

# Overland Flow simulation in ParFlow

ParFlow Short Course

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We solve the mixed form of Richards' and Shallow Water equations

$$S_S S_W(h) \frac{\partial h}{\partial t} + \theta \frac{\partial S_W(h)}{\partial t} = \nabla \cdot \mathbf{q} + q_r(\mathbf{x}, z) \quad \text{Mixed form of Richards' we solve for } h$$

Upper boundary condition (Neumann type) combined with Shallow Water Equation (same  $h$ )

$$\mathbf{k} \cdot (-\mathbf{K}_S(\mathbf{x}) k_r(h) \cdot \nabla(h+z)) = \frac{\partial ||h, 0||}{\partial t} - \nabla \cdot ||h, 0|| v_{sw} + q_r(\mathbf{x})$$

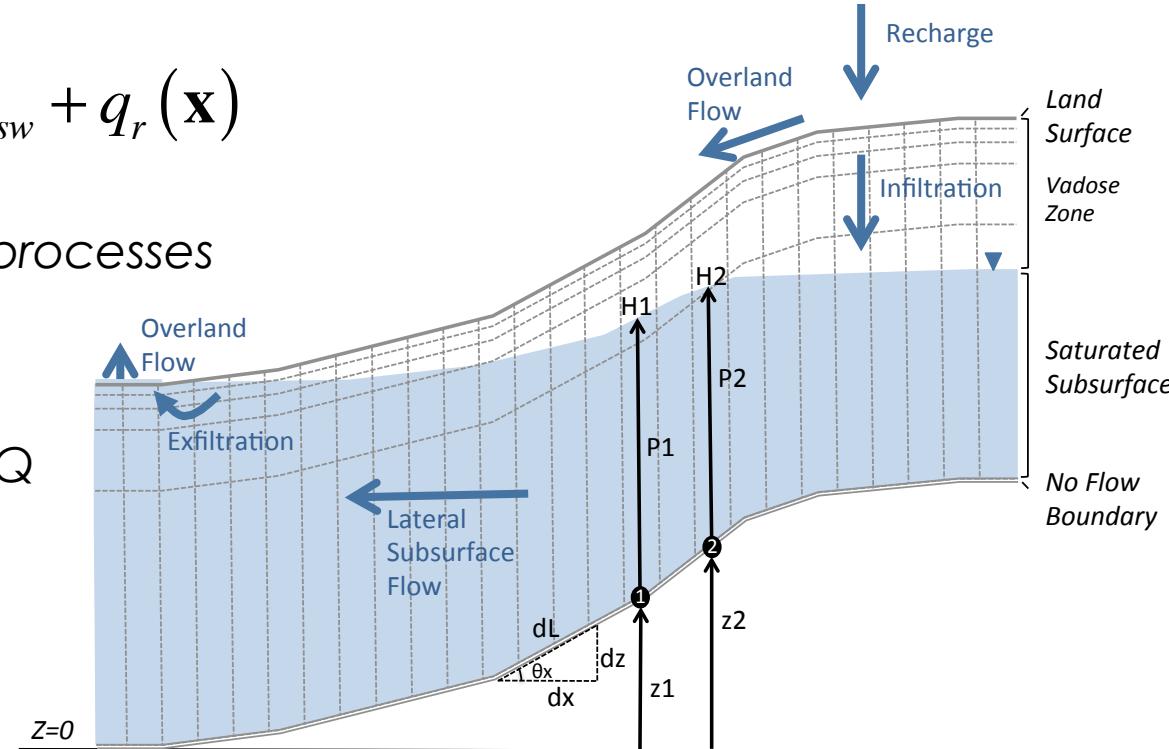
Boundary source/sink, from weather and land surface processes

$$q_r(x) = P(x) - E(x) \quad q_r(x, z) = -E_T(x, z)$$

Flux Relationships from modified Darcy and Mannings EQ

$$\mathbf{q} = -\mathbf{K}_S(\mathbf{x}) k_r(h) [\nabla(h+z) \cos \beta_x + \sin \beta_x]$$

