4, 6]. Each group should create a poster for display and prepare to present their explanation to the rest of the class in the plenary.

Extension

You may wish to leave the group with the challenge of what other pairs of numbers generate which multiples – for example which, if any, powers of 3 and 2 will give multiples of 5.

Plenary

Have pupils present and discuss their justifications. This is an ideal opportunity for peer evaluation – which explanation did the group like the best and why?

If time allows, the group can explore the problem 'What an odd fact(or)' on problem sheet 2 to reinforce this style of reasoning.

Solution notes

Power crazy

The justification for multiples of 10 using the properties of cycles is more than adequate.

What an odd fact(or)

The sum can be rewritten as $(1^{99}+9^{99}) + (2^{99}+8^{99}) + (3^{99}+7^{99}) + (4^{99}+6^{99}) + 5^{99}$. Each bracket contains a multiple of 10 – from the arguments in the main part of the lesson – and you are adding a multiple of 5 so the total must be a multiple of 5.

Further ideas

Expand $x^n + (10-x)^n$ to see why odd values of n will produce a result that has 10 as a factor. Allow the '10' to become a variable to generalise for multiples other than 10.