Overland Flow simulation in ParFlow

ParFlow Short Course

Reed Maxwell

We solve the mixed form of Richards' and Shallow Water equations

$$S_S S_W(h)\frac{\partial h}{\partial t}+\theta\frac{\partial S_W(h)}{\partial t}=\nabla\bullet\mathbf{q}+q_r(\mathbf{x},z)$$

Mixed form of Richards' we solve for h

Upper boundary condition (Neumann type) combined with Shallow Water Equation (same h)

$$\mathbf{k}\bullet\left(-\mathbf{K}_S(\mathbf{x})k_r(h)\bullet\nabla(h+z)\right)=\frac{\partial\|h,0\|}{\partial t}-\nabla\bullet\|h,0\|v_{sw}+q_r(\mathbf{x})$$

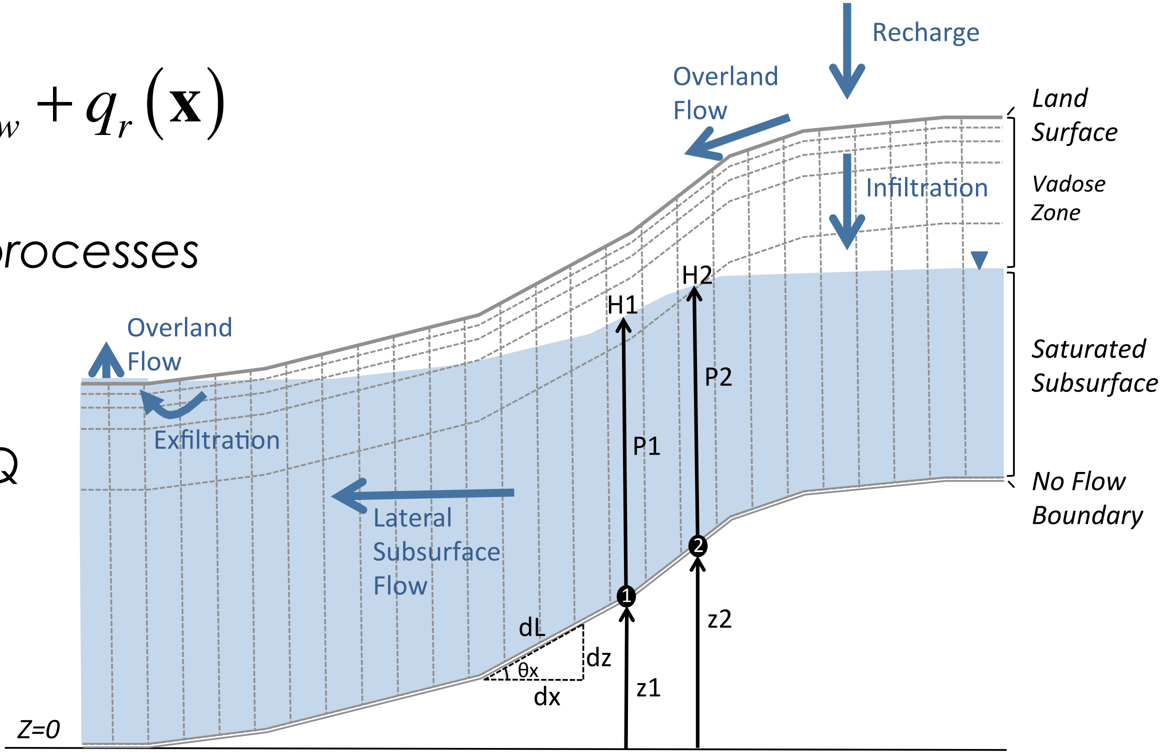
Boundary source/sink, from weather and land surface processes

$$q_r(x)=P(x)-E(x) \qquad q_r(x,z)=-E_T(x,z)$$

Flux Relationships from modified Darcy and Mannings EQ

$$\mathbf{q}=-\mathbf{K}_S(\mathbf{x})k_r(h)\left[\nabla(h+z)\cos\beta_x+\sin\beta_x\right]$$

$$v_x=-\frac{\sqrt{S_{f,x}}}{n}\psi_s^{2/3}$$



ParFlow uses a standard suite of solver packages and is very efficient

- Designed to be **parallel** from the ground-up using an object-oriented framework

- Newton Krylov nonlinear approach via the Kinsol solver package (**globally implicit**=*integrated*)

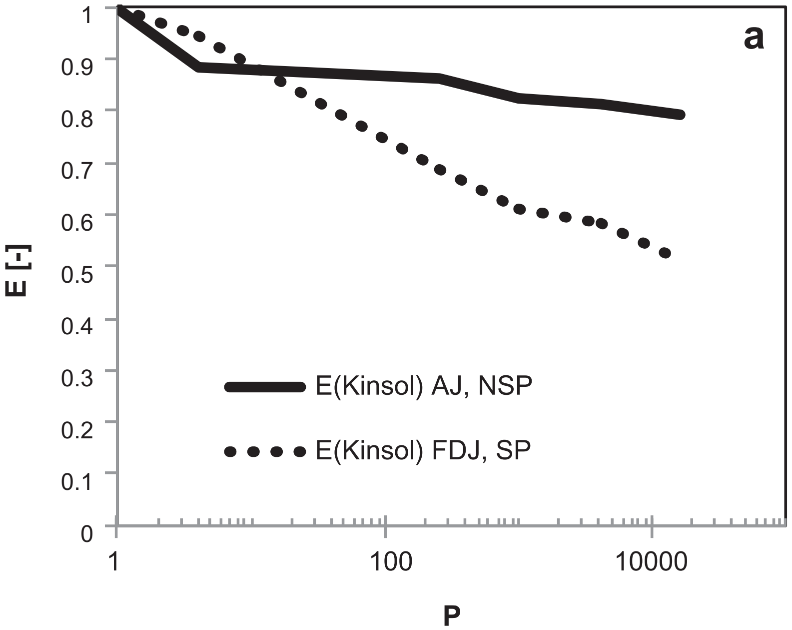
- Multigrid-preconditioned linear solver using the Hypre package

- **Physics-based** preconditioning based on analytical Jacobian to accelerate convergence

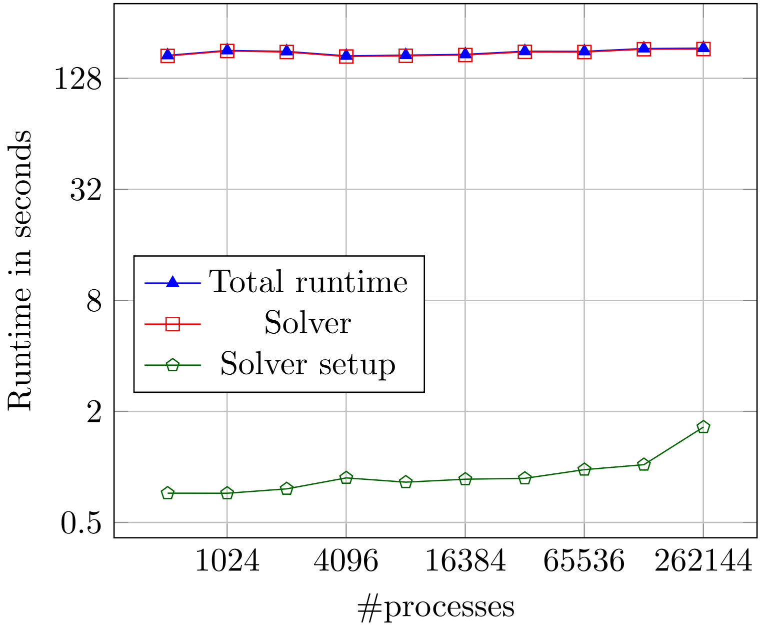
ParFlow demonstrates excellent parallel scalability

