

Discrete 1. Jan 05.

1. ~~6~~ 5 9 4 5 2. ~~①~~
 5 6 9 4 5 2 ①.
 5 6 9 4 5 2 ②.
 5 6 4 9 5 2
 5 4 6 9 5 2
 4 5 6 9 5 2. ③.
 4 5 6 5 9 2
 4 5 5 6 9 2
 4 5 5 6 9 2 ④.
 4 5 5 6 2 9
 4 5 5 2 6 9
 4 5 2 5 6 9
4 2 5 5 6 9
 2 4 5 5 6 9 ⑤.

2 i no of arcs = $\frac{1}{2}$ (sum of orders)
 as each ~~node~~ arc has 2 nodes on ends

i.e. $3+3+4+4+4+4 = 22$ So 11 arcs.

ii Semi Eulerian as 2 odd nodes.

iii Simple graph \Rightarrow no ~~cycles~~ loops
 no double connectors.

Vertices of order 4 joins to 4 other nodes
 so S def connected

2 order 3 ~~all~~ must connect to each other
 as well as into even nodes so all connected.

3 i Shortest cycle - U is $10 + ~~13~~ + 20$
 $= ~~43~~ 43.$
 $+ U \text{ in and out} = 21$

Shortest cycle = 64

ii Start at Q $\rightarrow U \rightarrow R \rightarrow T \rightarrow S \times$
 $T \rightarrow R \rightarrow Q \rightarrow U \times$
 $U \rightarrow Q \rightarrow R \rightarrow T \rightarrow S \times$
 $S \rightarrow T \rightarrow R \rightarrow Q \rightarrow U \times$

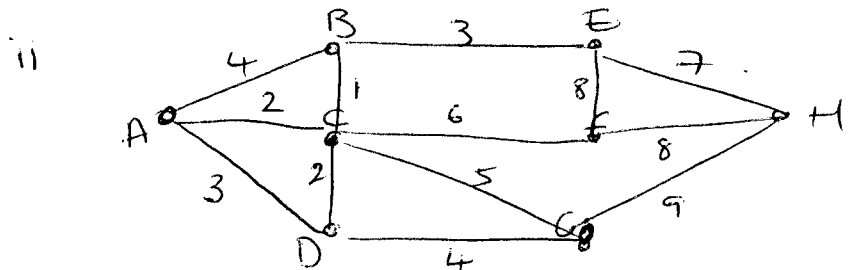
$R \rightarrow Q \rightarrow U \rightarrow T \rightarrow S \rightarrow R.$
 $= 10 + 9 + 18 + 20 + 32 = 89$

4.

	1	3	2	4	5	7	6	8
	↓	↓	↓	↓	↓	↓	↓	↓
	A	B	C	D	E	F	G	H
A	-	4	2	3	-	-	-	-
B	4	-	①	-	3	-	-	-
C	②	1	-	2	-	6	5	-
D	3	-	②	-	-	-	4	-
E	-	③	-	-	-	8	-	7
F	-	-	⑥	-	8	-	8	-
G	-	-	5	④	-	-	9	-
H	-	-	-	-	⑦	8	9	-

$$1 + 2 + 2 + 3 + 6 + 4 + 7 = 25.$$

Shortest time is 25 hours.



iii AC can't be used or either B or D can't be visited.

iv If E is first you must have come from B. So F is the next choice.

If F is first you must have come from C. So E is the next choice.

v ADGCB EFH $3+4+5+1+3+8+8 = 32$ ✗.
or
ABEFC DGH. $4+3+8+6+2+4+9 = 36$.

5. i. $y = 2x + 1$ is line $y - 2x \leq 1$ ✗
 $y = -4/3x + 4$ is line $y + 4/3x \leq 4$.
 $x \geq 0, y \geq 0$.

ii $y = 2x + 1$
 $y = -4/3x + 4$

$$2x + 1 = -4/3x + 4.$$

$$3\frac{1}{3}x = 3.$$

$$10x = 9.$$

$$x = 9/10$$

$$y = 18/10 + 1$$

$$y = 2.8$$

$$x = 0.9$$

$$P = 5x + 3y$$

$$P = 4.5 + 2.7 = 7.2$$

or $(0, 1) \Rightarrow 3$ $(3, 0) \Rightarrow 15$ $(0, 0) = 0$.
 Max P is 15 when $x = 3, y = 0$.

5. iii $3 = ax + 3y$

$3y = 3 - ax \quad y = 1 - \frac{1}{3}ax$

Grad of profit line is $-\frac{1}{3}a$.

Grad of constraints are $-\frac{4}{3}$ and 2

ie $-\frac{1}{3}a = 2 \Rightarrow a = -6$.

$-\frac{1}{3}a = -\frac{4}{3} \Rightarrow a = 4$.

