

Finite Element Analysis of Flat Slab

Using numerical formulation of Bogner-Fox-Schmit (BFS) plate element

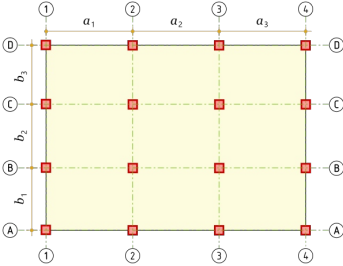
Input data

Span lengths

$r = \mathbf{hp}([3.6; 4.2; 4.2; 3.6]) = [3.6 \ 4.2 \ 4.2 \ 3.6] \text{ m}$

$r = \mathbf{hp}([3; 3.6; 3]) = [3 \ 3.6 \ 3] \text{ m}$

Number of axes - $n_{sa} = \text{len}(r) + 1 = 5$, $n_{sb} = \text{len}(r) + 1 = 4$



Axis coordinates - $\alpha_s = [0 \ 3.6 \ 7.8 \ 12 \ 15.6] \text{ m}$, $\beta_s = [0 \ 3 \ 6.6 \ 9.6] \text{ m}$

Slab dimensions - $I_a \Rightarrow x_{s,5} = 15.6 \text{ m}$, $I_b \Rightarrow y_{s,4} = 9.6 \text{ m}$

Thickness - $t = 0.2 \text{ m}$

Load - $q = 10 \text{ kN/m}^2$

Modulus of elasticity - $E = 35000 \text{ MPa}$

Poisson's ratio - $\nu = 0.2$

Finite element mesh

We will use Bogner-Fox-Schmit rectangular finite element with $n_{DOF5} = 16$

Element dimensions - $a_1 = 0.6 \text{ m}$, $b_1 = 0.6 \text{ m}$

Number of elements and joints along a and b -

$n_a = \text{ceiling}(\frac{a}{a_1}) = \text{ceiling}(\frac{a}{0.6}) = [6 \ 7 \ 7 \ 6]$, $n_{ea} = \text{sum}(e_a) = 26$, $n_{ja} = n_{ea} + 1 = 26 + 1 = 27$

$n_b = \text{ceiling}(\frac{b}{b_1}) = \text{ceiling}(\frac{b}{0.6}) = [5 \ 6 \ 5]$, $n_{eb} = \text{sum}(e_b) = 16$, $n_{jb} = n_{eb} + 1 = 16 + 1 = 17$

Total number of elements - $n_e = n_{ea} \cdot n_{eb} = 26 \cdot 16 = 416$

Total number of joints - $n_j = n_{ja} \cdot n_{jb} = 27 \cdot 17 = 459$

Supported joints count - $n_s = n_{sa} \cdot n_{sb} = 5 \cdot 4 = 20$

Joint coordinates

$j = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.6 \ 0.6 \ 0.6 \dots 15.6] \text{ m}$

$j = [0 \ 0.6 \ 1.2 \ 1.8 \ 2.4 \ 3 \ 3.6 \ 4.2 \ 4.8 \ 5.4 \ 6 \ 6.6 \ 7.2 \ 7.8 \ 8.4 \ 9 \ 9.6 \ 0 \ 0.6 \ 1.2 \dots 9.6] \text{ m}$

Numbers of joints at elements' corners

$\text{transp}(e_j) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 18 & 19 & 20 & 21 & \dots & 441 \\ 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 35 & 36 & 37 & 38 & \dots & 458 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 36 & 37 & 38 & 39 & \dots & 459 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 19 & 20 & 21 & 22 & \dots & 442 \end{bmatrix}$

Supported joints

$j = [1 \ 6 \ 12 \ 17 \ 103 \ 108 \ 114 \ 119 \ 222 \ 227 \ 233 \ 238 \ 341 \ 346 \ 352 \ 357 \ 443 \ 448 \ 454 \ 459]$

