

Subject: Power Systems - II

Code: EEC 503

Broad Topics:

1. Protective Relays,
2. Protective Relaying Schemes,
3. Short Circuit Calculations,
4. Systems of Bus-bars and
5. Circuit Interruption Devices.

Books:

1. The Art and Science of Protective Relaying

By: C. R. Mason I

2. Protective Relays: Their Theory and Practice

By: A. R. Van C. Warrington

3. Switchgear and Protection

By: S. S. Rao

4. Power System Protection

By: Patra, Basu and Choudhuri

Basic requirements of Relay Protection

- Energy is generated in the generating stations and this energy is made available to the consumers by the supply system.
- We usually think of a power system in terms of its more impressive parts – big generating units, transformers, high voltage transmission lines, low voltage distribution lines, loads etc.
- Apart from these elements there are many other important and fascinating components and **Protective Relay** is one of them.

Functions of Protective Relaying

- The basic function of protective relaying is to cause prompt removal of any element of a power system from service that has suffered fault or has started to operate in abnormal manner. If it is not removed, then
 - Damage of the element itself.
 - Damage of other element(s) connected with it.
 - Interference with the effective operation of the power system.
- However,

The relay alone can't remove the faulty element. It is aided in this task by the circuit breakers.

Contd

- A second function of protective relay is to provide the location of the fault and type of failure.
- To expedite the repair and maintenance process.

What is a circuit breaker? I

- Basically, it is a switching element.
- It is capable of disconnecting a faulty element.
- It operates only when it receives a signal from the associated relay.

Location of relays & circuit breakers

- They are located such that each generator, transformer, transmission line, bus-bar etc. can completely be disconnected from the rest of the system.

I

The circuit breakers must have sufficient current capacity, so that,

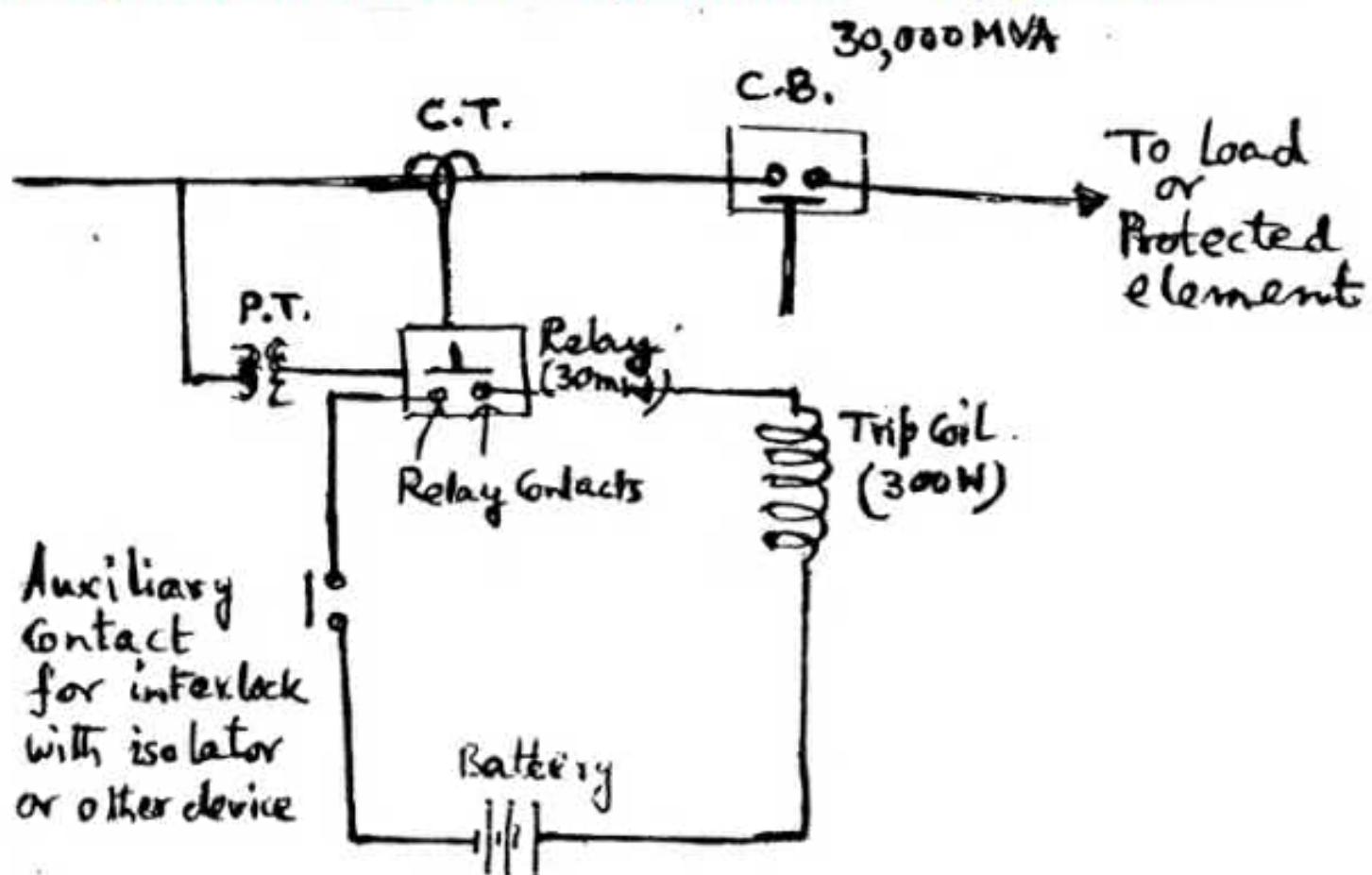
- They can carry momentarily the large short circuit current before interrupting it.
- They can close on a short circuit and to then disconnect it.

*** Important to remember**

- A protective relay does not prevent the occurrence of a fault. It can take action only after the fault has occurred.



Relay & C.B. work in conjunction – “Trip Circuit”



Contd

On occurrence of a fault:

- The circuit current increases.
- The system voltage decreases.
- These changes are sensed by the relay through CT & PT.
- The relay closes its contacts.
- Then, current flows through the trip circuit.
- This energizes the trip coil of the C.B.
- The C.B. operating mechanism is actuated.
- The C.B. contacts open to disconnect the protected element.
- The auxiliary contacts also open with the opening of the C.B. contacts to interrupt the trip circuit current.

Definition of 'Fault'

- A fault in an electrical equipment is defined as a defect in its electrical circuit due to which the flow of current is diverted from the intended path.

- Fault is due to failure of insulation between
 - Phase conductor and ground.
 - Between different phases.

Contd ...

- Consequences of a fault
 - Large flow of current.
 - Reduction of system voltage.
 - Damage of power system element(s).
 - Influence on system operation.



Fault Statistics

Helps us in considering the level and design of protection.

P. S. Element	Cause of fault	% of Occurrence
Overhead lines	<ol style="list-style-type: none"> 1. Lightning strokes. 2. Storms, earthquakes. 3. Internal over-voltages. 4. Birds, trees. 	30-40
Underground cables	<ol style="list-style-type: none"> 1. Damage during digging. 2. Insulation failure due to temperature rise. 3. Failure of joints. 	8-10
Alternators	<ol style="list-style-type: none"> 1. Stator faults. 2. Rotor faults. 3. Faults in associated equipment. 4. Faults in protective systems. 	6-8

Contd	1. Insulation failures. 2. Faults in tap-changers. 3. Faults in bushing. 4. Over voltage, over load. 5. Faults in protective systems.	
Transformers		10-12
C.T. & P.T.	1. Over voltage. 2. Insulation failures. 3. Breaking of conductors. 4. Wrong connections.	15-20
Switchgear	1. Insulation failures. 2. Mechanical defects. 3. Leakage of air/oil/gas. 4. Inadequate rating. 5. Lack of maintenance.	10-12

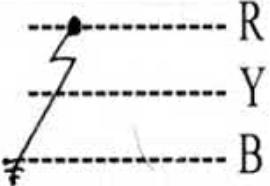
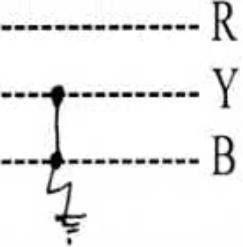
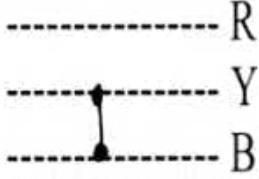
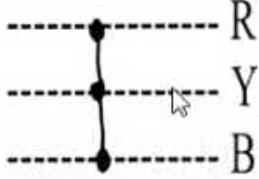
Faults in overhead lines – types & % occurrence

1. Shunt faults – across the line.
2. Series faults – in series with the line.
3. Simultaneous faults.

Shunt faults - involving

- a) One phase and ground (L-G) - 85%
- b) Two phases and ground (L-L-G) – 5%
- c) Two phases (L-L) – 8%
- d) Involving three phases (L-L-L) – 2%



Type of Fault	Symbolic Representation (all these are shunt faults)	Percentage occurrence
1. 1-phase to ground (L-G)		85
2. 2-phase to ground (L-L-G)		5
3. 2-phase (L-L)		8
4. 3-phase (L-L-L)		2 or less

Series faults



- a) One conductor open &
- b) Two open conductors.
- They occur very rarely.

1. One open conductor	<p>Open</p> <p>-----o o----- R</p> <p>----- Y</p> <p>----- B</p>
2. Two open conductors	<p>-----o o----- R</p> <p>Open</p> <p>-----o o----- Y</p> <p>----- B</p>

Simultaneous faults



These may occur in various combinations, - e.g.

- Shunt faults involving same/different phases
 - L-G faults in two different locations.
 - L-L faults between two phases and L-G fault in the other at other place.
- Simultaneous occurrence of series and shunt faults- e.g. breaking of one conductor near insulator – causes
 - one end to hang in air – one conductor open and
 - The other end touches the ground – L-G fault.

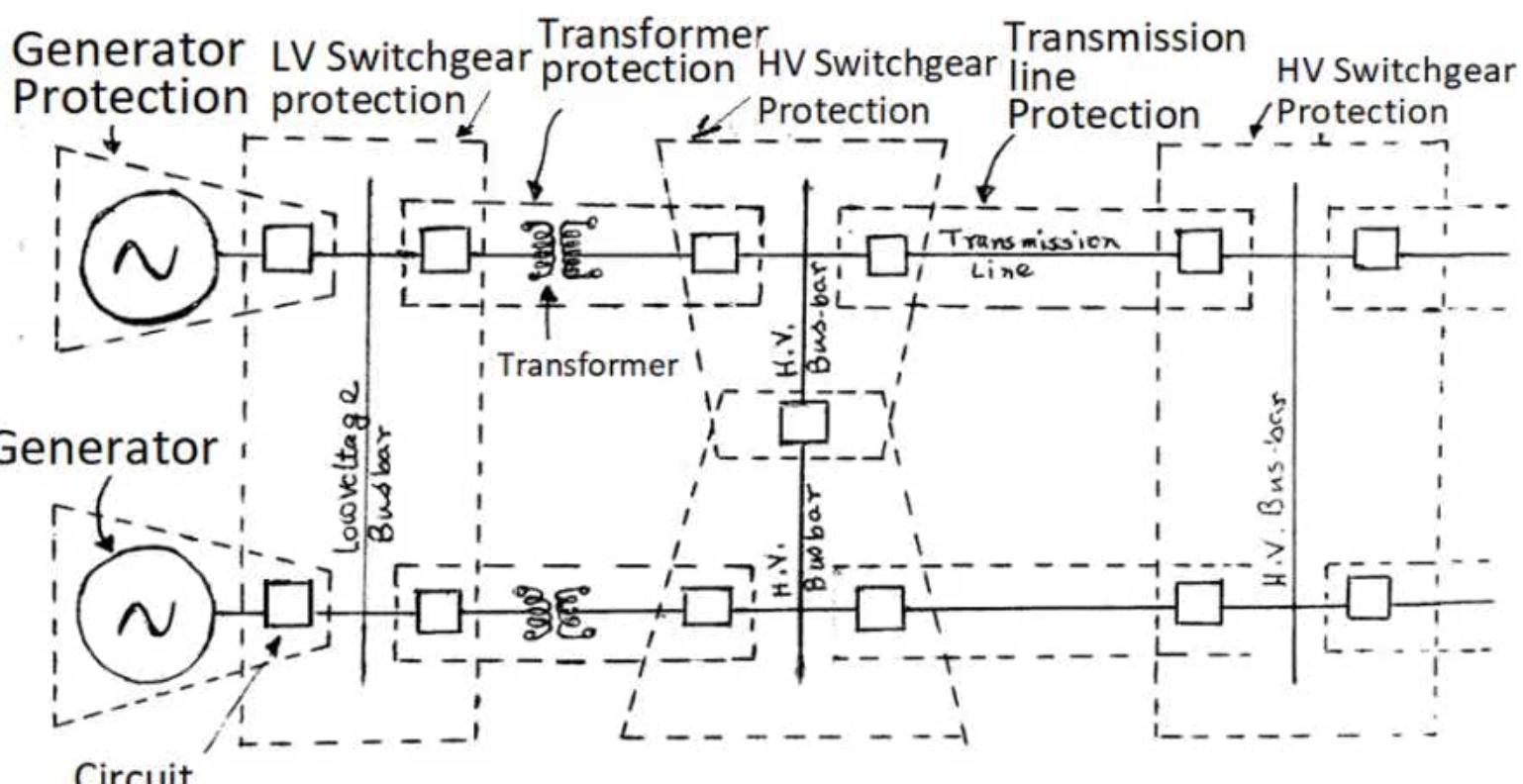
Primary and Back-up Relaying

- Protective relaying may categorized into two types:
 1. Primary relaying – first line of defense, responsible for protection of all power system elements.
 2. Back-up relaying – operates only when primary relaying fails, disconnects much larger area of power system.

Primary relaying

- Features –
 - C.B.s located in connection to each power system element.
 - A separate zone of protection created around each element.
 - Two adjacent zones overlap each other around a C.B.

Contd



Circuit
breaker

Fig. One line diagram of a portion of an electric power system illustrating primary relaying

Primary relaying may fail due to -



- Failure of the C.B.
- Failure of the relay itself.
- Failure of the d.c. tripping voltage.
- Discontinuity of the trip circuit.
- Failure of C.T. or P.T.
- Loss of voltage or current supply to relay due to fuse failure.

Back-up relaying

It should be arranged such that anything might cause
failure of primary relaying should not cause failure of
back-up relaying.

- The above condition may be satisfied if primary and back-up relays are located at different substations – may not always be possible.

Back-up relaying types

1. Remote back-up and
2. Local back-up

Remote back-up

- Features -
 - It operates from neighbouring substation.
 - It is entirely independent of local supplies.
 - It disconnects larger area of power system.

Illustration of remote back-up

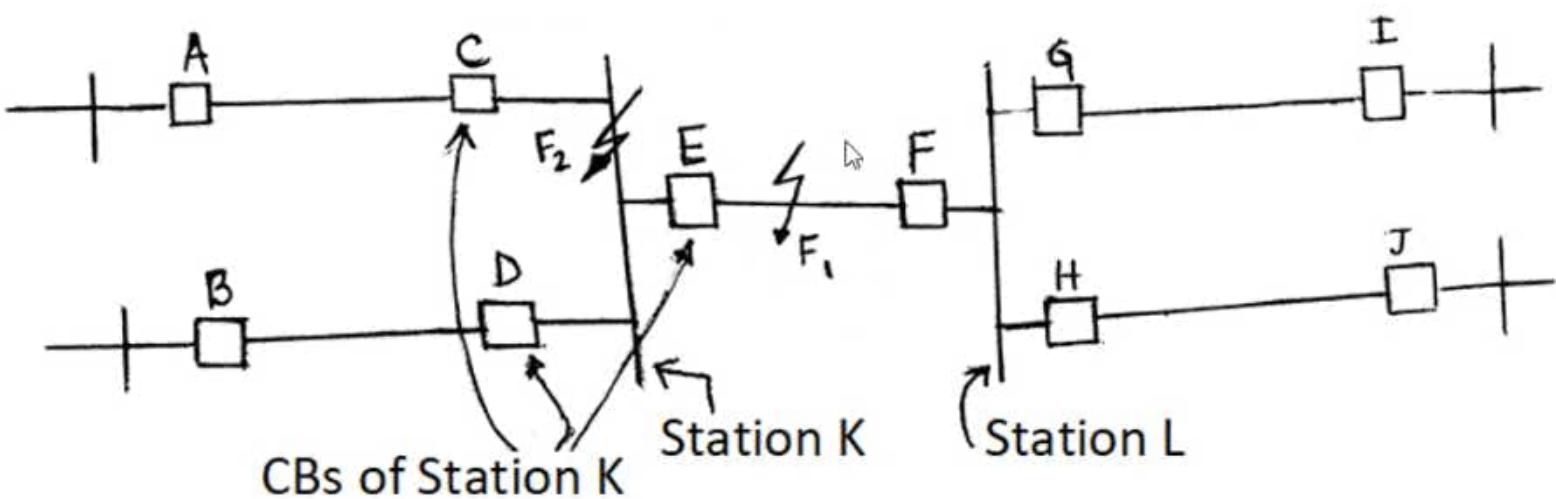


Fig. Illustration of remote back-up protection

Local back-up

Types of Local Back-up

1. Relay back-up and
2. Breaker back-up

Relay back-up

- Features –
 - Two relays work in parallel.
 - Their contacts connected in parallel to the trip circuit of same C.B.
 - Operating principle different for the two relays.

Illustration of Local Back-up

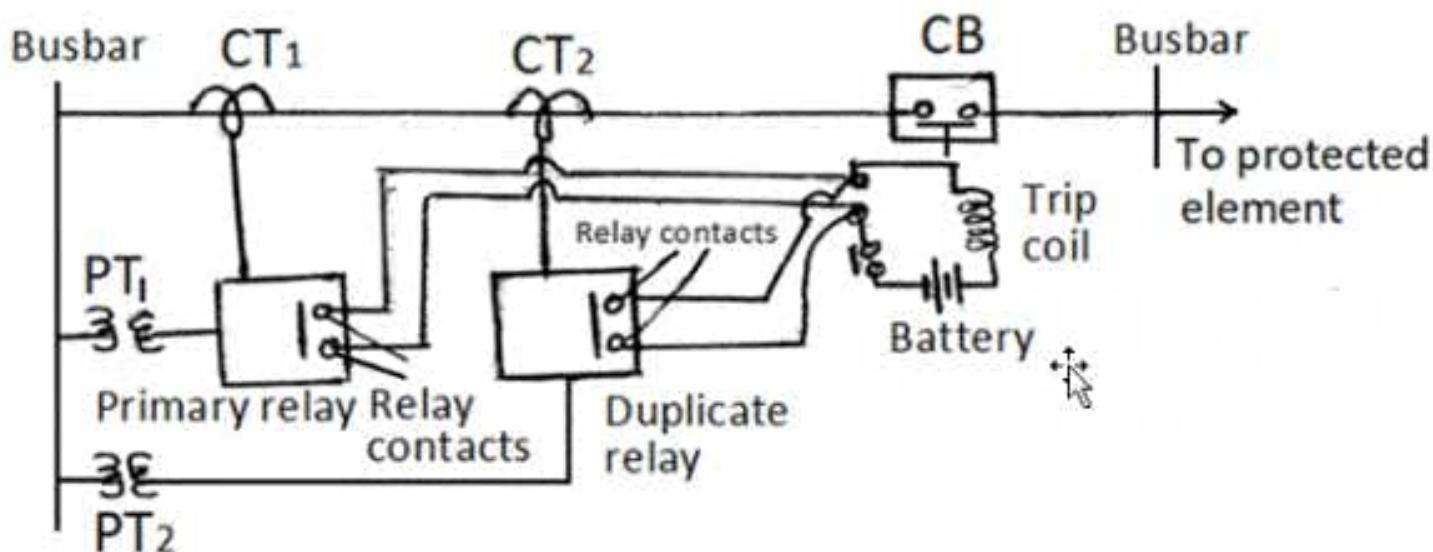


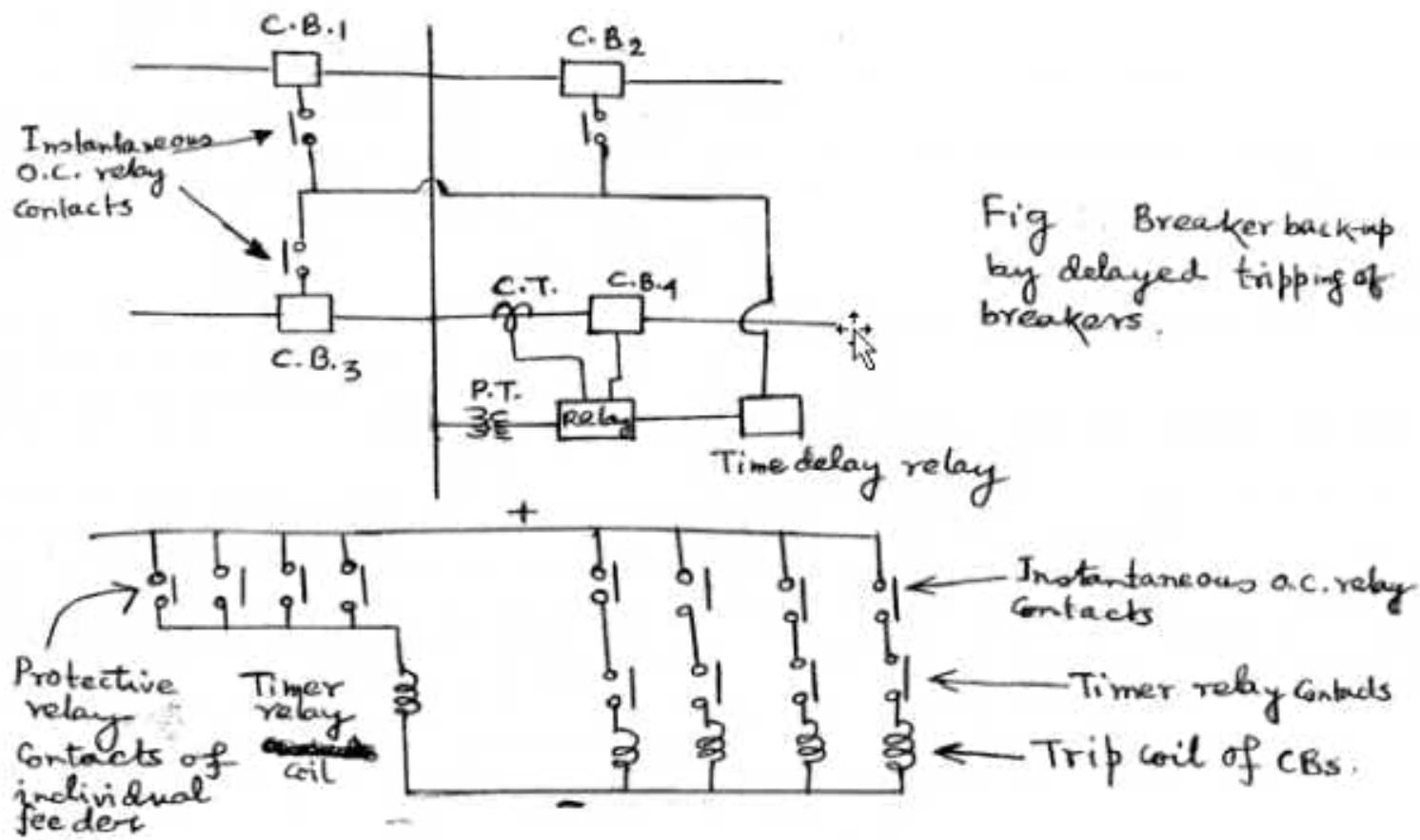
Fig. Relay back-up scheme using duplicate relay
(Duplicate CT & PT have been used)

Breaker Back-up

- Features –
- When a C.B. fails to operate after its own relay has operated, it is treated as bus fault.
- The same relay contacts are used to energize a time delay relay or timer relay.
- The timer relay operates all other circuit breakers connected to that bus after a requisite time.
- The Instantaneous o.c. relay takes care any malfunction of the timer relay.



Illustration of Breaker Back-up



Classification of Electrical Protective Relays

May be classified in two categories

1. Electromagnetic Relays and
2. Static Relays.

Principal Types of Electromagnetic Relays

1. Attracted Armature Type and
2. Induction Type.

Attracted Armature Type Relays

- Points to note –
- Simplest type of relays.
- These relays have a coil or electromagnet, actuated by system current or voltage.
- A plunger or a rotating vane is subjected to action of magnetic field.
- These are basically single actuating quantity relays.

Types of Attracted Armature Relays

1. Plunger type,
2. Hinged armature type,
3. Balanced beam type and
4. Polarized moving iron type.

Fig. Plunger type construction.

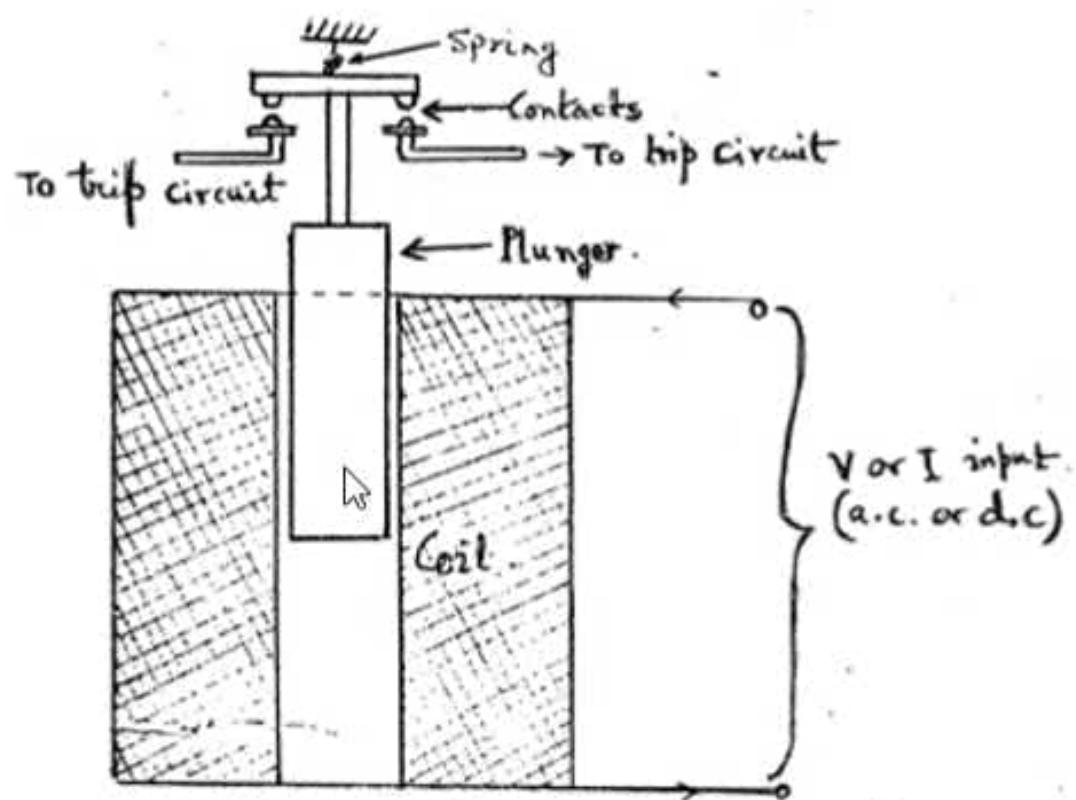


Fig. Hinged Armature type construction no. 1.

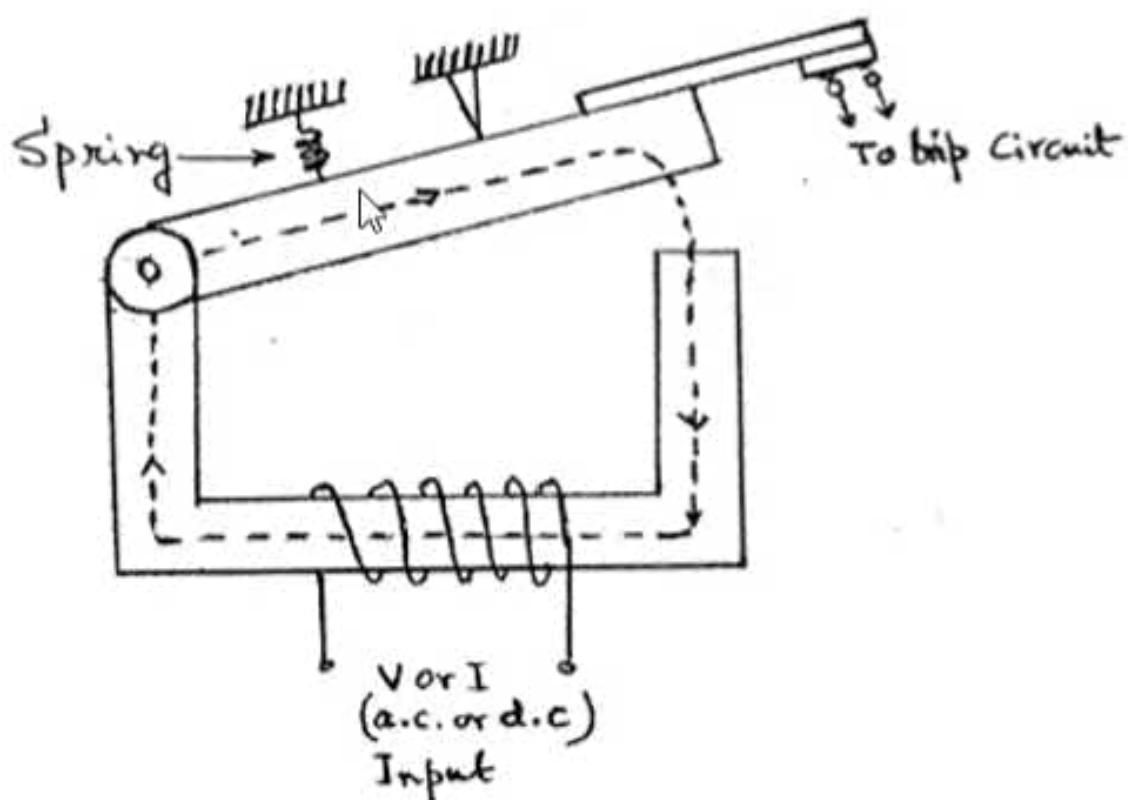


Fig. Hinged Armature type construction no. 2.

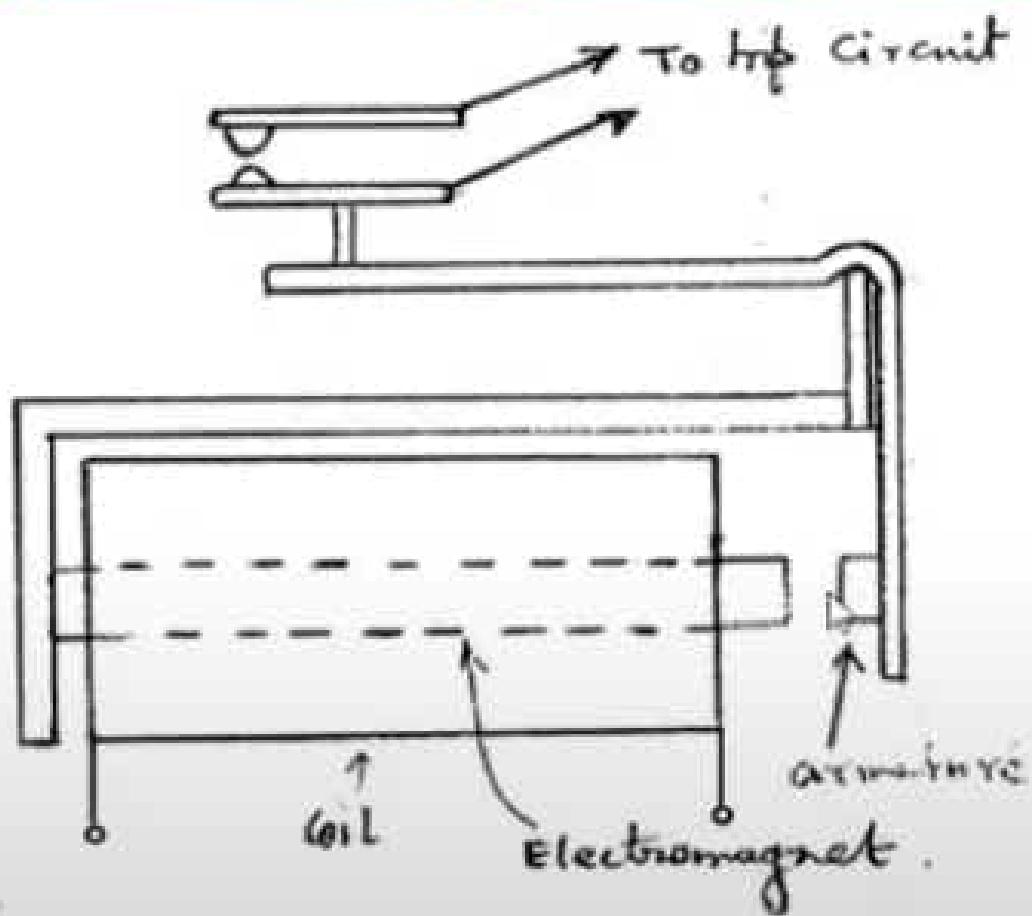


Fig. Balanced Beam type construction.

