Beyond the Kalman Filter: Particle filters for tracking applications

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Contents

What is tracking?

- Use models of the real world to
  - estimate the past and present
  - predict the future
- Achieved by extracting underlying information from sequence of noisy/uncertain observations
- Perform inference on-line
- Evaluate evolving sequence of probability distributions

Recursive filter

System model

$$x_t = f_t(x_{t-1}, \epsilon_t) \quad \leftrightarrow \quad p\,(x_t \,|\, x_{t-1})$$

Measurement model

$$y_t = h_t(x_t, \nu_t) \quad \leftrightarrow \quad p\,(y_t \,|\, x_t)$$

Information available

$$y_{1:t} = (y_1, \ldots, y_t)$$

$$p(x_0)$$

Want

$$p\,(x_{0:t+i} \,|\, y_{1:t})$$

and especially

$$p\,(x_t \,|\, y_{1:t})$$

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Recursive filter

Prediction

$$p\left(x_t \mid y_{1:t-1}\right) = \int p\left(x_t \mid x_{t-1}\right) p\left(x_{t-1} \mid y_{1:t-1}\right) dx_{t-1}$$

Update

$$p\left(x_t \mid y_{1:t}\right) = \frac{p\left(y_t \mid x_t\right) p\left(x_t \mid y_{1:t-1}\right)}{p\left(y_t \mid y_{1:t-1}\right)}$$

$$p\left(y_t \mid y_{1:t-1}\right) = \int p\left(y_t \mid x_t\right) p\left(x_t \mid y_{1:t-1}\right) dx_t$$

Alternatively ...

$$p\left(x_{0:t} \mid y_{1:t}\right) = p\left(x_{0:t-1} \mid y_{1:t-1}\right) \frac{p\left(y_t \mid x_t\right) p\left(x_t \mid x_{t-1}\right)}{p\left(y_t \mid y_{1:t-1}\right)}$$

$$p\left(x_t \mid y_{1:t}\right) = \int p\left(x_{0:t} \mid y_{1:t}\right) dx_{0:t-1}$$

Tracking : what are the problems?

- On-line processing
- Target manoeuvres
- Missing measurements
- Spurious measurements
- Multiple objects and/or sensors
- Finite sensor resolution
- Prior constraints
- Signature information

Nonlinear/non-Gaussian models

Analytic approximations

- EKF and variants : linearisation, Gaussian approx, unimodal
- Score function moment approximation : Masreliez (75), West (81), Fahrmeir (92), Pericchi, Smith (92)
- Series based approximation to score functions : Wu, Cheng (92)

Numerical approximations

- Discrete grid: Pole, West (88)
- Piecewise pdf: Kitagawa (87), Kramer, Sorenson (88)
- Series expansion: Sorenson, Stubberud (68)
- Gaussian mixtures: Sorenson, Alspach (72), West (92)
- Unscented filter: Julier, Uhlman (95)

Monte Carlo Approximations

- Sequential Importance Sampling (SIS): Handschin & Mayne, *Automatica*, 1969.
- Improved SIS: Zaritskii *et al.*, *Automation and Remote Control*, 1975.
- Rao-Blackwellisation: Akashi & Kumamoto, *Automatica*, 1977...

$\Rightarrow$ Too computationally demanding 20-30 years ago

Sequential Monte Carlo (SMC)

- SMC methods lead to estimate of the complete probability distribution
- Approximation centred on the pdf rather than compromising the state space model
- Known as Particle filters, SIR filters, bootstrap filters, Monte Carlo filters, Condensation etc

Why random samples?

- nonlinear/non-Gaussian
- whole pdf
- moments/quantiles
- HPD interval
- re-parameterisation
- constraints
- association hypotheses independent over time
- multiple models trivial
- scalable - how big is ∞
- parallelisable

Comparison

Kalman filter

- analytic solution
- restrictive assumptions
- deduce state from measurement
- KF “optimal”
- EKF “sub-optimal”

Particle filter

- sequential MC solution
- based on simulation
- no modelling restrictions
- predicts measurements from states
- optimal (with ∞ computational resources)

Book Advert

Sequential Monte Carlo methods in practice

Editors: Doucet, de Freitas, Gordon

Springer-Verlag (2001)

- Theoretical Foundations
- Efficiency Measures
- Applications :
  - Target tracking, missile guidance, image tracking, terrain referenced navigation, exchange rate prediction, portfolio allocation, in-situ ellipsometry, pollution monitoring, communications and audio engineering.

Useful Information

Books

- "Sequential Monte Carlo methods in practice", Doucet, de Freitas, Gordon, Springer, 2001.
- "Monte Carlo strategies in scientific computing", Liu, Springer, 2001.
- "Beyond the Kalman filter : Tracking applications of particle filters", Ristic, Arulampalam, Gordon, Artech House, 2003?

Papers

- "On sequential Monte Carlo sampling methods for Bayesian filtering", Statistics in Computing, Vol 10, No 3, pgs 197-208, 2000.
- IEEE Trans. Signal Processing special issue, February 2002.

Web site

- www.cs.ubc.ca/ nando/smc/index.html (includes software)

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Particle Filter

- Represent uncertainty over $x_{1:t}$ using diversity of weighted particles

$$\left\{x^i_{1:t}, w^i_t\right\}_{i=1}^N$$

- Ideally:

$$x^i_{1:t} \sim p(x_{1:t} \mid y_{1:t}) = \frac{p(x_{1:t}, y_{1:t})}{p(y_{1:t})}$$

where

$$p(y_{1:t}) = \int p(x_{1:t}, y_{1:t}) dx_{1:t}$$

- What if we can't sample $p(x_{1:t} \mid y_{1:t})$?

Particle Filter - Importance Sampling

- Sample from a convenient proposal distribution $q(x_{1:t} \mid y_{1:t})$
- Use importance sampling to modify weights

$$\begin{aligned}\int p(x_{1:t} \mid y_{1:t}) f(x_{1:t}) dx_{1:t} &= \int \frac{p(x_{1:t} \mid y_{1:t})}{q(x_{1:t} \mid y_{1:t})} q(x_{1:t} \mid y_{1:t}) f(x_{1:t}) dx_{1:t} \\ &\approx \sum_{i=1}^{N} w_t^i f(x_{1:t}^i)\end{aligned}$$

where

$$\begin{aligned} x_{1:t}^i &\sim q(x_{1:t} \mid y_{1:t}) \\ w_t^i &= \frac{p(x_{1:t} \mid y_{1:t})}{q(x_{1:t} \mid y_{1:t})}\end{aligned}$$

