100-square

Stepping into a problem

Prerequisite knowledge

- Understanding of place value
- Mental addition and subtraction strategies

Why do this problem?

This problem builds on an understanding of place value, starting from a familiar setting and leading on to generalisations.

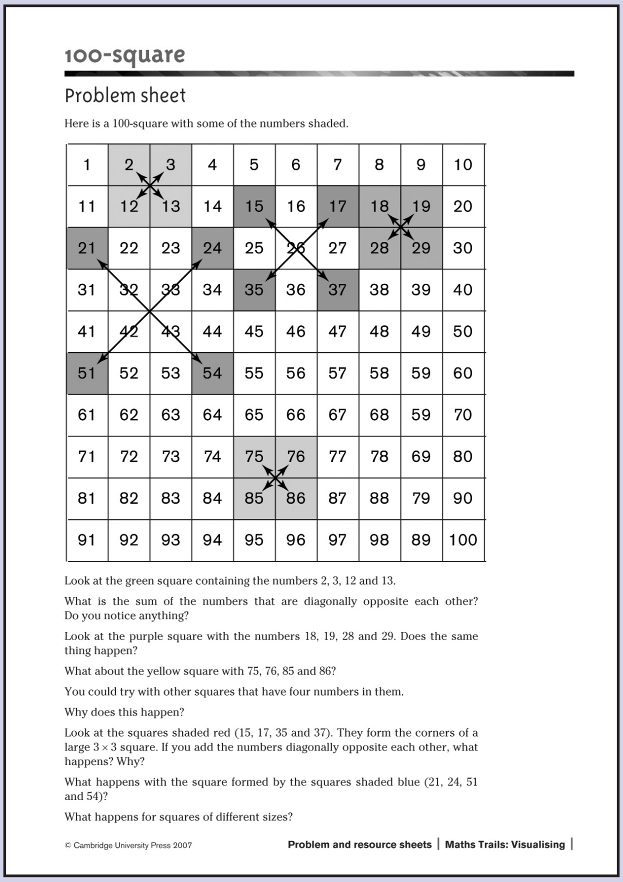
The problem also gives opportunities to focus on the comprehension phase of the problem-solving model.

Time

One lesson

Resources

Squared paper
CD-ROM: problem sheet; resource sheet
NRICH website (optional): www.nrich.maths.org, June 2005, 'Diagonal sums'; a challenging extension or follow-up activity is 'Multiply the addition square' (August 2005)



100-square

Problem sheet

Here is a 100-square with some of the numbers shaded.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	59	70
71	72	73	74	75	76	77	78	69	80
81	82	83	84	85	86	87	88	79	90
91	92	93	94	95	96	97	98	89	100

Look at the green square containing the numbers 2, 3, 12 and 13.

What is the sum of the numbers that are diagonally opposite each other? Do you notice anything?

Look at the purple square with the numbers 18, 19, 28 and 29. Does the same thing happen?

What about the yellow square with 75, 76, 85 and 86?

You could try with other squares that have four numbers in them.

Why does this happen?

Look at the squares shaded red (15, 17, 35 and 37). They form the corners of a large 3×3 square. If you add the numbers diagonally opposite each other, what happens? Why?

What happens with the square formed by the squares shaded blue (21, 24, 51 and 54)?

What happens for squares of different sizes?

Introducing the problem

Ask pupils to imagine a square array of numbers starting with a 1 in the top left-hand corner and then 2, 3, 4, ..., 10 going along the top row. Then say:

- Imagine the row beneath, starting at 11 and continuing. What would be under the 10? [20] How about under the ... ?
- Now imagine a third row starting at 21 and so on until you get to 100. How many rows are there? [10]
- Find 56 in your mind's eye. Go up two rows and then right two squares. What number do you arrive at? [38]

Repeat this process with several other numbers and different sets of two instructions (going up and down rows, and right and left).

Main part of the lesson

Ask pupils to imagine the same square array of numbers.

- Can you see the number 2? What number is immediately to the right of the 2? [3] What number is below the 2? [12] What number is below the 3? [13]
- What shape do these four numbers make? [a square]
- Which number is diagonally opposite the 2 in this shape? [13] What is the sum of these two numbers? [15]
- And what is diagonally opposite the 3? [12] What is the sum of these two numbers? [15]
- What do you notice? Do you think this is always the case? How might we find out?

At this point, you could put up the image of the 100-square on the problem sheet to talk about other examples of 2×2 squares.

Ask pupils to investigate 2×2 squares and to come up with a convincing argument to share with the class explaining why the diagonal sums of a square are always equal.

After about 10–15 minutes, bring the class together to talk about their findings. Pupils will have some idea of why the two totals are always the same, but they may need help in making links to the introductory activity and articulating their arguments. An algebraic explanation is not expected. If appropriate, ask pupils to spend a couple of minutes trying to explain their ideas to a partner (just for consolidation).

Ask pupils to investigate sums of numbers in diagonally opposite corners of different-sized squares. Note that this also works with tilted squares.

Plenary

Invite groups of pupils (or individuals) to come to the front and explain their findings using a blank 100-square (see resource sheet).

Solution notes

Since diagonal numbers form a square, the difference between the numbers in the upper corners is the same as the difference between the numbers in the lower corners, and likewise the difference between the upper and lower corners on either side is the same. Therefore, when we add the top left and bottom right, and the top right and bottom left, we get the same total.

For example, in a 2×2 square, the number in the top right is 1 more than the number in the top left. The number in the bottom left is 1 less than the number in the bottom right.

In general:

N	$N+1$
$N+10$	$N+11$

If we add the diagonals we get $2N + 11$ each time.

For a 3×3 square the result is the same (and the sum is twice the number in the middle):

N		$N+2$
	$N+11$	
$N+20$		$N+22$

The sum is $2N + 22$, which is $2(N + 11)$.

And so on ...