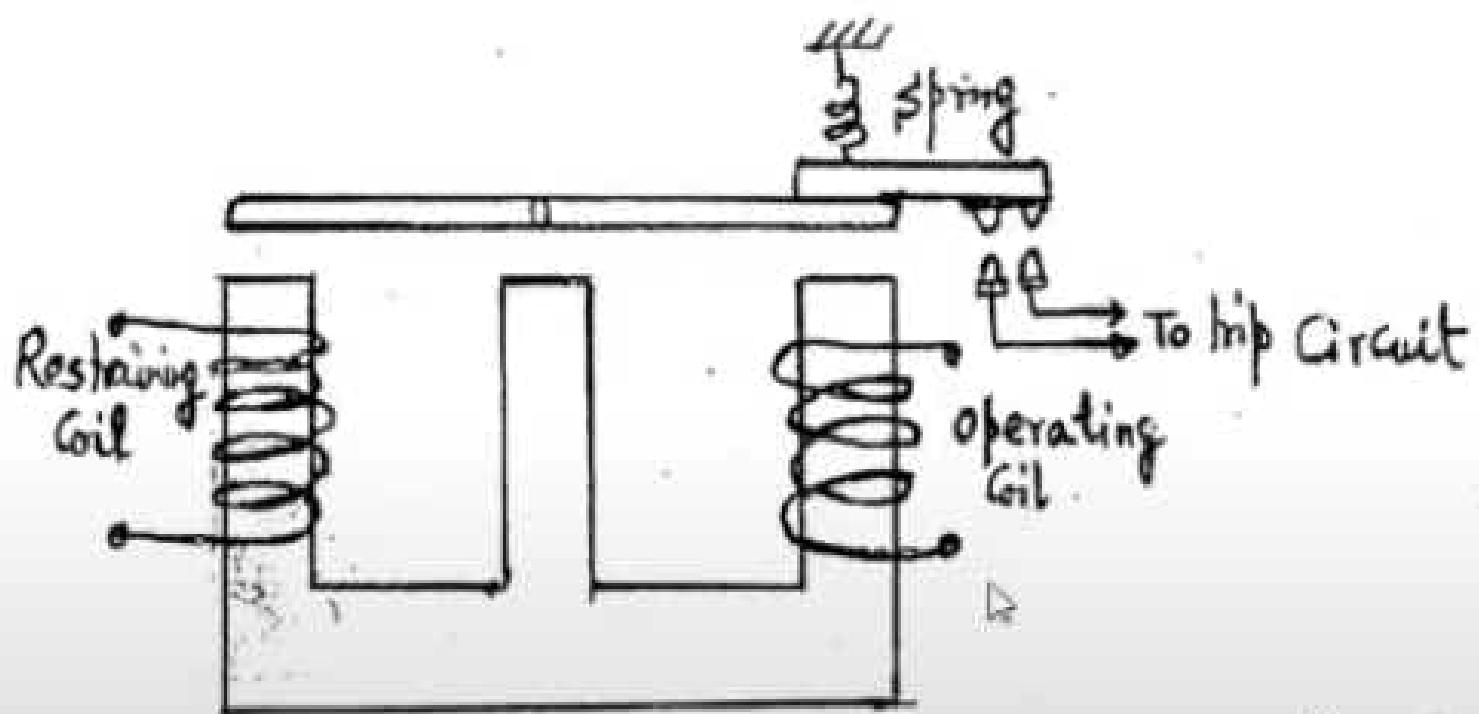
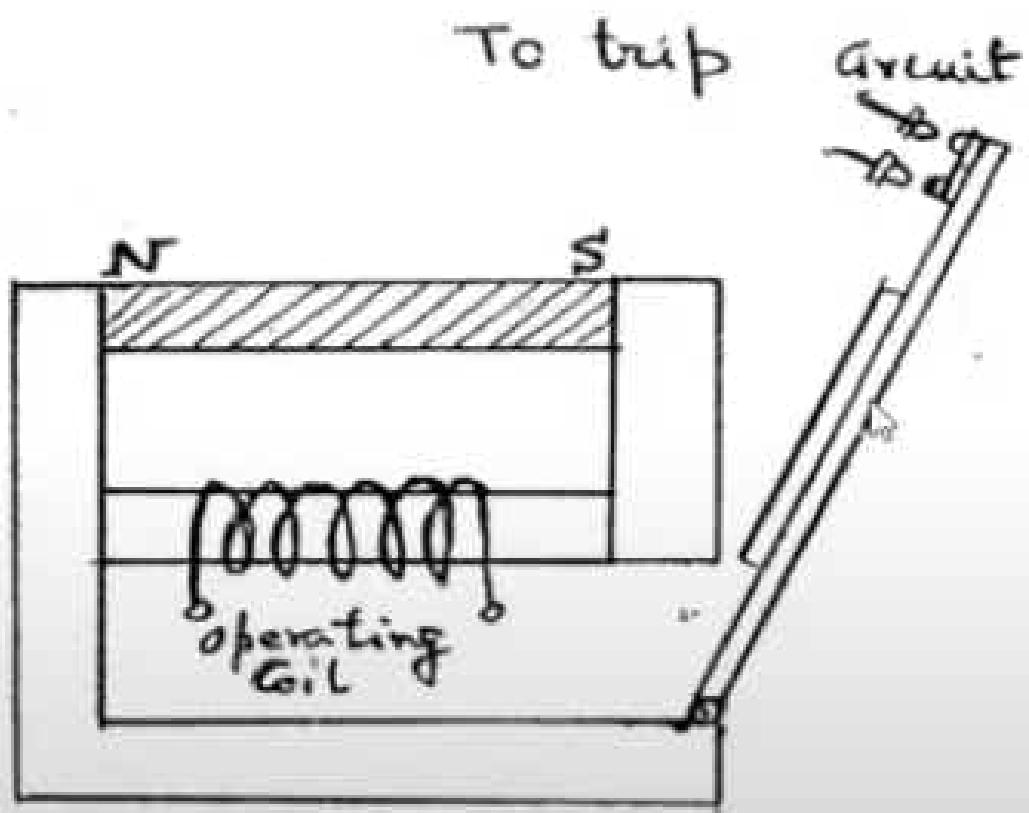


Fig. Polarized Moving Iron type construction



Features -

- Respond to both a.c. and d.c.
- Very fast in operation – described as instantaneous type – modern relay may have operating time of 0.5ms.
- Operating time doesn't vary with current in operating coil of the relay.
- Do not have directional features.
- Small VA burden, depends on construction, settings etc. – typically 0.2VA - 0.6VA, for current range 0.1A - 0.4A.
- Modern attracted armature relays are compact, robust and reliable.

Operating Principle



Electromagnetic force exerted on the moving element is proportional to the square of the flux in the air gap. If saturation is neglected it is proportional to the square of the operating current.

Thus,

$$F = F_e - F_r = K_1 I^2 - K_2$$

where,

- F = Net force,

Contd ...

- F_e = Electromagnetic force = $K_1 I^2$ = Operating force,
- K_1 = a constant,
- F_r = Restraining or opposing force = K_2 = a constant.

When the relay is on the verge of operation, the Electromagnetic force (F_e) and the Restraining force (F_r) are equal, i.e. 

$$F = F_e - F_r = K_1 I^2 - K_2 = 0$$

Contd ...

$$\text{Or, } I = \sqrt{\frac{K_2}{K_1}} = \text{Constant.}$$

Conclusions,



- For the above value of I , the operating and restraining forces are equal.
- If value of I becomes more than $\sqrt{\frac{K_2}{K_1}}$, the relay operates.

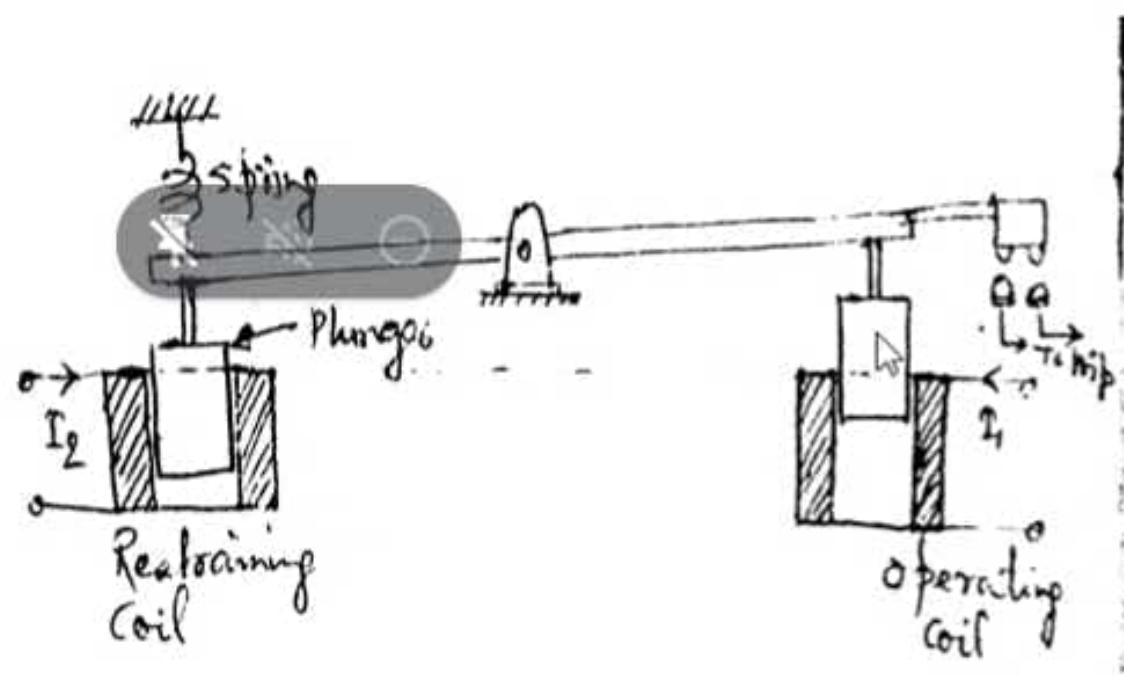


Balanced Beam Relay

Points to note –

- Consists of a horizontal beam pivoted centrally.
- One coil attached to each side of the beam.
- One coil develops operating force and the other develops restraining force.
- The beam remains horizontal till the operating force becomes more than restraining force.
- The action is similar to “See Saw” in children’s park – where the heavier side goes down.
- The coils act like playing children – if the operating force is more, that side goes down to close the relay contacts.

Fig. Balanced Beam Relay



Operating Principle

Neglecting the effect of spring, the net force F is,

$$F = F_e - F_r = K_1 I_1^2 - K_2 I_2^2$$

where,

I_1 = Current in operating coil,

I_2 = Current in restraining coil and

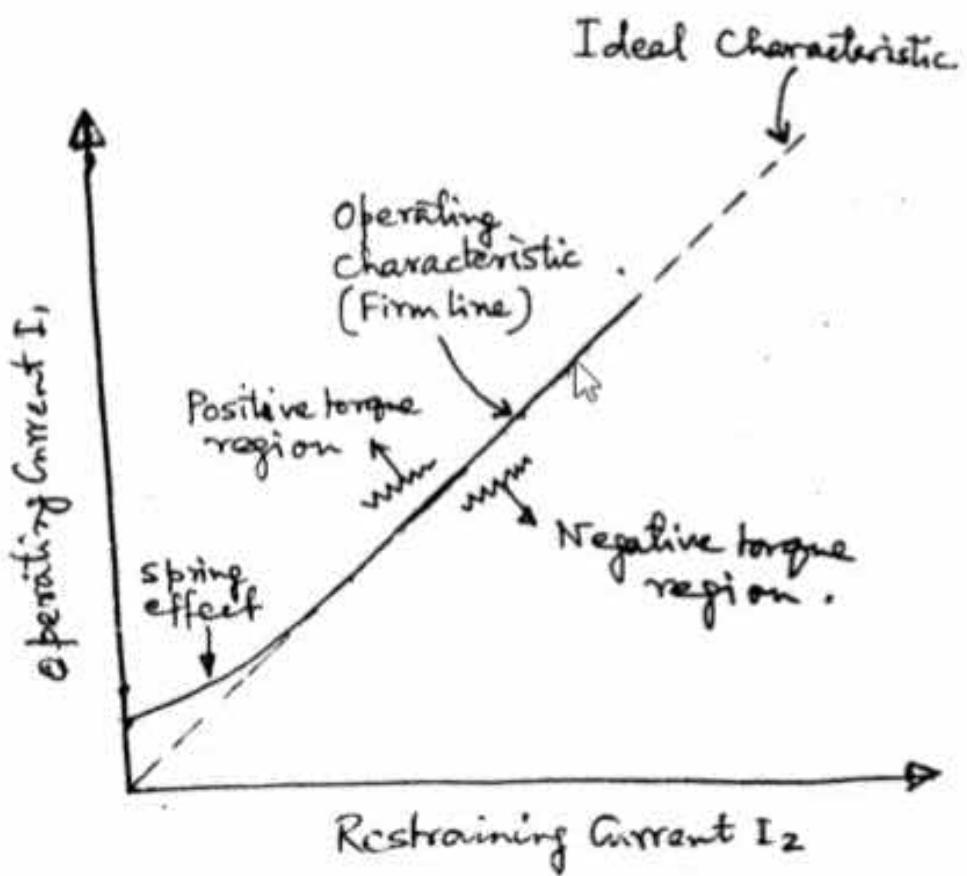
K_1, K_2 = constants.

When the relay is on the verge of operation,

$$F = F_e - F_r = K_1 I_1^2 - K_2 I_2^2 = 0$$

Or, $\frac{I_1}{I_2} = \sqrt{\frac{K_2}{K_1}} = \text{Constant.}$



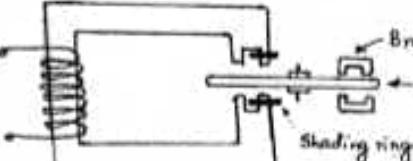


Induction Relays

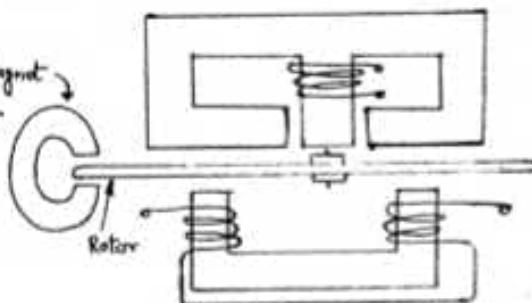
Operating Principle

- Two magnetic fluxes φ_1 and φ_2 differing in time phase penetrate through a disc.
- φ_1 and φ_2 induce e.m.f.s e_1 and e_2 in the disc, which lag behind respective flux by 90° .
- e_1 and e_2 lead to flow eddy currents i_1 and i_2 respectively.
- i_1 and i_2 lag behind the e_1 and e_2 by impedance angle of the disc.
- By the interaction of φ_2 with i_1 and φ_1 with i_2 a driving force is produced.

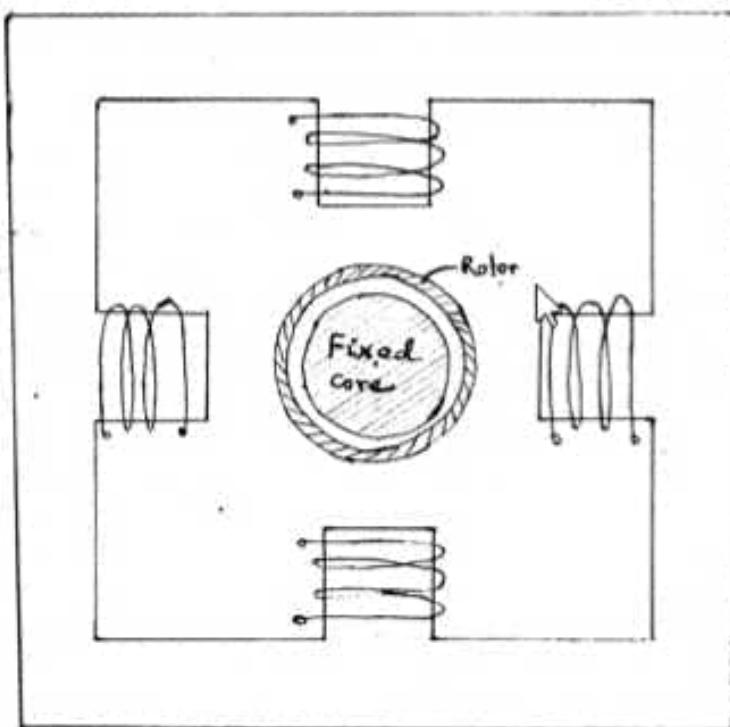
Types:



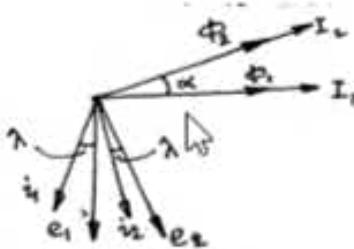
Shaded pole type induction
disc-relay



Wattmetric type induction
disc-relay



Induction
Cup type
relay.



Let, $\phi_1 = \phi_{m_1} \sin \omega t$

$$\phi_2 = \phi_{m_2} \sin(\omega t + \alpha)$$

where, α is the phase angle by which ϕ_2 leads ϕ_1 .

$$\therefore e_1 \propto \frac{d\phi_1}{dt} \propto \omega \cdot \phi_{m_1} \cos \omega t$$

$$\& e_2 \propto \frac{d\phi_2}{dt} \propto \omega \cdot \phi_{m_2} \cos(\omega t + \alpha)$$

$$\therefore i_1 \propto \omega \phi_{m_1} \cos(\omega t - \lambda)$$

$$\& i_2 \propto \omega \phi_{m_2} \cos(\omega t + \alpha - \lambda)$$

Therefore the resultant force is given by:

$$F = F_2 - F_1 \propto (\phi_2 \cdot i_1 - \phi_1 \cdot i_2)$$

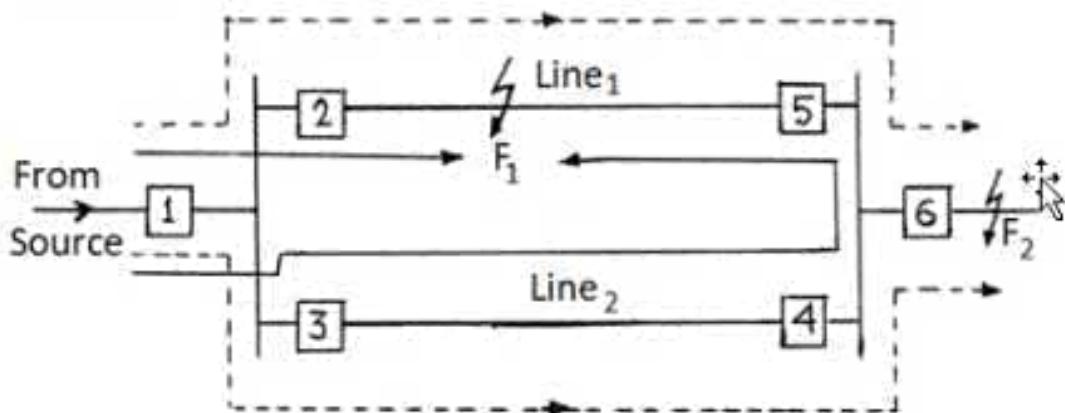
$$\propto \omega \phi_{m_1} \cdot \phi_{m_2} \left[\sin(\omega t + \alpha) \cos(\omega t - \lambda) - \sin \omega t \cdot \cos(\omega t + \alpha - \lambda) \right]$$

$$\propto \omega \cdot \phi_{m_1} \cdot \phi_{m_2} \cdot \sin \alpha \cdot \cos \lambda$$

$$\text{or, } F \propto \phi_{m_1} \cdot \phi_{m_2} \cdot \sin \alpha.$$

Directional Relay

- Consider the part of the Power System, with two parallel feeders/lines.



- Further, consider two conditions.
 - For normal condition or for a fault at F_2 , the current has two parallel paths –
 - From the source – through CB1- CB2- CB5- CB6 and the other
 - From the source – through CB1– CB3 – CB4 – CB6.

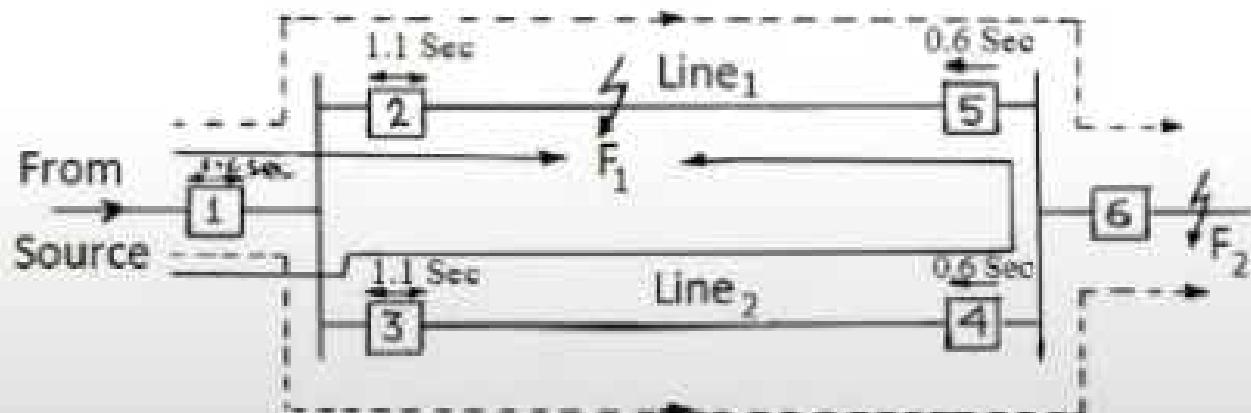
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2. For a fault at F_1 , again, the I current has two parallel paths –
- a) From the source – through CB1- CB2 – to the fault F_1 and the other
 - b) From the source – through CB1– CB3 – CB4 – CB5 - to the fault F_1 .

It is desirable to disconnect the faulty line only.

Contd...

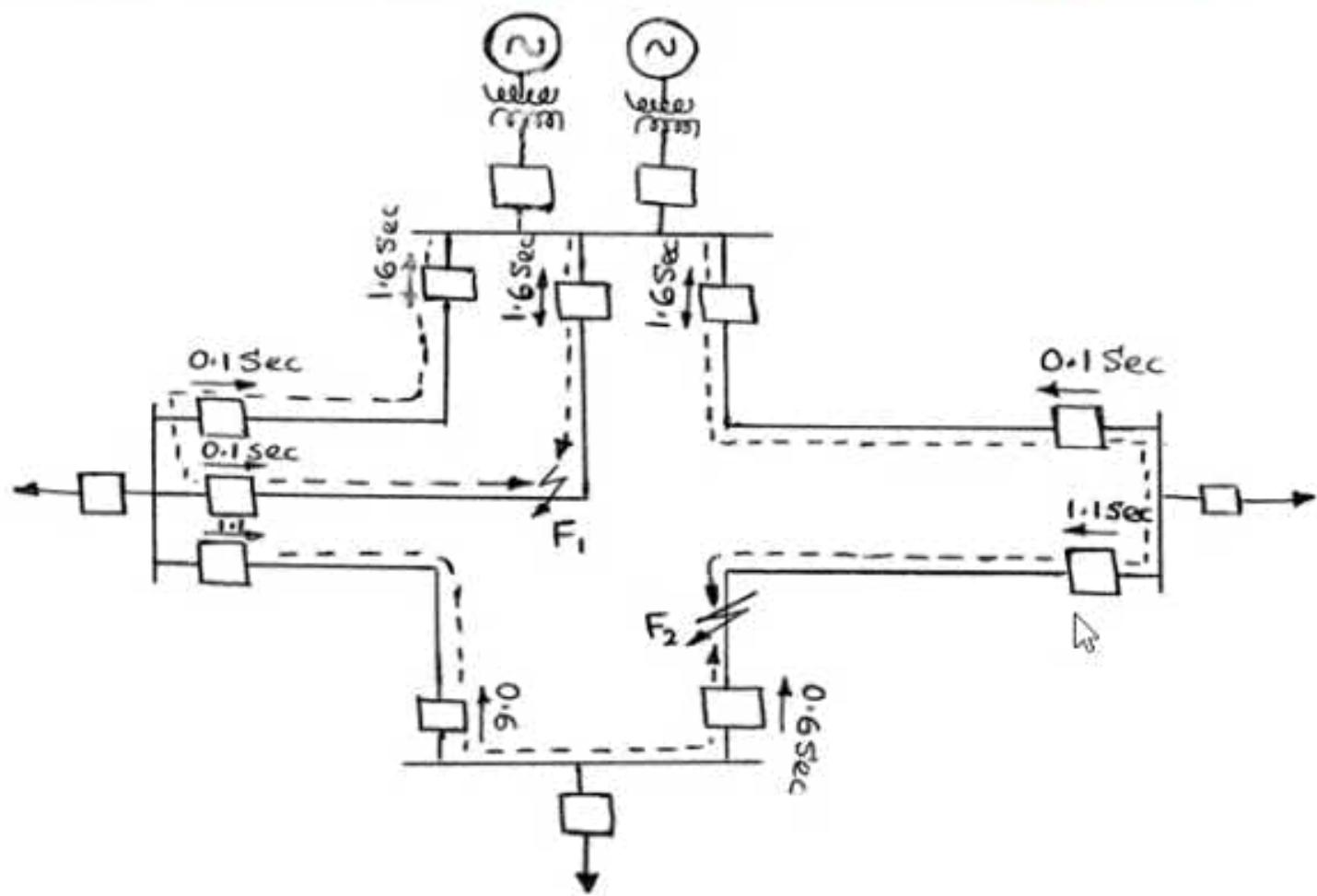
- This problem can be avoided by –
 - 1. Use of Directional Relays at receiving end and
 - 2. Use of suitable time setting of the relays.



Contd....

- Main feature of Directional Relay –
 - a) Doesn't operate for flow of current in normal direction.
 - b) Operates only when flow of current reverses through it, i.e. when current flows from bus bar to the line -> See the arrows with CB4 and CB5.
 - c) It also senses the magnitude of current.

Use of Directional Relay in interconnected System



Types of Directional Relays

Directional relays may be of three types -

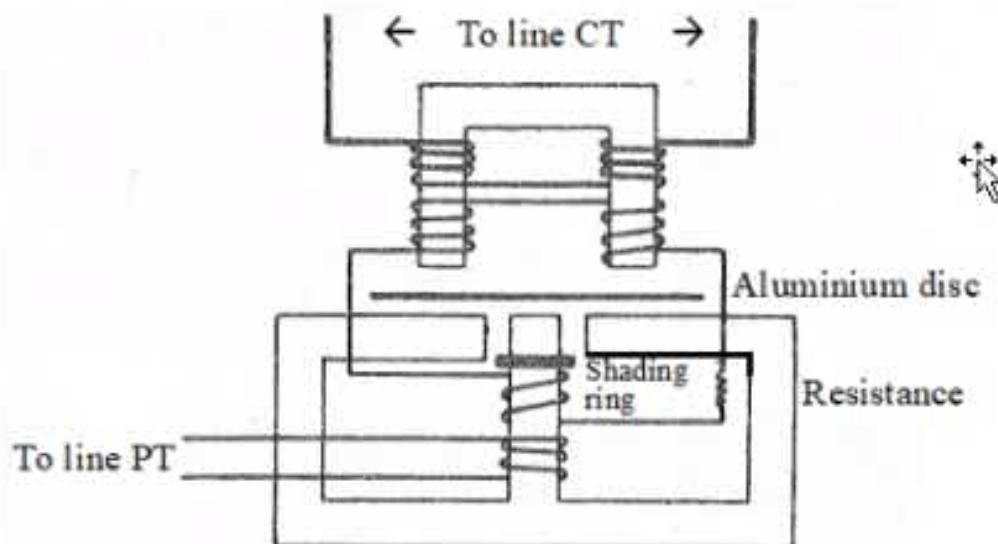
1. Directional over current,
2. Directional over power and
3. Directional earth fault.



In directional relay, there are two coils

1. Current coil – connected across secondary of C.T. and
2. Voltage coil - connected across secondary of P.T.

Hence, "Induction Disc, Watthour Meter" type construction is generally used.



Theory of Directional Relay

Let,

V = Voltage applied to the voltage coil of the relay,

z_v = Impedance of the voltage coil,

$I_V = V/z_v$ = Current in the voltage coil,

φ_V = Flux produced by I_V ,

I = Current in the current coil of the relay,

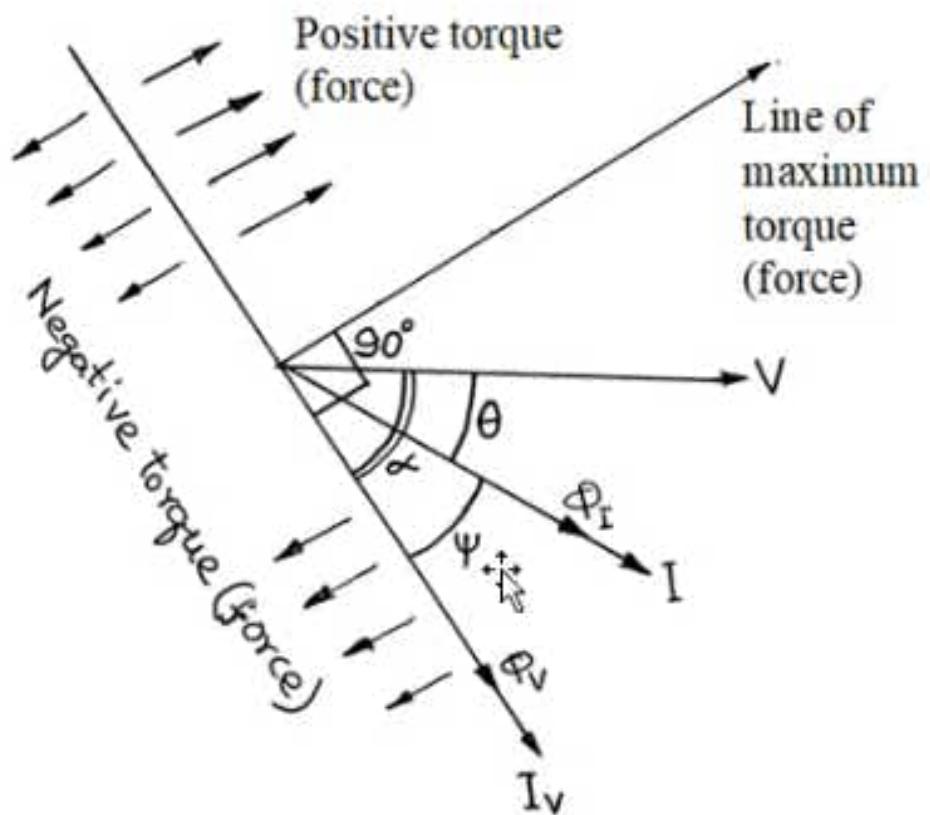
φ_I = Flux produced by I ,

α = Impedance angle of the voltage coil and

ψ = Angle between I and I_V .

I

Vector Diagram of Directional Relay



According to theory of Induction Relay, force due to interaction of two fluxes is given by

$$\begin{aligned} F &\propto \varphi_I \varphi_V \sin\psi \\ &\propto I V \sin\psi \\ &\propto I V \sin(\alpha - \theta) \end{aligned}$$

Hence, operating force of Directional Relay is proportional to

- Voltage applied to voltage coil of the relay,
- Current through the current coil of the relay and
- Sine of the angle $\psi = (\alpha - \theta)$.

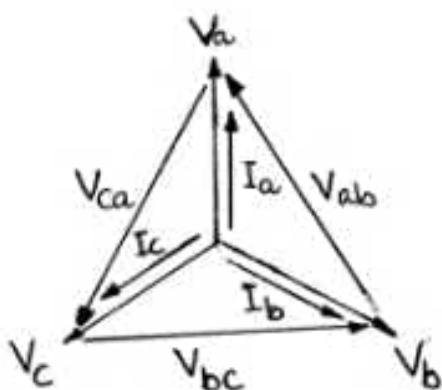
Points to note –

1. Operating force is positive if ψ remains between 0° to 180° ,
2. Operating force is negative if ψ remains between 180° to 360° ,
3. Operating force is maximum if $\psi = (\alpha - \theta) = 90^\circ$,
4. Relay does not operate if V or I is zero,
5. Changing the internal angle α relay characteristic can be varied –
 - a) If $\alpha = 0^\circ$, $F \propto V I \underline{\sin\theta} \rightarrow$ Acts as reactive power relay.
 - b) If $\alpha = 90^\circ$, $F \propto V I \underline{\sin(90^\circ - \theta)}$
 $\propto V I \underline{\cos\theta} \rightarrow$ Acts as active power relay.
 - c) If $0^\circ < \alpha < 90^\circ$, \rightarrow Relay responds to both active & reactive powers.

