What do you see?

Multipurpose

Prerequisite knowledge

- Ability to find areas of squares and right-angled triangles

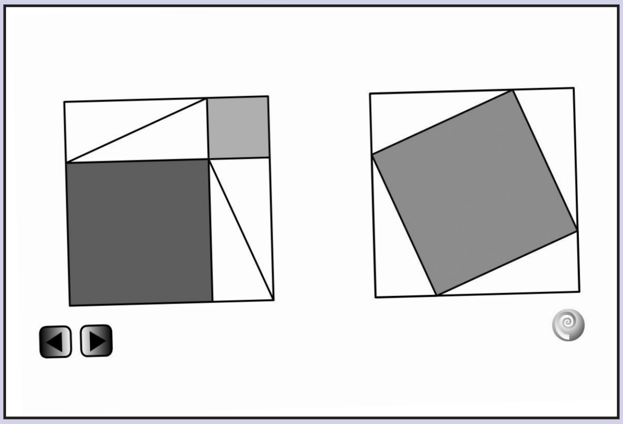
Why do this problem?

There is always the danger of leaping into proofs without giving pupils time to engage with and become familiar with the setting. Here we give an example of developing a proof of Pythagoras' theorem, based on the work of Bhaskara. Pupils spend time visualising what is happening before formalising findings (in the following lesson).

This problem also gives opportunities to pay particular attention to the evaluation phase of the problem-solving model.

Time

One lesson



Resources

Range of sizes of coloured sheets of paper, scissors, glue, rulers, pencils
CD-ROM: storyboards (resource sheets from 'Cubes within cubes'); animation
NRICH website (optional): www.nrich.maths.org; as a supporting activity you might want to look at the problem 'Tilted squares' (September 2004)

Introducing the problem

Explain that you will be showing pupils an animation that they will be asked to describe in detail afterwards. It is best if pupils focus on the animation and do not try to write things down; the aim is for them to have a clear view of the structure of what is happening as well as of the detail.

After you have shown the animation, ask pupils to describe what they saw. After each part of their description it may be useful to play and pause the animation before moving on to the next step.

- How did the animation start? [a right-angled triangle]
- Do you think it was a particular right-angled triangle? Does its size matter? [no, but at this stage you might want to say that you will return to this point later in the lesson]
- What happened next?
- How was the large square formed?
- What was coloured at the end? [two squares]
- How do you know they were squares? [draw out the connection between the sides of the triangle and the sides of the squares – two shorter sides]
- What happened next? [a copy of the square was taken and moved to one side]
- And then? [the triangles were rearranged inside the large square]
- What do you think the connection was between the three shaded squares? [area of red = area of blue + area of green]
- Can you explain why? [they (blue + green, and also red) are equivalent to the remaining area of the large square when the four right-angled triangles are removed]

This last point should be revisited when pupils have created their own constructions of the film.

Main part of the lesson

Hand out equipment and challenge pupils, working in pairs, to reproduce the animation using storyboards (see resource sheets for

'Cubes within cubes'). They will need to plan very carefully what will be included in each stage. They may wish to add a sentence to some of the stages to aid the audience's understanding.

Before starting, return to the issues of the size of the right-angled triangle – encourage some variation within the range limited by the space they have available!

The outcome is to demonstrate that the sum of the areas of the two smaller squares is equal to the area of the larger square.

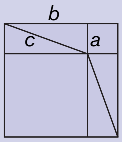
As pupils work, use the opportunity to check their understanding of the intended outcomes and that they have planned sufficiently before starting to cut up pieces of paper.

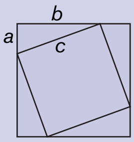
Plenary

Share some of the most effective representations, emphasising that the sizes of the right-angled triangles did not appear to affect the outcome concerning equal areas.

You might wish to talk to the class about Pythagoras' theorem and Bhaskara, and say you will be following this up in coming lessons.

Solution notes





$c^2 = b^2 + a^2$