

IDEA 20

TAKE THREE NUMBERS

This idea is similar to the 'classic' problem about finding five numbers (or weights) and the operation of addition only to produce all the possible values from 1 to 31. This is an interesting task to work on, not least of all because the solution set is 16, 8, 4, 2, and 1 which are the binary (base 2) column headings.

The 'Take three numbers' idea is similar to this problem but allows the use of subtraction as well as addition. The problem is to find the 'best' three numbers that can be used to make all the possible values from 1 to a value somewhere in between 10 and 20. I have not revealed what this highest value is in order to offer the reader a challenge and engage in the pleasure of puzzling.

For example, if the three numbers were 3, 4 and 6 then I can make the following:

$$\begin{aligned} 1 &= 4 - 3 \\ 2 &= 6 - 4 \\ 3 &= 3 \\ 4 &= 4 \\ 5 &= 6 + 3 - 4 \\ 6 &= 6 \\ 7 &= 3 + 4 \end{aligned}$$

Eight cannot be made, so with 3, 4 and 6 the highest value, including all the possible values from 1 upwards, is 7.

With four numbers all the values from 1 to 40 can be made and once students have solved the earlier problem this one will be easy!

This problem can provide an opportunity for students to find negative as well as positive solutions.

There are many mathematical tasks that can be created using strips of paper; the following idea was suggested to me by Anna McCafferty a PGCE student.

- Fold a strip of paper into a number of equal portions (say a minimum of four and a maximum of eight).
- On top of the first portion write a number between one and ten.
- On the second portion write a function, for example: +2.
- On the third portion write another function, for example: $\times 4$, and so on.

The idea is for students to carry out some mental arithmetic to try to work out the final answer as each of the functions are applied to the previous value.

For example, the start number could be 3 and the functions +2, $\times 4$, -8 , $\div 2$ and +5.

Students can work in pairs, each making a strip, choosing the number of portions and the start number and calculating the answer. Partners then have to work out what each others' answers are once all the functions have been applied.

Expressing the above as a sequence of functions using a start number (s):

s	$s + 2$	$4s + 8$	$2s$	$2s + 5$
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will provide an opportunity for students to write expressions using brackets, for example,
 $[(s + 2) \times 4 - 8] \div 2 + 5$.

The task could also be turned around and used as a context for solving equations. For example, if the start number s is unknown and the answer a is given, the idea is for a partner to work out the value of s .

For example, what is the value of s if $[(s + 2) \times 4 - 8] \div 2 + 5 = 11$?

IDEA 21

TAKE A STRIP OF PAPER