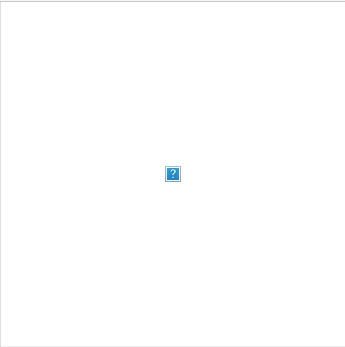


Finite Element Analysis of Flat Slab

Using analytical formulation of Bogner-Fox-Schmit (BFS) plate element

Input data

Span lengths
r = hp([3.6; 4.2; 4.2; 3.6]) = [3.6 4.2 4.2 3.6] m
r = hp([3; 3.6; 3]) = [3 3.6 3] m
Number of axes - nsa = len(r) + 1 = 5 , nsb = len(r) + 1 = 4



Axis coordinates -xs = [0 3.6 7.8 12 15.6] m, ys = [0 3 6.6 9.6] m

Slab dimensions - la = xs5,5 = 15.6 m, lb = ys4,4 = 9.6 m

Thickness - t = 0.2 m

Load - q = 10 kN/m²

Modulus of elasticity - E = 35000 MPa

Poisson's ratio - ν = 0.2

Finite element mesh

We will use BFS rectangular finite element with nDOFs = 16

Element dimensions - a1 = 0.6 m, b1 = 0.6 m

Number of elements and joints along a and b -

ia = ceiling(a/a1) = ceiling(3.6/0.6) = [6 7 7 6] , nea = sum(ia) = 26 , nja = nea + 1 = 26 + 1 = 27

ib = ceiling(b/b1) = ceiling(9.6/0.6) = [5 6 5] , neb = sum(ib) = 16 , njb = neb + 1 = 16 + 1 = 17

Total number of elements - ne = nea · neb = 26 · 16 = 416

Total number of joints - nj = nja · njb = 27 · 17 = 459

Supported joints count - ns = nsa · nsb = 5 · 4 = 20

Joint coordinates

j = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.6 0.6 ... 15.6] m

j = [0 0.6 1.2 1.8 2.4 3 3.6 4.2 4.8 5.4 6 6.6 7.2 7.8 8.4 9 9.6 0 0.6 1.2 ... 9.6] m

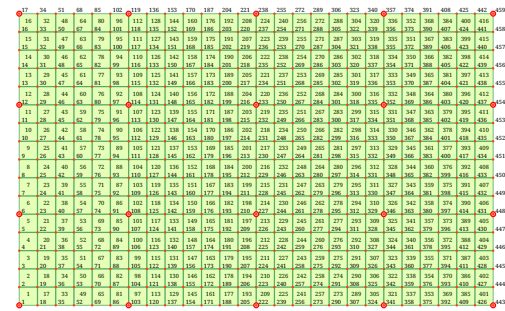
Numbers of joints at elements' corners

transp(ej) = [1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18 19 20 21 ... 441]
[18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 35 36 37 38 ... 458]
[19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 36 37 38 39 ... 459]
[2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 19 20 21 22 ... 442]

Supported joints

j = [1 6 12 17 103 108 114 119 222 227 233 238 341 346 352 357 443 448 454 459]

Joints for element e - je(e) = row(ej; e)



Finite element formulation

Shape functions

Along dimension a

Base functions First derivatives Second derivatives

ϕ1a(ξ) = 1 - ξ² · (3 - 2 · ξ) ϕ'1a(ξ) = -6 · ξ / a1 · (1 - ξ) ϕ''1a(ξ) = - 6 / a1² · (1 - 2 · ξ)

ϕ2a(ξ) = ξ · a1 · (1 - ξ · (2 - ξ)) ϕ'2a(ξ) = 1 - ξ · (4 - 3 · ξ) ϕ''2a(ξ) = - 2 / a1 · (2 - 3 · ξ)

ϕ3a(ξ) = ξ² · (3 - 2 · ξ) ϕ'3a(ξ) = 6 · ξ / a1 · (1 - ξ) ϕ''3a(ξ) = 6 / a1² · (1 - 2 · ξ)

