Mark Scheme 4724
January 2007

1	Factorise numerator and denominator	M1		<u>or</u> Attempt long division
	Num = $(x+6)(x-4)$ or denom = $x(x-4)$	A1		Result = $1+\dfrac{6x-24}{x^2-4x}$
	Final answer = $\dfrac{x+6}{x}$ or $1+\dfrac{6}{x}$	A1	3	= $1+\dfrac{6}{x}$
2	Use parts with $u=\ln x, \mathrm{d}v = x$	M1		& give 1st stage in form $\mathrm{f}(x) +/- \int \mathrm{g}(x)(\mathrm{d}x)$
	Obtain $\frac{1}{2}x^2\ln x - \int \frac{1}{x}.\frac{1}{2}x^2(\mathrm{d}x)$	A1		or $\frac{1}{2}x^2\ln x - \int \frac{1}{2}x(\mathrm{d}x)$
	= $\frac{1}{2}x^2\ \ln x - \frac{1}{4}x^2$ (+c)	A1		
	Use limits correctly	M1		
	Exact answer $2\ln 2 - \frac{3}{4}$	A1	5	AEF ISW
3	(i) Find $\boldsymbol{a}-\boldsymbol{b}$ or $\boldsymbol{b}-\boldsymbol{a}$ irrespective of label	M1		(expect $11\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ or $-11\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$)
	Method for magnitude of any vector	M1		
	$\sqrt{161}$ or $12.7(12.688578)$	A1	3	
	(ii) Using $(\overline{\mathrm{AO}}$ or $\overline{\mathrm{OA}})$ and $(\overline{AB}$ or $\overline{\mathrm{BA}})$	B1		Do not class angle AOB as MR
	$\cos\theta = \dfrac{\text{scalar product of any two vectors}}{\text{product of their moduli}}$	M1		
	43 or better (42.967…), 0.75 or better (0.7499218..)	A1	3	If 137 obtained, followed by 43, award A0 Common answer 114 probably $\rightarrow$ B0 M1 A0
4	Attempt to connect $\mathrm{d}x$ and $\mathrm{d}u$	M1		but not just $\mathrm{d}x = \mathrm{d}u$
	For $\mathrm{d}u = 2\,\mathrm{d}x$ AEF correctly used	A1		sight of $\frac{1}{2}$ ($\mathrm{d}u$) necessary
	$\int u^8 + u^7\ (\mathrm{d}u)$	A1		or $\int u^7(u+1)(\mathrm{d}u)$
	Attempt new limits for u at any stage (expect 0,1)	M1		or re-substitute & use ($\frac{5}{2}$,3)
	$\frac{17}{72}$	A1	5	**AG** WWW
	S.R. If M1 A0 A0 M1 A0, award S.R. B1 for answer $\frac{68}{72}, \frac{34}{36}$ or $\frac{17}{18}$			ISW
5	(i) Show clear knowledge of binomial expansion	M1		$-3x$ should appear but brackets can be missing; $-\frac{1}{3}.-\frac{4}{3}$ should appear, not $-\frac{1}{3}.\frac{2}{3}$
	= $1+x$	B1		Correct first 2 terms; not dep on M1
	$+2x^2$	A1		
	$+\frac{14}{3}x^3$	A1	4	
	(ii) Attempt to substitute $x+x^3$ for x in (i)	M1		Not just in the $\frac{14}{3}x^3$ term
	Clear indication that $(x+x^3)^2$ has no term in x^3	A1		
	$\frac{17}{3}$	√A1	3	f.t. $\mathrm{cf}(x)$ + $\mathrm{cf}(x^3)$ in part (i)
6	(i) $2x+1 = / \equiv A(x-3)+B$	M1		
	$A=2$	A1		
	$B=7$	A/B 1	3	Cover-up rule acceptable for B1
	(ii) $\int \frac{1}{x-3}(\mathrm{d}x) = \ln(x-3)$ or $\ln\lvert x-3\rvert$	B1		Accept A or $\frac{1}{A}$ as a multiplier
	$\int \frac{1}{(x-3)^2}(\mathrm{d}x) = -\frac{1}{x-3}$	B1		Accept B or $\frac{1}{B}$ as a multiplier
	6 + 2 ln 7 Follow-through $\dfrac{6}{7}B + A\ln 7$	√B2	4	