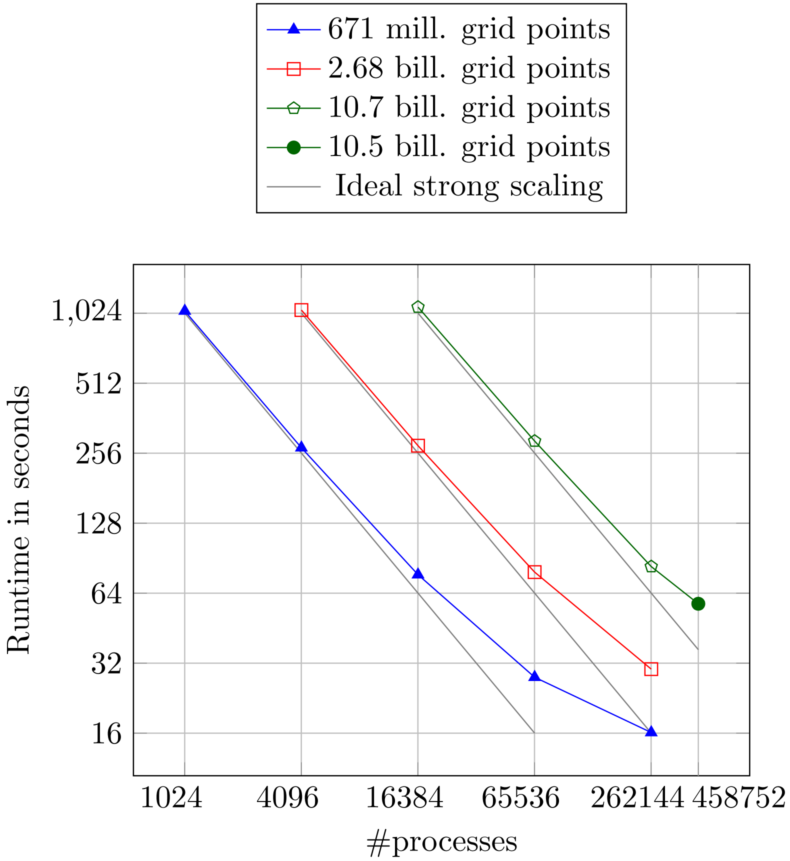
Weak scaling, where the problem size increases with the number of processors

Strong scaling, where the problem size remains constant



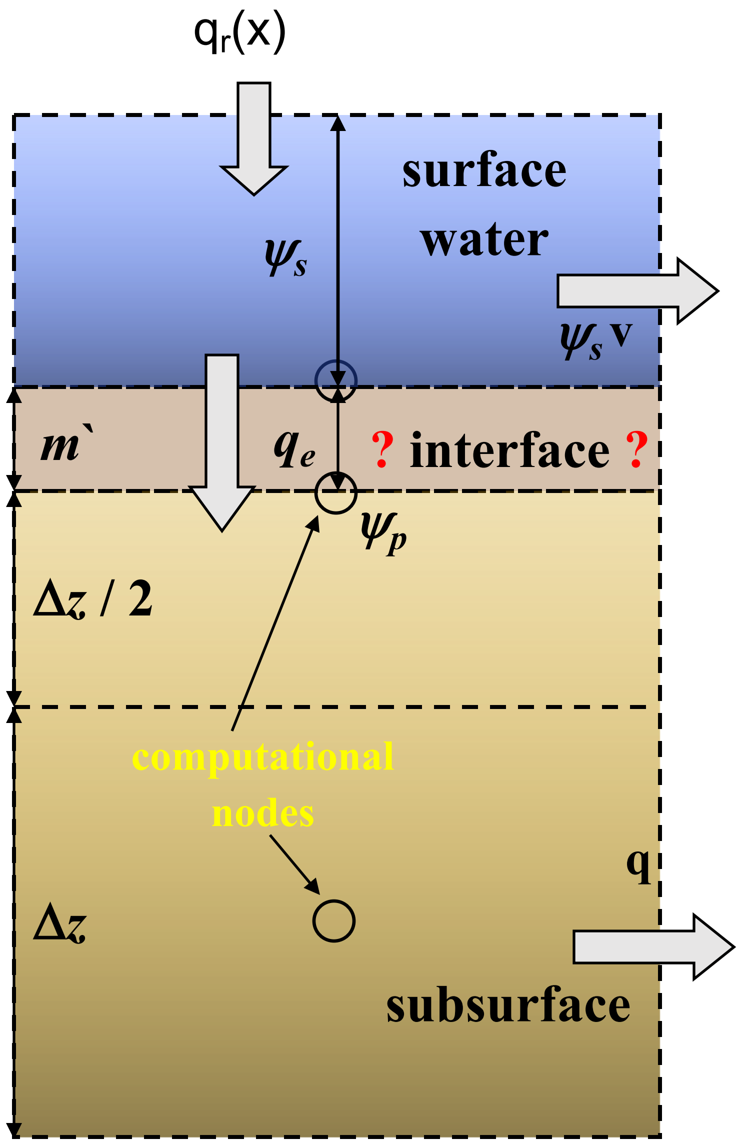
Maxwell, AWR 2013





Fonseca, arXiv:1702.06898 [cs.MS]

Overland Flow: The Conductance Concept



Kinematic wave eq

$$\frac{\partial \psi_s}{\partial t} = \nabla \cdot \psi_s \mathbf{v} - q_r(x) - q_e(x)$$

$$q_e(x) = \lambda(x)\left(\psi_s - \psi_p\right)$$

Exchange Flux

$$S_s S_w \frac{\partial \psi_p}{\partial t} + \phi \frac{\partial S_w\left(\psi_p\right)}{\partial t} = \nabla \cdot \mathbf{q} + q_s + m' q_e$$

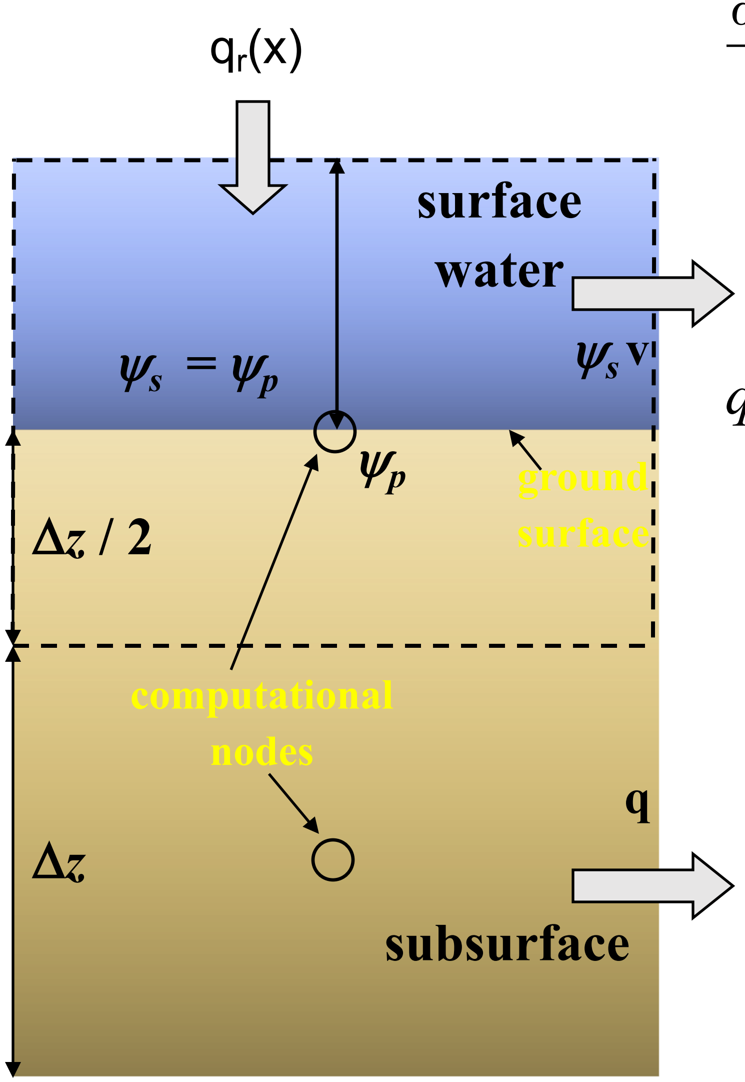
Richards' eq

e.g. VanderKwaak and Loague (2001); Panday and Huyakorn (2004)

Problems

- Conceptual
  - Distinct interface?
  - Interface properties?
  - Interface dynamics?
- Technical
  - Different time scales
  - Iterative solution?
  - Numerical instability
  - Mass balance
  - Uncertainty in parameter estimates

Overland Flow: General Pressure Formulation



$$\frac{\partial \psi_s}{\partial t} = \nabla \cdot \psi_s \mathbf{v} - q_r(x) - q_e(x)$$ Kinematic wave eq

$$\psi_s = \psi_p = \psi$$

The greater of ψ and 0

$$q_e(x) = \frac{\partial \|\psi, 0\|}{\partial t} - \nabla \cdot \|\psi, 0\| \mathbf{v} - q_r(x)$$

