OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4723**

Core Mathematics 3

MARK SCHEME

Specimen Paper

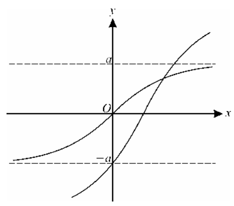
MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

1 *EITHER*: $4x^2+4x+1>x^2-2x+1$	M1		For squaring both sides
i.e. $3x^2+6x>0$	A1		For reduction to correct quadratic
So $x(x+2)>0$	M1		For factorising, or equivalent
Hence $x<-2$ or $x>0$	A1		For both critical values correct
	A1		For completely correct solution set
OR: Critical values where $2x+1=\pm(x-1)$	M1		For considering both cases, or from graphs
i.e. where $x=-2$ and $x=0$	B1		For the correct value -2
	A1		For the correct value 0
Hence $x<-2$ or $x>0$	M1		For any correct method for solution set using two critical values
	A1	**5**	For completely correct solution set
		$\boxed{5}$	
2 **(i)** $\sin x(\frac{1}{2}\sqrt{3})+\cos x(\frac{1}{2})+(\sqrt{3})(\cos x(\frac{1}{2}\sqrt{3})-\sin x(\frac{1}{2}))$	M1		For expanding both compound angles
	A1		For completely correct expansion
	M1		For using exact values of $\sin 30°$ and $\cos 30°$
$=\frac{1}{2}\cos x+\frac{3}{2}\cos x=2\cos x$, as required	A1	**4**	For showing given answer correctly
(ii) $\sin 45°+(\sqrt{3})\cos 45°=2\cos 15°$	M1		For letting $x=15°$ throughout
Hence $\cos 15°=\dfrac{1+\sqrt{3}}{2\sqrt{2}}$	A1	**2**	For any correct exact form
		$\boxed{6}$	
3 **(i)** $x_2=\sqrt[3]{7}=1.9129...$	B1		For 1.91… seen or implied
$x_3=1.9517...$, $x_4=1.9346...$	M1		For continuing the correct process
$\alpha=1.94$ to 2dp	A1	**3**	For correct value reached, following x_5 and x_6 both 1.94 to 2dp
(ii) $x=\sqrt[3]{(17-5x)}\Rightarrow x^3+5x-17=0$	M1		For letting $x_n=x_{n+1}=x$ (or α)
	A1	**2**	For correct equation stated
(iii) *EITHER*: Graphs of $y=x^3$ and $y=17-5x$ only cross once	M1		For argument based on sketching a pair of graphs, or a sketch of the cubic by calculator
Hence there is only one real root	A1√		For correct conclusion for a valid reason
OR: $\dfrac{d}{dx}(x^3+5x-17)=3x^2+5>0$	M1		For consideration of the cubic's gradient
Hence there is only one real root	A1√	**2**	For correct conclusion for a valid reason
		$\boxed{7}$	
4 **(i)** $\int_0^2(4x+1)^{-\frac{1}{2}}\,dx=\left[\frac{1}{2}(4x+1)^{\frac{1}{2}}\right]_0^2=\frac{1}{2}(3-1)=1$	M1		For integral of the form $k(4x+1)^{\frac{1}{2}}$
	A1		For correct indefinite integral
	M1		For correct use of limits
	A1	**4**	For given answer correctly shown
(ii) $\pi\int_0^2\dfrac{1}{4x+1}\,dx=\pi\left[\frac{1}{4}\ln(4x+1)\right]_0^2=\frac{1}{4}\pi\ln 9$	M1		For integral of the form $k\ln(4x+1)$
	A1		For correct $\frac{1}{4}\ln(4x+1)$, with or without π
	M1		Correct use of limits and π
	A1	**4**	For correct (simplified) exact value
		$\boxed{8}$	

