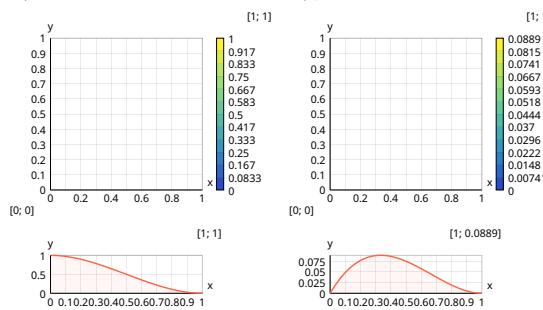


For vertical displacements w	For rotations θ_x	For rotations θ_y
$N_{1,w}(\xi, \eta) = \Phi_{1a}(\xi) \cdot \Phi_{1b}(\eta)$	$N_{1,0,}(\xi, \eta) = \Phi_{2a}(\xi) \cdot \Phi_{1b}(\eta)$	$N_{1,0,}(\xi, \eta) = \Phi_{1a}(\xi) \cdot \Phi_{2b}(\eta)$
$N_{2,w}(\xi, \eta) = \Phi_{3a}(\xi) \cdot \Phi_{1b}(\eta)$	$N_{2,0,}(\xi, \eta) = \Phi_{4a}(\xi) \cdot \Phi_{1b}(\eta)$	$N_{2,0,}(\xi, \eta) = \Phi_{3a}(\xi) \cdot \Phi_{2b}(\eta)$
$N_{3,w}(\xi, \eta) = \Phi_{3a}(\xi) \cdot \Phi_{3b}(\eta)$	$N_{3,0,}(\xi, \eta) = \Phi_{4a}(\xi) \cdot \Phi_{3b}(\eta)$	$N_{3,0,}(\xi, \eta) = \Phi_{3a}(\xi) \cdot \Phi_{4b}(\eta)$
$N_{4,w}(\xi, \eta) = \Phi_{1a}(\xi) \cdot \Phi_{3b}(\eta)$	$N_{4,0,}(\xi, \eta) = \Phi_{2a}(\xi) \cdot \Phi_{3b}(\eta)$	$N_{4,0,}(\xi, \eta) = \Phi_{1a}(\xi) \cdot \Phi_{4b}(\eta)$

$$\begin{aligned} \text{For twist } \Psi \\ N_{1,\Psi}(\xi; \eta) &= \Phi_{2a}(\xi) \cdot \Phi_{2b}(\eta) \\ N_{2,\Psi}(\xi; \eta) &= \Phi_{4a}(\xi) \cdot \Phi_{2b}(\eta) \\ N_{3,\Psi}(\xi; \eta) &= \Phi_{4a}(\xi) \cdot \Phi_{4b}(\eta) \\ N_{4,\Psi}(\xi; \eta) &= \Phi_{2a}(\xi) \cdot \Phi_{4b}(\eta) \end{aligned}$$



$N_{1,w}$ shape function plot



Constitutive matrix (stress - strain relationship)

$$D_{1,1} = \frac{E \cdot t^3}{12 \cdot (1 - \nu^2)} = \frac{35000 \cdot 0.2^3}{12 \cdot (1 - 0.2^2)} = 24.305556 \text{ kNm}$$

$$D = D_{1,1} \cdot \mathbf{hp}\left(\begin{bmatrix} 1; v_1; 0 & | & v_2; 1; 0 & | & 0; 0; \frac{1-v}{2} \end{bmatrix}\right) = 24.305556 \cdot \mathbf{hp}\left(\begin{bmatrix} 1; 0; 2; 0 & | & 0; 2; 1; 0 & | & 0; 0; \frac{1-0.2}{2} \end{bmatrix}\right) =$$

$$\begin{bmatrix} 24.305556 & 4.861111 & 0 & | & \text{kNm} \\ 4.861111 & 24.305556 & 0 & | & \\ 0 & 0 & 9.722222 & | & \end{bmatrix}$$

