

# OI 模板

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# 1 图的 DFS 树

## 1.1 强连通分量

一有向图上每个点有非负权值，求一条路径，使得路径上点权值和最大。点和边都可以多次经过，但是权值只计入答案一次。

Solution: 缩点后直接在 DAG 上 DP.

GraphTheory/TarjanSCC.cpp

```
1 #include <bits/stdc++.h>
2 #define fst first
3 #define snd second
4 using namespace std;
5
6 typedef pair<int, int> pii;
7 const int MAXN = 1e5, INF = 0x3f3f3f3f;
8
9 struct Graph {
10     struct Edge {
11         int v, next;
12     };
13
14     int n, m, e_ptr = 1, head[MAXN+10]; Edge E[(MAXN+10)<<1];
15
16     void add_edge(int u, int v) {
17         E[++e_ptr] = (Edge) { v, head[u] }; head[u] = e_ptr;
18     }
19 } G1, G2;
20
21 int dfs_clock, scc_cnt, sccno[MAXN+10], dfn[MAXN+10], low[MAXN+10];
22 int ans, topo_cnt, topo_seq[MAXN+10], w[MAXN+10],
23     tot[MAXN+10], vis[MAXN+10], dp[MAXN+10];
24
25 stack<int> S;
26 void dfs(int u) {
27     dfn[u] = low[u] = ++dfs_clock;
28     S.push(u);
29     for(int j=G1.head[u]; j; j=G1.E[j].next) {
30         int v = G1.E[j].v;
31         if(!dfn[v]) {
32             dfs(v);
33             low[u] = min(low[u], low[v]);
34         } else if(!sccno[v])
35             low[u] = min(low[u], dfn[v]);
36     }
37     if(low[u] == dfn[u]) {
38         int v; ++scc_cnt;
39         do {
40             v = S.top(); S.pop();
41             sccno[v] = scc_cnt;
42             tot[scc_cnt] += w[v];
43         } while(u != v);
44     }
45 }
46
47 void Tarjan() {
```

```

48     for(int u = 1; u <= G1.n; u++)
49         if(!dfn[u]) dfs(u);
50 }
51
52 void scc_graph() {
53     set<pii> evis;
54     for(int u = 1; u <= G1.n; u++)
55         for(int j=G1.head[u]; j; j=G1.E[j].next) {
56             int v = G1.E[j].v;
57             if(sccno[u] == sccno[v] || evis.count(make_pair(sccno[u], sccno[v])))
58                 continue;
59             else {
60                 evis.insert(make_pair(sccno[u], sccno[v]));
61                 G2.add_edge(sccno[u], sccno[v]);
62             }
63         }
64     G2.n = scc_cnt;
65 }
66
67 bool topo_dfs(int u) {
68     vis[u] = -1;
69     for(int j=G2.head[u]; j; j=G2.E[j].next) {
70         int v = G2.E[j].v;
71         if(vis[v] == -1 || (vis[v] == 0 && !topo_dfs(v)))
72             return false;
73     }
74     vis[u] = 1;
75     topo_seq[topo_cnt--] = u;
76     return true;
77 }
78
79 bool toposort() {
80     topo_cnt = G2.n;
81     for(int u = G2.n; u >= 1; u--)
82         if(vis[u] == 0 && !topo_dfs(u)) return false;
83     return true;
84 }
85
86 inline int readint() {
87     int f=1, r=0; char c=getchar();
88     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
89     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
90     return f*r;
91 }
92
93 void init() {
94     int u, v;
95     G1.n = readint(); G1.m = readint();
96     for(int i = 1; i <= G1.n; i++)
97         w[i] = readint();
98     for(int i = 1; i <= G1.m; i++) {
99         u = readint(); v = readint();
100         G1.add_edge(u, v);
101     }
102     Tarjan(); scc_graph();
103     assert(toposort());
104 }

```

```

105
106 void work() {
107     for(int i = G2.n; i >= 1; i--) {
108         int u = topo_seq[i], maxv = 0;
109         for(int j=G2.head[u]; j; j=G2.E[j].next) {
110             int v = G2.E[j].v;
111             if(dp[v] > maxv) maxv = dp[v];
112         }
113         dp[u] = tot[u] + maxv;
114         ans = max(ans, dp[u]);
115     }
116     printf("%d", ans);
117 }
118
119 int main() {
120     init(); work();
121     return 0;
122 }

```

## 1.2 桥和割点

注意 child 代表 DFS 树中的儿子数目，且只在走完 DFS 树中某个儿子后判断割点条件。  
桥只需要把 34 行 `>=` 改为 `>` 即可。

GraphTheory/CutVertex.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct Edge{ int v, next; };
5
6 const int MAXN = 1e5, MAXM = 1e5;
7 int n, m, cnt, e_ptr = 1, head[MAXN+10]; Edge E[(MAXM+10)<<1];
8 int dfs_clk, iscut[MAXN+10], dfn[MAXN+10], low[MAXN+10];
9
10 void add_edge(int u, int v) {
11     E[++e_ptr] = (Edge) { v, head[u] }; head[u] = e_ptr;
12 }
13
14 void add_pair(int u, int v) {
15     add_edge(u, v); add_edge(v, u);
16 }
17
18 inline int readint() {
19     int f=1, r=0; char c=getchar();
20     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
21     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
22     return f*r;
23 }
24
25 void dfs(int u, int fa) {
26     int child = 0;
27     dfn[u] = low[u] = ++dfs_clk;
28     for(int j=head[u]; j; j=E[j].next) {
29         int v = E[j].v;
30         if(v == fa) continue;

```

```

31     if(!dfn[v]) {
32         dfs(v, u); ++child;
33         low[u] = min(low[u], low[v]);
34         if(low[v] >= dfn[u]) iscut[u] = true;
35     } else if(dfn[v] < dfn[u] && v != fa)
36         low[u] = min(low[u], dfn[v]);
37 }
38 if(child == 1 && fa == -1)
39     iscut[u] = false;
40 }
41
42 int main() {
43     int u, v;
44     n = readint(); m = readint();
45     for(int i = 1; i <= m; i++) {
46         u = readint(); v = readint();
47         add_pair(u, v);
48     }
49     for(int i = 1; i <= n; i++)
50         if(!dfn[i]) dfs(i, -1);
51     for(int i = 1; i <= n; i++)
52         if(iscut[i]) ++cnt;
53     printf("%d\n", cnt);
54     for(int i = 1; i <= n; i++)
55         if(iscut[i]) printf("%d ", i);
56     return 0;
57 }

```

### 1.3 点双连通分量

GraphTheory/BCCVertex.cpp

```

1  /*
2      [UVa0J1364] Knights of the Round Table
3      好题。
4      首先，问题可以转化成求无向图中不属于任何一个奇圈的点的数目。
5      补集转换一下，变为求至少属于一个奇圈的点数目。
6      和圈相关的问题，可以考虑BCC。和圈和点都有关，考虑点双连通分量。
7      如果一个点双里面没有奇圈，那么它里面任何一个点显然都不属于任何一个奇圈。
8      只要一个点双里面有一个奇圈，那么点双中任何一个点都至少属于一个奇圈，因为我们
9      可以利用已有的奇圈来"包含"这个点双内的某个点。
10     奇=奇+偶。如果奇圈上有点v1, v2，这个奇圈外有点u，不妨假设有u->v1, u->v2的路径
11     （由双连通性质一定存在这样的v1, v2），则不管v1->u->v2含点数的奇偶性如何，总可以构造
12     一个u->v2->现有奇圈一部分->v1->u的新奇圈！
13     于是只需要找出所有BCC，然后对每个BCC二分图染色，即可得出答案。
14     （注意割点bccno无意义）
15 */
16 #include <bits/stdc++.h>
17 #define CLEAR(x) memset((x), 0, sizeof(x))
18 using namespace std;
19
20 struct Edge {
21     int u, v, next;
22 };
23
24 const int MAXN = 1e3, MAXM = 1e6;

```

```

25 int n, m, e_ptr = 1, head[MAXN+10], hate[MAXN+10][MAXN+10];
26 Edge E[(MAXM+10)<<1];
27 int dfs_clk, bcc_cnt, dfn[MAXN+10], low[MAXN+10], bccno[MAXN+10], iscut[MAXN+10];
28 vector<int> bcc[MAXN+10]; int clr[MAXN+10];
29
30 void add_edge(int u, int v) {
31     E[++e_ptr] = (Edge) { u, v, head[u] }; head[u] = e_ptr;
32 }
33
34 void add_pair(int u, int v) {
35     add_edge(u, v); add_edge(v, u);
36 }
37
38 inline int readint(){
39     int f=1, r=0; char c=getchar();
40     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
41     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
42     return f*r;
43 }
44
45 stack<int> S;
46 void dfs(int u, int fa) {
47     int child = 0;
48     dfn[u] = low[u] = ++dfs_clk;
49     for(int j=head[u]; j; j=E[j].next) {
50         int v = E[j].v; if(v == fa) continue;
51         if(!dfn[v]) {
52             S.push(j);
53             dfs(v, u); ++child;
54             low[u] = min(low[u], low[v]);
55             if(low[v] >= dfn[u]) {
56                 iscut[u] = true;
57                 ++bcc_cnt; int cur;
58                 do {
59                     cur = S.top(); S.pop();
60                     if(bccno[E[cur].u] != bcc_cnt) {
61                         bccno[E[cur].u] = bcc_cnt;
62                         bcc[bcc_cnt].push_back(E[cur].u);
63                     }
64                     if(bccno[E[cur].v] != bcc_cnt) {
65                         bccno[E[cur].v] = bcc_cnt;
66                         bcc[bcc_cnt].push_back(E[cur].v);
67                     }
68                 } while(E[cur].u != u || E[cur].v != v);
69             }
70             else if(dfn[v] < dfn[u] && v != fa) {
71                 S.push(j);
72                 low[u] = min(low[u], dfn[v]);
73             }
74         }
75         if(child == 1 && fa == -1)
76             iscut[u] = false;
77     }
78
79 void find_bcc() {
80     for(int i = 1; i <= n; i++)
81         if(!dfn[i]) dfs(i, -1);

```



```

82 }
83
84 bool bipartite(int u, int b) {
85     for(int j=head[u]; j; j=E[j].next) {
86         int v = E[j].v; if(bccno[v] != b) continue;
87         if(clr[v] == clr[u]) return false;
88         if(!clr[v]) {
89             clr[v] = 3 - clr[u];
90             if(!bipartite(v, b)) return false;
91         }
92     }
93     return true;
94 }
95
96 bool init() {
97     int u, v;
98     n = readint(); m = readint();
99     if(!n && !m) return false;
100     for(int i = 1; i <= m; i++) {
101         u = readint(); v = readint();
102         hate[u][v] = hate[v][u] = true;
103     }
104     for(u = 1; u <= n; u++)
105         for(v = u + 1; v <= n; v++)
106             if(!hate[u][v]) add_pair(u, v);
107     return true;
108 }
109
110 void work() {
111     int ans = n;
112     find_bcc();
113     for(int i = 1; i <= bcc_cnt; i++) {
114         for(int j = 0; j < (int)bcc[i].size(); j++)
115             bccno[bcc[i][j]] = i; // 割点 bccno 无意义
116         CLEAR(clr);
117         clr[bcc[i][0]] = 1;
118         if(!bipartite(bcc[i][0], i))
119             ans -= bcc[i].size();
120     }
121     printf("%d\n", ans);
122 }
123
124 void clear() {
125     for(int i = 1; i <= bcc_cnt; i++) bcc[i].clear();
126     n = m = 0; e_ptr = 1; CLEAR(head); CLEAR(hate);
127     dfs_clk = bcc_cnt = 0;
128     CLEAR(dfn); CLEAR(low); CLEAR(bccno); CLEAR(iscut); CLEAR(clr);
129 }
130
131 int main() {
132     while(true) {
133         if(!init()) break;
134         work(); clear();
135     }
136     return 0;
137 }

```

## 1.4 边双连通分量

找出割边后 DFS，同时避免经过割边，即可求出边双连通分量。

## 2 最短路

### 2.1 负环

GraphTheory/NegCycle.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct Edge {
5     int v, len, next;
6 };
7
8 const int MAXN = 2e5, MAXM = 2e5, INF = 0x3f3f3f3f;
9
10 int T, cz, e_ptr = 1, n, m, head[MAXN+10], ins[MAXN+10]; Edge E[(MAXM+10)<<1];
11 int dist[MAXN+10];
12
13 void add_edge(int u, int v, int len) {
14     E[++e_ptr] = (Edge) { v, len, head[u] }; head[u] = e_ptr;
15 }
16
17 void add_pair(int u, int v, int len) {
18     add_edge(u, v, len); add_edge(v, u, len);
19 }
20
21 bool spfa(int u) {
22     ins[u] = true;
23     for(int j=head[u]; j; j=E[j].next) {
24         int v = E[j].v, len = E[j].len;
25         if(dist[v] > dist[u] + len) {
26             dist[v] = dist[u] + len;
27             if(ins[v] || (!ins[v] && !spfa(v)))
28                 return false;
29         }
30     }
31     ins[u] = false; // 回溯
32     return true;
33 }
34
35 bool neg_cycle() {
36     memset(ins, 0, sizeof(ins));
37     fill(dist + 1, dist + n + 1, .0);
38     for(int i = 1; i <= n; i++)
39         if(!spfa(i)) return true;
40     return false;
41 }
42
43 void init() {
44     int u, v, w;
45     scanf("%d%d", &n, &m);
46     for(int i = 1; i <= m; i++) {
47         scanf("%d%d%d", &u, &v, &w);
48         if(w < 0)
49             add_edge(u, v, w);
50         else
51             add_pair(u, v, w);
52     }
```

```

52     }
53 }
54
55 void work() {
56     puts(neg_cycle() ? "YES" : "N0");
57 }
58
59 void clear() {
60     e_ptr = 2;
61     memset(head, 0, sizeof(head));
62 }
63
64 int main() {
65     int T;
66     scanf("%d", &T);
67     while(T--) {
68         init(); work(); clear();
69     }
70     return 0;
71 }

```

## 2.2 Dijkstra

### 2.2.1 Using std::priority\_queue

GraphTheory/Dijkstra-STL.cpp

```

1  #include <bits/stdc++.h>
2  #define fst first
3  #define snd second
4  using namespace std;
5
6  typedef pair<int, int> HeapNode;
7
8  struct Edge {
9      int v, len, next;
10 };
11
12 const int MAXN = 1e4, MAXM = 5e5, INF = 0x3f3f3f3f;
13 int n, m, s, e_ptr = 1, head[MAXN+10]; Edge E[(MAXM+10)<<1];
14 int dist[MAXN+10], done[MAXN+10];
15
16 void add_edge(int u, int v, int len) {
17     E[++e_ptr] = (Edge) { v, len, head[u] }; head[u] = e_ptr;
18 }
19
20 void add_pair(int u, int v, int len) {
21     add_edge(u, v, len); add_edge(v, u, len);
22 }
23
24 void Dijkstra() {
25     priority_queue<HeapNode, vector<HeapNode>, greater<HeapNode> > pq;
26     memset(done, 0, sizeof(done));
27     memset(dist, 0x3f, sizeof(dist));
28     dist[s] = 0; pq.push(make_pair(dist[s], s));
29     while(!pq.empty()) {

```

```

30     HeapNode p = pq.top(); pq.pop();
31     int u = p.snd;
32     if(done[u]) continue;
33     done[u] = true;
34     for(int j=head[u]; j; j=E[j].next) {
35         int v = E[j].v, len = E[j].len;
36         if(dist[v] > dist[u] + len) {
37             dist[v] = dist[u] + len;
38             pq.push(make_pair(dist[v], v));
39         }
40     }
41 }
42 }
43
44 inline int readint() {
45     int f=1, r=0; char c=getchar();
46     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
47     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
48     return f*r;
49 }
50
51 int main() {
52     int u, v, w;
53     n = readint(); m = readint(); s = readint();
54     for(int i = 1; i <= m; i++) {
55         u = readint(); v = readint(); w = readint();
56         add_edge(u, v, w);
57     }
58     Dijkstra();
59     for(int i = 1; i <= n; i++) {
60         if(dist[i] < INF)
61             printf("%d ", dist[i]);
62         else
63             printf("%d ", INT_MAX);
64     }
65     return 0;
66 }

