Flood Frequency Analysis: Multi-model and distribution Ensemble using Bayesian Model Averaging

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Abstract

The current practice in flood frequency analysis is to rely on a single model based on one specific type of distribution. The log-Pearson Type III distribution is still a standard for flood frequency analysis in seven countries including U.S. (IACWD, 1982).

The purpose of this study is to answer the following four questions :

1. Should one use the mandatory LP3 or the best-fit distributions identified by L-moments in flood frequency analysis?
2. Can the characteristics of floods (uncertainty) be fully represented by only one method based on one specific distribution?
3. Does the more complex flood frequency model give the better predictability?
4. Can the ensemble approaches help improve the predictability of flood frequency analysis?

Study Area and Data Selection

The study area consists of three 8-digit hydrologic unit code (HUC) watersheds located in the southern Columbia River Basin (CRB) (Figure 1). The urban land use in our three study watersheds is less than 4%, so our study area meets the rural area condition (natural flow) well.

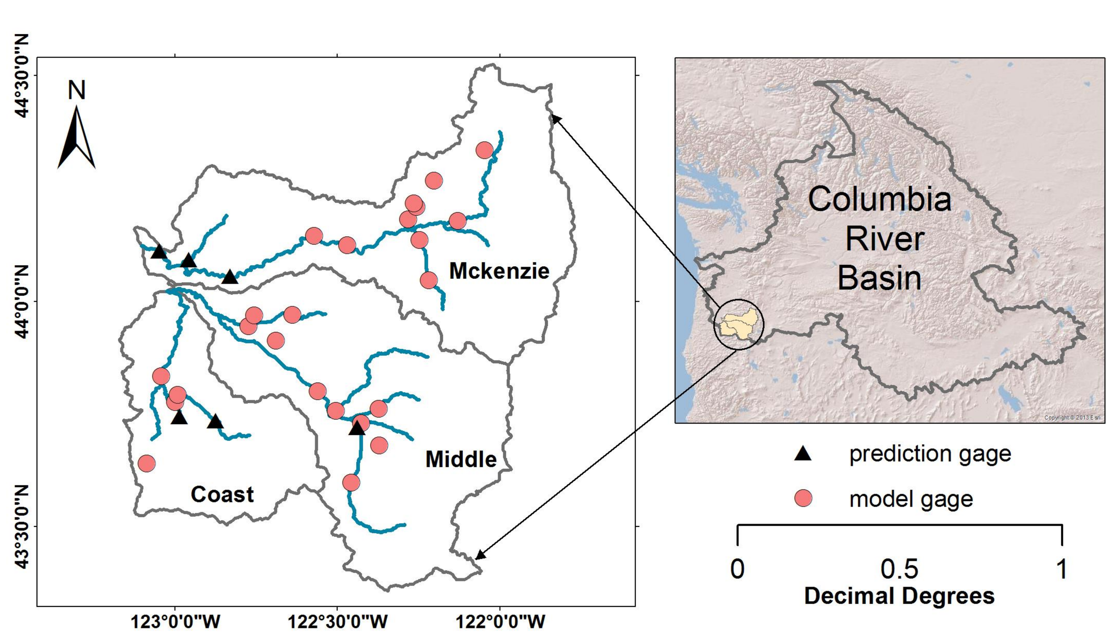

Figure 1. The location of the study area in CRB.

The database of information is derived from 30 USGS stream gages up through the water year 2012.

Table 1. The distribution of annual peak flows in 30 gages.

Drainage area (mi²)	Number of gages	Average length of records (years)
11.5 - 52.7	8	33
52.7 - 118.0	7	36
118.0 - 270.0	8	43
270.0 - 529.0	3	31
529.0 - 1337.0	4	26

In order to test the prediction of our approach, 6 gages were randomly selected and left aside as prediction gages.

Candidate Distributions Identification

Hosking and Wallis (1997) provided a comprehensive study of L-moments. The L-moments diagram is an effective way to identify the best-fit distribution for the given data.