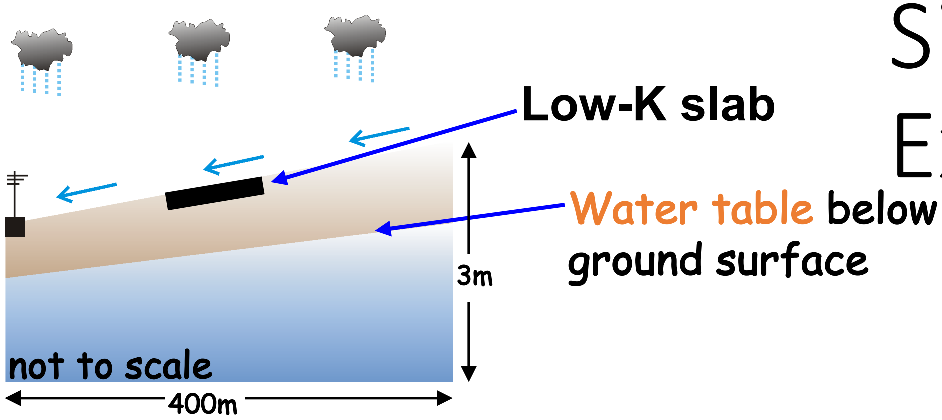
$$q_{bc} = -\mathbf{K}_s k_r(\psi) \nabla(\psi - z)$$ Neumann type BC

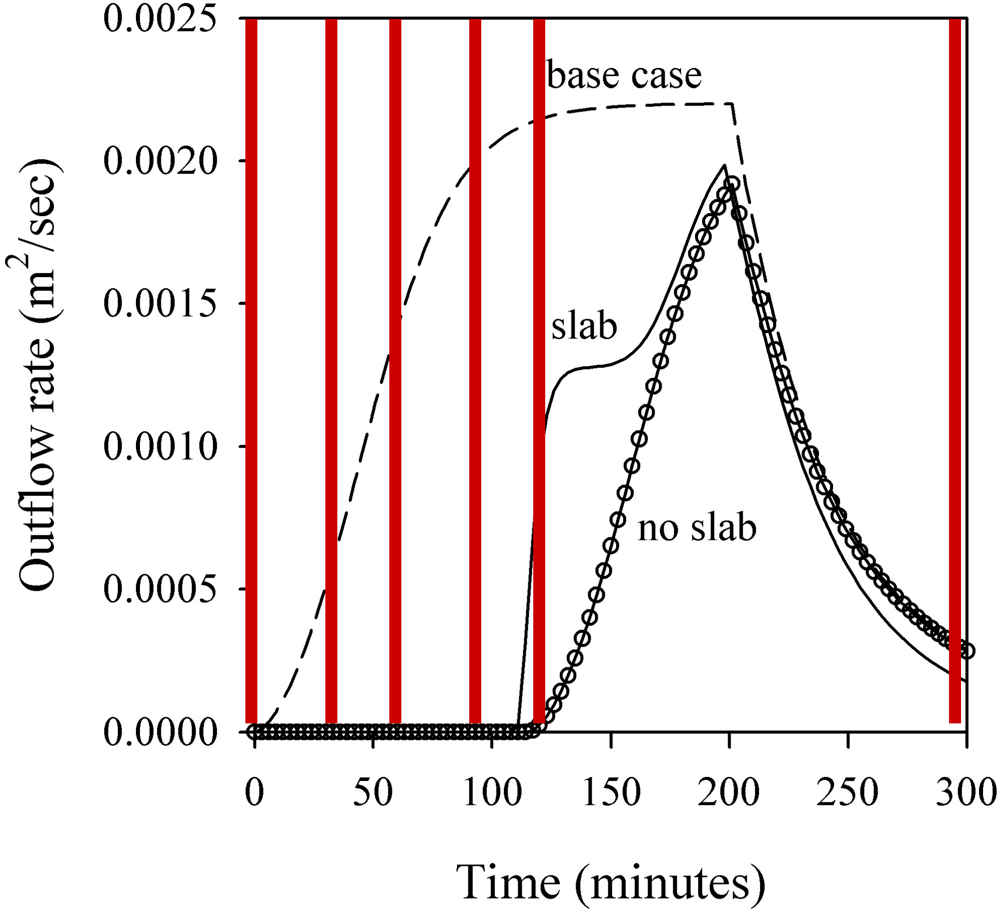
$$q_{bc} = q_e$$

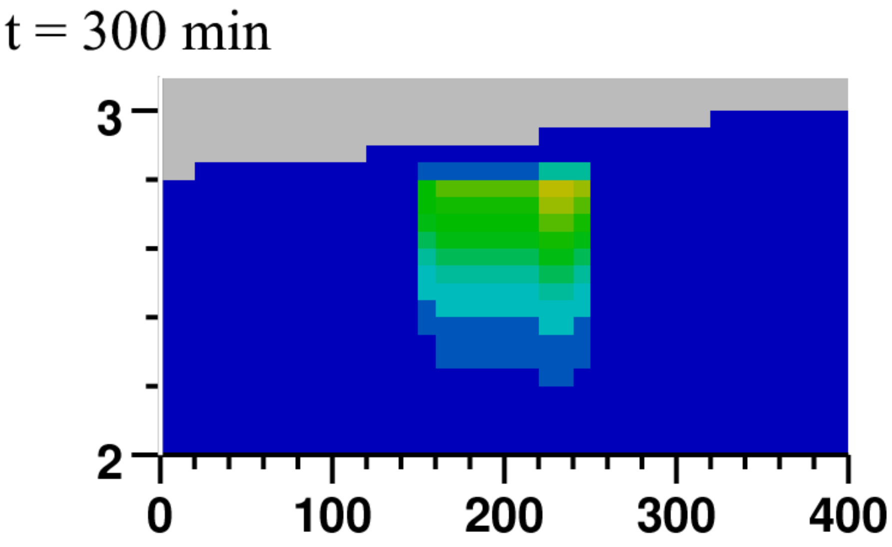
$$\boxed{-\mathbf{K}_s k_r(\psi) \nabla(\psi - z) = \frac{\partial \|\psi, 0\|}{\partial t} - \nabla \cdot \|\psi, 0\| \mathbf{v} - q_r(x)}$$

Kollet and Maxwell, AWR (2006)

