

Finite Element Analysis of Flat Slab

Using numerical formulation of Bogner-Fox-Schmit (BFS) plate element

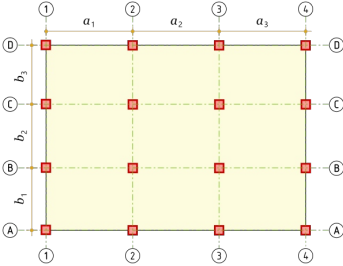
Input data

Span lengths

$r = \mathbf{hp}([3.6; 4.2; 4.2; 3.6]) = [3.6 \ 4.2 \ 4.2 \ 3.6] \text{ m}$

$r = \mathbf{hp}([3; 3.6; 3]) = [3 \ 3.6 \ 3] \text{ m}$

Number of axes - $n_{sa} = \text{len}(r) + 1 = 5$, $n_{sb} = \text{len}(r) + 1 = 4$



Axis coordinates - $\alpha_s = [0 \ 3.6 \ 7.8 \ 12 \ 15.6] \text{ m}$, $\eta_s = [0 \ 3 \ 6.6 \ 9.6] \text{ m}$

Slab dimensions - $l_a \Rightarrow x_{s,5} = 15.6 \text{ m}$, $l_b \Rightarrow y_{s,4} = 9.6 \text{ m}$

Thickness - $t = 0.2 \text{ m}$

Load - $q = 10 \text{ kN/m}^2$

Modulus of elasticity - $E = 35000 \text{ MPa}$

Poisson's ratio - $\nu = 0.2$

Finite element mesh

We will use Bogner-Fox-Schmit rectangular finite element with $n_{\text{DOF}_5} = 16$

Element dimensions - $a_1 = 0.6 \text{ m}$, $b_1 = 0.6 \text{ m}$

Number of elements and joints along a and b -

$n_a = \text{ceiling}(\frac{a}{a_1}) = \text{ceiling}(\frac{a}{0.6}) = [6 \ 7 \ 7 \ 6]$, $n_{ea} = \text{sum}(n_a) = 26$, $n_{ja} = n_{ea} + 1 = 26 + 1 = 27$

$n_b = \text{ceiling}(\frac{b}{b_1}) = \text{ceiling}(\frac{b}{0.6}) = [5 \ 6 \ 5]$, $n_{eb} = \text{sum}(n_b) = 16$, $n_{jb} = n_{eb} + 1 = 16 + 1 = 17$

Total number of elements - $n_e = n_{ea} \cdot n_{eb} = 26 \cdot 16 = 416$

Total number of joints - $n_j = n_{ja} \cdot n_{jb} = 27 \cdot 17 = 459$

Supported joints count - $n_s = n_{sa} \cdot n_{sb} = 5 \cdot 4 = 20$

Joint coordinates

$j = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.6 \ 0.6 \ 0.6 \dots \ 15.6] \text{ m}$

$j = [0 \ 0.6 \ 1.2 \ 1.8 \ 2.4 \ 3 \ 3.6 \ 4.2 \ 4.8 \ 5.4 \ 6 \ 6.6 \ 7.2 \ 7.8 \ 8.4 \ 9 \ 9.6 \ 0 \ 0.6 \ 1.2 \dots \ 9.6] \text{ m}$

Numbers of joints at elements' corners

$\text{transp}(e_j) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 18 & 19 & 20 & 21 & \dots & 441 \\ 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 35 & 36 & 37 & 38 & \dots & 458 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 36 & 37 & 38 & 39 & \dots & 459 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 19 & 20 & 21 & 22 & \dots & 442 \end{bmatrix}$

Supported joints

$j = [1 \ 6 \ 12 \ 17 \ 103 \ 108 \ 114 \ 119 \ 222 \ 227 \ 233 \ 238 \ 341 \ 346 \ 352 \ 357 \ 443 \ 448 \ 454 \ 459]$

