

Finite Element Analysis of Flat Slab

Using numerical formulation of Bogner-Fox-Schmit (BFS) plate element

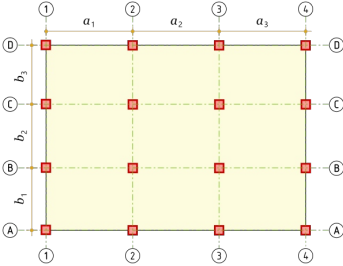
Input data

Span lengths

$r = \mathbf{hp}([3.6; 4.2; 4.2; 3.6]) = [3.6 \ 4.2 \ 4.2 \ 3.6] \text{ m}$

$r = \mathbf{hp}([3; 3.6; 3]) = [3 \ 3.6 \ 3] \text{ m}$

Number of axes -  $n_{sa} = \text{len}(r) + 1 = 5$ ,  $n_{sb} = \text{len}(r) + 1 = 4$



Axis coordinates -  $\alpha_s = [0 \ 3.6 \ 7.8 \ 12 \ 15.6] \text{ m}$ ,  $\eta_s = [0 \ 3 \ 6.6 \ 9.6] \text{ m}$

Slab dimensions -  $l_a \Rightarrow x_{s,5} = 15.6 \text{ m}$ ,  $l_b \Rightarrow y_{s,4} = 9.6 \text{ m}$

Thickness -  $t = 0.2 \text{ m}$

Load -  $q = 10 \text{ kN/m}^2$

Modulus of elasticity -  $E = 35000 \text{ MPa}$

Poisson's ratio -  $\nu = 0.2$

Finite element mesh

We will use Bogner-Fox-Schmit rectangular finite element with  $n_{DOF5} = 16$

Element dimensions -  $a_1 = 0.6 \text{ m}$ ,  $b_1 = 0.6 \text{ m}$

Number of elements and joints along  $a$  and  $b$  -

$n_a = \text{ceiling}(\frac{a}{a_1}) = \text{ceiling}(\frac{a}{0.6}) = [6 \ 7 \ 7 \ 6]$ ,  $n_{ea} = \text{sum}(e_a) = 26$ ,  $n_{ja} = n_{ea} + 1 = 26 + 1 = 27$

$n_b = \text{ceiling}(\frac{b}{b_1}) = \text{ceiling}(\frac{b}{0.6}) = [5 \ 6 \ 5]$ ,  $n_{eb} = \text{sum}(e_b) = 16$ ,  $n_{jb} = n_{eb} + 1 = 16 + 1 = 17$

Total number of elements -  $n_e = n_{ea} \cdot n_{eb} = 26 \cdot 16 = 416$

Total number of joints -  $n_j = n_{ja} \cdot n_{jb} = 27 \cdot 17 = 459$

Supported joints count -  $n_s = n_{sa} \cdot n_{sb} = 5 \cdot 4 = 20$

Joint coordinates

$j = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.6 \ 0.6 \ 0.6 \dots \ 15.6] \text{ m}$

$j = [0 \ 0.6 \ 1.2 \ 1.8 \ 2.4 \ 3 \ 3.6 \ 4.2 \ 4.8 \ 5.4 \ 6 \ 6.6 \ 7.2 \ 7.8 \ 8.4 \ 9 \ 9.6 \ 0 \ 0.6 \ 1.2 \dots \ 9.6] \text{ m}$

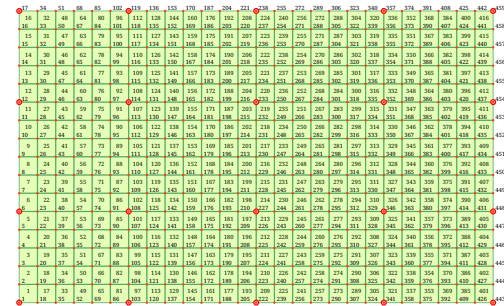
Numbers of joints at elements' corners

$\text{transp}(e_j) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 18 & 19 & 20 & 21 & \dots & 441 \\ 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 35 & 36 & 37 & 38 & \dots & 458 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 36 & 37 & 38 & 39 & \dots & 459 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 19 & 20 & 21 & 22 & \dots & 442 \end{bmatrix}$