ϕ4a(ξ) = ξ² · a1 · (-1 + ξ) ϕ'4a(ξ) = -ξ · (2 - 3 · ξ) ϕ''4a(ξ) = - 2 / a1 · (1 - 3 · ξ)

Along dimension b

Base functions First derivatives Second derivatives

ϕ1b(η) = 1 - η² · (3 - 2 · η) ϕ'1b(η) = -6 · η / b1 · (1 - η) ϕ''1b(η) = - 6 / b1² · (1 - 2 · η)

ϕ2b(η) = η · b1 · (1 - η · (2 - η)) ϕ'2b(η) = 1 - η · (4 - 3 · η) ϕ''2b(η) = - 2 / b1 · (2 - 3 · η)

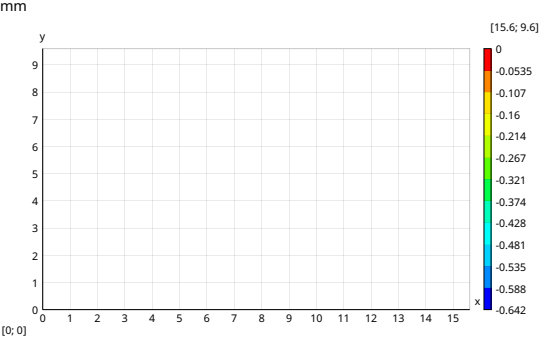
ϕ3b(η) = η² · (3 - 2 · η) ϕ'3b(η) = 6 · η / b1 · (1 - η) ϕ''3b(η) = 6 / b1² · (1 - 2 · η)

ϕ4b(η) = η² · b1 · (-1 + η) ϕ'4b(η) = -η · (2 - 3 · η) ϕ''4b(η) = - 2 / b1 · (1 - 3 · η)

Joint displacements

transp(*W*₂) =

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	...	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	...	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	...	0.299
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0



Bending moments

$$\mathbf{Z_j}(j) = \text{slice}(\mathbf{\varphi}; k_1 \cdot (j - 1) + 1; k_1 \cdot j)$$

$$\mathbf{Z_e}(e) = \text{hp}([\mathbf{Z_j}(e_{j,e}, 1); \mathbf{Z_j}(e_{j,e}, 2); \mathbf{Z_j}(e_{j,e}, 3); \mathbf{Z_j}(e_{j,e}, 4)])$$

Average bending moments at joints, kNm/m

*M*_j =

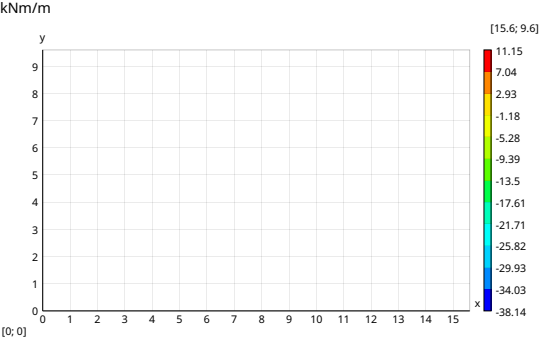
1.500275	0.30977747	0.2197379	0.1563434	0.1570478	0.9983225	0.1564233	0.1519636	0.1942465	0.151963	0.1564238	0.9983224	0.1570487	0.1563432	0.2197373	0.3097501	1.500258	8.50221	6.47933	5.777678	...	1.500075
1.561323	7.805992	9.338126	7.6683	3.333101	-28.361989	3.146397	7.323883	8.895047	7.323882	3.146398	-28.361989	3.333102	7.6683	9.338126	7.805978	1.56134	0.3220403	5.382484	7.126847	...	1.561519
8.088697	3.766245	0.4861465	-2.479522	-4.459403	0.1550078	4.780402	2.842763	-2.410353×10 ⁻⁸	-2.842763	-4.780402	-0.1550078	4.459403	2.479522	-0.4861463	-3.766235	-8.088697	4.109025	2.572475	0.3669336	...	8.088698