$$v_x = -\frac{\sqrt{S_{f,x}}}{n} \psi_s^{2/3}$$

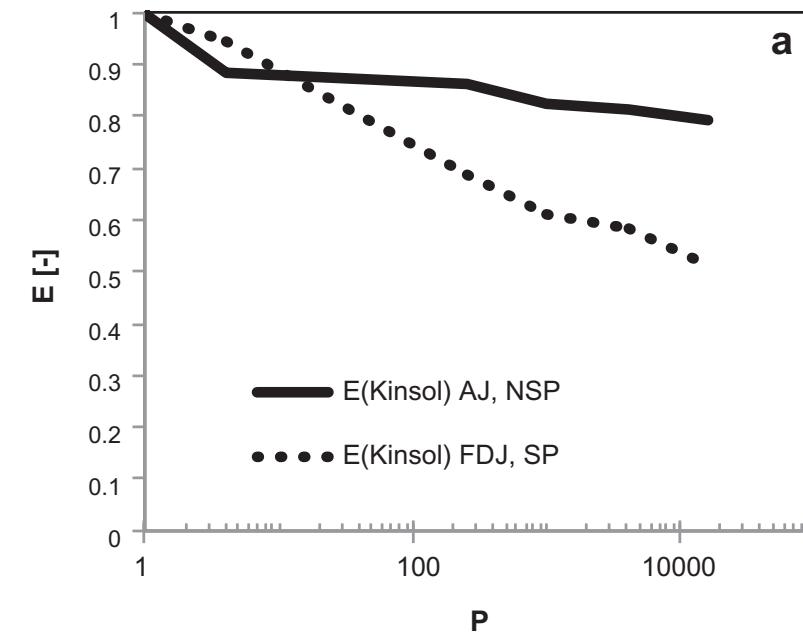


ParFlow uses a standard suite of solver packages and is very efficient

- Designed to be **parallel** from the ground-up using an object-oriented framework
- Newton Krylov nonlinear approach via the Kinsol solver package (**globally implicit**=*integrated*)
- Multigrid-preconditioned linear solver using the Hypre package
- **Physics-based** preconditioning based on analytical Jacobian to accelerate convergence

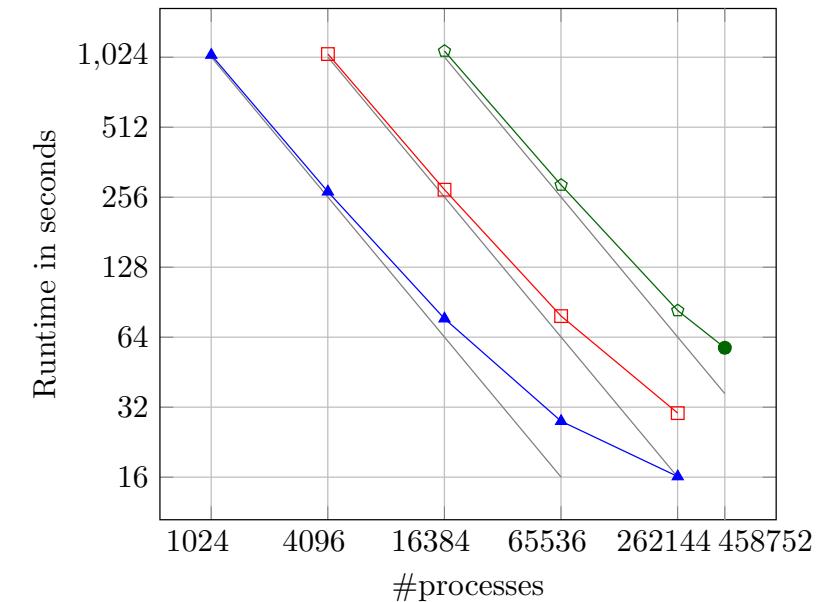
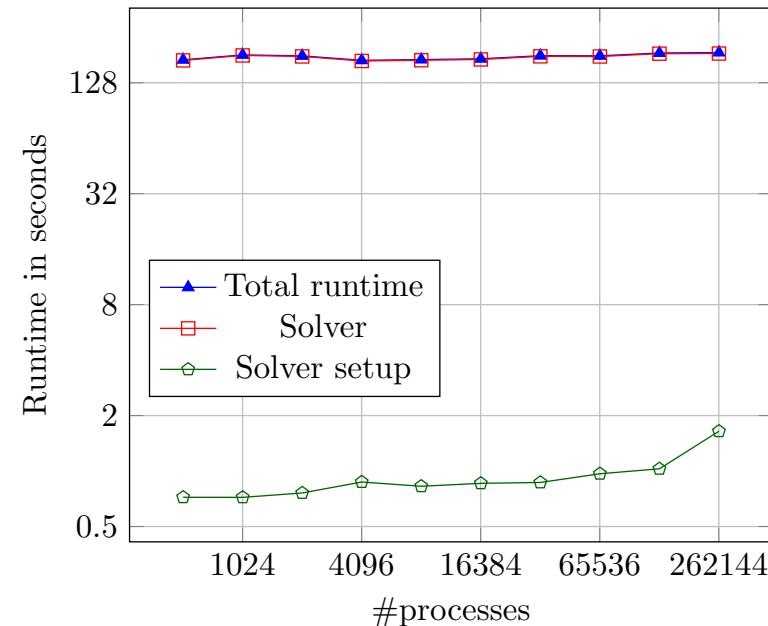
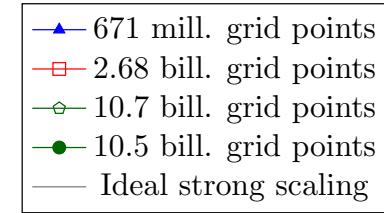
# ParFlow demonstrates excellent parallel scalability