7	$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1		
	$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
	$4x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$	B1		
	Put $\frac{dy}{dx} = 0$	*M1		
	Obtain $4x + y = 0$ AEF	A1		and no other (different) result
	Attempt to solve simultaneously with eqn of curve	dep*M1		
	Obtain $x^2 = 1$ or $y^2 = 16$ from $4x + y = 0$	A1		
	$(1,-4)$ and $(-1,4)$ and no other solutions	A1	**8**	Accept $(\pm 1, \mp 4)$ but not $(\pm 1, \pm 4)$
8	(i) Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $-\frac{1}{m}$ for grad of normal	M1		or change to cartesian.,diff & use $-\frac{1}{m}$
	$= -p$ **AG** WWW	A1	**2**	Not $-t$.
	(ii) Use correct formula to find gradient of line	M1		
	Obtain $\frac{2}{p+q}$ **AG** WWW	A1	**2**	Minimum of denom = $2(p-q)(p+q)$
	(iii) State $-p = \frac{2}{p+q}$	M1		Or find eqn normal at P & subst $(2q^2, 4q)$
	Simplify to $p^2 + pq + 2 = 0$ **AG** WWW	A1	**2**	With sufficient evidence
	(iv) (8,8) $\rightarrow t$ or p or $q = 2$ only	B1		No possibility of -2
	Subst $p = 2$ in eqn (iii) to find q_1	M1		Or eqn normal,solve simult with cartes/param
	Subst $p = q_1$ in eqn (iii) to find q_2	M1		Ditto
	$q_2 = \frac{11}{3} \rightarrow \left(\frac{242}{9}, \frac{44}{3}\right)$	A1	**4**	No follow-through; accept (26.9 , 14.7)
9	(i) Separate variables as $\int \sec^2 y \, dy = 2\int \cos^2 2x \, dx$	M1		seen or implied
	LHS = tan y	A1		
	RHS; attempt to change to double angle	M1		
	Correctly shown as 1 + cos 4x	A1		
	$\int \cos 4x \, dx = \frac{1}{4}\sin 4x$	A1		
	Completely correct equation (other than +c)	A1		$\tan y = x + \frac{1}{4}\sin 4x$
	+c on either side	A1	**7**	<u>not</u> on both sides unless c_1 and c_2
	(ii) Use boundary condition	M1		provided a sensible outcome would ensue
	c (on RHS) = 1	A1		or $c_2 - c_1 = 1$; not fortuitously obtained
	Substitute $x = \frac{1}{6}\pi$ into their eqn, produce y = 1.05	A1	**3**	or 4.19 or 7.33 etc. Radians only
10	(i) For (either point) + t(diff between posn vectors)	M1		"**r** =" not necessary for the M mark
	r = (either point) + t(**i** -2**j** – 3**k** or –**i** + 2**j** + 3**k**)	A1	**2**	**…** but it is essential for the A mark
	(ii) **r** = s(**i** + 2**j** – **k**) or (**i** + 2**j** – **k**) + s(**i** + 2**j** – **k**)	B1		Accept any parameter, including t
	Eval scalar product of **i**+2**j**-**k** & their dir vect in (i)	M1		
	Show as (1x1 or 1)+(2x-2 or -4)+(-1x-3 or 3)	A1		This is just one example of numbers involved
	= 0 <u>and</u> state perpendicular **AG**	A1	**4**	
	(iii) For at least two equations with diff parameters	M1		e.g. $5 + t = s$, $2 - 2t = 2s$, $-9 - 3t = -s$
	Obtain $t = -2$ or $s = 3$ (possibly -3 or 2 or -2)	A1		Check if $t = 2, 1$ or -1
	Subst. into eqn AB or OT and produce 3**i** + 6**j** – 3**k**	A1	**3**	
	(iv) Indicate that $\lvert OC \rvert$ is to be found	M1		where C is their point of intersection
	$\sqrt{54}$;f.t. $\sqrt{a^2 + b^2 + c^2}$ from a**i** + b**j** + c**k** in (iii)	√A1	**2**	

<u>In the above question, accept any vectorial notation</u>
t and s may be interchanged, and values stated above need to be treated with caution.
In (iii), if the point of intersection is correct, it is more than likely that the whole part is correct – but check.