Joints for element e - $j_e(e) = \text{row}(e_j; e)$

17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425	442	459
18	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400	416	432
19	33	47	61	75	89	103	117	131	145	159	173	187	201	215	229	243	257	271	285	299	313	327	341	355	369	433
20	35	43	51	59	67	75	83	91	99	107	115	123	131	139	147	155	163	171	179	187	195	203	211	219	227	235
21	36	46	56	66	76	86	96	106	116	126	136	146	156	166	176	186	196	206	216	226	236	246	256	266	276	286
22	37	49	61	73	85	97	109	121	133	145	157	169	181	193	205	217	229	241	253	265	277	289	301	313	325	337
23	38	50	62	74	86	98	110	122	134	146	158	170	182	194	206	218	230	242	254	266	278	290	302	314	326	338
24	39	51	63	75	87	99	111	123	135	147	159	171	183	195	207	219	231	243	255	267	279	291	303	315	327	339
25	40	52	64	76	88	100	112	124	136	148	160	172	184	196	208	220	232	244	256	268	280	292	304	316	328	340
26	41	53	65	77	89	101	113	125	137	149	161	173	185	197	209	221	233	245	257	269	281	293	305	317	329	341
27	42	54	66	78	90	102	114	126	138	150	162	174	186	198	210	222	234	246	258	270	282	294	306	318	330	342
28	43	55	67	79	91	103	115	127	139	151	163	175	187	199	211	223	235	247	259	271	283	295	307	319	331	343
29	44	56	68	80	92	104	116	128	140	152	164	176	188	200	212	224	236	248	260	272	284	296	308	320	332	344
30	45	57	69	81	93	105	117	129	141	153	165	177	189	201	213	225	237	249	261	273	285	297	309	321	333	345
31	46	58	70	82	94	106	118	130	142	154	166	178	190	202	214	226	238	250	262	274	286	298	310	322	334	346
32	47	59	71	83	95	107	119	131	143	155	167	179	191	203	215	227	239	251	263	275	287	299	311	323	335	347
33	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300	312	324	336	348
34	49	61	73	85	97	109	121	133	145	157	169	181	193	205	217	229	241	253	265	277	289	301	313	325	337	349
35	50	62	74	86	98	110	122	134	146	158	170	182	194	206	218	230	242	254	266	278	290	302	314	326	338	350
36	51	63	75	87	99	111	123	135	147	159	171	183	195	207	219	231	243	255	267	279	291	303	315	327	339	351
37	52	64	76	88	100	112	124	136	148	160	172	184	196	208	220	232	244	256	268	280	292	304	316	328	340	352
38	53	65	77	89	101	113	125	137	149	161	173	185	197	209	221	233	245	257	269	281	293	305	317	329	341	353
39	54	66	78	90	102	114	126	138	150	162	174	186	198	210	222	234	246	258	270	282	294	306	318	330	342	354
40	55	67	79	91	103	115	127	139	151	163	175	187	199	211	223	235	247	259	271	283	295	307	319	331	343	355
41	56	68	80	92	104	116	128	140	152	164	176	188	200	212	224	236	248	260	272	284	296	308	320	332	344	356
42	57	69	81	93	105	117	129	141	153	165	177	189	201	213	225	237	249	261	273	285	297	309	321	333	345	357
43	58	70	82	94	106	118	130	142	154	166	178	190	202	214	226	238	250	262	274	286	298	310	322	334	346	358
44	59	71	83	95	107	119	131	143	155	167	179	191	203	215	227	239	251	263	275	287	299	311	323	335	347	359
45	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300	312	324	336	348	360
46	61	73	85	97	109	121	133	145	157	169	181	193	205	217	229	241	253	265	277	289	301	313	325	337	349	361
47	62	74	86	98	110	122	134	146	158	170	182	194	206	218	230	242	254	266	278	290	302	314	326	338	350	362
48	63	75	87	99	111	123	135	147	159	171	183	195	207	219	231	243	255	267	279	291	303	315	327	339	351	363
49	64	76	88	100	112	124	136	148	160	172	184	196	208	220	232	244	256	268	280	292	304	316	328	340	352	364
50	65	77	89	101	113	125	137	149	161	173	185	197	209	221	233	245	257	269	281	293	305	317	329	341	353	365
51	66	78	90	102	114	126	138	150	162	174	186	198	210	222	234	246	258	270	282	294	306	318	330	342	354	366
52	67	79	91	103	115	127	139	151	163	175	187	199	211	223	235	247	259	271	283	295	307	319	331	343	355	367
53	68	80	92	104	116	128	140	152	164	176	188	200	212	224	236	248	260	272	284	296	308	320	332	344	356	368
54	69	81	93	105	117	129	141	153	165	177	189	201	213	225	237	249	261	273	285	297	309	321	333	345	357	369
55	70	82	94	106	118	130	142	154	166	178	190	202	214	226	238	250	262	274	286	298	310	322	334	346	358	370
56	71	83	95	107	119	131	143	155	167	179	191	203	215	227	239	251	263	275	287	299	311	323	335	347	359	371
57	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300	312	324	336	348	360	372
58	73	85	97	109	121	133	145	157	169	181	193	205	217	229	241	253	265	277	289	301	313	325	337	349	361	373
59	74	86	98	110	122	134	146	158	170	182	194	206	218	230	242	254	266	278	290	302	314	326	338	350	362	374
60	75	87	99	111	123	135	147	159	171	183	195	207	219	231	243	255	267	279	291	303	315	327	339	351	363	375