Supported joints

$j = [1 \ 6 \ 12 \ 17 \ 103 \ 108 \ 114 \ 119 \ 222 \ 227 \ 233 \ 238 \ 341 \ 346 \ 352 \ 357 \ 443 \ 448 \ 454 \ 459]$

Joints for element  $e$  -  $j_e(e) = \text{row}(e_j; e)$



Finite element formulation

Shape functions

Along dimension  $a$

Base functions First derivatives Second derivatives

$\Phi_{1a}(\xi) = 1 - \xi^2 \cdot (3 - 2 \cdot \xi)$   $\Phi'_{1a}(\xi) = -6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$   $\Phi''_{1a}(\xi) = -\frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$

$\Phi_{2a}(\xi) = \xi \cdot a_1 \cdot (1 - \xi \cdot (2 - \xi))$   $\Phi'_{2a}(\xi) = 1 - \xi \cdot (4 - 3 \cdot \xi)$   $\Phi''_{2a}(\xi) = -\frac{2}{a_1} \cdot (2 - 3 \cdot \xi)$

$\Phi_{3a}(\xi) = \xi^2 \cdot (3 - 2 \cdot \xi)$   $\Phi'_{3a}(\xi) = 6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$   $\Phi''_{3a}(\xi) = \frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$

$\Phi_{4a}(\xi) = \xi^2 \cdot a_1 \cdot (-1 + \xi)$   $\Phi'_{4a}(\xi) = -\xi \cdot (2 - 3 \cdot \xi)$   $\Phi''_{4a}(\xi) = -\frac{2}{a_1} \cdot (1 - 3 \cdot \xi)$

Along dimension  $b$

Base functions First derivatives Second derivatives

$\Phi_{1b}(\eta) = 1 - \eta^2 \cdot (3 - 2 \cdot \eta)$   $\Phi'_{1b}(\eta) = -6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$   $\Phi''_{1b}(\eta) = -\frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$

$\Phi_{2b}(\eta) = \eta \cdot b_1 \cdot (1 - \eta \cdot (2 - \eta))$   $\Phi'_{2b}(\eta) = 1 - \eta \cdot (4 - 3 \cdot \eta)$   $\Phi''_{2b}(\eta) = -\frac{2}{b_1} \cdot (2 - 3 \cdot \eta)$

$\Phi_{3b}(\eta) = \eta^2 \cdot (3 - 2 \cdot \eta)$   $\Phi'_{3b}(\eta) = 6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$   $\Phi''_{3b}(\eta) = \frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$

$\Phi_{4b}(\eta) = \eta^2 \cdot b_1 \cdot (-1 + \eta)$   $\Phi'_{4b}(\eta) = -\eta \cdot (2 - 3 \cdot \eta)$   $\Phi''_{4b}(\eta) = -\frac{2}{b_1} \cdot (1 - 3 \cdot \eta)$

For vertical displacements  $w$  For rotations  $\theta_x$  For rotations  $\theta_y$

$N_{1,w}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{1,\theta_x}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{1,\theta_y}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{2,w}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{2,\theta_x}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{2,\theta_y}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{2b}(\eta)$

$$N_{3,w}(\xi; \eta) = \phi_{3a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{3,\theta_1}(\xi; \eta) = \phi_{4a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{3,\theta_1}(\xi; \eta) = \phi_{3a}(\xi) \cdot \phi_{4b}(\eta)$$

$$N_{4,w}(\xi; \eta) = \phi_{1a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{4,\theta_1}(\xi; \eta) = \phi_{2a}(\xi) \cdot \phi_{3b}(\eta) \quad N_{4,\theta_1}(\xi; \eta) = \phi_{1a}(\xi) \cdot \phi_{4b}(\eta)$$

