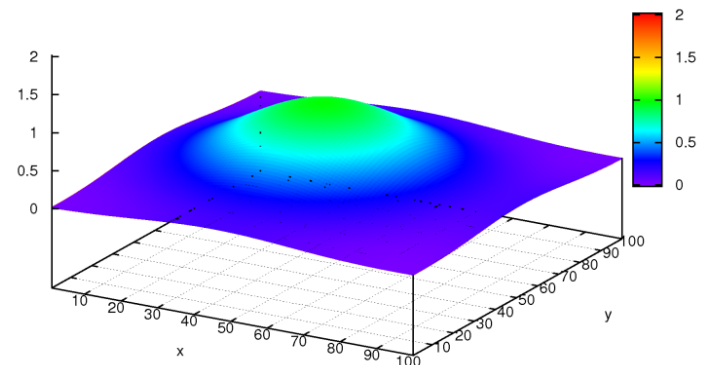
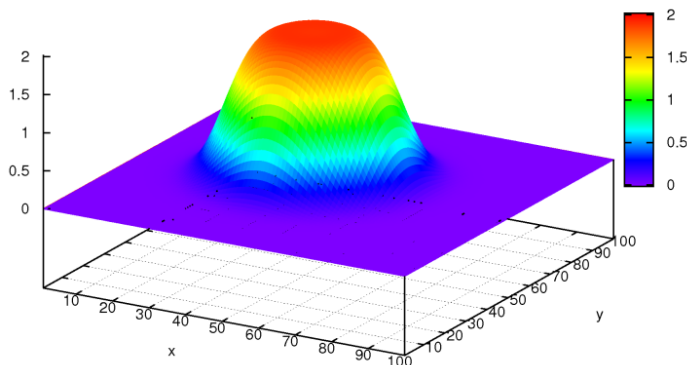


The Heat Conduction Example – Overview

- Heat equation solver as debugging example
- Heat equation describes heat distribution in a region over time
- Equation (2D):
$$\frac{\partial u}{\partial t} = k \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$
- Example implements solution on a 2D grid
- Time step for a grid cell (x,y):

$$\frac{\Delta u[x,y]}{\Delta t} = k \left(\frac{u[x-1,y] + u[x+1,y] - 2u[x,y]}{\Delta x^2} + \frac{u[x,y-1] + u[x,y+1] - 2u[x,y]}{\Delta y^2} \right)$$

- Visualization as 3D chart:

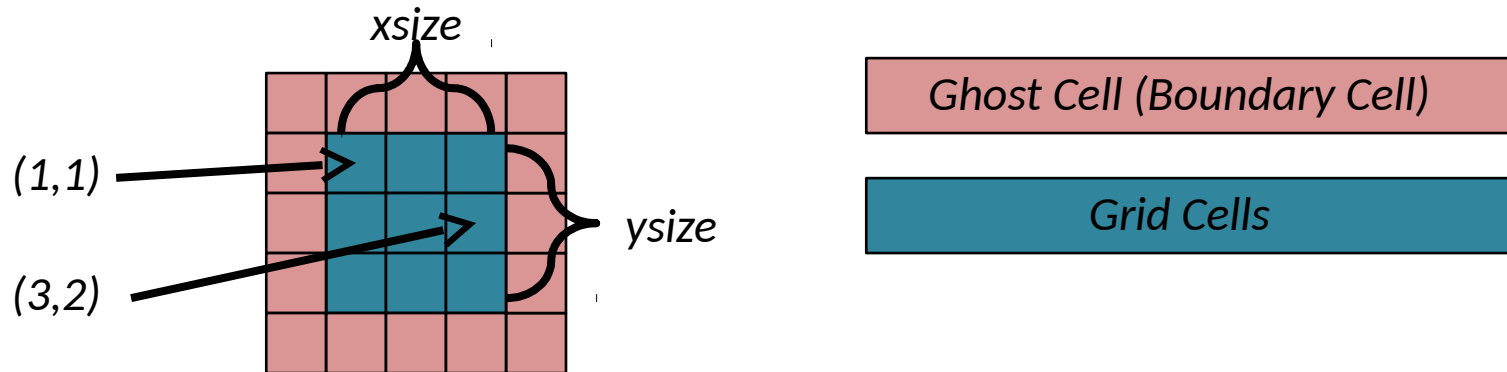


The Heat Conduction Example – Source Code

- Available in C + Fortran90
- Parallel versions with MPI + OpenMP
- Key object is the structure for the 2D grid
- Functions:
 - heatAllocate & heatDeallocate – Creates/Frees the grid
 - heatInitialize & heatInitFunc – Sets the initial heat distribution
 - heatPrint – Prints the grid to stdout
 - heatTimestep – Calculates one timestep for the full grid
 - heatBoundary – Exchanges boundary data
 - heatTotalEnergy – Calculates overall energy amount
 - Main function – Contains main loop