$$N_{3,w}(\xi; \eta) = \phi_{3a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{3,\theta_1}(\xi; \eta) = \phi_{4a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{3,\theta_1}(\xi; \eta) = \phi_{3a}(\xi) \cdot \phi_{4b}(\eta)$$

$$N_{4,w}(\xi; \eta) = \phi_{1a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{4,\theta_1}(\xi; \eta) = \phi_{2a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{4,\theta_1}(\xi; \eta) = \phi_{1a}(\xi) \cdot \phi_{4b}(\eta)$$

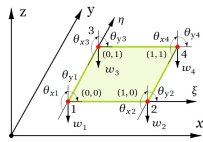
For twist ψ

$$N_{1,\psi}(\xi; \eta) = \phi_{2a}(\xi) \cdot \phi_{2b}(\eta)$$

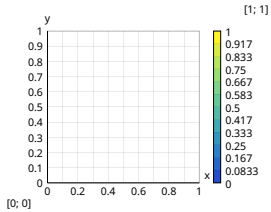
$$N_{2,\psi}(\xi; \eta) = \phi_{4a}(\xi) \cdot \phi_{2b}(\eta)$$

$$N_{3,\psi}(\xi; \eta) = \phi_{4a}(\xi) \cdot \phi_{4b}(\eta)$$

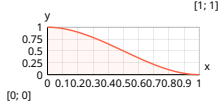
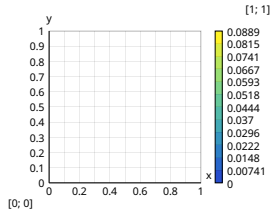
$$N_{4,\psi}(\xi; \eta) = \phi_{2a}(\xi) \cdot \phi_{4b}(\eta)$$



$N_{1,w}$ shape function plot



N_{1,θ_1} shape function plot



Shape functions vector

$$N(i; \xi; \eta) = \text{take}(i; N_{1,w}(\xi; \eta); N_{1,\theta_1}(\xi; \eta); N_{1,\psi}(\xi; \eta); N_{2,w}(\xi; \eta); N_{2,\theta_1}(\xi; \eta); N_{2,\psi}(\xi; \eta);$$

$$\eta); N_{3,w}(\xi; \eta); N_{3,\theta_1}(\xi; \eta); N_{3,\psi}(\xi; \eta); N_{4,w}(\xi; \eta); N_{4,\theta_1}(\xi; \eta); N_{4,\psi}(\xi; \eta))$$

Constitutive matrix (stress - strain relationship)

$$D = \frac{E \cdot \nu^3}{12 \cdot (1 - \nu^2)} \cdot \text{hp}\left(\begin{bmatrix} 1; \nu; 0 & \nu; 1; 0 & 0; 0; 0; 0; 0 \\ 0; 0; 1 - \nu & 0 & 0; 0; 1 - \nu & 0 & 0; 0; 0 \end{bmatrix}\right) = \frac{35000 \cdot 0.2^3}{12 \cdot (1 - 0.2^2)} \cdot \text{hp}\left(\begin{bmatrix} 1; 0.2; 0 & 0.2; 1; 0 & 0; 0; 0; 0 \end{bmatrix}\right)$$

Strain-displacement matrix

$$B_1(j; \xi; \eta) = \text{take}(j; \phi'_{1a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{4b}(\eta))$$

$$B_2(j; \xi; \eta) = \text{take}(j; \phi_{1a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{1a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{4b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{4b}(\eta); \phi_{1a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{1a}(\xi) \cdot \phi'_{4b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{4b}(\eta))$$

$$B_3(j; \xi; \eta) = 2 \cdot \text{take}(j; \phi'_{1a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{4b}(\eta))$$

$$B(j; \xi; \eta) = \text{hp}([B_1(j; \xi; \eta); B_2(j; \xi; \eta); B_3(j; \xi; \eta)])$$