Particle Filter - Importance Sampling

- Pick a convenient proposal
- Define the un-normalised weight:

$$\tilde{w}_t^i = \frac{p(x_{1:t}, y_{1:t})}{q(x_{1:t} \mid y_{1:t})}$$

- Can then calculate approximation to $p(y_{1:t})$

$$p(y_{1:t}) \approx \sum_{i=1}^{N} \tilde{w}_t^i$$

- Normalised weight is

$$w_t^i = \frac{p(x_{1:t} \mid y_{1:t})}{q(x_{1:t} \mid y_{1:t})} = \frac{p(x_{1:t}, y_{1:t})}{q(x_{1:t} \mid y_{1:t})} \frac{1}{p(y_{1:t})} = \frac{\tilde{w}_t^i}{\sum_{i=1}^{N} \tilde{w}_t^i}$$

Particle Filter - SIS

- To perform **Sequential Importance Sampling**, SIS

$$q(x_{1:t} \mid y_{1:t}) \triangleq \underbrace{q(x_{1:t-1} \mid y_{1:t-1})}_{\text{Keep existing path}} \underbrace{q(x_t \mid x_{t-1}, y_t)}_{\text{Extend path}}$$

- The un-normalised weight then takes the appealing form

$$\begin{aligned}
\tilde{w}_t^i &= \frac{p(x_{1:t}^i, y_t \mid y_{1:t-1})}{q(x_{1:t}^i \mid y_{1:t})} \\
&= \frac{p(x_{1:t-1}^i \mid y_{1:t-1})}{q(x_{1:t-1}^i \mid y_{1:t-1})} \frac{p(x_t^i, y_t \mid x_{1:t-1}^i)}{q(x_t^i \mid x_{t-1}^i, y_t)} \\
&= w_{t-1}^i \underbrace{\frac{p(y_t \mid x_{t-1}^i) p(x_t^i \mid x_{t-1}^i)}{q(x_t^i \mid x_{t-1}^i, y_t)}}_{\text{Incremental weight}}
\end{aligned}$$

Illustration of SIS

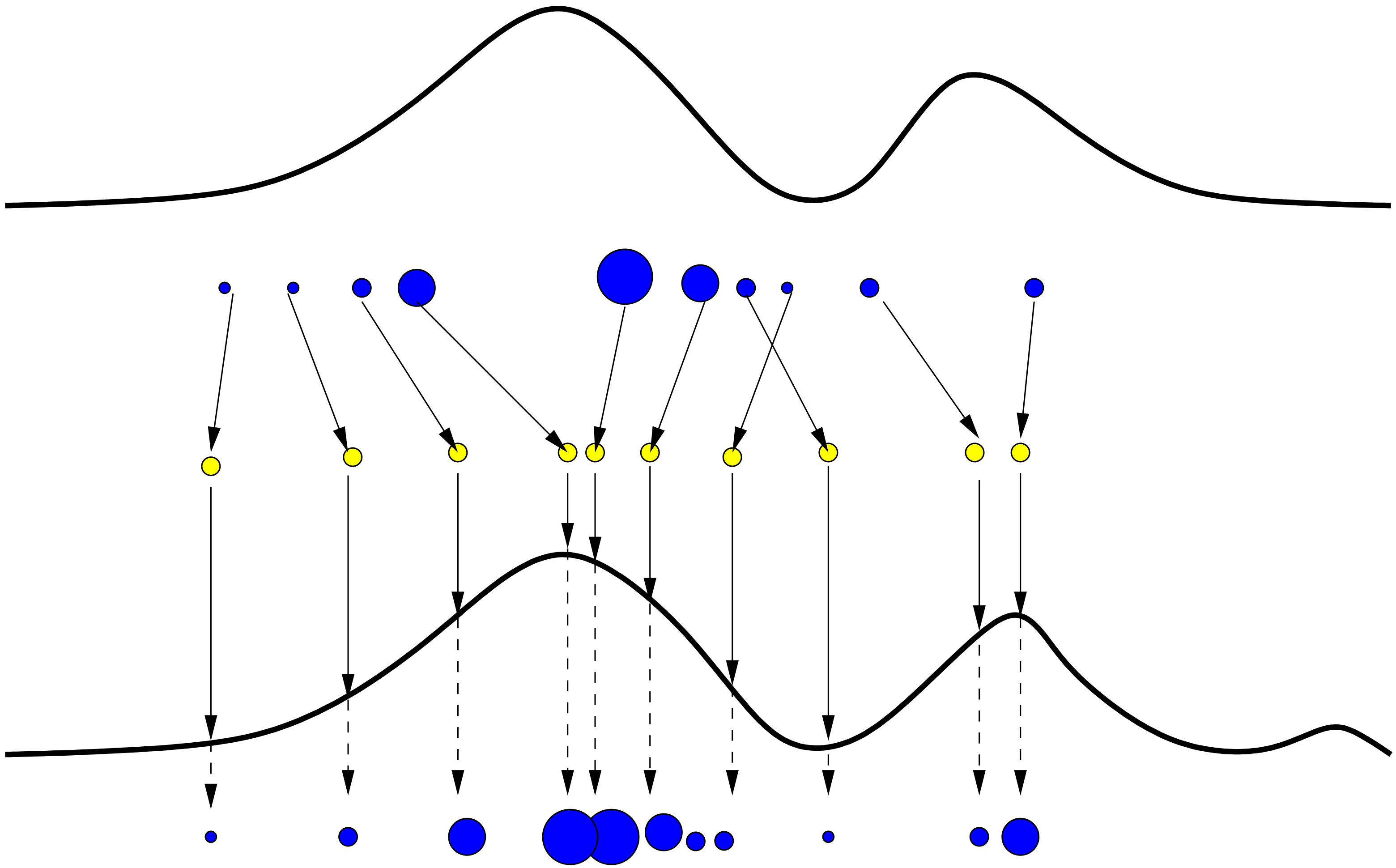
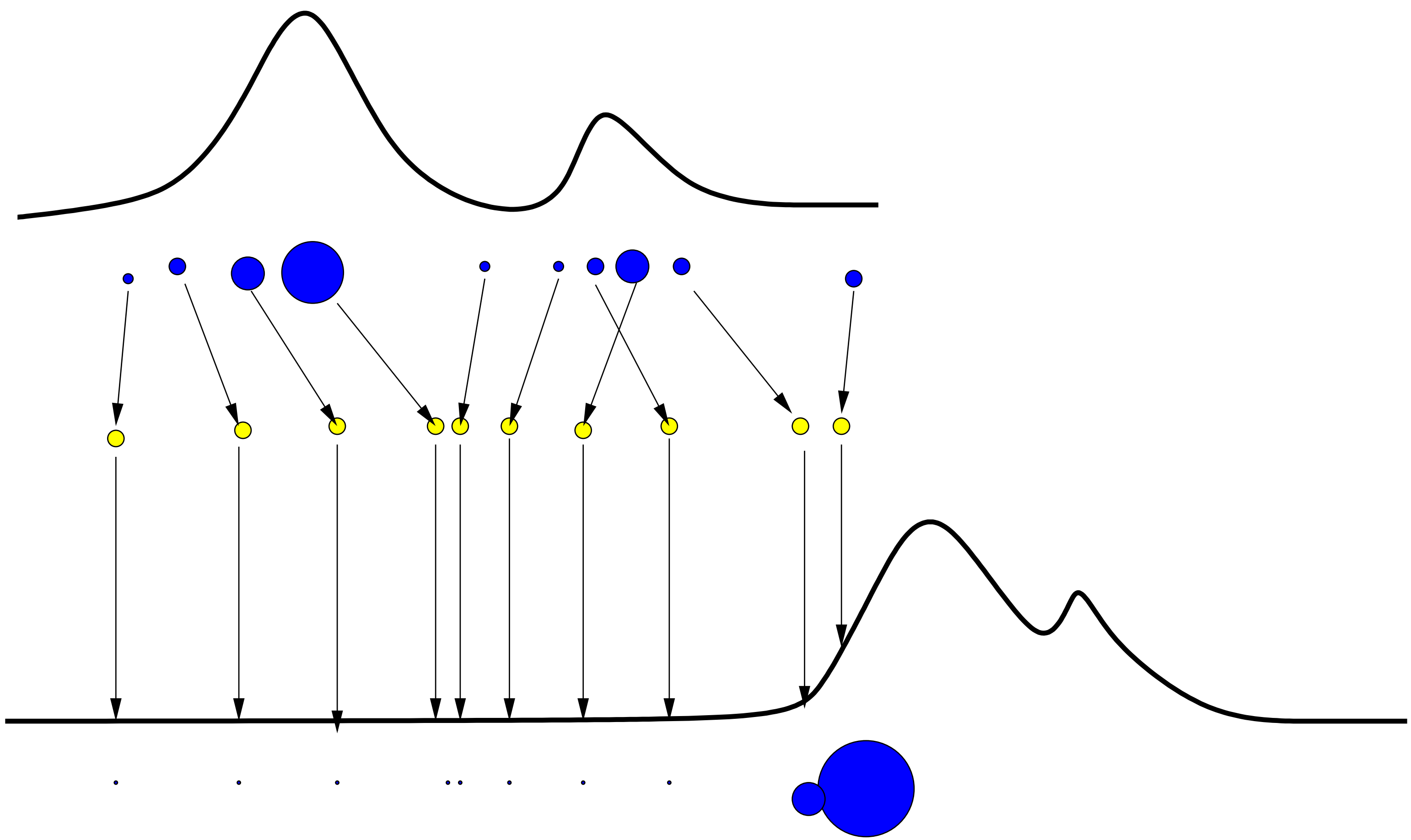