Simulation Example





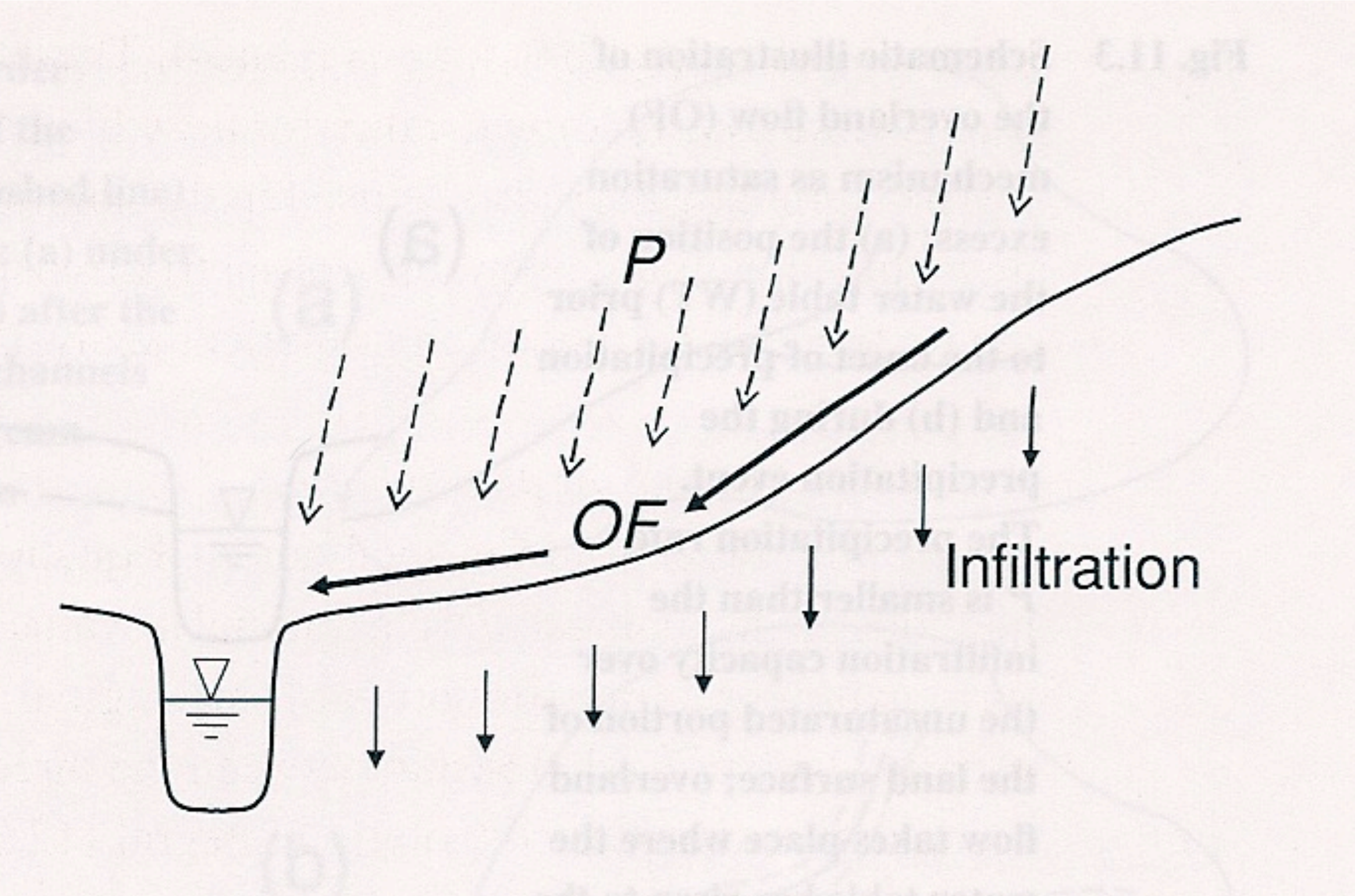


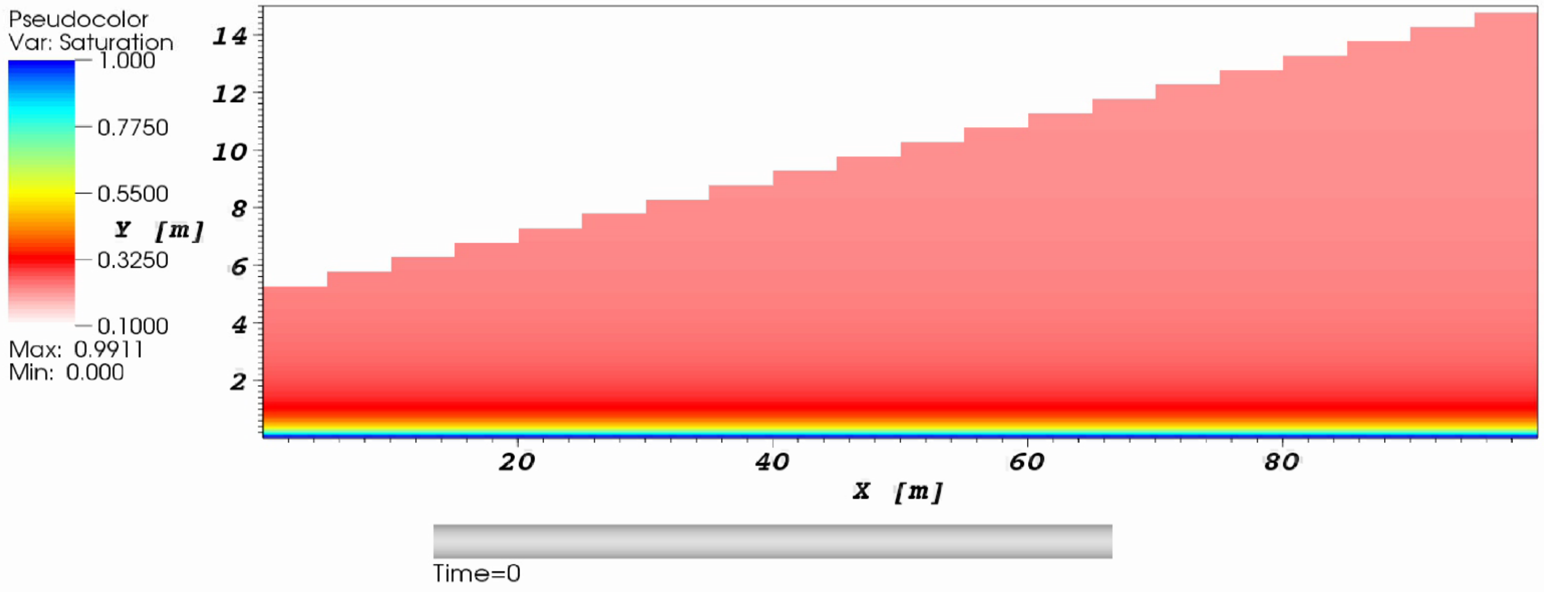
Kollet & Maxwell, AWR, 2006

Runoff Generation Mechanisms

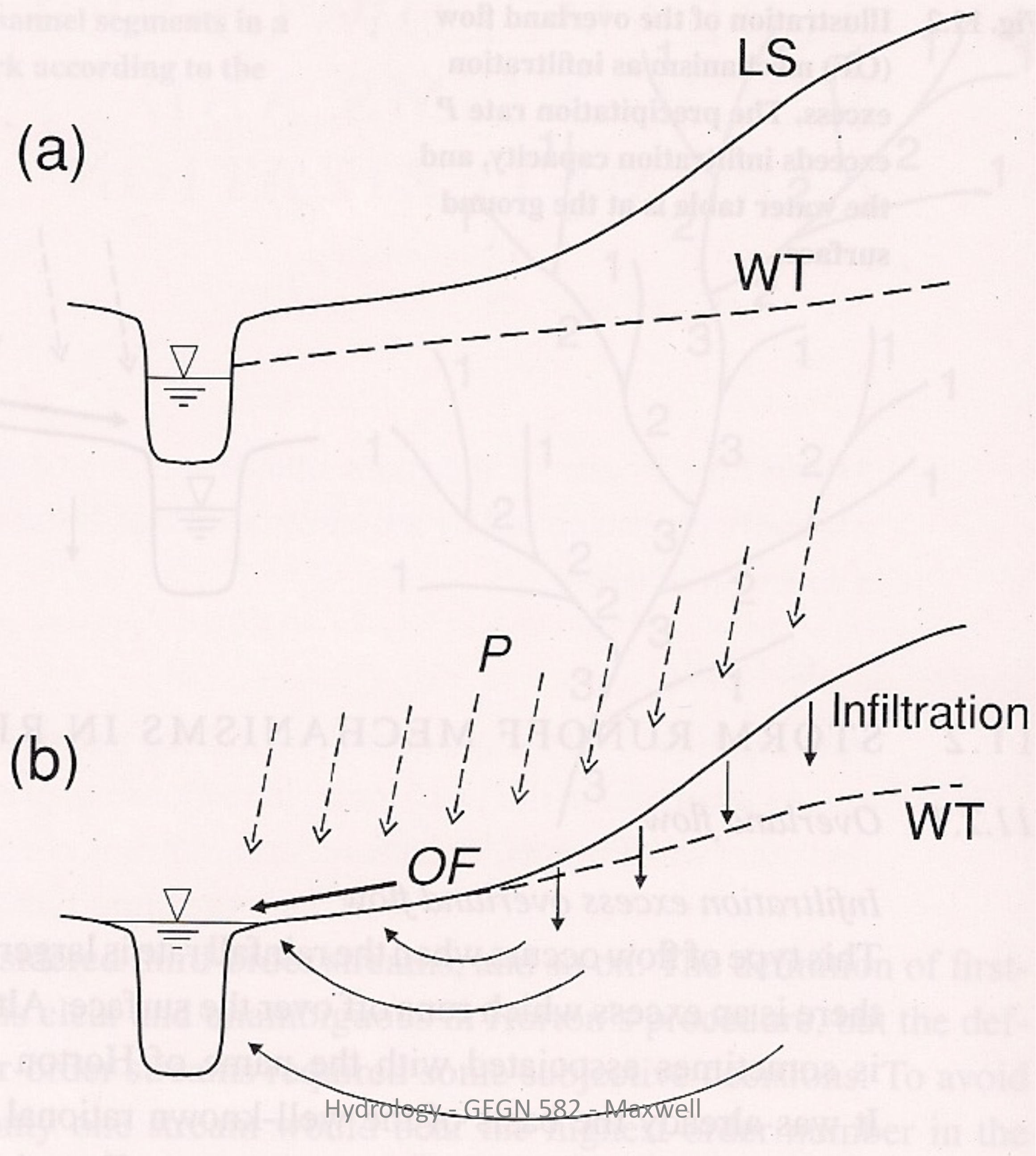
- Runoff is commonly thought to be generated via three different mechanisms:
  - Infiltration Excess
  - Saturation Excess
  - Subsurface Stormflow