The coefficients of the stiffness matrix will be calculated by using the equation

$$K_{e,ij} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 B_i(\xi; \eta)^T \cdot D \cdot B_j(\xi; \eta) \, d\xi \, d\eta$$

Element stiffness matrix

(above the main diagonal only)

$$BTDB_e(i; j; \xi; \eta) = \text{transp}(B(i; \xi; \eta)) \cdot D \cdot B(j; \xi; \eta)$$

$$K_e(i; j) = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 BTDB_e(i; j; \xi; \eta) \, d\xi \, d\eta$$

$$\text{\$Repeat}\{\text{\$Repeat}\{K_{e,i,j} = K_e(i; j) \text{ for } j = i..n\} \text{ for } i = 1..n\} = 0.9777823$$

$K_e =$

796.296296	135.185185	135.185185	16.736111	-391.203704	84.953704	-13.657407	4.097222	-13.888889	36.574074	36.574074	-8.541667	-391.203704	-13.657407	84.953704	4.097222
0	46.666667	21.597222	4.666667	-84.953704	14.027778	-4.094824	0.7083333	-36.574074	10.277778	8.541667	-1.624997	-13.657407	1.944444	4.097222	-0.5833333
0	0	46.666667	4.666667	-13.657407	4.094824	1.944444	-0.5833333	-36.574074	8.541667	10.277778	-1.625	-84.953704	-4.097222	14.027778	0.7083333
0	0	0	0.9777823	-4.097222	0.7083333	0.5833333	-0.1611111	-8.541667	1.624997	1.625	-0.2305541	-4.097222	0.5833333	0.7083333	-0.1611111
0	0	0	0	796.296296	-135.185185	135.185185	-16.736111	-391.203704	13.657407	84.953704	-4.097222	-13.888889	-36.574074	36.574074	8.541667
0	0	0	0	0	46.666667	-21.597222	4.666667	13.657407	1.944444	-4.097222	-0.5833333	36.574074	10.277778	-8.541667	-1.624997
0	0	0	0	0	0	46.666667	-4.666667	-84.953704	4.097222	14.027778	-0.7083333	-36.574074	-8.541667	10.277778	1.625
0	0	0	0	0	0	0	0.9777823	4.097222	0.5833333	-0.7083333	-0.1611111	8.541667	1.624997	-1.625	-0.2305541
0	0	0	0	0	0	0	0	796.296296	-135.185185	-135.185185	16.736111	-391.203704	-84.953704	13.657407	4.097222
0	0	0	0	0	0	0	0	0	46.666667	21.597222	-4.666667	84.953704	14.027778	-4.094824	-0.7083333
0	0	0	0	0	0	0	0	0	46.666667	-4.666667	13.657407	4.094824	1.944444	0.5833333	0.7083333
0	0	0	0	0	0	0	0	0	0	0	0.9777823	-4.097222	-0.7083333	-0.5833333	-0.1611111
0	0	0	0	0	0	0	0	0	0	0	0	796.296296	135.185185	-135.185185	-16.736111
0	0	0	0	0	0	0	0	0	0	0	0	46.666667	-21.597222	-4.666667	0.7083333
0	0	0	0	0	0	0	0	0	0	0	0	0	0	46.666667	4.666667
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9777823

Element load vector

$$F_{e,i} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 N_i(\xi; \eta)^T \cdot q \, d\xi \, d\eta$$

$$r_e = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 0.9 \ -0.09 \ 0.09 \ -0.009 \ 0.9 \ -0.09 \ -0.09 \ 0.009 \ 0.9 \ 0.09 \ -0.09 \ -0.009] \text{ kN}$$

Solution

Global stiffness matrix

$$K =$$

Global load vector

$$\vec{r} = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ \dots \ 0.009] \text{ kN}$$

Solution of the system of equations

$$\vec{r} = \text{solve}(\mathbf{K}, \vec{F}) = [0 \ 0.5523613 \ 0.3827392 \ -0.4161287 \ 0.2028064 \ 0.3732832 \ 0.2648463 \ -0.1937261 \ 0.2989047 \\ 0.3091182 \ 0.04830843 \ -0.02501454 \ 0.261207 \ 0.3426386 \ -0.1651493 \ 0.1275073 \ 0.1211489 \\ 0.4681287 \ -0.2671179 \ 0.2293302 \ \dots \ -0.4161286] \text{ mm}$$