*Weak scaling, where the problem size increases with the number of processors*



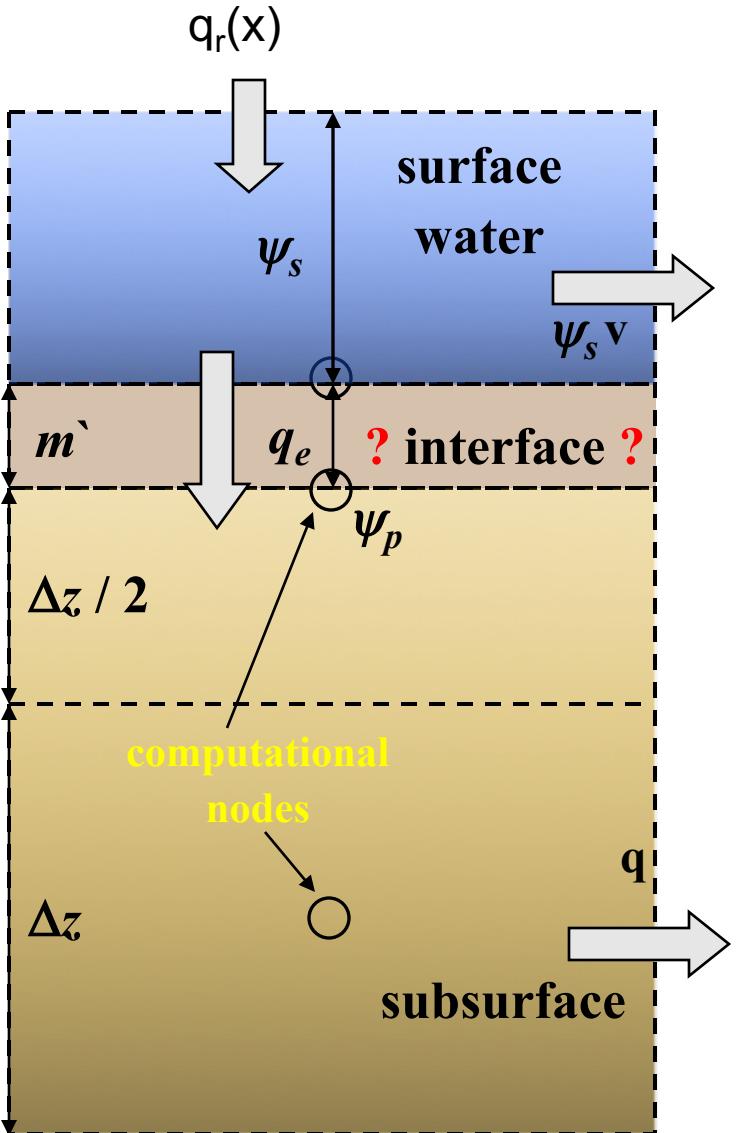
Maxwell, AWR 2013

*Strong scaling, where the problem size remains constant*



Fonseca, arXiv:1702.06898 [cs.MS]

# Overland Flow: The Conductance Concept



Kinematic wave eq

$$\frac{\partial \psi_s}{\partial t} = \nabla \cdot \psi_s \mathbf{v} - q_r(x) - q_e(x)$$

$$q_e(x) = \lambda(x)(\psi_s - \psi_p)$$

Exchange Flux

$$S_s S_w \frac{\partial \psi_p}{\partial t} + \phi \frac{\partial S_w(\psi_p)}{\partial t} = \nabla \cdot \mathbf{q} + q_s + m' q_e$$