The Heat Conduction Example – 2D Grid

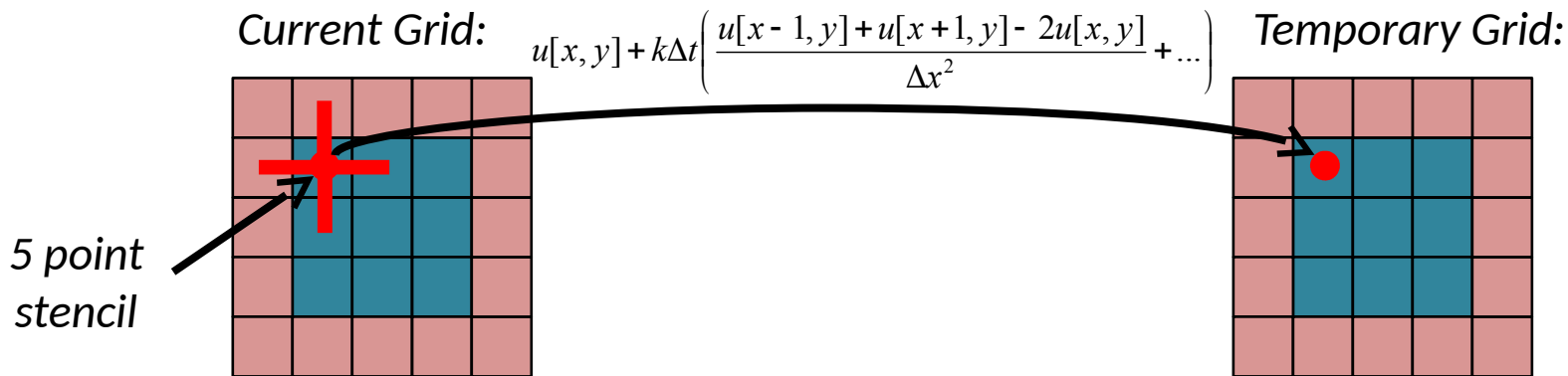
- Grid Structure contains 2 grids and the grid size
- Grid:



- Ghost cells used as neighbors, needed for border cells of actual grid

The Heat Conduction Example – Time Steps

- Time steps are calculated for all actual grid cells, results stored in second grid:



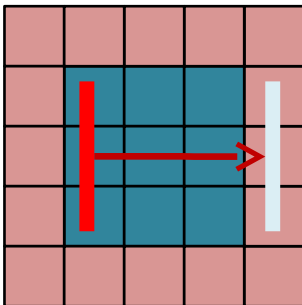
- Application switches grid after calculation for all grid cells

The Heat Conduction Example – Boundary

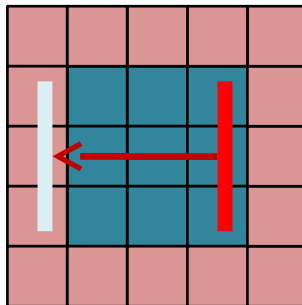
- 5 point stencil needs left, right, up, down neighbor for each grid point
- Ghost cells serve as neighbors
- Code uses periodic ghost cells
- After time step: copy operation to update ghost cells
(=> “heatBoundary”)
- Update:

Grid:

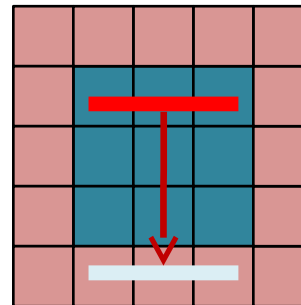
(1) Left border



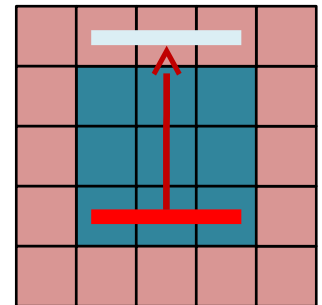
(2) Right border



(3) Top border



(4) Bottom border



The Heat Conduction Example – Main Function

- Starts initialization
- Iterates time steps and boundary exchange (heatTimestep, heatBoundary)
- Uses 20 iterations
- Prints initial and final grid to standard output
- Calculates total energy at beginning and end (simple basic verification)