Bending moments for the plate

Bending moments - *M*_x

transp(*M*_x) =

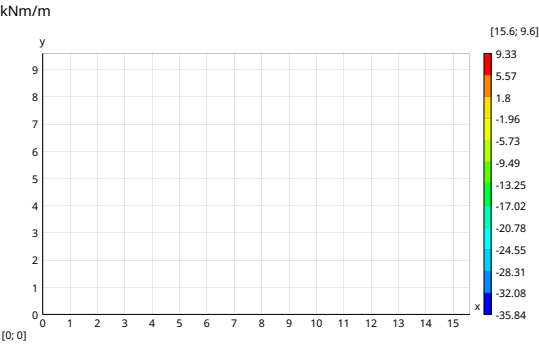
1.500275	8.50221	11.065783	10.476631	6.796938	0.8844119	-31.234001	0.3020629	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860142	5.919519	8.876859	8.779239	5.643938	0.3020627	...	1.500259
0.3097747	6.47933	9.431828	8.876831	4.922389	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360952	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	7.249978	3.808562	-3.260011	...	0.3097374
0.2197379	5.777678	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	...	0.2197445
0.1563434	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.832661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.832661	...	0.1563406
0.1570478	7.26966	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	...	0.1570494
0.9983225	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	...	0.9983223
0.1564233	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	...	0.1564268
0.1519636	5.873203	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.626467	6.469771	2.594271	-4.009025	...	0.1519571
0.1942465	5.514146	8.380234	7.692958	3.468366	-2.491073	-5.742609	-3.07336	2.310482	5.979232	6.142664	2.786926	-2.376796	-5.118358	-2.376796	2.786926	6.142664	5.979232	2.310482	-3.07336	...	0.194254
0.151963	5.873204	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.626467	6.469771	2.594271	-4.009025	...	0.1519564
0.1564238	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	...	0.1564266
0.9983224	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	...	0.9983221
0.1570487	7.269659	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	...	0.1570582
0.1563432	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.832661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.832661	...	0.156342
0.2197373	5.777677	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	...	0.2197303
0.3097501	6.479335	9.43183	8.876831	4.922388	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360953	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	7.249978	3.808562	-3.260012	...	0.3094814
1.500258	8.502224	11.065783	10.47663	6.796939	0.8844112	-31.234001	0.302063	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860143	5.919519	8.876859	8.779239	5.643938	0.3020627	...	1.500075