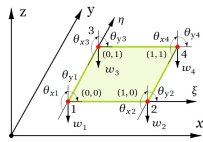
For twist  $\psi$

$$N_{1,\psi}(\xi; \eta) = \phi_{2a}(\xi) \cdot \phi_{2b}(\eta)$$

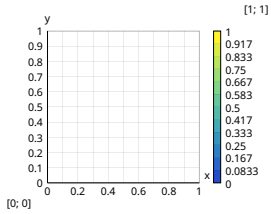
$$N_{2,\psi}(\xi; \eta) = \phi_{4a}(\xi) \cdot \phi_{2b}(\eta)$$

$$N_{3,\psi}(\xi; \eta) = \phi_{4a}(\xi) \cdot \phi_{4b}(\eta)$$

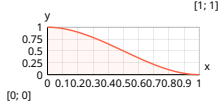
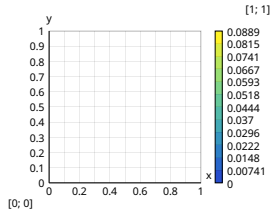
$$N_{4,\psi}(\xi; \eta) = \phi_{2a}(\xi) \cdot \phi_{4b}(\eta)$$



$N_{1,w}$  shape function plot



$N_{1,\theta_1}$  shape function plot



Shape functions vector

$$N(i; \xi; \eta) = \text{take}(i; N_{1,w}(\xi; \eta); N_{1,\theta_1}(\xi; \eta); N_{1,\psi}(\xi; \eta); N_{2,w}(\xi; \eta); N_{2,\theta_1}(\xi; \eta); N_{2,\psi}(\xi; \eta);$$

$$\eta); N_{3,w}(\xi; \eta); N_{3,\theta_1}(\xi; \eta); N_{3,\psi}(\xi; \eta); N_{4,w}(\xi; \eta); N_{4,\theta_1}(\xi; \eta); N_{4,\psi}(\xi; \eta))$$

Constitutive matrix (stress - strain relationship)

$$D = \frac{E \cdot \nu^3}{12 \cdot (1 - \nu^2)} \cdot \text{hp}\left(\begin{bmatrix} 1; \nu; 0 & \nu; 1; 0 & 0; 0; 0; 0 \\ 0; 0; 1 - \nu & 0 & 0; 0; 1 - \nu & 0 & 0; 0; 0 \end{bmatrix}\right) = \frac{35000 \cdot 0.2^3}{12 \cdot (1 - 0.2^2)} \cdot \text{hp}\left(\begin{bmatrix} 1; 0.2; 0 & 0.2; 1; 0 & 0; 0; 0; 0 \end{bmatrix}\right) = \begin{bmatrix} 24.305556 & 4.861111 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4.861111 & 24.305556 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.722222 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ kNm}$$

Strain-displacement matrix

$$B_1(j; \xi; \eta) = \text{take}(j; \phi'_{1a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{2a}(\xi) \cdot \phi_{4b}(\eta))$$

$$B_2(j; \xi; \eta) = \text{take}(j; \phi_{1a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{1a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{2b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{4b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{4b}(\eta); \phi_{1a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{1a}(\xi) \cdot \phi'_{4b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{4b}(\eta))$$

$$B_3(j; \xi; \eta) = 2 \cdot \text{take}(j; \phi'_{1a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{4b}(\eta))$$

$$B(j; \xi; \eta) = \text{hp}([B_1(j; \xi; \eta); B_2(j; \xi; \eta); B_3(j; \xi; \eta)])$$

The coefficients of the stiffness matrix will be calculated by using the equation

$$K_{e,ij} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 B_i(\xi; \eta)^T \cdot D \cdot B_j(\xi; \eta) \, d\xi \, d\eta$$

Element stiffness matrix

(above the main diagonal only)

$$BTDB_e(i; j; \xi; \eta) = \text{transp}(B(i; \xi; \eta)) \cdot D \cdot B(j; \xi; \eta)$$

$$K_e(i; j) = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 BTDB_e(i; j; \xi; \eta) \, d\xi \, d\eta$$

$$\text{\$Repeat}\{\text{\$Repeat}\{K_{e,i,j} = K_e(i; j) \text{ for } j = i..n\} \text{ for } i = 1..m\} = 0.9777823$$