```

## 2.2.2 Using `__gnu_pbds::priority_queue`

使用了扩展库 `pb_ds` 中的配对堆，自带修改堆内元素操作，速度更快。仅在允许使用 STL 扩展时才使用。

### GraphTheory/Dijkstra-pb\_ds.cpp

```

1 #include <bits/stdc++.h>
2 #include <bits/extc++.h>
3 #define fst first
4 #define snd second
5 using namespace std;
6
7 typedef pair<int, int> HeapNode;
8 typedef __gnu_pbds::priority_queue<HeapNode, greater<HeapNode>,
9     __gnu_pbds::pairing_heap_tag> PairingHeap;
10
11 struct Edge {

```

```

12     int v, len, next;
13 };
14
15 const int MAXN = 1e4, MAXM = 5e5, INF = 0x3f3f3f3f;
16 int n, m, s, e_ptr = 1, head[MAXN+10]; Edge E[(MAXM+10)<<1];
17 int dist[MAXN+10]; PairingHeap pq; PairingHeap::point_iterator it[MAXN+10];
18
19 void add_edge(int u, int v, int len) {
20     E[++e_ptr] = (Edge) { v, len, head[u] }; head[u] = e_ptr;
21 }
22
23 void add_pair(int u, int v, int len) {
24     add_edge(u, v, len); add_edge(v, u, len);
25 }
26
27 void Dijkstra() {
28     memset(it, 0, sizeof(it));
29     memset(dist, 0x3f, sizeof(dist));
30     dist[s] = 0; it[s] = pq.push(make_pair(dist[s], s));
31     while(!pq.empty()) {
32         HeapNode p = pq.top(); pq.pop();
33         int u = p.snd;
34         for(int j=head[u]; j; j=E[j].next) {
35             int v = E[j].v, len = E[j].len;
36             if(dist[v] > dist[u] + len) {
37                 dist[v] = dist[u] + len;
38                 if(it[v] == NULL)
39                     it[v] = pq.push(make_pair(dist[v], v));
40                 else
41                     pq.modify(it[v], make_pair(dist[v], v));
42             }
43         }
44     }
45 }
46
47 inline int readint() {
48     int f=1, r=0; char c=getchar();
49     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
50     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
51     return f*r;
52 }
53
54 int main() {
55     int u, v, w;
56     n = readint(); m = readint(); s = readint();
57     for(int i = 1; i <= m; i++) {
58         u = readint(); v = readint(); w = readint();
59         add_edge(u, v, w);
60     }
61     Dijkstra();
62     for(int i = 1; i <= n; i++) {
63         if(dist[i] < INF)
64             printf("%d ", dist[i]);
65         else
66             printf("%d ", INT_MAX);
67     }
68     return 0;

```

## 3 网络流

### 3.1 最大流

### 3.2 Dinic

NetworkFlow/Dinic.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct Edge {
5     int v, flow, cap, next;
6 };
7
8 const int MAXN = 1e4, MAXM = 1e5, INF = 0x3f3f3f3f;
9 int n, m, s, t, e_ptr = 1, head[MAXN+10]; Edge E[(MAXM+10)<<1];
10 int d[MAXN+10], cur[MAXN+10];
11
12 void AddEdge(int u, int v, int cap) {
13     E[++e_ptr] = (Edge) { v, 0, cap, head[u] }; head[u] = e_ptr;
14     E[++e_ptr] = (Edge) { u, 0, 0, head[v] }; head[v] = e_ptr;
15 }
16
17 bool BFS() {
18     queue<int> Q;
19     memset(d, 0xff, sizeof(d));
20     Q.push(s); d[s] = 0;
21     while(!Q.empty()) {
22         int u = Q.front(); Q.pop();
23         for(int j=head[u]; j; j=E[j].next) {
24             int v = E[j].v, f = E[j].flow, c = E[j].cap;
25             if(f < c && d[v] == -1) {
26                 d[v] = d[u] + 1;
27                 if(v == t) return true;
28                 else Q.push(v);
29             }
30         }
31     }
32     return false;
33 }
34
35 int DFS(int u, int flow) {
36     if(u == t || flow == 0) return flow; // !!!!!
37     int res = flow;
38     for(int &j=cur[u]; j; j=E[j].next) { // !!!!!
39         int v = E[j].v, f = E[j].flow, c = E[j].cap;
40         if(f < c && d[v] == d[u] + 1) {
41             int aug = DFS(v, min(res, c-f));
42             E[j].flow += aug; E[j^1].flow -= aug;
43             res -= aug;
44             if(res == 0) break; // !!!!!
45         }
46     }
47     return res;
48 }

```

```

46     }
47     return flow - res;
48 }
49
50 int Dinic() {
51     int MaxFlow = 0, CurFlow = 0;
52     while(BFS()) {
53         memcpy(cur, head, sizeof(head));
54         while((CurFlow = DFS(s, INF)))
55             MaxFlow += CurFlow;
56     }
57     return MaxFlow;
58 }
59
60 int main() {
61     int u, v, c;
62     scanf("%d%d%d", &n, &m, &s, &t);
63     for(int i = 1; i <= m; i++) {
64         scanf("%d%d%d", &u, &v, &c);
65         AddEdge(u, v, c);
66     }
67     printf("%d", Dinic());
68     return 0;
69 }

```

### 3.3 最小费用最大流

#### 3.3.1 zkw 费用流

NetworkFlow/zkw.cpp

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  typedef long long int64;
5  struct Edge {
6      int u, v;
7      int64 flow, cap, cost;
8      int next;
9  };
10
11 const int MAXN = 5e3, MAXM = 5e4;
12 const int64 LL_INF = 0x3f3f3f3f3f3f3fLL;
13 int n, m, s, t, e_ptr = 1, head[MAXN+10]; Edge E[(MAXM+10)<<1]; // ** E[(MAXM+10)<<1] **
14 int64 MaxFlow, MinCost, dist[MAXN+10], inq[MAXN+10], vis[MAXN+10];
15
16 void add_edge(int u, int v, int64 cap, int64 cost) {
17     E[++e_ptr] = (Edge) { u, v, 0, cap, cost, head[u] }; head[u] = e_ptr;
18     E[++e_ptr] = (Edge) { v, u, 0, 0, -cost, head[v] }; head[v] = e_ptr;
19 }
20
21 bool spfa() {
22     queue<int> Q;
23     memset(dist, 0x3f, sizeof(dist));
24     Q.push(t); dist[t] = 0; inq[t] = true;
25     while(!Q.empty()) {

```



```

26     int u = Q.front(); Q.pop(); inq[u] = false;
27     for(int j=head[u]; j; j=E[j].next) {
28         int v = E[j].v; int64 f = E[j^1].flow, c = E[j^1].cap, len = E[j^1].cost;
29         if(f < c && dist[v] > dist[u] + len) {
30             dist[v] = dist[u] + len;
31             if(!inq[v]) {
32                 inq[v] = true; Q.push(v);
33             }
34         }
35     }
36 }
37 return dist[s] != LL_INF;
38 }
39
40 int64 dfs(int u, int64 flow) {
41     if(u == t || flow == 0) return flow;
42     vis[u] = true;
43     int64 res = flow;
44     for(int j=head[u]; j; j=E[j].next) {
45         int v = E[j].v; int64 f = E[j].flow, c = E[j].cap, len = E[j].cost;
46         if(f < c && !vis[v] && dist[v] == dist[u] - len) {
47             int64 aug = dfs(v, min(res, c-f));
48             E[j].flow += aug; E[j^1].flow -= aug;
49             res -= aug;
50             if(res == 0LL) break;
51         }
52     }
53     return flow - res;
54 }
55
56 void zkw() {
57     int64 CurFlow = 0LL;
58     while(spfa()) {
59         while(memset(vis, 0, sizeof(vis)),
60             CurFlow = dfs(s, LL_INF)) {
61             MaxFlow += CurFlow;
62             MinCost += dist[s] * CurFlow;
63         }
64     }
65 }
66
67 template<typename T>
68 inline void readint(T &x) {
69     T f=1, r=0; char c=getchar();
70     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
71     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
72     x = f*r;
73 }
74
75 int main() {
76     int u, v; int64 w, c;
77     readint(n); readint(m); readint(s); readint(t);
78     for(int i = 1; i <= m; i++) {
79         readint(u); readint(v); readint(w); readint(c);
80         add_edge(u, v, w, c);
81     }
82     zkw();

```

```

83     printf("%lld %lld", MaxFlow, MinCost);
84     return 0;
85 }

```

### 3.3.2 Primal Dual

NetworkFlow/PrimalDual.cpp

```

1  #include <bits/stdc++.h>
2  #include <bits/extc++.h>
3  #define fst first
4  #define snd second
5
6  using namespace std;
7
8  typedef long long int64;
9  typedef pair<int64, int> HeapNode;
10 typedef __gnu_pbds::priority_queue<HeapNode, greater<HeapNode>,
11     __gnu_pbds::pairing_heap_tag> PairingHeap;
12 const int MAXN = 5e3, MAXM = 5e4;
13 const int64 LL_INF = 0x3f3f3f3f3f3f3fLL;
14
15 struct Edge {
16     int u, v;
17     int64 flow, cap, cost;
18     int next;
19 };
20
21 int n, m, s, t, e_ptr = 1, head[MAXN+10]; Edge E[(MAXM+10)<<1];
22 int64 MaxFlow, MinCost, delta, dist[MAXN+10], vis[MAXN+10], inq[MAXN+10];
23 PairingHeap::point_iterator it[MAXN+10];
24
25 void add_edge(int u, int v, int64 cap, int64 cost) {
26     E[++e_ptr] = (Edge) { u, v, 0, cap, cost, head[u] }; head[u] = e_ptr;
27     E[++e_ptr] = (Edge) { v, u, 0, 0, -cost, head[v] }; head[v] = e_ptr;
28 }
29
30 void Reduce() {
31     for(int i = 2; i <= e_ptr; i++)
32         E[i].cost -= (dist[E[i].u] - dist[E[i].v]);
33     delta += dist[s];
34 }
35
36 bool BellmanFord() {
37     queue<int> Q;
38     memset(dist, 0x3f, sizeof(dist));
39     Q.push(t); dist[t] = 0; inq[t] = true;
40     while(!Q.empty()) {
41         int u = Q.front(); Q.pop(); inq[u] = false;
42         for(int j=head[u]; j; j=E[j].next) {
43             int v = E[j].v; int64 f = E[j^1].flow, c = E[j^1].cap, len = E[j^1].cost;
44             if(f < c && dist[v] > dist[u] + len) {
45                 dist[v] = dist[u] + len;
46                 if(!inq[v]) {
47                     inq[v] = true; Q.push(v);
48                 }

```

```

49     }
50 }
51 }
52 return dist[s] != LL_INF;
53 }
54
55 bool Dijkstra() {
56     PairingHeap pq;
57     memset(dist, 0x3f, sizeof(dist));
58     memset(it, 0, sizeof(it));
59     dist[t] = 0; it[t] = pq.push(make_pair(dist[t], t));
60     while(!pq.empty()) {
61         HeapNode t = pq.top(); pq.pop();
62         int u = t.snd;
63         for(int j=head[u]; j; j=E[j].next) {
64             int v = E[j].v; int64 f = E[j].flow, c = E[j].cap, len = E[j].cost;
65             if(f < c && dist[v] > dist[u] + len) {
66                 dist[v] = dist[u] + len;
67                 if(it[v] == NULL)
68                     it[v] = pq.push(make_pair(dist[v], v));
69                 else
70                     pq.modify(it[v], make_pair(dist[v], v));
71             }
72         }
73     }
74     return dist[s] != LL_INF;
75 }
76
77 int64 dfs(int u, int64 flow) {
78     if(u == t || flow == 0) return flow;
79     vis[u] = true;
80     int64 res = flow;
81     for(int j=head[u]; j; j=E[j].next) {
82         int v = E[j].v; int64 f = E[j].flow, c = E[j].cap, len = E[j].cost;
83         if(f < c && !vis[v] && len == 0) {
84             int64 aug = dfs(v, min(res, c-f));
85             E[j].flow += aug; E[j^1].flow -= aug;
86             res -= aug;
87             if(res == 0) break;
88         }
89     }
90     return flow - res;
91 }
92
93 void Augment() {
94     int64 CurFlow = 0;
95     while( memset(vis, 0, sizeof(vis)),
96           (CurFlow = dfs(s, LL_INF)) ) {
97         MaxFlow += CurFlow;
98         MinCost += delta * CurFlow;
99     }
100 }
101
102 void PrimalDual() {
103     if(!BellmanFord()) return;
104     Reduce(); Augment();
105     while(Dijkstra()) {

```

```

106     Reduce(); Augment();
107 }
108 }
109
110 template<typename T>
111 inline void readint(T &x) {
112     T f=1, r=0; char c=getchar();
113     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
114     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
115     x = f*r;
116 }
117
118 int main() {
119     int u, v; int64 w, c;
120     readint(n); readint(m); readint(s); readint(t);
121     for(int i = 1; i <= m; i++) {
122         readint(u); readint(v); readint(w); readint(c);
123         add_edge(u, v, w, c);
124     }
125     PrimalDual();
126     printf("%lld %lld", MaxFlow, MinCost);
127     return 0;
128 }

```

## 4 树

### 4.1 倍增 LCA

Tree/DoublingLCA.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct Edge { int v, next; };
5
6 const int MAXN = 1e6, LOG = 20;
7 int n, q, s, e_ptr = 1, head[MAXN+10]; Edge E[(MAXN+10)<<1];
8 int dep[MAXN+10], anc[MAXN+10][LOG+1];
9
10 void add_edge(int u, int v) { E[++e_ptr] = (Edge) { v, head[u] }; head[u] = e_ptr; }
11 void add_pair(int u, int v) { add_edge(u, v); add_edge(v, u); }
12
13 void dfs(int u) {
14     for(int i = 1; i <= LOG; i++)
15         anc[u][i] = anc[anc[u][i-1]][i-1];
16     for(int j=head[u]; j; j=E[j].next) {
17         int v = E[j].v;
18         if(v == anc[u][0]) continue;
19         anc[v][0] = u; dep[v] = dep[u] + 1;
20         dfs(v);
21     }
22 }
23
24 int lca(int u, int v) {
25     if(dep[u] < dep[v]) swap(u, v);
26     for(int i = LOG; i >= 0; i--)
27         if(dep[anc[u][i]] >= dep[v])
28             u = anc[u][i];
29     if(u == v) return u;
30     for(int i = LOG; i >= 0; i--)
31         if(anc[u][i] != anc[v][i])
32             u = anc[u][i], v = anc[v][i];
33     u = anc[u][0], v = anc[v][0];
34     return u;
35 }
36
37 inline int readint() {
38     int f=1, r=0; char c=getchar();
39     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
40     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
41     return f*r;
42 }
43
44 int main() {
45     int u, v;
46     n = readint(); q = readint(); s = readint();
47     for(int i = 1; i <= n-1; i++) {
48         u = readint(); v = readint();
49         add_pair(u, v);
50     }
51     dep[s] = 1; dfs(s);
```