Bending moments *M*_y

transp(*M*_y) =

1.561323	0.3220403	0.2354335	0.2167291	0.1693836	0.1786538	1.012971	0.1781616	0.1678576	0.2114171	0.2108095	0.1672762	0.1764527	0.9864448	0.1764533	0.1672753	0.2108105	0.2114162	0.1678582	0.1781613	...	1.561345
7.805992	5.382484	4.309253	4.135769	4.742647	6.34008	8.230114	6.28276	4.612133	3.892868	3.868094	4.530706	6.1222	7.985279	6.1222	4.530706	3.868093	3.892869	4.612132	6.28276	...	7.80597
9.338126	7.126847	5.800492	5.553422	6.415097	8.11573	9.046504	8.014613	6.188934	5.145102	5.11703	6.090686	7.82112	8.77703	7.82112	6.090686	5.11703	5.145101	6.188934	8.014613	...	9.338132
7.6683	5.417781	4.262721	3.920623	4.233829	5.419878	6.344599	5.301132	3.979565	3.47814	3.481043	3.968664	5.222849	6.18144	5.222849	3.968664	3.481043	3.47814	3.979565	5.301132	...	7.668298
3.333101	-0.3279945	0.5397481	0.4057356	-1.174273	-3.412893	-0.3301173	-3.53113	-1.399477	0.09976086	0.1610125	-1.210479	-3.234594	-0.2183684	-3.234594	-1.210479	0.1610125	0.09976085	-1.399477	-3.53113	...	3.333102
-28.361989	-7.038398	-1.936124	-1.64658	-4.99389	-13.644591	-36.317421	-13.771031	-5.215778	-1.858062	-1.763919	-4.886758	-13.033067	-34.570998	-13.033067	-4.886758	-1.763919	-1.858062	-5.215778	-13.771031	...	-28.361989
3.146397	-0.5066875	0.3485373	0.1799829	-1.45618	-3.762354	-0.7232665	-3.922882	-1.765717	-0.2519029	-0.1967723	-1.596231	-3.661551	-0.6660923	-3.661551	-1.596231	-0.1967723	-0.2519029	-1.765717	-3.922882	...	3.1464
7.323883	5.094567	3.909	3.489181	3.675526	4.685937	5.498817	4.484059	3.254296	2.794477	2.788653	3.214598	4.35007	6.224849	4.35007	3.214598	2.788653	2.794477	3.254296	4.484059	...	7.323878
8.895047	6.740137	5.350891	4.961298	5.608992	6.913134	7.572611	6.693252	5.141881	4.180623	4.149738	5.027895	6.456963	7.213083	6.456963	5.027895	4.149738	4.180623	5.141881	6.693252	...	8.895053
7.323882	5.094567	3.908999	3.489181	3.675526	4.685937	5.498817	4.484059	3.254296	2.794477	2.788653	3.214598	4.35007	6.224849	4.35007	3.214598	2.788653	2.794477	3.254296	4.484059	...	7.323877
3.146398	-0.5066877	0.3485373	0.1799829	-1.45618	-3.762354	-0.7232665	-3.922882	-1.765717	-0.2519029	-0.1967723	-1.596231	-3.661551	-0.6660923	-3.661551	-1.596231	-0.1967723	-0.2519029	-1.765717	-3.922882	...	3.1464
-28.361989	-7.038398	-1.936124	-1.64658	-4.99389	-13.644591	-36.317421	-13.771031	-5.215778	-1.858062	-1.763919	-4.886758	-13.033067	-34.570998	-13.033067	-4.886758	-1.763919	-1.858062	-5.215778	-13.771031	...	-28.36199
3.333102	-0.3279946	0.5397482	0.4057355	-1.174273	-3.412893	-0.3301173	-3.53113	-1.399477	0.09976086	0.1610125	-1.210479	-3.234594	-0.2183684	-3.234594	-1.210479	0.1610125	0.09976085	-1.399477	-3.53113	...	3.333109
7.6683	5.417781	4.262721	3.920623	4.233829	5.419878	6.344599	5.301132	3.979565	3.47814	3.481043	3.968664	5.222849	6.18144	5.222849	3.968664	3.481043	3.47814	3.979565	5.301132	...	7.6683
9.338126	7.126845	5.800491	5.553422	6.415097	8.11573	9.046504	8.014613	6.188934	5.145102	5.11703	6.090686	7.82112	8.77703	7.82112	6.090686	5.11703	5.145101	6.188934	8.014613	...	9.338126
7.805978	5.382479	4.309254	4.13577	4.742647	6.340081	8.230114	6.28276	4.612133	3.892868	3.868094	4.530706	6.1222	7.985279	6.1222	4.530706	3.868093	3.892869	4.612132	6.28276	...	7.805828
1.56134	0.3220664	0.2354339	0.2167277	0.1693845	0.1786529	1.012971	0.1781618	0.1678575	0.2114172	0.2108094	0.1672763	0.1764527	0.9864448	0.1764534	0.167275	0.2108108	0.211416	0.1678581	0.1781614	...	1.561519