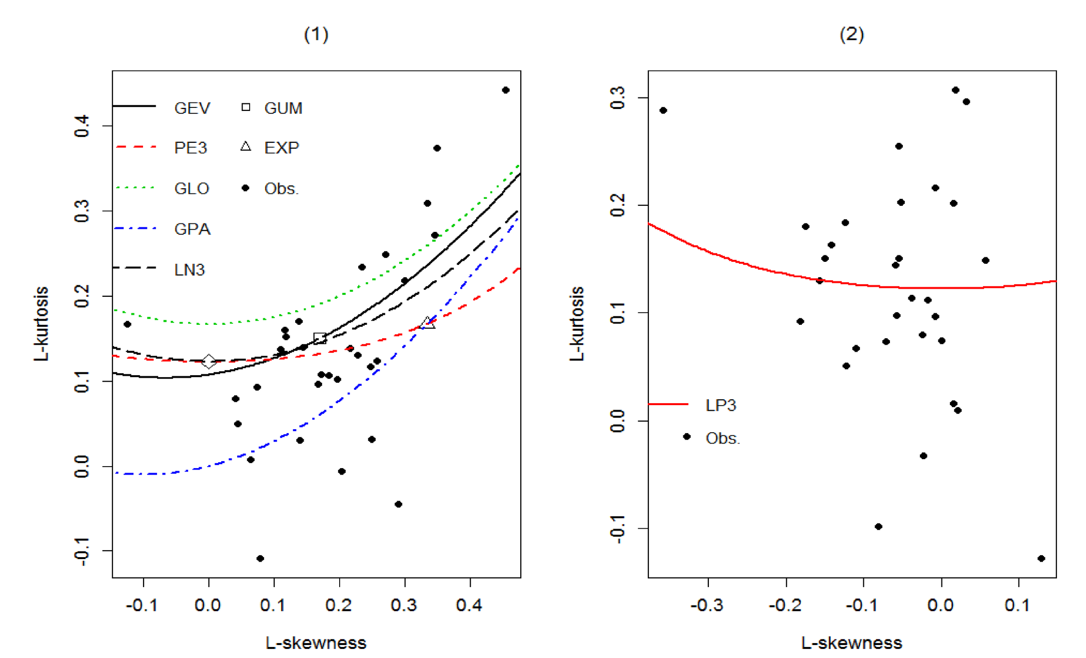

Figure 2. L-moments diagrams of L-skewness versus L-kurtosis for 30 gages.

The results identified 3 best-fit distributions for our area:

1. log-Pearson Type III (LP3)
2. generalized extreme value (GEV)
3. 3-parameter lognormal (LN3)

Bulletin-17B	Regional Frequency Analysis	Bayesian Hierarchical Model
LP3	LN3	GEV

Bulletin-17B: LP3

The standard procedure of Bulletin-17B method includes three steps:

1. Fitting the LP3 distribution to the annual peak flows of each selected gage. The equation is: $\log_{10} Q_T = \bar{X} + KS$
2. Estimating the flood magnitudes at chosen return intervals (quantiles) for each selected gage.
3. Correlating the quantiles with the watershed characteristics. The generalized least squares (GLS) method is used in this study.

Following Tasker and Stedinger (1989), the model equation for GLS is as follows:

$$\hat{\mathbf{Y}} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}, \quad \boldsymbol{\beta} = \left(\mathbf{X}^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\mathbf{Y}, \quad \hat{\boldsymbol{\Lambda}} = \hat{\sigma}_\delta^2\mathbf{I} + \hat{\boldsymbol{\Sigma}}$$

When estimating the lag zero cross-correlation in GLS, we use the non-linear regression model to smooth out the data:

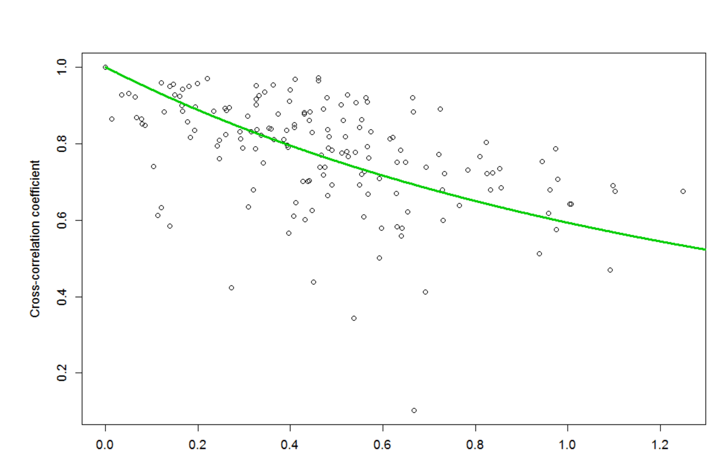

Figure 2. Cross-correlations of annual peak flows.