Element stiffness matrix calculation ... ▾

Element stiffness matrix coefficients (above the main diagonal only)

$$K_e = D_{1,1} \cdot K_e = 24.305556 \cdot K_e =$$

Element load vector

$$\begin{aligned} \mathbf{e} &= \frac{\mathbf{q} \cdot \mathbf{A}_1}{24} \cdot \left[6; \mathbf{a}_1; \mathbf{b}_1; \frac{\mathbf{A}_1}{6}; 6; -\mathbf{a}_1; \mathbf{b}_1; \frac{-\mathbf{A}_1}{6}; 6; -\mathbf{a}_1; -\mathbf{b}_1; \frac{\mathbf{A}_1}{6}; 6; \mathbf{a}_1; -\mathbf{b}_1; \frac{-\mathbf{A}_1}{6} \right] = \frac{10 \cdot 0.36}{24} \cdot \left[6; 0.6; 0.6; \frac{0.36}{6}; 6; -0.6; \right. \\ &\quad \left. 0.6; -\frac{0.36}{6}; 6; -0.6; -0.6; \frac{0.36}{6}; 6; 0.6; -0.6; -\frac{0.36}{6} \right] = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 0.9 \ -0.09 \ 0.09 \ -0.009 \end{aligned}$$

0.9 -0.09 -0.09 0.009 0.9 0.09 -0.09 -0.009] kN

Solution

Global stiffness matrix

L Global load vector

$$\vec{r} = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \dots 0.009] \text{ kN}$$

Solution of the system of equations

$\mathbf{z} = \text{slsolve}(\mathbf{K}\mathbf{F}) = [0 \ 0.5523352 \ 0.382696 \ -0.4159901 \ 0.2028057 \ 0.3732616 \ 0.2648531 \ -0.1936926 \ 0.2989084]$

0.3091046 0.04831304 -0.02500182 0.2612117 0.3426307 -0.1651507 0.1275183 0.1211517

0.4681261 -0.2671228 0.2293407 ... -0.4159902] mm

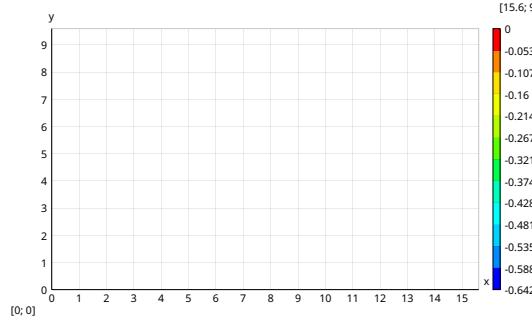
Results

joint displacements

transp(\mathbf{W}_2) =

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.139	0.398	0.556	0.56	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	...	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	...	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	...	0.299
0.139	0.398	0.556	0.56	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0

mm



Bending moments

$Z_j(j) = \text{slice}(\mathcal{Z}; k_1 \cdot (j-1) + 1; k_1 \cdot j)$

$Z_e(e) = \text{hp}([Z_j(e_{j,e}, 1); Z_j(e_{j,e}, 2); Z_j(e_{j,e}, 3); Z_j(e_{j,e}, 4)])$

Average bending moments at joints, KNm/m

$M_j =$

1.500275	0.3097747	0.2197379	0.1563434	0.1570478	0.9983225	0.1564233	0.1519636	0.1942465	0.151963	0.1564238	0.9983224	0.1570487	0.1563432	0.2197373	0.3097501	1.500258	8.50221	6.47933	5.777678	...	1.500075
1.561323	7.805992	9.338126	7.6683	3.333101	-28.361989	3.146397	7.323883	8.895047	7.323882	3.146398	-28.361989	3.333102	7.6683	9.338126	7.805978	1.56134	0.3220403	5.382484	7.126847	...	1.561519
0.088697	3.766245	4.0861465	-2.479522	-4.459403	0.1550078	4.780402	2.842763	-2.410353e+08	-2.842763	-4.780402	-0.1550078	4.459403	2.479522	-0.4861463	-3.766235	-0.088697	4.109025	2.572475	0.3669336	...	0.088698