$K_e =$

|            |            |            |           |             |             |            |            |             |             |             |            |             |            |             |            |
|------------|------------|------------|-----------|-------------|-------------|------------|------------|-------------|-------------|-------------|------------|-------------|------------|-------------|------------|
| 796.296296 | 135.185185 | 135.185185 | 16.736111 | -391.203704 | 84.953704   | -13.657407 | 4.097222   | -13.888889  | 36.574074   | 36.574074   | -8.541667  | -391.203704 | -13.657407 | 84.953704   | 4.097222   |
| 0          | 46.666667  | 21.597222  | 4.666667  | -84.953704  | 14.027778   | -4.094824  | 0.7083333  | -36.574074  | 10.277778   | 8.541667    | -1.624997  | -13.657407  | 1.944444   | 4.097222    | -0.5833333 |
| 0          | 0          | 46.666667  | 4.666667  | -13.657407  | 4.094824    | 1.944444   | -0.5833333 | -36.574074  | 8.541667    | 10.277778   | -1.625     | -84.953704  | -4.097222  | 14.027778   | 0.7083333  |
| 0          | 0          | 0          | 0.9777823 | -4.097222   | 0.7083333   | 0.5833333  | -0.1611111 | -8.541667   | 1.624997    | 1.625       | -0.2305541 | -4.097222   | 0.5833333  | 0.7083333   | -0.1611111 |
| 0          | 0          | 0          | 0         | 796.296296  | -135.185185 | 135.185185 | -16.736111 | -391.203704 | 13.657407   | 84.953704   | -4.097222  | -13.888889  | -36.574074 | 36.574074   | 8.541667   |
| 0          | 0          | 0          | 0         | 0           | 46.666667   | -21.597222 | 4.666667   | 13.657407   | 1.944444    | -4.097222   | -0.5833333 | 36.574074   | 10.277778  | -8.541667   | -1.624997  |
| 0          | 0          | 0          | 0         | 0           | 0           | 46.666667  | -4.666667  | -84.953704  | 4.097222    | 14.027778   | -0.7083333 | -36.574074  | -8.541667  | 10.277778   | 1.625      |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0.9777823  | 4.097222    | 0.5833333   | -0.7083333  | -0.1611111 | 8.541667    | 1.624997   | -1.625      | -0.2305541 |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 796.296296  | -135.185185 | -135.185185 | 16.736111  | -391.203704 | -84.953704 | 13.657407   | 4.097222   |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 0           | 46.666667   | 21.597222   | -4.666667  | 84.953704   | 14.027778  | -4.094824   | -0.7083333 |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 0           | 46.666667   | -4.666667   | 13.657407  | 4.094824    | 1.944444   | 0.5833333   | 0.7083333  |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 0           | 0           | 0           | 0.9777823  | -4.097222   | -0.7083333 | -0.5833333  | -0.1611111 |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 0           | 0           | 0           | 0          | 796.296296  | 135.185185 | -135.185185 | -16.736111 |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 0           | 0           | 0           | 0          | 46.666667   | -21.597222 | -4.666667   | 0.7083333  |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 0           | 0           | 0           | 0          | 0           | 0          | 46.666667   | 4.666667   |
| 0          | 0          | 0          | 0         | 0           | 0           | 0          | 0          | 0           | 0           | 0           | 0          | 0           | 0          | 0           | 0.9777823  |

Element load vector

$$F_{e,i} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 N_i(\xi; \eta)^T \cdot q \, d\xi \, d\eta$$

$$r_e = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 0.9 \ -0.09 \ 0.09 \ -0.009 \ 0.9 \ -0.09 \ -0.09 \ 0.009 \ 0.9 \ 0.09 \ -0.09 \ -0.009] \text{ kN}$$