Joints for element e - $j_e(e) = \text{row}(e_j; e)$

17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425	442	459
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400	416	432
15	31	47	63	79	95	111	127	143	159	175	191	207	223	239	255	271	287	303	319	335	351	367	383	399	415	431
14	30	46	62	78	94	110	126	142	158	174	190	206	222	238	254	270	286	302	318	334	350	366	382	398	414	430
13	29	45	61	77	93	109	125	141	157	173	189	205	221	237	253	269	285	301	317	333	349	365	381	397	413	429
12	28	44	60	76	92	108	124	140	156	172	188	204	220	236	252	268	284	300	316	332	348	364	380	396	412	428
11	27	43	59	75	91	107	123	139	155	171	187	203	219	235	251	267	283	299	315	331	347	363	379	395	411	427
10	26	42	58	74	90	106	122	138	154	170	186	202	218	234	250	266	282	298	314	330	346	362	378	394	410	426
9	25	41	57	73	89	105	121	137	153	169	185	201	217	233	249	265	281	297	313	329	345	361	377	393	409	425
8	24	40	56	72	88	104	120	136	152	168	184	200	216	232	248	264	280	296	312	328	344	360	376	392	408	424
7	23	39	55	71	87	103	119	135	151	167	183	199	215	231	247	263	279	295	311	327	343	359	375	391	407	423
6	22	38	54	70	86	102	118	134	150	166	182	198	214	230	246	262	278	294	310	326	342	358	374	390	406	422
5	21	37	53	69	85	101	117	133	149	165	181	197	213	229	245	261	277	293	309	325	341	357	373	389	405	421
4	20	36	52	68	84	100	116	132	148	164	180	196	212	228	244	260	276	292	308	324	340	356	372	388	404	420
3	19	35	51	67	83	99	115	131	147	163	179	195	211	227	243	259	275	291	307	323	339	355	371	387	403	419
2	18	34	50	66	82	98	114	130	146	162	178	194	210	226	242	258	274	290	306	322	338	354	370	386	402	418
1	17	33	49	65	81	97	113	129	145	161	177	193	209	225	241	257	273	289	305	321	337	353	369	385	401	417
0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400	416

Finite element formulation

Shape functions

Along dimension a

Base functions	First derivatives	Second derivatives
$\Phi_{1a}(\xi) = 1 - \xi^2 \cdot (3 - 2 \cdot \xi)$	$\Phi'_{1a}(\xi) = -6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$	$\Phi''_{1a}(\xi) = -\frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$
$\Phi_{2a}(\xi) = \xi \cdot a_1 \cdot (1 - \xi \cdot (2 - \xi))$	$\Phi'_{2a}(\xi) = 1 - \xi \cdot (4 - 3 \cdot \xi)$	$\Phi''_{2a}(\xi) = -\frac{2}{a_1} \cdot (2 - 3 \cdot \xi)$
$\Phi_{3a}(\xi) = \xi^2 \cdot (3 - 2 \cdot \xi)$	$\Phi'_{3a}(\xi) = 6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$	$\Phi''_{3a}(\xi) = \frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$
$\Phi_{4a}(\xi) = \xi^2 \cdot a_1 \cdot (-1 + \xi)$	$\Phi'_{4a}(\xi) = -\xi \cdot (2 - 3 \cdot \xi)$	$\Phi''_{4a}(\xi) = -\frac{2}{a_1} \cdot (1 - 3 \cdot \xi)$

Along dimension b

Base functions	First derivatives	Second derivatives
$\Phi_{1b}(\eta) = 1 - \eta^2 \cdot (3 - 2 \cdot \eta)$	$\Phi'_{1b}(\eta) = -6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$	$\Phi''_{1b}(\eta) = -\frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$
$\Phi_{2b}(\eta) = \eta \cdot b_1 \cdot (1 - \eta \cdot (2 - \eta))$	$\Phi'_{2b}(\eta) = 1 - \eta \cdot (4 - 3 \cdot \eta)$	$\Phi''_{2b}(\eta) = -\frac{2}{b_1} \cdot (2 - 3 \cdot \eta)$
$\Phi_{3b}(\eta) = \eta^2 \cdot (3 - 2 \cdot \eta)$	$\Phi'_{3b}(\eta) = 6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$	$\Phi''_{3b}(\eta) = \frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$
$\Phi_{4b}(\eta) = \eta^2 \cdot b_1 \cdot (-1 + \eta)$	$\Phi'_{4b}(\eta) = -\eta \cdot (2 - 3 \cdot \eta)$	$\Phi''_{4b}(\eta) = -\frac{2}{b_1} \cdot (1 - 3 \cdot \eta)$

For vertical displacements w

$N_{1,w}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{1,\theta_x}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{1,\theta_y}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{2,w}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{2,\theta_x}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{2,\theta_y}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{2b}(\eta)$

Global load vector

$$\vec{F} = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ \dots \ 0.009] \text{ kN}$$

Solution of the system of equations

$$\vec{f} = \text{solve}(\mathbf{K}, \vec{F}) = [0 \ 0.5523613 \ 0.3827392 \ -0.4161287 \ 0.2028064 \ 0.3732832 \ 0.2648463 \ -0.1937261 \ 0.2989047 \ 0.3091182 \ 0.04830843 \ -0.02501454 \ 0.261207 \ 0.3426386 \ -0.1651493 \ 0.1275073 \ 0.1211489 \ 0.4681287 \ -0.2671179 \ 0.2293302 \ \dots \ -0.4161286] \text{ mm}$$