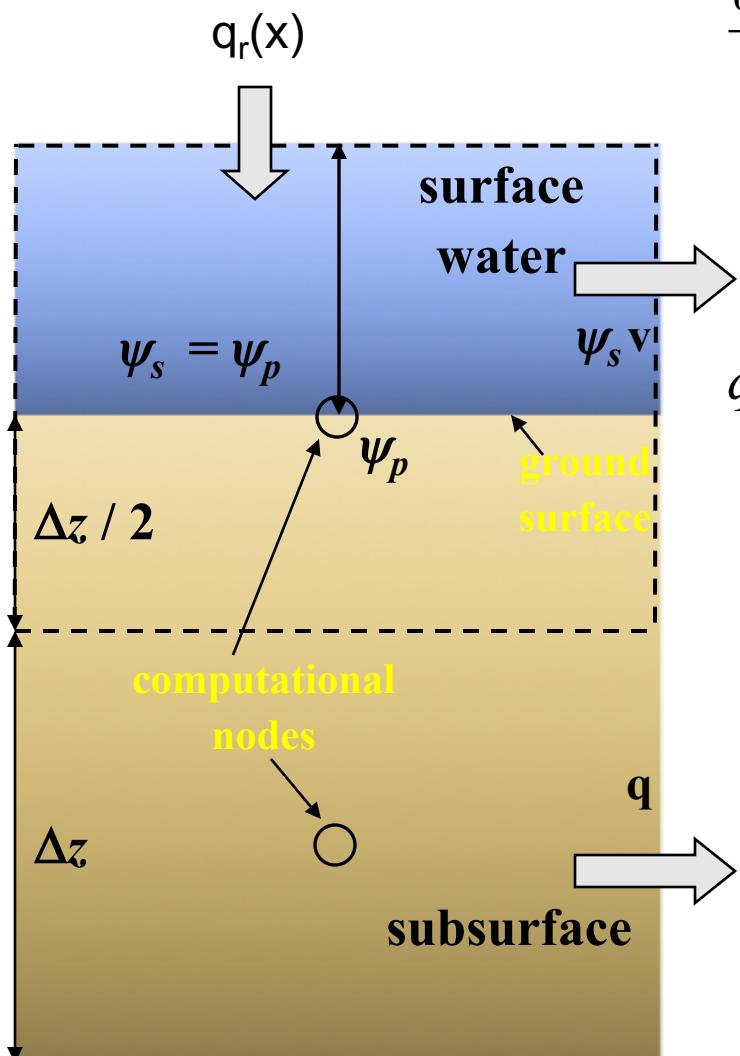
Richards' eq

e.g. VanderKwaak and Loague (2001); Panday and Huyakorn (2004)

# Problems

- Conceptual
  - Distinct interface?
  - Interface properties?
  - Interface dynamics?
- Technical
  - Different time scales
  - Iterative solution?
  - Numerical instability
  - Mass balance
  - Uncertainty in parameter estimates

# Overland Flow: General Pressure Formulation



$$\frac{\partial \psi_s}{\partial t} = \nabla \cdot \psi_s \mathbf{v} - q_r(x) - q_e(x) \quad \text{Kinematic wave eq}$$

$$\psi_s = \psi_p = \psi$$

$$q_e(x) = \frac{\partial \|\psi, 0\|}{\partial t} - \nabla \cdot \|\psi, 0\| \mathbf{v} - q_r(x)$$