Regional Frequency Analysis (RFA): LN3

The standard procedure of RFA includes three steps:
(1) delineating a homogeneous region (L-moments).
(2) deriving a dimensionless regional frequency curve (L-moments).
(3) estimating an index-flood through watershed characteristics (GLS).

The probability density function (pdf) of LN3 distribution is given as follows:

$$f(z) = \frac{\exp(\xi y - y^2/2)}{\sigma\sqrt{2\pi}}, \qquad y = \begin{cases} -\xi^{-1}\ln[1-\xi(z-\mu)/\sigma], & \xi \neq 0 \\ (z-\mu)/\sigma, & \xi = 0 \end{cases}$$

The uncertainties of the LN3 parameters can be quantified by bootstrapping method.

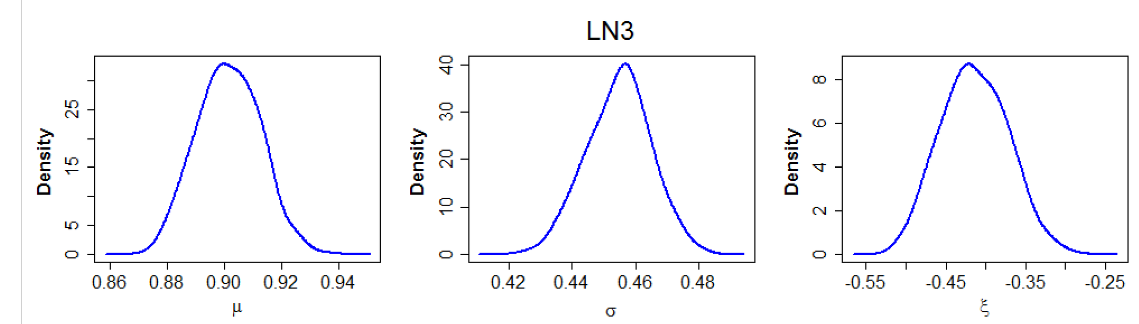

Figure 3. Distributions for the three parameters of LN3 distribution.

Bayesian Hierarchical Model: GEV

Bayes Law: $p(\boldsymbol{\theta}|\mathbf{z}) \propto p(\mathbf{z}|\boldsymbol{\theta}) \times p(\boldsymbol{\theta})$
Hierarchical: $p(\boldsymbol{\theta}|\mathbf{z}) \propto p_1(\mathbf{z}|\boldsymbol{\theta}_1) \times p_2(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2) \times p_3(\boldsymbol{\theta}_2)$

The classical Bayesian hierarchical model includes three layers: (1) data layer – the likelihood function, (2) process layer - multivariate Gaussian field, and (3) prior layer – prior information.

In process layer, the exponential geostatistics variogram model is used: $\Sigma = \beta_0 \times \exp(-\beta_1 \times \|\mathbf{z} - \mathbf{z}'\|)$