```

52     while(q--) {
53         u = readint(); v = readint();
54         printf("%d\n", lca(u, v));
55     }
56     return 0;
57 }

```

## 4.2 欧拉序列求 LCA

Tree/EulerTourLCA.cpp

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  const int MAXN = 1e6;
5
6  struct Edge {
7      int v, next;
8  };
9
10 int n, q, s, e_ptr = 1, dfs_clock, head[MAXN+10]; Edge E[(MAXN+10)<<1];
11 int dfn[MAXN+10], dfs_seq[MAXN+10], idx[MAXN+10], euler_seq[(MAXN+10)<<1], st[(MAXN+10)<<1][22];
12 /*
13     dfn: dfs-clock of vertex u
14     idx: the index of vertex u in euler-tour sequence
15     dfs_seq: the dfs sequence
16 */
17
18 void add_edge(int u, int v) {
19     E[++e_ptr] = (Edge) { v, head[u] }; head[u] = e_ptr;
20 }
21
22 void add_pair(int u, int v) {
23     add_edge(u, v); add_edge(v, u);
24 }
25
26 inline int readint() {
27     int f=1, r=0; char c=getchar();
28     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
29     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
30     return f*r;
31 }
32
33 void dfs(int u, int fa) {
34     euler_seq[++euler_seq[0]] = dfn[u] = ++dfs_clock;
35     idx[u] = euler_seq[0]; dfs_seq[dfs_clock] = u;
36     for(int j=head[u]; j; j=E[j].next) {
37         int v = E[j].v;
38         if(v == fa) continue;
39         dfs(v, u);
40         euler_seq[++euler_seq[0]] = dfn[u];
41     }
42 }
43
44 void init_lca() {
45     memset(st, 0x3f, sizeof(st));

```

```

46     for(int i = 1; i <= euler_seq[0]; i++)
47         st[i][0] = euler_seq[i];
48     for(int j = 1; j <= 21; j++)
49         for(int i = 1; i <= euler_seq[0] - (1<<j) + 1; i++) // bounds of sparse-table!
50             st[i][j] = min(st[i][j-1], st[i + (1<<(j-1))][j-1]);
51 }
52
53 int query(int l, int r) {
54     if(l > r) swap(l, r);
55     int j;
56     for(j = 0; (1<<(j+1)) <= (r-l+1); j++);
57     return min(st[l][j], st[r - (1<<j) + 1][j]);
58 }
59
60 int lca(int u, int v) {
61     return dfs_seq[query(idx[u], idx[v])];
62 }
63
64 int main() {
65     int u, v;
66     n = readint(); q = readint(); s = readint();
67     for(int i = 1; i <= n-1; i++) {
68         u = readint(); v = readint();
69         add_pair(u, v);
70     }
71     dfs(s, -1); init_lca();
72     while(q--) {
73         u = readint(); v = readint();
74         printf("%d\n", lca(u, v));
75     }
76     return 0;
77 }

```

### 4.3 树链剖分

Tree/HLD.cpp

```

1 // call Dfs1(1) and Dfs2(1, 1)
2 const int MAXN = 1e5;
3 int dfs_clock, Fa[MAXN+10], Son[MAXN+10], Sz[MAXN+10],
4     Dep[MAXN+10], Top[MAXN+10], Dfn[MAXN+10];
5
6 void Dfs1(int u) { // Fa Son Sz Dep
7     int maxsz = 0; Sz[u] = 1;
8     for(int j=head[u]; j; j=E[j].next) {
9         int v = E[j].v;
10        if(v == Fa[u]) continue;
11        Fa[v] = u; Dep[v] = Dep[u] + 1; // !
12        Dfs1(v); Sz[u] += Sz[v];
13        if(Sz[v] > maxsz) {
14            maxsz = Sz[v];
15            Son[u] = v;
16        }
17    }
18 }
19

```

```

20 void Dfs2(int u, int anc) { // Top Dfn
21     Dfn[u] = ++dfs_clock; Top[u] = anc;
22     if(Son[u]) Dfs2(Son[u], anc);
23     for(int j=head[u]; j; j=E[j].next) {
24         int v = E[j].v;
25         if(v == Fa[u] || v == Son[u]) continue;
26         Dfs2(v, v);
27     }
28 }
29
30 int LCA(int u, int v) {
31     while(Top[u] != Top[v]) {
32         if(Dep[Top[u]] < Dep[Top[v]]) swap(u, v);
33         u = Fa[Top[u]];
34     }
35     if(Dep[u] > Dep[v]) swap(u, v);
36     return u;
37 }
38
39 int HLDQuery(int u, int v) {
40     int ret = -INF;
41     while(Top[u] != Top[v]) {
42         if(Dep[Top[u]] < Dep[Top[v]]) swap(u, v);
43         ret = max(ret, st_query(1, 1, n, Dfn[Top[u]], Dfn[u]));
44         u = Fa[Top[u]];
45     }
46     if(Dep[u] > Dep[v]) swap(u, v);
47     ret = max(ret, st_query(1, 1, n, Dfn[u], Dfn[v]));
48     return ret;
49 }

```

## 4.4 点分治

Tree/DivConquerOnVertex.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct Edge { int v, len, next; };
5
6 const int MAXN = 1e4, MAXK = 1e7;
7
8 int n, q, k, e_ptr = 1, head[MAXN+10]; Edge E[(MAXN+10)<<1];
9 int ans, root, tosz, vis[MAXN+10], f[MAXN+10], sz[MAXN+10],
10    dist[MAXN+10], mp[MAXK+10], pths[MAXN+10];
11
12 void add_edge(int u, int v, int len) {
13     E[++e_ptr] = (Edge) { v, len, head[u] }; head[u] = e_ptr;
14 }
15
16 void add_pair(int u, int v, int len) {
17     add_edge(u, v, len); add_edge(v, u, len);
18 }
19
20 inline int readint() {
21     int f=1, r=0; char c=getchar();

```



```

22 while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
23 while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
24 return f*r;
25 }
26
27 void get_centroid(int u, int fa) {
28     f[u] = 0, sz[u] = 1;
29     for(int j=head[u]; j; j=E[j].next) {
30         int v = E[j].v;
31         if(vis[v] || v == fa) continue;
32         get_centroid(v, u); sz[u] += sz[v];
33         if(sz[v] > f[u]) f[u] = sz[v];
34     }
35     f[u] = max(f[u], totsiz - sz[u]);
36     if(f[u] < f[root]) root = u;
37 }
38
39 void count_nd(int u, int fa) {
40     sz[u] = 1;
41     for(int j=head[u]; j; j=E[j].next) {
42         int v = E[j].v;
43         if(vis[v] || v == fa) continue;
44         count_nd(v, u); sz[u] += sz[v];
45     }
46     return;
47 }
48
49 void get_dist(int u, int fa) {
50     pths[++pths[0]] = dist[u];
51     for(int j=head[u]; j; j=E[j].next) {
52         int v = E[j].v, len = E[j].len;
53         if(vis[v] || v == fa) continue;
54         dist[v] = dist[u] + len;
55         get_dist(v, u);
56     }
57 }
58
59 int calc(int u, int w) {
60     int ret = 0;
61     dist[u] = w; pths[0] = 0;
62     get_dist(u, -1);
63     sort(pths + 1, pths + pths[0] + 1);
64     for(int i = 1; i <= pths[0]; i++) {
65         if(pths[i] <= k)
66             ret += mp[k - pths[i]];
67         mp[pths[i]]++;
68     }
69     for(int i = 1; i <= pths[0]; i++)
70         mp[pths[i]]--;
71     return ret;
72 }
73
74 void solve(int u) {
75     ans += calc(u, 0);
76     vis[u] = true;
77     for(int j=head[u]; j; j=E[j].next) {
78         int v = E[j].v, len = E[j].len;

```

```

79     if(vis[v]) continue;
80     ans -= calc(v, len);
81     count_nd(v, -1); totsiz = sz[v];
82     root = 0;
83     get_centroid(v, -1);
84     solve(v);
85 }
86 }
87
88 int main() {
89     int a, b, c;
90     n = readint(); q = readint();
91     for(int i = 1; i <= n-1; i++) {
92         a = readint(); b = readint(); c = readint();
93         add_pair(a, b, c);
94     }
95     while(q--) {
96         k = readint();
97         f[root=0] = n;
98         memset(vis, 0, sizeof(vis));
99         get_centroid(1, -1);
100        ans = 0;
101        solve(root);
102        puts(ans ? "AYE" : "NAY");
103    }
104 }

```

## 5 单调数据结构

### 5.1 单调队列 (滑动窗口)

Monotonic/SlidingWindow.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 1e6;
5 int n, k, Hd, Tl, A[MAXN+10], Q[MAXN+10];
6
7 void SlideMin() {
8     Hd = 1, Tl = 0;
9     for(int i = 1; i <= k; i++) {
10         while(Hd <= Tl && A[Q[Tl]] >= A[i]) Tl--;
11         Q[++Tl] = i;
12     }
13     printf("%d ", A[Q[Hd]]);
14     for(int i = k+1; i <= n; i++) {
15         while(Hd <= Tl && Q[Hd] < i-k+1) Hd++;
16         while(Hd <= Tl && A[Q[Tl]] >= A[i]) Tl--;
17         Q[++Tl] = i;
18         printf("%d ", A[Q[Hd]]);
19     }
20 }
21
22 void SlideMax() {
23     Hd = 1, Tl = 0;
24     for(int i = 1; i <= k; i++) {
25         while(Hd <= Tl && A[Q[Tl]] <= A[i]) Tl--;
26         Q[++Tl] = i;
27     }
28     printf("%d ", A[Q[Hd]]);
29     for(int i = k+1; i <= n; i++) {
30         while(Hd <= Tl && Q[Hd] < i-k+1) Hd++;
31         while(Hd <= Tl && A[Q[Tl]] <= A[i]) Tl--;
32         Q[++Tl] = i;
33         printf("%d ", A[Q[Hd]]);
34     }
35 }
36
37 inline int readint() {
38     int f=1, r=0; char c=getchar();
39     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
40     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
41     return f*r;
42 }
43
44 int main() {
45     n = readint(); k = readint();
46     for(int i = 1; i <= n; i++) A[i] = readint();
47     SlideMin(); putchar(10); SlideMax();
48     return 0;
49 }
```

## 5.2 单调栈

[JSOI2008] 最大数 注意：下标从栈底到顶递增，而值则递减。（一个数字前面的比它小的数肯定不会成为询问的答案）还有：可能  $L = 0$ ，此时 `lower_bound` 传入空区间，返回  $L$ ！所以必须特判！

Monotonic/MaxNumber.cpp

```
1  /*
2   * [JSOI2008]最大数
3   */
4
5  #include <bits/stdc++.h>
6  using namespace std;
7
8  const int MAXN = 2e5;
9  int q, mod, n, last, a[MAXN+10], s[MAXN+10];
10
11 int main() {
12     char op; int x;
13     cin.sync_with_stdio(false);
14     cin.tie(NULL);
15     cin >> q >> mod;
16     while(q--) {
17         cin >> op >> x;
18         switch(op) {
19             case 'Q':
20                 if(x == 0)
21                     cout << (last = 0) << endl;
22                 else
23                     cout << (last = a[*lower_bound(s + 1, s + s[0] + 1, n-x+1)]) << endl;
24                 break;
25             case 'A':
26                 x = (x + last) % mod;
27                 while(s[0] && a[s[s[0]]] < x) --s[0];
28                 s[++s[0]] = ++n; a[n] = x;
29                 break;
30         }
31     }
32 }
```

## 6 线段树

### 6.1 Lazy-Tag

**Solution:** 暴力拆开式子后（或者根据《重难点手册》的结论），发现要维护区间的  $\sum x_i, \sum y_i, \sum x_i y_i, \sum x_i^2$ ，同时要支持区间加和区间设置为  $S + i$  和  $T_j$ 。在线段树上维护  $add_s, add_t, set_s, set_t$ ，然后推一推式子找出 Lazy-tag 更新主 Tag 的公式即可。几个坑点：

1.  $add_s, add_t$  标记在下推的时候，不能赋值，要累加!!! 累加!!! 累加!!!
2. 只有  $set_s, set_t$  用  $-\infty$  来标记不存在， $add_s, add_t$  必须用 0 标记不存在！不然是给自己找麻烦，多出来各种特判!!!

SegTree/CorrelationAnalyse.cpp

```
1  /*
2      [SD0I2017] 相关分析
3      Coded by panda_2134
4  */
5  #include <bits/stdc++.h>
6  #define LC(o) ((o)*2)
7  #define RC(o) ((o)*2+1)
8  #define Mid(x, y) (((x) + (y)) / 2)
9  using namespace std;
10
11 const double eps = 1e-6, NONE = -1e6;
12 const int MAXN = 1e5;
13
14 int dcmp(double x) {
15     return fabs(x) < eps ? 0 : (x > 0 ? 1 : -1);
16 }
17
18 struct Info {
19     double x, y, xy, x2;
20     Info() { x = y = xy = x2 = .0; }
21     Info(double a, double b, double c, double d):
22         x(a), y(b), xy(c), x2(d) {}
23     Info operator+(const Info &rhs) const {
24         return Info(x + rhs.x, y + rhs.y, xy + rhs.xy, x2 + rhs.x2);
25     }
26     Info operator+=(const Info &rhs) { return *this = *this + rhs; }
27 };
28
29 struct Node {
30     double x, y, xy, x2, add_s, add_t, set_s, set_t;
31     Node() {
32         x = y = xy = x2 = .0;
33         add_s = add_t = .0;
34         set_s = set_t = NONE;
35     }
36     void clear() { x = y = xy = x2 = .0; }
37 } nd[(MAXN+10)<<2];
38
39 int n, q;
40 double X[MAXN+10], Y[MAXN+10];
```

```

41
42 void Maintain(int o, double L, double R) {
43     if(dcmp(nd[o].set_s - NONE) == 0) {
44         // no set (if set_s exists, then set_t exists, and vice versa)
45         assert(dcmp(nd[o].set_t - NONE) == 0);
46         nd[o].clear();
47         if(L != R) {
48             nd[o].x = nd[LC(o)].x + nd[RC(o)].x;
49             nd[o].y = nd[LC(o)].y + nd[RC(o)].y;
50             nd[o].xy = nd[LC(o)].xy + nd[RC(o)].xy;
51             nd[o].x2 = nd[LC(o)].x2 + nd[RC(o)].x2;
52         }
53     } else {
54         nd[o].x2 = (R-L+1) * nd[o].set_s * nd[o].set_s
55             + R * (R+1) * (2*R+1) / 6 - L * (L-1) * (2*L-1) / 6
56             + nd[o].set_s * (L+R) * (R-L+1);
57         nd[o].xy = (R-L+1) * nd[o].set_s * nd[o].set_t
58             + (nd[o].set_s + nd[o].set_t) * (L+R) * (R-L+1) / 2
59             + R * (R+1) * (2*R+1) / 6 - L * (L-1) * (2*L-1) / 6;
60         nd[o].x = (R-L+1) * nd[o].set_s + (L+R) * (R-L+1) / 2;
61         nd[o].y = (R-L+1) * nd[o].set_t + (L+R) * (R-L+1) / 2;
62     }
63     nd[o].x2 += 2 * nd[o].add_s * nd[o].x + (R-L+1) * nd[o].add_s * nd[o].add_s;
64     nd[o].xy += nd[o].add_t * nd[o].x
65         + nd[o].add_s * nd[o].y + (R-L+1) * nd[o].add_s * nd[o].add_t;
66     nd[o].x += (R-L+1) * nd[o].add_s;
67     nd[o].y += (R-L+1) * nd[o].add_t; // update last
68 }
69
70 void Pushdown(int o) {
71     if(dcmp(nd[o].set_s - NONE) != 0) { // mark exist
72         assert(dcmp(nd[o].set_t - NONE) != 0);
73         nd[LC(o)].set_s = nd[RC(o)].set_s = nd[o].set_s;
74         nd[LC(o)].set_t = nd[RC(o)].set_t = nd[o].set_t;
75         nd[LC(o)].add_s = nd[RC(o)].add_s = .0;
76         nd[LC(o)].add_t = nd[RC(o)].add_t = .0;
77         nd[o].set_s = NONE;
78         nd[o].set_t = NONE;
79     }
80     if(dcmp(nd[o].add_s) != 0) {
81         nd[LC(o)].add_s += nd[o].add_s; //add 标记要累加!!!!!!!!!!!!
82         nd[RC(o)].add_s += nd[o].add_s;
83         nd[o].add_s = .0;
84     }
85     if(dcmp(nd[o].add_t) != 0) {
86         nd[LC(o)].add_t += nd[o].add_t;
87         nd[RC(o)].add_t += nd[o].add_t;
88         nd[o].add_t = .0;
89     }
90 }
91
92 Info Query(int o, int L, int R, int qL, int qR) {
93     Maintain(o, L, R);
94     if(qL <= L && R <= qR)
95         return Info(nd[o].x, nd[o].y, nd[o].xy, nd[o].x2);
96     else {
97         Info ret;