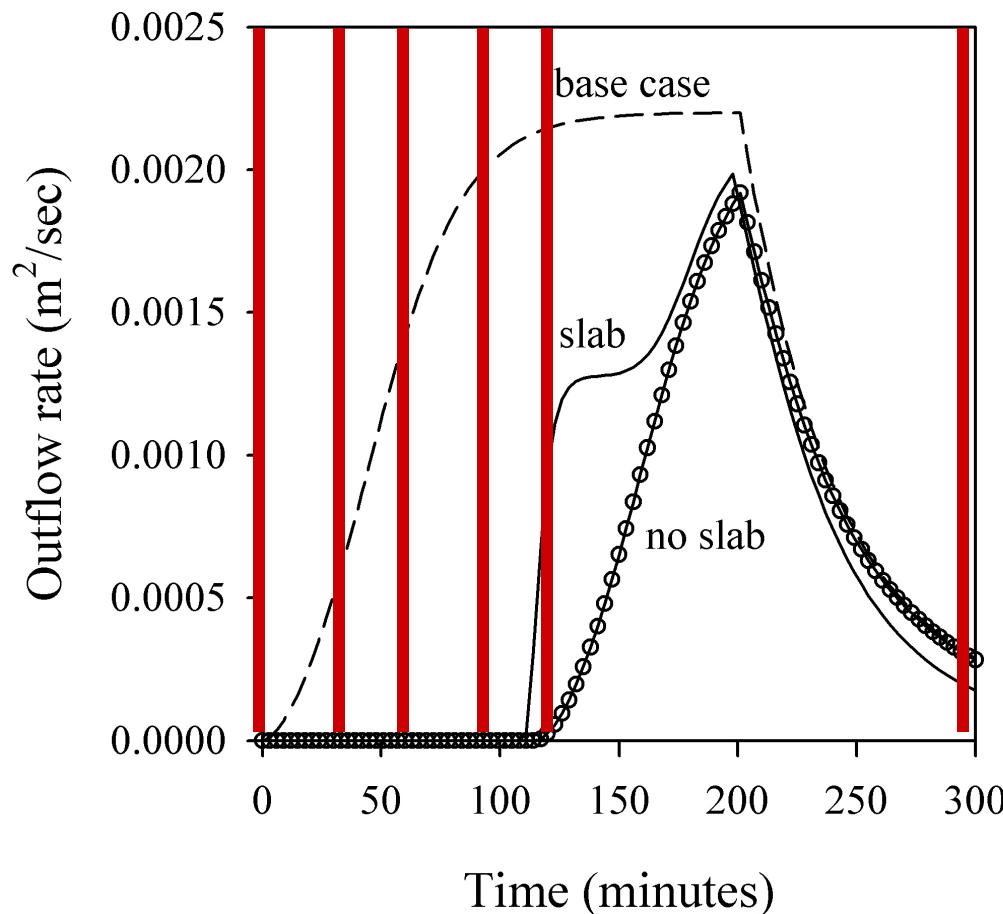
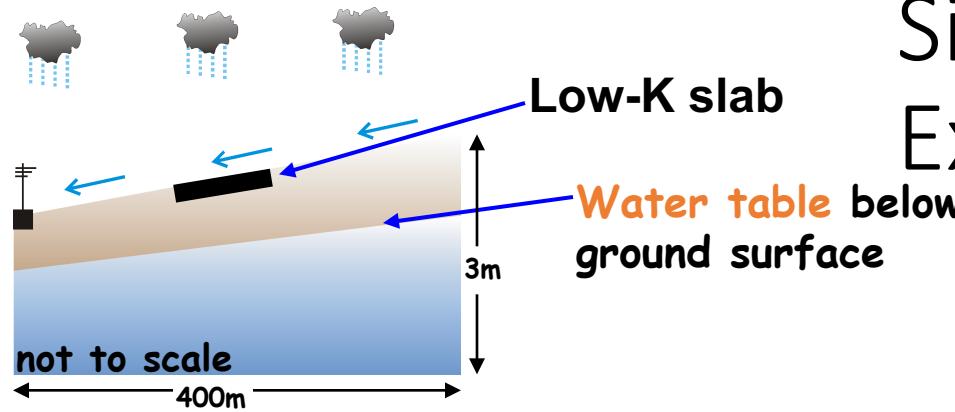
The greater of  $\psi$  and 0

$$q_{bc} = -\mathbf{K}_s k_r(\psi) \nabla(\psi - z) \quad \text{Neumann type BC}$$

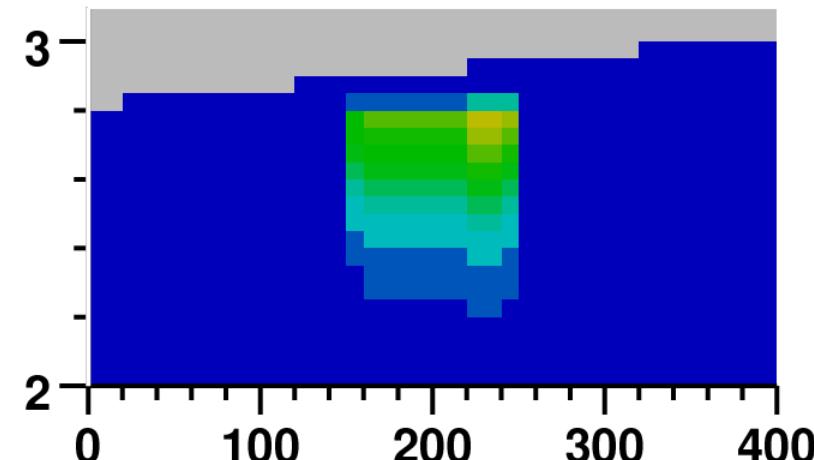
$$q_{bc} = q_e$$

$$-\mathbf{K}_s k_r(\psi) \nabla(\psi - z) = \frac{\partial \|\psi, 0\|}{\partial t} - \nabla \cdot \|\psi, 0\| \mathbf{v} - q_r(x)$$