Joint displacements

mm


$$Z_j(j) = \text{slice}(\mathcal{Z}; k_1 \cdot (j-1) + 1; k_1 \cdot j)$$

$$Z_e(e) = \mathbf{hp}([Z_j(e_{j,e,1}); Z_j(e_{j,e,2}); Z_j(e_{j,e,3}); Z_j(e_{j,e,4})])$$

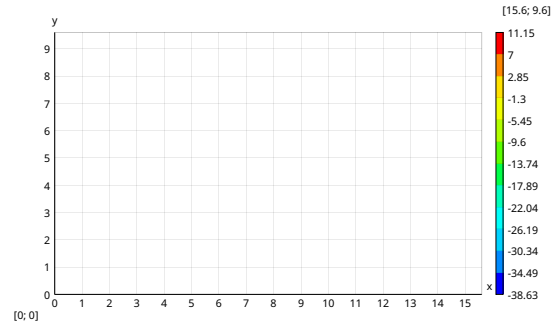
Average bending moments at joints, kNm/m

$$M_j =$$

Bending moments for the plate

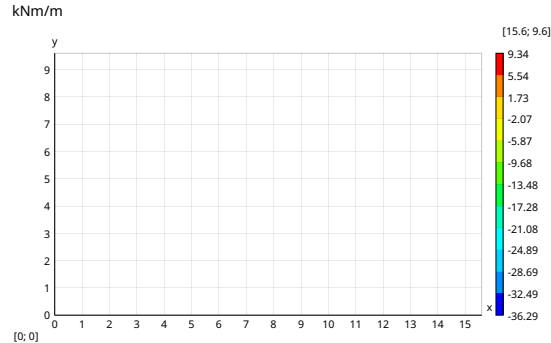
Bending moments - M_x

$$\text{transp}(Mx) =$$



Bending moments M_y

transp (M_y) =																								
1.566914	0.3205972	0.2333587	0.2147852	0.1682816	0.1798035	1.015789	0.1794301	0.1669894	0.2098191	0.2091906	0.1663467	0.1776119	0.9891482	0.1776123	0.1663459	0.2091915	0.2098181	0.1669903	0.1794296	...	1.566846			
7.805101	5.382201	4.308412	4.134952	4.742331	6.340395	8.230714	6.28316	4.611981	3.89228	3.867499	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805153			
9.33778	7.126518	5.800074	5.553032	6.414898	8.11577	9.046666	8.014704	6.188836	5.14485	5.116777	6.090589	7.821224	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337789			
7.668097	5.41756	4.262493	3.920413	4.233692	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093			
3.333	-0.3280779	0.5396346	0.405616	-1.17438	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964996	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964995	-1.399574	-3.531204	...	3.332998			
-28.361735	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736			
3.14639	-0.5066696	0.3485146	0.1799353	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146398			
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874			
8.895019	6.740122	5.350863	4.96126	5.608937	6.913063	7.572529	6.693158	5.141784	4.180525	4.149637	5.027791	6.456862	7.212973	6.456862	5.027791	4.149637	4.180525	5.141784	6.693158	...	8.895035			
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874			
3.14639	-0.5066695	0.3485145	0.1799354	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146397			
-28.361734	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736			
3.332997	-0.3280781	0.5396345	0.405616	-1.174379	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964994	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964996	-1.399574	-3.531204	...	3.333003			
7.668093	5.41756	4.262493	3.920414	4.233691	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093			
9.337791	7.126529	5.800079	5.553029	6.414899	8.115769	9.046667	8.014704	6.188836	5.14485	5.116777	6.090589	7.821225	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337788			
7.80518	5.382247	4.308397	4.134958	4.742326	6.340398	8.230713	6.283161	4.61198	3.892281	3.867498	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805067			
1.566813	0.3204128	0.2333748	0.2147746	0.1682944	0.1797983	1.015789	0.1794292	0.1669914	0.2098169	0.2091927	0.166345	0.1776128	0.9891482	0.177612	0.1663463	0.209191	0.2098187	0.1669898	0.1794301	...	1.56695			



