Different but same

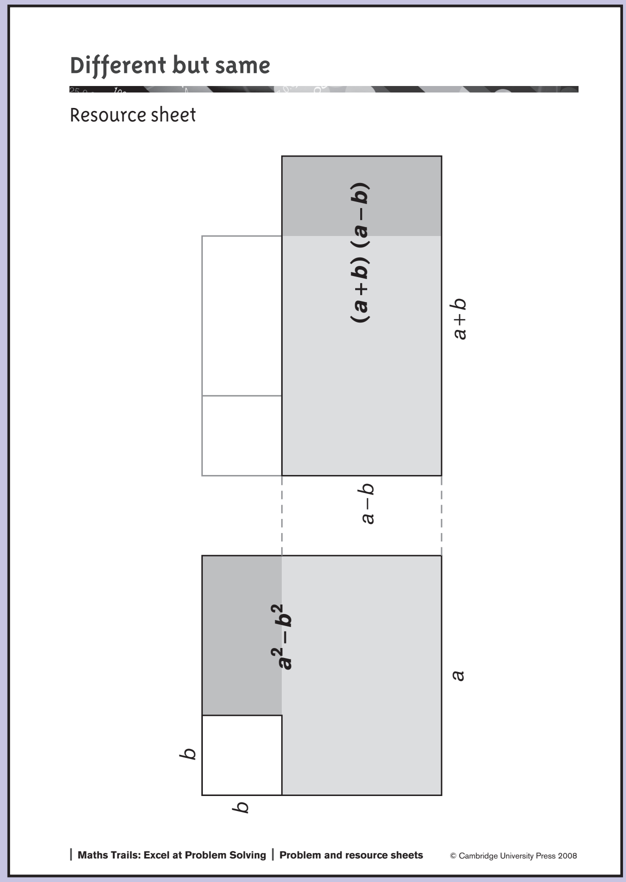
Modelling and optimisation

Prerequisite knowledge
- Square of a number
- Expansion of brackets

Why do this unit?
Pupils will learn the general skill of producing a table of results from two variables, applying it here to explore algebraic equivalence. The algebraic identity used here is the difference of two squares.

Time
One lesson

Resources
CD-ROM: spreadsheet, resource sheet



Introducing the unit

Show the group the sheet 'Grid 1' on the spreadsheet.

- What can you say about the red numbers? [square numbers]
- Can you see a connection between the blue numbers in the grid and the red numbers at the top and side? [sum of squares]
- Do you notice anything else, and can you account for it? [For example, the main diagonal top-left to bottom-right contains only even numbers, and parallel diagonals alternate all odd, all even. This is because two odd or two even squares produce an even sum, and an odd and an even square produce an odd sum.]

Main part of the unit

Show the group the sheet 'Grid 2' on the spreadsheet. Ask pupils to spend one minute on their own making a quick record of anything they notice. Pupils can then share in pairs, leading into a whole-group discussion which explores the patterns in diagonals or in rows and columns, as with 'Grid 1'. [For example, rows and columns show the same pattern of increase – going up in odd number steps. As well, diagonals consist of either all odd or all even numbers.]

- Do you see more than one connection between the blue numbers within the grid and the black numbers at the top and side of the grid? [a difference of squares, and a product of a sum with a difference, for example in cell K6: $8^2 - 3^2 = 55$ and also $(8+3)(8-3) = 55$]

The activity from this point forwards helps pupils to consolidate their understanding of this equivalence (that a difference of two squares is identical to the product of a sum and a difference, or in algebra: $a^2 - b^2 = (a + b)(a - b)$).

This equivalence will be reinforced through:

- constructing two versions of the table, one using the differences of two squares and the other using the product of a sum and a difference;
- looking at the algebraic equivalence of the two expressions (expanding brackets);
- considering diagrammatic representations (in the plenary).

Show pupils 'Blank grid 1' on the spreadsheet. The aim is to create a table for each of the two expressions, and check that they give the same result. First create the table for the difference of two squares.

- What formula should we type for cell D4 in the first grid? [pupils may say =D3–C4]

Type this formula into D4 and press **Enter** so that pupils can see the result. So far this looks good.

Copy the formula across the whole grid.

- This does not work – why? [Look at the values in the first row: for example the 11 in G4 should have been the difference between 16 and 1 but the actual calculation was the difference between 16 and 5.]

Explain that to overcome this problem we will use **absolute references** (see the sheet 'Help - Table of 2 variables'). Demonstrate the correct formula, =D$3–$C4. Discuss the negative values that appear in the grid (use the **ABS** command if you wish to make all results positive).

We have done this calculation using square (red) numbers. But here we need to work directly with the black numbers, so display sheet 'Blank grid 2'.

Discuss and derive the formula needed in the first table (difference of two squares). [=C$2^2–$B3^2]. Explain to the group that they need to produce both tables (difference of the two squares and product of a sum and a difference) in 'Blank grid 2'. [For the product table the formula is =(O$2+$N3)*(O$2–$N3).]

Producing grids with absolute references can be a demanding task. Judge when to draw pupils into discussion on issues relating to the process or understanding as they arise. When pupils have completed both grids move into the plenary, where a connection can be made with algebraic and/or diagrammatic representations of the relationship.

Plenary

- Why do you think the two grids contain the same numbers? [The formulae are equivalent – pupils might say 'We are doing the same thing in a different way'.]

Show and discuss the diagrammatic presentation of equivalence offered on the resource sheet and connect that with the 'traditional' algebraic approach involving the expansion of brackets.

Solution notes

$a^2 - b^2 = (a + b)(a - b)$