# Simulation Example



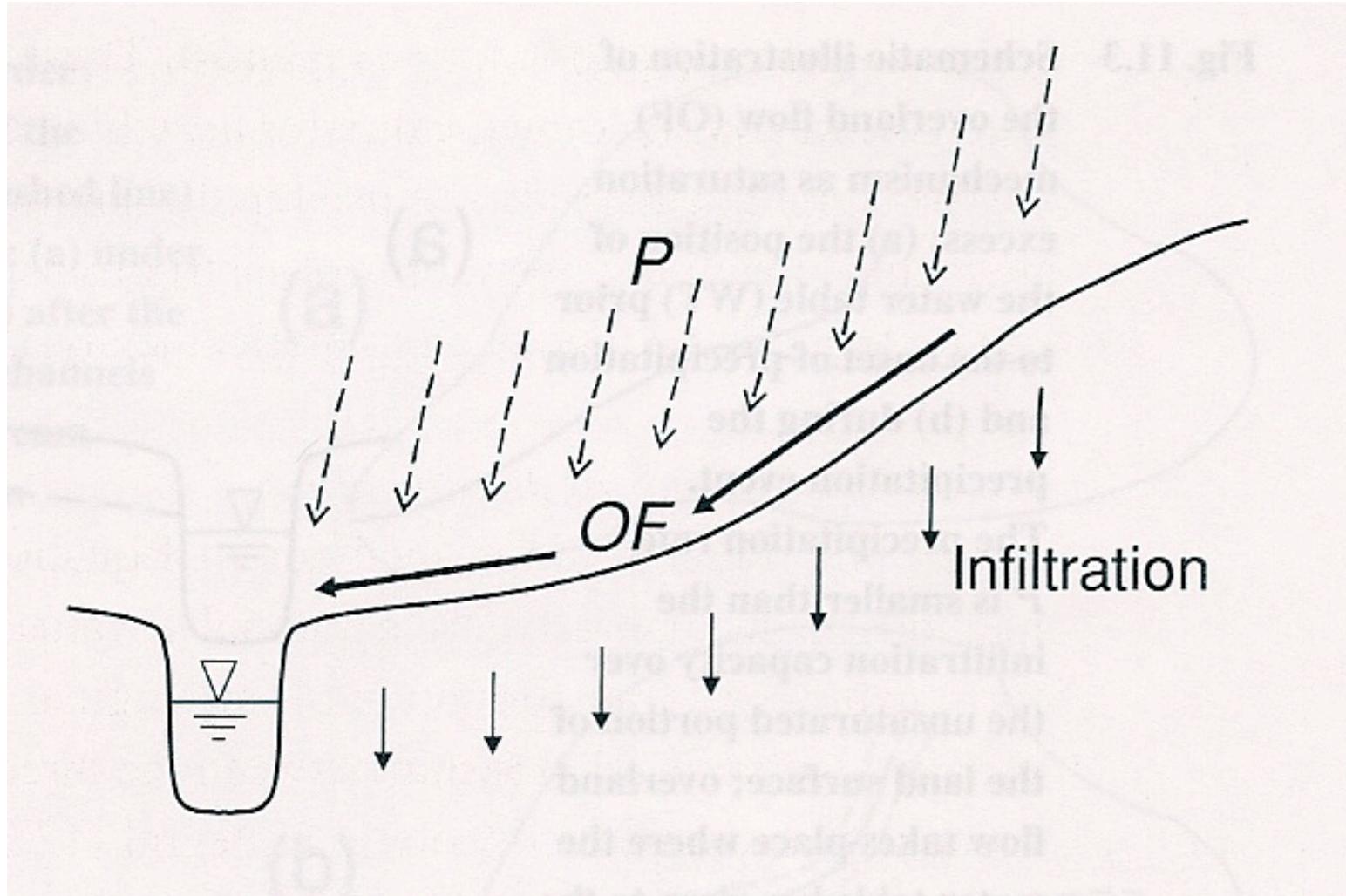
$t = 300 \text{ min}$

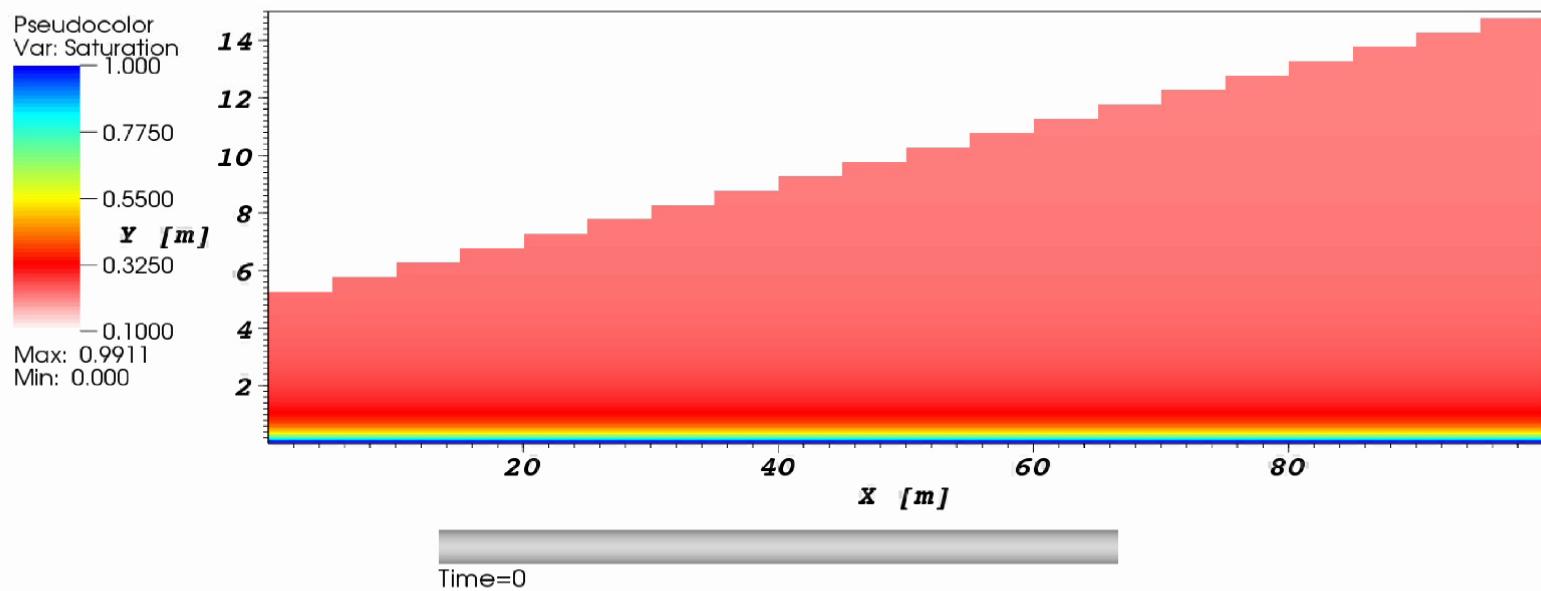


# Runoff Generation Mechanisms