Solution

Global stiffness matrix

$$K =$$

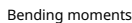
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Global load vector
r = [0.9 0.09 0.09 0.009 1.8 0.18 0 0 1.8 0.18 0 0 1.8 0.18 0 0 1.8 0.18 0 0 ... 0.009] kN
Solution of the system of equations
f = solve(K*f) = [0 0.5523613 0.3827392 -0.4161287 0.2028064 0.3732832 0.2648463 -0.1937261 0.2989047
0.3091182 0.04830843 -0.02501454 0.261207 0.3426386 -0.1651493 0.1275073 0.1211489
0.4681287 -0.2671179 0.2293302 ... -0.4161286] mm

```

## Joint displacements

mm

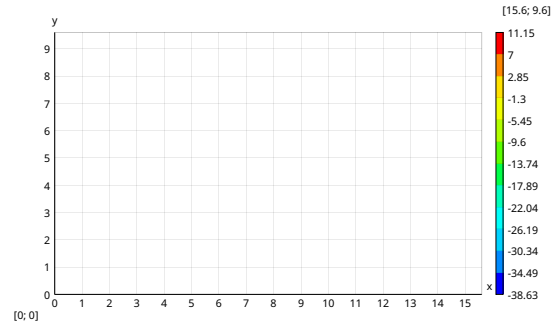

$$Z_j(j) = \text{slice}(\mathcal{Z}; k_1 \cdot (j - 1) + 1; k_1 \cdot j)$$

Average bending moments at joints, kNm/m

### Bending moments for the plate

Bending moments -  $M_x$

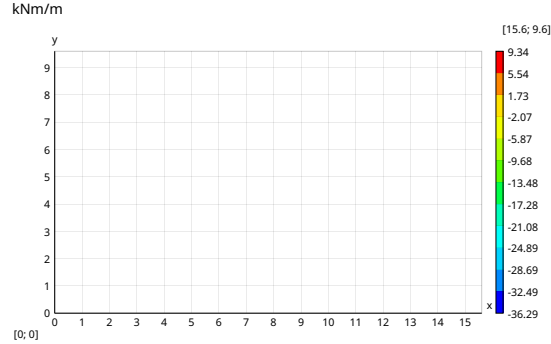
kNm/m



Bending moments  $M_y$

transp ( $M_y$ ) =

|            |            |           |           |           |            |            |            |           |            |            |           |            |            |            |           |            |            |           |            |     |            |
|------------|------------|-----------|-----------|-----------|------------|------------|------------|-----------|------------|------------|-----------|------------|------------|------------|-----------|------------|------------|-----------|------------|-----|------------|
| 1.566914   | 0.3205972  | 0.2333587 | 0.2147852 | 0.1682816 | 0.1798035  | 1.015789   | 0.1794301  | 0.1669894 | 0.2098191  | 0.2091906  | 0.1663467 | 0.1776119  | 0.9891482  | 0.1776123  | 0.1663459 | 0.2091915  | 0.2098181  | 0.1669903 | 0.1794296  | ... | 1.566846   |
| 7.805101   | 5.382201   | 4.308412  | 4.134952  | 4.742331  | 6.340395   | 8.230714   | 6.28316    | 4.611981  | 3.89228    | 3.867499   | 4.530534  | 6.12258    | 7.985896   | 6.12258    | 4.530534  | 3.867499   | 3.89228    | 4.611981  | 6.28316    | ... | 7.805153   |
| 9.33778    | 7.126518   | 5.800074  | 5.553032  | 6.414898  | 8.11577    | 9.046666   | 8.014704   | 6.188836  | 5.14485    | 5.116777   | 6.090589  | 7.821224   | 8.777225   | 7.821225   | 6.090589  | 5.116777   | 5.14485    | 6.188836  | 8.014704   | ... | 9.337789   |
| 7.668097   | 5.41756    | 4.262493  | 3.920413  | 4.233692  | 5.419822   | 6.344591   | 5.301104   | 3.979478  | 3.477998   | 3.480904   | 3.968582  | 5.222837   | 6.181459   | 5.222837   | 3.968582  | 3.480904   | 3.477998   | 3.979478  | 5.301104   | ... | 7.668093   |
| 3.333      | -0.3280779 | 0.5396346 | 0.405616  | -1.17438  | -3.412974  | -0.3301829 | -3.531204  | -1.399574 | 0.09964996 | 0.1609008  | -1.210571 | -3.234664  | -0.2184219 | -3.234664  | -1.210571 | 0.1609008  | 0.09964995 | -1.399574 | -3.531204  | ... | 3.332998   |
| -28.361735 | -7.038366  | -1.936162 | -1.646652 | -4.993974 | -13.644678 | -36.317523 | -13.771126 | -5.215881 | -1.858171  | -1.764027  | -4.886867 | -13.033174 | -34.571125 | -13.033174 | -4.886867 | -1.764027  | -1.858171  | -5.215881 | -13.771126 | ... | -28.361736 |
| 3.14639    | -0.5066696 | 0.3485146 | 0.1799353 | -1.456247 | -3.762435  | -0.7233495 | -3.922981  | -1.765819 | -0.2520091 | -0.1968807 | -1.596341 | -3.661657  | -0.6661947 | -3.661657  | -1.596341 | -0.1968807 | -0.2520091 | -1.765819 | -3.922981  | ... | 3.146398   |
| 7.323888   | 5.094548   | 3.908978  | 3.489139  | 3.675468  | 4.685865   | 5.49873    | 4.483969   | 3.254199  | 2.794376   | 2.788549   | 3.214495  | 4.349964   | 5.243651   | 4.349964   | 3.214495  | 2.788549   | 2.794376   | 3.254199  | 4.483969   | ... | 7.323874   |
| 8.895019   | 6.740122   | 5.350863  | 4.96126   | 5.608937  | 6.913063   | 7.572529   | 6.693158   | 5.141784  | 4.180525   | 4.149637   | 5.027791  | 6.456862   | 7.212973   | 6.456862   | 5.027791  | 4.149637   | 4.180525   | 5.141784  | 6.693158   | ... | 8.895035   |
| 7.323888   | 5.094548   | 3.908978  | 3.489139  | 3.675468  | 4.685865   | 5.49873    | 4.483969   | 3.254199  | 2.794376   | 2.788549   | 3.214495  | 4.349964   | 5.243651   | 4.349964   | 3.214495  | 2.788549   | 2.794376   | 3.254199  | 4.483969   | ... | 7.323874   |
| 3.14639    | -0.5066695 | 0.3485145 | 0.1799354 | -1.456247 | -3.762435  | -0.7233495 | -3.922981  | -1.765819 | -0.2520091 | -0.1968807 | -1.596341 | -3.661657  | -0.6661947 | -3.661657  | -1.596341 | -0.1968807 | -0.2520091 | -1.765819 | -3.922981  | ... | 3.146397   |