Directional Relay Connections

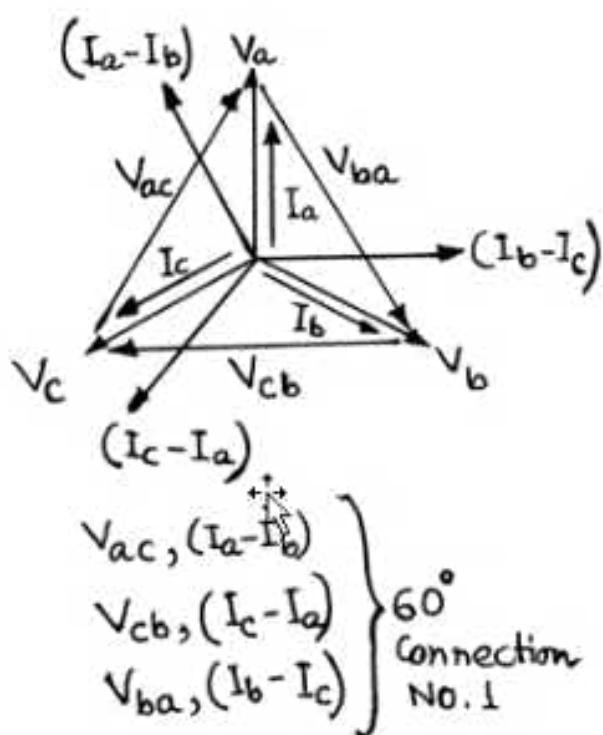
- Internal angle α being fixed for a relay, value of the power factor angle θ is important for production of operating force (torque) in the relay.
- During fault, system voltage usually becomes very low.
- During fault, current usually lags behind the voltage by almost 90° , which may not give high torque.
- So, some standard phase shift is deliberately introduced between V and I through suitable C.T. and P.T. connections to obtain maximum force (or torque) – these are 90° , 60° and 30° connections.

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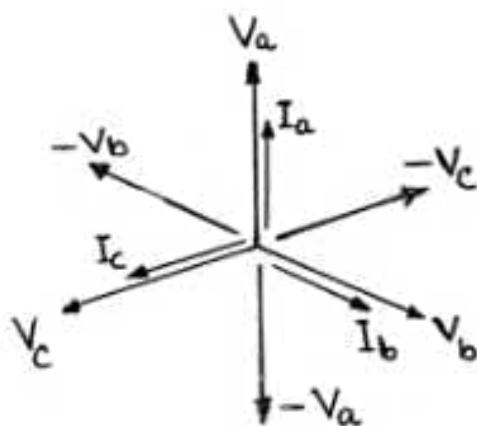


V_{ab}, I_c
 V_{bc}, I_a
 V_{ca}, I_b

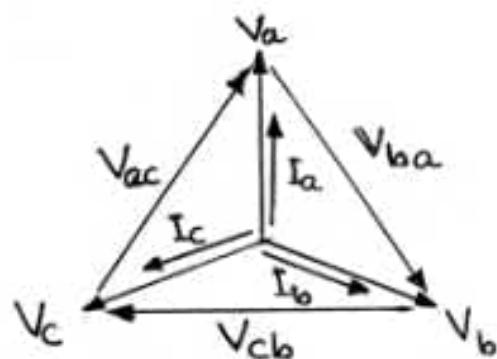
$\left. \begin{matrix} \\ \\ \end{matrix} \right\} 90^\circ \text{ Connections}$



Contd



$-V_a, I_b$
 $-V_b, I_c$
 $-V_c, I_a$
} 60°
 Connection
 NO. 2



V_{ac}, I_a
 V_{cb}, I_c
 V_{ba}, I_b
} 30°
 Connection

Differential Relay

- Differential Relay responds to vector difference of two or more similar quantities.

This means,

- 1) The differential relay has at least two actuating quantities, say I_1 & I_2 .
- 2) The two or more actuating quantities are of similar type, i.e. all currents/voltages.
- 3) The relay responds to vector difference, i.e. both magnitude and phase are responsible for relay operation.

Application of differential relay

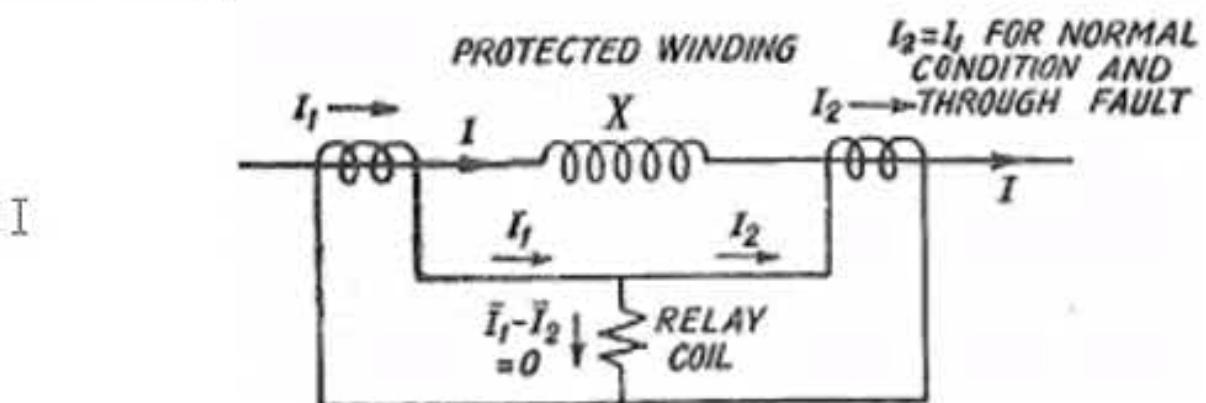
- Points to note -
- Differential protection is generally unit type of protection, i.e. protection zone is exactly determined by the location of C.T.s/P.T.s.
- The vector difference is obtained by suitable connections of C.T./ P.T. secondaries.
- Generally, differential relays are current differential relays, where vector difference between current entering the circuit and leaving the circuit are used for relay operation.

Differential relay is used for protection of -

1. Generator, generator-transformer,
2. Transformer, I
3. Feeder/line by pilot wire differential protection.
4. Transmission line by phase comparison carrier current protection.
5. Protection of large motors.
6. Bus-zone protection.

Principle of Circulating Current Differential Protection

Consider the diagram.



- Here, 'X' be the circuit to protected.
- It may be winding of a generator/transformer/ any other equipment.
- The current entering and leaving are monitored.
- It is done through C.T.s connected at two sides.

Consider, two conditions -

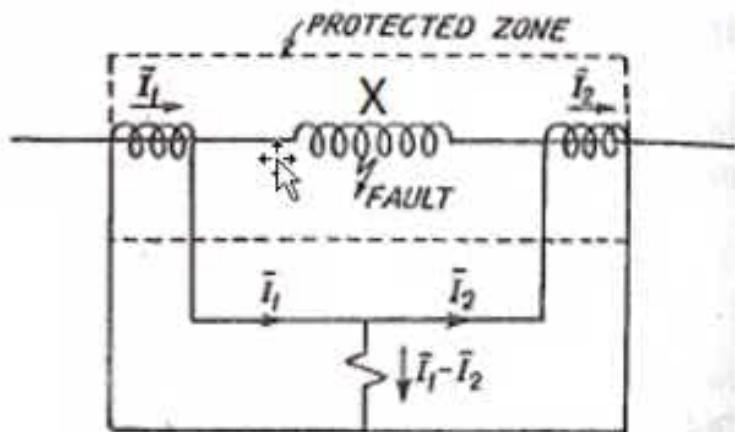
1. Normal condition/through fault.

- The two C.T. ratios are so chosen that they reflect equal currents at their secondary, i.e. $I_1 = I_2$.
- I_1 & I_2 circulate through the pilot wires.
- No current flows through the relay operating coil as $I_1 - I_2 = 0$.
- The relay does not operate.

I

2. Internal fault in the winding 'X'.

Consider, the next diagram,

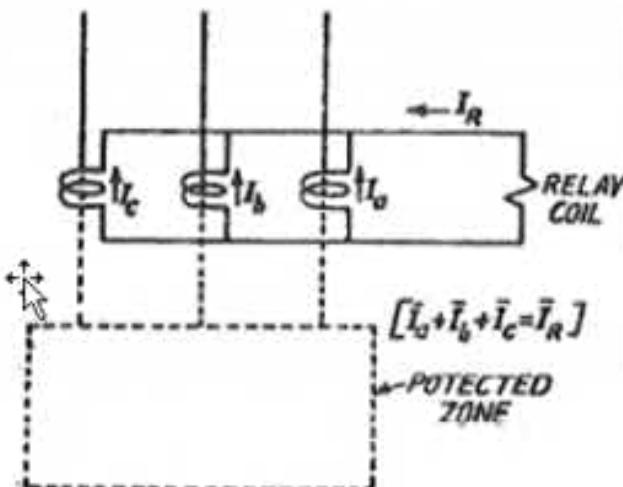


- Currents entering and leaving winding are not equal.
- Two C.T.s reflect unequal currents at their secondary, i.e. $I_1 \neq I_2$.
- Current $I_1 - I_2$ flows through the relay operating coil.
- The relay operates.

Differential Protection of Three Phase Circuits

Refer to the diagram. Here,

- Three C.T. secondaries are connected in parallel.
 - Relay is connected across it.
1. During normal condition, the three C.T. Secondary currents are balanced.
 - No current flows through relay.
 - The relay does not operate.
 2. During fault in the protected zone,
 - Three C.T. secondary currents are unbalanced.
 - Differential operating current flows through the relay.
 - The relay operates.



Difficulties - Circulating Current Differential Protection

1. Difference in pilot wire length → can be avoided by use of series resistors (adjusted onsite) with pilot wires.
2. C.T. ratio error during short circuits → avoided using Biased differential protection.
3. Magnetizing current inrush in transformer while switching on.
4. Tap changing of transformer.



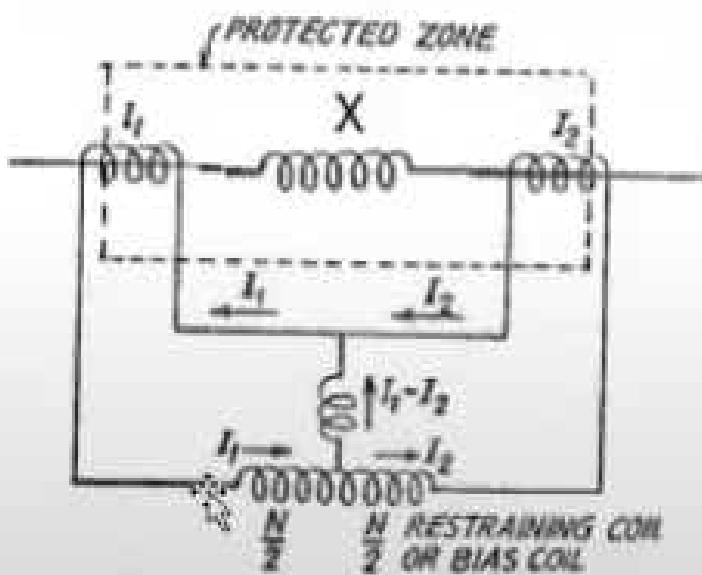
Biased or Percentage Differential Protection

- Reason for this modification – to overcome the difficulties of C.T. ratio error due to high value of short circuit current.
- C.T. ratio error may cause operation of the circulating current differential protection, even for external faults – not desirable.



Main Features of Biased Differential Protection

- This relay has an additional coil – restraining coil.
- Restraining coil is connected in series with pilot wires.
- Operating coil is connected at the mid point of the restraining coil.
- Total ampere-turns in restraining coil =
 $I_1 \cdot \frac{N}{2} + I_2 \cdot \frac{N}{2} = N \cdot (I_1 + I_2)/2$
- $(I_1 + I_2)/2$ is average restraining current in N turns.

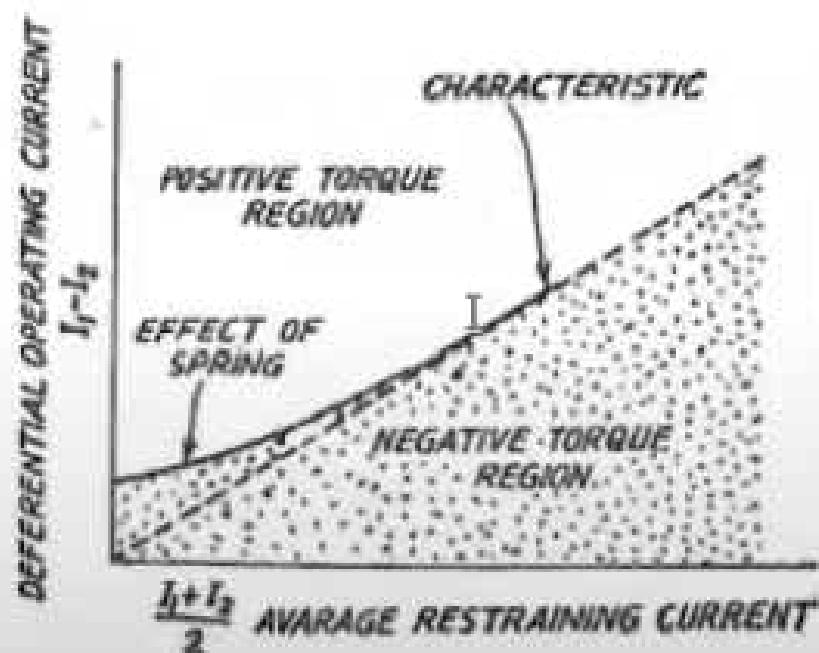


- For external faults both I_1 and I_2 increase.
- Thus, restraining force/torque increases to prevent mal operation.
- This relay is called Percentage Differential relay as the ratio of differential operating current ($I_1 - I_2$) to average restraining current $(I_1 + I_2)/2$ has a fixed percentage. Hence the name.

- This relay is also called Biased Differential relay because the restraining coil is also called bias coil as it provides additional flux.

Characteristic

The biased differential relay has rising pick-up characteristic, except for the initial portion due to the effect of spring.



Settings of Biased Differential Relay

1. Setting of operating coil circuit: The percentage setting (Basic Setting) of operating coil circuit is defined as the ratio:

% Basic Setting =

$$\frac{\text{Smallest current in operating coil to cause operation}}{\text{Rated current of operating coil}} \times 100$$

(when current in restraining coil is zero)

2. Setting of restraining coil circuit: It is defined as the ratio:

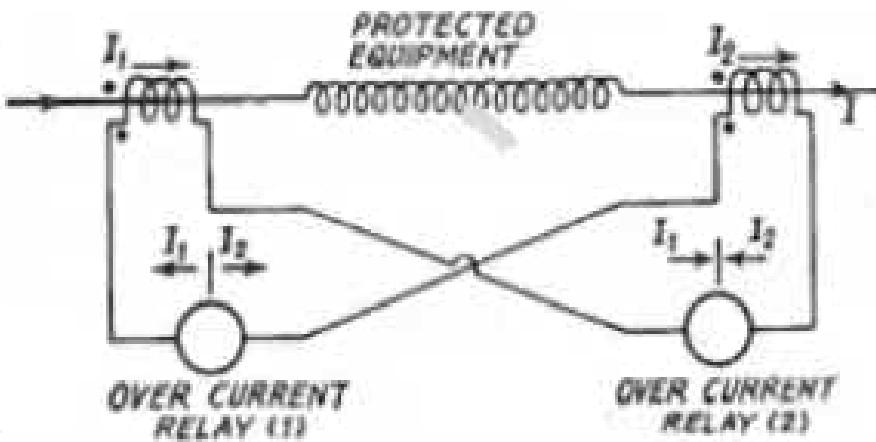
% Pick-up value=

$$\frac{\text{Current in operating coil to cause operation}}{\text{Current in restraining coil}} \times 100$$

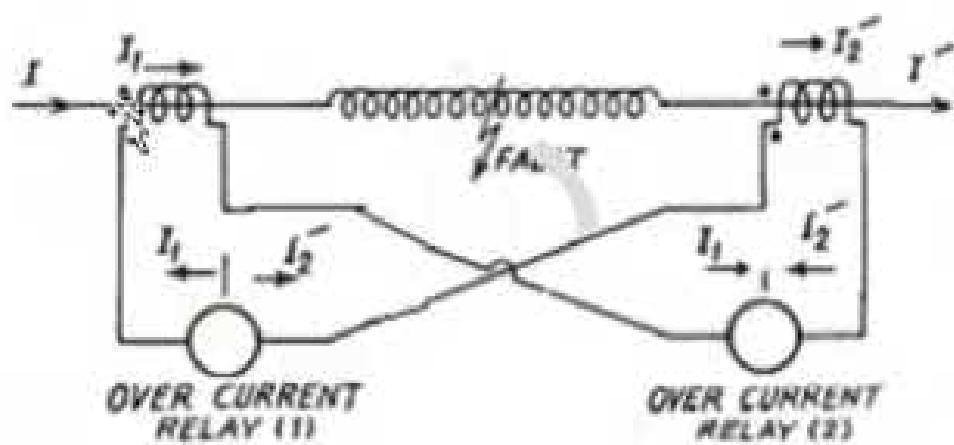
Balanced Voltage Differential Protection

Consider the figure.

- Secondaries of C.T.s are so connected that for normal condition or through fault the secondary currents on two sides are equal and oppose each other & also their voltages are balanced.
 - Relay does not operate.



- During internal fault the condition changes.
- Current entering the protected circuit, no longer equal to that leaving.
- Earlier balance is disturbed.
- C.T. secondary currents are different.
- C.T. secondary voltages are different.
- Current flows through the relays. Relays operate.
- ❖ C.T. used have air gap in the core.



Fault Calculations

Introduction:

- Fault in an electrical equipment is defined as a defect in its electrical circuit due to which flow of current is diverted from the intended path.
- Fault is due to reduction of insulation strength between phase conductors & Phase conductors and ground.
- Fault is associated with large flow of current & reduction in system voltage.
- If fault continues for longer period of time → power system equipment may be damaged with loss of system stability.
- Power system elements generators, transformers, transmission lines, underground cables, synchronous & induction motors develop fault sooner or later.

- To protect power system elements from damage suitable protective schemes are employed after ascertaining normal current, fault current and fault MVA.
- Protective relay is aided by CBs of suitable rating.

Definitions of Normal & Fault MVA

- At a particular point in a power system, if

V_n = Normal line to line voltage in kV &

I_n = Normal line current in kA, then

$$\text{Normal MVA} = \sqrt{3} \cdot V_n \cdot I_n$$

- At that point, if we consider a 3φ fault and

V_f = Line to line voltage in kV during fault &

I_f = Fault current in kA, then

$$\text{Fault MVA} = \sqrt{3} \cdot V_f \cdot I_f$$

➤ Generally, fault MVA is several times the normal MVA.

Fault Classification

1. Symmetrical fault: Involving all the three phases at same place \rightarrow three phase (L-L-L) fault.
2. Unsymmetrical faults: Involving one or more phases with/without ground. May be classified as
 - a) Single line to ground (L-G) fault,
 - b) Line to line (L-L) fault,
 - c) Double line to ground (L-L-G) fault &
 - d) Simultaneous faults.

Steps of Symmetrical Fault Calculations

1. Single line representation of the power system.
2. To draw impedance diagram → from which simplified reactance/positive sequence diagram.
3. To select suitable base kV & kVA from which base current & base impedance are calculated.
4. Calculation of fault current & fault MVA.

Single Line Representation of Power Systems

- In this diagram power system elements are represented by typical assigned symbols & connected by one line only.
- Neutral earthing is indicated.
- A balanced 3φ system is solved for symmetrical faults considering 1φ representation.

- Consider the following single line diagram.

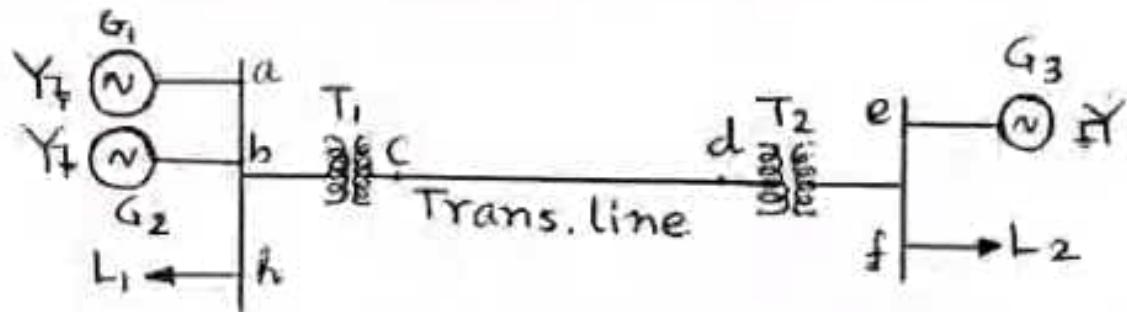
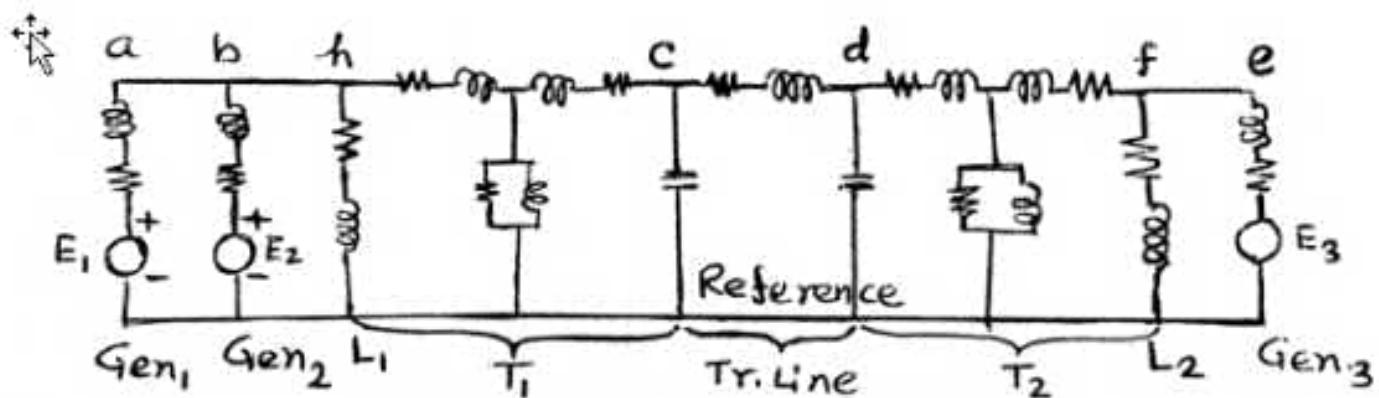


Fig. Single Line diagram of a typical system

To Draw Impedance/Reactance/Positive Sequence Network

- In this diagram each element represented by equivalent circuit. Then, to simply following assumptions are made:
 - System resistance & capacitance are neglected.
 - Magnetizing currents are neglected.
 - Rotating machines shown by emf source in series with reactance.
 - Static loads are omitted.
 - Induction motors are omitted.

- Thus, the impedance diagram is



- The simplified reactance diagram is

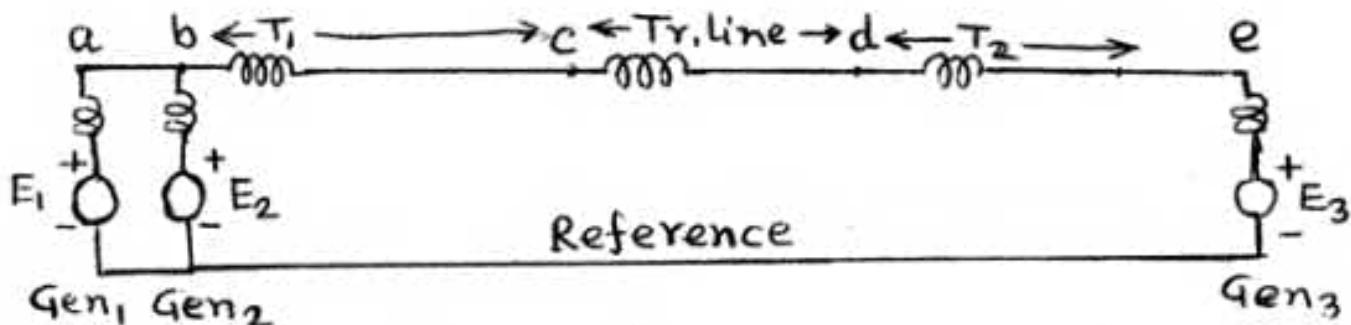


Fig. Reactance diagram/positive sequence network

Per Unit Method

- Quantities voltage (V), current (I), kVA (or MVA) and impedance are expressed as percentage or per unit of their selected base values.

Example: Base value (i.e. 100% or 1 p.u.) of voltage be 400kV.

$$\text{Then, } 200\text{kV} = \frac{200}{400} = 0.5 \text{ p.u. or } 50\%$$

$$\text{Similarly, } 40\text{kV} = \frac{40}{400} = 0.1 \text{ p.u. or } 10\%$$

Example: Base value (i.e. 100% or 1 p.u.) of current be 50kA.

$$\text{Then, } 10\text{kA} = \frac{10}{50} = 0.2 \text{ p.u. or } 20\%$$

Similarly for other quantities.

Advantages of P.U. Method

1. Simplified calculations.
2. For circuits connected by transformer → this method is particularly suitable. Choice of suitable base kVA & kV for both primary & secondary sides → p.u. reactance remains same when referred to any of the sides of the transformer → therefore, various circuits may be connected in simplified manner in reactance diagram.
3. Machine reactance is given in p.u.

Selection of Base Values

- Rule: Base kV & kVA are selected first and then base values of current (I) & impedance (Z) are determined → since later quantities are related to the earlier two.

- For circuits connected by transformer → same base kVA is chosen for both primary and secondary sides & choose voltages of the primary & secondary sides as respective side's base values → Such a choice gives same p.u. reactance of the transformer when referred to any of the circuits.

Determination of Base Current & Base Impedance

For Single Phase Circuits

(i) Select base kV & base kVA.

$$(ii) \text{Base current (in ampere)} = \frac{\text{Base kVA}}{\text{Base kV}}$$

$$(iii) \text{Base impedance (in ohm)} = \frac{\text{Base kV} \times 1000}{\text{Base current}} = \frac{\text{Base kV} \times 1000}{\frac{\text{Base kVA}}{\text{Base kV}}} \\ = \frac{(\text{Base kV})^2 \times 1000}{\text{Base kVA}}$$

(iv) Base power = Base kVA

Example: In a 1φ circuit, base kV = 19kV & base kVA = 5000kVA. Calculate base current & base impedance. Also convert 50A & 50Ω in p.u. values.

Soln: Base current = $\frac{\text{Base kVA}}{\text{Base kV}} = \frac{5000}{19} = 263.1579\text{A.}$

Base impedance = $\frac{(\text{Base kV})^2 \times 1000}{\text{Base kVA}} = \frac{(19)^2 \times 1000}{5000} = 72.2\Omega$

Hence, $50\text{A} = \frac{50}{263.1579} = 0.189\text{ p.u.}$

Similarly, $50\Omega = \frac{50}{72.2} = 0.6925\text{ p.u.}$

Change of base

I

Z p.u. referred to new base = Z p.u. referred to old base ×

$$\left(\frac{\text{Base kV Old}}{\text{Base kV New}} \right)^2 \times \left(\frac{\text{Base kVA New}}{\text{Base kVA Old}} \right)$$

Example: A 11kV, 15 MVA generator has reactance of 0.15 p.u. referred to its ratings as bases. New bases chosen for calculations are 110kV and 30MVA. Calculate the new p.u. reactance of the generator.

Soln:

Z p.u. referred to new base = Z p.u. referred to old base x

$$\left(\frac{\text{Base kV Old}}{\text{Base kV New}} \right)^2 \times \left(\frac{\text{Base kVA New}}{\text{Base kVA Old}} \right)$$

$$= 0.15 \times \left(\frac{11}{110} \right)^2 \times \left(\frac{30,000}{15,000} \right) = 0.003 \text{ p.u.}$$

Determination of Base Current & Base Impedance

(i) Solved on 1φ basis (When 3φ systems solved on 1φ basis)

Select –

$$(a) \text{Base kV} = \text{Line to neutral kV} \quad (b) \text{Base kVA} = \frac{3\phi \text{ kVA}}{3}$$

$$\text{Then, base current} = \frac{\text{Base kVA}}{\text{Base kV}}$$

$$\text{Base impedance (in ohm)} = \frac{(\text{Base kV})^2 \times 1000}{\text{Base kVA}}$$

(ii) Solved on 3φ basis (When 3φ systems solved on 3φ basis)

Select –

$$(a) \text{Base kV} = \text{Line to line (3φ) kV} \quad (b) \text{Base kVA} = 3\phi \text{ kVA}$$

$$\text{Then, base current} = \frac{\text{Base kVA}}{\sqrt{3} \text{ Base kV}}$$

$$\begin{aligned} \text{Base impedance (in ohm)} &= \frac{\left(\frac{3\phi \text{ Base kV}}{\sqrt{3}} \right)^2 \times 1000}{\text{Base kVA}} \\ &= \frac{(3\phi \text{ Base kV})^2 \times 1000}{3 \text{ Base kVA}} \end{aligned}$$

- Note: Expressions of base impedance for both the 1φ & 3φ systems are same.

Example: A generating station is feeding a 132kV system. Determine the total fault current, fault level (fault MVA) & fault current supplied by each alternator for a 3φ fault at the receiving end. The line is 200km long.

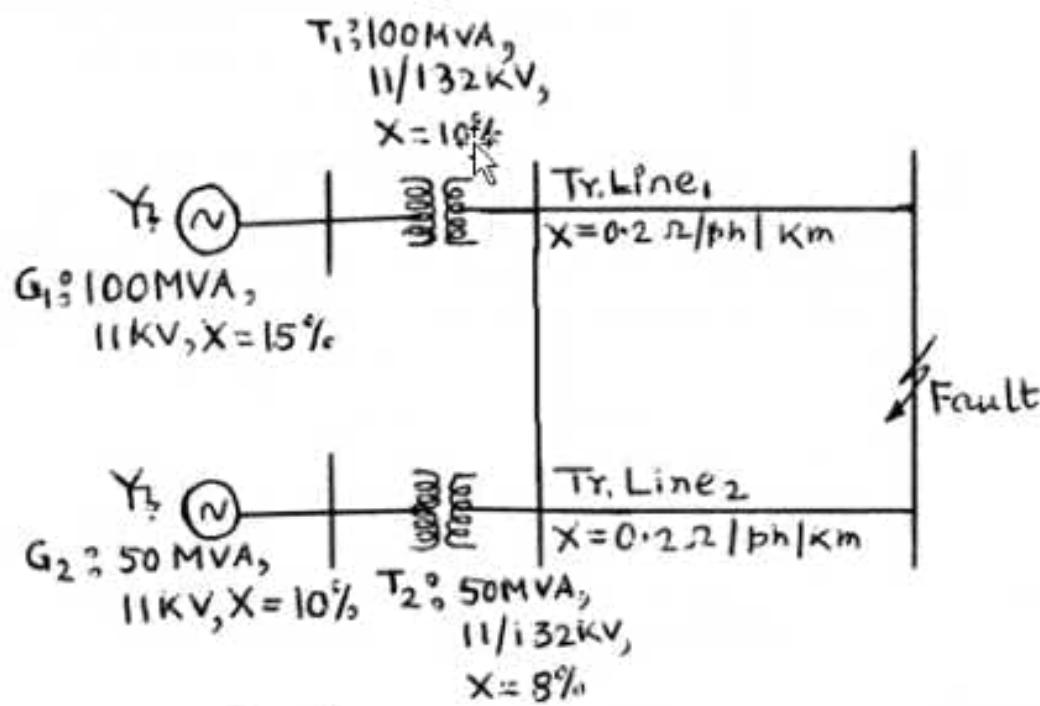


Fig. 1

Soln: Consider, (i) Base kVA for the whole circuit be 100,000 kVA
 (ii) For L.V. Circuit base KV = 11 KV, for H.V. Side
 base KV = $\frac{132}{\sqrt{3}}$ KV.

So, p.u. reactance of $G_1 = j0.15$

$$\text{p.u. reactance of } G_2 = j0.1 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{50}\right) = j0.2$$

p.u. reactance of $T_1 = j0.1$

$$\text{p.u. reactance of } T_2 = j0.08 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{50}\right) = j0.16$$

$$\text{Now, base impedance on H.V. side} = \frac{(132)^2 \times 1000}{100,000} = 174.24 \Omega$$

$$\text{So, p.u. reactance of each line} = \frac{0.2 \times 200}{174.24} = j0.230 \text{ p.u.}$$

The reactance diagram of the given system is,

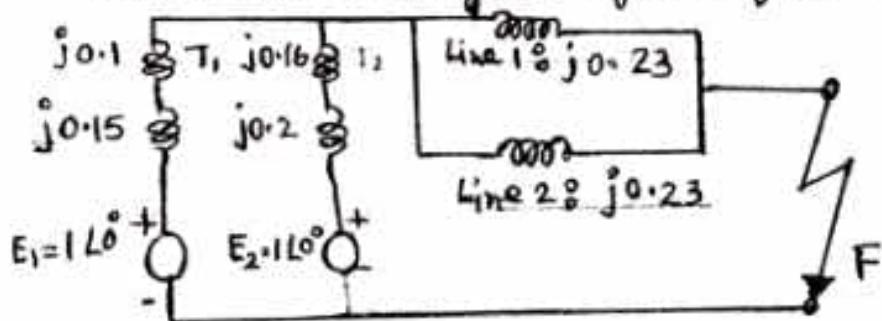


Fig 2 Reactance diagram of fig 1.

The next step is to find out the Thevenin's equivalent circuit between the fault points.

