4721 Core Mathematics 1

1	$\frac{4(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$ $=\frac{12+4\sqrt{7}}{9-7}$ $=6+2\sqrt{7}$	M1 B1	3 3	Multiply top and bottom by conjugate 9 ± 7 soi in denominator $6 + 2\sqrt{7}$
2(i) (ii)	$x^2 + y^2 = 49$			$x^2 + y^2 = 49$
	$x^{2} + y^{2} - 6x - 10y - 30 = 0$ $(x - 3)^{2} - 9 + (y - 5)^{2} - 25 - 30 = 0$ $(x - 3)^{2} + (y - 5)^{2} = 64$ $r^{2} = 64$	M1		3 ² 5 ² 30 with consistent signs soi
	r = 8	A1	2 3	8 cao
3	$a(x+3)^{2} + c = 3x^{2} + bx + 10$ $3(x^{2} + 6x + 9) + c = 3x^{2} + bx + 10$ $3x^{2} + 18x + 27 + c = 3x^{2} + bx + 10$ $c = -17$			$a = 3 \text{ soi}$ $b = 18 \text{ soi}$ $c = 10 - 9a \text{ or } c = 10 - \frac{b^2}{12}$ $c = -17$
4(i) (ii)	$p = -1$ $\sqrt{25k^2} = 15$ $25k^2 = 225$ $k^2 = 9$	B1 M1		$p = -1$ Attempt to square 15 or attempt to square root $25k^2$ $k = 3$
(iii)	$k = \pm 3$ $\sqrt[3]{t} = 2$ $t = 8$	M1 A1	3 2 6	$k = -3$ $\frac{1}{\frac{1}{t^3}} = \frac{1}{2} \text{ or } t^{\frac{1}{3}} = 2 \text{ soi}$ $t = 8$

		1	
5(i)	ر لا	B1	+ve cubic
	× ×	B1 2	+ve or -ve cubic with point of inflection at (0, 2) and no max/min points
(ii)	^y	B1	curve with correct curvature in +ve quadrant only
	×	B1 2	completely correct curve
(iii)	Stretch	B1	stretch
	scale factor 1.5 parallel to y-axis	B1 B1 3	factor 1.5 parallel to y-axis or in y-direction
6(i)	EITHER) / (I	
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	M1	Correct method to solve quadratic
	$x = \frac{-8 \pm \sqrt{64 - 40}}{2}$		
		A1	$x = \frac{-8 \pm \sqrt{24}}{2}$
	$x = \frac{-8 \pm \sqrt{24}}{2}$		2
	$x = \frac{-8 \pm 2\sqrt{6}}{2}$ $x = -4 \pm \sqrt{6}$		
	$x = -4 \pm \sqrt{6}$	A1 3	$x = -4 \pm \sqrt{6}$
	OR $(n+4)^2 = 16+10=0$		
	$(x+4)^2 - 16 + 10 = 0$ $(x+4)^2 = 6$		
	$x + 4 = \pm \sqrt{6}$ M1 A1		
	$x = \pm \sqrt{6} - 4 $ A1		
(ii)	\ 7	B1	+ve parabola
	10	B1	parabola cutting y-axis at (0, 10) where (0, 10) is not min/max point
		B1 3	parabola with 2 negative roots
		M1	$x \le \text{lower root} x \ge \text{higher root} (\text{allow} <,>)$
(iii)	$x \le -\sqrt{6} - 4, x \ge \sqrt{6} - 4$	A1 ft 2	Fully correct answer, ft from roots found in (i)
		8	

7(i)	Gradient = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$
(ii)	$y - 5 = -\frac{1}{2}(x - 6)$	M1 B1 ft	Equation of straight line through (6, 5) with any non-zero numerical gradient Uses gradient found in (i) in their equation of line
	2y - 10 = -x + 6 $x + 2y - 16 = 0$	A1 3	Correct answer in correct form (integer coefficients)
(iii)	EITHER $\frac{4-x}{2} = x^2 + x + 1$	*M1	Substitute to find an equation in x (or y)
	$4-x = 2x^{2} + 2x + 2$ $2x^{2} + 3x - 2 = 0$ $(2x-1)(x+2) = 0$	DM1	Correct method to solve quadratic
	$x = \frac{1}{2}, x = -2$	A1	$x = \frac{1}{2}, x = -2$ $y = \frac{7}{4}, y = 3$
	$y = \frac{7}{4}, y = 3$	A1 4	$y - \frac{1}{4}, y - 3$ SR one correct (x,y) pair www B1
	OR $y = (4-2y)^2 + (4-2y) + 1$ * N	4	
	$y = (4-2y)^{2} + (4-2y) + 1$ $y = 16-16y + 4y^{2} + 4-2y + 1$ $0 = 21-19y + 4y^{2}$	1	
	0 = (4y - 7)(y - 3) DM	[1	
	$y = \frac{7}{4}, y = 3$ A1		
	$x = \frac{1}{2}, x = -2$ A1	8	

