

4723 Core Mathematics 3

1	<u>Either</u> : Obtain $x = 0$ Form linear equation with signs of $4x$ and $3x$ different State $4x - 5 = -3x + 5$ Obtain $\frac{10}{7}$ and no other non-zero value(s) <u>Or</u> : Obtain $16x^2 - 40x + 25 = 9x^2 - 30x + 25$ Attempt solution of quadratic equation Obtain $\frac{10}{7}$ and no other non-zero value(s) Obtain 0	B1 ignoring errors in working M1 ignoring other sign errors A1 or equiv without brackets A1 or exact equiv <div>4</div> B1 or equiv M1 at least as far as factorisation or use of formula A1 or exact equiv B1 ignoring errors in working <div>4</div>												
2	(i) Show graph indicating attempt at reflection in $y = x$ Show correct graph with x -coord 2 and y -coord -3 indicated (ii) Show graph indicating attempt at reflection in x -axis Show correct graph with x -coord -3 indicated ... and y -coord -4 indicated [SC: Incorrect curve earning M0 but both correct intercepts indicated]	M1 with correct curvature and crossing negative y -axis and positive x -axis <div>A1</div> <div>2</div> M1 with correct curvature and crossing each negative axis <div>A1</div> <div>A1</div> <div>B1]</div> <div>3</div>												
3	Attempt use of product rule Obtain $2x \ln x + x^2 \cdot \frac{1}{x}$ Substitute e to obtain $3e$ for gradient Attempt eqn of straight line with numerical gradient Obtain $y - e^2 = 3e(x - e)$ Obtain $y = 3ex - 2e^2$	M1 ... + ... form A1 or equiv A1 or exact (unsimplified) equiv M1 allowing approx values A1 ✓ or equiv; following their gradient provided obtained by diffn attempt; allow approx values A1 in terms of e now and in requested form <div>6</div>												
4	(i) Differentiate to obtain form $kx(2x^2 + 9)^n$ Obtain correct $10x(2x^2 + 9)^{\frac{3}{2}}$ Equate to 100 and confirm $x = 10(2x^2 + 9)^{-\frac{3}{2}}$ (ii) Attempt relevant calculations with 0.3 and 0.4 Obtain at least one correct value Obtain two correct values and conclude appropriately	M1 any constant k ; any $n < \frac{5}{2}$ A1 or (unsimplified) equiv A1 AG ; necessary detail required <div>3</div> M1 A1 <table><tr><td>x</td><td>$f(x)$</td><td>$x - f(x)$</td><td>$f'(x)$</td></tr><tr><td>0.3</td><td>0.3595</td><td>-0.0595</td><td>83.4</td></tr><tr><td>0.4</td><td>0.3515</td><td>0.0485</td><td>113.8</td></tr></table> A1 noting sign change or showing $0.3 < f(0.3)$ and $0.4 > f(0.4)$ or showing gradients either side of 100 <div>3</div>	x	$f(x)$	$x - f(x)$	$f'(x)$	0.3	0.3595	-0.0595	83.4	0.4	0.3515	0.0485	113.8
x	$f(x)$	$x - f(x)$	$f'(x)$											
0.3	0.3595	-0.0595	83.4											
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(iii) Obtain correct first iterate Carry out correct process Obtain 0.3553	B1 M1 finding at least 3 iterates in all A1 answer required to exactly 4 dp 3
[0.3 \rightarrow 0.35953 \rightarrow 0.35497 \rightarrow 0.35534 \rightarrow 0.35531; 0.35 \rightarrow 0.35575 \rightarrow 0.35528 \rightarrow 0.35532 (\rightarrow 0.35531); 0.4 \rightarrow 0.35146 \rightarrow 0.35563 \rightarrow 0.35529 \rightarrow 0.35532]	
5 (a) Obtain expression of form $\frac{a \tan \alpha}{b + c \tan^2 \alpha}$	M1 any non-zero constants a, b, c
State correct $\frac{2 \tan \alpha}{1 - \tan^2 \alpha}$	A1 or equiv
Attempt to produce polynomial equation in $\tan \alpha$	M1 using sound process
Obtain at least one correct value of $\tan \alpha$	A1 $\tan \alpha = \pm \sqrt{\frac{4}{5}}$
Obtain 41.8	A1 allow 42 or greater accuracy; allow 0.73
Obtain 138.2 and no other values between 0 and 180	A1 allow 138 or greater accuracy
[SC: Answers only 41.8 or ... B1; 138.2 or ... and no others B1]	6
(b)(i) State $\frac{7}{6}$	B1 1
(ii) Attempt use of identity linking $\cot^2 \beta$ and $\operatorname{cosec}^2 \beta$	M1 or equiv retaining exactness; condone sign errors
Obtain $\frac{13}{36}$	A1 or exact equiv 2
6 Integrate $k_1 e^{nx}$ to obtain $k_2 e^{nx}$ Obtain correct indefinite integral of their $k_1 e^{nx}$ Substitute limits to obtain $\frac{1}{6} \pi (e^3 - 1)$ or $\frac{1}{6} (e^3 - 1)$ Integrate $k(2x - 1)^n$ to obtain $k'(2x - 1)^{n+1}$ Obtain correct indefinite integral of their $k(2x - 1)^n$ Substitute limits to obtain $\frac{1}{18} \pi$ or $\frac{1}{18}$ Apply formula $\int \pi y^2 dx$ at least once Subtract, correct way round, attempts at volumes y^2 Obtain $\frac{1}{6} \pi e^3 - \frac{2}{9} \pi$	M1 any constants involving π or not; any n A1 A1 or exact equiv perhaps involving e^0 M1 any constants involving π or not; any n A1 A1 or exact equiv B1 for $y = e^{3x}$ and/or $y = (2x - 1)^4$ M1 allow with π missing but must involve A1 or similarly simplified exact equiv 9
7 (i) State $A = 42$ State $k = \frac{1}{9}$ Attempt correct process for finding m Obtain $\frac{1}{9} \ln 2$ or 0.077	B1 B1 or 0.11 or greater accuracy M1 involving logarithms or equiv A1 or 0.08 or greater accuracy 4
(ii) Attempt solution for t using either formula Obtain 11.3	M1 using correct process (log's or T&I or ...) A1 or greater accuracy; allow 11.3 ± 0.1 2
(iii) Differentiate to obtain form Be^{mt} Obtain $3.235e^{0.077t}$ Obtain 47.9	M1 where B is different from A A1✓ or equiv; following their A and m A1 allow 48 or greater accuracy 3