Illustration of SIS - data conflict



SIS

Problem : Whatever the importance function, degeneracy is observed (Kong, Liu and Wong 1994).

- **Introduce a selection scheme to discard/multiply particles $x^i_{0:t}$ with respectively high/low importance weights**
- **Resampling maps the weighted random measure $(x^i_{0:t}, w_t)$ onto the equally weighted random measure $(x^i_{0:t}, N^{-1})$**
- **Scheme generates N_i children such that $\sum_{i=1}^{N} N_i = N$ and satisfies $E(N_i) = Nw^i_k$**

Illustration of SIR

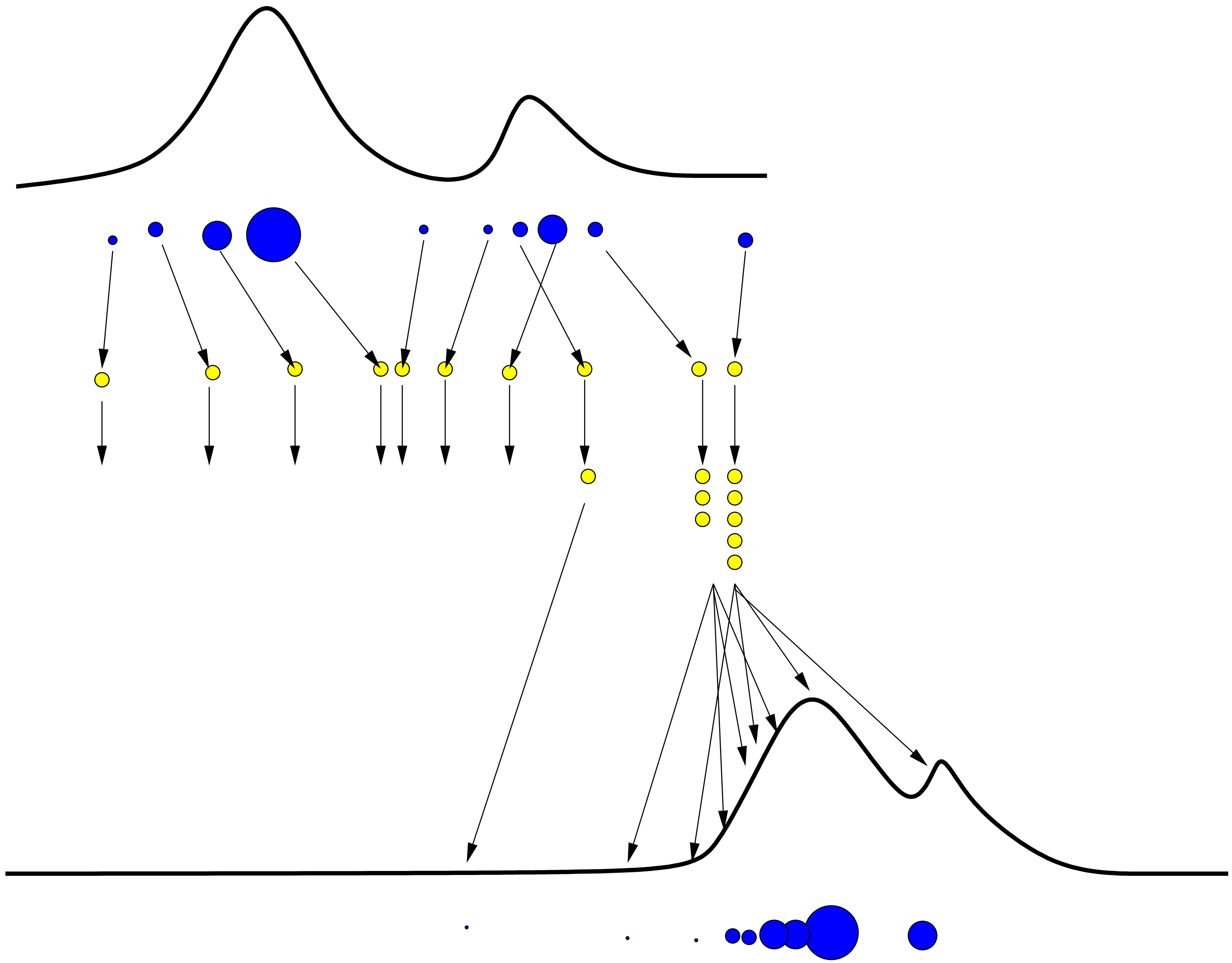
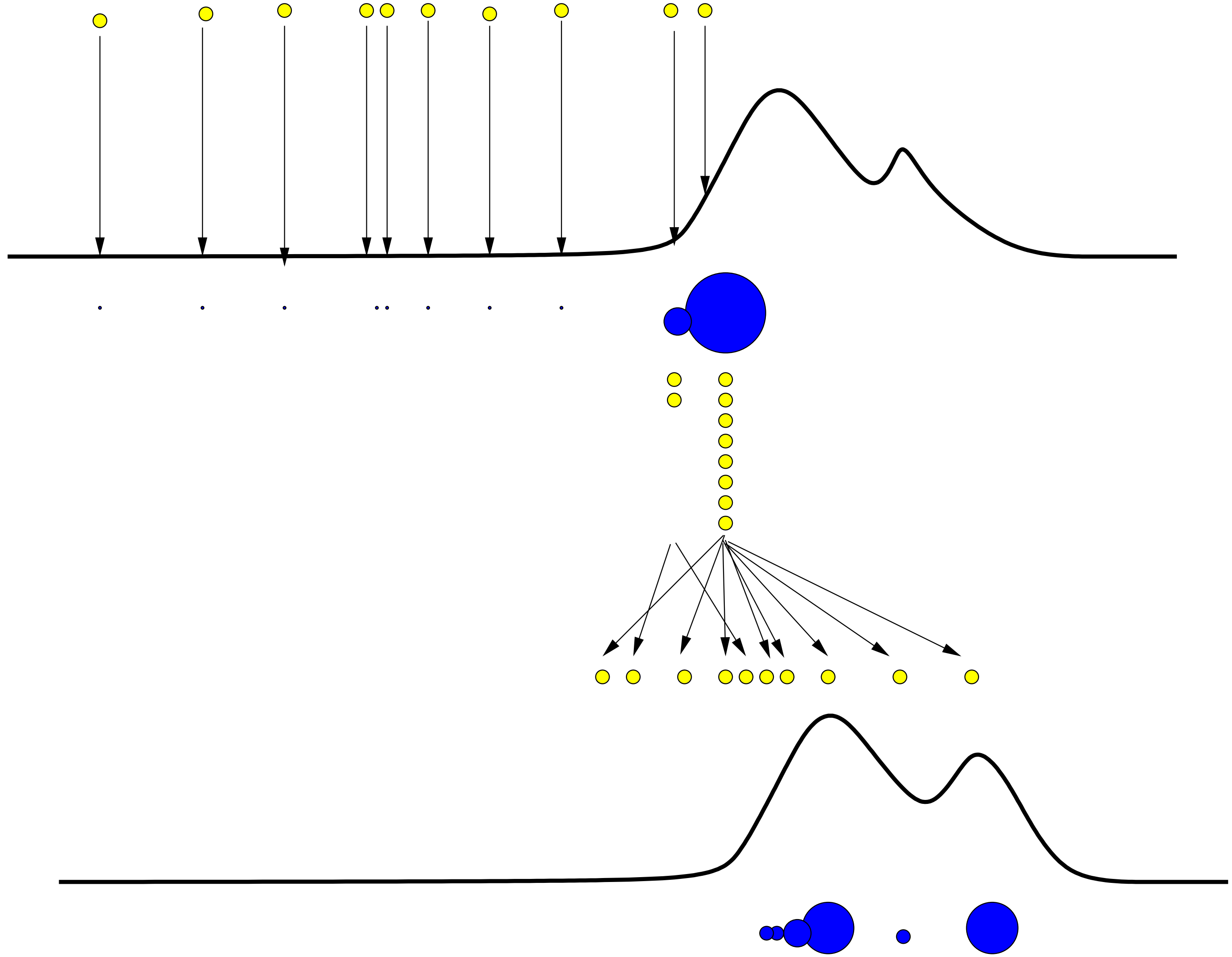


Illustration of SIR



Ingredients for Particle filter

- Importance sampling function
  - Prior $p(x_t \mid x_{t-1}^{(i)})$
  - Optimal $p(x_t \mid x_{t-1}^{(i)}, y_t)$
  - UKF, linearised EKF, ...
- Redistribution scheme
  - Multinomial
  - Deterministic
  - Residual
  - Stratified
- Careful initialisation procedure (for efficiency)

Improvements to SIR

To alleviate degeneracy problems many other methods have been proposed

- Local linearisation (Doucet, 1998; Pitt & Shephard, 1999) using the EKF to estimate the importance distribution or UKF (Doucet et al, 1999)
