



- Time: 1½ hours.
- You are advised to show all your working.
- Calculators may be used.



1 It is given that $f(x) = \sin^{-1} x + 1 - 9x^2$.

- (i) Show that there is a root of $f(x) = 0$ in the interval $0.3 \leq x \leq 0.4$. [2] 
 (ii) This root is to be estimated using the iterative formula 



$$x_{n+1} = \sqrt{\frac{\sin^{-1} x_n + 1}{9}}, \quad x_0 = 0.4.$$



Showing your values of x_1, x_2, x_3, \dots , obtain the value, to three decimal places, of the root. [4]

2 It is given that $\cot x + 3 - \operatorname{cosec}^2 x = 0$.

- (i) Show that this equation can be written in the form 
 $\cot^2 x - \cot x - 2 = 0$. [2]
 (ii) Hence solve the equation $\cot x + 3 - \operatorname{cosec}^2 x = 0$ giving all values of x , where appropriate, to one decimal place in the interval $0^\circ \leq x < 360^\circ$. [5] 

3 The curve C has the equation $y = \frac{x+3}{\sqrt{(x^2+9)}}$.

- (i) Find $\frac{dy}{dx}$ in terms of x . [5] 
 (ii) Show that C has a stationary point where $x = 3$, and deduce that the line $y = \sqrt{2}$ is the horizontal tangent to C . [3] 

- 4 (i) Describe a sequence of geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 3 \ln(x+2)$. [4] 
 (ii) Use Simpson's Rule with five ordinates (four strips) to find an approximate value for $\int_{-1}^3 [\ln(x+2)]^2 dx$, giving your answer to four significant figures. [5] 

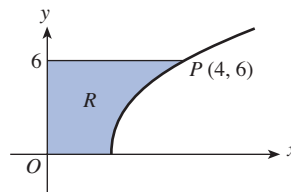
- 5 The value, £ V , of a car at age t months is modelled by the formula $V = Ae^{-kt}$, where k and A are positive constants. The value of the car when new was £9000. The value of the car is expected to decrease to £4500 after 36 months.

Write down the value of A , and show that $k = 0.019254$ approximately. [4]

Use the model to

- (i) calculate the value, to the nearest pound, of the car when it is 18 months old; [2]
 (ii) find the age of the car, to the nearest month, when its value first falls below £1800. [4]

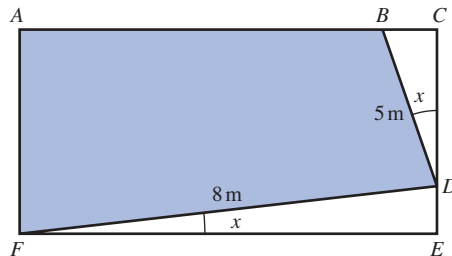
6



The diagram shows part of the curve with equation $y^2 = 12(x - 1)$. P is the point on the curve with coordinates $(4, 6)$. The finite region R is enclosed by the curve, the line $y = 6$, the x -axis and the y -axis.

The region R is rotated through 2π about the x -axis. Find the exact value of the volume of the solid generated. [10]




- 7 The diagram shows the plan of a rectangular garden $ACEF$.



The shaded area $ABDF$ represents the lawn, which has a perimeter of 26 m.

It is given that $BD = 5$ m, $DF = 8$ m and angle $DFE = \text{angle } BDC = x$, $x \neq 0$.

- (i) Show that $13 \cos x + 3 \sin x = 13$. [5]
 (ii) Express $13 \cos x + 3 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving your values of R and α to two decimal places. [3]
 (iii) Hence find the value of x , giving your answer to one decimal place. [3]

- 8 A function f is defined for all real values of x by $f(x) = e^{2x+3} - 1$.
- (i) Find the range of f . [1] 
- (ii) Sketch the curve with equation $y = f(x)$, showing the coordinates of any points at which the curve meets the coordinate axes. [4] 
- (iii) The curve with equation $y = f(x)$ has a gradient of 8 at the point P . Find the x -coordinate of P , giving your answer in the form $\ln a + b$, where a is an integer and b is a constant. [6] 
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END OF QUESTIONS