Rollin' rollin' rollin'

Stepping into a problem

Prerequisite knowledge

- Pupils will have considered the locus of the centre of a circle rolling around a square and a triangle (see the problem 'Roundabout')

Why do this problem?

The challenge in this problem is developing a convincing argument as the answer may feel counter-intuitive. Plenty of time spent considering the case of a coin rolling around polygons with increasing numbers of sides may support pupils with their understanding. The opportunity to discuss multiple solutions is also valuable. Some pupils may take the case of an 'infinitely' sided polygon as a way into the problem. Others may wish to consider the length of the locus of the centre of the outer coin as a way of 'seeing' the number of turns.

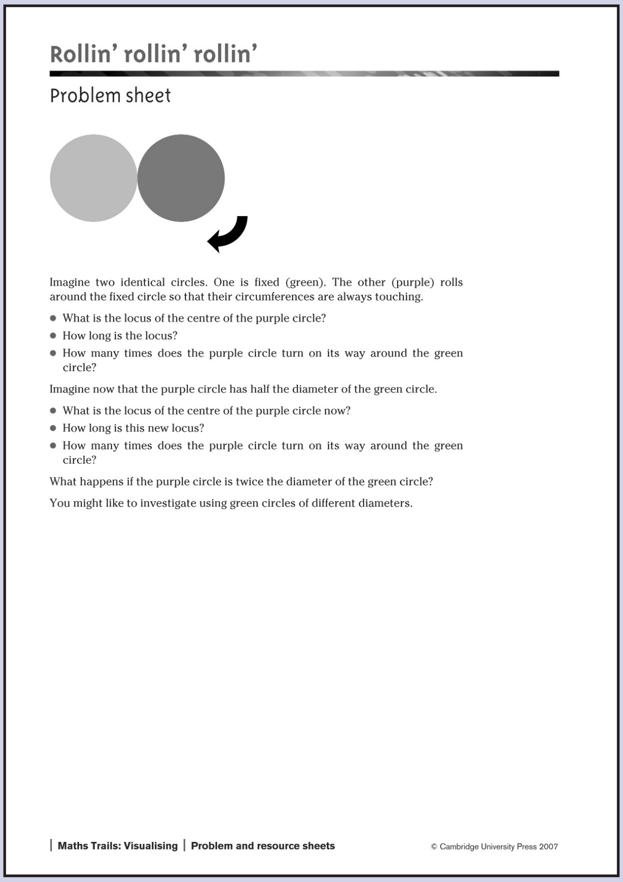
The problem also gives opportunities to pay particular attention to the evaluation phase of the problem-solving model.

Time

Half or one lesson

Resources

Optional: coins or counters of different sizes
CD-ROM: problem sheet; resource sheet; interactivity



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Problem sheet

Imagine two identical circles. One is fixed (green). The other (purple) rolls around the fixed circle so that their circumferences are always touching.

- What is the locus of the centre of the purple circle?
- How long is the locus?
- How many times does the purple circle turn on its way around the green circle?

Imagine now that the purple circle has half the diameter of the green circle.

- What is the locus of the centre of the purple circle now?
- How long is this new locus?
- How many times does the purple circle turn on its way around the green circle?

What happens if the purple circle is twice the diameter of the green circle?

You might like to investigate using green circles of different diameters.

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NRICH website (optional): www.nrich.maths.org, February 2004, 'Rollin' rollin' rollin''

Introducing the problem

Review the ideas developed while working on 'Roundabout'. Ask pupils to imagine a circle rolling around the circumference of an equilateral (regular) triangle.

- What is the locus of its centre? [a triangle with 'curved' corners]
- What sort of 'curves' are at the corners and how do you know? [arcs of a circle – definition of a circle]
- What is the relationship between the arcs of the circles found at each vertex? [together they form a full turn]
- What is the length of the locus? [perimeter of the triangle plus circumference of the circle]

Ask pupils to think about the number of times the circle turns as it moves around the triangle – once, more than once – and justify their ideas. [the circle completes a full turn just by going around each of the vertices so the answer must be 'more than once'; the number of revolutions then depends on the lengths of the sides of the triangle]

You might wish to extend this introduction to the discussion of squares and other polygons, emphasising the length of the arc and the amount the circle turns rather than the shape of the locus as was the focus in the lesson 'Roundabout'.

Main part of the lesson

The aim of the session is to investigate what happens to the locus as one circle rolls around another circle.

Start with two identical coins or circles. Ask pupils to describe:

- the locus of the centre of the rolling circle;
- how long it is;
- how many times the circle turns on its way around.

As an extension pupils can investigate the result for outer circles (or coins) of different sizes.

The instinctive answer is that, given two circles of equal sizes, the outer circle turns once on its way around the inner circle. However, reference to the interactivity (or a discussion of the introductory part of the lesson and what happens with polygons with more and more sides) should begin to convince pupils that the answer is 'more than once'.

Hand out coins or counters and problem sheets.

After a few minutes, stop the group to discuss findings. Many will say that the circle turns once. Demonstrating the interactivity or asking pupils to use the coins to see what happens to the image (that it is up the right way and facing the same direction as at the starting point when it is halfway around) suggests the coin turns twice. The pupils' task is to find an argument why:

- first by convincing themselves;
- then by convincing their partners;
- and, when you stop them again in a few minutes, by convincing the rest of the class.

Tell the class that you will ask anyone in a group to describe their argument to the rest of the class so it is important that everyone in the group feels confident to speak and not just the person with the original idea. In fact you will try not to ask this person as their job is to be able to describe their reasoning so well that everyone else in the group feels confident.

Leave pupils to work and when several can offer convincing arguments, stop the class to discuss findings.

If some are still not convinced then, the task is to go back and refine arguments.

Some pupils may be convinced but need to spend more time 'feeling comfortable' with the result and convincing themselves. Others may be ready to move on to the extension activities.

At this point you may wish to move pupils into slightly different groups.

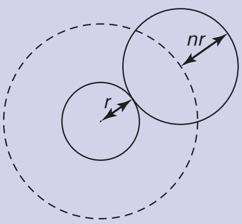
Plenary

One aim of the plenary is to share findings for circles of different sizes (if appropriate) but the key aim is to discuss what constituted convincing arguments within the pupils' groups. A list of features of convincing arguments may be useful, for example:

- small steps;
- diagrams;
- not making assumptions;
- describing things in more than one way.

Different people see things differently, so more than one way of seeing something can be useful.

Solution notes



The total distance the centre of the outer circle travels is equal to the circumference of a circle of radius $nr + r = (n + 1)r$.

That is $2(n + 1)r\pi$, so the outer circle will rotate:

$$\frac{\text{total distance}}{\text{circumference of outer circle}}$$

$$= \frac{2(n+1)\pi r}{2\pi rn} \text{ times}$$

$$= \frac{(n+1)}{n} \text{ times}$$

If the circles are the same size, $n = 1$ and the number of revolutions $= 2$.

If the outer circle is twice the size of the inner circle, $n = 2$ and the number of revolutions $= \frac{3}{2}$ $= 1.5$ times, etc.