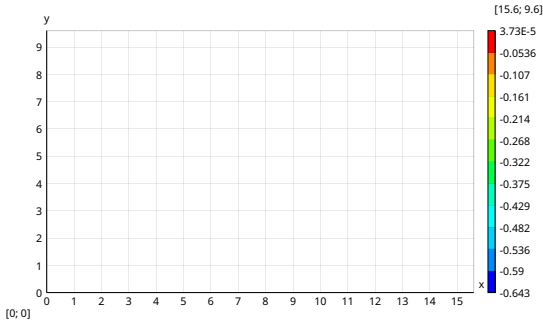
Results

Joint displacements

$$\text{transp}(W_z) =$$

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.435	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.435	0.325	...	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	...	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.435	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.435	0.325	...	0.299
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0

mm



Bending moments

$$Z_j(j) = \text{slice}(\mathcal{Z}; k_1 \cdot (j-1) + 1; k_1 \cdot j)$$

$$Z_e(e) = \mathbf{hp}([Z_j(e_{j,e,1}); Z_j(e_{j,e,2}); Z_j(e_{j,e,3}); Z_j(e_{j,e,4})])$$

Average bending moments at joints, kNm/m

$M_i =$

1.498465	0.3097372	0.2197322	0.1563433	0.1507519	0.9983302	0.1564193	0.1519741	0.19422	0.1519742	0.1564197	0.9983303	0.1570475	0.1563397	0.2197486	0.3098808	1.49858	8.502998	6.479199	5.777684	...	1.498434
1.566914	7.805101	9.33778	7.680907	3.333	-28.61735	3.14639	7.323888	8.895109	7.323888	3.14639	-28.61734	3.33297	7.668093	9.337791	7.80518	1.566813	8.5029972	5.382201	7.126518	...	1.56695
8.091391	3.766896	0.4863938	-2.479308	-4.459199	0.1550864	4.780359	-8.247273	1.18852-10 ⁻⁸	-8.247273	-4.780359	-0.1550866	4.459199	-2.479312	-0.4863957	-3.766697	-8.09139	4.110857	2.572942	0.367139	...	8.09133