- Rejection methods (Müller, 1991; Hürzeler & Künsch, 1998; Doucet, 1998; Pitt & Shephard, 1999)
- Auxiliary particle filters (Pitt & Shephard, 1999)
- Kernel smoothing (Gordon, 1993; Liu & West, 2000; Musso et al, 2000)
- MCMC methods (Müller, 1992; Gordon & Whitby, 1995; Berzuini et al, 1997; Gilks & Berzuini, 1999; Andrieu et al, 1999)
- Bridging densities : (Clapp & Godsill, 1999)

Auxiliary SIR - ASIR

- Introduced by Pitt and Shephard 1999.
- Use importance sampling function $q(x_t, i \mid y_{1:t})$
- Auxiliary variable i refers to index of particle at time $t-1$
- Importance distribution chosen to satisfy

$$q(x_t, i \mid y_{1:t}) \propto p(y_t \mid \mu_t^i) p(x_t \mid x_{t-1}^i) w_{t-1}^i$$

  - μ_t^i is some characterisation of x_t given x_{t-1}^i
  - eg, $\mu_t^i = \mathbb{E}(x_t \mid x_{t-1}^i)$ or $\mu_t^i \sim p(x_t \mid x_{t-1}^i)$
- This gives

$$w_t^j \propto w_{t-1}^{i^j} \frac{p(y_t \mid x_t^j) p(x_t^j \mid x_{t-1}^{i^j})}{q(x_t^j, i^j \mid y_{1:t})} = \frac{p(y_t \mid x_t^j)}{p(y_t \mid \mu_t^{i^j})}$$