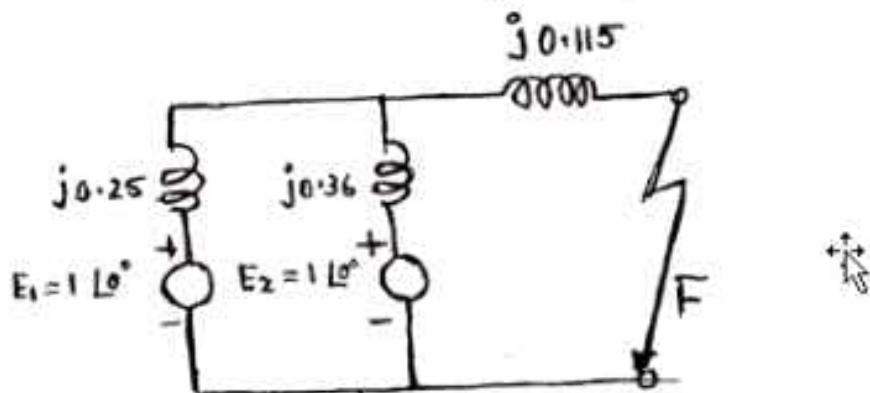


Fig 3. Equivalent (reduced) of fig 2.

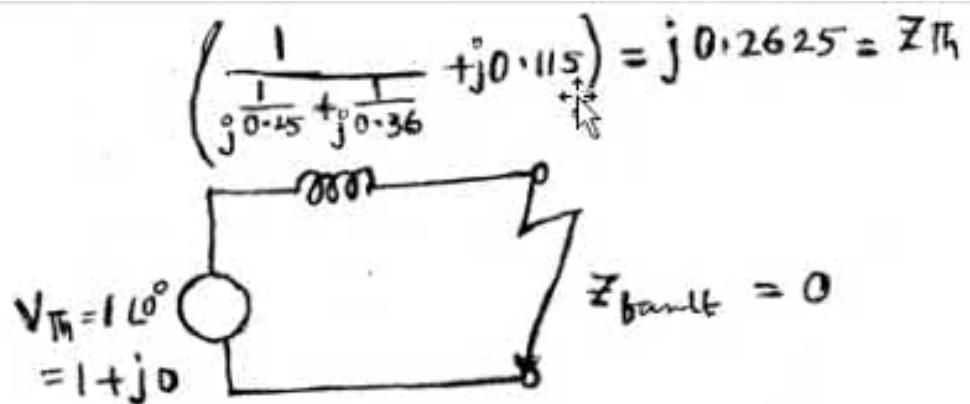


Fig. 4. Thévenin's equivalent circuit of fig. 3.

Thus, The fault current = $\frac{V_{Th}}{Z_{Th} + Z_{fault}} = \frac{1 + j0}{j0.2625 + 0} = -j3.8095 \text{ p.u.}$

Base current on H.V. side = $\frac{100,000}{\sqrt{3}, 132} = 437.3866 \text{ A}$

Hence fault current = $3.8095 \times 437.3866 \angle -90^\circ \text{ A}$
 $= 1666.2315 \angle -90^\circ \text{ A}$

$$\text{Fault level (Fault MVA)} = \sqrt{3} \cdot 132.1666.2345 \cdot 10^{-3} = 380.9524 \text{ MVA}$$

Current supplied by G_1 ,

$$= \frac{(1+j0) - (-j3.8095)(j0.115)}{j0.25} = \frac{1+j0 + j3.8095 \times j0.115}{j0.25} \\ = -j2.24763 \text{ p.u.}$$

Current supplied by G_2

$$= \frac{(1+j0) - (-j3.8095)(j0.115)}{j0.36} = \frac{1+j0 + j3.8095 \times j0.115}{j0.36} \\ = -j1.56085 \text{ p.u.}$$

$$\text{Base current on L.V. side} = \frac{100 \times 1000}{\sqrt{3} \cdot 11} = 5248.6388 \text{ A}$$

$$\text{So, current supplied by } G_1 = -j2.24763 \text{ p.u.}$$

$$= 2.24763 \times 5248.6388 \angle -90^\circ \text{ A}$$

$$= 11796.9980 \angle -90^\circ \text{ A}$$

$$\text{Current supplied by } G_2 = -j1.56085 \text{ p.u.} = 1.56085 \times 5248.6388 \angle -90^\circ \text{ A} \\ = 8192.3379 \angle -90^\circ \text{ A}$$

Fault Limiting Reactors

- Short circuit currents in large interconnected systems → very high → may damage power system elements.
- To limit short circuit currents to safe values → during faults → fault (current) limiting reactors are used at strategic locations in series → to protect the power system elements.
- The fault (current) limiting reactors also known as series reactors.

- The fault limiting reactors are
 - (i) large inductive coils → wound for high self inductance &
 - (ii) very small resistance.
- As the resistance is kept small in comparison to reactance, no appreciable amount of energy is wasted in them.



Fault limiting reactors are used for



- (i) Limiting the fault current in the existing system.
- (ii) When the existing system is extended by adding more generators (at existing plants) or new generating stations or new interconnections, inclusion of series reactors at strategic locations reduce the fault current & fault level → this enables us to continue with the existing CBs.

- (iii) Series reactors are used in short-circuit test plants.
- (iv) Series reactors are used to limit the arc-current in arc-furnaces.

Design Features of Fault Limiting Reactors

- (i) high self inductance.
- (ii) very small resistance.
- (iii) Inductance (or reactance) value should not reduce due to saturation under short circuit conditions.
 - ❖ So, reactors are built with non-magnetic cores/with iron cores with air gap included in magnetic circuit.

Locations of Series Reactors

Placement of series reactors should be such that

- (i) They limit feeding of large fault currents by generators.
- (ii) During normal operation voltage drop across reactor is less.

I

Accordingly, reactors are located judiciously in the following positions:

1. Generator Reactors

- Reactors are connected in series with generators.
- Reactors are not needed for modern generators

→ Modern generators have

large reactance obtained by deep slots & other design features. → high reactance is provided to safeguard generators against dead 3φ faults at the terminals.

- When new generators are installed in old power stations, reactors may be used with old generators.

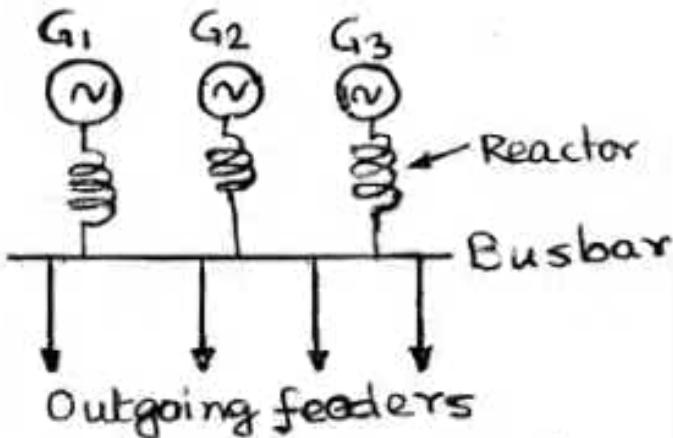


Fig. Generator reactor

- The main disadvantages of the scheme are
 - (i) In the event of a fault at the common busbar/close to it → busbar voltage may come down to very low value → causing loss of synchronism.
 - (ii) Continuous current flow through reactors cause power loss & heating of reactors.

2. Feeder Reactors

- Main disadvantage of generator reactor is overcome in this scheme.
- Reactors are connected in series with the feeders.
- In the event of a fault in any of the feeders, common bus bar (also other feeders) is (are) less affected.
- The main disadvantages of the scheme are
 - (i) Number of reactors is equal to number of feeders.
 - (ii) Here also, like the earlier scheme, continuous loss takes place in reactors.

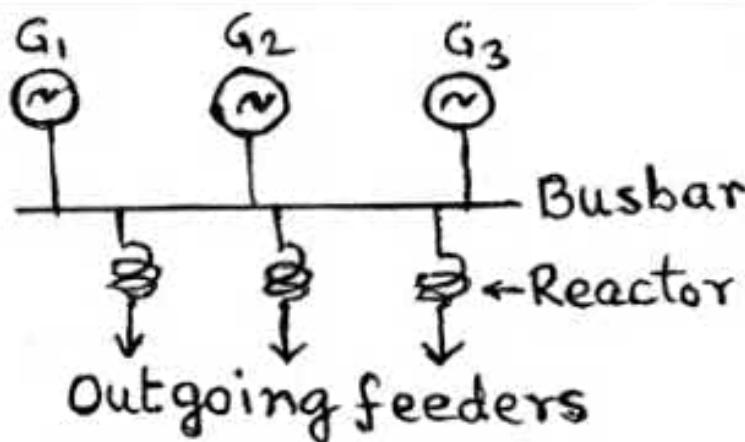
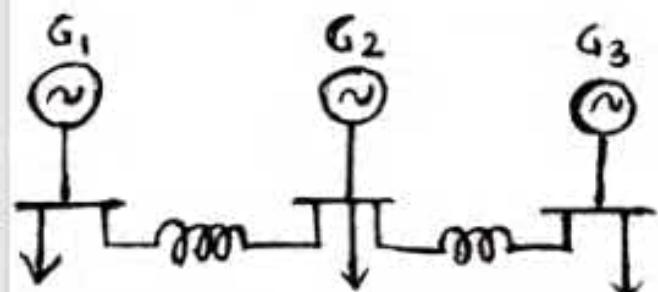
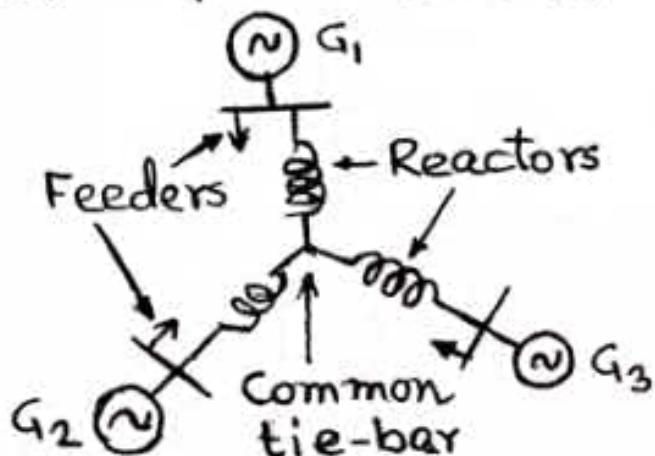


Fig. 2 Feeder reactor

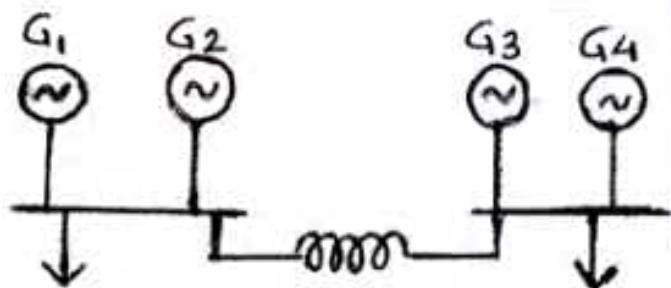
3. Busbar Reactors



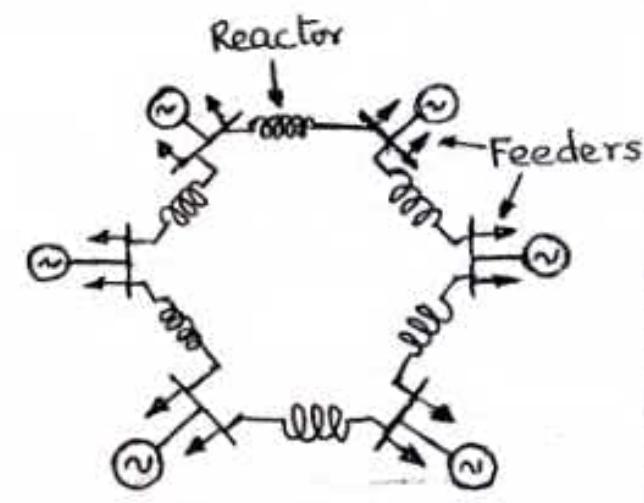
(a) Simple connection



(c) Star connection



(b) Sectionalized buses



(d) Ring connection

Fig. 3 Busbar reactor connections.

- Four methods are there.
- The first two methods, i.e. connections (a) & (b) are used in small & moderate output plants.
- The third & fourth methods, i.e. connections (c) & (d) are used in large output plants.
- Continuous power loss during normal operation is avoided by inter-connecting busbars by reactors → as during normal operation small current flows through reactors.
- During fault in one of the feeders → generators connected to that busbar directly feed to the fault & the others through reactors.

Formula for Fault MVA

$$\text{Fault MVA} = \frac{\text{Base MVA}}{X_{Th}}$$

Proof:

$$\begin{aligned}\text{Fault MVA} &= \sqrt{3} \cdot (\text{Base kV}) \cdot (\text{Fault current in Amp}) \cdot 10^{-3} \\ &= \sqrt{3} \cdot (\text{Base kV}) \cdot (\text{Base Current} \cdot \text{Fault current in p.u.}) \cdot 10^{-3}\end{aligned}$$

$$\begin{aligned}&= (\sqrt{3} \cdot \text{Base kV} \cdot \text{Base Current} \cdot 10^{-3}) \cdot \frac{1}{X_{Th}} \\ &= \frac{\text{Base MVA}}{X_{Th}} \quad (\text{Proved})\end{aligned}$$

Example: A generating station consists of three 8MVA, 0.15 p.u. alternators, which are individually connected to a bus. These three individual buses are connected to a common tie-bar through reactors of 5MVA, 0.1 p.u. Determine the fault MVA for a fault at one of the individual buses.

Soln: Let, base MVA = 8MVA

and base voltage to be same

For all generators & reactors.

Hence, p.u. reactance of the reactors referred to new

$$\text{Base} = 0.1 \times \frac{8}{5} = 0.16 \text{ p.u.}$$

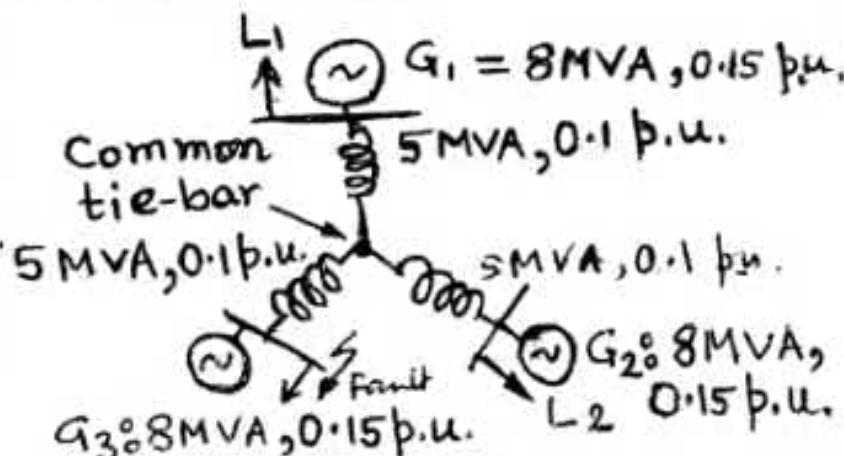


Fig. 1 Singleline diagram

Then reactance diagram of Fig. 1 is,

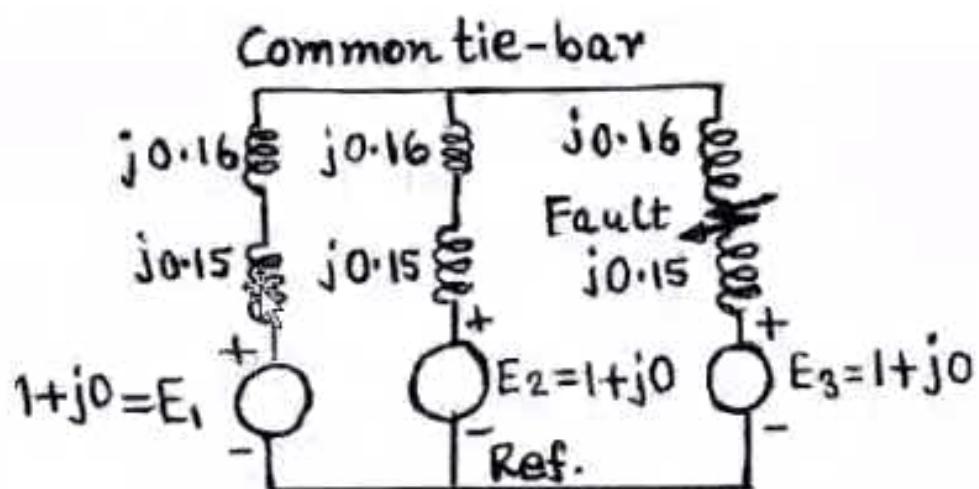


Fig. 2 Reactance diagram

Fig. 2 reduces to,

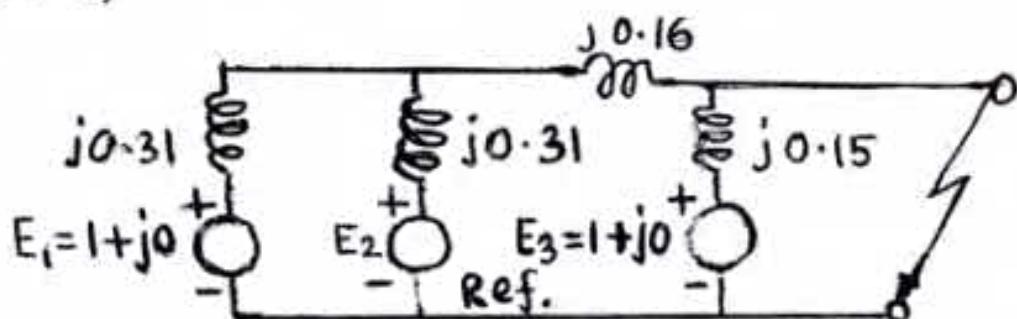


Fig. 3 Reduced diagram of fig. 2

Which further reduces to (Fig. 4),
Thevenin's equivalent of Fig. 4 is,

$$Z_{Th} = 0.1016129$$

$$\begin{aligned} V_{Th} &= 1 \angle 0^\circ \\ &= 1 + j0 \end{aligned}$$

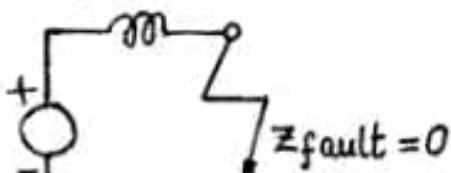


Fig. 5 Thevenin's equivalent

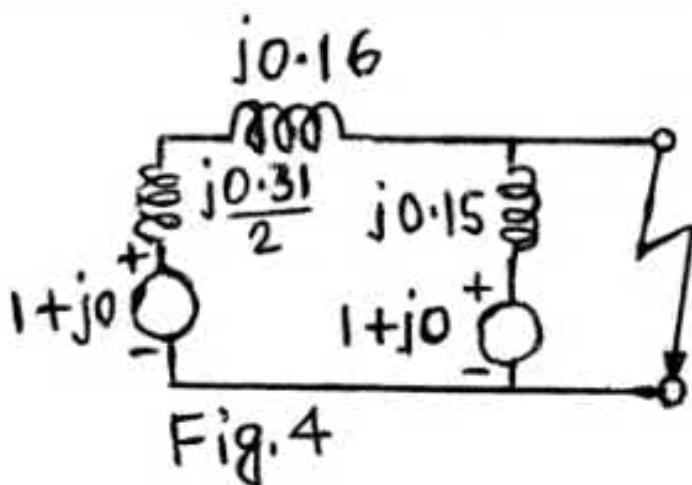


Fig. 4

$$Z_{Th} = \left[\frac{1}{\frac{j0.31}{2} + j0.16} + \frac{1}{j0.15} \right]^{-1} = 0.1016129$$

$$\text{Hence fault MVA} = \frac{\text{Base MVA}}{Z_{Th} + Z_{fault}} = \frac{8}{0.1016129} = 78.73016 \text{ MVA}$$

Example: A system having four alternators, each rated at 11kV, 50MVA and having a reactance of 15%, as shown in the fig. Find, (a) fault level for a fault on one of the feeders (near the bus) with 'zero' value of reactance X and (b) the reactance of the current limiting reactor X to limit the fault MVA to 800MVA for a fault at the same point.

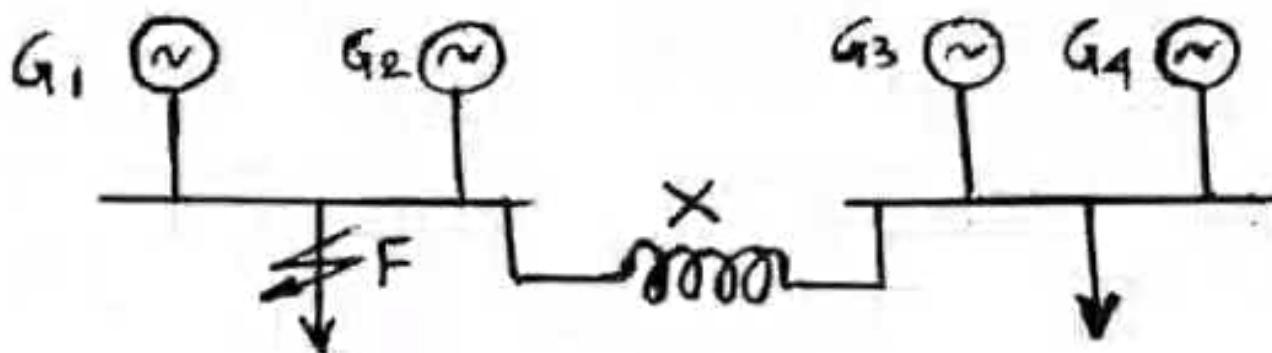


Fig.1
 $G_1, G_2, G_3, G_4 \stackrel{*}{\circ} 11\text{kV}, 50\text{MVA}$,
 reactance = 15%

Soln: (a) Take base MVA = 50 MVA, base KV = 11 KV.
 Reactance diagram is therefore,

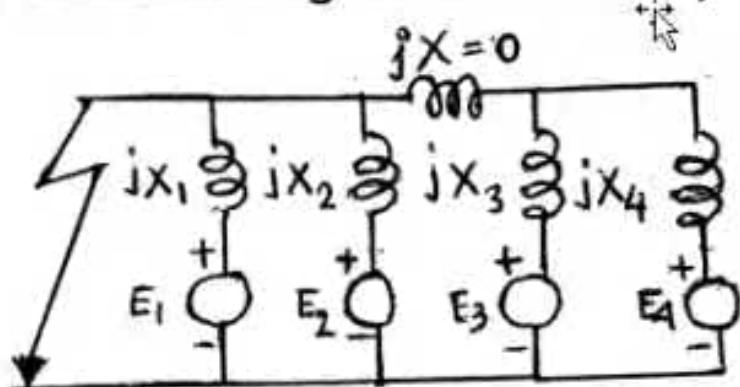


Fig. 2 Reactance diagram
 $jx_1 = jx_2 = jx_3 = jx_4 = j0.15$
 $E_1 = E_2 = E_3 = E_4 = 1 \angle 0^\circ$

Then, Thevenin's equivalent circuit of fig. 2 is

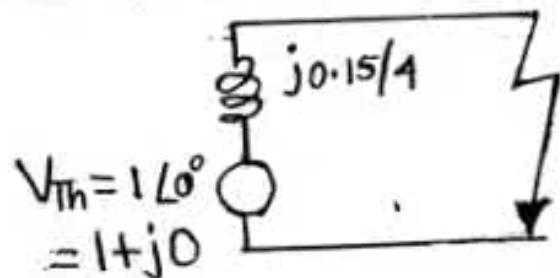


Fig. 3 Thevenin's equivalent circuit

$$\text{Therefore, fault current} = \frac{1+j0}{(j0.15)/4} = -j26.66 \text{ p.u.}$$

$$\text{Now base current} = \frac{\text{Base kVA}}{\sqrt{3} \cdot \text{Base kV}} = \frac{50 \times 1000}{\sqrt{3} \cdot 11} = 2624.3194 \text{ A}$$

$$\text{So, fault current} = 26.66 \times 2624.3194 \angle -90^\circ \\ = 69981.8508 \angle -90^\circ \text{ A.}$$

$$\text{Fault MVA} = \sqrt{3} \cdot 11 \cdot 69981.8508 \cdot 10^{-3} = 1333.3333 \text{ MVA}$$

(b) Alternators G_1 and G_2 will supply $0.5 \times 1333.3333 = 666.66 \text{ MVA}$ directly to the fault. Therefore, the fault MVA from G_3 and G_4 must be limited to $(800 - 666.66) = 133.33 \text{ MVA}$.

The reactance of G_3 and G_4 together is $j0.15/2 = j0.075 \text{ p.u.}$
Thus,

$$\sqrt{3} \cdot 11 \cdot \left(\frac{1}{X+0.075} \right) \times 2624.3194 \cdot 10^{-3} = 133.33$$

$$\omega, X + 0.075 = \frac{\sqrt{3} \cdot 11 \cdot 2624.3194 \cdot 10^{-3}}{133.33} \quad \omega, X = 0.3 \text{ p.u.}$$

$$\text{Now, base impedance} = \frac{(\text{Base KV})^2 \times 1000}{\text{Base KVA}} = \frac{(11)^2 \times 1000}{50000} = 2.42 \Omega$$

Hence, the reactor should be $0.3 \times 2.42 = 0.726 \Omega$



Example: Consider the ring connection of reactors for the four alternators, as shown in Fig. 1, where reactances are given in p.u. based on 100MVA. Neglecting the resistance, determine the fault MVA for a fault at F.

Soln: Consider the base MVA be 100MVA and base voltage to be same for all generators & reactors. Then, reactance diagram of Fig. 1 is given in Fig. 2.

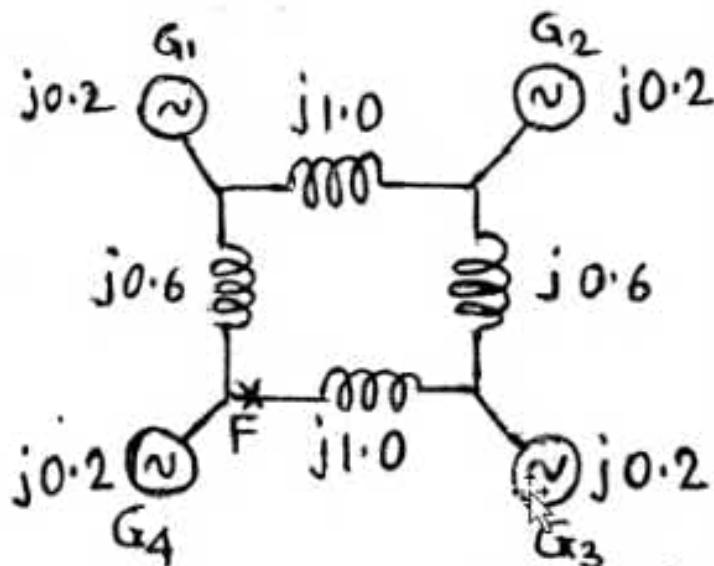


Fig.1 Ring connection of four alternators

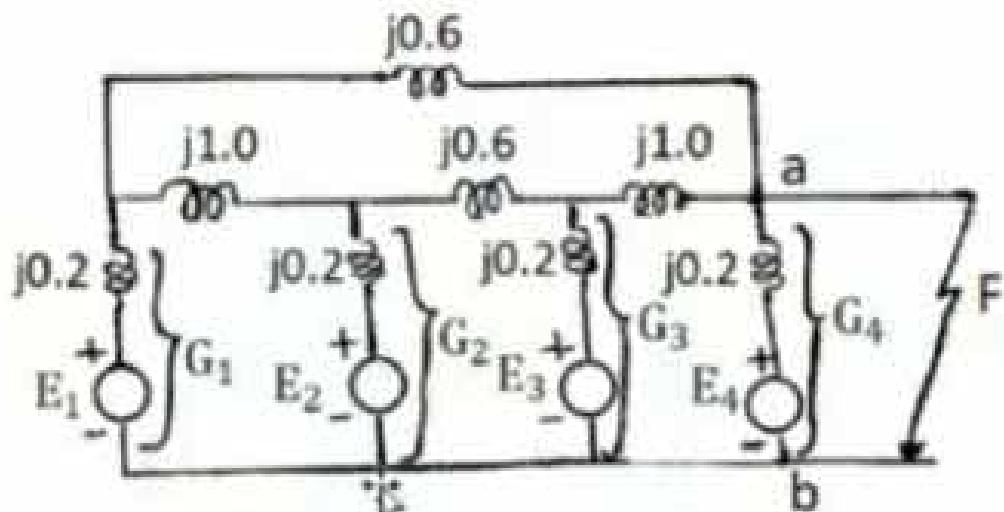


Fig. 2 Reactance diagram

To determine Thevenin's equivalent reactance between 'a' and 'b' fig. 2 is reproduced next with reactances only for simplicity.

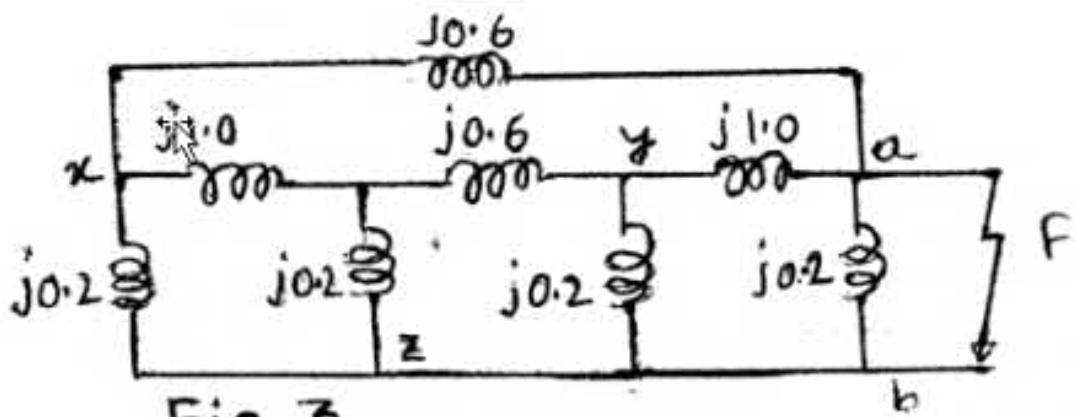
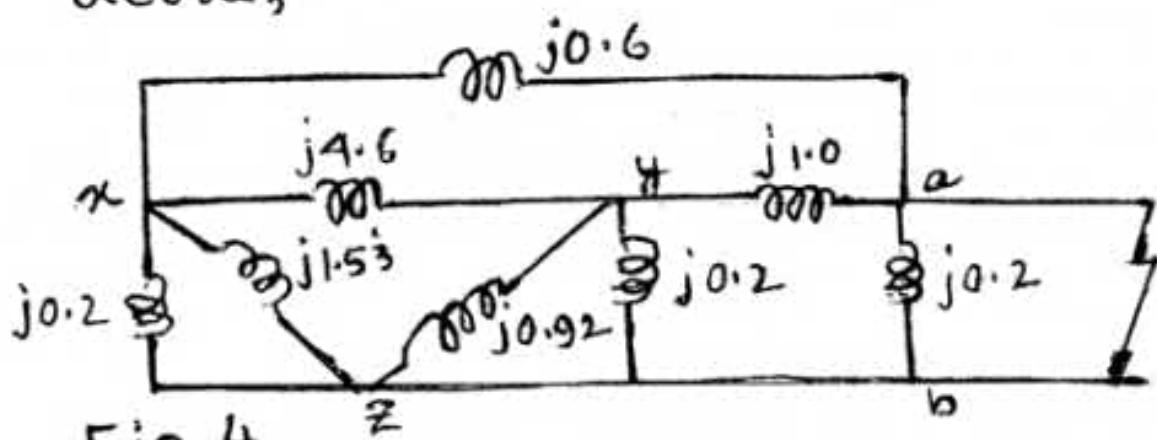


Fig. 3
Converting star 'xyz' in equivalent delta,



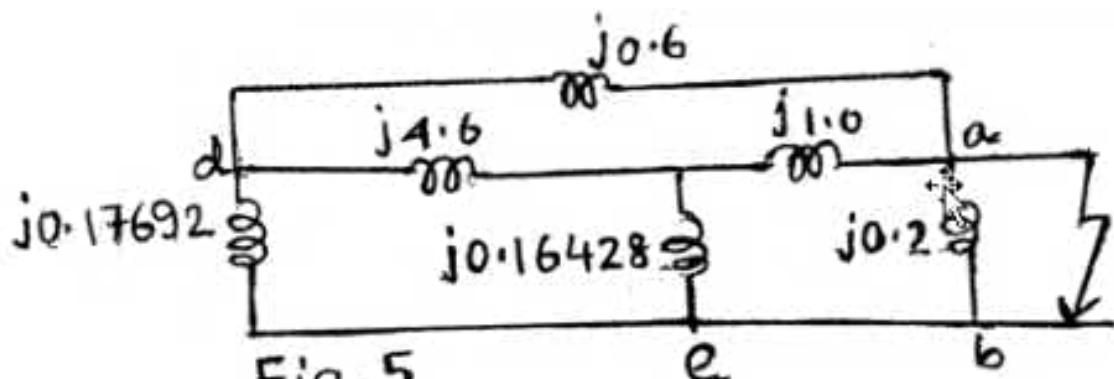
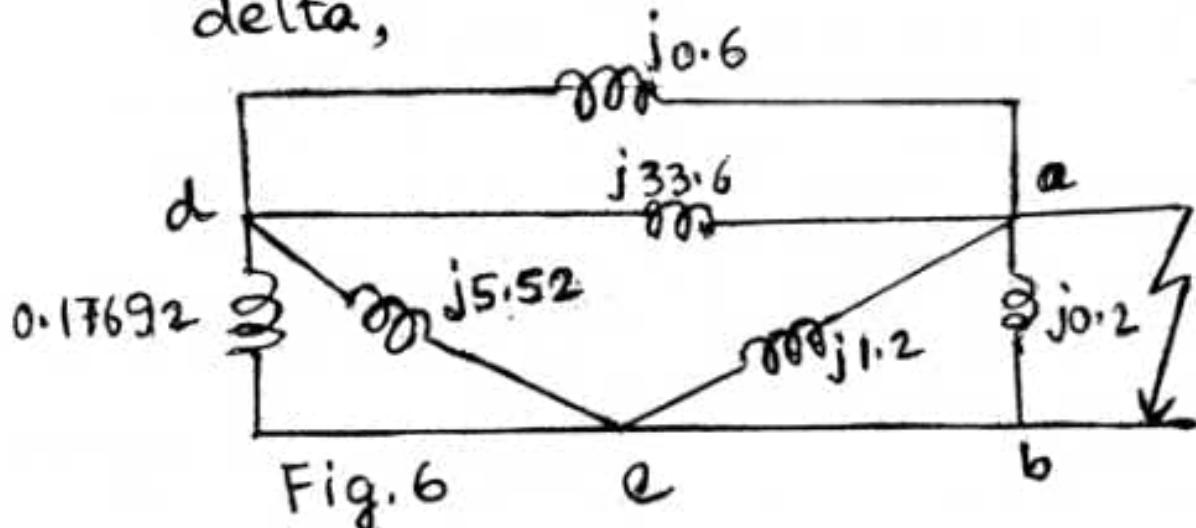


Fig. 5
Converting star 'dea' in equivalent delta,



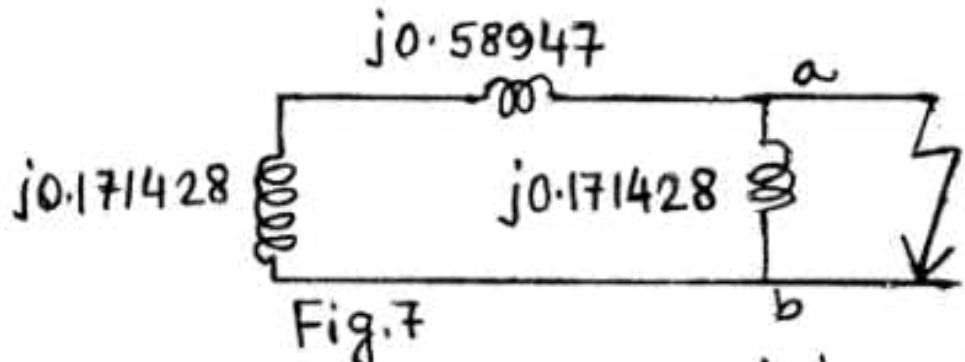


Fig.7

Fig.7 yields Thevenin's equivalent reactance between 'a' and 'b' as
 $j0.13991 \text{ p.u.}$

$$\text{Hence, fault MVA} = \frac{\text{Base MVA}}{X_{Th} + X_{\text{fault}}} = \frac{100}{0.13991} \\ = 714.7563 \text{ MVA}$$

Example: A small generating station has two alternators of 2.5MVA and 5MVA with percentage reactances of 8 and 6 respectively. The CBs are rated at 150MVA. Due to increase in system load, it is intended to extend the system by a supply from the grid via a transformer of 10MVA rating with 7.5 percent reactance. If the system voltage is 3.3kV, find the reactance necessary to protect the circuit breakers. The overall scheme is shown in Fig. 1.