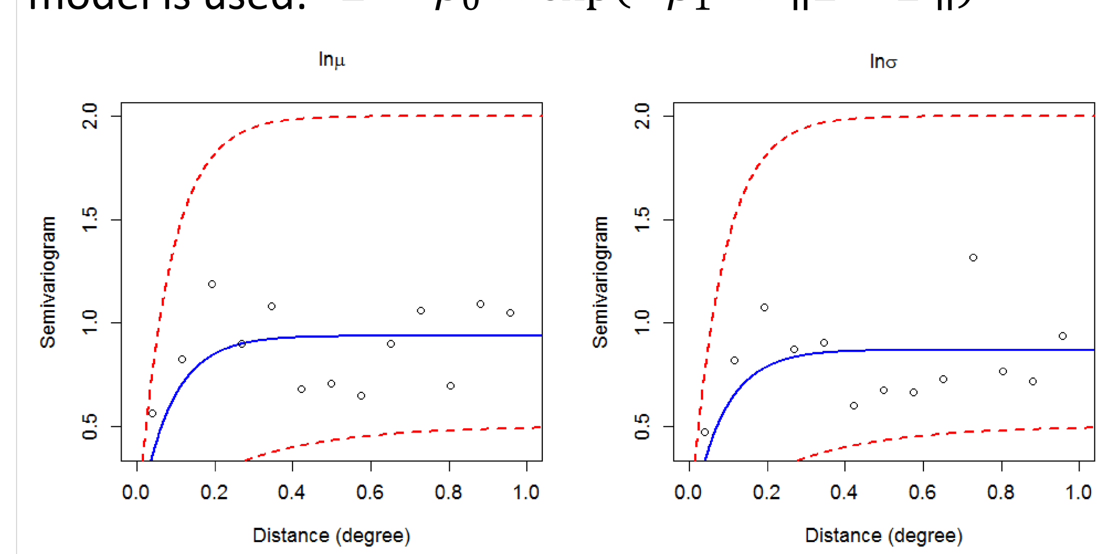

Figure 4. Semivariogram plots for empirical estimates with envelopes (dashed lines) reflecting the priors.

Bayesian Model Averaging (BMA)

Following Duan et al. (2007), Let **D** is a vector of observation data, $\mathbf{M}_k$ is the predictions of model k, then the p($\mathbf{M}_k$|**D**) is the posterior distribution. By the law of total probability, we have:

$$p(\boldsymbol{Y}) = \sum p(\boldsymbol{Y}|\boldsymbol{M}_k, \mathbf{D})\, p(\boldsymbol{M}_k|\mathbf{D})$$

$$E(\boldsymbol{Y}) = \sum w_k \boldsymbol{M}_k, \quad w_k = p(\boldsymbol{M}_k|\mathbf{D})$$

The Expectation Maximization algorithm is used to find w_k

Performance of Each Distribution

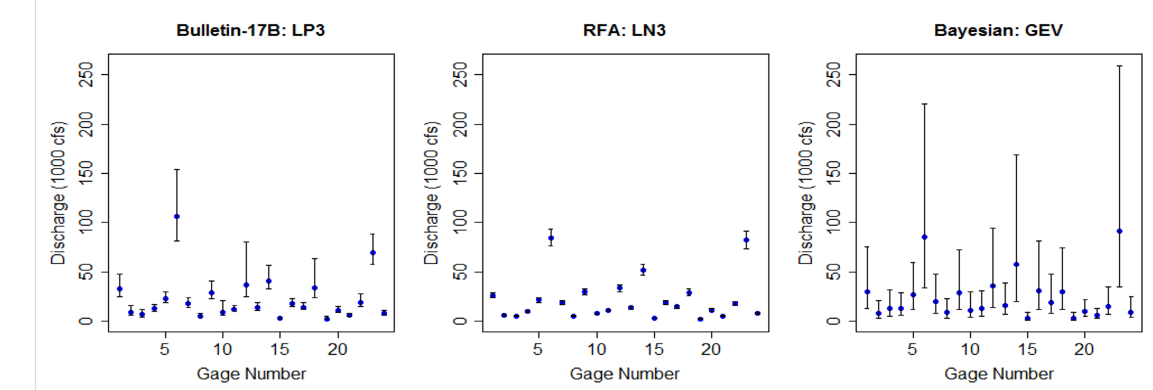

Figure 5. The mean and 95% confidence intervals for the 100 year floods for the 24 gages.