Bending moments for the plate

Bending moments - M_x

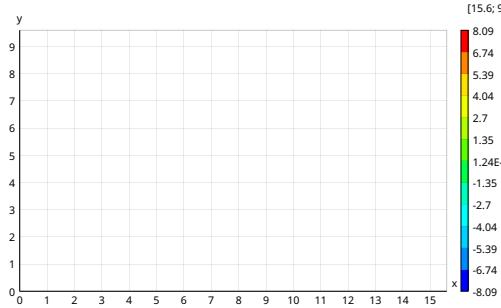
transp(\mathcal{M}_x) =

1.500275	8.50221	11.065783	10.476631	6.796938	0.8844119	-31.234001	0.3020629	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860142	5.919519	8.876859	8.779239	5.643938	0.3020627	...	1.500259	
0.3097747	6.47933	9.431828	8.876831	4.922389	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360952	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	7.249978	3.808562	-3.260011	...	0.3097374	
0.2197379	5.777678	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-6.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	...	0.2197445	
0.1563434	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.823661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-6.926775	3.304594	6.926775	6.797415	2.913644	-3.823661	...	0.1563406	
0.1570478	7.269659	10.37227	9.556617	4.972566	-5.268395	-3.257726	-16.118845	-8.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	...	0.1570494
0.9983225	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	...	0.9983223	
0.1564233	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	...	0.1564268	
0.1519636	5.873703	8.92797	8.229786	3.7733483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	6.626467	6.469771	2.594271	-4.009025	...	0.1519571		
0.1942465	5.514146	8.380234	7.692958	3.468366	-2.491073	-5.742609	-3.07333	2.310482	5.979232	6.142664	2.786926	-5.118358	-2.376799	-2.376799	6.142664	5.979232	2.310482	-3.07336	...	0.194254		
0.151963	5.873204	8.92797	8.229786	3.7733483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	6.626467	6.469771	2.594271	-4.009025	...	0.1519564		
0.1564238	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	...	0.1564266	
0.9983225	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	...	0.9983221	
0.1570487	7.269659	10.37227	9.556617	4.972566	-5.268395	-16.118845	-8.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	...	0.1570582	
0.1563432	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.823661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.823661	...	0.1563432	
0.2197373	5.777677	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	...	0.2197303	
0.3097501	6.479335	9.43183	8.876831	4.922388	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360953	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	7.249978	3.808562	-3.260012	...	0.3094814	
1.500258	8.502224	11.065783	10.476631	6.796939	0.8844112	-31.234001	0.302063	5.643937	8.779239													