```

```

98     Pushdown(o);
99     if(qL <= Mid(L, R)) ret += Query(LC(o), L, Mid(L, R), qL, qR);
100    else Maintain(LC(o), L, Mid(L, R));
101    if(qR >= Mid(L, R)+1) ret += Query(RC(o), Mid(L, R)+1, R, qL, qR);
102    else Maintain(RC(o), Mid(L, R)+1, R);
103    return ret;
104 }
105 }
106
107 void BuildTree(int o, int L, int R) {
108     if(L == R) {
109         nd[o].add_s = X[L];
110         nd[o].add_t = Y[L];
111     } else {
112         BuildTree(LC(o), L, Mid(L, R));
113         BuildTree(RC(o), Mid(L, R)+1, R);
114     }
115     Maintain(o, L, R);
116 }
117
118 void Add(int o, int L, int R, int qL, int qR, double S, double T) {
119     if(qL <= L && R <= qR) {
120         nd[o].add_s += S;
121         nd[o].add_t += T;
122     } else {
123         Pushdown(o);
124         if(qL <= Mid(L, R)) Add(LC(o), L, Mid(L, R), qL, qR, S, T);
125         else Maintain(LC(o), L, Mid(L, R));
126         if(qR >= Mid(L, R)+1) Add(RC(o), Mid(L, R)+1, R, qL, qR, S, T);
127         else Maintain(RC(o), Mid(L, R)+1, R);
128     }
129     Maintain(o, L, R);
130 }
131
132 void Set(int o, int L, int R, int qL, int qR, double S, double T) {
133     if(qL <= L && R <= qR) {
134         nd[o].add_s = nd[o].add_t = .0; // override 'add' mark
135         nd[o].set_s = S;
136         nd[o].set_t = T;
137     } else {
138         Pushdown(o);
139         if(qL <= Mid(L, R)) Set(LC(o), L, Mid(L, R), qL, qR, S, T);
140         else Maintain(LC(o), L, Mid(L, R));
141         if(qR >= Mid(L, R)+1) Set(RC(o), Mid(L, R)+1, R, qL, qR, S, T);
142         else Maintain(RC(o), Mid(L, R)+1, R);
143     }
144     Maintain(o, L, R);
145 }
146
147 void init() {
148     scanf("%d", &n, &q);
149     for(int i = 1; i <= n; i++)
150         scanf("%lf", &X[i]);
151     for(int i = 1; i <= n; i++)
152         scanf("%lf", &Y[i]);
153     BuildTree(1, 1, n);
154 }

```

```

155
156 void work() {
157     int op, L, R; double S, T;
158     Info res;
159     while(q-->) {
160         scanf("%d", &op);
161         switch(op) {
162             case 1:
163                 scanf("%d%d", &L, &R);
164                 res = Query(1, 1, n, L, R);
165                 printf("%.12lf\n",
166                     (res.xy - res.x * res.y / (R-L+1)) / (res.x2 - res.x * res.x / (R-L+1)));
167                 break;
168             case 2:
169                 scanf("%d%d%lf%lf", &L, &R, &S, &T);
170                 Add(1, 1, n, L, R, S, T);
171                 break;
172             case 3:
173                 scanf("%d%d%lf%lf", &L, &R, &S, &T);
174                 Set(1, 1, n, L, R, S, T);
175                 break;
176         }
177     }
178 }
179
180 int main() {
181     init(); work();
182     return 0;
183 }

```

## 6.2 动态开点线段树

[P3380] 二逼平衡树 树状数组套动态开点线段树。

线段树一般都不写指针的，容易错……

SegTree/2BBalancedTree.cpp

```

1 #include <bits/stdc++.h>
2 #define Mid(x, y) (((x)+(y)) >> 1)
3 using namespace std;
4
5 const int MAXN = 5e4, NOT_FOUND = 2147483647;
6
7 struct Query {
8     int type, a, b, c;
9 } qry[MAXN+10];
10
11 int n, q, cnt, lc[MAXN*300], rc[MAXN*300], sumv[MAXN*300];
12 int rt[MAXN+10], w[MAXN+10], nums[(MAXN+10)<<1];
13 // nums 要开成所有数字的种类的大小！或者直接开输入的 4 倍！第二次错了！
14
15 void maintain(int o, int L, int R) {
16     if(L != R)
17         sumv[o] = sumv[lc[o]] + sumv[rc[o]];
18 }
19

```



```

20 void st_add(int &o, int L, int R, int p, int val) {
21     if(!o) o = ++cnt;
22     if(L == R) sumv[o] += val;
23     else {
24         if(p <= Mid(L, R))
25             st_add(lc[o], L, Mid(L, R), p, val);
26         else
27             st_add(rc[o], Mid(L, R)+1, R, p, val);
28         maintain(o, L, R);
29     }
30 }
31
32 int st_kth(vector<int> &o, vector<int> &his, int L, int R, int k) {
33     if(L == R)
34         return L;
35     else {
36         int lc_sum = 0;
37         for(int &x : o) lc_sum += sumv[lc[x]];
38         for(int &x : his) lc_sum -= sumv[lc[x]];
39         if(k <= lc_sum) {
40             for(int &x : o) x = lc[x];
41             for(int &x : his) x = lc[x];
42             return st_kth(o, his, L, Mid(L, R), k);
43         } else {
44             k -= lc_sum;
45             for(int &x : o) x = rc[x];
46             for(int &x : his) x = rc[x];
47             return st_kth(o, his, Mid(L, R)+1, R, k);
48         }
49     }
50 }
51
52 int st_sum(vector<int> &o, vector<int> &his, int L, int R, int qL, int qR) {
53     int ret = 0;
54     if(qL <= L && R <= qR) {
55         for(int &x : o) ret += sumv[x];
56         for(int &x : his) ret -= sumv[x];
57     } else {
58         vector<int> o2 = o, his2 = his;
59         if(qL <= Mid(L, R)) {
60             for(int &x : o) x = lc[x];
61             for(int &x : his) x = lc[x];
62             ret += st_sum(o, his, L, Mid(L, R), qL, qR);
63         }
64         o = o2, his = his2;
65         if(qR >= Mid(L, R)+1) {
66             for(int &x : o) x = rc[x];
67             for(int &x : his) x = rc[x];
68             ret += st_sum(o, his, Mid(L, R)+1, R, qL, qR);
69         }
70     }
71     return ret;
72 }
73
74 inline int lowbit(int x) { return x & (-x); }
75
76 inline void bit_sum(int p, vector<int> &o) {

```

```

77     while(p > 0) {
78         o.push_back(rt[p]);
79         p -= lowbit(p);
80     }
81 }
82
83 inline void bit_add(int p, int w, int val) {
84     while(p <= n) {
85         st_add(rt[p], 1, nums[0], w, val);
86         p += lowbit(p);
87     }
88 }
89
90 inline int kth(int L, int R, int k) {
91     vector<int> o, his;
92     bit_sum(R, o); bit_sum(L-1, his);
93     return st_kth(o, his, 1, nums[0], k);
94 }
95
96 inline int getrank(int L, int R, int val) {
97     vector<int> o, his;
98     bit_sum(R, o); bit_sum(L-1, his);
99     if(val != 1)
100         return st_sum(o, his, 1, nums[0], 1, val - 1) + 1;
101     else
102         return 1;
103 }
104
105 inline int count(int L, int R, int val) {
106     vector<int> o, his;
107     bit_sum(R, o); bit_sum(L-1, his);
108     return st_sum(o, his, 1, nums[0], val, val);
109 }
110
111 inline void modify(int p, int val) {
112     bit_add(p, w[p], -1);
113     w[p] = val;
114     bit_add(p, w[p], 1);
115 }
116
117 inline int pre(int L, int R, int val) {
118     int rk = getrank(L, R, val);
119     if(rk == 1) return -NOT_FOUND;
120     return kth(L, R, rk-1);
121 }
122
123 inline int suf(int L, int R, int val) {
124     int rk = getrank(L, R, val), cnt = count(L, R, val);
125     if(rk + cnt - 1 == R - L + 1) return NOT_FOUND;
126     return kth(L, R, rk + cnt);
127 }
128
129 inline int readint() {
130     int f=1, r=0; char c=getchar();
131     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
132     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
133     return f*r;

```

```

134 }
135
136 int main() {
137     int ans;
138     n = readint(); q = readint();
139     for(int i = 1; i <= n; i++) {
140         w[i] = readint();
141         nums[++nums[0]] = w[i];
142     }
143     for(int i = 1; i <= q; i++) {
144         qry[i].type = readint();
145         switch(qry[i].type) {
146             case 1: case 2: case 4: case 5:
147                 qry[i].a = readint(); qry[i].b = readint(); qry[i].c = readint();
148                 if(qry[i].type != 2) nums[++nums[0]] = qry[i].c;
149                 break;
150             case 3:
151                 qry[i].a = readint(); qry[i].b = readint();
152                 nums[++nums[0]] = qry[i].b;
153                 break;
154         }
155     }
156
157     sort(nums + 1, nums + nums[0] + 1);
158     nums[0] = unique(nums + 1, nums + nums[0] + 1) - &nums[1];
159
160     for(int i = 1; i <= n; i++) {
161         w[i] = lower_bound(nums + 1, nums + nums[0] + 1, w[i]) - nums;
162         bit_add(i, w[i], 1);
163     }
164
165     for(int i = 1; i <= q; i++) {
166         switch(qry[i].type) {
167             case 1: case 4: case 5:
168                 qry[i].c = lower_bound(nums + 1, nums + nums[0] + 1, qry[i].c) - nums;
169                 break;
170             case 3:
171                 qry[i].b = lower_bound(nums + 1, nums + nums[0] + 1, qry[i].b) - nums;
172                 break;
173         }
174     }
175     for(int i = 1; i <= q; i++) {
176         switch(qry[i].type) {
177             case 1:
178                 printf("%d\n", getrank(qry[i].a, qry[i].b, qry[i].c));
179                 break;
180             case 2:
181                 printf("%d\n", nums[kth(qry[i].a, qry[i].b, qry[i].c)]);
182                 break;
183             case 3:
184                 modify(qry[i].a, qry[i].b);
185                 break;
186             case 4:
187                 ans = pre(qry[i].a, qry[i].b, qry[i].c);
188                 if(ans != -NOT_FOUND) ans = nums[ans];
189                 printf("%d\n", ans);
190                 break;

```

```

191         case 5:
192             ans = suf(qry[i].a, qry[i].b, qry[i].c);
193             if(ans != NOT_FOUND) ans = nums[ans];
194             printf("%d\n", ans);
195             break;
196         }
197     }
198     return 0;
199 }

```

### 6.3 可持久化线段树

SegTree/LongtermSegtree.cpp

```

1 #include <bits/stdc++.h>
2 #define Mid(x, y) (((x) + (y)) >> 1)
3 using namespace std;
4
5 const int MAXN = 1e6, BKT = 4e7;
6
7 int n, q, cnt, ver, w[MAXN+10], rt[MAXN+10], lc[BKT], rc[BKT], v[BKT];
8
9 inline int readint() {
10     int f=1, r=0; char c=getchar();
11     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
12     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
13     return f*r;
14 }
15
16 void build_tree(int &o, int L, int R) {
17     o = ++cnt;
18     if(L == R) v[o] = w[L];
19     else {
20         build_tree(lc[o], L, Mid(L, R));
21         build_tree(rc[o], Mid(L, R)+1, R);
22     }
23 }
24
25 void modify(int &o, int his, int L, int R, int p, int val) {
26     o = ++cnt;
27     if(L == R) v[o] = val;
28     else {
29         if(p <= Mid(L, R)) {
30             rc[o] = rc[his];
31             modify(lc[o], lc[his], L, Mid(L, R), p, val);
32         } else {
33             lc[o] = lc[his];
34             modify(rc[o], rc[his], Mid(L, R)+1, R, p, val);
35         }
36     }
37 }
38
39 int query(int o, int L, int R, int p) {
40     if(!o) return 0;
41     if(L == R) return v[o];
42     else {

```

```

43     if(p <= Mid(L, R))
44         return query(lc[o], L, Mid(L, R), p);
45     else
46         return query(rc[o], Mid(L, R)+1, R, p);
47 }
48 }
49
50 void init() {
51     n = readint(); q = readint();
52     for(int i = 1; i <= n; i++)
53         w[i] = readint();
54     build_tree(rt[++ver], 1, n);
55 }
56
57 void work() {
58     int op, prv, idx, val;
59     while(q--) {
60         ++ver;
61         prv = readint() + 1;
62         op = readint();
63         switch(op) {
64             case 1:
65                 idx = readint(); val = readint();
66                 modify(rt[ver], rt[prv], 1, n, idx, val);
67                 break;
68             case 2:
69                 idx = readint();
70                 printf("%d\n", query(rt[prv], 1, n, idx));
71                 rt[ver] = rt[prv];
72                 break;
73         }
74     }
75 }
76
77 int main() {
78     init(); work();
79     return 0;
80 }

```

## 7 离线二维数点

### 7.1 带修改

#### 7.1.1 静态：线段树 + 扫描线

（未实现）

#### 7.1.2 动态：CDQ 分治

陌上花开：三维数点 = 动态二维数点

注意去重处理的坑点：

1. 在分治统计的时候，无论是加点还是查询答案，都一定要考虑到多个重复点的贡献！

2. 注意去重方法：用map比较方便。

2D/cdq.cpp

```
1 #include <bits/stdc++.h>
2 #define fst first
3 #define snd second
4 using namespace std;
5
6 struct Point {
7     int x, y, z, idx;
8     bool operator<(const Point &rhs) const {
9         return x == rhs.x ?
10            (
11                y == rhs.y ?
12                z < rhs.z : y < rhs.y
13            ) : x < rhs.x;
14     }
15     bool operator==(const Point &rhs) const {
16         return x == rhs.x && y == rhs.y && z == rhs.z;
17     }
18 };
19
20 struct Query {
21     int x, y, z, idx, type;
22     bool operator<(const Query &rhs) const {
23         return y == rhs.y ? type < rhs.type : y < rhs.y;
24     }
25 };
26
27 const int MAXN = 3e5;
28 map<Point, int> p_cnt;
29 int n, k, q_cnt, totv[MAXN+10], ans[MAXN+10], ansCnt[MAXN+10], bit[MAXN+10];
30 Query qry[MAXN+10], T[MAXN+10];
31
32 inline int readint() {
33     int f=1, r=0; char c=getchar();
34     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
35     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
36     return f*r;
37 }
38
39 void init() {
40     int x, y, z;
41     n = readint(); k = readint();
42     for(int i = 1; i <= n; i++) {
43         x = readint(); y = readint(); z = readint();
44         p_cnt[(Point){ x, y, z, i } ]++;
45     }
46     for(auto p : p_cnt) {
47         totv[p.fst.idx] = p.snd; ans[p.fst.idx] = -p.snd;
48         qry[++q_cnt] = { p.fst.x, p.fst.y, p.fst.z, p.fst.idx, 1 };
49         qry[++q_cnt] = { p.fst.x, p.fst.y, p.fst.z, p.fst.idx, 2 };
50     }
51 }
52
53 inline int lowbit(int x) {
54     return x & (-x);
```

```

55 }
56
57 void add(int p, int val) {
58     while(p <= k) {
59         bit[p] += val;
60         p += lowbit(p);
61     }
62 }
63
64 int sum(int p) {
65     int ret = 0;
66     while(p > 0) {
67         ret += bit[p];
68         p -= lowbit(p);
69     }
70     return ret;
71 }
72
73 void solve(int L, int R) {
74     if(L + 1 >= R) return;
75
76     int pl, pr, M, p;
77     M = L + (R - L) / 2;
78     pl = L, pr = M; p = L;
79
80     solve(L, M); solve(M, R);
81
82     while(pl < M || pr < R) {
83         if(pr >= R || (pl < M && qry[pl] < qry[pr])) {
84             if(qry[pl].type == 1)
85                 add(qry[pl].z, totv[qry[pl].idx]);
86             T[p++] = qry[pl++];
87         } else {
88             if(qry[pr].type == 2)
89                 ans[qry[pr].idx] += totv[qry[pr].idx] * sum(qry[pr].z);
90             T[p++] = qry[pr++];
91         }
92     }
93
94     pl = L, pr = M;
95     while(pl < M || pr < R) {
96         if(pr >= R || (pl < M && qry[pl] < qry[pr])) {
97             if(qry[pl].type == 1)
98                 add(qry[pl].z, -totv[qry[pl].idx]);
99             pl++;
100         } else pr++;
101     }
102     assert(!sum(k));
103     for(int i = L; i < R; i++) qry[i] = T[i];
104 }
105
106 void work() {
107     sort(qry + 1, qry + q_cnt + 1, [](const Query &lhs, const Query &rhs) {
108         return lhs.x == rhs.x ? lhs.type < rhs.type : lhs.x < rhs.x;
109     });
110     solve(1, q_cnt + 1);
111     for(int i = 1; i <= q_cnt; i++) {