6 i $P - 2x + 5y + z$
 $5x + 3y - 8z + s = 15$
 $2x + 6y + 8z + t = 24$

ii

	P	x	y	z	s	t	=	
1	-2	5	1	0	0	0	15	r_1
0	5	3	-5	1	0	15	15	r_2
0	2	8	8	0	1	24	24	r_3

Pivot on x $15 \div 5 = 3 \leftarrow r_2$
 $24 \div 2 = 12$

1	0	$5\frac{4}{5}$	-1	$\frac{2}{5}$	0	15 6	$r_4 = r_1 + 2r_5$
0	1	$\frac{3}{5}$	-1	$\frac{1}{5}$	0	3	$r_5 = r_2 \div 5$
0	0	$4\frac{4}{5}$	10	$-\frac{2}{5}$	1	18	$r_6 = r_3 - 2r_5$

iii Pivot on z $\times r_6$

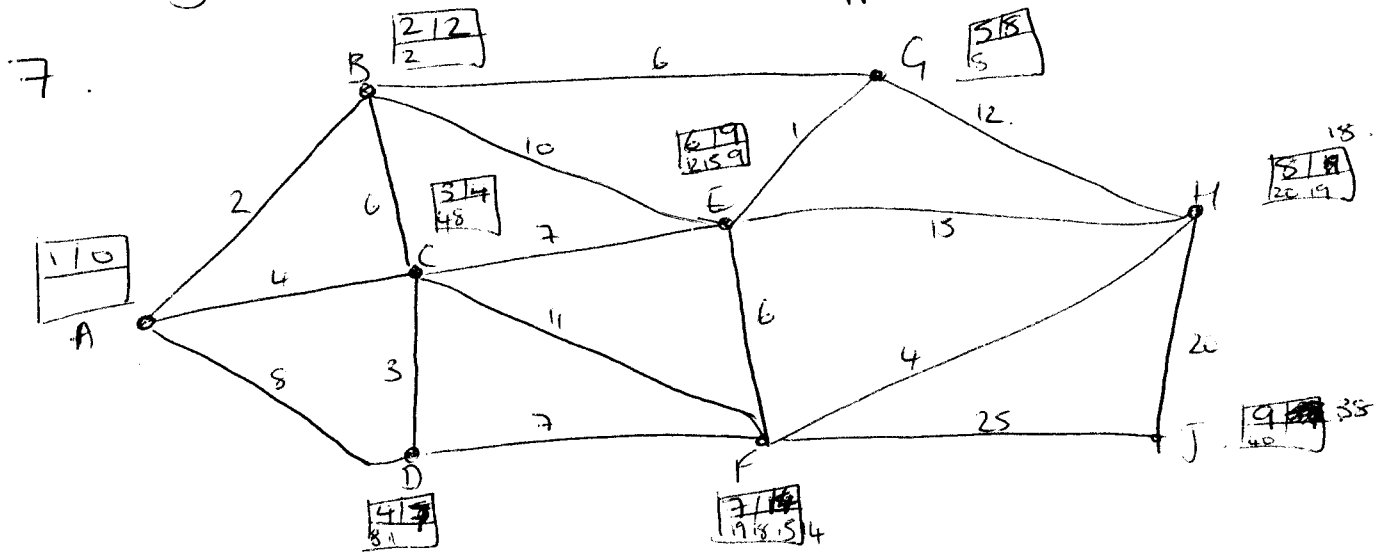
1	0	6.68	0	0.36	0.1	7.8	$r_7 = r_4 + r_9$
0	1	1.08	0	0.16	0.1	4.8	$r_8 = r_5 + r_9$
0	0	0.48	1	-0.04	0.1	1.8	$r_9 = r_6 \div 10$

$P = 7.8 \quad x = 4.8 \quad y = 0 \quad z = 1.8$

iv. New pivot row is r_3 .

	P	x	y	z	s	t	=	
1	-2	5	1	0	0	0	15	r_1
0	5	3	-5	1	0	N	15	r_2
0	2	8	8	0	1	24	24	r_3
1	0	11	9	0	1	12	12	$r_4 = r_1 + 2r_3$
0	0	12	-25	1	-2.5	N-60	15	$r_5 = r_2 - 5r_3$
0	1	3	4	0	0.5	12	12	$r_6 = r_3 \div 2$

Solution is now $P=12$ $x=12$ $y=0$ $z=0$.
 y is still 0 so no effect.



Shortest route to $E = 900$ m. $A \rightarrow B \rightarrow G \rightarrow E$
to $J = 3800$ m $A \rightarrow C \rightarrow D \rightarrow F \rightarrow H \rightarrow J$

ii $3800 + 900 = 4700$ m. $E \rightarrow G \rightarrow B \rightarrow A \rightarrow C \rightarrow D \rightarrow F \rightarrow H \rightarrow J$.

iii $G \rightarrow A \Rightarrow G \rightarrow B \rightarrow A$. } misses E & J .
 $A \rightarrow H \Rightarrow A \rightarrow C \rightarrow D \rightarrow F \rightarrow H$

b. Ignore A & E as starting at A and ending at E .

C D F G .

Poss pairs

$$CD \quad FG = 3 + 7 = 10$$

$$CF \quad DG = 10 + 11 = 21$$

$$CA \quad FD = 8 + 7 = 15$$

Shortest route is CD & FG .

$$2 + 6 + 4 + 5 + 3 + 7 + 11 + 7 + 10 + 6 + 1 + 15 + 6 + 4 + 25 + 20 + 12$$

$$= \underline{147} + 10 = 15700 \text{ m}$$