Bending moments M_{xy} transp(\mathbf{M}_{xy}) =

8.088697	4.109025	1.416336	-0.9453079	-3.232331	-4.781353	0.02176366	4.838587	3.334748	1.141499	-1.025238	-3.194104	-4.673215	9.790543e-9	4.673215	3.194104	1.025238	-1.141499	-
3.766245	2.572475	0.9829256	-0.515595	-2.056503	-3.228613	0.05825511	3.354524	2.212955	0.7309473	-0.6696087	-2.124156	-3.225638	-2.241521e-8	3.225638	2.124156	0.6696087	-0.7309473	-
0.4861465	0.3669336	0.21034	0.08576266	-0.1194038	-0.3136824	0.0734795	0.4611397	0.2665259	0.05675809	-0.07980332	-0.2737226	-0.431664	4.761034e-9	0.431664	0.2737226	0.07980332	-0.05675809	-0
-2.479522	-1.796141	-0.601364	0.6167145	1.746922	2.09269	-0.07626484	-1.948268	-1.634146	-0.5815058	0.4919668	1.536331	1.879395	-1.027996e-9	-1.879395	-1.536331	-0.4919668	0.5815058	1
-4.459403	-3.240091	-0.8978376	0.6826157	2.370906	4.525	0.07575709	-4.382355	-2.264704	-0.6817866	0.6007781	2.152138	4.210913	2.395547e-10	-4.210913	-2.152138	-0.6007781	0.6817866	2
0.1550078	0.1611012	0.1491877	0.1245521	0.09635607	0.07794845	0.07385991	0.06986752	0.05123571	0.0210446	-0.008412859	-0.02659053	-0.02344541	-5.804372e-11	0.02344541	0.02659053	0.008412859	-0.0210446	-0
4.780402	3.565976	1.18935	-0.4505739	-2.206794	-4.399329	0.06346754	4.535463	2.379375	0.7252099	-0.6264028	-2.225368	-4.279295	1.444385e-11	4.279295	2.225368	0.6264028	-0.7252099	-
2.842763	2.134393	0.8682899	-0.4377384	-1.679619	-2.084094	0.03777839	2.168358	1.796138	0.6296579	-0.5417111	-1.677943	-2.037693	-3.548115e-12	2.037693	1.677943	0.5417111	-0.6296579	-1
-2.410353e-8	3.062399e-8	-4.397001e-9	3.066824e-10	-8.12632e-11	3.636829e-10	-1.325189e-11	4.764349e-12	-7.14584e-13	1.790365e-13	-3.8029e-13	2.910935e-13	0	1.375317e-13	-9.949168e-13	2.393e-12	-3.712996e-12	2.380218e-12	3.02
-2.842763	-2.134393	-0.8682899	0.4377384	1.679619	2.084094	-0.03777839	-2.168358	-1.796138	-0.6296579	0.5417111	1.677943	2.037693	4.127238e-12	-2.037693	-1.677943	-0.5417111	0.6296579	1
-4.780402	-3.565976	-1.18935	0.4505739	2.206794	4.399329	-0.06346754	-4.535463	-2.379375	-0.7252099	0.6264028	2.225368	4.279295	-1.911611e-11	-4.279295	-2.225368	-0.6264028	0.7252099	2
-0.1550078	-0.1611012	-0.1491877	-0.1245521	-0.09635607	-0.07794845	-0.07385991	-0.06986752	-0.05123571	-0.0210446	0.008412859	0.02659053	0.02344541	7.954439e-11	-0.02344541	-0.02659053	-0.008412859	0.0210446	0
4.459403	3.240091	0.8978376	-0.6826157	-2.370906	-4.525	-0.07575709	4.382355	2.264704	0.6817866	-0.6007781	-2.152138	-4.210913	-3.335117e-10	4.210913	2.152138	0.6007781	-0.6817866	-
2.479522	1.796141	0.601364	-0.6167145	-1.746922	-2.09269	-0.07626484	1.948268	1.634146	0.5815058	-0.4919668	1.536331	1.879395	1.414276e-9	1.879395	1.536331	0.4919668	-0.5815058	-1
-0.4861463	-0.3669336	-0.21034	-0.08576274	0.1194038	0.3136824	-0.07347951	-0.4611397	-0.2665259	-0.05675809	0.07980332	0.2737226	0.431664	-6.33653e-9	-0.431664	-0.2737226	-0.07980332	0.05675809	0
-3.766245	-2.572475	-0.9829263	0.5155949	2.056503	3.228613	-0.05825506	-3.354524	-2.212955	-0.7309473	0.6696087	2.124156	3.225638	2.942834e-8	-3.225638	-2.124156	-0.6696087	0.7309473	2
-8.088697	-4.109025	-1.416336	0.9453084	3.232331	4.781353	-0.0217637	-4.838587	-3.334748	-1.141499	1.025238	3.194104	4.673215	-1.247931e-8	-4.673215	-3.194104	-1.025238	1.141499	3

KNm/m



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