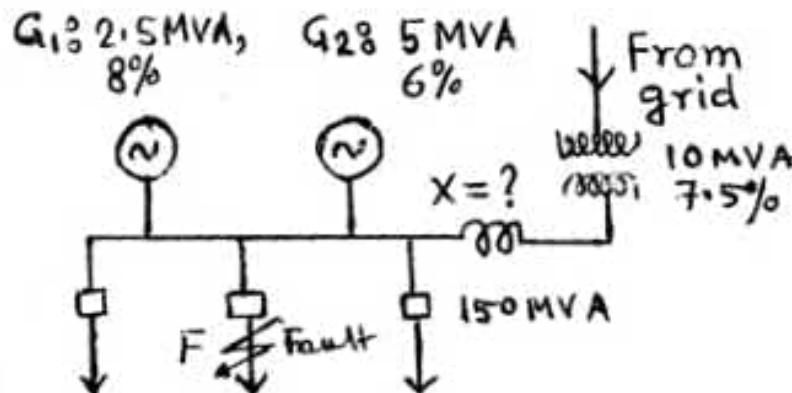


Fig.1. Single line diagram

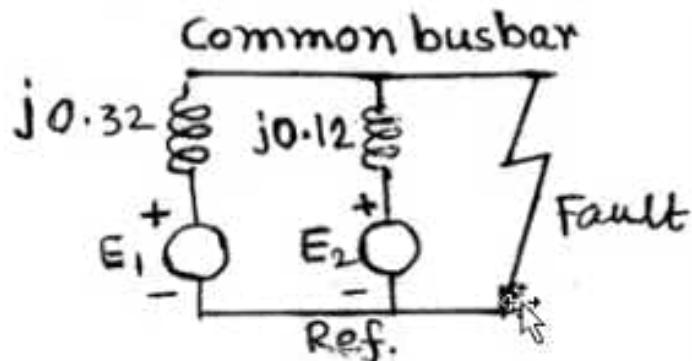
Soln: Take 10MVA as base MVA and 3.3kV as base kV. Then p.u. reactances of the two generators referred to new bases are

$$\text{For generator } G_1 = 0.08 \times \left(\frac{10}{2.5} \right) = 0.32 \text{ p.u.}$$

$$\text{For generator } G_2 = 0.06 \times \left(\frac{10}{5} \right) = 0.12 \text{ p.u.}$$

In the old system (prior to increase of load) for a fault at F, the reactance diagram is

Fig. 2 Reactance diagram before expansion of the system.



Thevenin's equivalent reactance X_{Th}

$$= \frac{1}{\frac{1}{j0.32} + \frac{1}{j0.12}} = j0.087272 \text{ p.u}$$

$$\text{Short Circuit MVA} = \frac{\text{Base MVA}}{X_{Th} + X_{fault}}$$

$$= \frac{10}{0.087272} = 114.583 \text{ MVA}$$

If supply is taken from the grid through transformer, without the busbar reactor, then fault MVA contributed by the grid through the transformer is $10/0.075 = 133.3333$ MVA

Hence, total MVA to fault without reactance is $(114.5833 + 133.3333) = 247.9166$ MVA.

If the system has to continue with the old CBs rated at 150MVA, then fault contributed by grid should be reduced due to reactor to $150 - 114.5833 = 35.4166$ MVA.

Thus, if X be the value of the reactance in p.u., then,

$$\frac{10}{0.075+X} = 35.416$$

$$\therefore X = 0.20735294 \text{ p.u.}$$

Now, base impedance = $\frac{(3.3)^2 \times 1000}{10000} = 1.089\Omega$

So, the value of the reactance in ohm = $0.20735294 \times 1.089 \approx 0.22581 \Omega$

Selection of Current Limiting Reactors

The following points are considered:

1. Rated voltage,
2. Normal current rating,
3. Short time current rating,
4. kVA/MVA rating,
5. Rated impedance in ohm/p.u.,
6. Frequency,
7. Single phase/three phase,
8. Dry/oil immersed; if oil immersed → then iron cored/not.
9. Type of cooling → natural/air-cooled.
10. Indoor/outdoor.

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- Home Work: (i) Learn about constructions of different types of series reactors &
(ii) Prepare a short note on “Saturated Reactors.”

Symmetrical Components

For unsymmetrical faults, such as,

- (a) Single line to ground (L-G) fault,
- (b) Line to line (L-L) fault &
- (c) Double line to ground (L-L-G) fault

- Simple 1ϕ representation of the system is not valid.
- Under such circumstances method of symmetrical components is used.

Principle of Symmetrical Components

- A system of 'n' unbalanced vectors can be resolved into 'n' balanced systems → each of which consists of 'n' balanced vectors.
- These balanced vectors are called symmetrical components of the original unbalanced vectors.

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Symmetrical Components of 3φ system

- An unbalanced set of 3φ voltages V_a , V_b & V_c (or currents I_a , I_b & I_c) can be resolved into three balanced set of voltages (or currents) as:
 1. Positive sequence components,
 2. Negative sequence components and
 3. Zero sequence components.



Positive sequence components

A set of three symmetrical voltages V_{a_1} , V_{b_1} & V_{c_1} (or currents I_{a_1} , I_{b_1} & I_{c_1}), equal in magnitude & displaced mutually by 120° , having phase sequence same as that of original unbalanced system.

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Negative sequence components:

A set of three symmetrical voltages V_{a2} , V_{b2} & V_{c2} (or currents I_{a2} , I_{b2} & I_{c2}), equal in magnitude & displaced mutually by 120° , having phase sequence opposite to that of original unbalanced system.



Zero sequence components:

A set of three symmetrical voltages V_{a_0} , V_{b_0} & V_{c_0} (or currents I_{a_0} , I_{b_0} & I_{c_0}), equal in magnitude & in phase with each other (i.e. no phase displacement amongst them).

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❖ Thus, if the original unbalanced system has phase sequence $a-b-c$, then positive sequence components have phase sequence $a_1 - b_1 - c_1$ but negative sequence components have phase sequence $a_2 - c_2 - b_2$.



Proof of the Method

- It is a very complicated process to resolve a set of three unbalance vectors into its positive, negative & zero sequence components.
- So, the method is proved for 3ϕ system by considering sets of positive, negative & zero sequence components & adding them to obtain a set of unbalanced system.

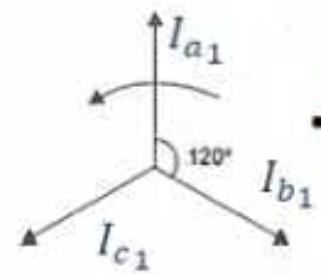


Fig. Positive sequence components

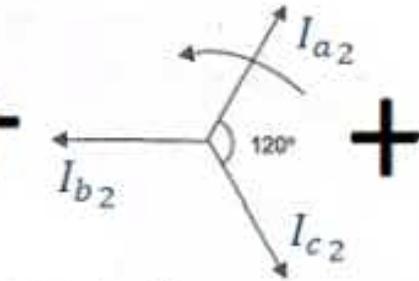


Fig. Negative sequence components

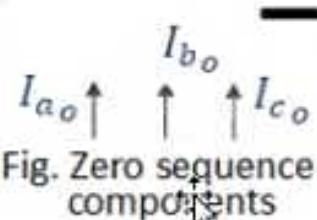


Fig. Zero sequence components

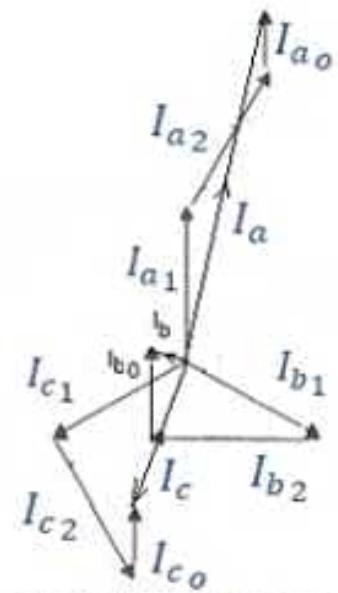


Fig. Original Unbalance System

Hence,

$$I_a = I_{a_0} + I_{a_1} + I_{a_2}$$

$$I_b = I_{b_0} + I_{b_1} + I_{b_2}$$

$$I_c = I_{c_0} + I_{c_1} + I_{c_2} \dots\dots\dots(1)$$

So, for a set of 3 unbalanced voltages V_a , V_b & V_c ,

$$V_a = V_{a_0} + V_{a_1} + V_{a_2}$$

$$V_b = V_{b_0} + V_{b_1} + V_{b_2}$$

$$V_c = V_{c_0} + V_{c_1} + V_{c_2} \dots\dots\dots(2)$$

The Operator 'a'

The letter 'a' is commonly used to designate the operator that causes a counter clockwise rotation of 120° . Thus,

$$a = 1 \angle 120^\circ$$

$$= -0.5 + j 0.866$$

$$a^2 = 1 \angle 240^\circ$$

$$= -0.5 - j 0.866$$

$$a^3 = 1 \angle 360^\circ$$

$$= 1 + j 0$$

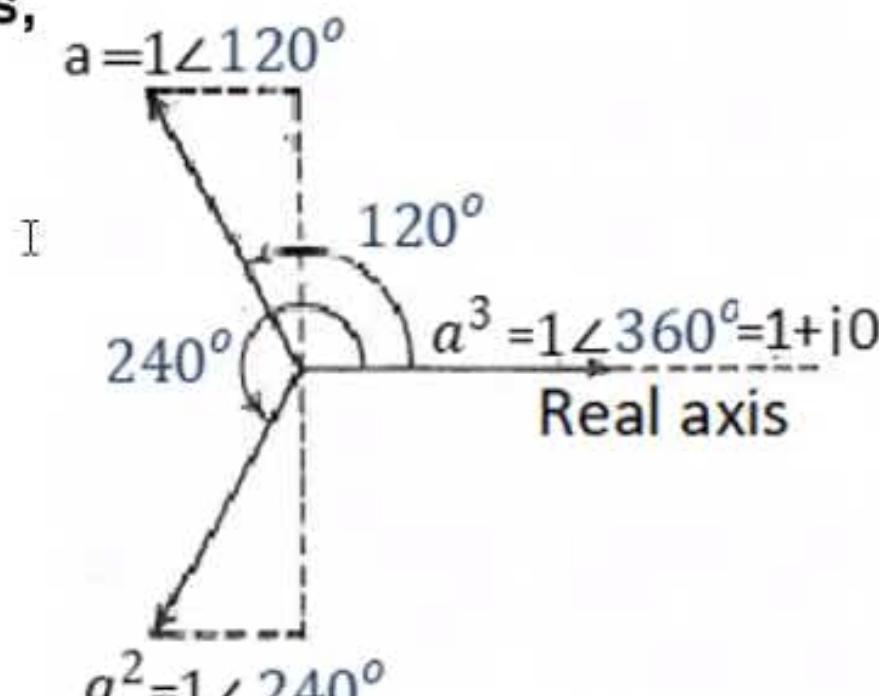


Fig. Operator 'a'

Using operator 'a' different symmetrical components can be expressed as,

<u>Positive Sequence</u>	<u>Negative Sequence</u>	<u>Zero Sequence</u>
V_{a_1}	V_{a_2}	V_{a_0}
$V_{b_1} = a^2 V_{a_1}$	$V_{b_2} = a V_{a_2}$	$V_{b_0} = V_{a_0}$
$V_{c_1} = a V_{a_1}$	$V_{c_2} = a^2 V_{a_2}$	$V_{c_0} = V_{a_0}$

Thus, from the set of eqns. (2)

$$\left. \begin{aligned} V_a &= V_{a_0} + V_{a_1} + V_{a_2} = V_{a_0} + aV_{a_1} + a^2V_{a_2} \\ V_b &= V_{b_0} + V_{b_1} + V_{b_2} = V_{a_0} + a^2V_{a_1} + aV_{a_2} \\ V_c &= V_{c_0} + V_{c_1} + V_{c_2} = V_{a_0} + aV_{a_1} + a^2V_{a_2} \end{aligned} \right\} \dots\dots\dots(3)$$

The set of eqns. (3) can be written in terms of vectors & matrix as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \dots \quad (4)$$

Define, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$

Then,

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \dots\dots\dots(5)$$

Also, we can write,

$$\left. \begin{aligned} V_{a_0} &= \frac{1}{3}(V_a + V_b + V_c) \\ V_{a_1} &= \frac{1}{3}(V_a + aV_b + a^2V_c) \\ V_{a_2} &= \frac{1}{3}(V_a + a^2V_b + aV_c) \end{aligned} \right\} \dots\dots\dots(6)$$

So, eqns. (5) or (6) can be used to determine the symmetrical components.

Similarly, for a set of unbalanced currents I_a , I_b & I_c , symmetrical components can be calculated as

$$\left. \begin{aligned} I_{a_0} &= \frac{1}{3}[I_a + I_b + I_c] \\ I_{a_1} &= \frac{1}{3}[I_a + a I_b + a^2 I_c] \\ I_{a_2} &= \frac{1}{3}[I_a + a^2 I_b + a I_c] \end{aligned} \right\} \dots\dots\dots(7)$$

Zero Sequence Currents

In a 3 ϕ system, when there is a neutral return path for the currents,

$$I_n = I_a + I_b + I_c$$

So, the zero sequence current, I_{a0} is,

$$I_{a0} = \frac{1}{3} [I_a + I_b + I_c] = \frac{I_n}{3}$$

If load is delta connected or star connected with neutral not connected to ground, then, $I_n = 0$. So,
 $I_{a0} = 0$.

Example: The line currents in a 3φ supply due to an unbalanced load are $I_R = 12 + j24$, $I_Y = 16 - j2$ & $I_B = -4 - j6$ amperes. If the phase sequence is RYB, calculate the sequence currents.

Soln: $I_R = 12 + j24 = 26.8328 \angle 63.4349^\circ$

$$I_Y = 16 - j2 = 16.1245 \angle -7.1250^\circ$$

$$I_B = -4 - j6 = 7.2111 \angle -123.6901^\circ$$

$$\begin{aligned} I_{R0} &= \frac{1}{3} [I_R + I_Y + I_B] = \frac{1}{3} [12 + j24 + 16 - j2 - 4 - j6] \\ &= 8 + j 5.333 = 9.6148 \angle 33.6901^\circ \text{ Amps.} \end{aligned}$$

$$I_{R_1} = \frac{1}{3}[I_R + a I_Y + a^2 I_B] = \frac{1}{3}[12 + j24 + 1 \angle 120^\circ \cdot 16.1245 \angle -7.1250^\circ + 1 \angle 240^\circ \cdot 7.2111 \angle -123.6901^\circ] \\ = 0.8453 + j 15.0168 \\ = 15.1305 \angle 86.7974^\circ \text{Amp.}$$

$$I_{R_2} = \frac{1}{3}[I_R + a^2 I_Y + a I_B] = \frac{1}{3}[12 + j 24 + 1 \angle 240^\circ \cdot 16.1245 \angle -7.1250^\circ + 1 \angle 120^\circ \cdot 7.2111 \angle -123.6901^\circ] \\ = 3.1547 + j 3.5598 \\ = 4.7565 \angle 48.4528^\circ \text{Amp.}$$

Sequence Impedance

- Impedance offered by an equipment to +ve seq. component of current is called +ve seq. impedance of that equipment.
- Similarly, the -ve & zero seq. impedances are defined.

Sequence Networks of Unloaded Alternator

- Generator e.m.f.s are balanced voltages → considered to be +ve seq. e.m.f.s.
- Generators don't induce -ve seq. or zero seq. e.m.f.s.
- So, phase seq. of +ve seq. components is the same as the phase seq. of induced e.m.f.s.

1. Positive Seq. Network of Generator

- For each phase consists of an e.m.f. source in series with +ve seq.

Impedance Z_1 .

- E_a is induced e.m.f. of one phase.
- Z_1 is replaced by jX_1 , neglecting resistance. It is the +ve seq. reactance of the generator.

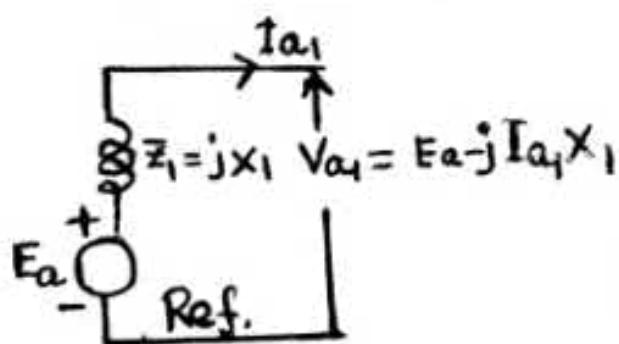


Fig. Positive sequence network for phase 'a'

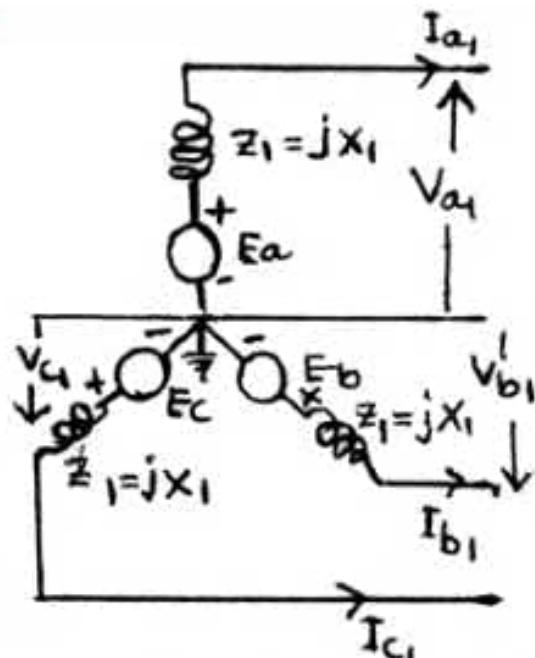


Fig. Positive sequence network for three phases

$$\text{Voltage eqn: } V_{a1} = E_a - j I_{a1} X_1$$

2. Negative Seq. Network of Generator

- As there is no -ve seq. e.m.f. induced by alternator,
- ve seq. network consists of -ve seq. impedance Z_2 (or reactance jX_2) only.

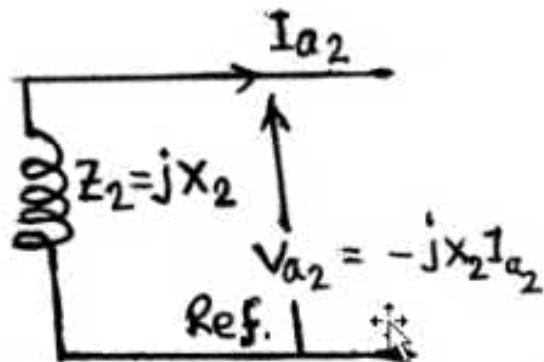


Fig. Negative sequence network for phase 'a'

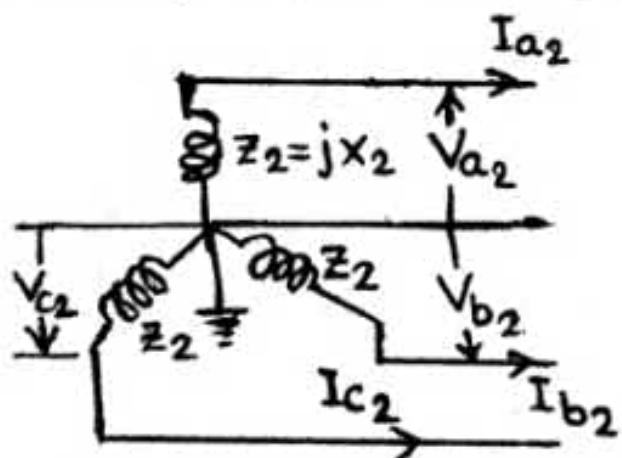


Fig. Negative sequence network for three phases

$$\text{Voltage equn. : } V_{a2} = -j I_{a2} X_2$$

3. Zero Sequence Network of Generator

- As generators do not induce any zero seq. e.m.f. →
- Zero seq. network consists of zero seq. impedance Z_{g_0} (or reactance jX_{g_0}) of the generator plus three times the impedance i.e. $3Z_n$ (or reactance $j3X_n$) connected in the neutral to ground circuit. This means,

$$Z_0 = Z_{g_0} + 3Z_n \quad (\text{or } jX_0 = jX_{g_0} + j3X_n)$$

The above can be explained as,

$$I_{a0} = \frac{1}{3} [I_a + I_b + I_c] = \frac{I_n}{3} \quad (\text{as } I_n = I_a + I_b + I_c)$$

or, $I_n = 3I_{a0}$



The current I_n flowing through neutral to ground impedance Z_n , produces a voltage drop of $I_n \cdot Z_n = 3I_{a_0} \cdot Z_n$. This is equivalent to flow of I_{a_0} through $3Z_n$. Hence,

Zero seq. impedance is

$$Z_0 = Z_{g_0} + 3Z_n.$$

If $Z_n = 0$, then $Z_0 = Z_{g_0}$.

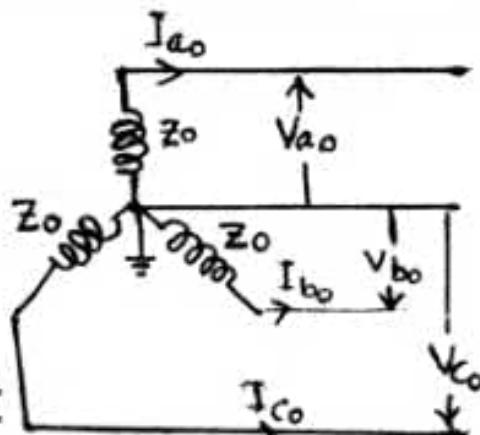


Fig. Zero sequence network for three phases

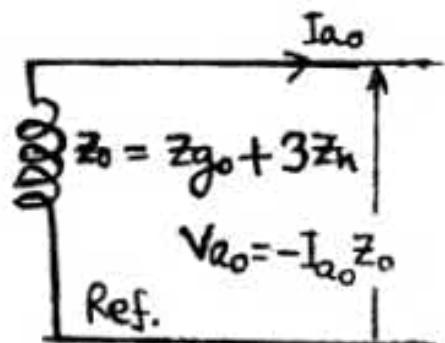
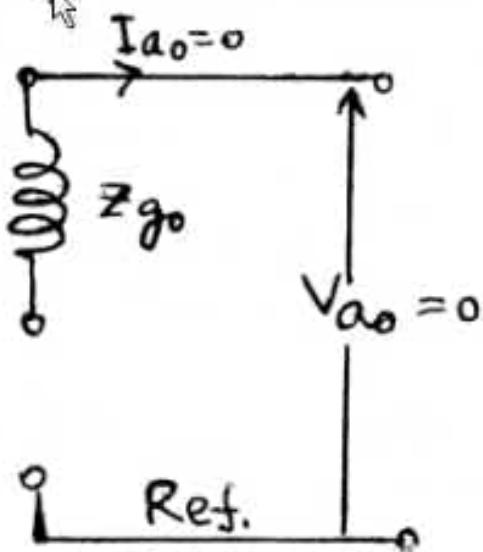


Fig. Zero seq. network for phase 'a'

If neutral is not grounded, then $Z_{z_0} = \infty$ and the zero seq. network is discontinuous.

Fig. Zero seq. network for insulated neutral



$$\text{Voltage eqn. : } V_{a0} = -j I_{a0} (x_{g0} + 3x_n)$$

L-G Fault on an unloaded 3φ Alternator

- Consider, L-G fault in phase 'a.'
- It is needed to find the fault currents & voltages of various Phases.
- Neglecting voltage drop in Z_n ,

$$V_a = 0; V_b, V_c \neq 0$$

$$I_a \neq 0; I_b, I_c = 0$$

So.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\Rightarrow I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \quad \dots \dots \dots (1)$$

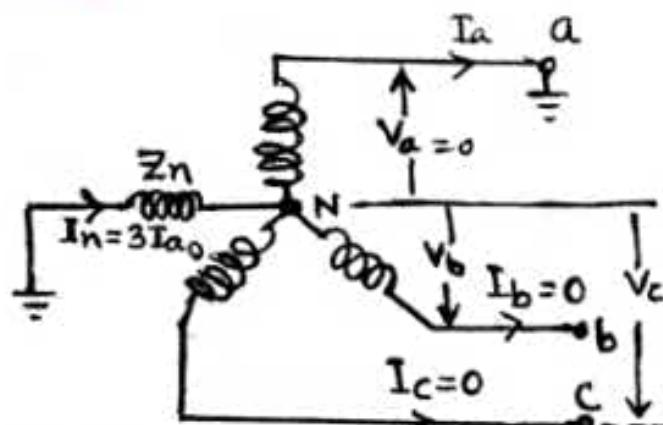


Fig. Circuit Condition for L-G fault

$$\begin{aligned}
 V_{a1} &= E_a - I_{a1}Z_1 \\
 V_{a2} &= -I_{a2}Z_2 \\
 V_{a0} &= -I_{a0}Z_0 \quad \dots\dots\dots (2)
 \end{aligned}$$

So, $V_a = V_{a1} + V_{a2} + V_{a0} = E_a - I_{a1}(Z_1 + Z_2 + Z_0) = 0$

Therefore,

$$I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2} = I_{a2} = I_{a0} = \frac{1}{3} I_a \dots(3)$$

- Since, the three seq. currents are equal & $V_a = 0$,
- ❖ the three seq. networks are connected in series & the end terminals are short circuited.

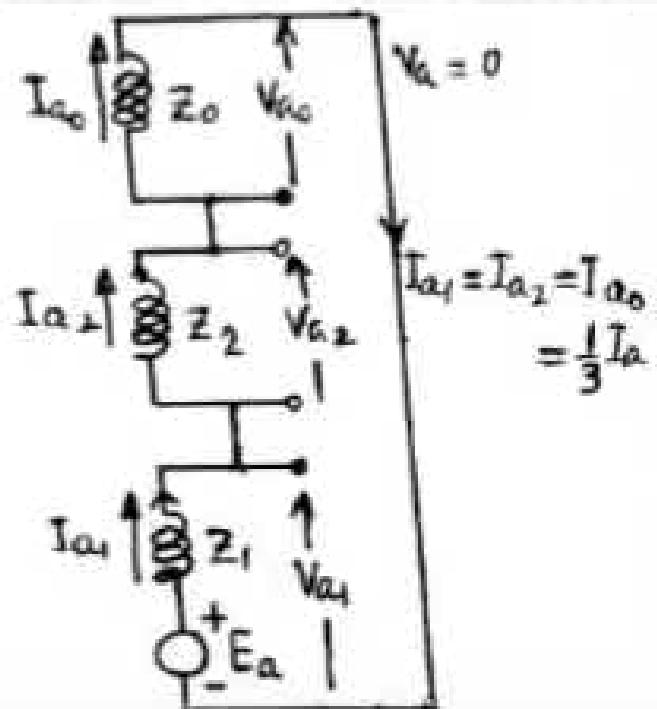
Next we calculate all the phase voltages & currents.

From eqn. (3),

$$I_a = I_{a_0} + I_{a_1} + I_{a_2} = \frac{3 E_a}{(Z_1 + Z_2 + 2r)} \quad (4)$$

$$I_b = I_c = 0 \quad (5)$$

$$\begin{aligned} V_a &= V_{a_0} + V_{a_1} + V_{a_2} \\ &= E_a - \frac{I_a}{3} (Z_1 + Z_2 + 2r) \\ &= 0 \quad (\text{check}) \end{aligned} \quad (6)$$



$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} \quad (7)$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} \quad (8)$$

Line to line Voltages :

$$\left. \begin{aligned} V_{ab} &= V_a - V_b \\ V_{bc} &= V_b - V_c \\ V_{ca} &= V_c - V_a \end{aligned} \right\} \quad (9)$$

Double Line to Ground Fault

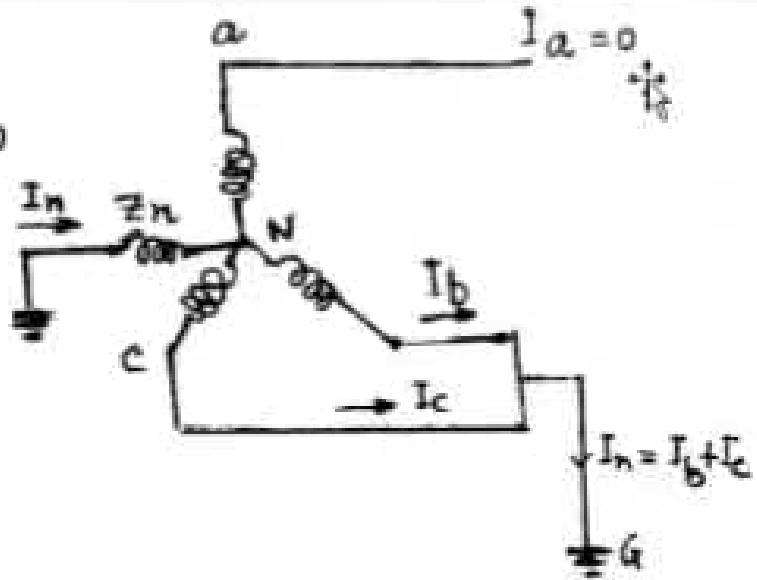
Assume, phases 'b' and 'c' to develop fault with the ground 'G.'

Here,

$$I_a = 0, I_b, I_c \neq 0.$$

$$V_b = V_c = 0, V_a \neq 0.$$

(Neglecting drop in Z_n)



Hence,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

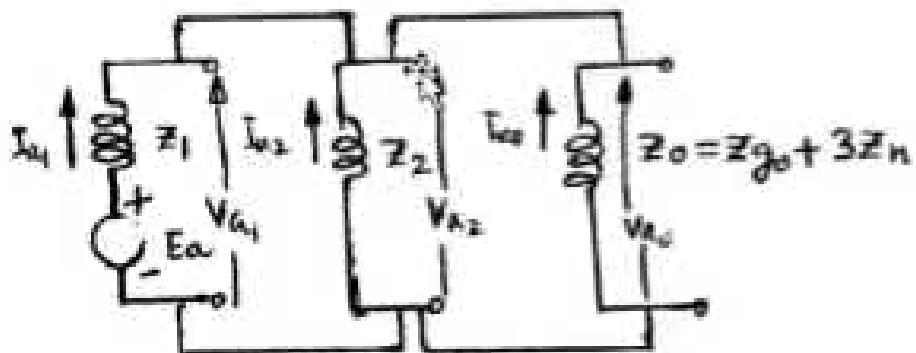
So,

$$V_{a0} = V_{a1} = V_{a2} = \frac{1}{3} V_a \quad \text{--- (1)}$$

Further,

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 \quad \text{--- (2)}$$

So, from eqns. (1) & (2), it may be concluded that the three seq. networks be connected in parallel.



From the figure,

$$I_{a1} = \frac{E_a}{Z_1 + 1/(Z_2 + Z_0)} = \frac{E_a}{Z_1 + \frac{Z_2 Z_0}{(Z_1 + Z_0)}} \quad \text{--- (3)}$$

As, $V_{a_1} = V_{a_4} = V_{a_2}$,

$$I_{a_2} = - \frac{V_{a_1}}{Z_2} \quad \text{--- (4)}$$

[V_{a_1} is calculated as: $V_{a_4} = E_a - I_{a_1} Z_1$]

$$I_{a_0} = - \frac{V_{a_1}}{Z_0} \quad \text{--- (5)}$$

Check that,

$$I_a = I_{a_0} + I_{a_1} + I_{a_2} = 0 \quad \text{--- (6)}$$

$$I_b = I_{a_0} + \alpha^2 I_{a_1} + \alpha I_{a_2} \quad \text{--- (7)}$$

$$I_c = I_{a_0} + \alpha I_{a_1} + \alpha^2 I_{a_2} \quad \text{--- (8)}$$

$$V_a = V_{a_0} + V_{a_1} + V_{a_2} = 3V_{a_1} \quad (9)$$

Where, $V_{a_1} = E_a - I_a \cdot Z_1$

$$V_b = V_{a_0} + a^2 V_{a_1} + a V_{a_2} = 0 \text{ (Check)} \quad (10)$$

$$V_c = V_{a_0} + a V_{a_1} + a^2 V_{a_2} = 0 \text{ (Check)} \quad (11)$$

Line to line voltages:

$$\left. \begin{aligned} V_{ab} &= V_a - V_b \\ V_{bc} &= V_b - V_c \\ V_{ca} &= V_c - V_a \end{aligned} \right\} \dots (12)$$

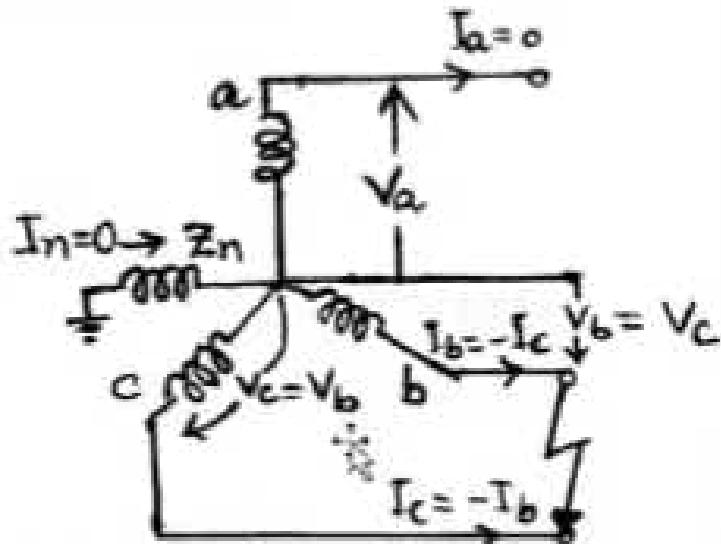
Line to Line Fault on an unloaded Alternator

Assume a line to line fault between phases 'b' and 'c.'

So,

$$I_a = 0, I_b = -I_c, I_c = -I_b$$

$$V_a \neq 0, V_b = V_c$$



In this case the fault does not involve the ground.

So, neutral current $I_n = 0$. This means,

$$I_{a_0} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c) = \frac{1}{3}(I_n) = 0 \quad \dots(1)$$

Now,

$$I_{a_1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$= \frac{1}{3}(\alpha I_b - \alpha^2 I_b) \quad [\text{Since, } I_a = 0 \text{ and } I_c = -I_b]$$

$$\alpha, I_a = \frac{1}{3} a (1-a) I_b \quad (2)$$

$$\begin{aligned} I_{a_2} &= \frac{1}{3} [I_a + a^2 I_b + a I_c] \\ &= \frac{1}{3} [a^2 I_b - a I_b] \quad [\text{Since, } I_a = 0 \text{ & } I_c = -I_b] \\ &= -\frac{1}{3} a (1-a) I_b \quad (3) \end{aligned}$$

Using eqns. (2) & (3),

$$I_{a_1} = -I_{a_2} \quad (4)$$

Considering eqn. (1),

$$V_{a_0} = -I_{a_0} Z_0 = 0 \quad (5)$$

Also,

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$
$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

◎ [As, $V_b = V_c$]

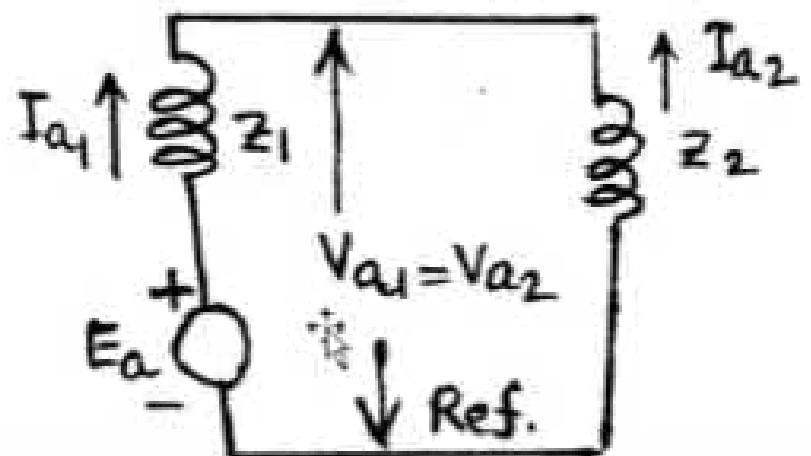
$$V_{a_1} = \frac{1}{3} [V_a + a V_b + a^2 V_b] = \frac{1}{3} [V_a + (a+a^2) V_b] \quad (6)$$

$$V_{a_2} = \frac{1}{3} [V_a + a^2 V_b + a V_b] = \frac{1}{3} [V_a + (a+a^2) V_b] \quad (7)$$

Considering eqns. (6) & (7),

$$V_{a_1} = V_{a_2} \quad (8)$$

Considering eqns. (4) & (8), +ve & -ve seq. networks are connected in parallel.



From the fig.,

$$I_{a1} = -I_{a2} = \frac{E_a}{Z_1 + Z_2} \quad (9)$$

Since I_{a_0} , I_{a_1} & I_{a_2} are known from eqns. (1) & (9), I_a , I_b & I_c can be calculated as,

$$I_a = I_{a_0} + I_{a_1} + I_{a_2} = 0 \text{ (Check)} \quad \dots\dots(10)$$

$$I_b = I_{a_0} + \alpha^2 I_{a_1} + \alpha I_{a_2} \quad \dots\dots(11)$$

$$I_c = I_{a_0} + \alpha I_{a_1} + \alpha^2 I_{a_2} \quad \dots\dots(12)$$

Now,

$$V_{a_1} = E_a - I_{a_1} \cdot Z_1 = V_{a_2} \quad \dots\dots(13)$$

Since V_{a_0} , V_{a_1} & V_{a_2} are known from eqns. (5) & (13), V_a , V_b & V_c can be calculated as,

$$V_a = V_{a_0} + V_{a_1} + V_{a_2} \quad (14)$$

$$V_b = V_{a_0} + a^2 V_{a_1} + a V_{a_2} \quad (15)$$

$$V_c = V_{a_0} + a V_{a_1} + a^2 V_{a_2} \quad (16)$$

Line to line voltages:

$$\left. \begin{aligned} V_{ab} &= V_a - V_b \\ V_{bc} &= V_b - V_c \\ V_{ca} &= V_c - V_a \end{aligned} \right\} \quad (17)$$

Example: A 3φ, 11kV, 50MVA alternator has $x_1=15\%$, $x_2=12\%$ & $x_0=8\%$. The neutral of the alternator is solidly grounded. Calculate the currents in all the phases due to (i) 3φ symmetrical fault, (ii) L-G fault, (iii) L-L-G fault & (iv) L-L fault.

Hints: Take base kV = 11kV & base MVA = 50 MVA

$$(i) I_a = \frac{E_a}{jx_1} = \frac{1.0}{j0.15} = -j 6.6666 \text{ p.u.}$$

Then, calculate the base current \rightarrow calculate the fault current in ampere.

(ii) Consider, the L-G fault to occur in phase 'a.'

$$I_a = \frac{3E_a}{jx_1 + jx_2 + jx_0} = \frac{3}{j0.15 + j0.12 + j0.08} = -j 8.5714 \text{ p.u.}$$

Then, as the base current is known \rightarrow calculate the fault current in ampere.

(iii) Consider L-L-G fault involving phases 'b' & 'c.' So, in this case $I_a=0$, I_b , $I_c = ?$

→ Complete as discussed.

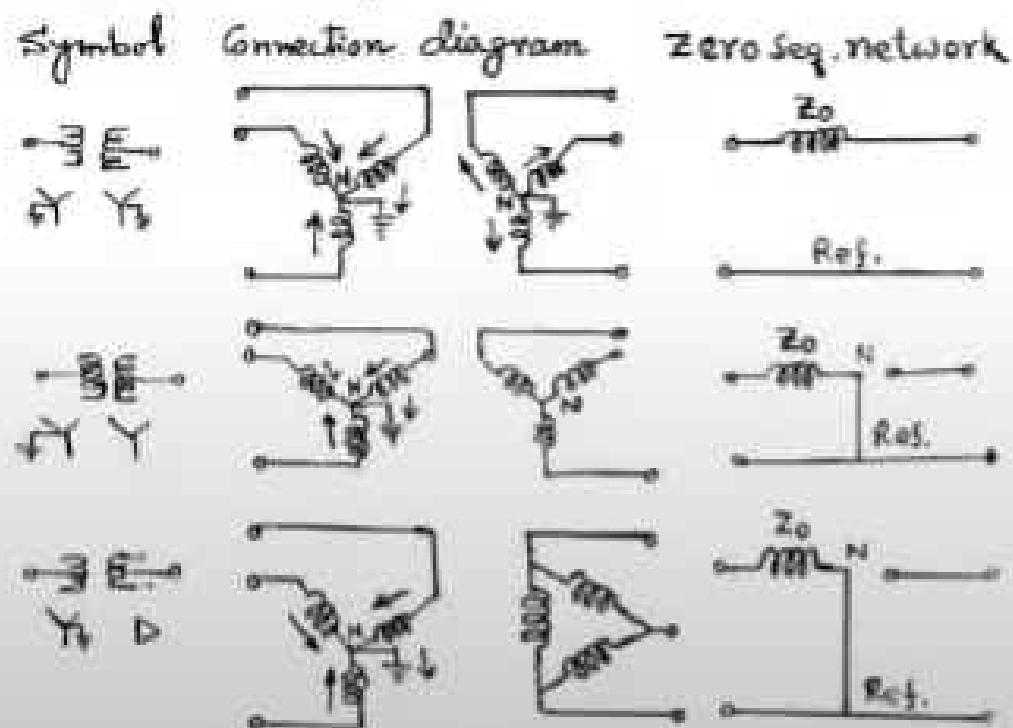
(iv) Consider L-L fault involving phases 'b' & 'c.' So, in this case

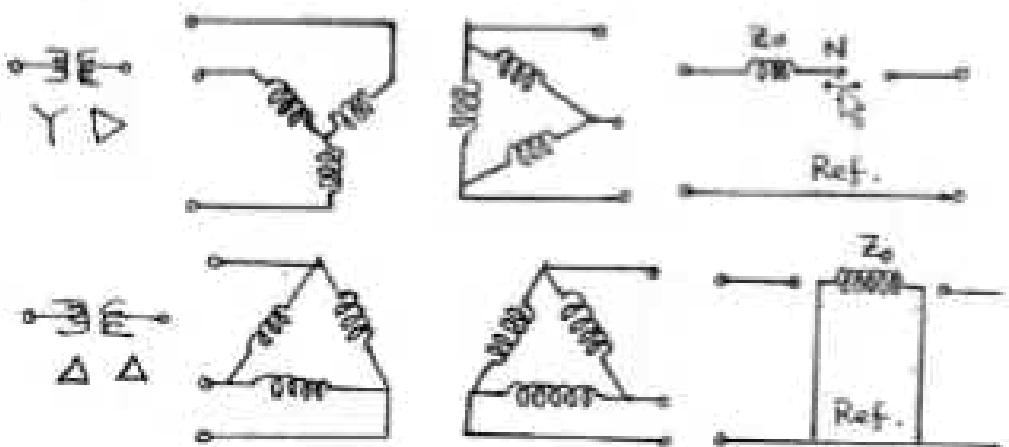
$I_a=0$, I_b , $I_c = ?$ I

→ Complete as discussed.

Zero Seq. Networks for Different 3Φ Transformer Connections

- Zero seq. current can only flow through ground connection.
- So, if neutral is not grounded \rightarrow zero seq. network is open \rightarrow zero seq. current can't flow.
- Hence, zero seq. networks are different for different types of transformer connections. I





- In the last connection, zero seq. current can flow in the delta, if any zero seq. voltage is produced within the delta. But this current can't come out of the delta. So, the typical zero seq. network is as shown in the fig.

Connections of Seq. Networks for Faults in Power Systems

- Connections of seq. networks due to different types of faults on an unloaded alternator are different.
- Same concept is extended towards different types of faults in power systems.
- Three seq. networks are drawn separately using the data (+ve, -ve & zero seq. reactances) given for different power system elements.
- The fault point is indicated in these networks.
- Then, the Thevenin's equivalent circuit is drawn for each of the networks.
- Next, the three Thevenin's equivalents circuits are connected in the same manner as is connected for an unloaded generator for the same type of fault.

- The three Thevenin's equivalent circuits are connected in series if it is a L-G fault.
- The three Thevenin's equivalent circuits are connected in parallel if it is a L-L-G fault.
- The +ve & -ve seq. Thevenin's equivalent circuits are connected in parallel if it is a L-L fault.

Example: A single line to ground fault has occurred at point 'P' of the system given in fig. 1. The two **generators** are rated at **1000kVA, 400V** with reactances of $x_1 = x_2 = x_0 = 10\%$. Each transformer is rated at **1000kVA/ 400/3300V** with leakage reactance of 5%. Reactances of the transmission line are of $x_1 = x_2 = 15\% , x_0 = 50\%$ on **1000kVA & 3300V base**. Neglect the pre-fault current & calculate the sub-transient fault current.

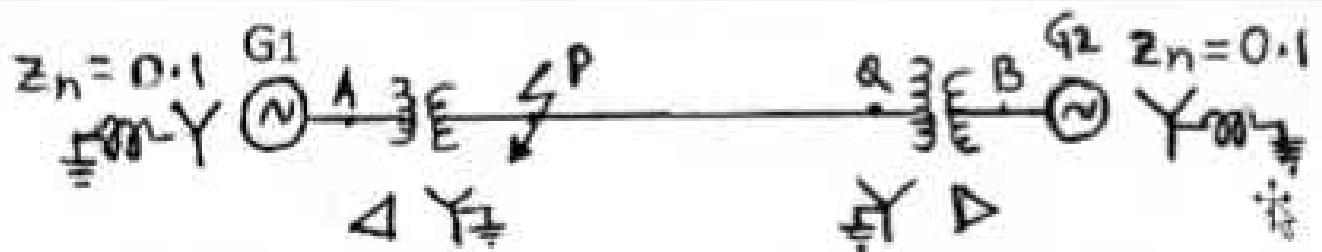


Fig. 1 Sample power system with L-G fault at P.

Soln: Take base kVA as 1000kVA & base kV on L.V. side as 0.4kV (i.e. 400V) & same on H.V. side as 3.3kV (i.e. 3300kV).

Positive Seq. network

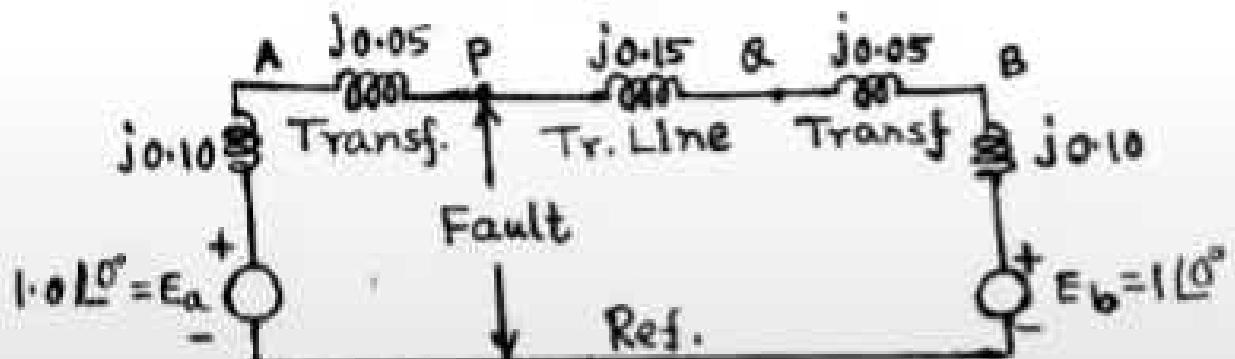


Fig. 2 Positive seq. network for L-G Fault

The Thevenin's equivalent circuit for the fault at P is,

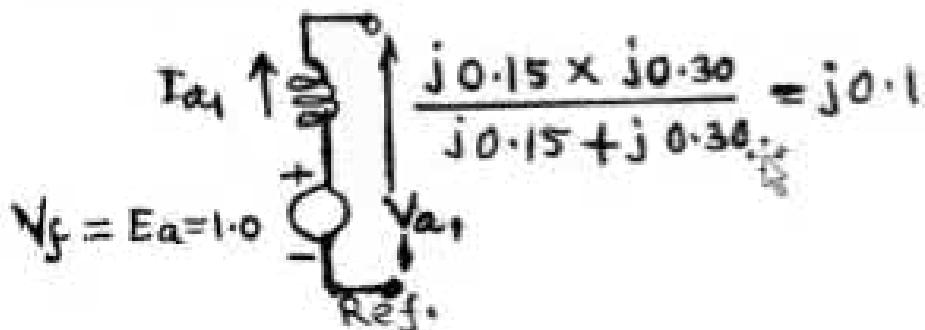


Fig. 3 Thevenin's equivalent of fig. 2

Negative Seq. Network

As there is no e.m.f. source in negative seq. network,

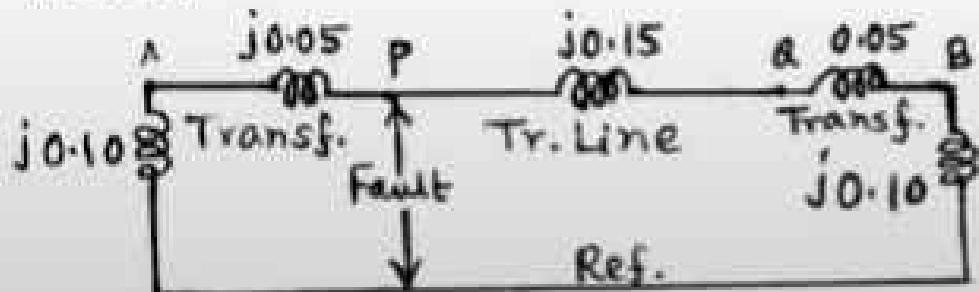


Fig. 4 The negative sequence network for L-G fault.

Thevenin's equivalent circuit for L-G fault at P is,

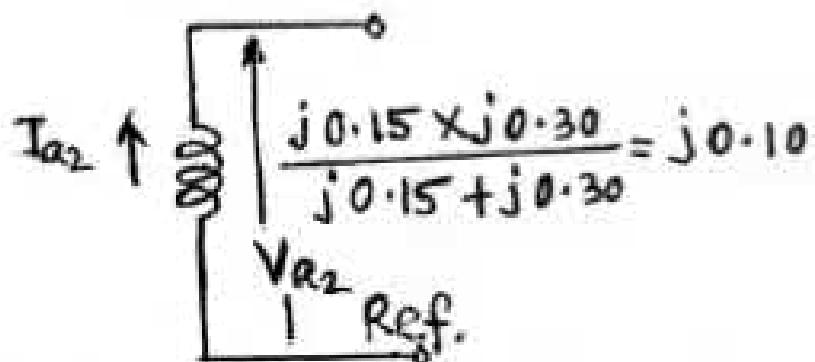


Fig. 5 Thevenin's equivalent circuit of fig. 4.

Zero seq. Network

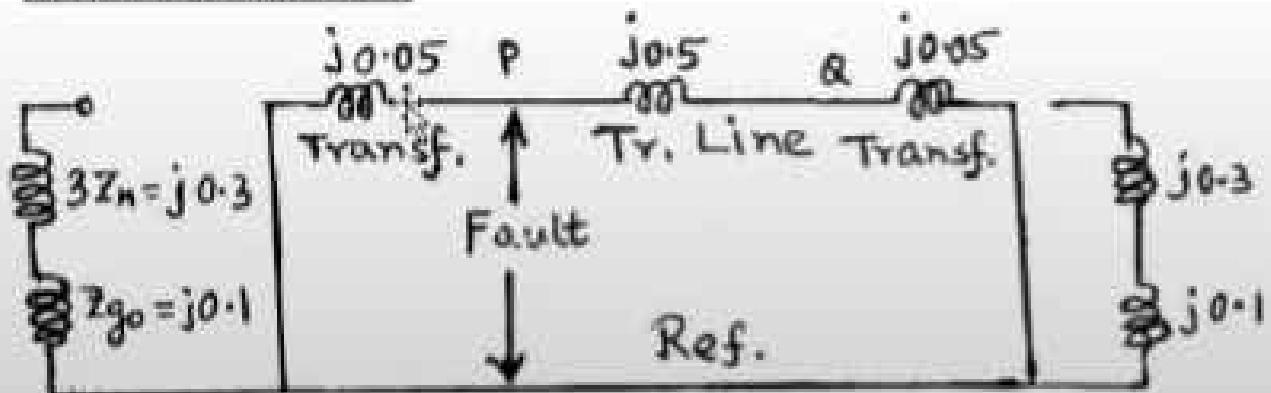


Fig. 6 Zero seq. network for L-G fault

Thevenin's equivalent circuit for a fault at P is,

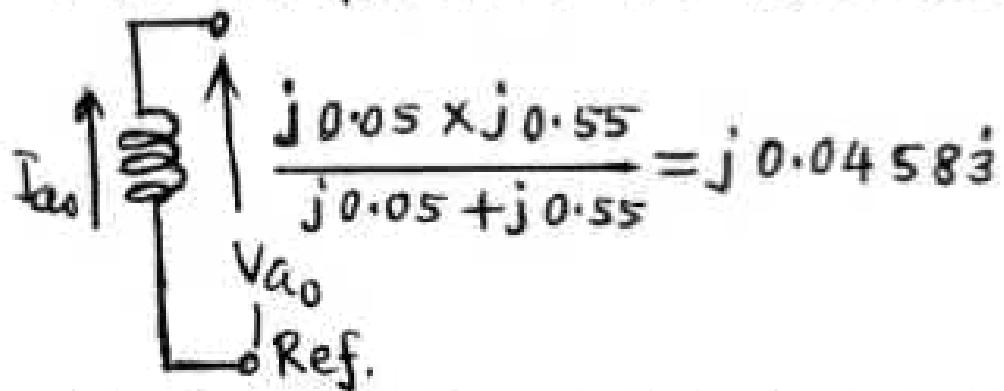


Fig. 7 Thevenin's equivalent for fig. 6

For L-G fault Thevenin's equivalent circuit for the three seq. networks are connected in series.

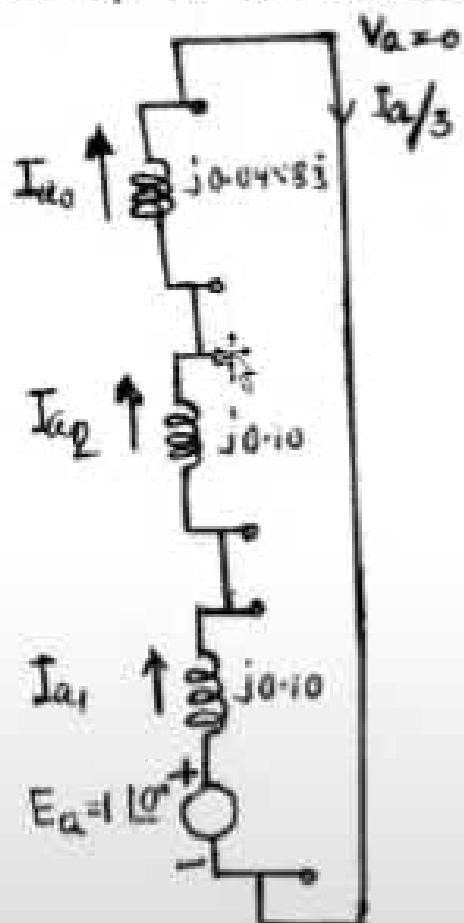


Fig. 8 Series connection of three Thevenin's equivalent circuits.

We know that for L-G fault,

$$I_{a0} = I_{a2} = I_{a1} = \frac{I_a}{3} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

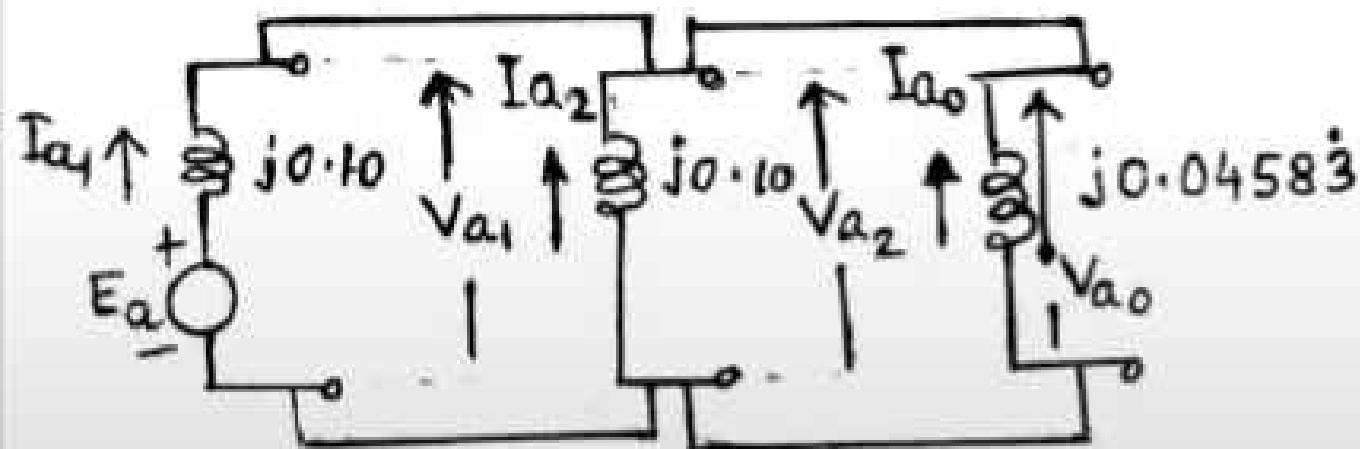
$$\text{So, } I_a = \frac{3}{(j0.10 + j0.10 + j0.0458)} = -j12.203389 \text{ p.u.}$$

$$\text{Now, base current} = \frac{\text{Base kVA}}{\sqrt{3} \cdot \text{Base KV}} = \frac{1000}{\sqrt{3} \cdot 3} \\ = 174.9546 \text{ A}$$

$$\text{Hence, fault current} = 12.203389 \cdot 174.9546 \\ = 2135.03937 \text{ A}$$

Example: Consider the system of the last example. Calculate the sub-transient fault currents for (a) L-L-G fault & (b) L-L fault at point 'P'.

Soln.: (a) L-L-G fault between phases b & C with ground is assumed. In this case Thevenin's equivalent circuits of the three seq. networks are to be connected in parallel.



$$\text{So, } I_{a_1} = \frac{E_a}{Z_1 + \frac{1}{(Z_2 + Z_0)}} = \frac{1+j0}{j0.10 + \frac{1}{(j0.10 + \frac{1}{0.04583})}} \\ = -j7.6087 \text{ p.u.}$$

$$V_{a_1} = E_a - I_{a_1} Z_1 = (1+j0) - (-j7.6087)(j0.10) = 0.2391 \text{ p.u.}$$

$$I_{a_2} = -\frac{V_{a_1}}{Z_2} = -\frac{0.2391}{j0.10} = j2.3913 \text{ p.u.}$$

$$I_{a_0} = -\frac{V_{a_1}}{Z_0} = -\frac{0.2391}{j0.04583} = j5.2174 \text{ p.u.}$$

$$I_b = I_{a_0} + \alpha^2 I_{a_1} + \alpha I_{a_2} = j5.2174 + (-0.5 - j0.866)(-j7.6087) \\ + (-0.5 + j0.866)(j2.3913) \\ = j5.2174 + j3.8043 - 6.5891 - j1.1957 - 2.0709 \\ = -8.6600 + j7.8260 = 11.6722 \angle 137.8959^\circ$$

$$\begin{aligned}
 I_c &= I_{a_1} + \alpha I_{a_2} + \alpha^2 I_{a_3} = j 5.2174 + (-0.5 + j 0.866) (-j 7.6087) \\
 &\quad + (-0.5 - j 0.866) (j 2.3913) \\
 &= j 5.2174 + j 3.8044 + 6.5891 - j 1.1957 + 2.0709 \\
 &= 8.6600 + j 7.8260 \\
 &= 11.6722 \angle 42.1044^\circ \text{ pu.}
 \end{aligned}$$

Now, The fault current to the ground = $I_h = (I_b + I_c)$

$$\begin{aligned}
 &= (-8.6600 + j 7.8260) + (8.6600 + j 7.8260) \\
 &= j 15.6520 \text{ pu.}
 \end{aligned}$$

Hence, fault current = $15.6520 \times 174.9546 \text{ A}$
 $= 2738.389399 \text{ A}$

3Φ Power in Terms of Symmetrical Components

- Power consumed in 3Φ circuits can be computed directly from symmetrical components of voltages & currents.
- Consider, 'S' be the total complex power for three phases.
- Also consider, V_a , V_b & V_c be the 3Φ voltages with corresponding currents of I_a , I_b & I_c .

$$\begin{aligned} \text{Then, } \overline{P} &= P + j Q = V_a \cdot I_a^* + V_b \cdot I_b^* + V_c \cdot I_c^* \\ &= [V_a \ V_b \ V_c] \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} \quad (1) \end{aligned}$$

But, it is known that,

$$\begin{aligned} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \\ &= A \cdot \mathbf{v} \quad (2) \end{aligned}$$

But, it is known that,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = A.V \quad \dots\dots\dots(2)$$

I

where, $\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = V \quad \dots\dots\dots(3)$

So, from eqn. (2),

$$[V_a \ V_b \ V_c] = (A \cdot V)^T = V^T \cdot A^T \quad \dots \dots \dots (4)$$

Similarly,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Define,

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = I \quad \dots \dots \dots (5)$$

Therefore,

$$\begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} = A^* \cdot I^* \quad \dots \dots \dots (6)$$

Utilising eqns. (4) & (6),

$$S = P + jQ = V^T \cdot A^T \cdot A^* \cdot I^* \quad \dots\dots\dots(7)$$

Now,

$A^T = A$ and since 'a' & ' a^2 ' are conjugates,

$$A^T \cdot A^* = A \cdot A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = 3 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots\dots\dots(8)$$

So,

$$S = 3 \cdot [V_{a_0} \ V_{a_1} \ V_{a_2}] \cdot \begin{bmatrix} I_{a_0}^* \\ I_{a_1}^* \\ I_{a_2}^* \end{bmatrix} = 3 \cdot [V_{a_0} \cdot I_{a_0}^* + V_{a_1} \cdot I_{a_1}^* + V_{a_2} \cdot I_{a_2}^*] \quad \dots\dots\dots(9)$$

Electrical Layouts & Busbar Arrangements

Any power system may be divided into:

1. Generating stations,
2. Transmission systems,
3. Receiving stations,
4. Distribution systems &
5. Final load points.

❖ In all the above stations or systems the important & necessary items are:

- (a) Busbar,
- (b) Circuit breaker,
- (c) Isolator,
- (d) Earthing switch,
- (e) C.T. & P.T. and
- (f) Lightning arrester.

Fig. 1 shows these items in an electrical system.