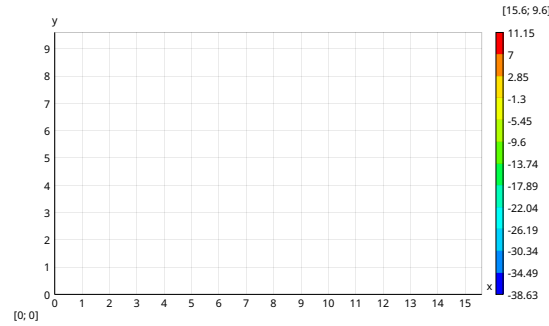
Bending moments for the plate

Bending moments - M_x

$$\text{transp}(\mathbf{M}\mathbf{x}) =$$

1.498465	8.502998	11.06652	10.47741	6.797447	0.888327	-1.235271	0.301501	5.644442	8.779998	8.877612	5.920011	0.685447	-30.133041	0.685445	9.920011	8.877618	8.779995	5.644443	0.3015014	...	1.498453	
0.3097372	6.479199	9.44272	8.87725	4.922637	-2.695317	-1.042892	-3.26094	3.808797	7.250442	7.361393	1.277052	-7.264349	-9.733242	-2.764349	4.127752	7.361393	7.250426	3.808797	-3.260293	...	0.3098324	
0.2197322	5.777684	8.716293	8.135055	4.039483	-2.257879	-6.060711	-1.2819291	2.932144	6.516758	6.640543	3.291674	-2.294253	-5.616624	-2.294253	3.291674	6.640546	6.516758	2.932144	-1.2819291	...	0.2197447	
0.1563433	4.953176	9.125993	0.946465	4.972785	4.075777	-3.257751	-7.98846	-3.832677	2.913674	6.979469	6.926822	3.304597	-3.205198	-7.381383	-3.205198	3.304597	6.926822	6.979469	2.913674	-3.832677	...	0.1563395
0.1570519	7.269644	10.372249	9.566604	4.289253	-5.268409	-6.118864	-8.565491	3.766832	7.713228	7.838243	1.457472	-5.092084	-10.086013	-5.096284	1.457472	7.838243	7.713228	3.766832	-8.565491	...	0.1570491	
0.9983302	9.038525	11.080418	10.122777	5.76385	-2.135317	-38.650059	-2.745557	4.52179	8.190804	8.315041	4.904088	-2.088413	-36.536458	-2.088413	4.904088	8.315041	8.190804	4.52179	-2.745557	...	0.9983301	
0.1564193	7.221047	10.282001	9.436465	4.848424	-3.589009	-6.18287	-9.56302	3.624262	7.560111	6.698855	4.05327	-5.180058	-10.915925	-5.183805	4.05327	6.698855	7.560111	3.624262	-5.96302	...	0.1564295	
0.1519741	8.873186	8.92795	8.229955	3.775343	-3.419119	-8.032747	-4.009042	2.594259	6.469761	6.628453	3.062913	-3.269947	-7.306587	-3.269947	3.062913	6.628453	6.469761	2.594259	-4.009042	...	0.1519596	
0.19422	5.514447	8.830213	7.692391	3.468337	-2.491105	-5.742638	-3.073384	2.310468	5.979222	6.14265	2.786902	-3.76825	-5.118402	-3.76825	2.786902	6.14265	5.979222	2.310468	-3.073384	...	0.1942406	
0.1519742	8.873186	8.92795	8.229755	3.774543	-3.419119	-8.032747	-4.009042	2.594259	6.469761	6.628453	3.062913	-3.269947	-7.306587	-3.269947	3.062913	6.628453	6.469761	2.594259	-4.009042	...	0.1519563	
0.1564197	7.221047	10.282001	9.436465	4.848424	-3.589009	-6.18287	-9.56302	3.624262	7.560111	6.698855	4.05327	-5.180058	-10.915925	-5.183805	4.05327	6.698855	7.560111	3.624262	-5.96302	...	0.1564287	
0.9983303	9.038524	11.080418	10.122777	5.76385	-2.135317	-38.650059	-2.745557	4.52179	8.190804	8.315041	4.904088	-2.088413	-36.536458	-2.088413	4.904088	8.315041	8.190804	4.52179	-2.745557	...	0.9983303	
0.1570475	7.269644	10.372249	9.566604	4.972584	-5.268409	-6.118864	-8.565491	3.766832	7.713228	7.838243	1.457472	-5.096284	-10.086013	-5.096284	1.457472	7.838243	7.713228	3.766832	-8.565491	...	0.1570551	
0.1563397	5.993177	9.125994	8.942789	4.075777	-3.257751	-7.98846	-3.832677	2.913674	6.979469	6.926822	3.304597	-3.205198	-7.381383	-3.205198	3.304597	6.926822	6.979469	2.913674	-3.832677	...	0.1563387	
0.2197486	5.777685	8.716292	8.135052	4.039483	-2.257878	-6.060711	-1.2819291	2.932144	6.516758	6.640543	3.291674	-2.294253	-5.616624	-2.294253	3.291674	6.640543	6.516758	2.932144	-1.2819291	...	0.2197395	
0.3098808	6.479177	9.432178	8.877282	4.922634	-2.695318	-1.042892	-3.260929	3.808797	7.250446	7.361393	1.277052	-7.264349	-9.733242	-2.764349	4.127752	7.361393	7.250426	3.808797	-3.260293	...	0.309677	
1.49858	8.502893	11.066537	10.477401	6.797448	0.888363	-1.235227	0.301501	5.644443	8.779994	8.877613	5.920009	0.6854455	-30.133041	0.6854449	9.920011	8.87761	8.779995	5.644442	0.3015017	...	1.498434	

