

4722 Core Mathematics 2

1 $(2 - 3x)^6 = 2^6 + 6 \cdot 2^5 \cdot (-3x) + 15 \cdot 2^4 \cdot (-3x)^2$
 $\quad \quad \quad = 64 - 576x + 2160x^2$

OR

M1 Attempt (at least) first two terms - product of binomial coefficient and powers of 2 and (-

A1 Obtain $64 - 576x$

M1 Attempt third term - binomial coefficient and powers of 2 and $(-3x)$

A1 Obtain $2160x^2$

M1 Attempt expansion involving all 6 brackets

A1 Obtain 64

A1 Obtain $-576x$

A1 Obtain $2160x^2$

SR if the expansion is attempted in descending order, and the required terms are never seen, then **B1 B1 B1** for $4860x^4, -2916x^5, 729x^6$

4

2 (i) $u_2 = \frac{2}{3}$
 $u_3 = \frac{-1}{2}$
 $u_4 = 3$

B1 Obtain correct u_2

B1✓ Obtain correct u_3 from their u_2

B1✓ Obtain correct u_4 from their u_3

3

(ii) sequence is periodic / cyclic / repeating

B1 Any equivalent comment

1

3 (i) $\frac{1}{2} \times 8^2 \times \theta = 48$
Hence $\theta = 1.5$ radians

M1 State or imply $(\frac{1}{2}) 8^2 \theta = 48$

A1 Obtain $\theta = 1.5$ (or 0.477π), or equiv

2

(ii) area = $48 - \frac{1}{2} \times 8^2 \times \sin 1.5$
 $= 48 - 31.9$
 $= 16.1$

M1* Attempt area of Δ using $(\frac{1}{2}) 8^2 \sin \theta$

M1d* Attempt $48 - \text{area of } \Delta$

A1 Obtain 16.1 cm^2

3

4 (i) $f(3) = 27a - 36 - 21a + 12 = 0$
 $6a = 24$
 $a = 4$

M1* Attempt $f(3)$

M1d* Equate attempt at $f(3)$ to 0 and attempt to solve

A1 Obtain $a = 4$

OR

M1* Attempt complete division / matching coeffs

M1d* Equate remainder to 0

A1 Obtain $a = 4$

3

(ii) $f(-2) = -32 - 16 + 56 + 12$
 $= 20$

M1 Attempt $f(-2)$

A1✓ Obtain 20 (or $6a - 4$, following their a)

2

<p>5 (i) $\int xdy = \int ((y-3)^2 - 2)dy$ $= \int (y^2 - 6y + 7)dy$ A.G. $3 + \sqrt{(2+2)} = 5, \quad 3 + \sqrt{(14+2)} = 7$</p>	<p>B1 Show $x = y^2 - 6y + 7$ convincingly B1 State or imply that required area $= \int xdy$ B1 Use $x = 2, 14$ to show new limits of $y = 5, 7$</p>
<p>(ii) $\left[\frac{1}{3}y^3 - 3y^2 + 7y\right]_5^7$ term $= ({}^{343}/_3 - 147 + 49) - ({}^{125}/_3 - 75 + 35)$ $= 16\frac{1}{3} - 1\frac{2}{3}$ $= 14\frac{2}{3}$</p>	<p>3 M1 Integration attempt, with at least one correct A1 All three terms correct M1 Attempt $F(7) - F(5)$ A1 Obtain $14\frac{2}{3}$, or exact equiv 4</p>
<p>6 (i) $ABC = 360 - (150 + 110) = 100^\circ$ A.G.</p>	<p>B1 Show convincingly that angle ABC is 100° 1</p>
<p>(ii) $CA^2 = 15^2 + 27^2 - 2 \times 15 \times 27 \times \cos 100^\circ$ $= 1094.655\dots$ $CA = 33.1$</p>	<p>M1 Attempt use of correct cosine rule A1 Obtain 33.1 km 2</p>
<p>(iii) $\frac{\sin C}{15} = \frac{\sin 100}{33.1}$ or $\frac{\sin A}{27} = \frac{\sin 100}{33.1}$ $C = 26.5^\circ$ $A = 53.5^\circ$ Hence bearing is 263°</p>	<p>M1 Attempt use of sine rule to find angle C or A (or equiv using cosine rule) A1✓ Correct unsimplified eqn, following their CA A1 Obtain $C = 26.5^\circ$ or $A = 53.5^\circ$ (allow 53.4°) A1✓ Obtain 263 or 264 (or $290^\circ -$ their angle C / $210 +$ their angle A) 4</p>
<p>7 (a) $\int (x^5 - x^4 + 5x^3)dx$ $= \frac{1}{6}x^6 - \frac{1}{5}x^5 + \frac{5}{4}x^4 (+c)$</p>	<p>M1 Expand brackets and attempt integration, or other valid integration attempt A1 Obtain at least one correct term A1 Obtain a fully correct expression B1 For $+c$, and no \int or dx (can be given in (b)(i) if not given here) 4</p>
<p>(b) (i) $-6x^{-3} (+c)$</p>	<p>M1 Obtain integral of the form kx^{-3} A1 Obtain $-6x^{-3} (+c)$ 2</p>
<p>(ii) $\left[-6x^{-3}\right]_2^\infty$ $= \frac{3}{4}$</p>	<p>B1* State or imply that $F(\infty) = 0$ (for $kx^n, n < -1$) B1d* Obtain $\frac{3}{4}$ (or equiv) 2</p>