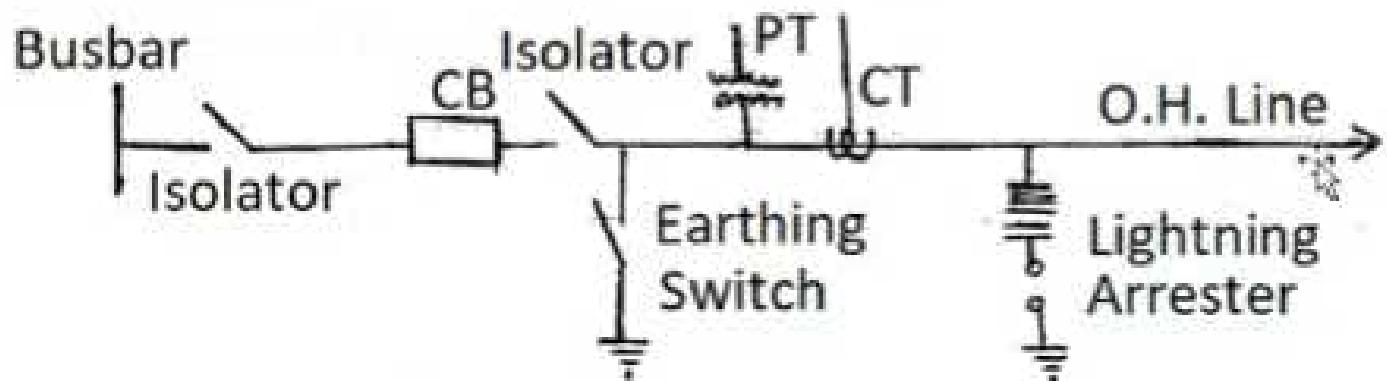


Fig. Equipment/items in an electrical scheme

Busbar

- Conducting bars made up of copper/aluminium to which generators/ transformers/ lines/ loads etc. are connected.
- Busbars of different phases & neutral are insulated from each other & ground.

Circuit Breaker

- It disconnects any item/element of power system that has suffered fault/has started to operate in abnormal manner.
- CB is aided in this task by protective relay.

Isolator

- It is also called disconnecting switch/simply disconnector.
- It is basically a switch, designed to be operated under no load condition.
- So, while opening → isolator operates only after CB has operated.
- While closing → isolator closes first, then the CB closes.
- For this type of operation → interlocks are used.

Earthing Switch

- High voltage/extra voltage lines have significant system capacitance between lines & between lines & earth.
- In such cases, even after lines are disconnected by the operation of CBs & isolators → there is some voltage to which these capacitances remain charged → dangerous for repair & maintenance works.
- This voltage is discharged to earth by closing the earthing switch.

C.T. & P.T.

- These are required for protection & measurement purposes.

Lightning Arrester

- Overhead lines are exposed to lightning strikes.
- High voltage surges due to lightning can cause breakdown of insulation of different power system items → eventually causing fault.
- Lightning arrester is connected between the line & the ground → to divert the high voltage surges to earth.
- It is placed such that → high voltage surges meet the lightning arrester first before meeting any power system element.

Load Break (Interrupt) Switch (L.B.S.)

- In distribution systems maximum of 33kV is used.
- In such cases fault level may not be high enough to justify the use of CBs economically.
- L.B.S. is used then in conjunction with fuses & CBs.
- It combines the functions of isolator & switch.
- L.B.S. is capable to make & break normal load currents.
- It is capable to make but not break the fault currents.
- It is also used to interrupt small inductive & capacitive currents.

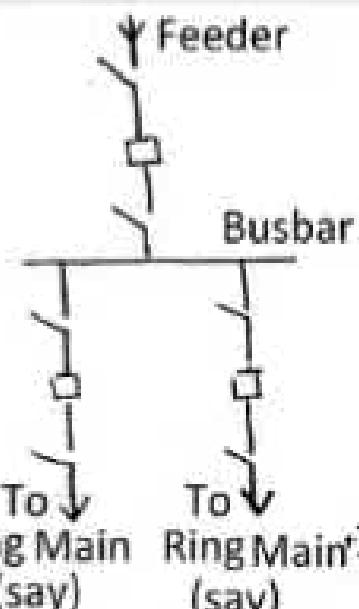


Fig. 1 Scheme using CBs & isolators

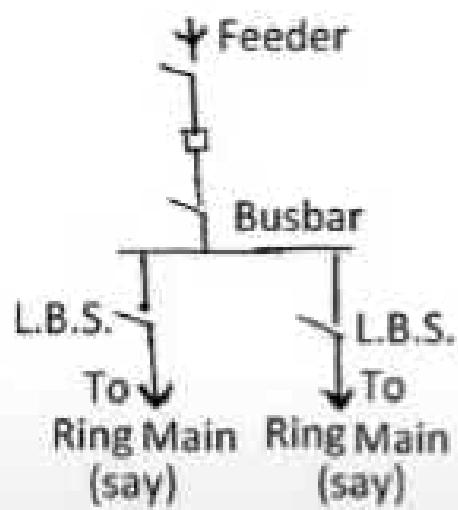


Fig. 2 Scheme of Fig.1 using L.B.S. & incoming CB

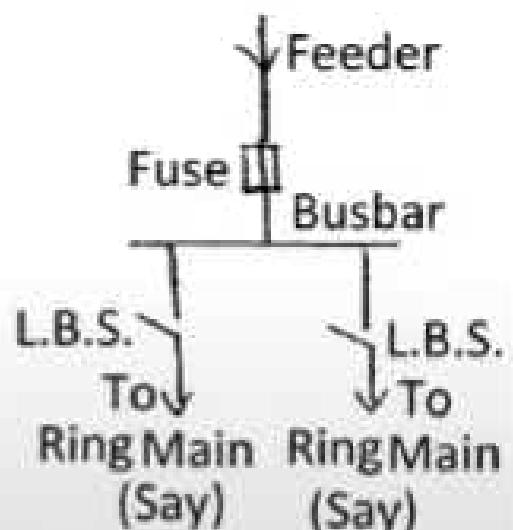


Fig. 3 Scheme of Fig.1 using L.B.S. & fuse

Busbar Arrangements

- There are different possible arrangements.
- Choice of a particular arrangement depends on various factors –
 - (i) System voltage,
 - (ii) Position of the station in the system,
 - (iii) Flexibility/simplicity of operation,
 - (iv) Maintenance without interruption of supply or danger to operating personnel.
 - (v) Reliability of supply,
 - (vi) Alternative arrangements in the event of an outage of any item.
 - (vii) Provisions for future expansions of the system.
 - (viii) Economical aspects etc.

1. Single Busbar Arrangement

- There is only one bus for each Phase to which various feeders & distributors are connected.
- This arrangement has least flexibility & immunity from total shut-down. I
- In case of a fault/maintenance of the bus bar, the entire system has to be de-energized → leading to complete shut-down.
- However, this scheme is the most simple & economical.
- Used in switchyards of small & medium sized substations/small power stations.

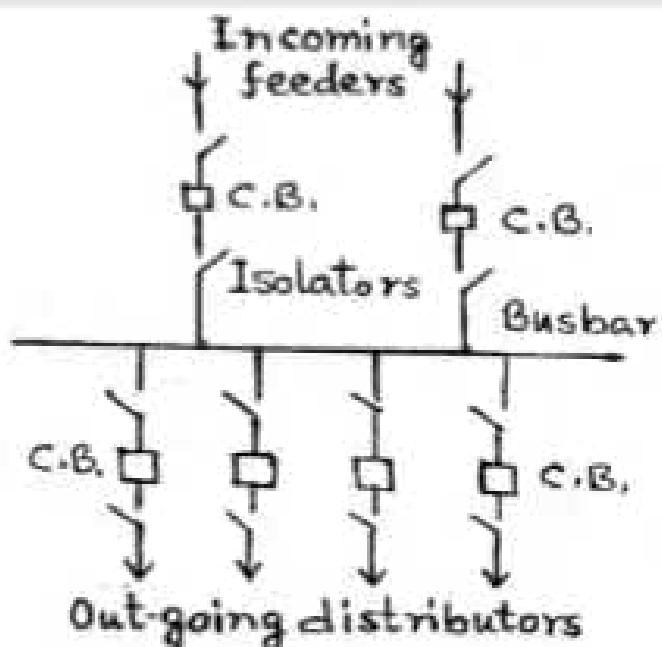


Fig. 1 Single busbar arrangement

2. Duplicate Busbar Arrangement

This type of arrangement provides –

- Flexibility of operation
- Continuity of supply.
- Periodic maintenance without shutdown.
- For a fault in one of the bus bars, the other one is used.

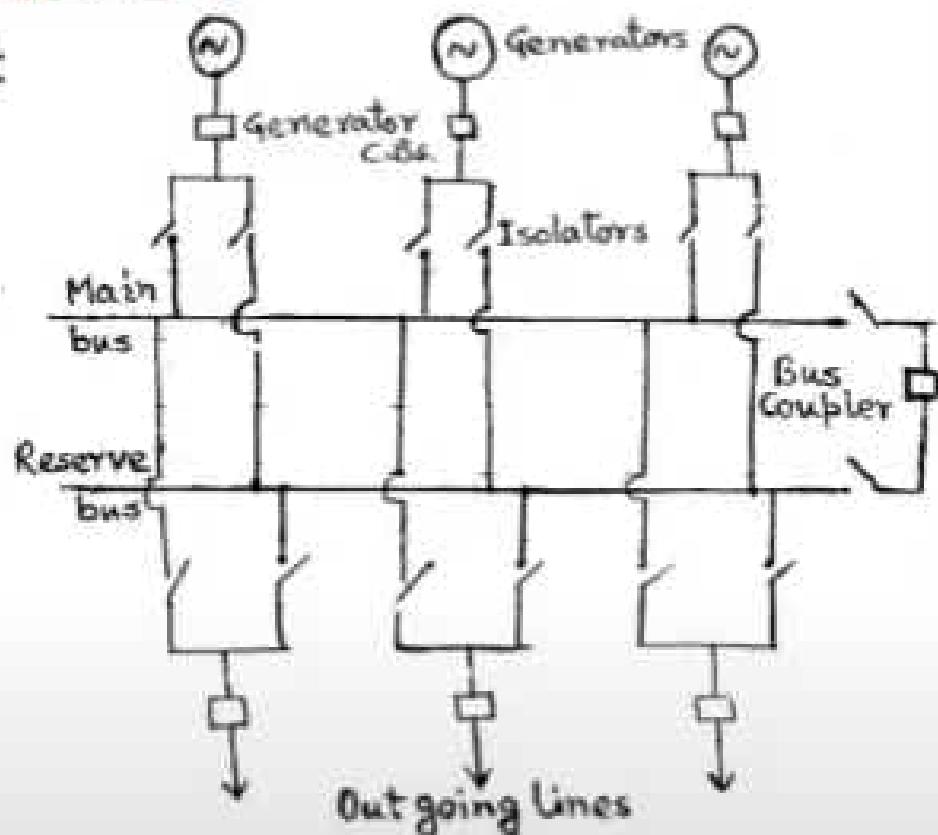


Fig.2. Duplicate bus-bar arrangement

Transfer of Operation from Main-bus to Reserve-bus

The following steps are followed:

1. Close the bus coupler → the two buses are now at same potential.
2. Close the isolators of the reserve-bus → now the loads are supplied from both the buses.
3. Open the isolators of the main-bus → load is now supplied only from the reserve-bus.
4. Open the bus coupler.

3. Sectionalization of Bus

This arrangement provides added advantages.

- One section can be completely shut-down

for maintenance &

repair → while other sections continue to operate.

- By adding current limiting reactors between sections, fault MVA can be considerably reduced.

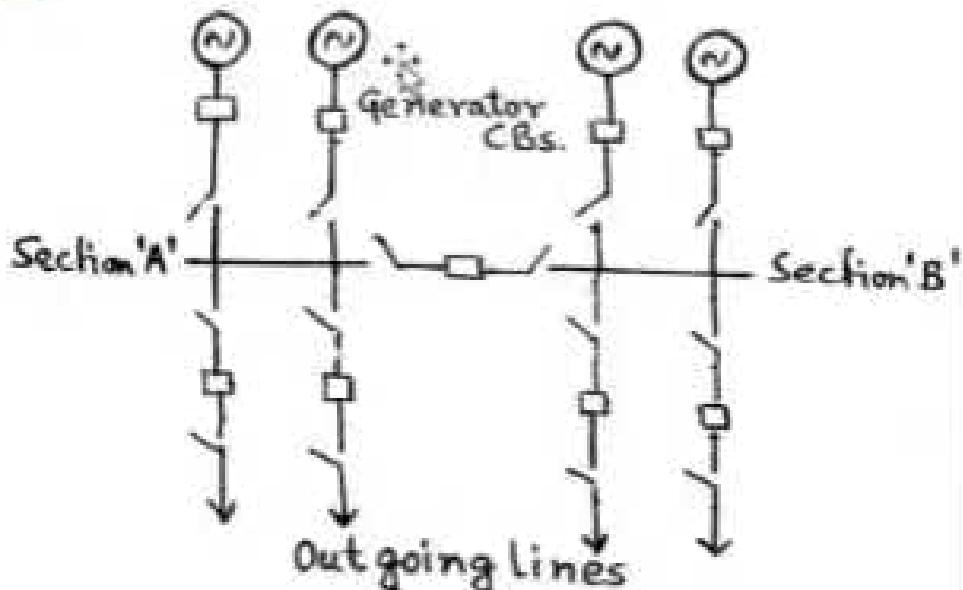


Fig.3 Sectionalization of bus

4. Ring Bus

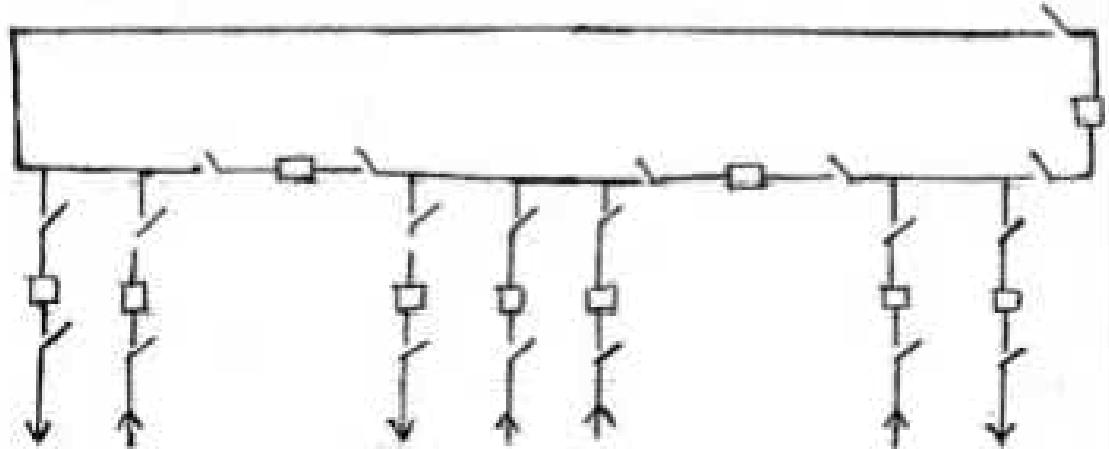


Fig.4. Ring bus arrangement

This arrangement provides greater flexibility-

- Power can be drawn from any adjacent section.
- Effect of fault is localized to that section only → other sections continue to operate.

5. One-And-A-Half Breaker Arrangement

This type of arrangement provides –

- Use of only three CBs for two circuits.
- Any CB can be made off for maintenance, without provision of by pass.
- No. of CBs per circuit is $1\frac{1}{2}$, hence name.
- Circuits I & II can get power from any bus.
- This arrangement gives high security against loss of supply.
- Suitable for switchyards of big generating stations & 400kV & 750kV substations.

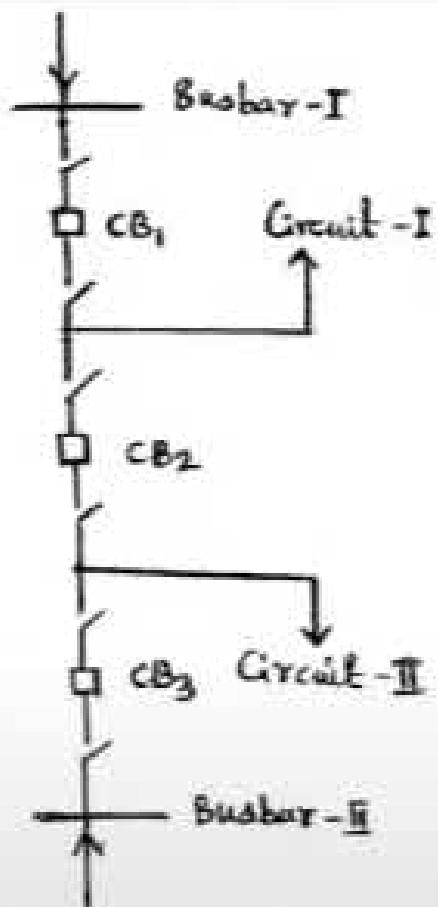


Fig.5.0ne-and-a-half
breaker arrangement

6. Mesh Arrangement

This arrangement provides –

- Economic use of CBs in substations.
- CBs are installed in the mesh formed by buses.
- Different circuits are connected at the nodes (buses) of the mesh.
- Here, four CBs control eight circuits.
- For fault in any circuit two CBs open → causing opening of the mesh.

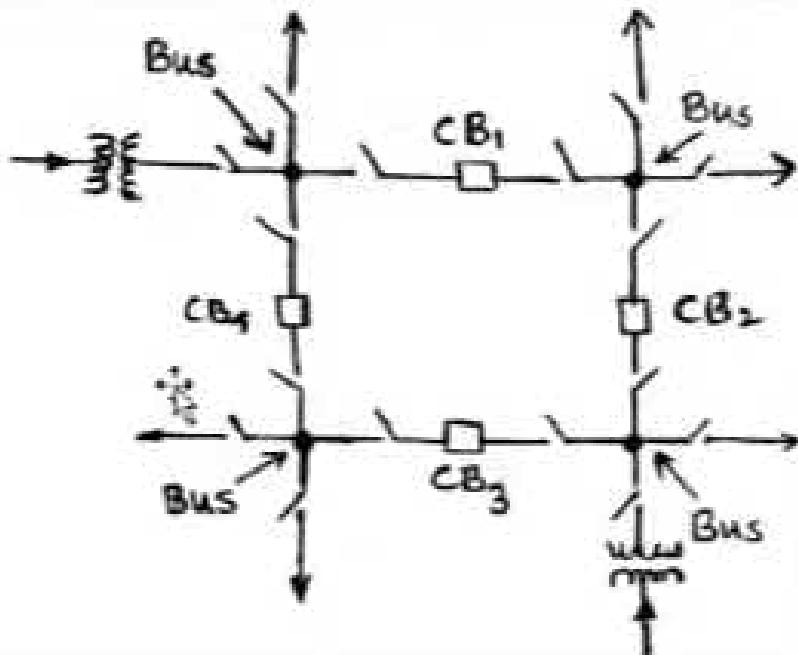


Fig. 6 Mesh arrangement (Four CBs control eight circuits)

Generator Connections

- Generated voltages are much lower in comparison to transmission voltages.
- These are limited to 33kV for design considerations.
- There are several types of generator connections.

1. Classical Method

- A number of units are connected to a common “Generator Bus” through generator CBs.
- This scheme is used for small & medium size power stations.

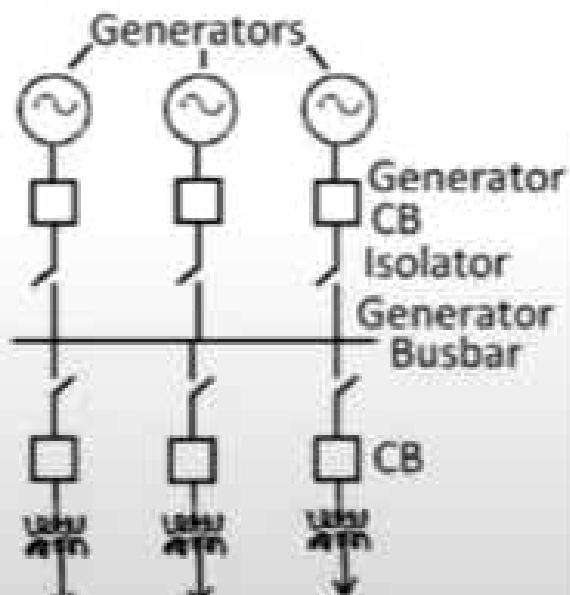


Fig. 1 Classical method of generator connection

2. Unit System of Generator Connection

- The scheme emerges from economical point of view.
- Generator is directly connected to transformer to form one unit, avoiding high capacity generator CB.
- The unit is connected to H.V. bus through H.V. CB.
- The H.V. bus is connected to live external supply system.
- There is a starting transformer to start the unit.
- The station service supply busbar initially receives power from the H.V. bus through the starting transformer.

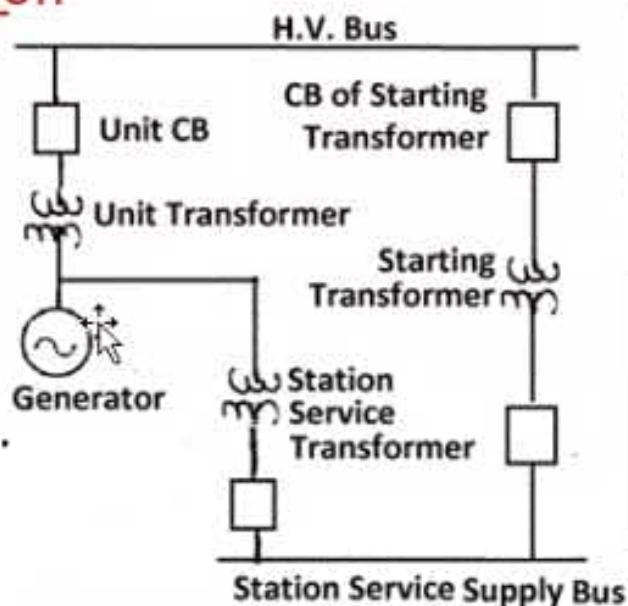


Fig. 2 Unit System of generator connection

- This power is necessary for the auxiliary items of the unit.
- As soon as the unit starts operating it is synchronized to the H.V. bus by closing of the H.V. CB.
- Then, the station service supply bus is switched over from the starting transformer to the station service transformer.
- This method is used in medium sized power stations.
- However, the starting transformer becomes idle after the unit is synchronized to the H.V. bus.

3. Scheme using Generator CB

- Here, the starting transformer is avoided by introducing the generator CB.
- Initially the generator CB is open & unit CB is closed.
- The station service supply bus receives power from HV bus through the unit transformer.
- This power is necessary for the auxiliary items of the unit.
- As soon as the unit starts operating it is synchronized to the H.V. bus by closing of the generator CB.

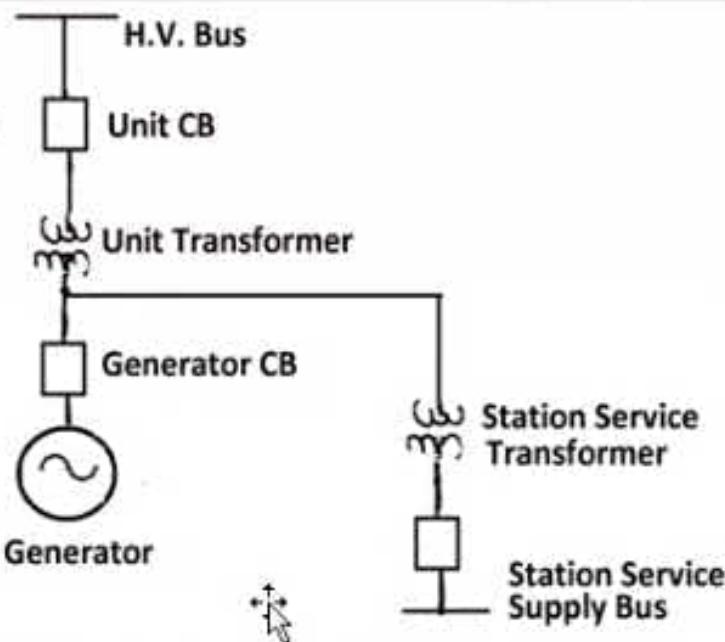


Fig. 3 Unit system of generator connection- with generator CB

Substations

Several substations between generating station & final load points.

Components of substation

1. Incoming lines,
2. Outgoing lines,
3. Transformers,
4. H.V. & L.V switchgears, relays, CBs, isolators etc.,
5. C.T.s & P.T.s, metering panels etc.,
6. Drop-out fuses,
7. Substation service equipment → lighting, auxiliary battery supply, transformer oil purification set, compressed air system.
8. Lightning arresters, overhead earth wires,
9. Station earthing systems.
10. Communication equipment → carrier current equipment, telephone.

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Layout of Substation Equipment

- Fig. shows layout of switchyard equipment of a typical receiving station.
- It has two incoming & three outgoing lines.
- It uses a typical single busbar Arrangement.

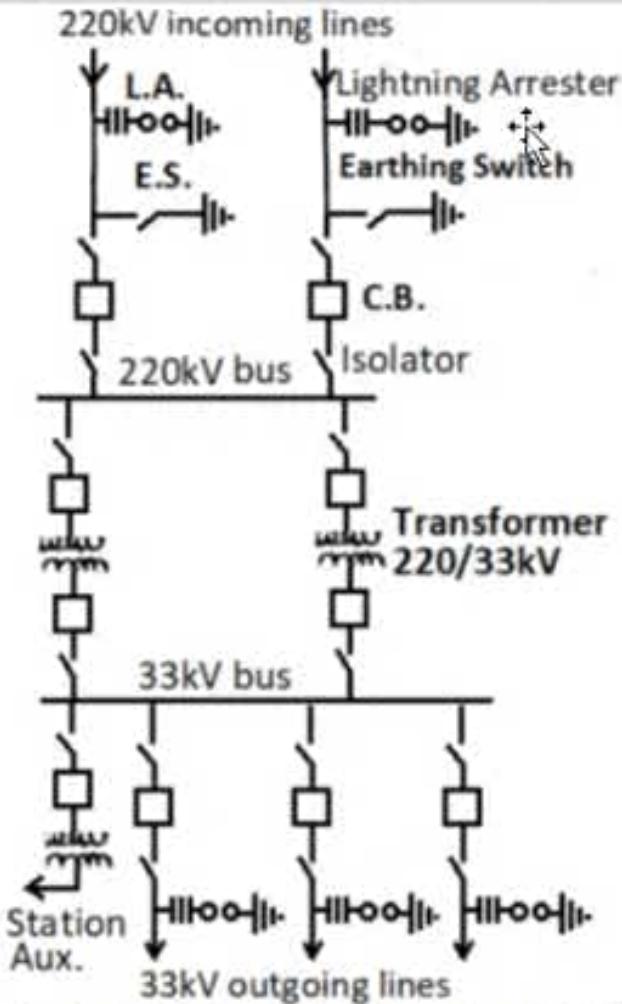


Fig. A typical receiving end substation

Layout of Typical Station in Distribution System

- In low voltage distribution systems elaborate protection may not be justified for protecting transformers up to 500kVA.
- These substations are generally unattended type.
- In such cases, H.V. fuses, like drop-out fuses are the only protection on the H.V. side.
- In L.V. side also fuses are used.
- Here, the fuses are suitably coordinated.

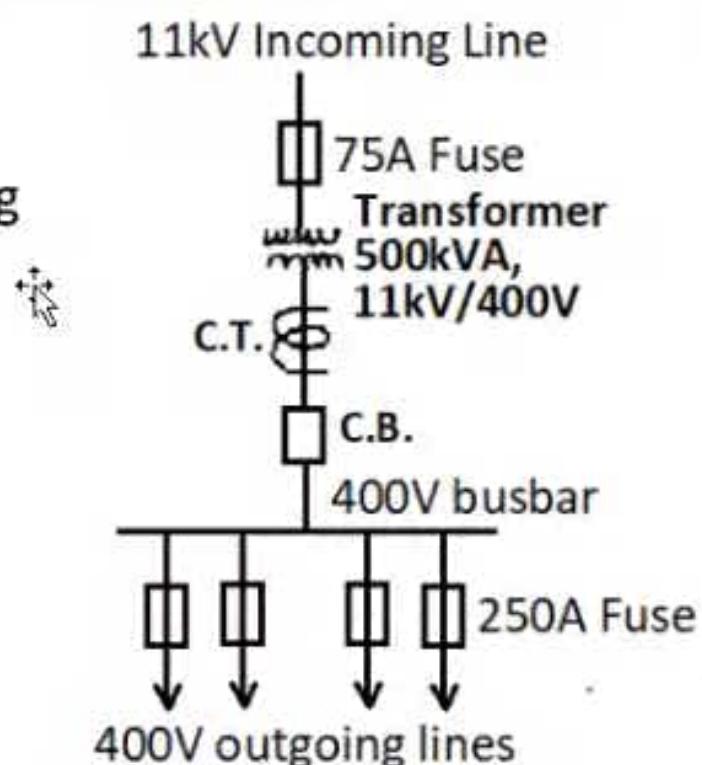


Fig. A typical distribution substation

Switching Station

- Substations generally comprise of transformers & switchgears.
- In case, no transformer is involved → the station is called the switching station.

Examples:

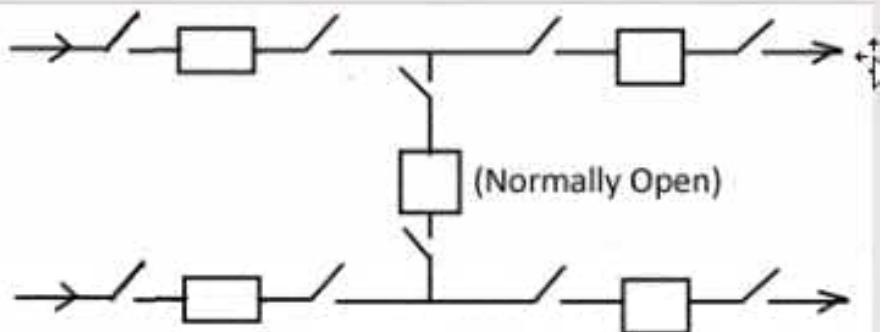


Fig. A switching station with two incoming & two outgoing lines using five CBs

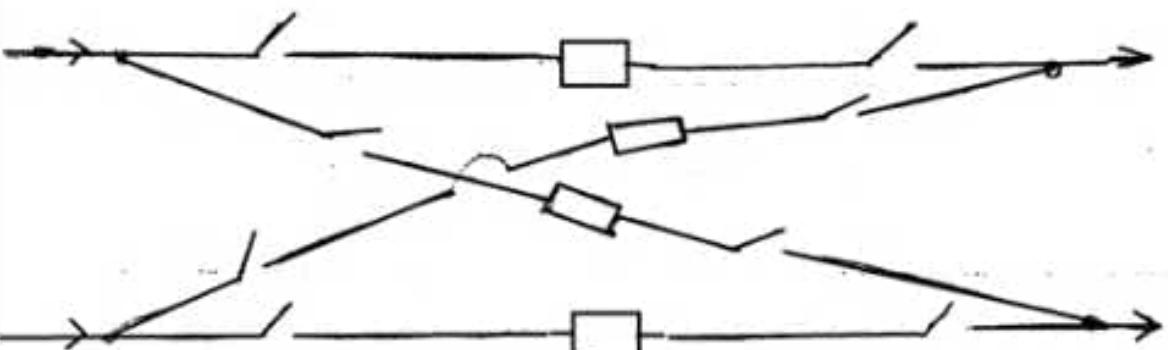


Fig. A switching station for two incoming & two outgoing lines using four CBs

Busbar Ratings

1. **Rated current:** RMS value of current that busbar can carry continuously with temperature rise within specified limits → Some of the rated values are 200A, 400A, 600A, 800A, 1200A, 1600A, 2000A, 2400A, 3000A.
2. **Rated voltage:** RMS value of the voltage between busbars → 0.415kV, 3.3kV, 6.6kV, 11kV, 33kV, 66kV, 110kV, 132kV, 220kV, 400kV, 765kV.
3. **Rated frequency:** 50Hz.
4. **Rated short time current:** Corresponds to short time current rating of CB → RMS value of current that busbar can carry with temperature rise within specified limits, for a specified duration.

Specified temperature limits are:

- Ambient temperature average: 35°C
- Ambient temperature peak (1 hour): 40°C
- Temperature rise permitted: 40°C

Short time duration: 1 Sec / 3 Sec.

Circuit Interruption Devices

I

Switchgear

- It is a general term, covering a wide range of equipment concerned with switching & protection.
- In other words, all items associated with fault clearing process are covered by the term switchgear.
- Switchgear includes → switches, fuses, CBs, isolators, relays, control panels, lightning arresters, CTs, PTs & various associated equipment.

Circuit Breaker (CB)

- It is one of the most important items of the switchgear.
- The CB performs:
 1. Making & breaking of normal currents.
 2. Making & breaking of fault currents.
- The part of the CB connected in one phase is called 'Pole.'
- A 3φ CB is called a "Three Pole" CB.
- In 1φ traction system single pole CB is used.

Arc Due to Opening of CB Contacts

- When two current carrying contacts are closed, actual contact is made at a number of points on the two surfaces.
- Number of contacts being a function of the pressure applied to keep the contacts together.
- When this pressure is reduced to separate the contacts → number of contacts reduces → this increases the contact resistance → this increases the contact temperature due to increased I^2R losses.
- This temperature is high enough to cause thermionic emission from the contact surfaces → thus tending to maintain the flow of current.
- Further, as soon as the CB opening is complete → contacts are physically separated → voltage across the contacts → causes further ionization due to potential gradient.
- Both these effects produce sufficient electrons to continue the current flow across the opened contacts by drawing an arc.
- Arc further intensifies due to ionization by collision.