Quantile-Quantile (QQ) Plots

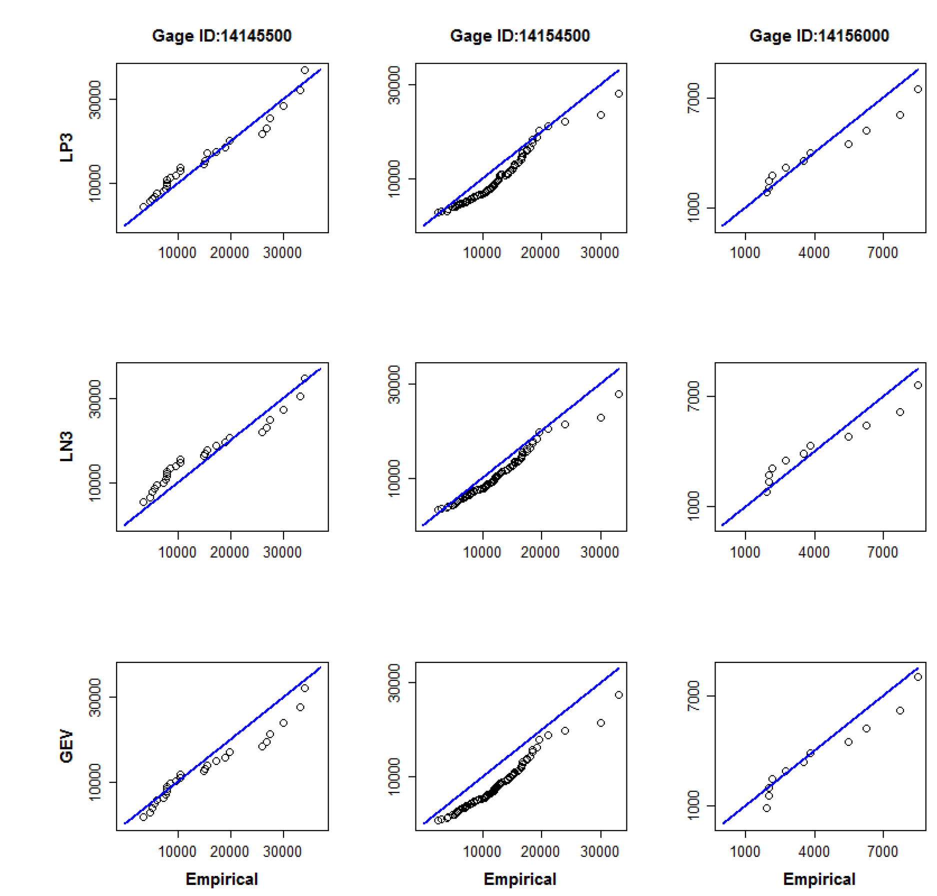

Figure 6. QQ-plots for the three prediction gages.

Multi-distribution Ensemble

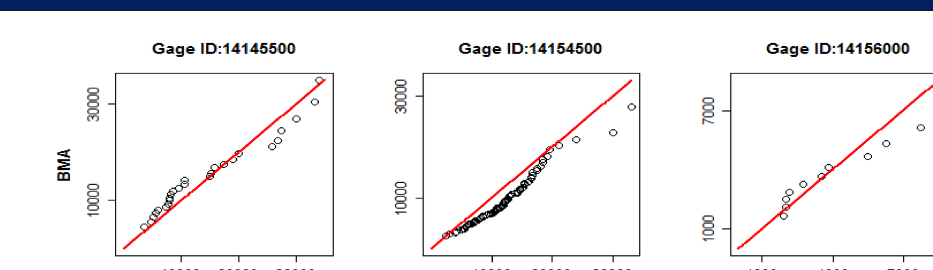

Figure 7. QQ-plots for the BMA predictions.

Table 2. NSE values for three distributions and BMA.

Type	NSE			Avg.
	14145500	14154500	14156000	
LP3	0.96	0.79	0.89	0.88
LN3	0.89	0.86	0.89	0.88
GEV	0.89	0.45	0.91	0.75
BMA	0.94	0.78	0.91	0.88

Conclusion

- Multi-distribution gives a more comprehensive estimation of the flood percentiles and uncertainty.
- No one method with one specific distribution can give the best performance for all the six prediction gages. Several distributions should be equally acceptable for flood frequency analysis (the equifinality concept).
- The most complex Bayesian model doesn't give a better performance.
- The BMA predictions outperform than any single model.