DOCUMENTATION OF LIQUINET PROGRAM

Basic Concepts

The Program is based on paper “Application of Finite Element computer Program to Water Distribution System Analysis” by Anthony G. Collins. In this program, we have used Darcy Weisbach Equation. Attempts have been made to take into account the effects of Partial closure of Valves and effects of Booster/Pumps on a Pipe Network.

Use of Units in Program

Use of units in the program need to be consistent throughout. That means if m/s^2 is used as units for acceleration, and kg/m^3 is used as units for density, then $N\ s/m^2$ must be used for dynamic viscosity. A table is provided below as a guide as to what units may be used for which kind of values.

Type of Unit	Gravitational Acceleration	Dynamic Viscosity	Flow Rate	Internal Diameter	Roughness	Length	Booster Head	Opening Percentage of Valve	Head at Node
SI units	m/s^2	Ns/m^2	m^3/s	m	m	m	m	%	m
Recomme-nded value if Any	9.81	1.002×10^{-3} (20° C, water)	--	--	--	--	--	--	--
Imperial units	ft/s^2	$lb_f\ s/ft^2$	ft^3/s	ft	ft	ft	ft	%	ft
Recomme-nded value if Any	32.174	2.035×10^{-5} (70° F, water)	--	--	--	--	--	--	--

Formulae Used:

For Pipe Segments

$$h = \frac{fLq^2}{2\,gdA^2} + \frac{kq^2}{2\,gA^2}$$

Here, h=Head Loss,
f=friction factor,
L=Length,
q=Flow Rate,
g=Gravitational Acceleration,
d=Diameter,
A=Area of Pipe calculated by $\Pi \frac{d^2}{4}$,
k=Total k factor of Pipe taking into account bend, reducers, valves, etc.

For Boosters

$$h=\frac{q^2}{2gA^2}-H_d$$

Here, h=Static Head Difference,
H_d=Total Head created by the Booster,
q=Flow Rate,
g=Gravitational Acceleration,
A=Area of Pipe calculated by $\Pi\frac{d^2}{4}$,

For Flow Control Valves

$$h=\frac{kq^2}{2gA^2}\times\left(\frac{100}{o}\right)^{2n}$$

Here, h=Head Loss,
k=Total k factor of Pipe taking into account bend, reducers, etc.,
g=Gravitational Acceleration,
d=Diameter,
A=Area of Pipe calculated by $\Pi\frac{d^2}{4}$,
o=opening percentage,
n=non-linearity factor.

Background of using Flow Control Valve Formula

For Valves, it is not necessary that flow within the valve be directly proportional to the opening percentage. It may be linear or non-linear (including quick opening and equal percentage).
Solving for q in the formula provided for flow control valve we have,