```

```

112     if(!totv[i]) continue;
113     ansCnt[ans[i] / totv[i]] += totv[i];
114 }
115 for(int i = 0; i < n; i++)
116     printf("%d\n", ansCnt[i]);
117 }
118
119 int main() {
120     init(); work();
121     return 0;
122 }

```

## 8 在线二维数点

### 8.0.1 动态：二维线段树

时间复杂度 插入 $O(\lg^2 n)$  – 查询 $O(\lg n)$  空间复杂度  $O(n^2)$

### 8.0.2 动态：树状数组套动态开点线段树

(见上方二逼平衡树)

### 8.0.3 动态：树状数组套平衡树

BalancedTree/DynamicInversion.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 typedef long long int64;
5
6 const int MAXN = 1e5;
7
8 //-----Treap-----
9 struct Node *null;
10 struct Node {
11     Node *ch[2];
12     int v, r, sz;
13     void init(int v_) {
14         v = v_; r = rand(); sz = 1;
15         ch[0] = ch[1] = null;
16     }
17     Node() {
18         init(0);
19     }
20     int cmp(int x) {
21         return (x == v ? -1 : (x > v ? 1 : 0));
22     }
23     void maintain() {
24         if(this != null)
25             sz = ch[0]->sz + ch[1]->sz + 1;
26     }
27 };
28

```



```

29 int n, m, w[MAXN+10], mp[MAXN+10], bitval[MAXN+10];
30 Node* bit[MAXN+10]; int64 ans;
31
32 const int alloc_size = 65536;
33 queue<Node*> pool;
34 void renew() {
35     Node* pit = new Node[alloc_size];
36     for(int i = 0; i < alloc_size; i++)
37         pool.push(pit++);
38 }
39
40 Node* newnode(int v) {
41     if(pool.empty()) renew();
42     Node* ret = pool.front(); pool.pop();
43     ret->init(v);
44     return ret;
45 }
46
47 void delnode(Node* &o) {
48     pool.push(o); o = null;
49 }
50
51 void rotate(Node* &o, int d) {
52     Node* k = o->ch[d^1];
53     o->ch[d^1] = k->ch[d];
54     k->ch[d] = o;
55     o->maintain(); k->maintain();
56     o = k;
57 }
58
59 void insert(Node* &o, int val) {
60     if(o == null)
61         o = newnode(val);
62     else {
63         int d = o->cmp(val);
64         if(d == -1) return;
65         insert(o->ch[d], val);
66         o->maintain();
67         if((o->r) > (o->ch[d]->r))
68             rotate(o, d^1);
69     }
70 }
71
72 void erase(Node* &o, int val) {
73     if(o == null) return;
74     int d = o->cmp(val);
75     if(d == -1) {
76         if(o->ch[1] == null) {
77             Node* lhs = o->ch[0];
78             delnode(o); o = lhs;
79         } else if(o->ch[0] == null) {
80             Node* rhs = o->ch[1];
81             delnode(o); o = rhs;
82         } else {
83             int d = (o->ch[0]->r) < (o->ch[1]->r) ? 1 : 0;
84             rotate(o, d);
85             erase(o->ch[d], val);

```

```

86     }
87 } else
88     erase(o->ch[d], val);
89 o->maintain();
90 }
91
92 int getrank(Node* o, int val) {
93     if(o == null) return 0;
94     int d = o->cmp(val);
95     if(d == -1) return o->ch[0]->sz; // 'ĖĖ±rank = <valµĀĖĖ,öĖý
96     return getrank(o->ch[d], val) + d * (o->ch[0]->sz + 1);
97 }
98
99 //-----
100
101 void init_null() {
102     null = new Node(); null->sz = 0;
103     for(int i = 0; i <= MAXN; i++)
104         bit[i] = null;
105 }
106
107 inline int lowbit(int x) { return x & (-x); }
108
109 int bit_sum(int p) {
110     int ret = 0;
111     while(p > 0) {
112         ret += bitval[p];
113         p -= lowbit(p);
114     }
115     return ret;
116 }
117
118 void bit_add(int p, int val) {
119     while(p <= n) {
120         bitval[p] += val;
121         p += lowbit(p);
122     }
123 }
124
125 void nd_bit_sum(int p, int &sz, Node* o[]) {
126     while(p > 0) {
127         o[sz++] = bit[p];
128         p -= lowbit(p);
129     }
130 }
131
132 void nd_bit_add(int p, int val) {
133     while(p <= n) {
134         insert(bit[p], val);
135         p += lowbit(p);
136     }
137 }
138
139 void nd_bit_del(int p, int val){
140     while(p <= n) {
141         erase(bit[p], val);
142         p += lowbit(p);

```

```

143     }
144 }
145
146 int query(int x, int y) {
147     int ret = 0, sz = 0; Node* vec[50];
148     nd_bit_sum(x, sz, vec);
149     for(int i = 0; i < sz; i++) {
150         Node* ptr = vec[i];
151         ret += getrank(ptr, y);
152     }
153     return ret;
154 }
155
156 inline int readint() {
157     int f=1, r=0; char c=getchar();
158     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
159     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
160     return f*r;
161 }
162
163 void init() {
164     n = readint(); m = readint();
165     for(int i = 1; i <= n; i++) {
166         w[i] = readint(); mp[w[i]] = i;
167         bit_add(w[i], 1); ans += bit_sum(n) - bit_sum(w[i]);
168         nd_bit_add(i, w[i]);
169     }
170 }
171
172 void work() {
173     int i;
174     while(m--) {
175         i = mp[readint()];
176         printf("%lld\n", ans);
177         ans -= query(i-1, n+1) - query(i-1, w[i]+1);
178         ans -= query(n, w[i]) - query(i, w[i]);
179         nd_bit_del(i, w[i]);
180     }
181 }
182
183 int main() {
184     srand(66623333);
185     init_null();
186     init(); work();
187 }

```

## 9 平衡树

### 9.1 Treap

BalancedTree/Treap.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct Node *null, *rt;
5 struct Node {
6     int v, r, sz, cnt;
7     Node *ch[2];
8     Node(int v_) {
9         v = v_; r = rand(); sz = cnt = 1;
10        ch[0] = ch[1] = null;
11    }
12    int cmp(int val) {
13        return val == v ? -1 : (val > v ? 1 : 0);
14    }
15    void maintain() {
16        if(this == null) return;
17        sz = ch[0]->sz + ch[1]->sz + cnt;
18    }
19 };
20
21 void init_null() {
22     null = new Node(0); null->sz = null->cnt = 0;
23     rt = null;
24 }
25
26 void rotate(Node* &o, int d) {
27     Node* k = o->ch[d^1];
28     o->ch[d^1] = k->ch[d];
29     k->ch[d] = o;
30     o->maintain(); k->maintain();
31     o = k;
32 }
33
34 void insert(Node* &o, int val) {
35     if(o == null) {
36         o = new Node(val);
37         return;
38     } else {
39         int d = o->cmp(val);
40         if(d == -1) {
41             ++o->cnt; o->maintain();
42         } else {
43             insert(o->ch[d], val);
44             o->maintain();
45             if(o->ch[d]->r < o->r) rotate(o, d^1);
46         }
47     }
48 }
49
50 void erase(Node* &o, int val) {
51     int d = o->cmp(val);
```

```

52     if(d == -1) {
53         if(o->cnt == 1) {
54             if(o->ch[1] == null) {
55                 Node* lhs = o->ch[0];
56                 delete o;
57                 o = lhs;
58             } else if(o->ch[0] == null) {
59                 Node* rhs = o->ch[1];
60                 delete o;
61                 o = rhs;
62             } else {
63                 int d2 = (o->ch[0]->r) > (o->ch[1]->r);
64                 rotate(o, d2^1);
65                 erase(o->ch[d2^1], val);
66             }
67         } else
68             --o->cnt;
69     } else
70         erase(o->ch[d], val);
71     o->maintain();
72 }
73
74 Node* kth(Node* o, int k) {
75     int d = (k >= o->ch[0]->sz + 1 && k <= o->ch[0]->sz + o->cnt) ? -1 :
76         (k <= o->ch[0]->sz ? 0 : 1);
77     if(d == -1) return o;
78     if(d == 1) k -= (o->sz - o->ch[1]->sz);
79     return kth(o->ch[d], k);
80 }
81
82 int get_rank(Node* o, int val) {
83     if(o == null) return 1;
84     int d = o->cmp(val);
85     if(d == -1) return o->ch[0]->sz + 1;
86     return get_rank(o->ch[d], val) + d * (o->sz - o->ch[1]->sz);
87 }
88
89 Node* find(Node* o, int val) {
90     if(o == null) return o;
91     int d = o->cmp(val);
92     if(d == -1) return o;
93     else return find(o->ch[d], val);
94 }
95
96 Node* pre(int val) {
97     int rk = get_rank(rt, val);
98     return rk != 1 ? kth(rt, rk-1) : null;
99 }
100
101 Node* succ(int val) {
102     int rk = get_rank(rt, val); // !!!!!!!!
103     return rk != (rt->sz) ? kth(rt, rk+find(rt, val)->cnt) : null;
104 }
105
106 inline int readint() {
107     int f=1, r=0; char c=getchar();
108     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }

```

```

109 while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
110 return f*r;
111 }
112
113 int main() {
114     srand(66623333);
115     int q, op, x;
116     init_null();
117     q = readint();
118     while(q--) {
119         op = readint();
120         switch(op) {
121             case 1:
122                 x = readint(); insert(rt, x);
123                 break;
124             case 2:
125                 x = readint(); erase(rt, x);
126                 break;
127             case 3:
128                 x = readint(); insert(rt, x);
129                 printf("%d\n", get_rank(rt, x));
130                 erase(rt, x);
131                 break;
132             case 4:
133                 x = readint();
134                 printf("%d\n", kth(rt, x)->v);
135                 break;
136             case 5:
137                 x = readint(); insert(rt, x);
138                 assert(pre(x) != null);
139                 printf("%d\n", pre(x)->v);
140                 erase(rt, x);
141                 break;
142             case 6:
143                 x = readint(); insert(rt, x);
144                 assert(succ(x) != null);
145                 printf("%d\n", succ(x)->v);
146                 erase(rt, x);
147                 break;
148         }
149     }
150     return 0;
151 }

```

## 9.2 Splay

BalancedTree/Splay.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 1e5;
5
6 struct Node *null, *rt;
7 struct Node {
8     int v, sz; bool flip;

```

```

9   Node* ch[2];
10  Node(int v_) { v = v_, sz = 1; flip = false; ch[0] = ch[1] = null; }
11  int cmp(int k) {
12      return k == ch[0]->sz + 1 ? -1 : (k > ch[0]->sz + 1 ? 1 : 0);
13  }
14  void rev() {
15      if(this == null) return;
16      flip ^= 1;
17  }
18  void maintain() {
19      if(this == null) return;
20      sz = ch[0]->sz + ch[1]->sz + 1;
21  }
22  void pushdown() {
23      if(flip) {
24          flip = false;
25          ch[0]->rev(); ch[1]->rev();
26          swap(ch[0], ch[1]);
27      }
28  }
29 };
30 int n, m;
31
32 void init_null() {
33     null = new Node(0); null->sz = 0;
34     rt = null;
35 }
36
37 void rotate(Node* &o, int d) {
38     Node* k = o->ch[d^1];
39     o->pushdown(); k->pushdown();
40     o->ch[d^1] = k->ch[d];
41     k->ch[d] = o;
42     o->maintain(); k->maintain();
43     o = k;
44 }
45
46 void splay(Node* &o, int k) {
47     o->pushdown();
48     int d = o->cmp(k);
49     if(d == 1) k -= (o->ch[0]->sz + 1);
50     if(d != -1) {
51         Node* p = o->ch[d];
52         p->pushdown();
53         int d2 = p->cmp(k);
54         if(d2 == 1) k -= (p->ch[0]->sz + 1);
55         if(d2 != -1) {
56             splay(p->ch[d2], k);
57             if(d == d2) rotate(o, d^1);
58             else rotate(o->ch[d], d);
59         }
60         rotate(o, d^1);
61     }
62 }
63
64 Node* merge(Node* lhs, Node* rhs) {
65     splay(lhs, lhs->sz);

```

