May/June 2006.

1) i) negative, as gradient of each line is negative
or, negative as negative coefficient of x

ii) Two equations are equal at $\bar{x}, \bar{y}$,

$$y = -0.6x + 13.0 \qquad (1)$$

$$x = -1.6y + 21 \qquad (2)$$

Sub (2) in (1)

$$y = -0.6(-1.6y + 21) + 13$$

$$y = 0.96y + (-12.6) + 13$$

$$0.04y = 0.4$$
$$y = 10$$ Sub in (2)

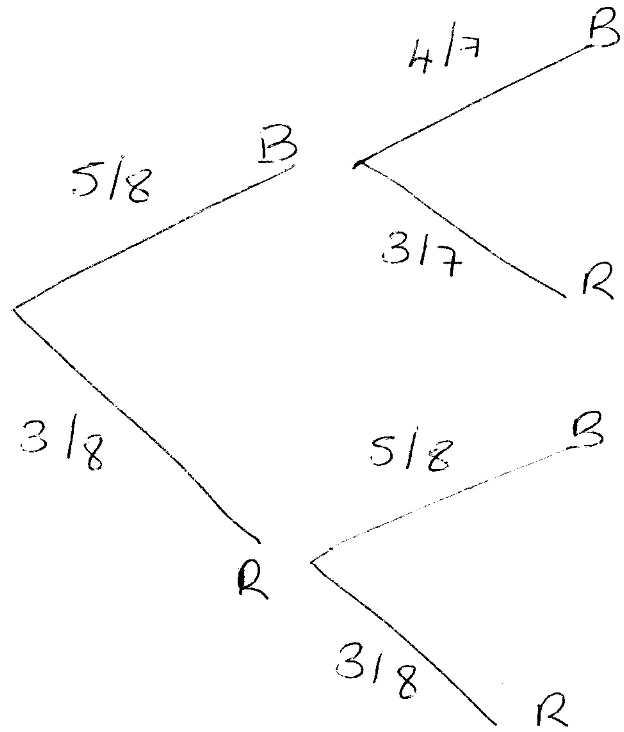
$$x = -1.6 \times 10 + 21$$
$$x = 5$$

So $\bar{x} = 5$, $\bar{y} = 10$

2. 5 black, not replaced

3 red replaced

i)



i) P(2nd disc black given first disc black) $= \underline{4/7}$

ii) P(2nd disc black)

(B B) or (R B)

$$\left(\frac{5}{8} \times \frac{4}{7}\right) + \left(\frac{3}{8} \times \frac{5}{8}\right) = \frac{265}{448} = \underline{0.592}$$

iii) P(different colours)

RB or BR

$$\left(\frac{3}{8} \times \frac{5}{8}\right) + \left(\frac{5}{8} \times \frac{3}{7}\right) = \frac{225}{448} = \underline{0.502}$$

3 DIVIDED

7 letters

D's $\Rightarrow$ 3
I's $\Rightarrow$ 2
E's $\Rightarrow$ 1

i) $\frac{7!}{3!\,2!\,1!} = 420$

ii) 3D's altogether - count as 1

So now have 5 letters

DDD's $\Rightarrow$ 1
I's $\Rightarrow$ 2
E's $\Rightarrow$ 1

$\frac{5!}{2!\,1!} = 60$

iii) P(at least 1 D) = 1 - P(no D's)

7cards choose 2 $\rightarrow$ total combinations

P(no D's) = $\frac{\text{No. no D combinations}}{\text{Total combinations}} = \frac{{}^4C_2}{{}^7C_2}$

$\therefore$ p(at least 1 D) $= 1 - \frac{{}^4C_2}{{}^7C_2} = 0.714$

4) i) $X \sim B(25, 0.2)$

$p(X \geq 5) = 1 - p(X \leq 4)$

$= 1 - 0.4207$

$= 0.579$ (3sf)

ii) $Y \sim B(10, 0.27)$ $p = 0.27$ $q = 0.73$

$p(Y = 3) = 10C_3 \times 0.27^3 \times 0.73^7$

$= 0.261$ (3sf)

