Two up

Developing construction skills

Prerequisite knowledge

- Symbolic notation to represent a mathematical idea
- Enter and change data on a spreadsheet

Why do this unit?

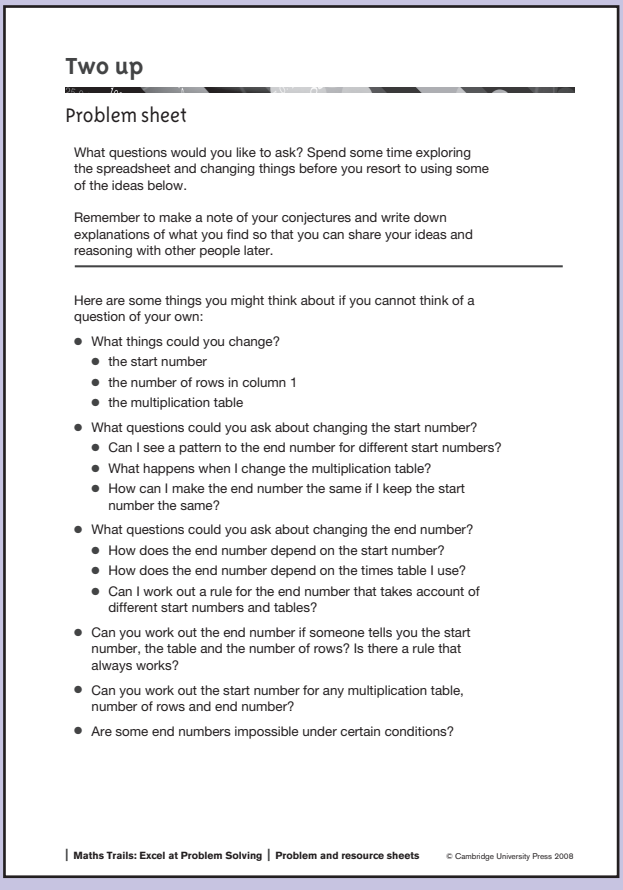
This activity offers pupils opportunities to pose their own problems and to employ algebra to aid justification. The use of the spreadsheet and the feedback it offers enables pupils to focus on patterns and relationships without being overwhelmed by the calculations.

Time

One lesson

Resources

CD-ROM: spreadsheet, problem sheet



Two up

Problem sheet

What questions would you like to ask? Spend some time exploring the spreadsheet and changing things before you resort to using some of the ideas below.

Remember to make a note of your conjectures and write down explanations of what you find so that you can share your ideas and reasoning with other people later.

Here are some things you might think about if you cannot think of a question of your own:

- What things could you change?
  - the start number
  - the number of rows in column 1
  - the multiplication table
- What questions could you ask about changing the start number?
  - Can I see a pattern to the end number for different start numbers?
  - What happens when I change the multiplication table?
  - How can I make the end number the same if I keep the start number the same?
- What questions could you ask about changing the end number?
  - How does the end number depend on the start number?
  - How does the end number depend on the times table I use?
  - Can I work out a rule for the end number that takes account of different start numbers and tables?
- Can you work out the end number if someone tells you the start number, the table and the number of rows? Is there a rule that always works?
- Can you work out the start number for any multiplication table, number of rows and end number?
- Are some end numbers impossible under certain conditions?

Maths Trails: Excel at Problem Solving | Problem and resource sheets

Introducing the unit

Model how to create the run of 'counting numbers' on a spreadsheet and how to create multiples of 2 using the counting numbers (see 'Two up (T)' on the spreadsheet (red tab)).

Use your own spreadsheet or 'Two up' on the spreadsheet (blue tab) and change the value of cell A2 several times. Ask the group to account for some of the new values appearing in the other cells.

- What will the final number in the first column be if I change the 1 in cell A2 to a 5, a 10, a 101? [24, 29, 120]
- What would be the first number if the final number was …?
- What will be the final number in the second column if I change the 1 in A2 to 4, 31, 1000? [46, 100, 2038]

Ask questions about how the spreadsheet was constructed.

- How else could I make the two times table in column B using the numbers from column A? [e.g. add each number to itself – demonstrate the equivalence on the spreadsheet in column C.]
- Will it have the same effect on the rest of the spreadsheet if I change the number in cell A2?
- What about other ways?

Ask pupils to work in pairs to create their own copy of 'Two up'. You may wish to make the original sheet available for the pupils to refer to. Both pupils in a pair should be able to demonstrate that they can confidently create this sheet.

Pairs should then test each other's understanding by asking questions about the final number in the second column for different starting numbers (and vice versa).

Main part of the unit

- What number would I have to put in cell A2 to make the final number in the second column 162? [62 – an interesting result as both numbers end with the same two digits]
- Are there other start numbers for which the last number is 100 plus the start number? [See solution notes.]

Ask the pupils to investigate this and to seek a generalisation of their results.

The problem sheet contains further conjectures and ideas to pursue. Encourage pupils to pose their own problems by giving them a few minutes to explore and think about questions of their own before letting them see the suggestions on the problem sheet.

- What other questions do you want to ask?
- What things can you change? [start number, number of rows in column 1, which multiplication table to use]
- What questions could you ask about changing the start number?
- Can you find a rule for describing the end number whatever the start number and the table that is used?

Ask pupils how they would like to share their findings in the plenary. They could:

- note each conjecture they decide to focus on;
- record their findings and arguments related to the conjecture.

Focus on things that surprised them and what information they needed to keep on the way.

Plenary

Select one or two 'surprising' findings to share with the class. Pupils do not have to be able to describe generalisations algebraically but identifying two or more different explanations pupils have for the same generalisation during the lesson and sharing these will encourage pupils to think that it is possible to have more than one correct solution in mathematics.

Solution notes

The only start value which produces an end value greater by one hundred is 62.
$2(x+19)=100+x$ leads to $x=62$.

However, changing the column length from 20 and looking for $x \geqslant 100+x$ gives
$2(x+n-1)=100+x$ where x is the start number and n is the numbers added on.

This leads to $n=(100-x+2)/2$ or $x=100-2n+2$.

Changing the tables from 2 to 3 times yields:
$3(x+n-1)=100+x$
$2x=100-3n+3$
$x=(100-3n+3)/2$ (for this to work n must be odd)

For example, if $n=21$, $x=20$ and the end number is 120.