| -28.361734 | -7.038366  | -1.936162 | -1.646652 | -4.993974 | -13.644678 | -36.317523 | -13.771126 | -5.215881 | -1.858171  | -1.764027  | -4.886867 | -13.033174 | -34.571125 | -13.033174 | -4.886867 | -1.764027  | -1.858171  | -5.215881 | -13.771126 | ... | -28.361736 |
| 3.332997   | -0.3280781 | 0.5396345 | 0.405616  | -1.174379 | -3.412974  | -0.3301829 | -3.531204  | -1.399574 | 0.09964994 | 0.1609008  | -1.210571 | -3.234664  | -0.2184219 | -3.234664  | -1.210571 | 0.1609008  | 0.09964996 | -1.399574 | -3.531204  | ... | 3.333003   |
| 7.668093   | 5.41756    | 4.262493  | 3.920414  | 4.233691  | 5.419822   | 6.344591   | 5.301104   | 3.979478  | 3.477998   | 3.480904   | 3.968582  | 5.222837   | 6.181459   | 5.222837   | 3.968582  | 3.480904   | 3.477998   | 3.979478  | 5.301104   | ... | 7.668093   |
| 9.337791   | 7.126529   | 5.800079  | 5.553029  | 6.414899  | 8.115769   | 9.046667   | 8.014704   | 6.188836  | 5.14485    | 5.116777   | 6.090589  | 7.821225   | 8.777225   | 7.821225   | 6.090589  | 5.116777   | 5.14485    | 6.188836  | 8.014704   | ... | 9.337788   |
| 7.80518    | 5.382247   | 4.308397  | 4.134958  | 4.742326  | 6.340398   | 8.230713   | 6.283161   | 4.61198   | 3.892281   | 3.867498   | 4.530534  | 6.12258    | 7.985896   | 6.12258    | 4.530534  | 3.867499   | 3.89228    | 4.611981  | 6.28316    | ... | 7.805067   |
| 1.566813   | 0.3204128  | 0.2333748 | 0.2147746 | 0.1682944 | 0.1797983  | 1.015789   | 0.1794292  | 0.1669914 | 0.2098169  | 0.2091927  | 0.166345  | 0.1776128  | 0.9891482  | 0.177612   | 0.1663463 | 0.209191   | 0.2098187  | 0.1669898 | 0.1794301  | ... | 1.56695    |



Bending moments  $M_{xy}$

|                          |                            |                             |                            |                            |                             |                          |                             |                            |                            |                             |                            |                             |                             |                             |                            |                             |                            |                             |
|--------------------------|----------------------------|-----------------------------|----------------------------|----------------------------|-----------------------------|--------------------------|-----------------------------|----------------------------|----------------------------|-----------------------------|----------------------------|-----------------------------|-----------------------------|-----------------------------|----------------------------|-----------------------------|----------------------------|-----------------------------|
| transp ( $M_{xy}$ ) =    |                            |                             |                            |                            |                             |                          |                             |                            |                            |                             |                            |                             |                             |                             |                            |                             |                            |                             |
| 8.091391                 | 4.110857                   | 1.417137                    | -0.9458016                 | -3.233923                  | -4.783472                   | 0.02175397               | 4.840696                    | 3.336369                   | 1.142106                   | -1.025808                   | -3.195694                  | -4.675291                   | 8.203632×10 <sup>-9</sup>   | 4.675291                    | 3.195694                   | 1.025808                    | -1.142106                  | -4.675291                   |
| 3.766896                 | 2.572942                   | 0.9831798                   | -0.5157529                 | -2.056961                  | -3.229076                   | 0.05823395               | 3.354953                    | 2.213405                   | 0.7311327                  | -0.6697977                  | -2.124616                  | -3.226088                   | -1.924384×10 <sup>-8</sup>  | 3.226088                    | 2.124615                   | 0.6697977                   | -0.731132                  | -3.226088                   |