8 (i) Show at least correct $\cos \theta \cos 60 + \sin \theta \sin 60$ or $\cos \theta \cos 60 - \sin \theta \sin 60$ Attempt expansion of both with exact numerical values attempted Obtain $\frac{1}{2}\sqrt{3} \sin \theta + \frac{5}{2} \cos \theta$	B1 M1 and with $\cos 60 \neq \sin 60$ A1 or exact equiv <div style="border: 1px solid black; padding: 2px; display: inline-block;">3</div>
(ii) Attempt correct process for finding R Attempt recognisable process for finding α Obtain $\sqrt{7} \sin(\theta + 70.9)$	M1 whether exact or approx M1 allowing sin / cos muddles A1 allow 2.65 for R ; allow 70.9 ± 0.1 for α <div style="border: 1px solid black; padding: 2px; display: inline-block;">3</div>
(iii) Attempt correct process to find any value of θ + their α Obtain any correct value for $\theta + 70.9$ Attempt correct process to find θ + their α in 3rd quadrant Obtain 131 [SC for solutions with no working shown: Correct answer only B4; 131 with other answers B2]	M1 A1 -158, -22, 202, 338, ... M1 or several values including this A1 or greater accuracy and no other <div style="border: 1px solid black; padding: 2px; display: inline-block;">4</div>
9 (i) Attempt use of quotient rule Obtain $\frac{75 - 15x^2}{(x^2 + 5)^2}$ Equate attempt at first derivative to zero and rearrange to solvable form Obtain $x = \sqrt{5}$ or 2.24 Recognise range as values less than y -coord of st pt Obtain $0 \leq y \leq \frac{3}{2}\sqrt{5}$	*M1 or equiv; allow u / v muddles A1 or (unsimplified) equiv; this M1A1 available at any stage of question M1 dep *M A1 or greater accuracy M1 allowing < here A1 any notation; with \leq now; any exact equiv <div style="border: 1px solid black; padding: 2px; display: inline-block;">6</div>
(ii) State $\sqrt{5}$	B1 ✓ following their x -coord of st pt; condone answer $x \geq \sqrt{5}$ but not inequality with k <div style="border: 1px solid black; padding: 2px; display: inline-block;">1</div>
(iii) Equate attempt at first derivative to -1 and attempt simplification Obtain $x^4 - 5x^2 + 100 = 0$ Attempt evaluation of discriminant or equiv Obtain -375 or equiv and conclude appropriately	*M1 and dependent on first M in part (i) A1 or equiv involving 3 non-zero terms M1 dep *M A1 <div style="border: 1px solid black; padding: 2px; display: inline-block;">4</div>