

Pass the fraction

Modelling and optimisation

Prerequisite knowledge

- Fractions of an amount

Why do this unit?

This activity is designed to support the discussion of problem-solving methods. The problem 'Pass the fraction' can be solved algebraically but a systematic approach quickly narrows down the possibilities. The activity emphasises that there is often value in recording on paper when using ICT.

Time

One lesson

Resources

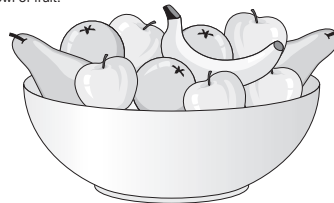
Counters or plastic coins may be useful
CD-ROM: spreadsheet and resource sheets 1 and 2
NRICH website (optional):
www.nrich.maths.org, March 2002, 'A bowl of fruit'; November 2004, 'Ben's game'; March 2002, 'In the money'

Pass the fraction

Resource sheet 1

Fruit in a bowl

Here is a bowl of fruit.



Half of the pieces of fruit in the bowl are apples.
There are also 3 oranges, 2 pears and a banana.
How many apples are there in the bowl?

If, instead, one quarter are apples and one quarter are oranges and there are 4 bananas, 3 pears and 3 plums, how many are apples?

In the money

There are a number of coins on a table.
One quarter of the coins show heads.



If I turn over 2 coins showing tails, then one third of the coins show heads. How many coins are there altogether?

Maths Trails: Excel at Problem Solving | Problem and resource sheets | © Cambridge University Press 2008

Introducing the unit

Use the problem 'Fruit in a bowl' on resource sheet 1 as a starting point for discussion.

- What do we know?
- What do we want to know?
- What can we find out?

Now look at the problem 'In the money' on the same resource sheet.

Ask the group to think on their own about how they might tackle this problem. Then ask pupils to discuss their ideas in pairs and get ready to share their ideas with the rest of the group. Where pupils suggest trying some numbers, develop this into a fully systematic approach.

- What do you know about the number of coins on the table? [multiple of 4]

Encourage pupils to try possibilities systematically. Give them some time to work in

pairs to find a solution and feed this back to the rest of the group.

Some pupils may wish to use counters or coins to look at the problem practically.

It is possible that a pupil may offer an algebraic approach or one involving direct manipulation of fractions. Each method suggested can be compared with a systematic approach, discussing the merits of each and leaving pupils satisfied that any one of them leads to a solution, although some methods may be more efficient than others.

Main part of the unit

Display the problem 'Pass the fraction' on resource sheet 2.

Tell pupils to spend two minutes working on the problem on their own before sharing their ideas with a partner. After a time stop the

group to share ideas and possible approaches.

- What do they know? [Ben must have a multiple of 3 counters, Ravi a multiple of 4...]

If pupils are not offering this insight, turn to the spreadsheet which calculates the number of counters left when pupils input different start values for Ben, Ravi and Emma.

- What is the spreadsheet doing? [Look at each cell to see what it contains.]

Input some possible start values for each person.

- Why are numbers in the grid sometimes decimals? [If the start number is not a multiple of the divisor you will get an answer which is not a whole number.]
- What does this tell us about the numbers of counters each person must have? [They must be multiples of 3, 4 and 5 respectively.]
- Also, what can we say about the total number of counters in use if Ben, Ravi and Emma all have the same at the end? [The total is a multiple of 3.]

The group can now return to the problem. They may still need support as they consider systematic approaches to finding a solution.

- How many different things can you change? [Three – the starting number of counters for each of the children.]

- Can you see how to fix the value of some things while you let one thing go through all its possible values?
- How do you know you have tried all the possibilities?

Explain that the spreadsheet can deal with the calculation but we need to record results in a way that lets us be sure we have all the solutions.

Plenary

Ask pupils, in pairs or groups, to explain what the problem was, what they tried, what their preferred method is and how confident they are that they have every solution (in other words that there is one, and only one, solution here).

The aim now is to give pupils a taste for how a given context can become, by degrees, more general. There is scope here for generalising. Ask pupils to make up similar problems of their own. ‘In the money’ can be posed with other differences than ‘two more’, or the fractions themselves might be different. In ‘Pass the fraction’ the fractions could be different or the number of participants might increase from three to four. It is enough to point out these possibilities, and invite other suggestions from the group, and not necessary to pursue the new problems at this time.

Solution notes

Fruit in a bowl

6 apples
5 apples and 5 oranges

In the money

A systematic approach might be to test what happens if you start with:

HTTT
HHTTTTTT
HHHTTTTTTTTT
...
Answer: 24 coins.

Pass the fraction

Emma 10, Ravi 8 and Ben 12 counters.