| 0.4863938                | 0.367139                   | 0.2104715                   | 0.08573235                 | -0.1195606                 | -0.3138301                  | 0.07348128               | 0.4612966                   | 0.2667083                  | 0.05683469                 | -0.07988453                 | -0.2739108                 | -0.4318273                  | 4.135681×10 <sup>-9</sup>   | 0.4318273                   | 0.2739109                  | 0.07988453                  | -0.0568346                 | -4.135681×10 <sup>-9</sup>  |
| -2.479308                | -1.795977                  | -0.6012575                  | 0.6167458                  | 1.746893                   | 2.092654                    | 0.07628287               | -1.948193                   | -1.634066                  | -0.5814737                 | 0.4919345                   | 1.536254                   | 1.879329                    | -9.399297×10 <sup>-10</sup> | -1.879329                   | -1.536254                  | -0.4919345                  | 0.5814737                  | 9.399297×10 <sup>-10</sup>  |
| -4.459199                | -3.239942                  | -0.8977427                  | 0.6826669                  | 2.370922                   | 4.524997                    | 0.07578146               | -4.382303                   | -2.264662                  | -0.6817699                 | 0.6007663                   | 2.152105                   | 4.210878                    | 2.347034×10 <sup>-10</sup>  | -4.210878                   | -2.152105                  | -0.6007663                  | 0.6817699                  | -2.347034×10 <sup>-10</sup> |
| 0.1550864                | 0.1611718                  | 0.1492474                   | 0.1245987                  | 0.09639145                 | 0.07797703                  | 0.07388198               | 0.06988651                  | 0.05125056                 | 0.02105121                 | -0.008413693                | -0.02659669                | -0.02344966                 | -6.046931×10 <sup>-11</sup> | 0.02344966                  | 0.02659669                 | 0.008413693                 | -0.0210512                 | 6.046931×10 <sup>-11</sup>  |
| 4.780359                 | 3.565968                   | 1.189369                    | -0.4505428                 | -2.206755                  | -4.399285                   | 0.06348492               | 4.535454                    | 2.37937                    | 0.7252095                  | -0.6263983                  | -2.225358                  | -4.279277                   | 1.578955×10 <sup>-11</sup>  | 4.279277                    | 2.225358                   | -0.6263983                  | -0.725209                  | -1.578955×10 <sup>-11</sup> |
| 2.842723                 | 2.13438                    | 0.8682924                   | -0.4377229                 | -1.679594                  | -2.084069                   | 0.03778704               | 2.16835                     | 1.796129                   | 0.6296557                  | -0.5417072                  | -1.677932                  | -2.037682                   | -4.744667×10 <sup>-12</sup> | 2.037682                    | 1.677932                   | 0.5417072                   | -0.629655                  | 4.744667×10 <sup>-12</sup>  |
| 1.18852×10 <sup>-8</sup> | -6.576042×10 <sup>-9</sup> | -7.016847×10 <sup>-10</sup> | 1.244056×10 <sup>-10</sup> | 2.191792×10 <sup>-10</sup> | -7.501514×10 <sup>-11</sup> | 1.8954×10 <sup>-11</sup> | -1.177769×10 <sup>-13</sup> | 3.047327×10 <sup>-12</sup> | 1.092988×10 <sup>-12</sup> | -4.777445×10 <sup>-12</sup> | 5.435255×10 <sup>-12</sup> | -3.554444×10 <sup>-12</sup> | 3.228026×10 <sup>-12</sup>  | -2.990397×10 <sup>-12</sup> | 1.375859×10 <sup>-12</sup> | -1.298123×10 <sup>-12</sup> | 7.488428×10 <sup>-13</sup> | -3.228026×10 <sup>-12</sup> |