iii) $B(n, 0.27)$ smallest value of n

$p(Z \geq 1) > 0.95$

$1 - p(X = 0) > 0.95$

$1 - {}^4C_0 \times 0.27^0 \times 0.73^{n-0} > 0.95$

$1 - {}^4C_0 \times 1 \times 0.73^n > 0.95$

$1 - 0.73^n > 0.95$

$0.05 > 0.73^n$

try $n = 5$, $0.73^5 = 0.2$

$n = 8$ $0.73^8 = 0.08$

$n = 9$ $0.73^9 = 0.0588$

$n = 10$ $0.73^{10} = 0.043$

$\therefore$ 10 is smallest value of n to satisfy $0.05 > 0.73^n$ hence, satisfys $p(Z \geq 1) > 0.95$.

5) i) $\frac{1}{3} + 1/4 + p + q = 1$

$$p + q = 5/12 \quad \text{—} ①$$

$E(x) = 1\,1/4 = 1/4 + 2p + 3q$

$$1 = 2p + 3q \quad \text{—} ②$$

from ① $p = 5/12 - q$ sub in ②

$$1 = 2(5/12 - q) + 3q$$

$$1 = 5/6 - 2q + 3q$$

$$\underline{q = 1/6}$$

$\therefore$ from ① $p + 1/6 = 5/12$

$$\underline{p = 1/4}$$

ii) $Var(x) = (1^2 \times 1/4) + (2^2 \times 1/4) + (3^2 \times 1/6) - (1\,1/4)^2$

$Var(x) = 1.1875$

Standard deviation $= \sqrt{var}$

$= \sqrt{1.1875}$

$= 1.0897$

$= 1.09$ (3sf)

6.

i) a)

Distance	Commission	Rank distance	Rank Commission	d	d^2
18	18	2	7	-5	25
15	45	4	1	3	9
12	19	7	6	1	1
14	24	5	3	2	4
16	27	3	2	1	1
24	22	1	5	-4	16
13	23	6	4	2	4
					$\Sigma = 60$

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 60}{7(49-1)}$$

$$= -0.071 \quad (3dp)$$

b) r_s shows little connection between the distance travelled & the commission.

c) No difference, as the ranking remained the same.

ii)

a) $r_s = -1$ (as each x increases, y decreases — so reverse of each other)

b) Pmcc, r — Strong negative value, ~ -0.9.

7.

Time	Smokers	midpoint (x)	fx	x^2f
1-4	31% = 620	2.5	1550	3875
4-14	27% = 540	9.5	5130	48735
15-29	19% = 380	22	8360	183920
30-59	14% = 280	44.5	12460	554470
60-239	9% = 180	149.5	26910	4023045