The Heat Conduction Example – OpenMP Version

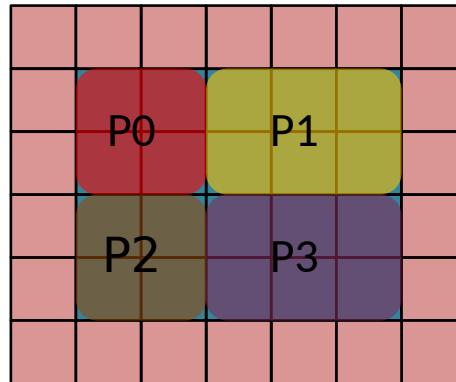
- Parallel for loop around the timestep function:

```
void heatTimestep(heatGrid* grid, double dt, double* dthetamax)
{
    ...
    #pragma omp parallel private(dtheta, x, y)
    {
        #pragma omp for
        for (x=1; x <= grid->xsize; x++)
        {
            for (y=1; y <= grid->ysize; y++)
            {
                dtheta = ...;
                grid->thetaneu[x][y] = ...;
            }
        }
    }
}
```

- ▲ Each thread covers part of the 2D grid

The Heat Conduction Example – MPI Version

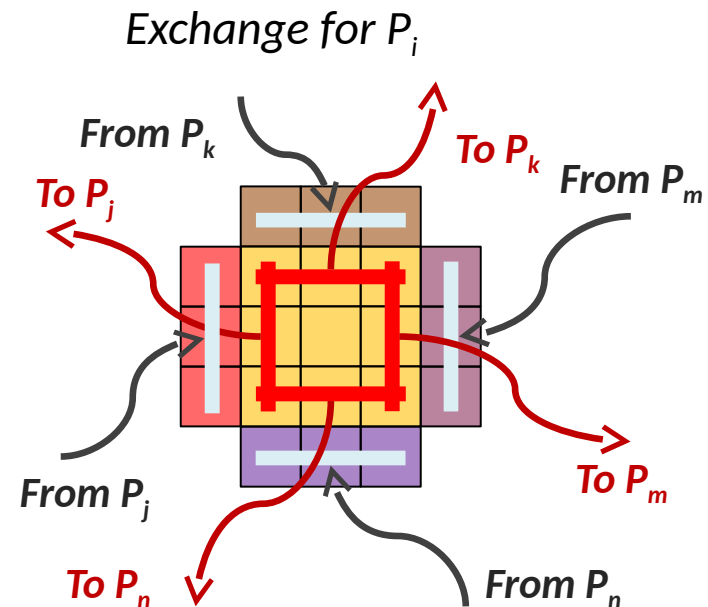
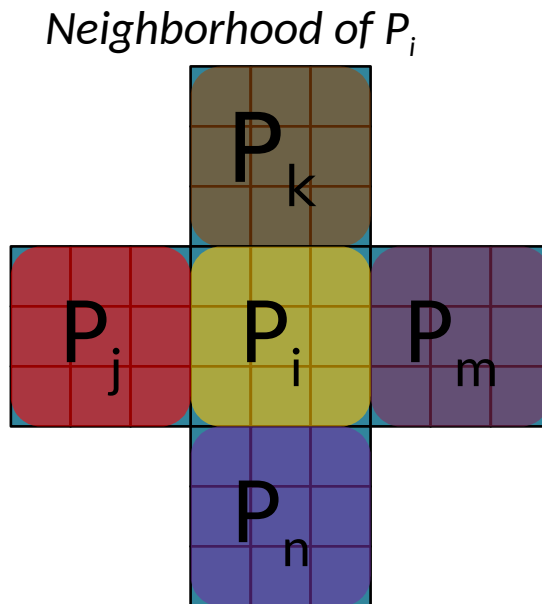
- Each process has a copy of both grids
- Initialization and set-up done by all processes
- Each process calculates on a part of the overall grid:



- Exchange with neighbors uses 2D Cartesian communicator
- Communications of a data columns uses a derived datatype
- Consequences:
 - New border exchange
 - Gathering of data necessary for output

The Heat Conduction Example – MPI Boundary

- MPI processes exchange borders with neighbors
- Cartesian grid provides up, down, left, right neighbors
- Periodic boundaries => grid is a donut
- Boundary exchange for process " P_i ":



The Heat Conduction Example – MPI Gather

- Data distributed between processes
- Printing requires I/O or gathering data on one process
- Implementation gathers data on process 0:

