Two and two

Mixed methods

Prerequisite knowledge

- Good understanding of place value

Why do this problem?

The multiple solutions can only be found if a systematic and well-reasoned argument is developed. Being convinced that you have all the solutions is at the core of this problem.

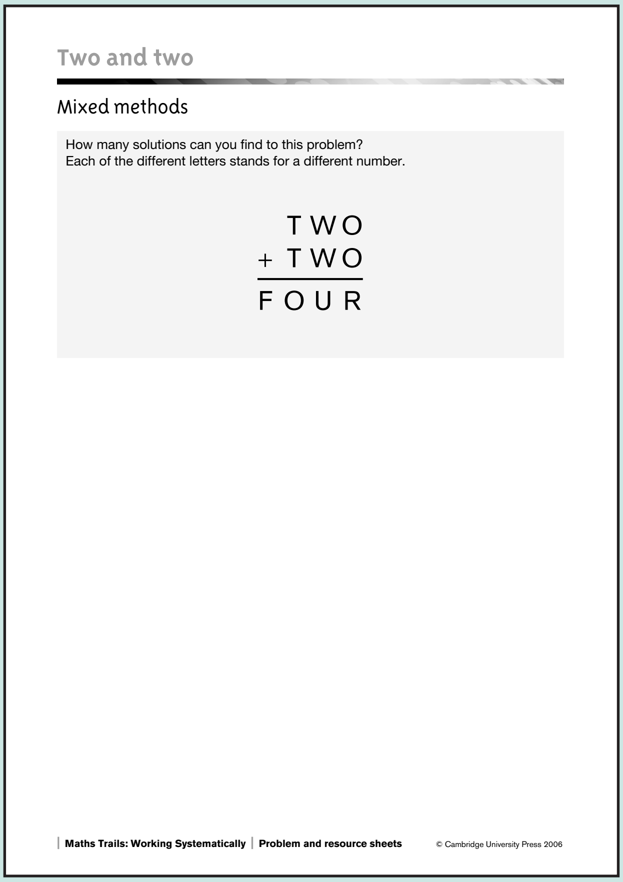
Time

One lesson

Resources

CD-ROM: problem sheet

NRICH website (optional):
www.nrich.maths.org June 2001 ‘Two and two’


Two and two

Mixed methods

How many solutions can you find to this problem?
Each of the different letters stands for a different number.

$$\begin{array}{rcccc} & & T & W & O \\ + & & T & W & O \\ \hline & F & O & U & R \end{array}$$

Maths Trails: Working Systematically | Problem and resource sheets

Introducing the problem

You might like to consider using ‘Reach 100’ as a precursor to this problem. Alternatively, if the class has needed a lot of support with this problem, ‘Reach 100’ would be a good way of consolidating what they have learnt.

To start with, present a simpler problem for discussion:

$$\begin{array}{cccc} & & b & b \\ + & & c & b \\ \hline & a & c & c \\ \hline \end{array}$$

What can the letters be? A class discussion and ideas might lead to:

- a has to be 1 because it is the largest carry you can get by adding two digits.
- We need to go systematically through all the possible values of b and therefore c.

Pupils will discover that the only possible answer is $b = 9$ and $c = 8$.

Main activity

This can be a difficult problem to get started with, but having looked at a simpler case, pupils should have a good idea about the need to be systematic.

Put the problem on the board and leave the pupils to investigate on their own for five minutes, searching for solutions or things they notice.

Lucky choices on the part of pupils may mean they find some solutions even if they have not been systematic. However, you need to encourage pupils to move to an approach which involves systematically trying all possibilities and ensuring that no solutions slip through the net. It would be useful to wander round the class at this early stage and give some nudges in the right direction:

- Can you use the same approach that we used in the first part of the lesson?

- How can you convince me that you have all the solutions or that there is only one solution?

After this short time stop the class and seek findings, which may include:

- F has to be 1;
- R has to be even;
- T has to be 5 or more (this links to a maximum carry of 1);
- some solutions.

Now you may need to discuss how to be more systematic in order to find all the solutions. How can they convince you that they have them all when they think they have finished? They need to approach the task systematically.

For example, start by substituting 5 for T. Systematically trying other numbers for each of the other letters, it soon emerges that you cannot obtain a solution. How about $T = 6$, 7, 8 or 9?

During the rest of the session pupils could write solutions on the board as they find them.

A gentle reminder that you need convincing of the completeness of the solution is worth making during the rest of the main part of the lesson, in anticipation of the plenary.

Plenary

It is not necessary to have all the solutions to produce a convincing argument that you can get them all.

Systematically trying values and keeping a note of what has been tried is a convincing argument. The group would be employing a method of proof by exhaustion! This should be the focus of the plenary discussion.

As suggested in the introduction, it may be appropriate to move on to the problem 'Reach 100' next.

Solution notes

There are seven different solutions.

$$\begin{array}{r} 734 \\ +\ 734 \\ \hline 1468 \end{array} \quad \begin{array}{r} 765 \\ +\ 765 \\ \hline 1530 \end{array} \quad \begin{array}{r} 836 \\ +\ 836 \\ \hline 1672 \end{array} \quad \begin{array}{r} 846 \\ +\ 846 \\ \hline 1692 \end{array}$$

$$\begin{array}{r} 867 \\ +\ 867 \\ \hline 1734 \end{array} \quad \begin{array}{r} 928 \\ +\ 928 \\ \hline 1856 \end{array} \quad \begin{array}{r} 938 \\ +\ 938 \\ \hline 1876 \end{array}$$