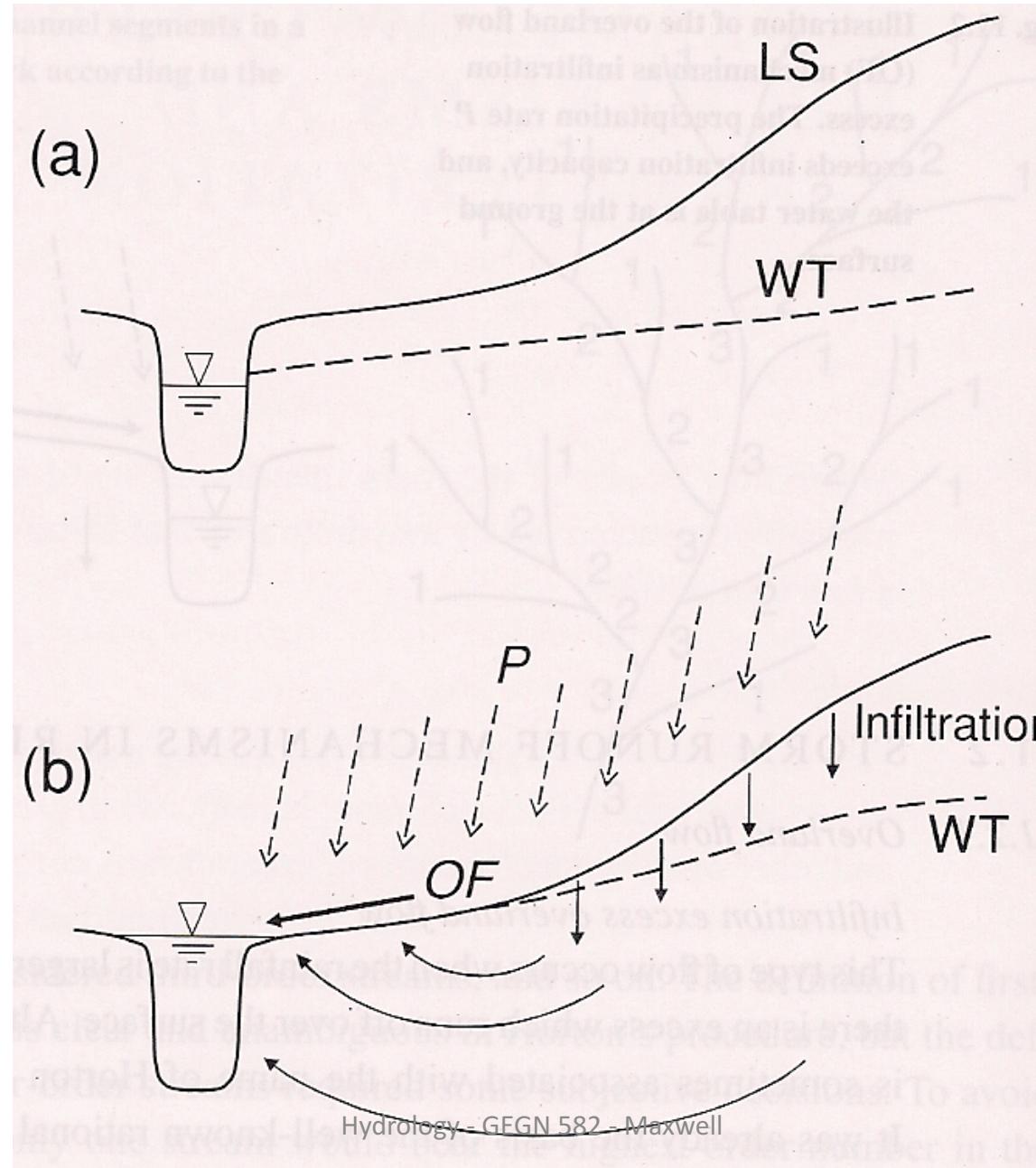
- Runoff is commonly thought to be generated via three different mechanisms:
  - Infiltration Excess
  - Saturation Excess
  - Subsurface Stormflow