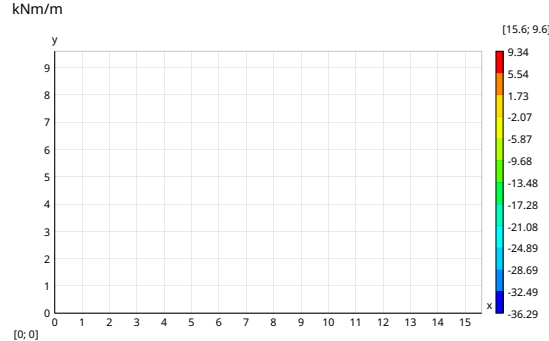
kNm/m



Bending moments M_y

transp (M_y) =

1.566914	0.3205972	0.2333587	0.2147852	0.1682816	0.1798035	1.015789	0.1794301	0.1669894	0.2098191	0.2091906	0.1663467	0.1776119	0.9891482	0.1776123	0.1663459	0.2091915	0.2098181	0.1669903	0.1794296	...	1.566846
7.805101	5.382201	4.308412	4.134952	4.742331	6.340395	8.230714	6.28316	4.611981	3.89228	3.867499	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805153
9.33778	7.126518	5.800074	5.553032	6.414898	8.11577	9.046666	8.014704	6.188836	5.14485	5.116777	6.090589	7.821224	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337789
7.668097	5.41756	4.262493	3.920413	4.233692	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
3.333	-0.3280779	0.5396346	0.405616	-1.17438	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964996	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964995	-1.399574	-3.531204	...	3.332998
-28.361735	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.14639	-0.5066696	0.3485146	0.1799353	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146398
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
8.895019	6.740122	5.350863	4.96126	5.608937	6.913063	7.572529	6.693158	5.141784	4.180525	4.149637	5.027791	6.456862	7.212973	6.456862	5.027791	4.149637	4.180525	5.141784	6.693158	...	8.895035
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
3.14639	-0.5066695	0.3485145	0.1799354	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146397
-28.361734	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.332997	-0.3280781	0.5396345	0.405616	-1.174379	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964994	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964996	-1.399574	-3.531204	...	3.333003
7.668093	5.41756	4.262493	3.920414	4.233691	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
9.337791	7.126529	5.800079	5.553029	6.414899	8.115769	9.046667	8.014704	6.188836	5.14485	5.116777	6.090589	7.821225	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337788
7.80518	5.382247	4.308397	4.134958	4.742326	6.340398	8.230713	6.283161	4.61198	3.892281	3.867498	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805067
1.566813	0.3204128	0.2333748	0.2147746	0.1682944	0.1797983	1.015789	0.1794292	0.1669914	0.2098169	0.2091927	0.166345	0.1776128	0.9891482	0.177612	0.1663463	0.209191	0.2098187	0.1669898	0.1794301	...	1.56695