$$q=A(o/100)^n\times(2\frac{hg}{k})^{1/2}$$

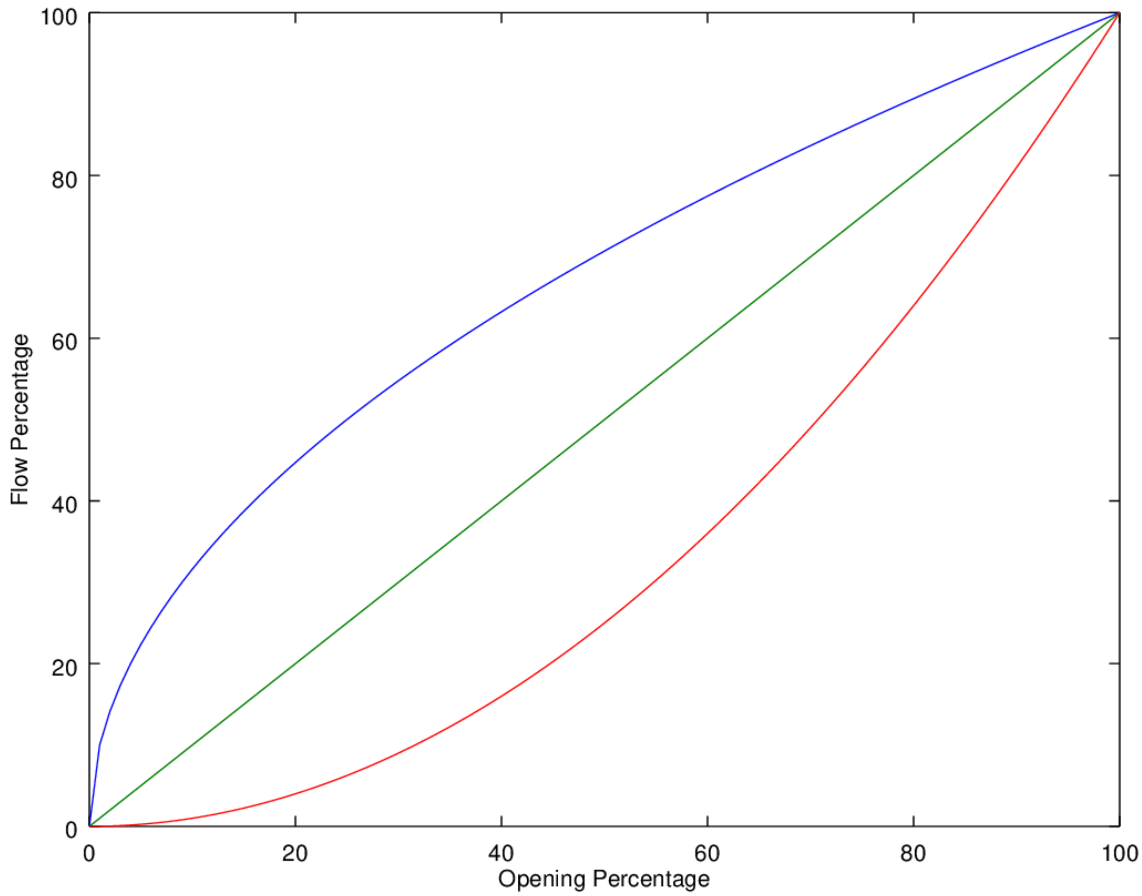
At 100 percent opening we have,

$$q_{100}=A(100/100)^n\times(2\frac{hg}{k})^{1/2}$$

Hence the flow percentage is,

$$\frac{q}{q_{100}}\times100\ \%$$

Using the above formula, by initializing ‘n’ as .5, 1, 2 each time, we plot a graph to find the following characteristics for different opening percentages.



Here, the blue line represents n=.5, green line represents n=1, and red line represents n=2.

An Octave/Matlab program "valve_characteristic.m" used to plot this has been provided in the source files, in the Documentation folder. In the program Diameter, Head Loss, k-factor and gravitational acceleration has been considered as 1 meter, 10 meters, 10 and 9.81 m^2/s respectively. Hence, a reasonable approximation of flow characteristics of the valve may be represented if adequate data is available.

Working Principle

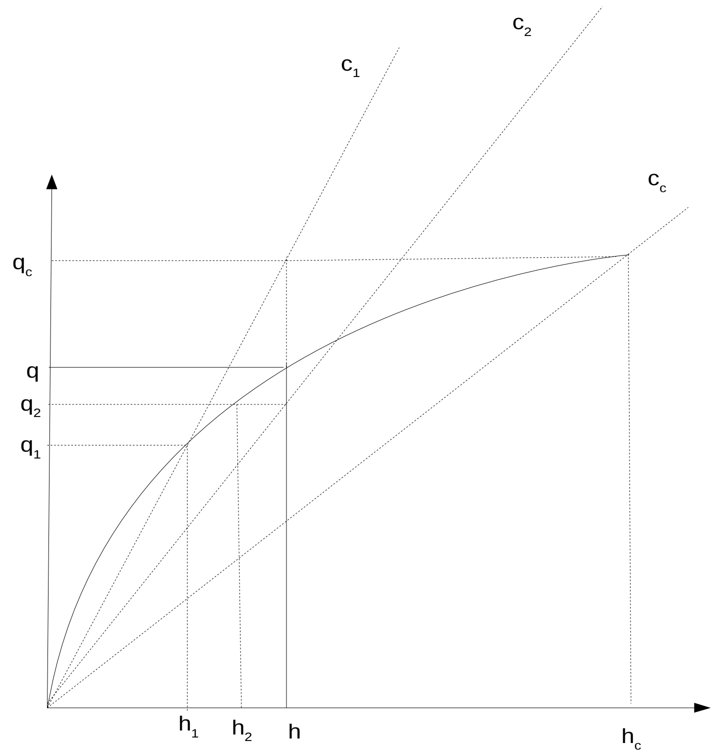
The program tries to approximate the solution by linearising the equation similar to the Matrix form $[F]=[K][x]$ as $[Q]=[C][H]$. Here Q is the Total amount of flow in or out of a particular node. H is the head at a certain node.