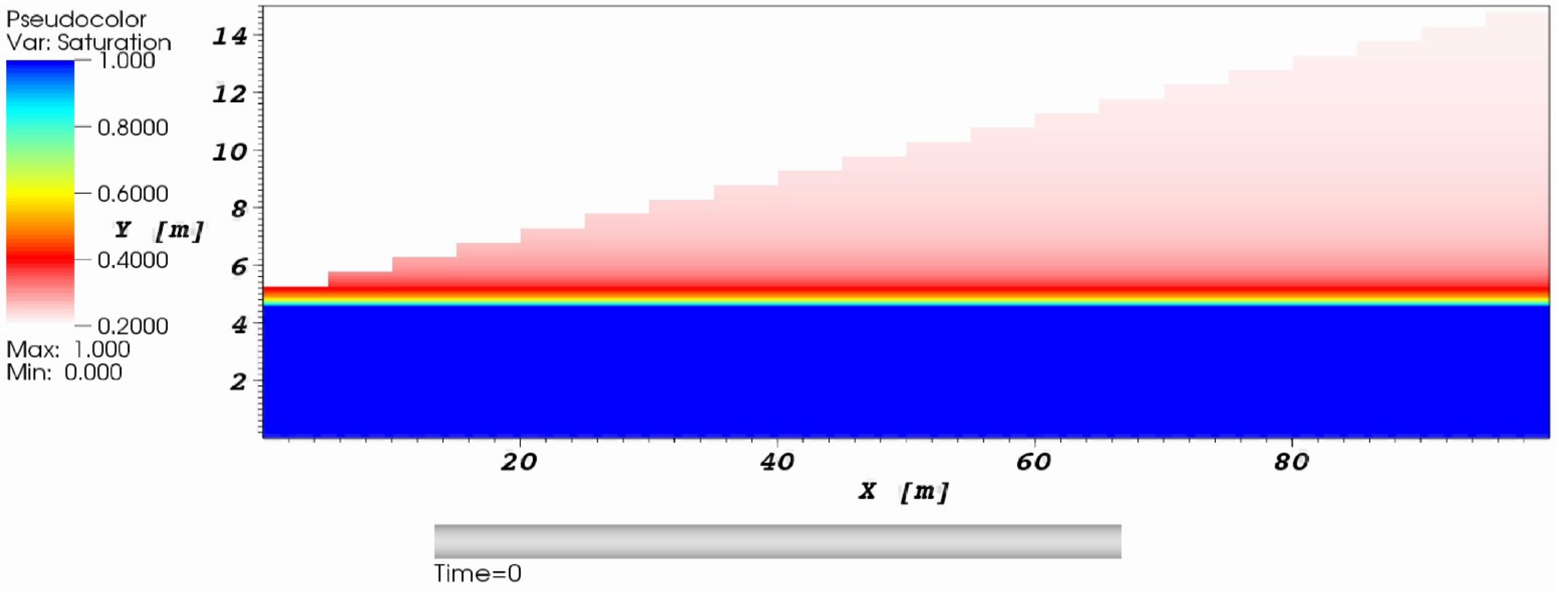
Infiltration Excess- “Hortonian” flow



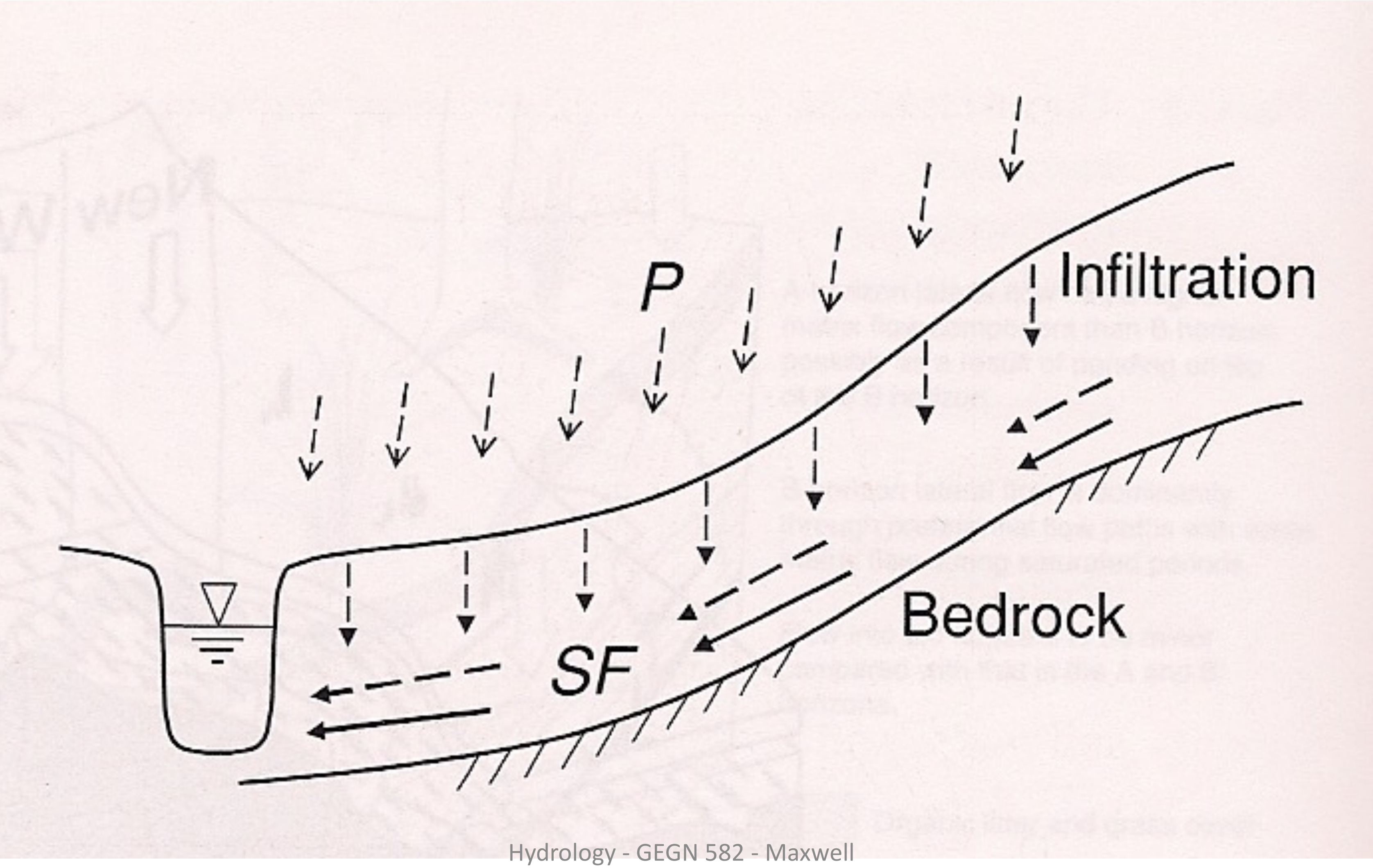

Pseudocolor
Var: Saturation
1.000
0.7750
0.5500
0.3250
0.1000
Max: 0.9911
Min: 0.000
Y [m]
14
12
10
8
6
4
2
20
40
60
80
X [m]
Time=0