$\Sigma = 2000$ $\quad\Sigma 54410$ $\quad\Sigma 4814045$

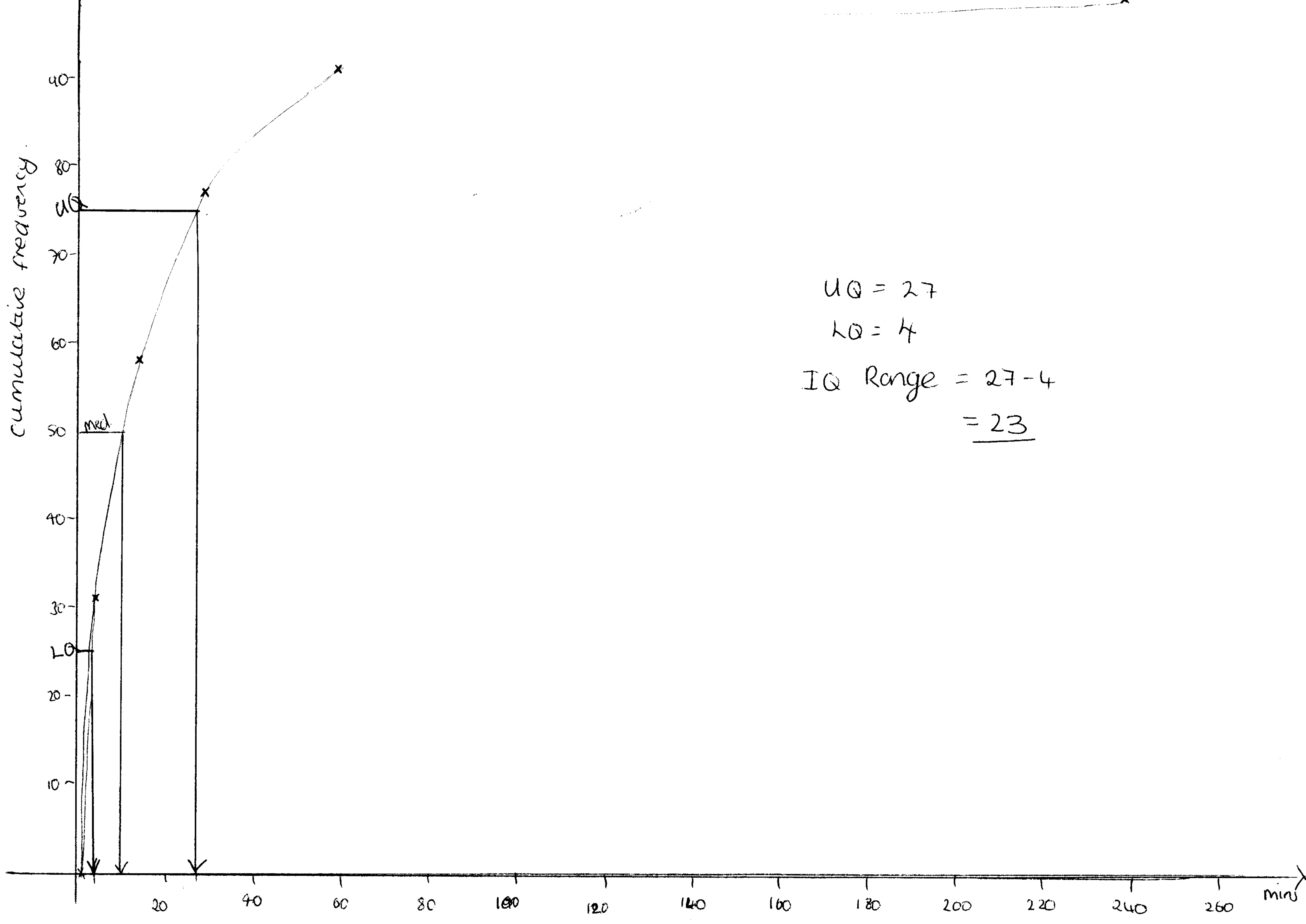
$$\bar{x} = \frac{54410}{2000} = \underline{\underline{27.2}} \text{ mins}$$

$$\text{Var} = \frac{\Sigma x^2 f}{n} - \bar{x}^2$$

$$= \frac{4814045}{2000} - 27.2^2$$

$$= 1667.1825$$

S. Deviation $= \sqrt{1667.1825} = 40.83$



UQ = 27

LQ = 4

IQ Range = 27 - 4

= <u>23</u>

iii) a) increase

b) increase

c) no change

8. $P(\text{Sucess})$ = fire lighting = $1/3$

X = NO. attempts

i) geometric distribution, need each event to be independent.

ii) a) $Geo(1/3)$ $\quad p = 1/3 \quad q = 2/3$

$p(X = 4) = pq^{x-1}$

$= \frac{1}{3} \times \frac{2}{3}^{3} = \frac{8}{81} = 0.0988$

b) $p(x < 4) = p(x \leq 3)$

$= p(x=1) + p(x=2) + p(x=3)$

$= \frac{1}{3} \times \frac{2}{3}^{0} + \frac{1}{3} \times \frac{2}{3}^{1} + \frac{1}{3} \times \frac{2}{3}^{2}$

$= \frac{19}{27}$

iii) $E(x) = 1/p = 1/{1/3} = 3$

iv) $p(x < 4) = 19/27$ = fewer 4 attempts

So march 1st more 4 attempts and
March 2nd more 4 attempts and
march 3rd fewer than 4 attempts

p(more than 4 attempts) $= p(x \geqslant 4)$

$= 1 - p(x \leqslant 3)$

$= 1 - 19/27 = 8/27$

$\therefore$ p(march 3rd fewer than 4 attempts)

$= \frac{8}{27} \times 8/27 \times 19/27 = \frac{1216}{19683} = 0.6618$