Bending moments M_{xy}

transp (M_{xy}) =																			
8.091391	4.110857	1.417137	-0.9458016	-3.233923	-4.783472	0.02175397	4.840696	3.336369	1.142106	-1.025808	-3.195694	-4.675291	8.203632×10 ⁻⁹	4.675291	3.195694	1.025808	-1.142106	-4.675291	-8.203632×10 ⁻⁹
3.766896	2.572942	0.9831798	-0.5157529	-2.056961	-3.229076	0.05823395	3.354953	2.213405	0.7311327	-0.6697977	-2.124616	-3.226088	-1.924384×10 ⁻⁸	3.226088	2.124615	0.6697977	-0.731132	-3.226088	-1.924384×10 ⁻⁸
0.4863938	0.367139	0.2104715	0.08573235	-0.1195606	-0.3138301	0.07348128	0.4612966	0.2667083	0.05683469	-0.07988453	-0.2739108	-0.4318273	4.135681×10 ⁻⁹	0.4318273	0.2739109	0.07988453	-0.0568346	-4.135681×10 ⁻⁹	
-2.479308	-1.795977	-0.6012575	0.6167458	1.746893	2.092654	0.07628287	-1.948193	-1.634066	-0.5814737	0.4919345	1.536254	1.879329	-9.399297×10 ⁻¹⁰	-1.879329	-1.536254	-0.4919345	0.5814737	1.879329	9.399297×10 ⁻¹⁰
-4.459199	-3.239942	-0.8977427	0.6826669	2.370922	4.524997	0.07578146	-4.382303	-2.264662	-0.6817699	0.6007663	2.152105	4.210878	2.347034×10 ⁻¹⁰	-4.210878	-2.152105	-0.6007663	0.6817699	4.210878	2.347034×10 ⁻¹⁰
0.1550864	0.1611718	0.1492474	0.1245987	0.09639145	0.07797703	0.07388198	0.06988651	0.05125056	0.02105121	-0.008413693	-0.02659669	-0.02344966	-6.046931×10 ⁻¹¹	0.02344966	0.02659669	0.008413693	-0.0210512	-0.02344966	-6.046931×10 ⁻¹¹
4.780359	3.565968	1.189369	-0.4505428	-2.206755	-4.399285	0.06348492	4.535454	2.37937	0.7252095	-0.6263983	-2.225358	-4.279277	1.578955×10 ⁻¹¹	4.279277	2.225358	-0.6263983	-0.725209	-4.279277	-1.578955×10 ⁻¹¹
2.842723	2.13438	0.8682924	-0.4377229	-1.679594	-2.084069	0.03778704	2.16835	1.796129	0.6296557	-0.5417072	-1.677932	-2.037682	-4.744667×10 ⁻¹²	2.037682	1.677932	0.5417072	-0.629655	-2.037682	-4.744667×10 ⁻¹²
1.18852×10 ⁻⁸	-6.576042×10 ⁻⁹	-7.016847×10 ⁻¹⁰	1.244056×10 ⁻¹⁰	2.191792×10 ⁻¹⁰	-7.501514×10 ⁻¹¹	1.8954×10 ⁻¹¹	-1.177769×10 ⁻¹³	3.047327×10 ⁻¹²	1.092988×10 ⁻¹²	-4.777445×10 ⁻¹²	5.435255×10 ⁻¹²	-3.554444×10 ⁻¹²	3.228026×10 ⁻¹²	-2.990397×10 ⁻¹²	1.375859×10 ⁻¹²	-1.298123×10 ⁻¹²	7.488428×10 ⁻¹³	-3.228026×10 ⁻¹²	3.554444×10 ⁻¹²
-2.842723	-2.13438	-0.8682924	0.4377229	1.679594	2.084069	-0.03778704	-2.16835	-1.796129	-0.6296557	0.5417072	1.677932	2.037682	-7.622464×10 ⁻¹²	-2.037682	-1.677932	-0.5417072	0.6296555	-2.037682	-7.622464×10 ⁻¹²
-4.780359	-3.565968	-1.189369	0.4505428	2.206755	4.399285	-0.06348492	-4.535454	-2.37937	-0.7252095	0.6263983	2.225358	4.279277	2.756043×10 ⁻¹¹	-4.279277	-2.225358	-0.6263983	0.725209	4.279277	2.756043×10 ⁻¹¹
-0.1550866	-0.1611716	-0.1492474	-0.1245987	-0.09639145	-0.07797703	-0.07388197	-0.06988651	-0.05125056	-0.02105121	0.008413693	0.02659669	0.02344966	-1.059221×10 ⁻¹⁰	-0.02344966	-0.02659669	-0.008413693	0.0210512	-1.059221×10 ⁻¹⁰	
4.459199	3.239941	0.8977427	-0.6826669	-2.370922	-4.524997	-0.07578147	4.382303	2.264662	0.6817699	-0.6007663	-2.152105	-4.210878	4.094768×10 ⁻¹⁰	4.210878	2.152105	0.6007663	-0.681769	-4.094768×10 ⁻¹⁰	
2.479312	1.795976	0.6012571	-0.6167458	-1.746893	-2.092654	-0.07628286	1.948193	1.634066	0.5814737	-0.4919345	-1.536254	-1.879329	-1.611647×10 ⁻⁹	1.879329	1.536254	0.4919345	-0.581473	-1.611647×10 ⁻⁹	
-0.4863957	-0.3671418	-0.2104716	-0.08573185	0.1195605	0.3138302	-0.07348132	-0.4612966	-0.2667083	-0.05683469	0.07988452	0.2739109	0.4318273	6.850159×10 ⁻⁹	-0.4318273	-0.2739109	-0.07988453	0.0568346	6.850159×10 ⁻⁹	
-3.766967	-2.57294	-0.9831755	0.5157536	2.056962	3.229076	-0.05823381	-3.354953	-2.213405	-0.7311327	0.6697977	2.124615	3.226088	-3.125512×10 ⁻⁸	-3.226088	-2.124615	-0.6697977	0.7311327	3.125512×10 ⁻⁸	
-8.09139	-4.110792	-1.417137	0.9457983	3.233922	4.783472	-0.02175395	-4.840696	-3.336369	-1.142106	1.025808	3.195694	4.675291	1.254052×10 ⁻⁸	-4.675291	-3.195694	-1.025808	1.142106	4.675291	1.254052×10 ⁻⁸