```

66     lhs->pushdown();
67     lhs->ch[1] = rhs;
68     lhs->maintain();
69     return lhs;
70 }
71
72 void split(Node* o, int k, Node* &lhs, Node* &rhs) {
73     splay(o, k);
74     o->pushdown();
75     lhs = o, rhs = o->ch[1];
76     o->ch[1] = null; o->maintain(); // 赋值后再断开和右儿子的连接, 并维护 sz!
77 }
78
79 void traverse(Node* o) {
80     if(o == null) return;
81     o->pushdown();
82     traverse(o->ch[0]);
83     if(o->v > 0) printf("%d ", o->v);
84     traverse(o->ch[1]);
85 }
86
87 inline int readint() {
88     int f=1, r=0; char c=getchar();
89     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
90     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
91     return f*r;
92 }
93
94 int main() {
95     int l, r; Node *a, *b, *c;
96     init_null();
97
98     n = readint(); m = readint();
99
100     rt = new Node(0); // dummy
101     for(int i = 1; i <= n; i++) rt = merge(rt, new Node(i));
102     rt = merge(rt, new Node(0)); // dummy
103
104     while(m--) {
105         l = readint() + 1, r = readint() + 1;
106         split(rt, l-1, a, b); split(b, r-l+1, b, c);
107         b->rev();
108         rt = merge(a, merge(b, c));
109     }
110
111     traverse(rt);
112     return 0;
113 }

```



## 10 动态树

### 10.1 Link-cut Tree

(似乎发现了以前模板里面判断边是否存在的一个错误……)

LCT/LCT.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 3e5;
5
6 struct Node *null;
7 struct Node {
8     int v, sumv; bool rev;
9     Node *fa, *ch[2];
10    Node(int v_) {
11        v = sumv = v_; rev = false;
12        fa = ch[0] = ch[1] = null;
13    }
14    bool splayrt() { return fa->ch[0] != this && fa->ch[1] != this; }
15    int rel() { return splayrt() ? -1 : (fa->ch[0] == this ? 0 : 1); }
16    void mark_rev() { rev ^= 1; }
17    void maintain() {
18        if(this == null) return;
19        sumv = ch[0]->sumv ^ v ^ ch[1]->sumv;
20    }
21    void pushdown() {
22        if(rev) {
23            rev = false;
24            ch[0]->mark_rev(); ch[1]->mark_rev();
25            swap(ch[0], ch[1]);
26        }
27    }
28 } *nd[MAXN+10];
29
30 set<pair<Node*, Node*> > edges;
31
32 void init_null() {
33     null = new Node(0);
34     for(int i = 0; i <= MAXN; i++)
35         nd[i] = null;
36 }
37
38 void rotate(Node* o) {
39     Node *x, *y, *k; int d, d2;
40     x = o->fa; y = x->fa;
41     d = o->rel(); d2 = x->rel();
42     k = o->ch[d^1];
43     if(!x->splayrt()) y->ch[d2] = o;
44     o->fa = y;
45     o->ch[d^1] = x; x->fa = o;
46     x->ch[d] = k; k->fa = x;
47     x->maintain(); o->maintain();
48 }
49
```

```

50 void splay(Node* o) {
51     static Node *x, *stk[MAXN+10]; int d, d2, p = 0;
52     for(stk[p=1] = o; !stk[p]->splayrt(); p++)
53         stk[p+1] = stk[p]->fa;
54     for(; p; p--) stk[p]->pushdown();
55     while(!o->splayrt()) {
56         x = o->fa;
57         d = o->rel(); d2 = x->rel();
58         if(d2 != -1) {
59             if(d == d2) rotate(x);
60             else rotate(o);
61         }
62         rotate(o);
63     }
64 }
65
66 void access(Node* o) {
67     for(Node* t = null; o != null; t = o, o = o->fa) {
68         splay(o); o->ch[1] = t; o->maintain();
69     }
70 }
71
72 Node* get_root(Node* o) {
73     access(o); splay(o);
74     while(o->ch[0] != null) o = o->ch[0];
75     splay(o); return o;
76 }
77
78 void make_root(Node* o) {
79     access(o); splay(o); o->mark_rev();
80 }
81
82 void add_edge(Node* u, Node* v) {
83     if(u > v) swap(u, v);
84     edges.insert(make_pair(u, v));
85 }
86
87 bool has_edge(Node* u, Node* v) {
88     if(u > v) swap(u, v); // 统一存储
89     return edges.count(make_pair(u, v)) > 0;
90 }
91
92 void link(Node *u, Node *v) {
93     if(get_root(u) == get_root(v)) return;
94     make_root(u); splay(u); u->fa = v;
95     add_edge(u, v);
96 }
97
98 void cut(Node *u, Node *v) {
99     if(get_root(u) != get_root(v)) return;
100    make_root(u); access(v); splay(u);
101    u->pushdown();
102    if(has_edge(u, v)) { // 不是 u->ch[1] == v!!!
103        u->ch[1] = null; v->fa = null; // v->fa !
104    }
105    u->maintain(); v->maintain();
106 }

```

```

107
108 int n, q;
109
110 inline int readint() {
111     int f=1, r=0; char c=getchar();
112     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
113     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
114     return f*r;
115 }
116
117 int main() {
118     int op, x, y;
119     init_null();
120     n = readint(); q = readint();
121     for(int i = 1; i <= n; i++)
122         nd[i] = new Node(readint());
123     while(q--) {
124         op = readint(); x = readint(); y = readint();
125         switch(op) {
126             case 0:
127                 assert(get_root(nd[x]) == get_root(nd[y]));
128                 make_root(nd[x]); access(nd[y]); splay(nd[x]);
129                 printf("%d\n", nd[x]->sumv);
130                 break;
131             case 1:
132                 link(nd[x], nd[y]);
133                 break;
134             case 2:
135                 cut(nd[x], nd[y]);
136                 break;
137             case 3:
138                 splay(nd[x]); nd[x]->v = y; nd[x]->maintain();
139                 break;
140         }
141     }
142     return 0;
143 }

```

## 11 字符串

### 11.1 KMP 字符串匹配

1-indexed

String/KMP.cpp

```
1 #include <bits/stdc++.h>
2 const int MAXN = 1000000;
3
4 int fail[MAXN+5];
5
6 void KMP(char* a, char* b){
7     int na = strlen(a+1), nb = strlen(b+1); //注意 +1
8     //Init
9     fail[1] = 0; //! static 在每次进入函数时保留上次修改的值
10    for(int i = 2; i <= nb; i++){
11        int j = fail[i-1]; //尝试扩展已经匹配部分
12        //无法匹配则缩短前后缀
13        while(j != 0 && b[j+1] != b[i]) j = fail[j];
14        //用已经匹配部分更新 fail 数组
15        if(b[j+1] == b[i]) fail[i] = j+1; // !!
16        //无法匹配前后缀
17        else fail[i] = 0;
18    }
19    //Match
20    for(int i = 1, j = 0; i <= na; i++){
21        //缩短前后缀
22        while(j != 0 && b[j+1] != a[i]) j = fail[j];
23        if(b[j+1] == a[i]) j++; //成功匹配
24        if(j == nb){
25            printf("%d\n", i - nb + 1);
26            j = fail[j];
27            //j = 0; //如果两个匹配部分不可重叠
28        }
29    }
30 }
31
32 int main() {
33     char a[MAXN+5], b[MAXN+5];
34     scanf("%s", a+1);
35     scanf("%s", b+1);
36     KMP(a, b); int t = strlen(b+1);
37     for(int i = 1; i <= t; i++)
38         printf("%d ", fail[i]);
39     return 0;
40 }
```

### 11.2 AC 自动机

0-indexed

String/ACAutomaton.cpp

```
1 #include <bits/stdc++.h>
```

```

2 #define CLEAR(x) memset((x), 0, sizeof(x))
3 using namespace std;
4
5 const int SIGMA = 26, MAX_TEMP_LEN = 70, MAXN = 150,
6 MAX_LEN = 1e6, MAX_NODE = MAXN * MAX_TEMP_LEN;
7
8 int N, sz, ch[MAX_NODE + 10][SIGMA + 2], f[MAX_NODE + 10], last[MAX_NODE+10],
9 val[MAX_NODE + 10], found_cnt[MAX_NODE+10];
10 char str[MAX_LEN+10], tpl[MAXN+10][MAX_TEMP_LEN+10];
11 unordered_map<string, int> ms;
12
13 inline int idx(char c) { return c - 'a' + 1; }
14
15 void insert(char *str) {
16     int u = 0, len = strlen(str);
17     for(int i = 0; i < len; i++) {
18         int c = idx(str[i]);
19         if(!ch[u][c]) ch[u][c] = ++sz;
20         u = ch[u][c];
21     }
22     ms[string(str)] = u;
23     ++val[u];
24 }
25
26 void get_fail() {
27     queue<int> Q;
28     f[0] = 0;
29     for(int c = 1; c <= SIGMA; c++) if(ch[0][c]) {
30         int v = ch[0][c];
31         f[v] = last[v] = 0;
32         Q.push(v);
33     }
34     while(!Q.empty()) {
35         int u = Q.front(); Q.pop();
36         for(int c = 1; c <= SIGMA; c++) {
37             int v = ch[u][c];
38             if(!v) {
39                 ch[u][c] = ch[f[u]][c];
40                 continue;
41             }
42
43             Q.push(v);
44
45             int u2 = f[u];
46             while(u2 && !ch[u2][c]) u2 = f[u2];
47             f[v] = ch[u2][c];
48             last[v] = val[f[v]] ? f[v] : last[f[v]];
49         }
50     }
51 }
52
53 void found(int u) {
54     for(; u; u = last[u])
55         found_cnt[u] += val[u];
56 }
57
58 void search(char *str) {

```

```

59     int u = 0, len = strlen(str);
60     for(int i = 0; i < len; i++) {
61         int c = idx(str[i]);
62         u = ch[u][c];
63         if(val[u]) found(u);
64         else if(last[u]) found(last[u]);
65     }
66 }
67
68 inline void readstr(char *str) {
69     char c=getchar(); int p=0;
70     while(!isalnum(c) && !ispunct(c)) c = getchar();
71     while(isalnum(c) || ispunct(c)) {
72         str[p++] = c;
73         c = getchar();
74     }
75     str[p++] = '\0';
76 }
77
78 int main() {
79     while(true) {
80         int ans = 0;
81         sz = 0; CLEAR(ch); CLEAR(f); CLEAR(found_cnt);
82         CLEAR(last); CLEAR(tpl); CLEAR(val); CLEAR(str);
83
84         scanf("%d", &N); if(N == 0) break;
85         for(int i = 1; i <= N; i++) {
86             readstr(tpl[i]); insert(tpl[i]);
87         }
88         get_fail();
89
90         readstr(str); search(str);
91
92         for(int i = 0; i <= sz; i++)
93             ans = max(ans, found_cnt[i]);
94         printf("%d\n", ans);
95         for(int i = 1; i <= N; i++)
96             if(found_cnt[ms[string(tpl[i])]] == ans)
97                 printf("%s\n", tpl[i]);
98     }
99     return 0;
100 }

```

## 11.3 后缀数组

0-indexed

String/SuffixArray.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXLEN = 1e6, SIGMA = 100;
5
6 inline int idx(char c) {
7     if(!c) return 0;

```

```

8     else if(isdigit(c)) return c - '0' + 1;
9     else if(isupper(c)) return c - 'A' + 1 + 10;
10    else if(islower(c)) return c - 'a' + 1 + 10 + 26;
11    else throw "Invalid Character";
12 }
13
14 struct SuffixArray {
15     int sa[MAXLEN+10], rk[MAXLEN+10], buf[3][MAXLEN+10], height[MAXLEN+10], c[MAXLEN+10];
16     void build_sa(char *s, int len) {
17         int m = SIGMA + 10, n = len + 1, *x = buf[0], *y = buf[1];
18         for(int i = 0; i < m; i++) c[i] = 0;
19         for(int i = 0; i < n; i++) ++c[x[i] = idx(s[i])];
20         for(int i = 1; i < m; i++) c[i] += c[i-1];
21         for(int i = n-1; i >= 0; i--) sa[--c[x[i]]] = i;
22         for(int k = 1; k <= n; k <= 1) {
23             int p = 0;
24             for(int i = n-k; i < n; i++) y[p++] = i;
25             for(int i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i] - k;
26             for(int i = 0; i < m; i++) c[i] = 0;
27             for(int i = 0; i < n; i++) ++c[x[y[i]]];
28             for(int i = 1; i < m; i++) c[i] += c[i-1];
29             for(int i = n-1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
30             swap(x, y);
31             p = 1, x[sa[0]] = 0;
32             for(int i = 1; i < n; i++)
33                 x[sa[i]] = (y[sa[i]] == y[sa[i-1]] && y[sa[i] + k] == y[sa[i-1] + k] ? p-1 : p++);
34             if(p >= n) break;
35             m = p;
36         }
37         memcpy(rk, x, sizeof(rk));
38         int k = 0;
39         for(int i = 0; i < n; i++) {
40             if(!rk[i]) continue;
41             if(k) k--;
42             int j = sa[rk[i]-1];
43             while(s[i+k] == s[j+k]) k++;
44             height[rk[i]] = k;
45         }
46     }
47 } SA;
48 inline void readstr(char* str) {
49     char c=getchar(); int p=0;
50     while(!isalnum(c) && !ispunct(c)) c=getchar();
51     while(isalnum(c) || ispunct(c)) {
52         str[p++] = c;
53         c = getchar();
54     }
55     str[p++] = '\0';
56 }
57
58 int len;
59 char str[MAXLEN+10];
60
61 int main() {
62     readstr(str); len = strlen(str);
63     SA.build_sa(str, len);
64     for(int i = 1; i <= len; i++)

```

```
65     printf("%d ", SA.sa[i]+1);  
66     return 0;  
67 }
```



## 12 Miscellaneous

### 12.1 ST 表

Misc/ST.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 1e5;
5
6 int n, q, a[MAXN+10], st[MAXN+10][22];
7
8 inline int readint() {
9     int f=1, r=0; char c=getchar();
10    while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
11    while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
12    return f*r;
13 }
14
15 void init_st() {
16     for(int i = 1; i <= n; i++) st[i][0] = a[i];
17     for(int j = 1; j <= 20; j++)
18         for(int i = 1; i <= n - (1<<j) + 1; i++)
19             st[i][j] = max(st[i][j-1], st[i+(1<<(j-1))][j-1]);
20 }
21
22 int query(int L, int R) {
23     if(L > R) return 0;
24     int j;
25     for(j = 0; (1<<(j+1)) <= (R-L+1); j++);
26     return max(st[L][j], st[R-(1<<j)+1][j]);
27 }
28
29 int main() {
30     int l, r;
31     n = readint(); q = readint();
32     for(int i = 1; i <= n; i++) a[i] = readint();
33     init_st();
34     while(q--) {
35         l = readint(); r = readint();
36         printf("%d\n", query(l, r));
37     }
38     return 0;
39 }
```

### 12.2 Fenwick Tree

Misc/BIT.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 5e5;
5
```

```

6 int n, q, a[MAXN+10];
7
8 inline int lowbit(int x) { return x & (-x); }
9
10 void add(int p, int val) {
11     while(p <= n) {
12         a[p] += val;
13         p += lowbit(p);
14     }
15 }
16
17 int query(int p) {
18     int ret = 0;
19     while(p > 0) {
20         ret += a[p];
21         p -= lowbit(p);
22     }
23     return ret;
24 }
25
26 inline int readint() {
27     int f=1, r=0; char c=getchar();
28     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
29     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
30     return f*r;
31 }
32
33 int main() {
34     n = readint(); q = readint();
35     for(int i = 1; i <= n; i++)
36         add(i, readint());
37     while(q--) {
38         int op, x, y;
39         op = readint(); x = readint(); y = readint();
40         switch(op) {
41             case 1:
42                 add(x, y);
43                 break;
44             case 2:
45                 printf("%d\n", query(y) - query(x-1));
46                 break;
47         }
48     }
49     return 0;
50 }

```

## 12.3 左偏树

Misc/LefiestTree.cpp

```

1 #include <bits/stdc++.h>
2 #define fst first
3 #define snd second
4 using namespace std;
5
6 typedef pair<int, int> pii;

```

```

7  const int MAXN = 1e5;
8
9  extern struct Node *null;
10 struct Node {
11     pii val; int dist;
12     Node *ch[2];
13     Node() {
14         ch[0] = ch[1] = null;
15         dist = -1; //!!!
16     };
17     Node(pii v_) {
18         ch[0] = ch[1] = null;
19         dist = -1; val = v_;
20     }
21 } Tnull, *null=&Tnull, *rt[MAXN+10];
22 int n, q, fa[MAXN+10], del[MAXN+10];
23
24 int get_fa(int x) { return x == fa[x] ? x : fa[x] = get_fa(fa[x]); }
25 void union_set(int x, int y) { fa[get_fa(y)] = get_fa(x); } // 顺序
26
27 Node* merge(Node* lhs, Node* rhs) {
28     if(lhs == null) return rhs;
29     else if(rhs == null) return lhs;
30     else {
31         if(lhs->val > rhs->val) swap(lhs, rhs);
32         lhs->ch[1] = merge(lhs->ch[1], rhs);
33         if(lhs->ch[0]->dist < lhs->ch[1]->dist)
34             swap(lhs->ch[0], lhs->ch[1]);
35         lhs->dist = lhs->ch[1]->dist + 1; // 距离应该是左右儿子的最小 dist + 1 (定义)
36         return lhs;
37     }
38 }
39
40 void pop(Node* &o) {
41     Node *lhs = o->ch[0], *rhs = o->ch[1];
42     delete o;
43     o = merge(lhs, rhs);
44 }
45
46 void push(Node* &o, pii val) {
47     o = merge(o, new Node(val));
48 }
49
50 inline int readint() {
51     int f=1, r=0; char c=getchar();
52     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
53     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
54     return f*r;
55 }
56
57 int main() {
58     int op, x, y;
59     n = readint(); q = readint();
60     for(int i = 1; i <= n; i++) {
61         fa[i] = i;
62         rt[i] = new Node(make_pair(readint(), i));
63     }

```