ASIR

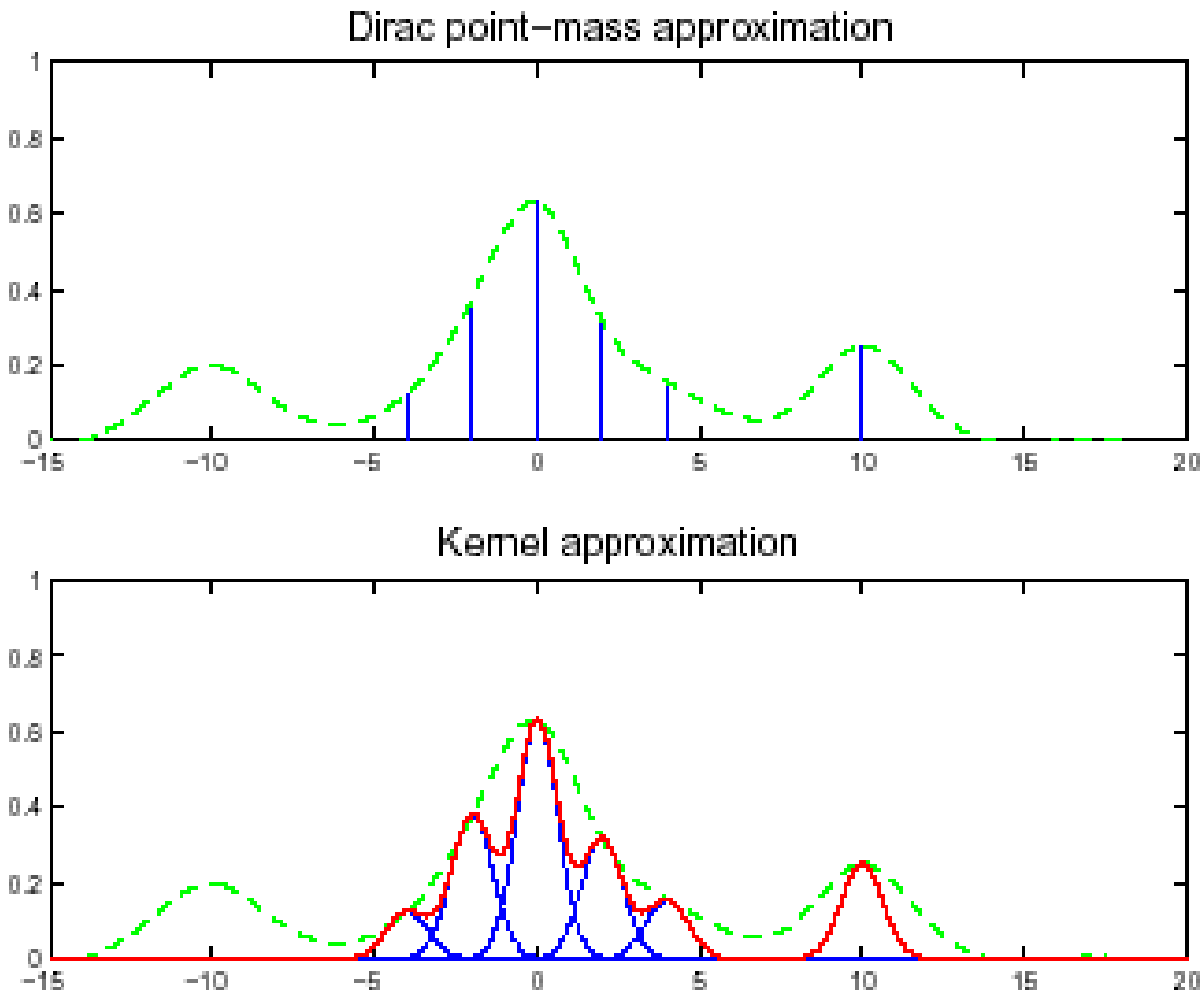
- Naturally uses points at $t-1$ which are "close" to measurement y_t
- If process noise is small then ASIR less sensitive to outliers than SIR
  - This is because single point μ_t^i characterises $p(x_t \mid x_{t-1})$ well
- But if process noise is large then ASIR can degrade performance
  - Since a single point μ_t^i does not characterise $p(x_t \mid x_{t-1})$

Regularised PF - RPF

- Resampling introduced to reduce degeneracy
- But, also reduces diversity
- RPF proposed as a solution
- Uses continuous Kernel based approximation

$$\hat{p}(x_{1:t} \mid y_{1:t}) \approx \sum_{i=1}^{N} w_t^i K_h \left(x_t - x_t^i \right)$$

RPF


Dirac point−mass approximation
Kernel approximation
0
0.2
0.4
1
−15
−10
−5
0
5
10
15
20

RPF

- Kernel K(.) and bandwidth h chosen to minimise MISE

$$MISE(\hat{p}) = \mathbb{E}\left[\int \{\hat{p}(x_t \mid y_{1:t}) - p(x_t \mid y_{1:t})\}^2 \, dx_t\right]$$

- For equally weighted samples, optimal choice is Epanechnikov kernel

$$K_{opt} = \begin{cases} \frac{n_x+2}{2c_{n_x}}(1 - \parallel x \parallel^2) & \textbf{if} \parallel x \parallel < 1 \\ 0 & \textbf{otherwise} \end{cases}$$

- Optimal bandwidth can be obtained as a function of underlying pdf
- Assume this is Gaussian with unit covariance matrix

$$h_{opt} = \left[8N(n_x+4)(2\sqrt{\pi})^{n_x} c_{n_x}^{-1}\right]^{1/(n_x+4)}$$

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MCMC Moves

- RPF moves are blind
- Instead, introduce Metropolis style acceptance step
  - Resampled x_k^R and created $x_{1:k}^R = \left\{ x_k^R, x_{1:k-1}^R \right\}$
  - Resampled x_k^R and then sampled from a proposal distribution $x_k^P \sim q(. \mid x_k^R)$ and created $x_{1:k}^P = \left\{ x_k^P, x_{1:k-1}^R \right\}$
- Assume $q(. \mid .)$ symmetric

$$x_{1:k} = \begin{cases} x_{1:k}^P & \text{with probability } \alpha \\ x_{1:k}^R & \text{otherwise} \end{cases}$$

$$\begin{aligned} \alpha &= \min\left(1, \frac{p(x_{1:k}^P \mid y_{1:k}) q(x_k^R \mid x_k^P))}{p(x_{1:k}^R \mid y_{1:k}) q(x_k^P \mid x_k^R)}\right) \\ &= \min\left(1, \frac{p(y_k \mid x_k^P) p(x_k^P \mid x_{k-1}^R)}{p(y_k \mid x_k^R) p(x_k^R \mid x_{k-1}^R)}\right) \end{aligned}$$

Tracking dim targets

- Detection and tracking of low SNR targets - better not to threshold the sensor data !
- The concept: track-before-detect
- Conventional TBD approaches:
  - Hough transform (Carlson et al, 1994)
  - dynamic programming (Barniv, 1990; Arnold et al, 1993)
  - maximum likelihood (Tonissen, 1994)
- The performance improved by 3-5 dB in comparison to the MHT (thresholded data).