8 (i)		M1 Attempt sketch of exponential graph (1 st quad) - if seen in 2 nd quad must be approx correct A1 Correct graph in both quadrants B1 State or imply (0, 2) only <div>3</div>
(ii)	$8^x = 2 \times 3^x$ $\log_2 8^x = \log_2 (2 \times 3^x)$ $x \log_2 8 = \log_2 2 + x \log_2 3$ $3x = 1 + x \log_2 3$ $x(3 - \log_2 3) = 1$, hence $x = \frac{1}{3 - \log_2 3}$ A.G.	M1 Form equation in x and take logs (to any consistent base, or no base) – could use \log_8 M1 Use $\log a^b = b \log a$ M1 Use $\log ab = \log a + \log b$, or equiv with $\log^{a/b}$ M1 Use $\log_2 8 = 3$ A1 Show given answer correctly
OR	$8^x = 2 \times 3^x$ $2^{3x} = 2 \times 3^x$ $2^{(3x-1)} = 3^x$ $\log_2 2^{(3x-1)} = \log_2 3^x$ $(3x-1) \log_2 2 = x \log_2 3$ $x(3 - \log_2 3) = 1$, hence $x = \frac{1}{3 - \log_2 3}$ A.G.	M1 Use $8^x = 2^{3x}$ M1 Attempt to rearrange equation to $2^k = 3^x$ M1 Take logs (to any base) M1 Use $\log a^b = b \log a$ A1 Show given answer correctly <div>5</div>
9 (a) (i)	$2 \sin x \frac{\sin x}{\cos x} - 5 = \cos x$ $2 \sin^2 x - 5 \cos x = \cos^2 x$ $2 - 2 \cos^2 x - 5 \cos x = \cos^2 x$ $3 \cos^2 x + 5 \cos x - 2 = 0$	M1 Use $\tan x \equiv \frac{\sin x}{\cos x}$ M1 Use $\sin^2 x \equiv 1 - \cos^2 x$ A1 Show given equation convincingly <div>3</div>
(ii)	$(3 \cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{3}$ $x = 1.23 \text{ rad}$ $x = 5.05 \text{ rad}$	M1 Attempt to solve quadratic in $\cos x$ M1 Attempt to find x from root(s) of quadratic A1 Obtain 1.23 rad or 70.5° A1✓ Obtain 5.05 rad or 289° (or $2\pi / 360^\circ$ - their solution) SR: B1 B1 for answer(s) only <div>4</div>
(b)	$0.5 \times 0.25 \times \{ \cos 0 + 2(\cos 0.25 + \cos 0.5 + \cos 0.75) + \cos 1 \}$ ≈ 0.837	M1 Attempt y -coords for at least 4 of the correct 5 x -coords M1 Use correct trapezium rule, any h , for their y values to find area between $x = 0$ and $x = 1$ M1 Correct h (soi) for their y values A1 Obtain 0.837 <div>4</div>

10 (i) $u_{15} = 2 + 14 \times 0.5$ $= 9 \text{ km}$	M1 Attempt use of $a + (n - 1)d$ A1 Obtain 9 km <div style="border: 1px solid black; padding: 2px; display: inline-block;">2</div>
(ii) $u_{20} = 2 \times 1.1^{19} = 12.2$ $u_{19} = 2 \times 1.1^{18} = 11.1$	B1 State, or imply, $r = 1.1$ M1 Attempt u_{20} , using ar^{n-1} A1 Obtain $u_{20} = 12.2$, and obtain $u_{19} = 11.1$
<p>OR</p>	B1 State, or imply, $r = 1.1$ M1 Attempt to solve $ar^{n-1} = 12$ A1 Obtain $n = 20$ (allow $n \geq 20$) <div style="border: 1px solid black; padding: 2px; display: inline-block;">3</div>
(iii) $\frac{2(1.1^n - 1)}{(1.1 - 1)} > 200$ $1.1^n > 11$ $n > \frac{\log 11}{\log 1.1}$ $n > 25.2$ ie Day 26	B1 State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1 - 1)}$ M1 Link (any sign) their attempt at S_N (of a GP) to 200 and attempt to solve A1 Obtain 26, or 25.2 or better A1 Conclude $n = 26$ only, or equiv eg Day 26 <div style="border: 1px solid black; padding: 2px; display: inline-block;">4</div>
(iv) $\text{swum} = 2 \times 30 = 60 \text{ km}$ $\text{run} = \frac{1}{2} \times 30 \times (4 + 29 \times 0.5)$ $= 277.5 \text{ km}$ $\text{cycle} = \frac{2(1.1^{30} - 1)}{(1.1 - 1)}$ $= 329.0 \text{ km}$ $\text{total} = 666 \text{ km}$	B1 Obtain 60 km, or $2 \times 30 \text{ km}$ M1 Attempt sum of AP, $d = 0.5$, $a = 2$, $n = 30$ M1 Attempt sum of GP, $r = 1.1$, $a = 2$, $n = 30$ A1 Obtain 666 or 667 km <div style="border: 1px solid black; padding: 2px; display: inline-block;">4</div>