Extinction of Arc

- If this arc is to be extinguished → each of these factors must be overcome → to reduce the supply of electrons.
- By increased contact separation → reduces potential gradient & increases contact resistance → lowering arc current & consequent temperature rise.
- However, contacts can't be separated indefinitely. So, other methods must be considered.
- By cooling the contact space. This reduces electron mobility & thus ionisation by collision. Then to assist the recombination of electrons with the surrounding medium.
- In case of an a.c. arc the problem is modified → current periodically passes through zero → when the arc gets extinguished automatically.
- Immediately after the arc is extinguished a voltage appears across the contacts.
- This voltage magnitude depends on mainly [the p.f. of the current immediately prior to interruption.
- This voltage tries to re-strike the arc.

Cont. from last slide-

- If this re-strike of arc is to be prevented \rightarrow dielectric strength of the contact space must increase sufficiently to withstand the potential gradient across it.
- Thus, there is a race between the rate at which the dielectric strength of the space between contacts improves and the rate of rise of the re-striking voltage.
- If the later is greater then arc continues till next current zero & same race will take place again \rightarrow else, the arc permanently extinguishes.

Energy Balance Theory

- Here, energy generated/liberated in the arc space due to flow of current through the arc resistance is compared with the rate at which energy is dissipated from the arc space. I
- If the former is greater then arc continues \rightarrow else extinguishes.

Current in the Arc

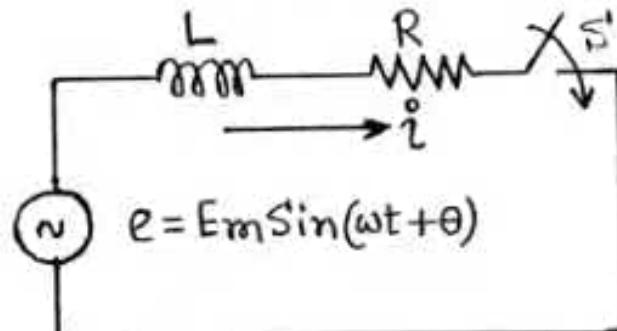
- One of the major factors involved in the interruption of arc is the current in the arc → since, it controls the heating & hence the temperature. I
- This current has two components:
 1. An a.c. component at system frequency → whose magnitude depends on impedance of the interrupted circuit → in majority of the cases, circuit impedance is almost entirely inductive & p.f. is low.
 2. **The second component is a d.c. transient**, superimposed on the alternating component → whose initial value may be any thing → from zero to the maxm. value of the a.c. component, depending on the instant in the cycle at which fault occurred & the type of fault.

Short Circuit in R-L Circuit

- The d.c. component depends on the type of fault & 3φ symmetrical fault is the most severe one – so, consider that this has occurred.
- In this case, all the phases are equally affected. So, only 1-φ needs to be considered.
- Consider the following circuit. Switch 'S' is closed to simulate a fault.

In this case,

$$L \cdot \frac{di}{dt} + R \cdot i = E_m \sin(\omega t + \theta) \quad (1)$$



Soln. of eqn. (1) gives the expression of current i .

The soln. of eqn. (1) is of the form:

$$i = A \cdot e^{-\frac{R}{L} \cdot t} + \frac{E_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \theta - \phi) \quad (2)$$

where, A is a constant and $\phi = \tan^{-1} \frac{\omega L}{R}$

Here, we put initial conditions to evaluate A.

In practice, $R \ll \omega L$.

$$\text{So, } \sqrt{R^2 + \omega^2 L^2} \approx \omega L \text{ and } \phi = \tan^{-1} \frac{\omega L}{R} = 90^\circ$$

Therefore, from eqn.(2),

$$\begin{aligned} i &= A \cdot e^{-\frac{R}{L} \cdot t} + \frac{E_m}{\omega L} \sin(\omega t + \theta - 90^\circ) \\ &= A \cdot e^{-\frac{R}{L} \cdot t} - \frac{E_m}{\omega L} \cos(\omega t + \theta) \end{aligned}$$

In this case, current is inductive, does not change instantaneously. So, at $t=0$, $i=0$.

Therefore, $i = A \cdot e^{-\frac{R}{L}t} - \frac{E_m}{\omega L} \cos(\omega t + \theta) = 0$

$$\text{OR, } A = \frac{E_m}{\omega L} \cos \theta \quad \text{--- (3)}$$

$$\text{So, } i = \frac{E_m}{\omega L} \cos \theta \cdot e^{-\frac{R}{L}t} - \frac{E_m}{\omega L} \cos(\omega t + \theta) \quad \text{--- (4)}$$

Case I : The switch is closed at $e=0$

This means at $t=0$, $e=0$

This implies, $\theta = 0^\circ$

$$\text{Hence, from eqn. (3), } A = \frac{E_m}{\omega L} \quad \text{--- (5)}$$

$$\text{So, } i = \frac{E_m}{\omega L} \cdot e^{-\frac{R}{L}t} - \frac{E_m}{\omega L} \overset{\uparrow}{\cos} \omega t \quad \text{--- (6)}$$

As the maxm. value of A is $\frac{E_m}{\omega L}$, the d.c. component of current is maxm. when the switch is closed at voltage zero. This is called the "Doubling Effect." This is due to the fact that the peak value of the resultant current is almost $2 \frac{E_m}{\omega L}$ at the first current loop. However, there is a slight drop in the d.c. Component at the time of resultant current peak and the peak value is considered to be $1.8 \frac{E_m}{\omega L}$.

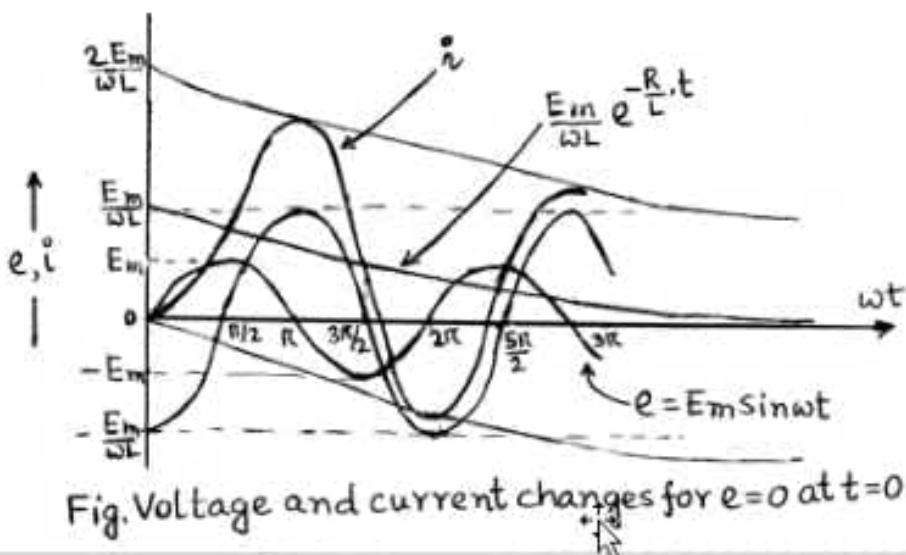


Fig. Voltage and current changes for $e=0$ at $t=0$.

Case II : The switch is closed at $e = E_m$ at $t=0$.
 This implies $\theta = 90^\circ$.

So, from eqn.(3), $A = \frac{E_m}{\omega L} \cos \theta = 0$ — (7)

So, from eqn.(4), $i = \frac{E_m}{\omega L} \sin \omega t$ — (8)

So, in this case the d.c. component is zero and the resultant current is symmetrical w.r.t. the horizontal axis.

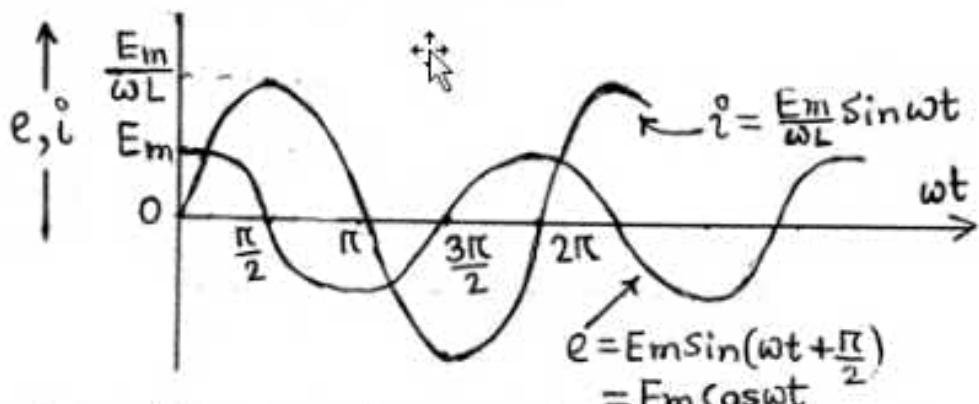


Fig. Voltage and current changes for $e = E_m$ at $t=0$.

Sub-transient, Transient & Steady State

- Short circuit analysis of R-L circuit may be extended for short circuit of 3φ alternator.
- If armature reaction & field current variation neglected → wave forms are similar.
- However, in alternator waveforms get modified due to armature reaction & field current variation.
- An oscillogram of 3φ currents is shown in the figure.
- Current wave forms are different due to different d.c. components.
- D.C. component depends on voltage at the time of short circuit.

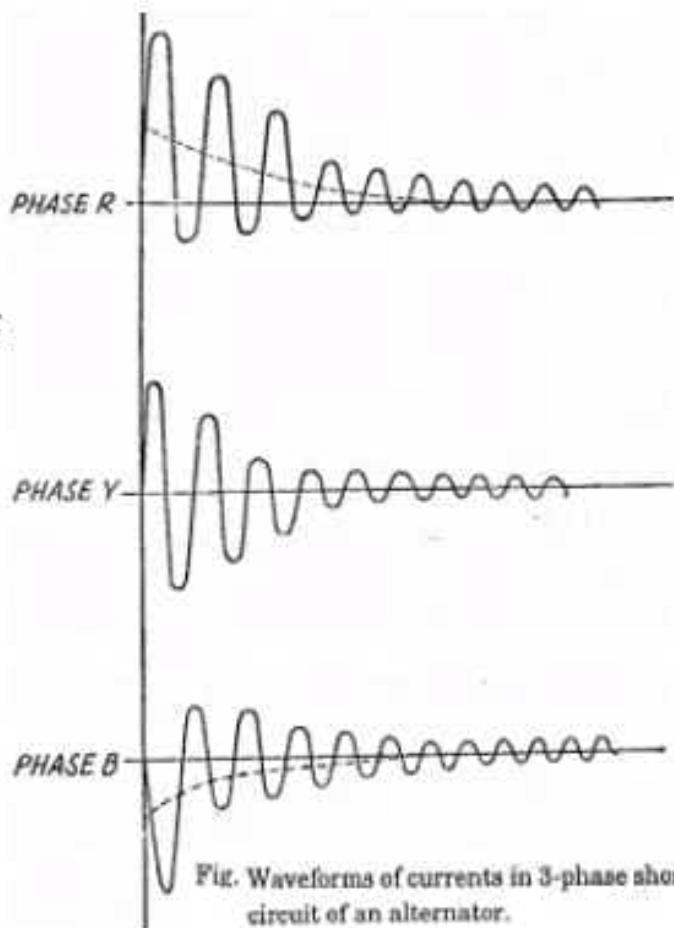


Fig. Waveforms of currents in 3-phase short-circuit of an alternator.

- On short circuit \rightarrow currents in three phases rise rapidly I to a very high value, 10-18 times rated current during the first quarter cycle.
- Flux passing through the air gap is large during first couple of cycles & reactance is also least.
- So, short circuit current is also high.

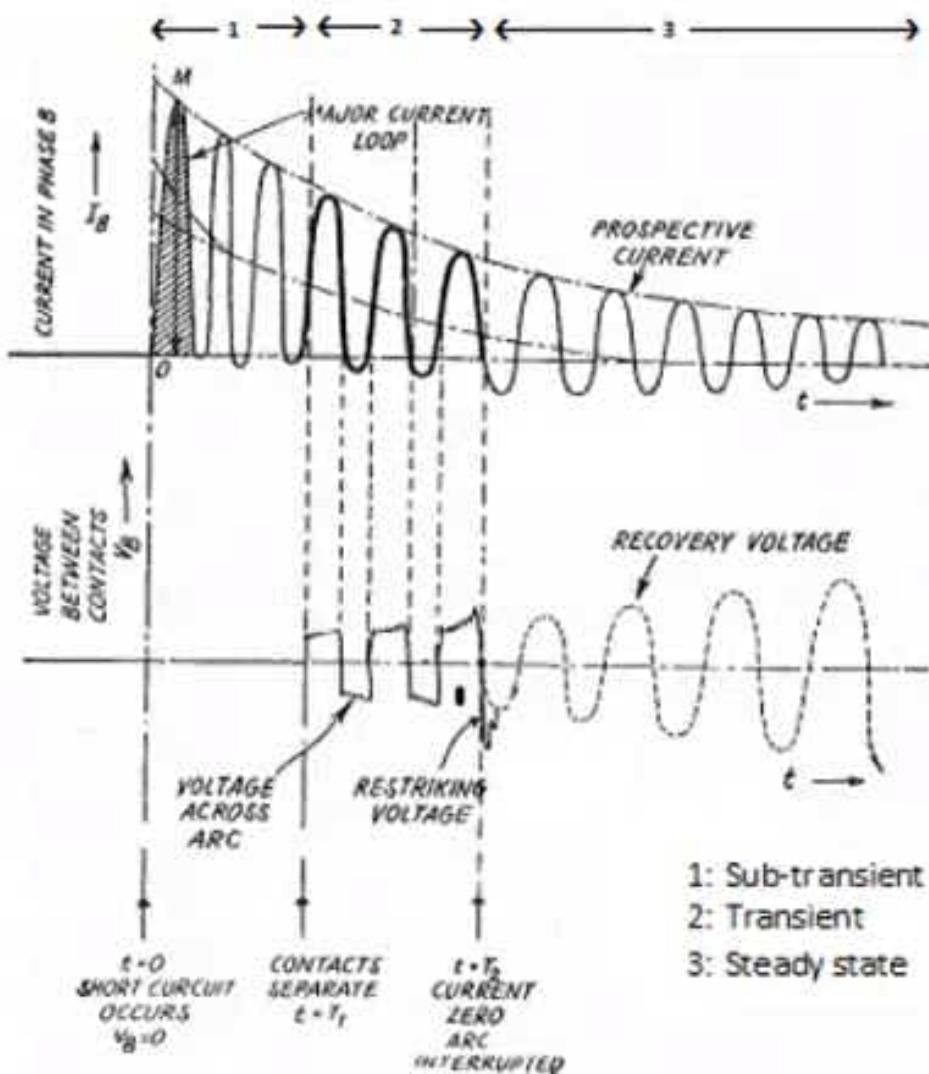


Fig. Oscillogram of current and voltage during fault-clearing.

- This reactance is known as **sub-transient reactance** & denoted by X'' .
- First few cycles come under **sub-transient state**.
- After first few cycles, decrement in the r.m.s. value of the short circuit current is less rapid – compared to first few cycles.
- This state is called the **transient state** & the reactance is known as **transient reactance** & denoted by X' .
- **CB contacts open in transient state.**
- Finally the transient dies out and current becomes steady & this state is known as **steady state**.
- This reactance is known as **steady state reactance** & denoted by X_d .

Making current: Peak value of the first major current loop 'OM' is the maxm. instantaneous value → known as the **making current** of the CB → expressed in kA peak.

Breaking current: R.M.S. value of short circuit current at the instant of CB contact separation is termed as **breaking current**. The CB contacts open at $t=t_1$.

- After contact separation an arc is drawn between the contacts.
- Arc current varies sinusoidally for a few cycles.
- At $t=t_2$, at some current zero \rightarrow dielectric strength of arc space builds up sufficiently, so as to prevent the continuation of arc.
- At this current zero, the arc is extinguished & is interrupted.
- But, what about the voltage between the contacts?
- Before $t=t_1$, contacts are closed \rightarrow voltage between contacts is zero.
- After $t=t_1$, contacts are separated \rightarrow arc drawn between contacts \rightarrow a voltage appears across the contacts \rightarrow voltage drop in the arc.
- This voltage drop is in phase with the arc current \rightarrow arc is resistive.
- Typical shape of voltage waveform is the result of volt-ampere characteristic of arc discharge.

- At $t=t_2$, at current zero \rightarrow arc is extinguished.
- Then a high frequency voltage transient appears across the contacts.
- It gets superimposed on power frequency system voltage.
- This high frequency voltage transient tries to restrike the arc.
- Hence, it is called “**Re-striking Voltage**” or “**Transient Recovery Voltage**” (TRV).
- Power frequency voltage system voltage appearing between the poles after arc extinction is called “**Recovery Voltage**.”
- Current that would flow in the circuit if the CB is replaced by a solid conductor is called “**Prospective Current**.”

- ❖ TRV causes a high dielectric stress across the CB contacts. If the dielectric strength of the medium does not build up faster than the rate of rise of the TRV, break down takes place causing reestablishment of the arc.
- ❖ If the dielectric strength of the medium builds up very rapidly, so that its rate is more than the rate of rise of the TRV, the CB interrupts the current successfully.
- ❖ The rate of rise of TRV generally depends on the circuit parameters & the type of switching duty involved.
- ❖ The rate of building up of dielectric strength depends on effective design of the CB.

Restriking Voltage

- It is one of the major factors considered in CB design.
- In diagram L is inductance between CB & source of emf & C is the capacitance to earth of the circuit including between CB contacts, when open.

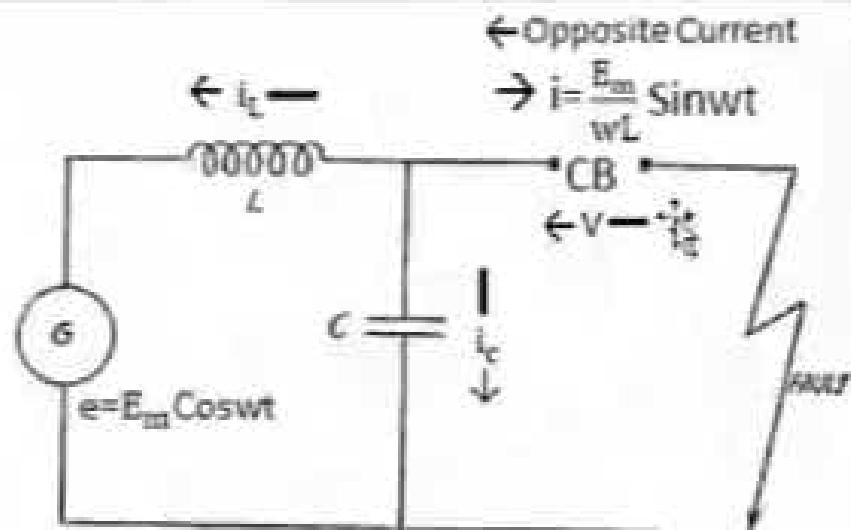


Fig. Simple equivalent circuit for calculation of TRV & RRRV

- Circuit resistance is neglected.
- Consider, fault has occurred (i.e. $t=0$) when voltage wave is passing through its maxm. value $\rightarrow e = E_m \cos \omega t$.
- This causes a fault current of $\frac{E_m}{wL} \sin \omega t$, as effect of C is not there before opening of CB contacts.
- Consider, the CB is open & the fault current is to be interrupted.

- For this, a cancelling current, equal & opposite to the original current can be simulated, being injected to the CB.
- Voltage necessary, to inject this current is the TRV → that appears across the contacts.

Thus, eqn. of the cancelling current is $i = i_L + i_c$

$$\text{or, } i = \frac{1}{L} \int v dt + C \frac{dv}{dt} \dots\dots\dots(1)$$

where v is the voltage across CB contacts, i.e. re-striking voltage.

From eqn. (1), $\frac{di}{dt} = \frac{v}{L} + C \frac{d^2v}{dt^2} \dots\dots\dots(2)$

Soln. of eqn. (2) depends on the current & if interruption takes place at a current zero, when 't' is very small,

$$i = \frac{E_m}{\omega L} \sin \omega t = \frac{E_m}{\omega L} \cdot \omega t = \frac{E_m \cdot t}{L} \quad (\text{as, } \sin \omega t = \omega t \text{ when } t \text{ or } \omega t \text{ is small})$$

$$\text{So, } \frac{di}{dt} = \frac{E_m}{L}$$

Substituting $\frac{di}{dt}$ value in eqn. (2),

$$\frac{E_m}{L} = \frac{V}{L} + C \frac{d^2 V}{dt^2} \dots\dots\dots(3)$$

Soln. of eqn. (3) is of the form,

$$V = E_m (1 - \cos \omega_0 t) \dots\dots\dots(4)$$

where, $\omega_0 = \frac{1}{\sqrt{LC}}$, or, $f_n = \frac{1}{2\pi\sqrt{LC}}$

The maxm. value of re-striking voltage is $2E_m$ (when $\omega_0 t = \pi$).

- ❖ Breakdown of the gap between the contacts depends on this maxm. value $2E_m$ & its rate of rise. I

Rate of Rise of Re-striking Voltage (R.R.R.V.)

$$R.R.R.V. = \frac{dv}{dt} = E_m w_0 \sin w_0 t \dots\dots\dots(5)$$

R.R.R.V. is maxm. when its derivative is zero, i.e.,

$$\frac{d^2v}{dt^2} = E_m w_0^2 \cos w_0 t = 0 \rightarrow w_0 t = \frac{\pi}{\sqrt{LC}} = \frac{\pi}{2}$$

Or, $t = \sqrt{LC} \cdot \frac{\pi}{2} \dots\dots\dots(6)$

The maxm. R.R.R.V. is the value of $\frac{dv}{dt}$ at $t = \sqrt{LC} \cdot \frac{\pi}{2}$, i.e.,

$$R.R.R.V._{Max} = E_m w_0 = \frac{E_m}{\sqrt{LC}} \dots\dots\dots(7)$$

$$\text{Now, as } f_n = \frac{1}{2\pi\sqrt{LC}} \rightarrow \frac{1}{\sqrt{LC}} = 2\pi f_n \dots\dots\dots(8)$$

$$\text{So, } R.R.R.V._{Max} = 2\pi f_n E_m \dots\dots\dots(9)$$

Resistance Switching, Damping of TRV & Opening Resistor

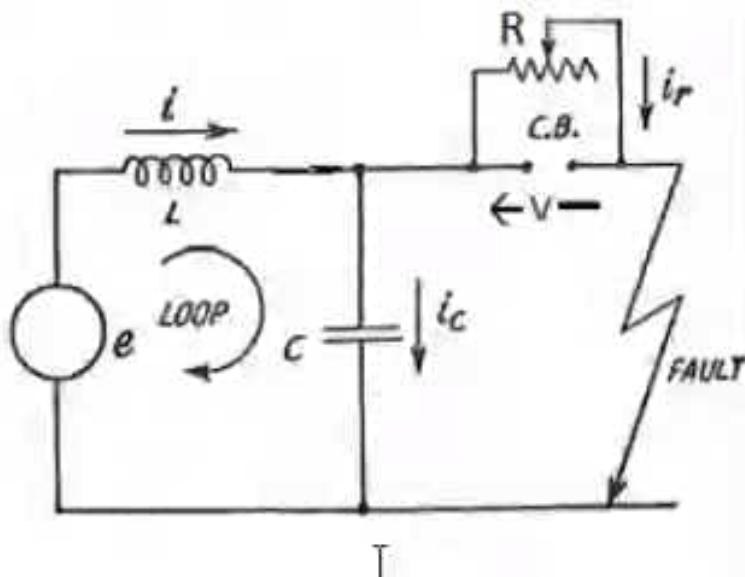
- Deliberate connection of a resistance in parallel with the contact space (arc) is called "Resistance Switching."
- It is used in CBs having high post zero resistance of contact space.
- To see effect of such a resistance on frequency of re-striking voltage transient, consider the circuit:

In the circuit,

$$e = L \frac{di}{dt} + \frac{1}{C} \int i_C dt \dots\dots\dots(1)$$

$$\frac{1}{C} \int i_C dt = i_R R \dots\dots\dots(2)$$

$$i = i_C + i_R \dots\dots\dots(3)$$



- Substituting eqns. (2) & (3) in eqn. (1)

$$e = L \cdot \frac{d}{dt} (i_c + i_r) + i_r \cdot R$$

$$= L \cdot \frac{di_c}{dt} + L \cdot \frac{di_r}{dt} + i_r \cdot R \dots\dots\dots(4)$$

$$\text{Since, } i_r \cdot R = \frac{1}{C} \int i_c \cdot dt \quad \rightarrow \quad R \cdot \frac{di_r}{dt} = \frac{i_c}{C} \quad \rightarrow i_c = RC \cdot \frac{di_r}{dt}$$

So, from eqn. (4),

$$e = RLC \cdot \frac{d^2 i_r}{dt^2} + L \cdot \frac{di_r}{dt} + i_r \cdot R$$

$$\text{or, } \frac{d^2 i_r}{dt^2} + \frac{1}{RC} \cdot \frac{di_r}{dt} + \frac{1}{LC} \cdot i_r = \frac{e}{RLC} \dots\dots\dots(5)$$

If interruption takes place at some current zero, when t is very small, then, $e = E_m \cos \omega t = E_m$.

So, from eq. (5),

$$\frac{d^2 i_r}{dt^2} + \frac{1}{RC} \cdot \frac{di_r}{dt} + \frac{1}{LC} \cdot i_r = \frac{E_m}{RLC} \dots\dots\dots(6)$$

Solution of eqn. is of the form,

$$i_r = \frac{E_m}{R} [1 - e^{-at} (\cos \sqrt{b} \cdot t + \frac{a}{b} \cdot \sin \sqrt{b} \cdot t)] \dots\dots\dots(7)$$

If $v = i_r$. R be the voltage across the contacts, then,

$$v = E_m \cdot [1 - e^{-at} (\cos \sqrt{b} \cdot t + \frac{a}{b} \cdot \sin \sqrt{b} \cdot t)] \dots\dots\dots(8)$$

$$\text{where, } a = \frac{1}{2CR} \text{ & } b = \frac{1}{LC} - \left(\frac{1}{2CR}\right)^2 \dots\dots\dots(9)$$

So, natural frequency of oscillation is,

$$f_n = \frac{1}{2\pi} \cdot \sqrt{\frac{1}{LC} - \left(\frac{1}{2CR}\right)^2} \dots\dots\dots(10)$$

- Value of resistance R, at which the freq. of TRV becomes zero is called the "Critical Damping Resistance."
- The resistance connected in parallel with CB for opening operation is called "Opening Resistance."

That is, $\frac{1}{LC} - \left(\frac{1}{2CR}\right)^2 = 0$ or, $4R^2.C^2 = LC$

$$R = \frac{1}{2} \cdot \sqrt{\frac{L}{C}} \dots\dots\dots(11)$$

- Thus, resistance switching relieves the CB from severe voltage transients.
- This resistance also diverts a part of the arc current.
- ❖ Resistance switching is needed for CBs having high post-zero resistance of the contact space after interruption of arc.
 - In this type of CBs, severe voltage transients may be produced due to current chopping (interruption of current before natural zero).
 - ❖ Post-zero resistance of air blast CB is high. So, resistance switching is used.
 - ❖ In oil CBs post zero resistance of contact space is low. Hence, resistance switching is not necessary.
 - ❖ However, resistance switching assists CBs in interruption of magnetizing & capacitive currents.

Switching of Transformer Magnetizing Current

- An unloaded transformer may be represented in equivalent circuit as a high reactance.
- When CB is opened, it appears in parallel with the system capacitance.
- Here, source reactance is neglected.
- When circuit is interrupted at CB, L & C forms an oscillatory circuit.
- This may result in a high voltage across the transformer terminals → particularly in a resonance condition.

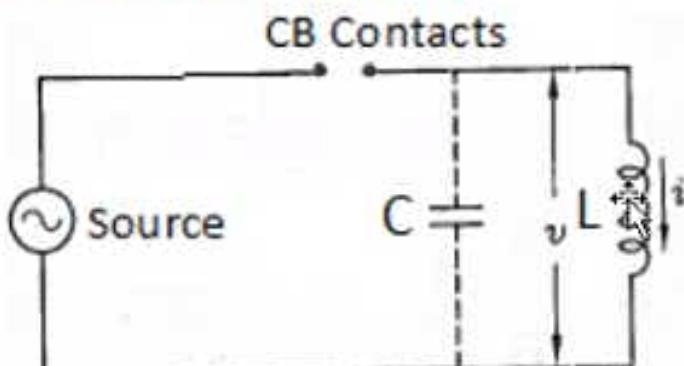


Fig. Circuit diagram illustrating interruption of low inductive current.

- This may cause damage/breakdown of the transformer insulation.
- If the arc re-strikes, then some of the oscillatory energy is absorbed in the CB & in system resistance.
- This relieves stress on the transformer.
- So, it is preferable to take a number of cycles before final interruption → in contrast to usual cases where interruption is desirable as quickly as possible.

Example: A 3φ shunt reactor of 5H is connected in an 132kV system to reduce the voltage at the receiving end of a long transmission line during low load conditions. The line to earth capacitance of the system is $0.01\mu F$. Calculate the voltage across the reactor terminals for interrupting its magnetizing current of 10A. Also, calculate the value of the resistance to be used across the CB contact space to eliminate the frequency re-striking voltage transient.

Soln.: Inductance of reactor = $L = 5H$;

Line to earth system capacitance = $C = 0.01\mu F$;

Magnetizing current = $i = 10A$. I

$$\text{Here, } \frac{1}{2} L i^2 = \frac{1}{2} C v^2;$$

$$\begin{aligned}\text{or, } v &= i \sqrt{\frac{L}{C}} = 10 \times \sqrt{\frac{5}{0.01 \times 10^{-6}}} \\ &= 223606.7978V \\ &= 223.6068kV\end{aligned}$$

$$\begin{aligned}\text{Necessary resistance } R &= \frac{1}{2} \cdot \sqrt{\frac{L}{C}} = \frac{1}{2} \sqrt{\frac{5}{0.01 \times 10^{-6}}} \\ &= 11180.3399 \Omega = 11.1803 k\Omega\end{aligned}$$

Switching of Capacitive Currents

- This situation arises during disconnecting a long unloaded transmission line or capacitor bank.

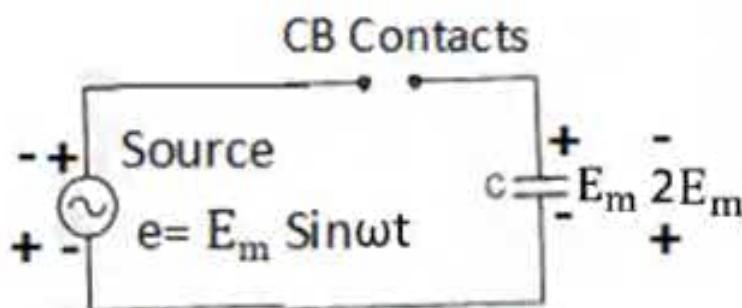


Fig. Circuit diagram illustrating
Interruption of capacitive currents

- Interrupted current is the charging current of the line or the capacitor bank.
- After first interruption at some current zero, charge held by C causes a voltage E_m across it (as p.f. is zero).

- However, the CB contact of the source side follows the normal system freq. voltage.
- Half a cycle later, the polarity of the supply reverses → causing a voltage of $2E_m$ across the CB contacts.
- If the arc restrikes at this point an oscillation takes place of this amplitude, I
- If the arc extinguishes for a second time at current zero, voltage held by C is $2E_m$, with reverse polarity.
- Half a cycle later, the polarity of the supply reverses → causing a voltage of $3E_m$ across the CB contacts.

- If the arc restrikes for the second time due to voltage $3E_m$ across it an oscillation takes place of this amplitude.
- If the arc extinguishes for a third time at current zero, voltage held by C is $3E_m$, with reverse polarity.
- Continued extinguishing & restrike of arc in this way may cause voltage build up to a value high enough to destroy the system insulation.
- So, in this case final interruption should take place as soon as possible.
- This is in ^Idirect contrast with the requirements for switching a transformer magnetizing current.

A.C. Circuit Breaking

Generally two modes are employed for arc extinction.

1. Low resistance or zero point interruption &
2. High resistance interruption.

Low Resistance or Zero Point Interruption

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- In this method the arc resistance is kept low & measures are taken to prevent the arc re-striking after it has extinguished at a natural current zero.
- The problem is to remove the electrons & ions from the contact space immediately after current zero by:

- Recombining them into neutral molecules → by cooling and increasing pressure, or
- swiping them away completely → by quickly replacing this ionized gases by a dielectric.

High Resistance Interruption

- In this method the arc resistance is increased by lengthening & cooling it to such an extent that the system voltage is no longer able to maintain the arc & arc gets extinguished.
- This technique is employed in air blast CB & DC CB.

Classification of CBs

May be classified in two ways:

1. On the basis of rated voltage

- CBs, rated below 1000V or 1kV are called low voltage CB.
- CBs, rated above 1000V or 1kV are called high voltage CB.



2. On the basis of arc quenching medium

- Usually, CBs are identified in this manner & can be classified as,
 - (i) Air-break CB,
 - (ii) Bulk oil CB,
 - (iii) Minimum oil CB,
 - (iv) Air blast CB,
 - (v) Sulphur Hexafluoride (SF_6) CB,
 - (vi) Vacuum CB.

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