Recursive Bayesian TBD

- Drawbacks of conventional TBD approaches: batch processing; prohibit or penalise deviations from the straight line motion; require enormous computational resources.
- A recursive Bayesian TBD (Salmond, 2001), implemented as a particle filter
  - no need to store/process multiple scans
  - target motion - stochastic dynamic equation
  - valid for non-gaussian and structured background noise
  - the effect of point spread function, finite resolution, unknown and fluctuating SNR are accommodated
  - target presence and absence explicitly modelled
- Run Demo

Mathematical formulation

- Target state vector

$$\mathbf{x}_k = [x_k \quad \dot{x}_k \quad y_k \quad \dot{y}_k \quad I_k]^T.$$

- State dynamics:

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{v}_k),$$

- Target existence - two state Markov chain, $E_k \in \{0, 1\}$ with TPM

$$\Pi = \begin{bmatrix} 1 - P_b & P_b \\ P_d & 1 - P_d \end{bmatrix}.$$

Mathematical formulation (Cont'd)

- **Sensor model: 2D map, image of a region $\Delta_x \times \Delta_y$.**
- **At each resolution cell (i,j) measured intensity:**

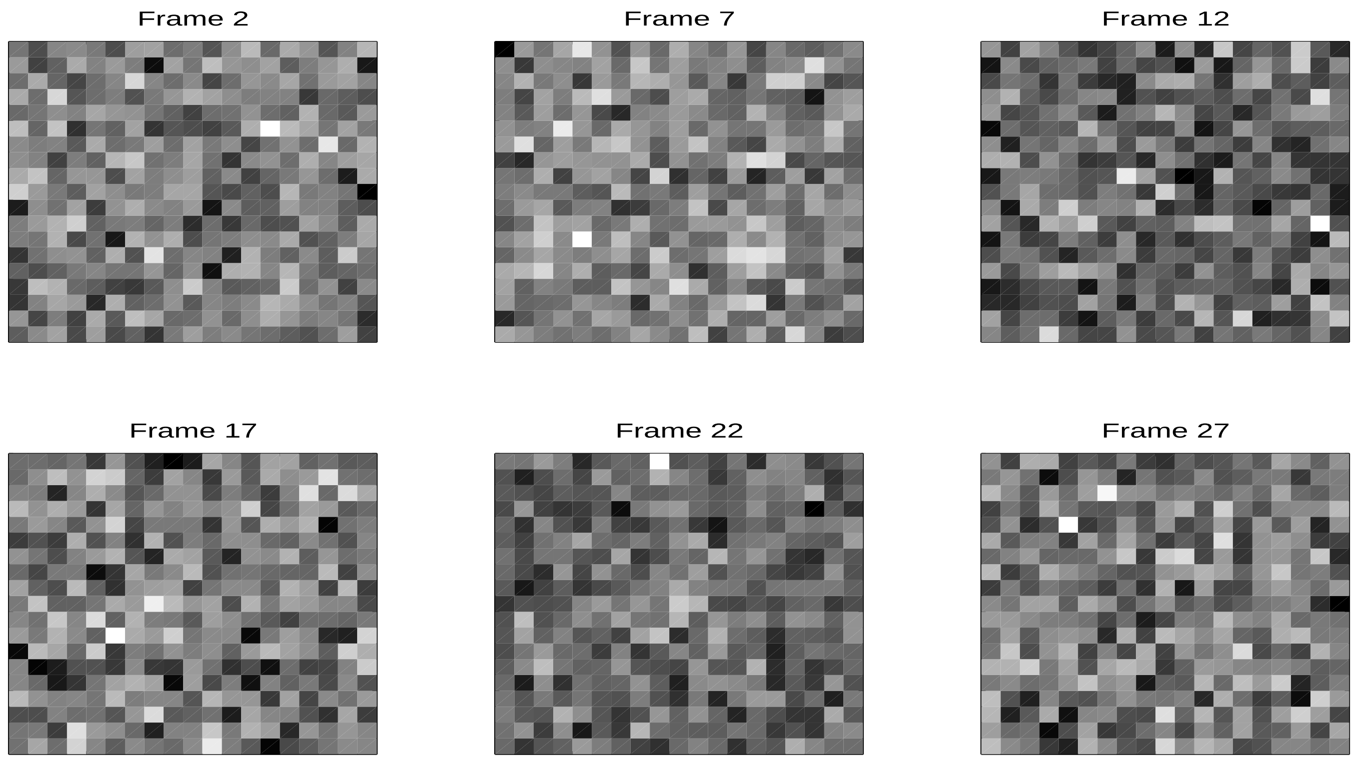
$$z_k^{(i,j)} = \begin{cases} h_k^{(i,j)}(\mathbf{x}_k) + w_k^{(i,j)} & \textbf{if target present} \\ \\ w_k^{(i,j)} & \textbf{if target absent} \end{cases}$$

where

$$h_k^{(i,j)}(\mathbf{x}_k) \approx \frac{\Delta_x \Delta_y I_k}{2\pi\Sigma^2} \exp\left\{ -\frac{(i\Delta_x - x_k)^2 + (j\Delta_y - y_k)^2}{2\Sigma^2} \right\}$$

Example

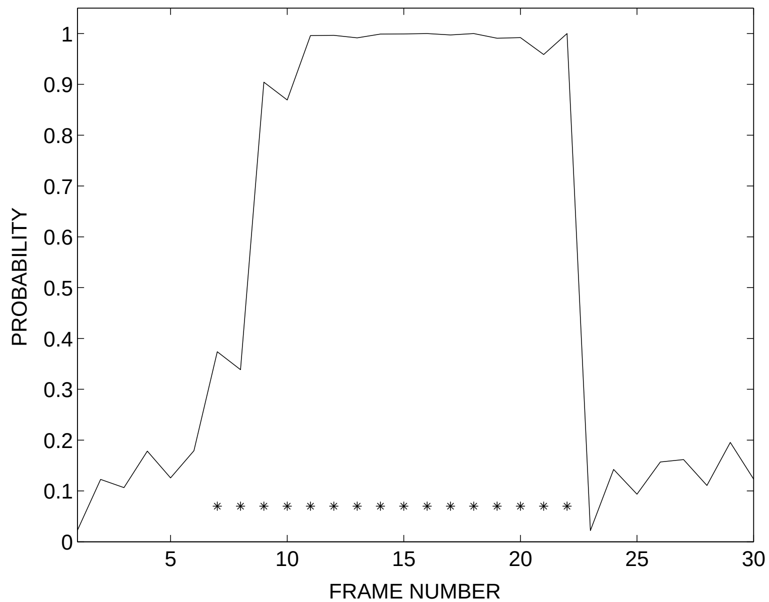
- 30 frames and target present in 7 to 22
- SNR = 6.7 dB (unknown)
- 20×20 cells

