

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MATHEMATICS**

**4736**

Decision Mathematics 1

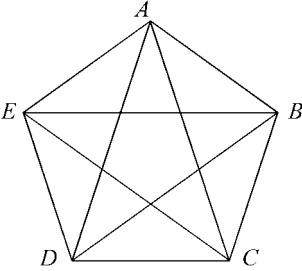
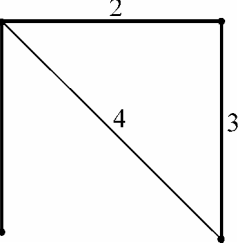
MARK SCHEME

**Specimen Paper**

<b>MAXIMUM MARK</b>	<b>72</b>
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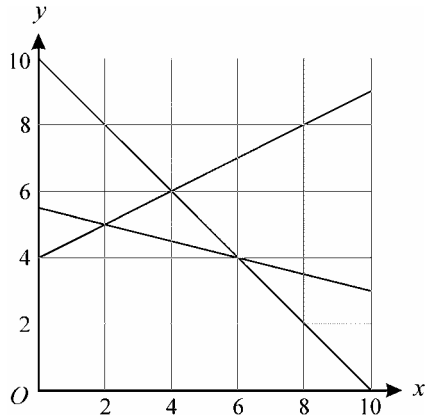
**This mark scheme consists of 4 printed pages.**

<p><b>1</b> (i)</p>  <p><math>K_5</math> is Eulerian since every node is even</p> <p>(ii) A path is (e.g.) <math>A-B-C</math></p> <p>(iii) A cycle is (e.g.) <math>A-B-C-A</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p><b>4</b></p>	<p>For correct graph</p> <p>For a correct statement</p> <p>For any correct path</p> <p>For any correct cycle</p>
<p><b>2</b> (i) Using Kruskal's algorithm, the arc of least weight is chosen first and so is certainly included The arc of second least weight is chosen next since just two arcs cannot form a cycle</p> <p>(ii)</p> 	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p><b>7</b></p>	<p>For identifying the first choice</p> <p>For identifying the second choice</p> <p>For correct justification</p> <p>For any connected graph with 4 nodes and at least 3 arcs</p> <p>For including a cycle</p> <p>For a network having the required property</p> <p>For making the minimum connector clear</p>
<p><b>3</b> (i) 1st pass: <math>\underline{6\ 3}\ 8\ 3\ 2</math> giving <math>3\ 6\ 8\ 3\ 2</math> 2nd pass: <math>3\ \underline{6\ 8}\ 3\ 2</math> giving <math>3\ 6\ 8\ 3\ 2</math> 3rd pass: <math>3\ 6\ \underline{8\ 3}\ 2</math> <math>3\ \underline{6\ 3}\ 8\ 2</math> <math>\underline{3\ 3}\ 6\ 8\ 2</math> giving <math>3\ 3\ 6\ 8\ 2</math> 4th pass: <math>3\ 3\ 6\ \underline{8\ 2}</math> <math>3\ 3\ \underline{6\ 2}\ 8</math> <math>3\ \underline{3\ 2}\ 6\ 8</math> <math>\underline{3\ 2}\ 3\ 6\ 8</math> giving <math>2\ 3\ 3\ 6\ 8</math></p> <p>(ii) The number of operations to be carried out, and thus the time to complete the algorithm, is (approximately) proportional to the square of the number of items to be sorted</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p><b>8</b></p>	<p>For correct result of first pass</p> <p>For correct result of second pass</p> <p>For correct shuttle process in third pass</p> <p>For correct shuttle process in final pass</p> <p>For shuttle sort completed correctly</p> <p>For idea of dependency on 'size' of problem</p> <p>For number of operations, or time required</p> <p>For square of list size</p>