Method of Calculation for Each Element

In local form, similar to the form $f=kx$, $q=ch$ has been considered where q is the flow through the particular element and h is the head loss.

We know from Darcy Weisbach equation that $h=\frac{flq^2}{2gdA^2}$. For sake of better understanding, lets consider the previous equation as $h=\frac{q^2}{k}$.

Now consider that h is known and q is to be found by using $q=ch$.



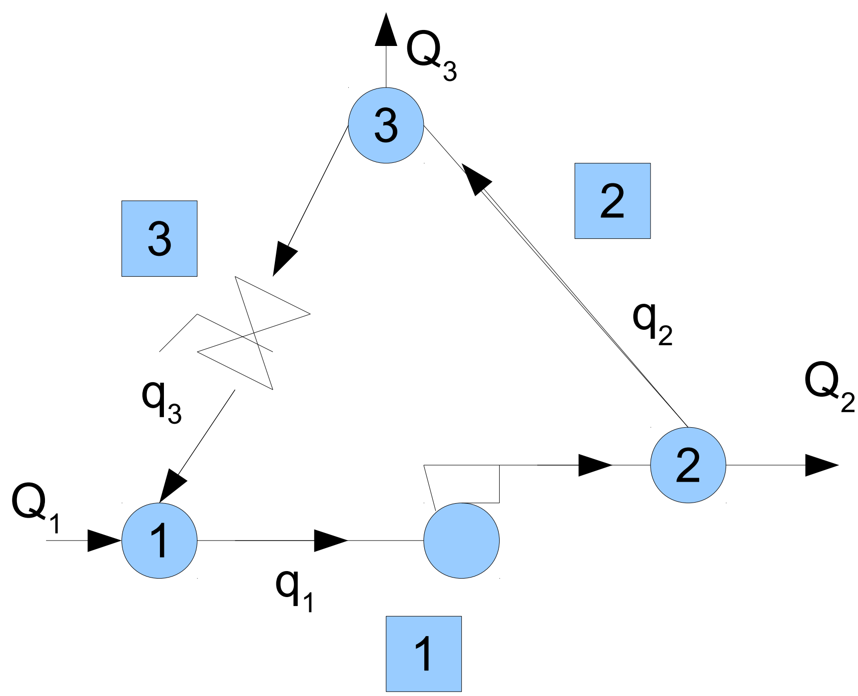
Refer to the graph above. We start of with an approximated value of q_1 In the program, this accomplished by asking the user for an approximate value of Reynold's number from where q_1 is calculated.

- Step 1.
  - h_1 Is calculated by $h_1=\frac{q_1^2}{k}$ and slope $c_1=\frac{q_1}{h_1}$.
- Step 2
  - Using the slope of c_1 we calculate q_c as $q_c=c_1\times h$.
- Step 3
  - $h_c=\frac{q_c^2}{k}$ And slope $c_c=\frac{q_c}{h_c}$
- Step 4
  - If h_c is within approximation range, we stop calculating. Else, we find new slope $c_2=\frac{c_1+c_c}{2}$ and continue to Step 5.
- Step 5
  - $q_2=c_2\times h$. We go back to Step 1 and continue the later Steps. h_2 Is calculated by replacing $h_{1.}$ q_1 Is replaced by q_2, and c_1 is replaced by $c_{2.}$

These steps are repeated for every element in each iteration. The instance of the iteration runs till $h_{1,} h_2$ and h_3 for all elements are within the allowed approximation range.