| -2.842723                | -2.13438                   | -0.8682924                  | 0.4377229                  | 1.679594                   | 2.084069                    | -0.03778704              | -2.16835                    | -1.796129                  | -0.6296557                 | 0.5417072                   | 1.677932                   | 2.037682                    | -7.622464×10 <sup>-12</sup> | -2.037682                   | -1.677932                  | -0.5417072                  | 0.6296555                  | 7.622464×10 <sup>-12</sup>  |
| -4.780359                | -3.565968                  | -1.189369                   | 0.4505428                  | 2.206755                   | 4.399285                    | -0.06348492              | -4.535454                   | -2.37937                   | -0.7252095                 | 0.6263983                   | 2.225358                   | 4.279277                    | 2.756043×10 <sup>-11</sup>  | -4.279277                   | -2.225358                  | -0.6263983                  | 0.725209                   | -2.756043×10 <sup>-11</sup> |
| -0.1550866               | -0.1611716                 | -0.1492474                  | -0.1245987                 | -0.09639145                | -0.07797703                 | -0.07388197              | -0.06988651                 | -0.05125056                | -0.02105121                | 0.008413693                 | 0.02659669                 | 0.02344966                  | -1.059221×10 <sup>-10</sup> | -0.02344966                 | -0.02659669                | -0.008413693                | 0.0210512                  | 1.059221×10 <sup>-10</sup>  |
| 4.459199                 | 3.239941                   | 0.8977427                   | -0.6826669                 | -2.370922                  | -4.524997                   | -0.07578147              | 4.382303                    | 2.264662                   | 0.6817699                  | -0.6007663                  | -2.152105                  | -4.210878                   | 4.094768×10 <sup>-10</sup>  | 4.210878                    | 2.152105                   | 0.6007663                   | -0.681769                  | -4.094768×10 <sup>-10</sup> |
| 2.479312                 | 1.795976                   | 0.6012571                   | -0.6167458                 | -1.746893                  | -2.092654                   | -0.07628286              | 1.948193                    | 1.634066                   | 0.5814737                  | -0.4919345                  | -1.536254                  | -1.879329                   | -1.611647×10 <sup>-9</sup>  | 1.879329                    | 1.536254                   | 0.4919345                   | -0.581473                  | 1.611647×10 <sup>-9</sup>   |
| -0.4863957               | -0.3671418                 | -0.2104716                  | -0.08573185                | 0.1195605                  | 0.3138302                   | -0.07348132              | -0.4612966                  | -0.2667083                 | -0.05683469                | 0.07988452                  | 0.2739109                  | 0.4318273                   | 6.850159×10 <sup>-9</sup>   | -0.4318273                  | -0.2739109                 | -0.07988453                 | 0.0568346                  | -6.850159×10 <sup>-9</sup>  |
| -3.766967                | -2.57294                   | -0.9831755                  | 0.5157536                  | 2.056962                   | 3.229076                    | -0.05823381              | -3.354953                   | -2.213405                  | -0.7311327                 | 0.6697977                   | 2.124615                   | 3.226088                    | -3.125512×10 <sup>-8</sup>  | -3.226088                   | -2.124615                  | -0.6697977                  | 0.731132                   | 3.125512×10 <sup>-8</sup>   |
| -8.09139                 | -4.110792                  | -1.417137                   | 0.9457983                  | 3.233922                   | 4.783472                    | -0.02175395              | -4.840696                   | -3.336369                  | -1.142106                  | 1.025808                    | 3.195694                   | 4.675291                    | 1.254052×10 <sup>-8</sup>   | -4.675291                   | -3.195694                  | -1.025808                   | 1.142106                   | -1.254052×10 <sup>-8</sup>  |