8(i)	$\frac{dy}{dx} = 3x^2 + 2x - 1$	*M1 A1	Attempt to differentiate (at least one correct term) 3 correct terms
	At stationary points, $3x^2+2x-1=0$	M1	Use of $\frac{dy}{dx} = 0$
	(3x - 1)(x + 1) = 0	DM1	Correct method to solve 3 term quadratic
	$x = \frac{1}{3}, x = -1$	A1	$x = \frac{1}{3}, \ x = -1$
	$y = \frac{76}{27}, \ y = 4$	A1 6	$y = \frac{76}{27}$, 4
			SR one correct (x,y) pair www B1
(ii)	$\frac{d^2y}{dx^2} = 6x + 2$	M1	Looks at sign of $\frac{d^2y}{dx^2}$ for at least one of their x-values or other correct method
	$x = \frac{1}{3}, \frac{d^2y}{dx^2} > 0$	A1	$x = \frac{1}{3}$, minimum point CWO
	$x = -1, \ \frac{d^2y}{dx^2} < 0$	A1 3	x = -1, maximum point CWO
(iii)	$-1 < x < \frac{1}{3}$	M1 A1 2	Any inequality (or inequalities) involving both their x values from part (i) Correct inequality (allow $<$ or \le)
		11	

9(i)	Gradient of AB = $\frac{-2-1}{-5-3}$	B1
	2	

$$\frac{3}{8}$$
 oe

$$y-1=\frac{3}{8}(x-3)$$

Equation of line through either A or B, any nonzero numerical gradient

$$8y - 8 = 3x - 9$$
$$3x - 8y - 1 = 0$$

Correct equation in correct form

(ii)
$$\left(\frac{-5+3}{2}, \frac{-2+1}{2}\right) = (-1, -\frac{1}{2})$$

M1 Uses
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

A1 2
$$\left| (-1, -\frac{1}{2}) \right|$$

(iii)
$$AC = \sqrt{(-5+3)^2 + (-2-4)^2} = \sqrt{2^2 + 6^2}$$

M1 Uses
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{vmatrix} =\sqrt{2^2+6} \\ =\sqrt{40} \\ =2\sqrt{10} \end{vmatrix}$$

A1
$$\sqrt{40}$$

A1 3 Correctly simplified surd

(iv) Gradient of AC =
$$\frac{-2-4}{-5+3} = 3$$

Gradient of BC = $\frac{4-1}{3-2} = -\frac{1}{3}$

Gradient of BC =
$$\frac{4-1}{-3-3} = -\frac{1}{2}$$

B1
$$-\frac{1}{2}$$
 oe

$$3 \times -\frac{1}{2} \neq -1$$
 so lines are not perpendicular

M1 Attempts to check
$$m_1 \times m_2$$

Correct conclusion www

10(i)	$24x^2 - 3x^{-4}$	B1 B1 B1	$ \begin{array}{c} 24x^{2} \\ kx^{-4} \\ -3x^{-4} \end{array} $
	$48x + 12x^{-5}$	M1 A1 5	Attempt to differentiate their (i) Fully correct
(ii)	$8x^3 + \frac{1}{x^3} = -9$		
	$8x^{6} + 1 = -9x^{3}$ $8x^{6} + 9x^{3} + 1 = 0$	*M1	Use a substitution to obtain a 3-term quadratic
	Let $y = x^3$ $8y^2 + 9y + 1 = 0$ (8y+1)(y+1) = 0	DM1 A1	Correct method to solve quadratic $-\frac{1}{8}$, -1
	$y = -\frac{1}{8}, y = -1$	M1	Attempt to cube root at least one of their <i>y</i> -values
	$x = -\frac{1}{2}, x = -1$	A1 5	
			SR one correct x value www B1
			SR for trial and improvement: x = -1 B1
		10	$x = -\frac{1}{2}$ B2 Justification that there are no further solutions B2