Bending moments M_{xy}

transp(M_{xy}) =

8.088697	4.109025	1.416336	-0.9453079	-3.232331	-4.781353	0.02176366	4.838587	3.334748	1.141499	-1.025238	-3.194104	-4.673215	9.790543×10 ⁻⁹	4.673215	3.194104	1.025238	-1.141499	∞
3.766245	2.572475	0.9829256	-0.515595	-2.056503	-3.228613	0.05825511	3.354524	2.212955	0.7309473	-0.6696087	-2.124156	-3.225638	-2.241521×10 ⁻⁸	3.225638	2.124156	0.6696087	-0.7309473	∞
0.4861465	0.3669336	0.21034	0.08576266	-0.1194038	-0.3136824	0.0734795	0.4611397	0.2665259	0.05675809	-0.07980332	-0.2737226	-0.431664	4.761034×10 ⁻⁹	0.431664	0.2737226	0.07980332	-0.05675809	-0
-2.479522	-1.796141	-0.601364	0.6167145	1.746922	2.09269	0.07626484	-1.948268	-1.634146	-0.5815058	0.4919668	1.536331	1.879395	-1.027996×10 ⁻⁹	-1.879395	-1.536331	-0.4919668	0.5815058	1
-4.459403	-3.240091	-0.8978376	0.6826157	2.370906	4.525	0.07575709	-4.382355	-2.264704	-0.6817866	0.6007781	2.152138	4.210913	2.395547×10 ⁻¹⁰	-4.210913	-2.152138	-0.6007781	0.6817866	2
0.1550078	0.1611012	0.1491877	0.1245521	0.09635607	0.07794845	0.07385991	0.06986752	0.05123571	0.0210446	-0.008412859	-0.02659053	-0.02344541	-5.804372×10 ⁻¹¹	0.02344541	0.02659053	0.008412859	-0.0210446	-0.
2.842763	2.134393	0.8682899	-0.4377384	-1.679619	-2.084094	0.03777839	2.168358	1.796138	0.6296579	-0.5417111	-1.677943	-2.037693	-3.548115×10 ⁻¹²	2.037693	1.677943	0.5417111	-0.6296579	-1
-2.410353×10 ⁻⁸	3.062399×10 ⁻⁸	-4.397001×10 ⁻⁹	3.066824×10 ⁻¹⁰	-8.12632×10 ⁻¹¹	3.636829×10 ⁻¹¹	-1.325189×10 ⁻¹¹	4.764349×10 ⁻¹²	-7.14584×10 ⁻¹³	1.790365×10 ⁻¹³	-3.8029×10 ⁻¹³	2.910935×10 ⁻¹³	0	1.375317×10 ⁻¹³	-9.949168×10 ⁻¹³	2.393×10 ⁻¹²	-3.712996×10 ⁻¹²	2.380218×10 ⁻¹²	3.02
-2.842763	-2.134393	-0.8682899	0.4377384	1.679619	2.084094	-0.03777839	-2.168358	-1.796138	-0.6296579	0.5417111	1.677943	2.037693	4.127238×10 ⁻¹²	-2.037693	-1.677943	-0.5417111	0.6296579	1
-4.780402	-3.565976	-1.18935	0.4505739	2.206794	4.399329	-0.06346754	-4.535463	-2.379375	-0.7252099	0.6264028	2.225368	4.279295	-1.911611×10 ⁻¹¹	-4.279295	-2.225368	-0.6264028	0.7252099	2
-0.1550078	-0.1611013	-0.1491877	-0.1245521	-0.09635607	-0.07794845	-0.07385991	-0.06986752	-0.05123571	-0.0210446	0.008412859	0.02659053	0.02344541	7.954439×10 ⁻¹¹	-0.02344541	-0.02659053	-0.008412859	0.0210446	0.
4.459403	3.240091	0.8978376	-0.6826157	-2.370906	-4.525	-0.07575709	4.382355	2.264704	0.6817866	-0.6007781	-2.152138	-4.210913	-3.335117×10 ⁻¹⁰	4.210913	2.152138	0.6007781	-0.6817866	∞
2.479522	1.796141	0.601364	-0.6167145	-1.746922	-2.09269	-0.07626484	1.948268	1.634146	0.5815058	-0.4919668	-1.536331	-1.879395	1.414276×10 ⁻⁹	1.879395	1.536331	0.4919668	-0.5815058	-1
-0.4861463	-0.3669331	-0.21034	-0.08576274	0.1194038	0.3136824	-0.07347951	-0.4611397	-0.2665259	-0.05675809	0.07980332	0.2737226	0.431664	-6.33653×10 ⁻⁹	-0.431664	-0.2737226	-0.07980332	0.05675809	0.
-3.766235	-2.572475	-0.9829263	0.5155949	2.056503	3.228613	-0.05825506	-3.354524	-2.212955	-0.7309473	0.6696087	2.124156	3.225638	2.942834×10 ⁻⁸	-3.225638	-2.124156	-0.6696087	0.7309473	2
-8.088697	-4.109035	-1.416336	0.9453084	3.232331	4.781353	-0.0217637	-4.838587	-3.334748	-1.141499	1.025238	3.194104	4.673215	-1.247931×10 ⁻⁸	-4.673215	-3.194104	-1.025238	1.141499	3