5	**(i)** 200 °C	B1	**1**	For value 200
	(ii) $150 = 200 - 180e^{-0.1t} \Rightarrow e^{-0.1t} = \frac{50}{180}$	M1		For isolating the exponential term
	Hence $-0.1t = \ln\frac{5}{18} \Rightarrow t = 12.8$	M1		For taking logs correctly
		A1	**3**	For correct value 12.8 (minutes)
	(iii) $\frac{d\theta}{dt} = 18e^{-0.1t}$	M1		For differentiation attempt
		A1		For correct derivative
	Hence rate is $18e^{-0.1\times 12.8} = 5.0$ °C per minute	M1		For using their value from **(ii)** in their $\dot{\theta}$
		A1	**4**	For value 5.0(0)
			$\boxed{8}$	
6	**(i)** Domain of f^{-1} is $x \geqslant 1$	B1		For the correct set, in any notation
	Range is $x \geqslant 0$	B1	**2**	Ditto
	(ii) If $y = 1 + \sqrt{x}$, then $x = (y-1)^2$	M1		For changing the subject, or equivalent
	Hence $f^{-1}(x) = (x-1)^2$	A1	**2**	For correct expression in terms of x
	(iii) The graphs intersect on the line $y = x$	B1		For stating or using this fact
	Hence x satisfies $x = (x-1)^2$	B1		For either $x = f(x)$ or $x = f^{-1}(x)$
	i.e. $x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$	M1		For solving the relevant quadratic equation
	So $x = \frac{1}{2}(3 + \sqrt{5})$ as x must be greater than 1	A1	**4**	For showing the given answer fully
			$\boxed{8}$	
7	**(i)** $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	B1	**1**	For correct RHS stated
	(ii) $\frac{8t}{1-t^2} + 3 \times \frac{1}{t} \times (1 + t^2) = 0$	B1		For $\cot x = \frac{1}{t}$ seen
		B1		For $\sec^2 x = 1 + t^2$ seen
	Hence $8t^2 + 3(1-t^2)(1+t^2) = 0$	M1		For complete substitution in terms of t
	i.e. $3t^4 - 8t^2 - 3 = 0$, as required	A1	**4**	For showing given equation correctly
	(iii) $(3t^2 + 1)(t^2 - 3) = 0$	M1		For factorising or other solution method
	Hence $t = \pm\sqrt{3}$	A1		For $t^2 = 3$ found correctly
	So $x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$	A1		For any two correct angles
		A1	**4**	For all four correct and no others
			$\boxed{9}$	

[Turn over

8	**(i)**	$\frac{dy}{dx}=\frac{2\ln x}{x}$	M1	For relevant attempt at the chain rule
			A1	For correct result, in any form
		$\frac{d^2y}{dx^2}=\frac{x(2/x)-2\ln x}{x^2}=\frac{2-2\ln x}{x^2}$	M1	For relevant attempt at quotient rule
			A1 **4**	For correct simplified answer
	(ii)	For maximum gradient, $2-2\ln x=0\Rightarrow x=\mathrm{e}$	M1	For equating second derivative to zero
			A1	For correct value e
		Hence P is $(\mathrm{e},1)$	A1√	For stating or using the y-coordinate
		The gradient at P is $\frac{2}{\mathrm{e}}$	A1√	For stating or using the gradient at P
		Tangent at P is $y-1=\frac{2}{\mathrm{e}}(x-\mathrm{e})$	M1	For forming the equation of the tangent
		Hence, when $x=0$, $y=-1$ as required	A1 **6**	For correct verification of $(0,-1)$
			$\boxed{10}$	
9	**(i)**	$a=\frac{1}{2}\pi$	B1 **1**	For correct exact value stated
	(ii)	$x=\tan(\frac{1}{4}\pi)=1$	M1	For use of $x=\tan(\frac{1}{2}a)$
			A1√ **2**	For correct answer, following their a
	(iii)		B1	For x-translation of (approx) $+1$
			B1	For y-stretch with (approx) factor 2
		Asymptotes are $y=\pm 2a$	B1 **3**	For correct statement of asymptotes
	(iv)	x $\tan^{-1}x$ $2\tan^{-1}(x-1)$		
		1.535 0.993 0.983	M1	For relevant evaluations at 1.535, 1.545
		1.545 0.996 0.998		
		Hence graphs cross between 1.535 and 1.545	A1 **2**	For correct details and explanation
	(v)	Relevant values of $\left(\tan^{-1}x\right)^2$ are (approximately)	M1	For the relevant function values seen or
		0, 0.0600, 0.2150, 0.4141, 0.6169		implied; must be radians, not degrees
		$\frac{1}{12}\{0+4(0.0600+0.4141)+2\times0.2150+0.6169\}$	M1	For use of correct formula with $h=\frac{1}{4}$
		Hence required approximation is 0.245	A1 **3**	For correct (2 or 3sf) answer
			$\boxed{4}$	