OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4723/01**

MATHEMATICS

Core Mathematics 3

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 Functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = x^3 + 4 \quad \text{and} \quad \mathrm{g}(x) = 2x - 5.$$

Evaluate

(i) $\mathrm{fg}(1)$, [2]

(ii) $\mathrm{f}^{-1}(12)$. [3]

2 The sequence defined by

$$x_1 = 3, \qquad x_{n+1} = \sqrt[3]{31 - \tfrac{5}{2}x_n}$$

converges to the number α.

(i) Find the value of α correct to 3 decimal places, showing the result of each iteration. [3]

(ii) Find an equation of the form $ax^3 + bx + c = 0$, where a, b and c are integers, which has α as a root. [3]

3 **(a)** Solve, for $0° < \alpha < 180°$, the equation $\sec \tfrac{1}{2}\alpha = 4$. [3]

(b) Solve, for $0° < \beta < 180°$, the equation $\tan \beta = 7 \cot \beta$. [4]

4 Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

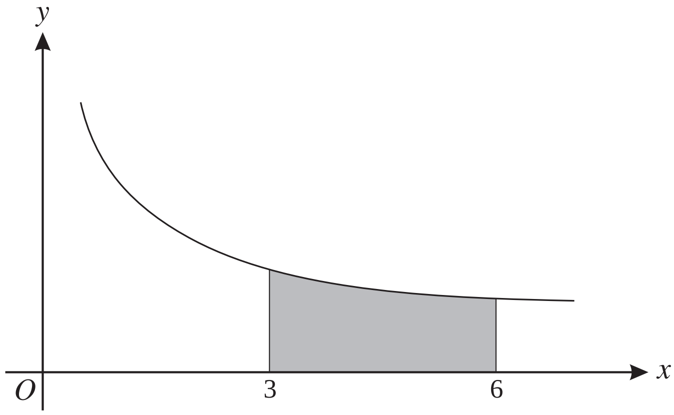
$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

(i) Find the value of $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 2$. [3]

(ii) The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate, in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures. [3]

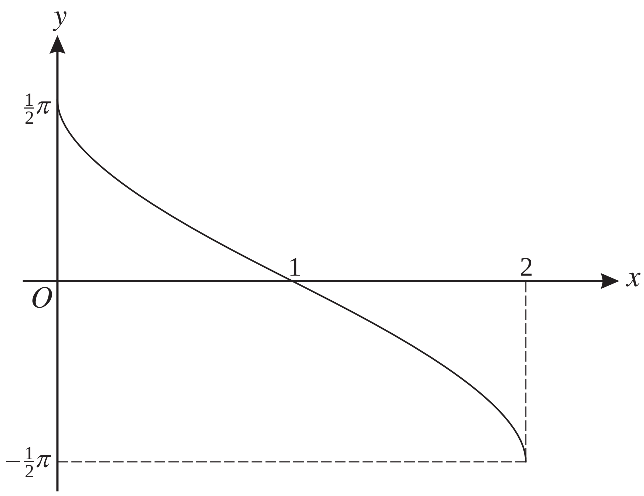
5 **(a)** Find $\int (3x+7)^9 \, dx$. [3]

(b)



The diagram shows the curve $y = \dfrac{1}{2\sqrt{x}}$. The shaded region is bounded by the curve and the lines $x = 3$, $x = 6$ and $y = 0$. The shaded region is rotated completely about the x-axis. Find the exact volume of the solid produced, simplifying your answer. [5]

6



The diagram shows the graph of $y = -\sin^{-1}(x-1)$.

(i) Give details of the pair of geometrical transformations which transforms the graph of $y = -\sin^{-1}(x-1)$ to the graph of $y = \sin^{-1} x$. [3]

(ii) Sketch the graph of $y = |-\sin^{-1}(x-1)|$. [2]

(iii) Find the exact solutions of the equation $|-\sin^{-1}(x-1)| = \frac{1}{3}\pi$. [3]

7 A curve has equation $y = \dfrac{xe^{2x}}{x+k}$, where k is a non-zero constant.

(i) Differentiate xe^{2x}, and show that $\dfrac{dy}{dx} = \dfrac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$. [5]

(ii) Given that the curve has exactly one stationary point, find the value of k, and determine the exact coordinates of the stationary point. [5]

8 The definite integral I is defined by

$$I = \int_0^6 2^x \, dx.$$

(i) Use Simpson's rule with 6 strips to find an approximate value of I. [4]

(ii) By first writing 2^x in the form e^{kx}, where the constant k is to be determined, find the exact value of I. [4]

(iii) Use the answers to parts **(i)** and **(ii)** to deduce that $\ln 2 \approx \frac{9}{13}$. [2]

9 **(i)** Use the identity for $\cos(A + B)$ to prove that

$$4\cos(\theta + 60^\circ)\cos(\theta + 30^\circ) \equiv \sqrt{3} - 2\sin 2\theta. \quad [4]$$

(ii) Hence find the exact value of $4\cos 82.5^\circ \cos 52.5^\circ$. [2]

(iii) Solve, for $0^\circ < \theta < 90^\circ$, the equation $4\cos(\theta + 60^\circ)\cos(\theta + 30^\circ) = 1$. [3]

(iv) Given that there are no values of θ which satisfy the equation

$$4\cos(\theta + 60^\circ)\cos(\theta + 30^\circ) = k,$$

determine the set of values of the constant k. [3]