```

64 while(q--) {
65     op = readint();
66     switch(op) {
67         case 1:
68             x = readint(); y = readint();
69             if(del[x] || del[y] || get_fa(x) == get_fa(y))
70                 continue;
71             rt[get_fa(x)] = merge(rt[get_fa(x)], rt[get_fa(y)]);
72             union_set(x, y);
73             break;
74         case 2:
75             x = readint();
76             if(del[x]) puts("-1");
77             else {
78                 pii u = rt[get_fa(x)]->val;
79                 printf("%d\n", u.fst);
80                 del[u.snd] = true;
81                 pop(rt[get_fa(x)]);
82             }
83             break;
84     }
85 }
86 return 0;
87 }

```

## 13 莫队

### 13.1 普通莫队

MoQueue/HH.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 5e4, MAXQ = 2e5, MAXC = 1e6;
5 int N, Q, BlkSize, L, R, NowAns, A[MAXN+10], M[MAXC+10], Ans[MAXQ+10];
6
7 struct Query {
8     int L, R, id;
9     Query() {}
10    Query(int l, int r, int i): L(l),R(r),id(i) {}
11    inline bool operator<(const Query& rhs) const {
12        return L/BlkSize == rhs.L/BlkSize ?
13            R < rhs.R : L/BlkSize < rhs.L/BlkSize;
14    }
15 } q[MAXQ+10];
16
17 template<typename T>
18 inline void readint(T& x) {
19     T f=1, r=0; char c=getchar();
20     while(!isdigit(c)){ if(c=='-')f=-1; c=getchar(); }
21     while(isdigit(c)){ r=r*10+c-'0'; c=getchar(); }
22     x = f*r;
23 }
24
25 inline char readc() {
26     char c=getchar();
27     while(!isalnum(c) && !ispunct(c))
28         c=getchar();
29     return c;
30 }
31
32 inline void readstr(char *str) {
33     char c=getchar(); int p=0;
34     while(!isalnum(c) && !ispunct(c)) c=getchar();
35     while(isalnum(c) || ispunct(c)) {
36         str[p++]=c;
37         c=getchar();
38     }
39     str[p]='\0';
40 }
41
42 void Init() {
43     int u, v;
44     readint(N); BlkSize = ceil(sqrt(N));
45     for(int i=1; i<=N; i++)
46         readint(A[i]);
47     readint(Q);
48     for(int i=1; i<=Q; i++) {
49         readint(u); readint(v);
50         q[i] = Query(u, v, i);
51     }
```

```

52     sort(q+1, q+Q+1);
53 }
54
55 inline void Add(int Clr) {
56     if(M[Clr]++ == 0) NowAns++;
57 }
58
59 inline void Sub(int Clr) {
60     if(--M[Clr] == 0) NowAns--;
61 }
62
63 void Work() {
64     L=1, R=0; NowAns=0;
65     for(int i=1; i<=Q; i++) {
66         while(R < q[i].R) Add(A[++R]);
67         while(L > q[i].L) Add(A[--L]);
68         while(R > q[i].R) Sub(A[R--]);
69         while(L < q[i].L) Sub(A[L++]);
70         Ans[q[i].id] = NowAns;
71     }
72     for(int i=1; i<=Q; i++)
73         printf("%d\n", Ans[i]);
74 }
75
76 int main() {
77     Init(); Work();
78     return 0;
79 }

```

## 13.2 带修改莫队

MoQueue/Color.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 1e4, MAXC = 1e6;
5 int N, Q, Q1, Q2, L, R, T, BlkSize, NowAns, M[MAXC+10], A[MAXN+10], B[MAXN+10], Ans[MAXN+10];
6
7 struct Query {
8     int L, R, T, id;
9     Query() {}
10    Query(int l, int r, int t, int id_): L(l), R(r), T(t), id(id_) {}
11    bool operator<(const Query& rhs) const {
12        if(L/BlkSize == rhs.L/BlkSize) {
13            if(R/BlkSize == rhs.R/BlkSize)
14                return T < rhs.T;
15            else return R/BlkSize < rhs.R/BlkSize;
16        } else return L/BlkSize < rhs.L/BlkSize;
17    }
18 } q[MAXN+10];
19
20 struct Modify {
21     int p, val, orig, id;
22     Modify() {}
23     Modify(int p_, int val_, int orig_, int id_): p(p_), val(val_), orig(orig_), id(id_) {}

```

```

24 } mod[MAXN+10];
25
26 template<typename T>
27 inline void readint(T& x) {
28     T f=1, r=0; char c=getchar();
29     while(!isdigit(c)){ if(c=='-')f=-1; c=getchar(); }
30     while(isdigit(c)){ r=r*10+c-'0'; c=getchar(); }
31     x = f*r;
32 }
33
34 inline char readc() {
35     char c=getchar();
36     while(!isalnum(c) && !ispunct(c))
37         c=getchar();
38     return c;
39 }
40
41 void Init() {
42     static int u, v; char op;
43     readint(N); readint(Q);
44     BlkSize = ceil(pow(N, 0.67));
45     for(int i=1; i<=N; i++) {
46         readint(A[i]); B[i] = A[i];
47     }
48     for(int i=1; i<=Q; i++) {
49         op = readc(); readint(u); readint(v);
50         switch(op) {
51             case 'Q':
52                 q[++Q1] = Query(u, v, Q2, i);
53                 break;
54             case 'R':
55                 mod[++Q2] = Modify(u, v, B[u], i);
56                 B[u] = v;
57                 break;
58         }
59     }
60     sort(q+1, q+Q1+1);
61 }
62
63 inline void add(int Clr) {
64     if(M[Clr]++ == 0) NowAns++;
65 }
66
67 inline void sub(int Clr) {
68     if(--M[Clr] == 0) NowAns--;
69 }
70
71 inline void goforth(int t) {
72     //先把修改点纳入当前区间!
73     while(L > mod[t].p) add(A[--L]);
74     while(R < mod[t].p) add(A[++R]);
75     A[mod[t].p] = mod[t].val;
76     sub(mod[t].orig); add(mod[t].val);
77 }
78
79 inline void goback(int t) {
80     while(L > mod[t].p) add(A[--L]);

```

```

81     while(R < mod[t].p) add(A[++R]);
82     A[mod[t].p] = mod[t].orig; //改回去!
83     sub(mod[t].val); add(mod[t].orig);
84 }
85
86 void Work() {
87     L=1, R=0, T=0;
88     for(int i=1; i<=Q1; i++) {
89         while(T < q[i].T) goforth(++T);
90         while(T > q[i].T) goback(T--); //先调整时间后调整区间
91         while(R < q[i].R) add(A[++R]);
92         while(L > q[i].L) add(A[--L]);
93         while(R > q[i].R) sub(A[R--]);
94         while(L < q[i].L) sub(A[L++]);
95         Ans[q[i].id] = NowAns;
96     }
97     for(int i=1; i<=Q; i++)
98         if(Ans[i]) {
99             printf("%d\n", Ans[i]);
100         }
101 }
102
103 int main() {
104     Init(); Work();
105     return 0;
106 }

```



## 14 分块相关

### 14.1 分块

例题：教主的魔法

Block/Magic.cpp

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAXN = 1e6, INF = 0x3f3f3f3f;
5 int N, Q, BlkSize, A[MAXN+10], B[MAXN+10], Blk[MAXN+10],
6     L[MAXN+10], R[MAXN+10], Addv[MAXN+10];
7
8 template<typename T>
9 inline void readint(T& x) {
10     T f=1, r=0; char c=getchar();
11     while(!isdigit(c)){ if(c=='-')f=-1; c=getchar(); }
12     while(isdigit(c)){ r=r*10+c-'0'; c=getchar(); }
13     x = f*r;
14 }
15
16 inline char readc() {
17     char c=getchar();
18     while(!isalnum(c) && !ispunct(c))
19         c=getchar();
20     return c;
21 }
22
23 inline void InitBlk() {
24     BlkSize = ceil(sqrt(N));
25     for(int i=0; i*BlkSize + 1 <= N; i++) { //注意在分块时考虑末尾块的情况!
26         L[i] = i*BlkSize + 1, R[i] = min((i+1)*BlkSize, N);
27         for(int j=L[i]; j<=R[i]; j++) {
28             Blk[j] = i; B[j] = A[j];
29         }
30         sort(B+L[i], B+R[i]+1);
31     }
32 }
33
34 inline void Maintain(int o) {
35     for(int i=L[o]; i<=R[o]; i++)
36         B[i] = A[i];
37     sort(B+L[o], B+R[o]+1);
38 }
39
40 inline void Add(int qL, int qR, int v) {
41     if(Blk[qL] == Blk[qR]) {
42         for(int i=qL; i<=qR; i++)
43             A[i] += v;
44         Maintain(Blk[qL]);
45     } else {
46         for(int i=qL; Blk[i] == Blk[qL]; i++)
47             A[i] += v;
48         for(int i=qR; Blk[i] == Blk[qR]; i--)
49             A[i] += v;
```

```

50     Maintain(Blk[qL]); Maintain(Blk[qR]);
51     for(int i=Blk[qL]+1; i<=Blk[qR]-1; i++)
52         Addv[i] += v;
53 }
54 }
55
56 inline int Query(int qL, int qR, int v) { //>=v
57     int ret = 0, p;
58     if(Blk[qL] == Blk[qR]) {
59         for(int i=qL; i<=qR; i++)
60             if(A[i] + Addv[Blk[i]] >= v) ret++;
61         return ret;
62     } else {
63         for(int i=qL; Blk[i] == Blk[qL]; i++)
64             if(A[i] + Addv[Blk[i]] >= v) ret++;
65         for(int i=qR; Blk[i] == Blk[qR]; i--)
66             if(A[i] + Addv[Blk[i]] >= v) ret++;
67         for(int i=Blk[qL]+1; i<=Blk[qR]-1; i++) {
68             p = lower_bound(B+L[i], B+R[i]+1, v-Addv[i]) - (B+L[i]);
69             ret += R[i] - L[i] + 1 - p;
70         }
71         return ret;
72     }
73 }
74
75 void Init() {
76     readint(N); readint(Q);
77     for(int i=1; i<=N; i++) readint(A[i]);
78     InitBlk();
79 }
80
81 void Work() {
82     int l, r, v; char op;
83     while(Q--) {
84         op = readc(); readint(l); readint(r); readint(v);
85         switch(op) {
86             case 'M':
87                 Add(l, r, v);
88                 break;
89             case 'A':
90                 printf("%d\n", Query(l, r, v));
91                 break;
92         }
93     }
94 }
95
96 int main() {
97     Init(); Work();
98     return 0;
99 }

```

## 14.2 区间众数

Block/Mode.cpp

```

1 #include <bits/stdc++.h>

```

```

2 using namespace std;
3
4 const int MAXN = 1e5, SQN = 316;
5 int N, Q, BlkSize, MxBlk, A[MAXN+10], Blk[MAXN+10], L[MAXN+10], R[MAXN+10];
6 int Num[MAXN+10], Mode[SQN+10][SQN+10], PreCnt[SQN+10][MAXN+10];
7
8 inline void InitBlk() {
9     static int Md, Bkt[MAXN+10];
10    BlkSize = ceil(sqrt(N)) + 0.5;
11    for(int i=0; i * BlkSize + 1 <= N; i++) {
12        MxBlk = i;
13        L[i] = i * BlkSize + 1, R[i] = min(N, (i+1) * BlkSize);
14        for(int j = L[i]; j <= R[i]; j++) Blk[j] = i;
15    }
16    for(int i = 1; i <= N; i++)
17        PreCnt[Blk[i]][A[i]]++;
18    for(int i = 1; i <= MxBlk; i++)
19        for(int j = 1; j <= N; j++)
20            PreCnt[i][j] += PreCnt[i-1][j];
21    for(int i = 0; i <= MxBlk; i++) {
22        for(int j = i; j <= MxBlk; j++) {
23            if(i < j) Md = Mode[i][j-1];
24            else Md = 0;
25            for(int k = L[j]; k <= R[j]; k++) {
26                Bkt[A[k]]++; // !!!
27                int lhs = Bkt[A[k]] + (i < j ? PreCnt[j-1][A[k]] - (i>=1 ? PreCnt[i-1][A[k]] : 0) : 0);
28                int rhs = Bkt[Md] + (i < j ? PreCnt[j-1][Md] - (i>=1 ? PreCnt[i-1][Md] : 0) : 0);
29                if(lhs > rhs || (lhs == rhs && A[k] < Md))
30                    Md = A[k];
31            }
32            Mode[i][j] = Md;
33            for(int k = L[j]; k <= R[j]; k++) Bkt[A[k]]--;
34        }
35    }
36 }
37
38 int Query(int qL, int qR) {
39     static int Md, Bkt[MAXN+10];
40     if(Blk[qL] == Blk[qR]) {
41         Md = 0;
42         for(int i = qL; i <= qR; i++) {
43             Bkt[A[i]]++;
44             if(Bkt[A[i]] > Bkt[Md] || (Bkt[A[i]] == Bkt[Md] && A[i] < Md))
45                 Md = A[i];
46         }
47         for(int i = qL; i <= qR; i++) Bkt[A[i]]--;
48         return Md;
49     } else {
50         if(Blk[qL] + 1 <= Blk[qR] - 1)
51             Md = Mode[Blk[qL]+1][Blk[qR]-1];
52         else Md = 0;
53         for(int i = qL; Blk[i] == Blk[qL]; i++) {
54             ++Bkt[A[i]];
55             int lhs = Bkt[A[i]] + (Blk[qL]+1 <= Blk[qR]-1 ?
56                 PreCnt[Blk[qR]-1][A[i]] - PreCnt[Blk[qL]][A[i]] : 0);
57             int rhs = Bkt[Md] + (Blk[qL]+1 <= Blk[qR]-1 ?
58                 PreCnt[Blk[qR]-1][Md] - PreCnt[Blk[qL]][Md] : 0);

```

```

59         if(lhs > rhs || (lhs == rhs && A[i] < Md))
60             Md = A[i];
61     }
62     for(int i = qR; Blk[i] == Blk[qR]; i--) {
63         ++Bkt[A[i]];
64         int lhs = Bkt[A[i]] + (Blk[qL]+1 <= Blk[qR]-1 ?
65             PreCnt[Blk[qR]-1][A[i]] - PreCnt[Blk[qL]][A[i]] : 0);
66         int rhs = Bkt[Md] + (Blk[qL]+1 <= Blk[qR]-1 ?
67             PreCnt[Blk[qR]-1][Md] - PreCnt[Blk[qL]][Md] : 0);
68         if(lhs > rhs || (lhs == rhs && A[i] < Md))
69             Md = A[i];
70     }
71     for(int i = qL; Blk[i] == Blk[qL]; i++) --Bkt[A[i]];
72     for(int i = qR; Blk[i] == Blk[qR]; i--) --Bkt[A[i]];
73     return Md;
74 }
75 }
76
77 template<typename T>
78 inline void readint(T& x) {
79     T f=1, r=0; char c=getchar();
80     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
81     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
82     x = f*r;
83 }
84
85 void Init() {
86     readint(N); readint(Q);
87     for(int i=1; i<=N; i++) {
88         readint(A[i]);
89         Num[++Num[0]] = A[i];
90     }
91     sort(Num+1, Num+Num[0]+1);
92     Num[0] = unique(Num+1, Num+Num[0]+1) - Num - 1;
93     for(int i=1; i<=N; i++)
94         A[i] = lower_bound(Num+1, Num+Num[0]+1, A[i]) - Num;
95     InitBlk();
96 }
97
98 void Work() {
99     int l, r, LastAns = 0;
100     for(int i=1; i<=Q; i++) {
101         readint(l); readint(r);
102         l = (l + LastAns - 1) % N + 1;
103         r = (r + LastAns - 1) % N + 1;
104         if(l > r) swap(l, r);
105         printf("%d\n", LastAns = Num[Query(l, r)]);
106     }
107 }
108
109 int main() {
110     Init(); Work();
111     return 0;
112 }

