

Core 1 May 2005

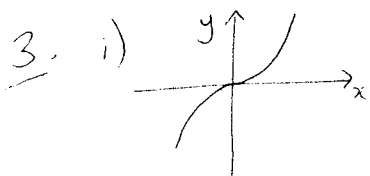
1. $x^2 - 6x - 40 \geq 0$
 $(x+4)(x-10) \geq 0$
 \therefore Critical values -4 and 10
if $x < -4$ then $(x+4) < 0$ and $(x-10) < 0$
so $(x+4)(x-10) > 0$

if $x > 10$ then $(x+4) > 0$ and $(x-10) > 0$
so $(x+4)(x-10) > 0$

so if $x \leq -4$ or $x \geq 10$ then $(x+4)(x-10) \geq 0$

2. i) $3(x^2 + 4x) + 7 = 3(x+2)^2 - 12 + 7$
 $= 3(x+2)^2 - 5$
 $\therefore a = 2 \quad b = -5$

ii) $x = -2$



ii) Either a reflection in y axis or a reflection in the x axis (as $-x^3 = (-x)^3$)

iii) $y = (x-p)^3$

4. This is a quadratic in x^3 . So let $k = x^3$

$$\therefore x^6 + 26x^3 - 27 = 0$$

$$= k^2 + 26k - 27 = 0$$

$$(k+27)(k-1) = 0 \quad \therefore k = -27, 1$$

$$\therefore x^3 = -27 \quad x^3 = 1$$

$$\text{So } x = -3 \quad \text{or } x = 1$$

5. a) $2x^{2/3} \times 3x^{-1} = 6x^{(2/3-1)} = 6x^{-1/3}$

b) $2^{40} \times 4^{30} = 2^{40} \times (2^2)^{30} = 2^{40} \times 2^{60} = 2^{100}$

c) $\frac{26}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{26(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})} = \frac{26(4+\sqrt{3})}{16-3}$
 $= \frac{26(4+\sqrt{3})}{13} = 2(4+\sqrt{3}) = \underline{\underline{8+2\sqrt{3}}}$

6. i) $(x^2 + 2x + 1)(3x - 4)$

$$3x^3 + 6x^2 + 3x - 4x^2 - 8x - 4$$
$$= 3x^3 + 2x^2 - 5x - 4$$

ii) $f'(x) = 9x^2 + 4x - 5$

iii) $f''(x) = 18x + 4$

7. i) a) $b^2 - 4ac \quad 6^2 - 4 \times 1 \times 9 = 0$

b) $(-10)^2 - 4 \times 1 \times 12 = 100 - 48 = 52$

c) $(-2)^2 - 4 \times 1 \times 5 = 4 - 20 = -16$

- a) Fig 3. - 1 root at $x = -3$
 b) Fig 2 - 2 roots both +ve
 c) Fig 5 - no real roots. does not meet x-axis.

i) Circle with centre $(0,0)$ radius 5.

ii) $x^2 + y^2 = 25$ $2x + y - 5 = 0 \Rightarrow y = 5 - 2x$

$$x^2 + (5 - 2x)^2 = 25$$

$$x^2 + 25 + 4x^2 - 20x = 25$$

$$5x^2 - 20x = 0$$

$$5(x^2 - 4x) = 0 \quad 5x(x - 4) = 0$$

$$\therefore x = 0 \text{ or } x = 4$$

$$(0, 5) \quad (4, -3)$$

i) $4x - 3y + 5 = 0 \Rightarrow 4x + 5 = 3y \Rightarrow y = \frac{4}{3}x + \frac{5}{3}$

\therefore Gradient is $\frac{4}{3}$

ii) l_2 has gradient $-\frac{3}{4}$ and passes through $(1, 2)$

$$\text{So } \frac{y-2}{x-1} = -\frac{3}{4} \quad y-2 = -\frac{3}{4}(x-1) \Rightarrow 4y-8 = -3x+3$$

$$\Rightarrow 3x + 4y - 11 = 0$$

iii) at x-axis $y=0$ so for l_1 , $4x - 3y + 5 = 0$

$$4x + 5 = 0$$

$$x = -\frac{5}{4}$$

at y-axis $x=0$

$$\text{So for } l_2 \quad 3x + 4y - 11 = 0$$

$$4y = 11$$

$$y = \frac{11}{4}$$

$$\therefore P = \left(-\frac{5}{4}, 0\right) \quad Q = \left(0, \frac{11}{4}\right)$$

$$\text{So midpoint is } \left(\frac{0 + (-5/4)}{2}, \frac{0 + 11/4}{2}\right) = \left(-\frac{5}{8}, \frac{11}{8}\right)$$

$$\text{iv) } \sqrt{\left(0 - \frac{5}{4}\right)^2 + \left(\frac{11}{4} - 0\right)^2} = \sqrt{\frac{25}{16} + \frac{121}{16}} = \sqrt{\frac{146}{16}} = \frac{\sqrt{146}}{4}$$

10. i) $y = \frac{1}{3}x^3 - 9x$ $\frac{dy}{dx} = \frac{1}{3}x^2 - 9 = x^2 - 9$

ii) $x^2 - 9 = 0$ $x = 3$ or -3 $\therefore y = \frac{1}{3}3^3 - 9 \cdot 3 = -18$

$$\text{or } y = \frac{1}{3}(-3)^3 + 9 \cdot 3 = 18$$

$$\text{So } (3, -18) \quad (-3, 18)$$

iii) $\frac{d^2y}{dx^2} = 2x$ if $x = 3$ $\frac{d^2y}{dx^2} = 6 \therefore (3, -18)$ is a minimum.

if $x = -3$ $\frac{d^2y}{dx^2} = -6 \therefore (-3, 18)$ is a maximum.

iv) Gradient of $24x + 3y + 2 = 0$ is -8

$$\text{as } 3y = -24x - 2 \Rightarrow y = -8x - 2/3$$

\therefore Equate gradient of curve to -8

$$x^2 - 9 = -8 \quad x^2 = 1 \quad \therefore x = 1 \text{ or } -1$$

on curve if $x = 1$ $y = \frac{1}{3} - 9 = -8\frac{2}{3}$

if $x = -1$ $y = -\frac{1}{3} + 9 = 8\frac{2}{3}$

on line if $x = 1$ $y = -8\frac{2}{3}$

if $x = -1$ $y = 8 - \frac{2}{3} = 7\frac{1}{3}$

$$\text{So } p = 1 \quad q = -8\frac{2}{3}$$

These points are on line and curve