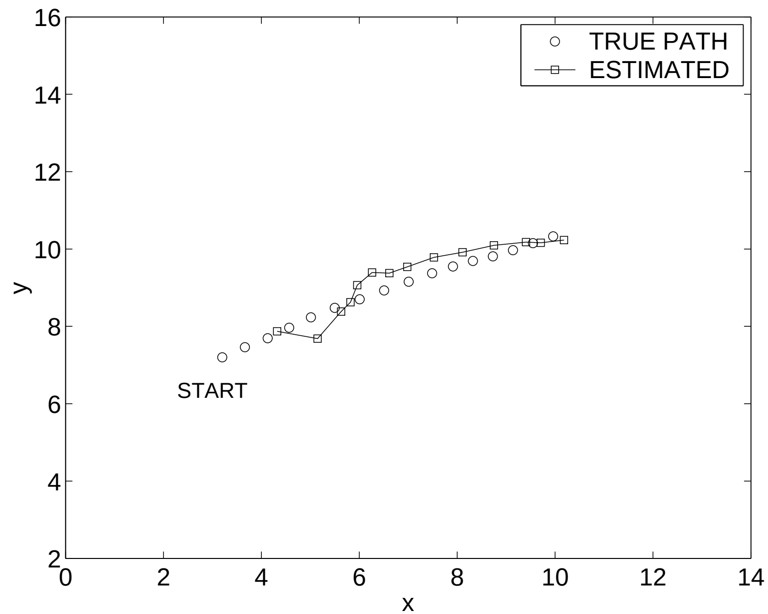


Particle filter output (6 states)

Figures suppressed to reduce file size.

Particle filter output (6 states)





PF based TBD - Performance

- Derived CRLBs (as a function of SNR)
- Compared PF-TBD to CRLBs
- Detection and Tracking reliable at 5 dB (or higher)

Sonobuoy and Submarine

- Noisy bearing measurements from drifting sensors
- Uncertainty in sensor locations
- Sensor loss
- High proportion of spurious bearing measurements
- Run demo

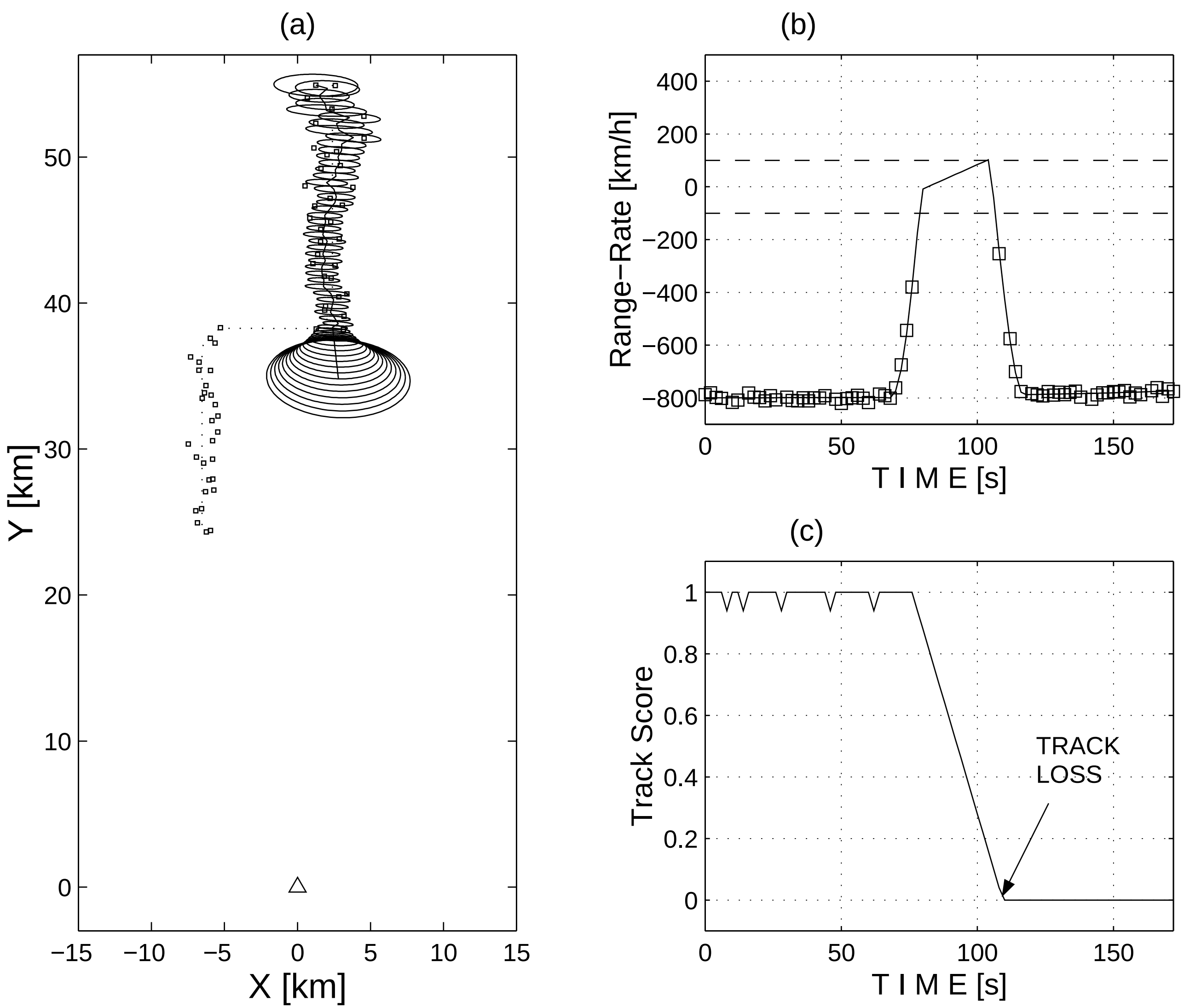
Blind Doppler

- Blind Doppler zone to to filter out ground clutter + ground moving targets
- A simple EP measure against any CW or pulse Doppler radar
- Causes track loss
- Aided by on-board RWR or ESM