Formation of Global Matrix

To demonstrate how the global matrix is formed we consider a simple problem given below.



Circles with numbers indicate nodes. Flows Q_1, Q_2 and Q_3 are going into the nodes 1, 2, and 3. Squares with numbers represent elements. In the program for flow going into the node (going into to the pipe network), the values of Q_1, Q_2 or Q_3 shall be positive. For flow coming out of the node (coming out of the pipe network), the values of Q_1, Q_2 or Q_3 shall be negative.

Flows within the pipe are represented by q_1, q_2 and q_3. Flow towards a particular node is taken as positive. Flow away from a node is negative.

It is to be noted that head loss within an element or static head difference across any element(as in case of Boosters) is the difference of the Heads of the two nodes at the end of the particular element. Hence, we may write the following as per the flow direction of the liquid.

$h_1 = H_1 - H_2$ .....(a)

$h_2 = H_2 - H_3$ .....(b)

$h_3 = H_3 - H_1$ .....(c)

Where $H_{1,} H_2$ and H_3 are the Heads at nodes 1, 2, 3 respectively.

As mentioned under heading “Method of calculation for each element”, we take the following form of consideration for each element.

$$q = ch + cH_d$$

where h is the head loss/static head difference

& H_d is the booster head contribution or equal to zero (0), if element is otherwise.

Hence, we may form the following equations for each element.

$$q_1 = c_1(H_1 - H_2) + c_1 H_{d1} \quad .....(1)$$

$$q_2 = c_2(H_2 - H_3) + c_2 H_{d2} \quad .....(2)$$

$$q_3 = c_3(H_3 - H_1) + c_3 H_{d3} \quad .....(3)$$

Above, H_{d2} and H_{d3} are equal to zero(0).
We have the following equations at each node. $Q_{1,} Q_2$ and Q_3 Are considered positive below, for the sake of explanation.

$$Q_1 - q_1 + q_3 = 0$$
$$\rightarrow Q_1 = q_1 - q_3 \quad .....(4)$$

$$-Q_2 - q_2 + q_1 = 0$$
$$\rightarrow -Q_2 = q_2 - q_1 \quad .....(5)$$

$$-Q_3 + q_2 - q_3 = 0$$
$$\rightarrow -Q_3 = q_3 - q_2 \quad .....(6)$$

Substituting Equations (1), (2) & (3) in (4), (5) & (6) as appropriate, we have the following.

$$Q_1 = (c_1 + c_3) H_1 - c_1 H_2 - c_3 H_3 + c_1 H_{d1} - c_3 H_{d3} \quad .....(7)$$

$$-Q_2 = -c_1 H_1 + (c_1 + c_2) H_2 - c_2 H_3 - c_1 H_{d1} + c_2 H_{d2} \quad .....(8)$$

$$-Q_3 = -c_3 H_1 - c_2 H_2 + (c_3 + c_2) H_3 + c_3 H_{d3} - c_2 H_{d2} \quad .....(9)$$

From equations (7), (8) & (9) we can form the following matrix.

$$\begin{bmatrix} Q_1 \\ -Q_2 \\ -Q_3 \end{bmatrix} = \begin{bmatrix} c_1 + c_3 & -c_1 & -c_3 \\ -c_1 & c_1 + c2 & -c2 \\ -c_3 & -c_2 & c_3 + c_2 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} + \begin{bmatrix} c_1 & 0 & -c3 \\ -c_1 & c_2 & 0 \\ 0 & -c_2 & c_3 \end{bmatrix} \begin{bmatrix} H_{d1} \\ H_{d2} \\ H_{d3} \end{bmatrix}$$

Next, $q_{1,} q_2$ and q_3 values are found out as per equations (1), (2), & (3). Values of $h_{1,} h_2$ and h_3 are found out as per equations (a), (b) & (c). Next, as stated under heading “Method of calculation for each element”, we check for values for all of $h_{1,} h_2$ and h_3 . This continues till all of $h_{1,} h_2$ and h_3 are with the allowed approximation range.