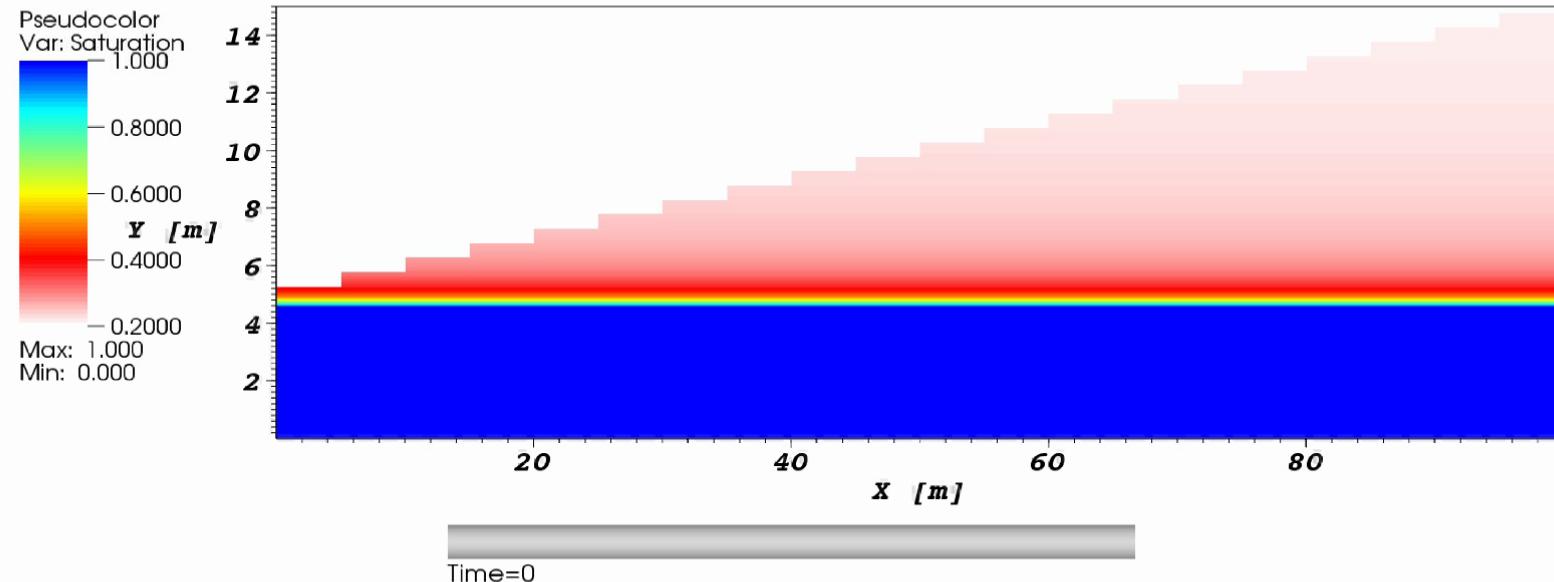
# Infiltration Excess- “Hortonian” flow





# Saturation Excess- “Dunne” flow





# Subsurface Stormflow