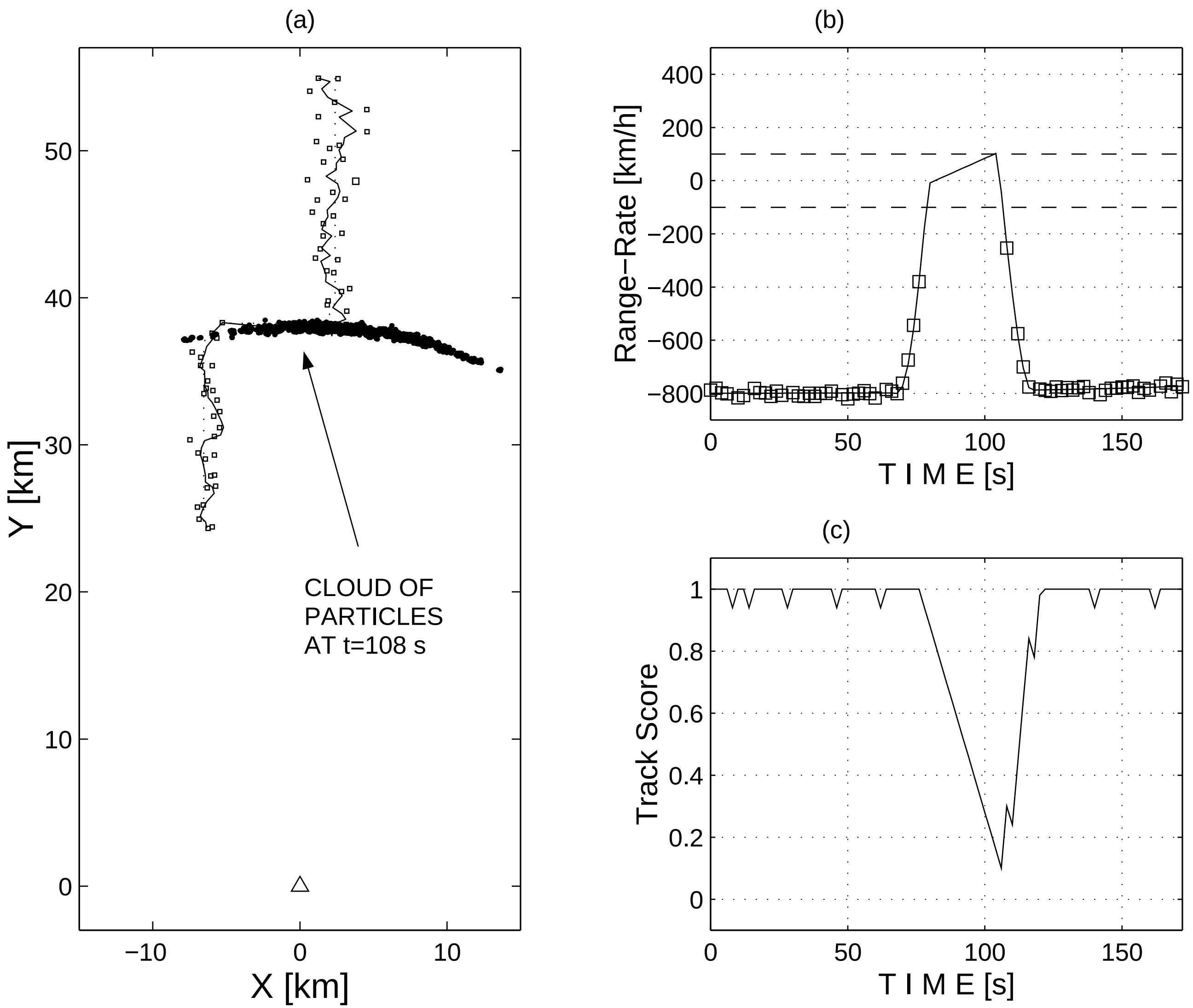
Tracking with hard constraints

- Prior information on sensor constraints
- Posterior pdf is truncated (non-Gaussian)
- Example :
  - 2-D tracking with CV model
  - $(r, \theta, \dot{r})$ measurements
  - $p_d < 1$
  - EKF and Particle Filter with identical gating and track scoring

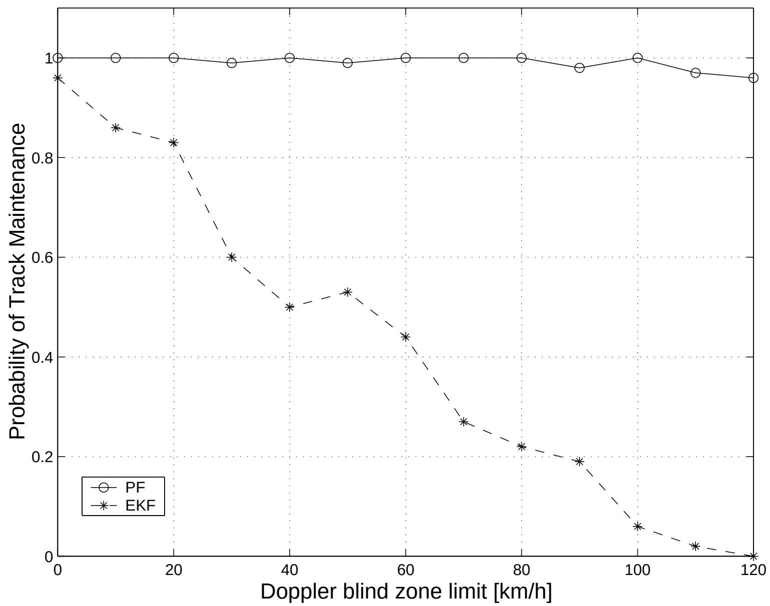
EKF


(a)
Y [km]
X [km]
(b)
Range−Rate [km/h]
T I M E [s]
(c)
Track Score
TRACK
LOSS
T I M E [s]

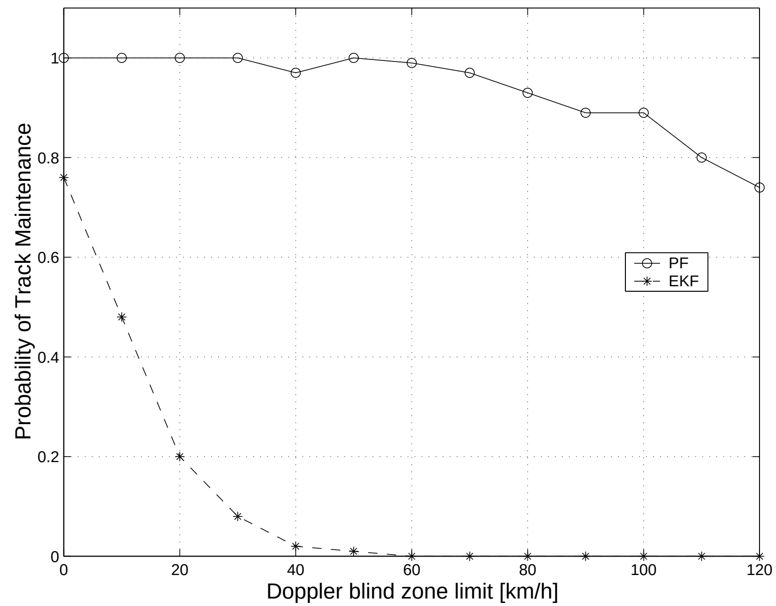
Particle Filter



Track continuity



T=2s



T=4s

Problems hindering SMC

- Convergence results
  - Becoming available (Del Moral, Crisan, Chopin, Lyons, Doucet ...)
- Communication bandwidth
  - Large particle sets impossible to send
- Interoperability
  - Need to integrate with varied tracking algorithms
- Computation
  - Expensive so look to minimise Monte Carlo
- Multi-target problems
  - Rao-Blackwellisation

Final comments

- Sequential Monte Carlo methods
  - “Optimal” filtering for nonlinear/non Gaussian models
  - Flexible/Parallelizable
  - Not a black-box: efficiency depends on careful design.
- If the Kalman filter is appropriate for your application use it
- A Kalman filter is a (one) particle filter