4	<p>(i)</p> <table border="1"> <thead> <tr> <th>STEP</th><th>A</th><th>B</th><th>C</th></tr> </thead> <tbody> <tr><td>1</td><td>6</td><td>13</td><td>0</td></tr> <tr><td>2</td><td>6</td><td>13</td><td>6</td></tr> <tr><td>4</td><td>12</td><td>6</td><td>6</td></tr> <tr><td>4</td><td>24</td><td>3</td><td>6</td></tr> <tr><td>2</td><td>24</td><td>3</td><td>30</td></tr> <tr><td>4</td><td>48</td><td>1</td><td>30</td></tr> <tr><td>2</td><td>48</td><td>1</td><td>78</td></tr> <tr><td>3</td><td>48</td><td>1</td><td>78</td></tr> <tr><td>6</td><td colspan="3">Output 78</td></tr> </tbody> </table>	STEP	A	B	C	1	6	13	0	2	6	13	6	4	12	6	6	4	24	3	6	2	24	3	30	4	48	1	30	2	48	1	78	3	48	1	78	6	Output 78			<p>B1 M1  M1 A1  A1</p>	<p>For assigning value to <math>C</math> in first Step 2 For updating <math>A</math> and <math>B</math> in first Step 4  For continuing algorithm and updating <math>C</math> For correct new value 30 for <math>C</math>  5 For correct output</p>
STEP	A	B	C																																								
1	6	13	0																																								
2	6	13	6																																								
4	12	6	6																																								
4	24	3	6																																								
2	24	3	30																																								
4	48	1	30																																								
2	48	1	78																																								
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(ii)	<table border="1"> <thead> <tr> <th>STEP</th><th>A</th><th>B</th><th>C</th></tr> </thead> <tbody> <tr><td>1</td><td><math>A</math></td><td>8</td><td>0</td></tr> <tr><td>4</td><td><math>2A</math></td><td>4</td><td>0</td></tr> <tr><td>4</td><td><math>4A</math></td><td>2</td><td>0</td></tr> <tr><td>4</td><td><math>8A</math></td><td>1</td><td>0</td></tr> <tr><td>2</td><td><math>8A</math></td><td>1</td><td><math>8A</math></td></tr> <tr><td>3</td><td><math>8A</math></td><td>1</td><td><math>8A</math></td></tr> <tr><td>6</td><td colspan="3">Output <math>8A</math></td></tr> </tbody> </table>	STEP	A	B	C	1	$A$	8	0	4	$2A$	4	0	4	$4A$	2	0	4	$8A$	1	0	2	$8A$	1	$8A$	3	$8A$	1	$8A$	6	Output $8A$			<p>M1 M1 A1</p>	<p>For values of <math>A</math> doubling For values of <math>B</math> halving For output <math>8A</math></p>								
STEP	A	B	C																																								
1	$A$	8	0																																								
4	$2A$	4	0																																								
4	$4A$	2	0																																								
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2	$8A$	1	$8A$																																								
3	$8A$	1	$8A$																																								
6	Output $8A$																																										
The output is the product of the inputs		B1	<p>4 9 For identifying multiplication</p>																																								
5	<p>(i) A minimum connector on reduced network has arcs <math>CE</math>, <math>ED</math>, <math>BD</math>, <math>AB</math>, giving length 23 km Two shortest arcs from <math>F</math> have weights 7, 8 Hence lower bound is <math>23 + 7 + 8 = 38</math> km</p> <p>(ii) The best upper bound is 47 km The best lower bound is 40 km</p> <p>(iii) Other orders are <math>CED</math>, <math>DCE</math>, <math>DEC</math>, <math>ECD</math>, <math>EDC</math> Shortest is <math>ABDCEFA</math>, of length 42 km</p>	<p>M1 A1 M1 A1  B1 B1  M1 A1 A1</p>	<p>For attempt at a relevant minimum connector For correct weight 23 For identifying the two shortest arcs at <math>F</math> 4 For showing given answer correctly  For the correct answer 2 For the correct answer  For calculation of at least one other length For any correct bound less than 47 km 3 For the correct value 42 9</p>																																								
6	<p>(i)</p> <p>Least travel time is 40 minutes Route is <math>A-B-C-D</math></p> <p>(ii) The Route Inspection algorithm is used <math>A, B, C</math> and <math>E</math> are odd nodes  <math>AB = 16</math>   <math>AC = 27</math>   <math>AE = 37</math>  <math>CE = \frac{10}{26}</math>   <math>BE = \frac{21}{48}</math>   <math>BC = \frac{11}{48}</math>  Double up on <math>AB</math> and <math>CE</math>  Sum of arcs is 172  Hence shortest time is <math>172 + 26 = 198</math> minutes</p> <p>(iii) Nearest neighbour algorithm gives <math>A-B-C-E-D-A</math> Hence required path is <math>A-B-C-E-D</math></p>	<p>M1 M1 A1 B1  B1 B1  B1 B1  M1 B1 M1 M1 A1  M1 A1 B1</p>	<p>For correct use of temporary labels For updating <math>E</math> and <math>D</math> For all permanent labels correct For correct order of assignment stated  For correct value 40 6 For correct route  For stating or implying the correct algorithm For identifying the odd nodes For pairing odd nodes correctly For selecting appropriate pair for doubling For adding weights on all the arcs 6 For correct value 198  For starting the algorithm correctly, up to <math>C</math> For the correct cycle <math>A-B-C-E-D-A</math> 3 For a correct path 15</p>																																								

7

(i)



Hence maximum  $P = 18$ , occurring at  $(2, 5)$

M1

For lines  $x + 4y = 22$  and  $x + y = 10$ 

M1

For line  $-x + 2y = 8$ 

A1

For correct diagram including shading

B1✓

For vertices  $(0, 0)$ ,  $(0, 4)$ ,  $(10, 0)$ 

B1✓

For vertex  $(2, 5)$ 

B1✓

For vertex  $(6, 4)$ 

B1

For the correct value 18

B1

8

For identifying the correct vertex

(ii)

$P$	$x$	$y$	$s$	$t$	$u$	
1	1	-4	0	0	0	0
0	1	4	1	0	0	22
0	-1	2	0	0	1	8

B1

For the correct pay-off row

M1

For the use of three slack variables

A1

For all constraints correct

Pivot on 2 in row 3

1	-1	0	0	0	2	16
0	3	0	1	0	-2	6
0	$1\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	6
0	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	4

M1

For choice of pivot

M1

For pivoting correctly

A1✓

For correct tableau

Now pivot on 3 in row 1

1	0	0	$\frac{1}{3}$	0	$1\frac{1}{3}$	18
0	1	0	$\frac{1}{3}$	0	$-\frac{2}{3}$	2
0	0	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	3
0	0	1	$\frac{1}{6}$	0	$\frac{1}{6}$	5

M1

For choice of pivot

M1

For pivoting correctly

A1

For correct tableau

Hence  $P = 18$  when  $x = 2$ ,  $y = 5$

B1✓

10

For reading off correctly from final tableau

(iii) Vertices  $(0, 0) \rightarrow (0, 4) \rightarrow (2, 5)$  indicated

M1

For indication of starting at the origin

A1

2

For the correct correspondence indicated

20