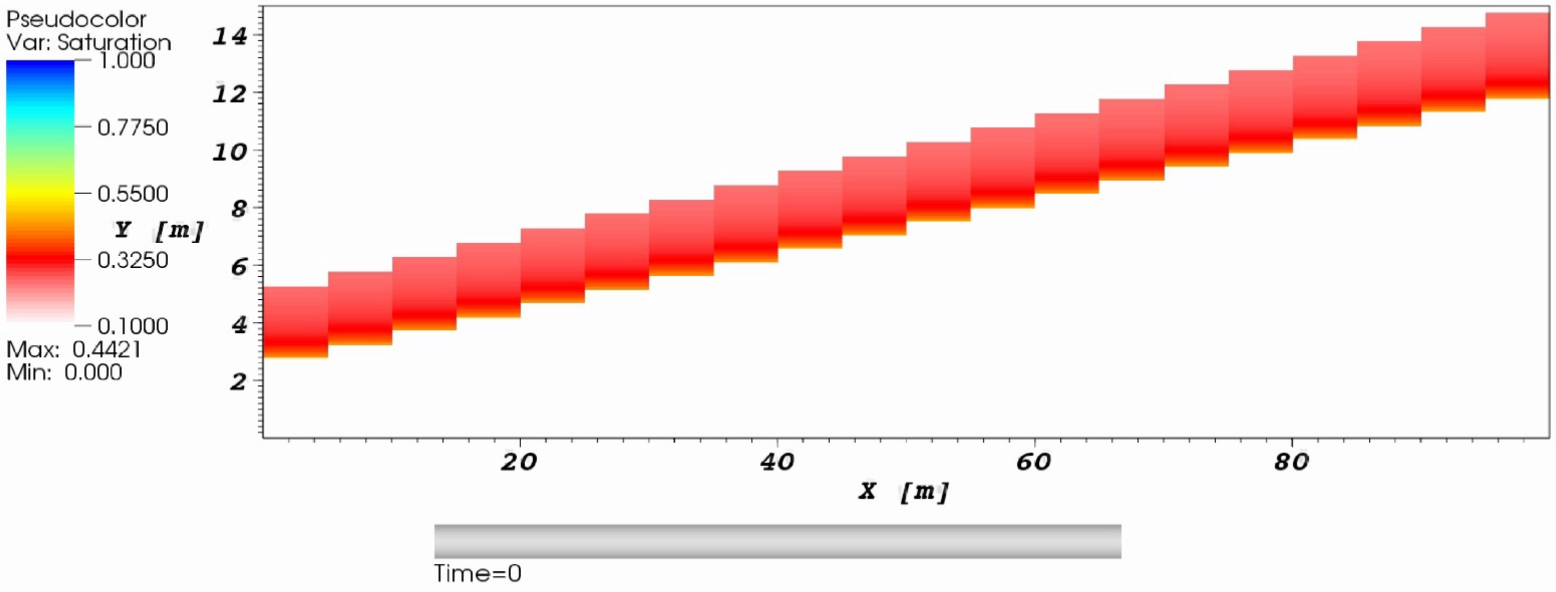
Saturation Excess- “Dunne” flow

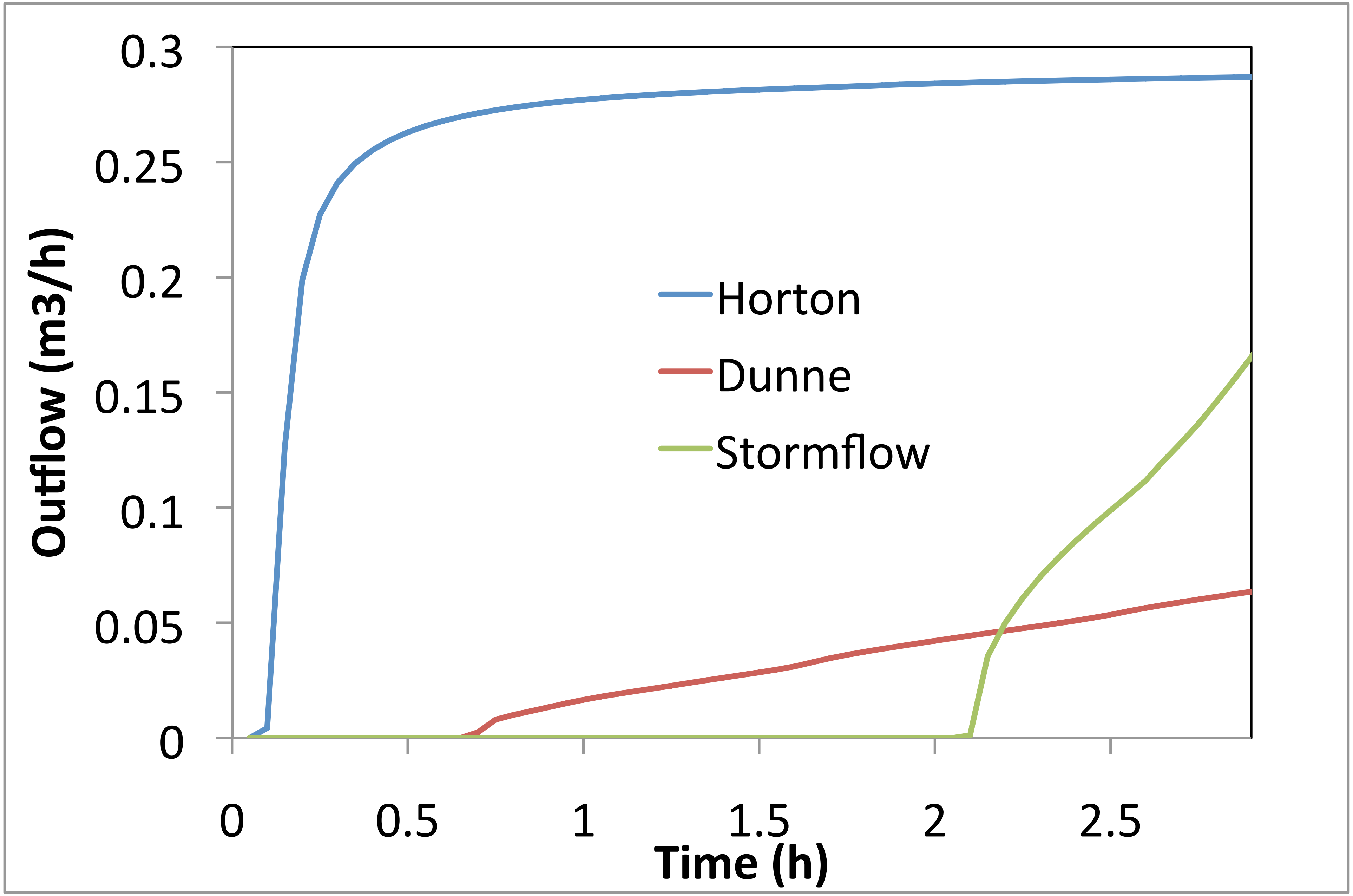


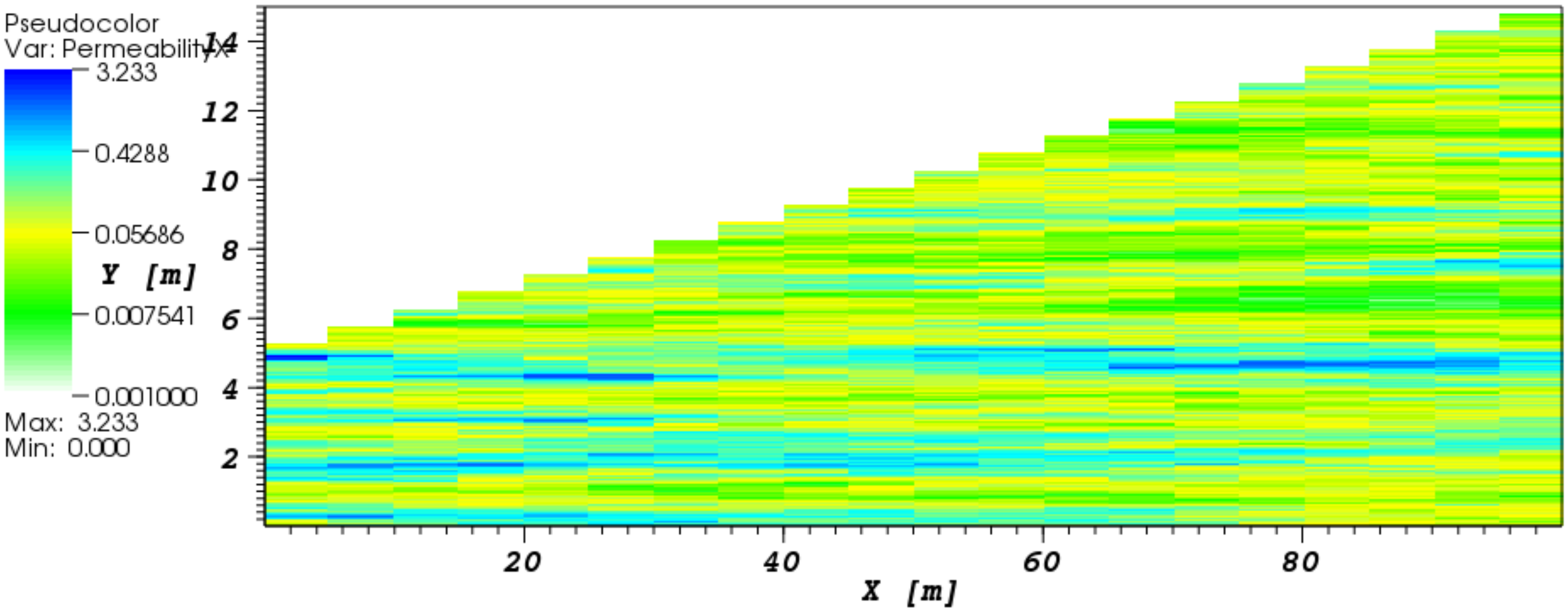

Pseudocolor
Var: Saturation
1.000
0.8000
0.6000
0.4000
0.2000
Max: 1.000
Min: 0.000
Y [m]
14
12
10
8
6
4
2
20
40
60
80
X [m]
Time=0