```

## 15 多项式

(为何比别人多了 4 倍常数……)

### 15.1 快速傅里叶变换

Poly/FFT.cpp

```
1 #include <bits/stdc++.h>
2
3 const int MAXN = 4e6;
4 const double PI = 3.14159265358979323846264338;
5
6 struct cpx {
7     double real, imag;
8     cpx() { real = imag = .0; }
9     cpx(double x) { real = x, imag = .0; }
10    cpx(double x, double y) { real = x, imag = y; }
11    friend cpx operator+(const cpx &lsh, const cpx &rsh) {
12        return cpx(lsh.real + rsh.real, lsh.imag + rsh.imag);
13    }
14    friend cpx operator-(const cpx &lsh, const cpx &rsh) {
15        return cpx(lsh.real - rsh.real, lsh.imag - rsh.imag);
16    }
17    friend cpx operator*(const cpx &lsh, const cpx &rsh) {
18        return cpx(lsh.real * rsh.real - lsh.imag * rsh.imag,
19            lsh.imag * rsh.real + lsh.real * rsh.imag);
20    }
21    cpx operator*=(const cpx &rsh) { return (*this) = (*this) * rsh; }
22    cpx conj() const { return cpx(real, -imag); }
23    friend cpx operator/(const cpx &lsh, double rsh) {
24        return cpx(lsh.real / rsh, lsh.imag / rsh);
25    }
26    friend cpx operator/(const cpx &lsh, const cpx &rsh) {
27        cpx ret = lsh * rsh.conj();
28        ret.real /= (rsh.real * rsh.real + rsh.imag * rsh.imag);
29        ret.imag /= (rsh.real * rsh.real + rsh.imag * rsh.imag);
30        return ret;
31    }
32    cpx operator/=(const cpx &rsh) { return (*this) = (*this) / rsh; }
33 };
34
35 int n, m, R[MAXN+10]; double A[MAXN+10], B[MAXN+10], C[MAXN+10];
36
37 inline cpx get_rt(int step, bool inv) { // rotation factor
38     return inv ? cpx(std::cos(2*PI / step), -std::sin(2*PI / step)) :
39         cpx(std::cos(2*PI / step), std::sin(2*PI / step));
40 }
41
42 void fft(cpx A[], int len, bool inv) {
43     for(int i = 0; i < len; i++)
44         if(R[i] > i) std::swap(A[i], A[R[i]]);
45     for(int step = 1; step < len; step <= 1) {
46         for(int i = 0; i < len; i += (step<<1)) {
47             cpx omega = 1, rt = get_rt(step<<1, inv);
48             for(int j = 0; j < step; j++, omega *= rt) {
```

```

49         cpx t = omega * A[i+j+step];
50         A[i+j+step] = A[i+j] - t;
51         A[i+j] = A[i+j] + t;
52     }
53 }
54 }
55 if(inv)
56     for(int i = 0; i < len; i++) A[i] /= len;
57 }
58
59 void conv(double dA[], double dB[], int deg1, int deg2, double dC[]) { // deg: 输入多项式的度数
60     int len;
61     static cpx A[MAXN+10], B[MAXN+10], C[MAXN+10];
62     for(len = 1; len < deg1+deg2+1; len <= 1); // 考虑乘完后的长度
63     for(int i = 0; i < len; i++) {
64         A[i] = dA[i], B[i] = dB[i];
65     }
66
67     R[0] = 0;
68     for(int i = 1; i < len; i++)
69         R[i] = ((R[i>>1]>>1) | (len >> (i&1))) & (len-1);
70
71     fft(A, len, false); fft(B, len, false);
72     for(int i = 0; i < len; i++) C[i] = A[i] * B[i];
73     fft(C, len, true);
74     for(int i = 0; i < len; i++) dC[i] = C[i].real;
75 }
76
77 inline int readint() {
78     int f=1, r=0; char c=getchar();
79     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
80     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
81     return f*r;
82 }
83
84 int main() {
85     n = readint(); m = readint();
86     for(int i = 0; i <= n; i++) A[i] = readint();
87     for(int i = 0; i <= m; i++) B[i] = readint();
88     conv(A, B, n, m, C);
89     for(int i = 0; i <= n+m; i++) std::printf("%d ", int(round(C[i])));
90     return 0;
91 }

```

## 15.2 快速数论变换

998244353 的原根是 3

Poly/NTT.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 typedef long long int64;
5 const int MAXN = 4e6, MOD = 998244353, G = 3;
6

```

```

7 int n, m, A[MAXN+10], B[MAXN+10], C[MAXN+10], R[MAXN+10];
8
9 int64 fastpow(int64 a, int64 x) {
10     int64 ret = 1; a %= MOD;
11     while(x) {
12         if(x & 1) ret = ret * a % MOD;
13         x >>= 1; a = a * a % MOD;
14     }
15     return ret;
16 }
17
18 int get_rt(int step, bool inv) {
19     return !inv ? fastpow(G, (MOD-1) / step) : fastpow(G, (MOD-1) / step * (step-1));
20 }
21
22 void ntt(int A[], int len, bool inv) {
23     for(int i = 0; i < len; i++)
24         if(R[i] > i) swap(A[R[i]], A[i]);
25     for(int step = 1; step < len; step <= 1) {
26         for(int i = 0; i < len; i += (step < 1)) {
27             int64 omega = 1, rt = get_rt(step < 1, inv);
28             for(int j = 0; j < step; j++, omega = (omega * rt) % MOD) {
29                 int t = omega * A[i+j+step] % MOD;
30                 A[i+j+step] = ((A[i+j] - t) % MOD + MOD) % MOD;
31                 A[i+j] = (A[i+j] + t) % MOD;
32             }
33         }
34     }
35     if(inv) {
36         int64 inv_ele = fastpow(len, MOD-2);
37         for(int i = 0; i < len; i++) A[i] = (A[i] * inv_ele) % MOD;
38     }
39 }
40
41 void conv(int A[], int B[], int deg1, int deg2, int C[]) {
42     int len; for(len = 1; len < deg1+deg2+1; len <= 1);
43     R[0] = 0;
44     for(int i = 1; i < len; i++)
45         R[i] = ((R[i>>1]>>1) | (len >> (i & 1))) & (len-1);
46     ntt(A, len, false); ntt(B, len, false);
47     for(int i = 0; i < len; i++) C[i] = int64(A[i]) * B[i] % MOD;
48     ntt(A, len, true); ntt(B, len, true); ntt(C, len, true);
49 }
50
51 template<typename T>
52 inline void readint(T &x) {
53     int f=1, r=0; char c=getchar();
54     while(!isdigit(c)) { if(c=='-')f=-1; c=getchar(); }
55     while(isdigit(c)) { r=r*10+c-'0'; c=getchar(); }
56     x = f*r;
57 }
58
59 template<typename T>
60 inline void writeint(T &x) {
61     static char buf[32];
62     char *ptr = buf;
63     if(x < 0) putchar('-'), x = -x;

```

```

64     do {
65         *ptr++ = (x % 10) + '0';
66         x /= 10;
67     } while(x);
68     do
69         putchar(*--ptr);
70     while(ptr != buf);
71 }
72
73 int main() {
74     readint(n); readint(m);
75     for(int i = 0; i <= n; i++) readint(A[i]);
76     for(int i = 0; i <= m; i++) readint(B[i]);
77     conv(A, B, n, m, C);
78     for(int i = 0; i <= n+m; i++) writeint(C[i]), putchar(' ');
79     return 0;
80 }

```

## 16 数论

### 16.1 线性求逆元

推导 令  $p = ki + r (0 \leq r < i)$

则  $0 \equiv ki + r \pmod{p}$

$\Rightarrow ki \cdot i^{-1}r^{-1} + r \cdot i^{-1}r^{-1} \equiv 0$

$\Rightarrow i^{-1} \equiv -k \cdot r^{-1} \equiv p - \left\lfloor \frac{p}{i} \right\rfloor + p \bmod i \pmod{p}$

Math/inv.cpp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int MAXN = 3e6;
4 int n, p, inv[MAXN+10];
5 int main() {
6     cin >> n >> p;
7     inv[1] = 1; printf("%d\n", inv[1]);
8     for(int i = 2; i <= n; i++)
9         printf("%d\n", inv[i] = (p - (long long)(p / i) * inv[p % i] % p) % p);
10    return 0;
11 }

```

### 16.2 线性筛

#### 16.2.1 求 $\varphi(n)$

Math/phi.cpp

```

1 int64 GetPhi(int64 x) { // 单个数的 Phi
2     int64 ret = x;
3     for(int i = 2; i <= x; i++) if(x % i == 0) {
4         ret = ret / i * (i-1);
5         while(x % i == 0) x /= i;
6     }

```



```

7   if(x > 1)
8       ret = ret / x * (x-1);
9   return ret;
10 }
11
12 int PrimeCnt, Phi[MAXNUM+10], PrimeLst[MAXNUM+10], NotPrime[MAXNUM+10];
13 void PhiSieve() { // 线性筛 Phi
14     NotPrime[1] = true;
15     Phi[1] = 1;
16     for(int i = 2; i <= MAXNUM; i++) {
17         if(!NotPrime[i]) {
18             PrimeLst[++PrimeCnt] = i;
19             Phi[i] = i-1;
20         }
21         for(int j = 1; j <= PrimeCnt; j++) {
22             if(i * PrimeLst[j] > MAXNUM) break;
23             NotPrime[i * PrimeLst[j]] = true;
24             if(i % PrimeLst[j] == 0) {
25                 Phi[i * PrimeLst[j]] = Phi[i] * PrimeLst[j];
26                 break;
27             } else
28                 Phi[i * PrimeLst[j]] = Phi[i] * Phi[PrimeLst[j]];
29         }
30     }
31 }

```

### 16.2.2 求 $\mu(n)$

Math/mu.cpp

```

1  int PrimeCnt, PrimeLst[MAXNUM+10], NotPrime[MAXNUM+10], Mu[MAXNUM+10];
2
3  void MuSieve() {
4      NotPrime[1] = true;
5      Mu[1] = 1;
6      for(int i = 2; i <= MAXNUM; i++) {
7          if(!NotPrime[i]) {
8              PrimeLst[++PrimeCnt] = i;
9              Mu[i] = -1;
10         }
11         for(int j = 1; j <= PrimeCnt; j++) {
12             if(i * PrimeLst[j] > MAXNUM) break;
13             NotPrime[i * PrimeLst[j]] = true;
14             if(i % PrimeLst[j] == 0) {
15                 Mu[i * PrimeLst[j]] = 0;
16                 break;
17             } else
18                 Mu[i * PrimeLst[j]] = -Mu[i];
19         }
20     }
21 }

```

## 16.3 扩展欧几里得定理

```

1 void exgcd(int a, int b, int &d, int &x, int &y) {
2     if(b == 0) { d = a, x = 1, y = 0; }
3     else {
4         exgcd(b, a % b, d, y, x); y -= x * (a / b);
5     }
6 }

```

## 16.4 中国剩余定理

[https://blog.csdn.net/ruoruo\\_cheng/article/details/52075213](https://blog.csdn.net/ruoruo_cheng/article/details/52075213)

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 typedef long long int64;
5 const int MAXN = 1e5;
6 int n; int64 a[MAXN+10], m[MAXN+10];
7
8 void exgcd(int64 a, int64 b, int64 &d, int64 &x, int64 &y) {
9     if(b == 0) { d = a, x = 1, y = 0; }
10    else {
11        exgcd(b, a % b, d, y, x); y -= x * (a / b);
12    }
13 }
14
15 int64 china(int n, int64 a[], int64 m[]) {
16     int64 M = 1, ret = 0, Mi, Minv, d, y;
17     for(int i = 1; i <= n; i++)
18         M *= m[i];
19     for(int i = 1; i <= n; i++) {
20         Mi = M / m[i];
21         exgcd(Mi, m[i], d, Minv, y);
22         assert(d == 1);
23         ret = (ret + Mi * Minv % M * a[i] % M) % M;
24     }
25     ret = (ret + M) % M;
26     return ret;
27 }
28
29 int main() {
30     scanf("%d", &n);
31     for(int i = 1; i <= n; i++)
32         scanf("%lld%lld", &a[i], &m[i]);
33     printf("%lld\n", china(n, a, m));
34     return 0;
35 }

```

## 16.5 扩展欧拉定理

$$a^b \equiv a^{b \bmod \varphi(n) + [b \geq \varphi(n)] \cdot \varphi(n)} \pmod{n}$$

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int MAXP = 1e7;
4
5 int T, p, phi[MAXP+10];
6
7 void InitPhi() {
8     phi[1] = 1; //do not set phi[2], for it is a prime
9     for(int i=2; i<=MAXP; i++)
10         if(!phi[i]) {
11             for(int j=i; j<=MAXP; j+=i) {
12                 if(!phi[j]) phi[j] = j;
13                 phi[j] = phi[j] / i * (i-1);
14             }
15         }
16 }
17
18 inline int fastmul(int a, int x, int mod) {
19     int ret = 0;
20     while(x) {
21         if(x&1) ret = ((ret%mod) + (a%mod))%mod;
22         x>>=1; a = ((a%mod) + (a%mod)) %mod;
23     }
24     return ret;
25 }
26
27 inline int fastpow(int a, int x, int mod) {
28     int ret = 1;
29     while(x) {
30         if(x&1) ret = fastmul(ret, a, mod) % mod;
31         x>>=1; a = fastmul(a, a, mod) % mod;
32     }
33     return ret;
34 }
35
36 int solve(int mod) {
37     if(mod == 1) return 0;
38     return fastpow(2, solve(phi[mod])+phi[mod], mod);
39 }
40
41 int main() {
42     InitPhi();
43     scanf("%d", &T);
44     while(T--) {
45         scanf("%d", &p);
46         printf("%d\n", solve(p));
47     }
48 }

```

## 16.6 Lucas 定理

```

1 #include <bits/stdc++.h>
2 using std::cin;

```

```

3 using std::cout;
4
5 typedef long long int64;
6 const int MAXN = 1e5;
7 int T, n, m, p, fact[MAXN+10], factinv[MAXN+10];
8
9 int C(int n, int m) {
10     if(m < 0 || m > n) return 0;
11     return ((int64)fact[n] * factinv[m]) % p * factinv[n-m] % p;
12 }
13
14 int lucas(int n, int m) {
15     if(n < p && m < p) return C(n, m);
16     return C(n % p, m % p) * (int64)lucas(n / p, m / p) % p;
17 }
18
19 inline int64 fastpow(int64 a, int64 x) {
20     int64 ret = 1;
21     while(x) {
22         if(x & 1) ret = ret * a % p;
23         x >>= 1; a = a * a % p;
24     }
25     return ret;
26 }
27
28 void init_inv() {
29     fact[0] = factinv[0] = 1;
30     for(int i = 1; i <= p-1; i++)
31         fact[i] = (int64)fact[i-1] * i % p;
32     factinv[p-1] = fastpow(fact[p-1], p-2);
33     for(int i = p-2; i >= 1; i--)
34         factinv[i] = factinv[i+1] * (int64)(i+1) % p;
35 }
36
37 int main() {
38     cin >> T;
39     while(T--) {
40         cin >> n >> m >> p;
41         init_inv();
42         cout << lucas(n+m, m) << '\n';
43     }
44     return 0;
45 }

```