Geoboard

Multipurpose

Prerequisite knowledge
- Properties of triangles
- Basic symmetry

Why do this problem?
This problem requires a systematic approach and is a good context for generalisation and symbolic representation of findings. In the longer term, it is a good way to get into circle theorems.

The problem also gives opportunities to pay particular attention to the analysis and synthesis phase of the problem-solving model.

Time
One lesson (but there is a lot of material on the NRICH website which can develop and extend these early ideas)
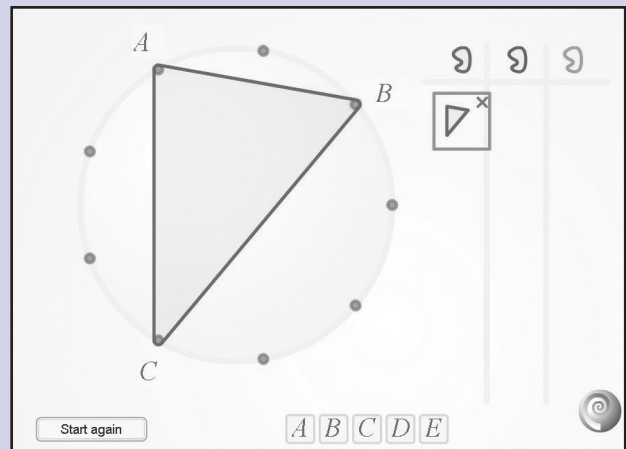
Resources
CD-ROM: resource sheets 1–3; solution notes; interactivity
NRICH website (optional):
www.nrich.maths.org; this problem is based on 'Nine-pin triangles' (July 2005); the problem 'Triangle pin-down' (July 2005) also has links to sheets with different 'point circles' in the hints

Introducing the problem

Invite pupils to imagine a circle with 7 equally spaced dots around the circumference.

- How many different triangles do you think it might be possible to draw by joining three of these dots?
- How might we go about finding out?

Pupils might suggest that they will draw them all, making sure all are found and none are counted twice.

Show them a 7-point circle (resource sheet 1 or use the interactivity). Ask them to imagine a triangle on this circle and to think about how they would describe their triangle to someone else.

Invite a few pupils to do exactly that.

One difficulty they will find is describing the orientation of the triangles. One way to deal with this is to number the points, which also helps with counting the number of 'steps' around the circumference between vertices.

For example, the triangle drawn below can be described as having vertices 2, 3, 5, or in terms of its starting points and step sizes (clockwise).
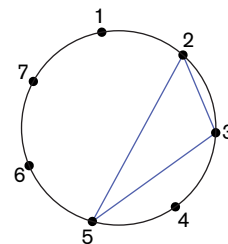
- How many other triangles could you make that look like this but have different vertices?

Ask a pupil who had imagined the same triangle but in a different position to describe it.

- Would you describe these triangles as being different or the same? [we will count them as identical]

Main part of the lesson

Refer back to the question of how many different triangles can be drawn on a 7-point circle. Say that the class has already talked about how to describe the triangles, but how can they be organised in the way they go about finding them all, so as to be sure they haven't missed any out or repeated any?

In the ensuing discussion, the aim is to use pupils' ideas to come up with systematic methods, such as the following:

- Find all the triangles with side 1 step first, then all with side 2 steps, and so on. (In this case, there are three variables corresponding to the vertices of the triangles; two of these need to be kept constant while you vary the third.)
- Some may identify that it is sufficient to use a numerical representation for steps and find all combinations, for example: 1, 1, 5; 1, 2, 4; 1, 3, 3; ...

Ask pupils to work in small groups to find the number of different types of triangles in a 7-point circle [there are 4].

After 5 minutes, stop and review findings. Deal with issues of systematisation and symmetry, using the interactivity to aid visualisation.

Raise the conjecture that you might be able to predict the number of different triangles in different point circles.

- How could we find out?

Set the class on the task. The process could be speeded up by asking different groups to investigate particular point circles (see resource sheets 1–3).

Plenary

Invite pupils to volunteer the number of triangles that can be drawn on a 7-point circle. There is likely to be some discrepancy, which is a good opportunity for groups to share their approaches and to consider whether their method really was systematic. Challenge groups to convince others that there are no other possibilities, using their approach as a justification.

If the class has been able to investigate other point circles, talk about similarities and differences between odd-numbered and even-numbered point circles.

Solution notes

See the CD-ROM.