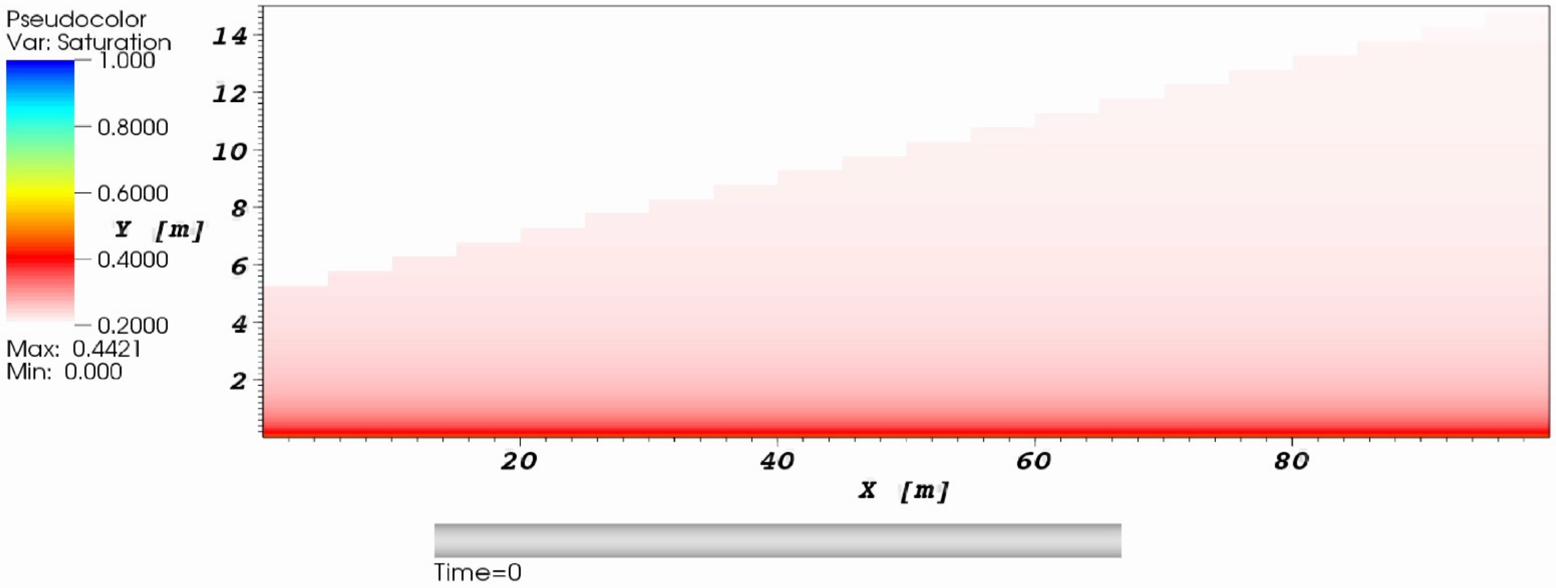
Subsurface Stormflow

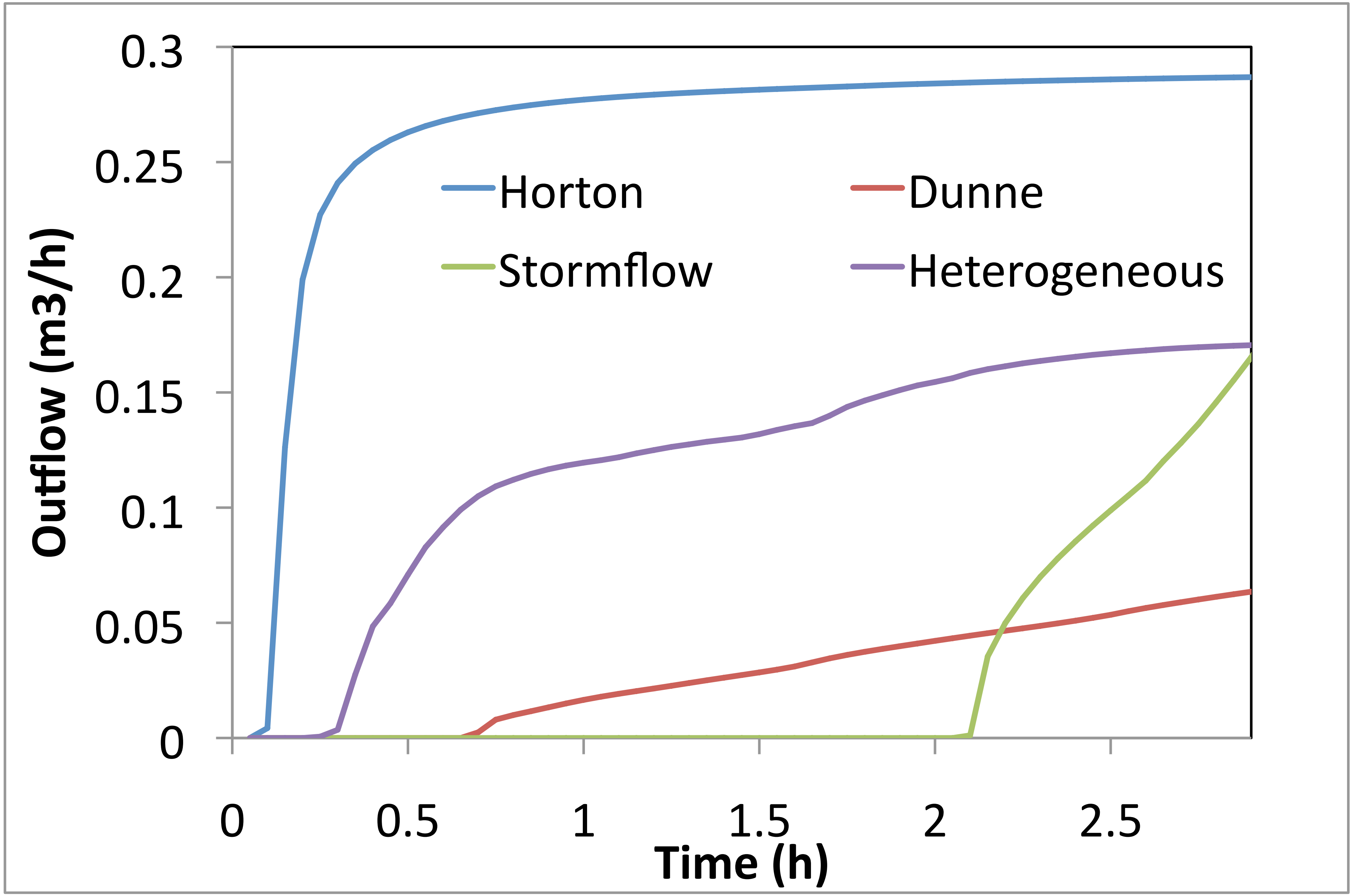


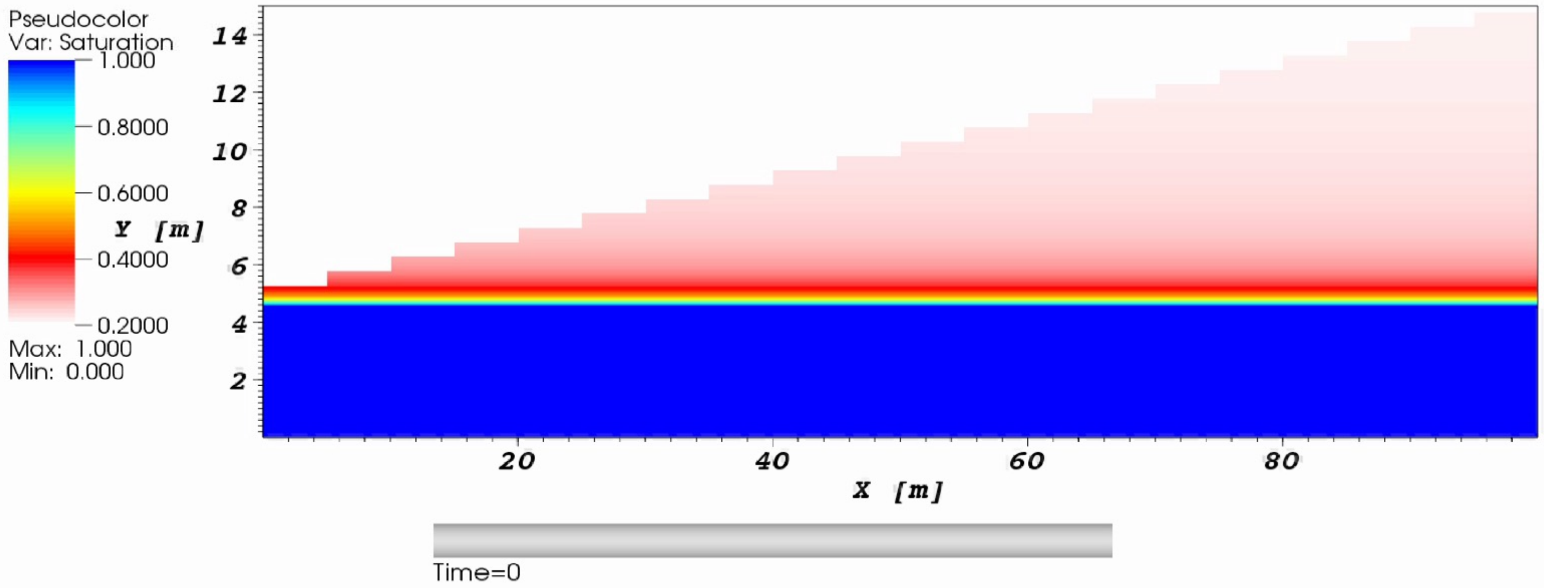

Pseudocolor
Var: Saturation
1.000
0.7750
0.5500
0.3250
0.1000
Max: 0.4421
Min: 0.000
Y [m]
14
12
10
8
6
4
2
20
40
60
80
X [m]
Time=0


Outflow (m3/h)
0.3
0.25
0.2
0.15
0.1
0.05
0
Horton
Dunne
Stormflow
0
0.5
1
1.5
2
2.5
Time (h)


Pseudocolor
Var: PermeabilityX
3.233
0.4288
0.05686
0.007541
0.001000
Max: 3.233
Min: 0.000
Y [m]
14
12
10
8
6
4
2
20
40
60
80
X [m]


Pseudocolor
Var: Saturation
1.000
0.8000
0.6000
0.4000
0.2000