Bending moments M_{xy}

transp (M_{xy}) =																								
8.091391	4.110857	1.417137	-0.9458016	-3.233923	-4.783472	0.02175397	4.840696	3.336369	1.142106	-1.025808	-3.195694	-4.675291	8.203632×10 ⁻⁹	4.675291	3.195694	1.025808	-1.142106							
3.766896	2.572942	0.9831798	-0.5157529	-2.056961	-3.229076	0.05823395	3.354953	2.213405	0.7311327	-0.6697977	-2.124616	-3.226088	-1.924384×10 ⁻⁸	3.226088	2.124615	0.6697977	-0.731132							
0.4863938	0.367139	0.2104715	0.08573235	-0.1195606	-0.3138301	0.07348128	0.4612966	0.2667083	0.05683469	-0.07988453	-0.2739108	-0.4318273	4.135681×10 ⁻⁹	0.4318273	0.2739109	0.07988453	-0.0568346							
-2.479308	-1.795977	-0.6012575	0.6167458	1.746893	2.092654	0.07628287	-1.948193	-1.634066	-0.5814737	0.4919345	1.536254	1.879329	-9.399297×10 ⁻¹⁰	-1.879329	-1.536254	-0.4919345	0.5814737							
-4.459199	-3.239942	-0.8977427	0.6826669	2.370922	4.524997	0.07578146	-4.382303	-2.264662	-0.6817699	0.6007663	2.152105	4.210878	2.347034×10 ⁻¹⁰	-4.210878	-2.152105	-0.6007663	0.6817699							
0.1550864	0.1611718	0.1492474	0.1245987	0.09639145	0.07797703	0.07388198	0.06988651	0.05125056	0.02105121	-0.008413693	-0.02659669	-0.02344966	-6.046931×10 ⁻¹¹	0.02344966	0.02659669	0.008413693	-0.0210512							
4.780359	3.565968	1.189369	-0.4505428	-2.206755	-4.399285	0.06348492	4.535454	2.37937	0.7252095	-0.6263983	-2.225358	-4.279277	1.578955×10 ⁻¹¹	4.279277	2.225358	-0.6263983	-0.725209							
2.842723	2.13438	0.8682924	-0.4377229	-1.679594	-2.084069	0.03778704	2.16835	1.796129	0.6296557	-0.5417072	-1.677932	-2.037682	-4.744667×10 ⁻¹²	2.037682	1.677932	0.5417072	-0.629655							
1.18852×10 ⁻⁸	-6.576042×10 ⁻⁹	-7.016847×10 ⁻¹⁰	1.244056×10 ⁻¹⁰	2.191792×10 ⁻¹⁰	-7.501514×10 ⁻¹¹	1.8954×10 ⁻¹¹	-1.177769×10 ⁻¹³	3.047327×10 ⁻¹²	1.092988×10 ⁻¹²	-4.777445×10 ⁻¹²	5.435255×10 ⁻¹²	-3.554444×10 ⁻¹²	3.228026×10 ⁻¹²	-2.990397×10 ⁻¹²	1.375859×10 ⁻¹²	-1.298123×10 ⁻¹²	7.488428×10 ⁻¹¹							
-2.842723	-2.13438	-0.8682924	0.4377229	1.679594	2.084069	-0.03778704	-2.16835	-1.796129	-0.6296557	0.5417072	1.677932	2.037682	-7.622464×10 ⁻¹²	-2.037682	-1.677932	-0.5417072	0.6296555							
-4.780359	-3.565968	-1.189369	0.4505428	2.206755	4.399285	-0.06348492	-4.535454	-2.37937	-0.7252095	0.6263983	2.225358	4.279277	2.756043×10 ⁻¹¹	-4.279277	-2.225358	-0.6263983	0.725209							
-0.1550866	-0.1611716	-0.1492474	-0.1245987	-0.09639145	-0.07797703	-0.07388197	-0.06988651	-0.05125056	-0.02105121	0.008413693	0.02659669	0.02344966	-1.059221×10 ⁻¹⁰	-0.02344966	-0.02659669	-0.008413693	0.0210512							
4.459199	3.239941	0.8977427	-0.6826669	-2.370922	-4.524997	-0.07578147	4.382303	2.264662	0.6817699	-0.6007663	-2.152105	-4.210878	4.094768×10 ⁻¹⁰	4.210878	2.152105	0.6007663	-0.681769							
2.479312	1.795976	0.6012571	-0.6167458	-1.746893	-2.092654	-0.07628286	1.948193	1.634066	0.5814737	-0.4919345	-1.536254	-1.879329	-1.611647×10 ⁻⁹	1.879329	1.536254	0.4919345	-0.581473							
-0.4863957	-0.3671418	-0.2104716	-0.08573185	0.1195605	0.3138302	-0.07348132	-0.4612966	-0.2667083	-0.05683469	0.07988452	0.2739109	0.4318273	6.850159×10 ⁻⁹	-0.4318273	-0.2739109	-0.07988453	0.0568346							
-3.766967	-2.57294	-0.9831755	0.5157536	2.056962	3.229076	-0.05823381	-3.354953	-2.213405	-0.7311327	0.6697977	2.124615	3.226088	-3.125512×10 ⁻⁸	-3.226088	-2.124615	-0.6697977	0.7311327							
-8.09139	-4.110792	-1.417137	0.9457983	3.233922	4.783472	-0.02175395	-4.840696	-3.336369	-1.142106	1.025808	3.195694	4.675291	1.254052×10 ⁻⁸	-4.675291	-3.195694	-1.025808	1.142106							