Max: 0.4421
Min: 0.000
Y [m]
X [m]
Time=0


0.3
0.25
0.2
0.15
0.1
0.05
0
Outflow (m3/h)
Horton
Dunne
Stormflow
Heterogeneous
0
0.5
1
1.5
2
2.5
Time (h)


Pseudocolor
Var: Saturation
1.000
0.8000
0.6000
0.4000
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Max: 1.000
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Y [m]
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X [m]
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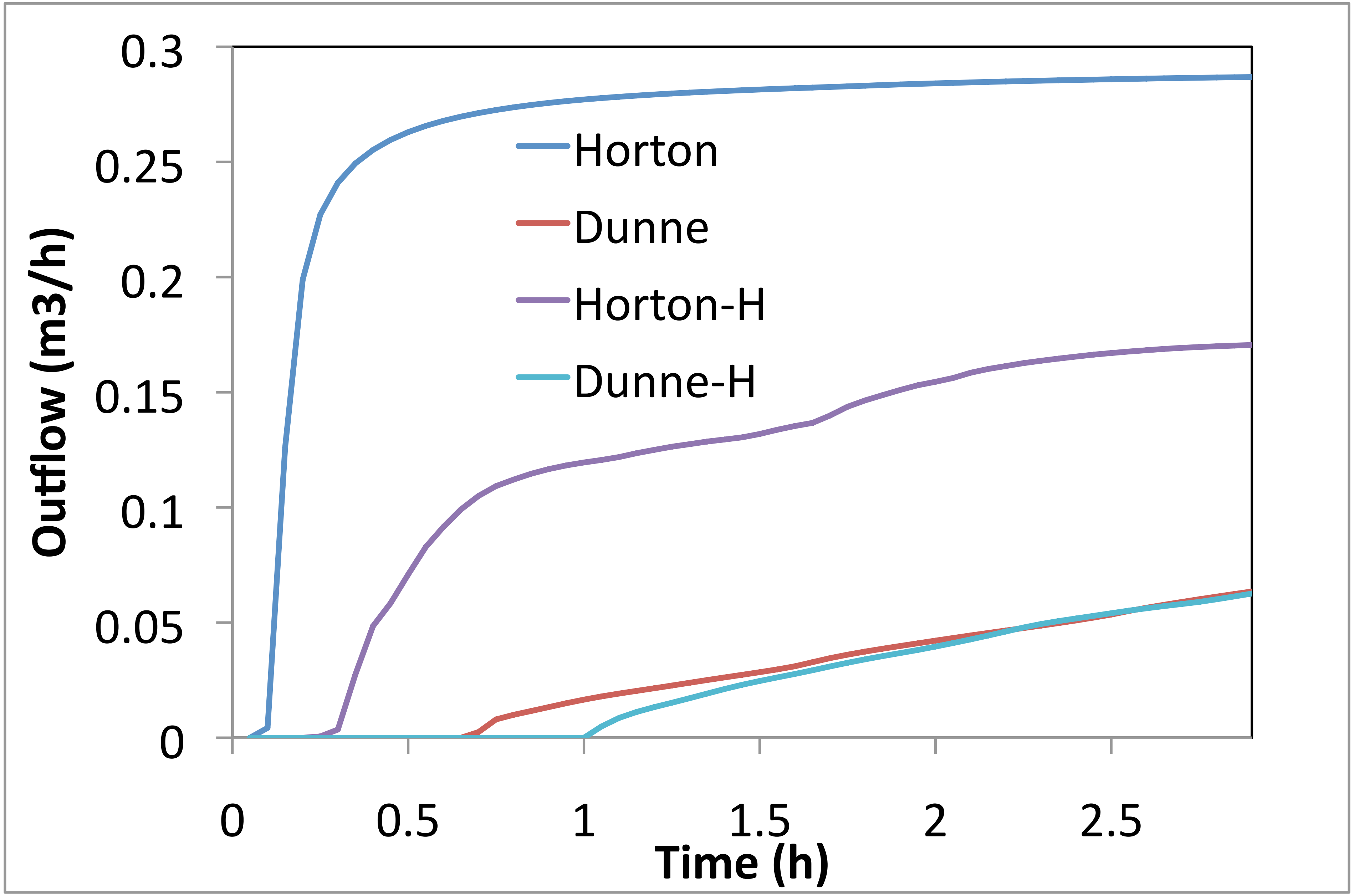
0.3
0.25
0.2
0.15
0.1
0.05
0
Outflow (m3/h)
0
0.5
1
1.5
2
2.5
Time (h)
Horton
Dunne
Horton-H
Dunne-H