Finite Element Analysis of Flat Slab

Using analytical formulation of Bogner-Fox-Schmit (BFS) plate element

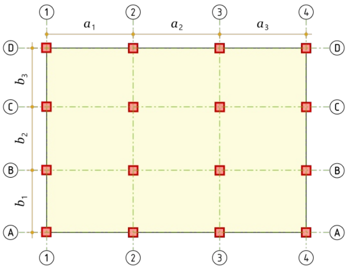
Input data

Span lengths

$a = \mathbf{hp}([3.6; 4.2; 4.2; 3.6]) = [3.6\ \ 4.2\ \ 4.2\ \ 3.6]$ m

$b = \mathbf{hp}([3; 3.6; 3]) = [3\ \ 3.6\ \ 3]$ m

Number of axes - $n_{sa} = \mathbf{len}(a) + 1 = 5$, $n_{sb} = \mathbf{len}(b) + 1 = 4$



Axis coordinates - $x_s = [0\ \ 3.6\ \ 7.8\ \ 12\ \ 15.6]$ m, $y_s = [0\ \ 3\ \ 6.6\ \ 9.6]$ m

Slab dimensions - $l_a = x_{s.5} = 15.6$ m, $l_b = y_{s.4} = 9.6$ m

Thickness - $t = 0.2$ m

Load - $q = 10$ kN/m²

Modulus of elasticity - $E = 35000$ MPa

Poisson`s ratio - $\nu = 0.2$

Finite element mesh

We will use BFS rectangular finite element with $n_{DOFs} = 16$

Element dimensions - $a_1 = 0.6$ m, $b_1 = 0.6$ m

Number of elements and joints along *a* and *b* -

$n_a = \mathbf{ceiling}\left(\dfrac{a}{a_1}\right) = \mathbf{ceiling}\left(\dfrac{a}{0.6}\right) = [6\ \ 7\ \ 7\ \ 6]$, $n_{ea} = \mathbf{sum}(n_a) = 26$, $n_{ja} = n_{ea} + 1 = 26 + 1 = 27$

$n_b = \mathbf{ceiling}\left(\dfrac{b}{b_1}\right) = \mathbf{ceiling}\left(\dfrac{b}{0.6}\right) = [5\ \ 6\ \ 5]$, $n_{eb} = \mathbf{sum}(n_b) = 16$, $n_{jb} = n_{eb} + 1 = 16 + 1 = 17$

Total number of elements - $n_e = n_{ea} \cdot n_{eb} = 26 \cdot 16 = 416$

Total number of joints - $n_j = n_{ja} \cdot n_{jb} = 27 \cdot 17 = 459$

Supported joints count - $n_s = n_{sa} \cdot n_{sb} = 5 \cdot 4 = 20$

Joint coordinates

$x_j = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.6\ 0.6\ 0.6\ \ldots\ 15.6]$ m

$y_j = [0\ 0.6\ 1.2\ 1.8\ 2.4\ 3\ 3.6\ 4.2\ 4.8\ 5.4\ 6\ 6.6\ 7.2\ 7.8\ 8.4\ 9\ 9.6\ 0\ 0.6\ 1.2\ \ldots\ 9.6]$ m

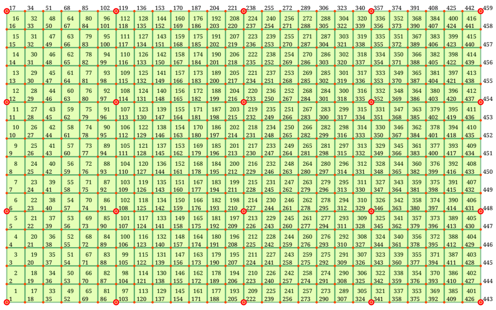
Numbers of joints at elements' corners

$$\mathbf{transp}(e_j) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 18 & 19 & 20 & 21 & \cdots & 441 \\ 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 35 & 36 & 37 & 38 & \cdots & 458 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 36 & 37 & 38 & 39 & \cdots & 459 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 19 & 20 & 21 & 22 & \cdots & 442 \end{bmatrix}$$

Supported joints

$s_j = [1\ \ 6\ \ 12\ \ 17\ \ 103\ \ 108\ \ 114\ \ 119\ \ 222\ \ 227\ \ 233\ \ 238\ \ 341\ \ 346\ \ 352\ \ 357\ \ 443\ \ 448\ \ 454\ \ 459]$

Joints for element e - $j_e(e) = \mathbf{row}(e_j; e)$



Finite element formulation

Shape functions

Along dimension *a*

Base functions

$\Phi_{1a}(\xi) = 1 - \xi^2 \cdot (3 - 2 \cdot \xi)$

$\Phi_{2a}(\xi) = \xi \cdot a_1 \cdot (1 - \xi \cdot (2 - \xi))$

$\Phi_{3a}(\xi) = \xi^2 \cdot (3 - 2 \cdot \xi)$

$\Phi_{4a}(\xi) = \xi^2 \cdot a_1 \cdot (-1 + \xi)$

First derivatives

$\Phi'_{1a}(\xi) = -6 \cdot \dfrac{\xi}{a_1} \cdot (1 - \xi)$

$\Phi'_{2a}(\xi) = 1 - \xi \cdot (4 - 3 \cdot \xi)$

$\Phi'_{3a}(\xi) = 6 \cdot \dfrac{\xi}{a_1} \cdot (1 - \xi)$

$\Phi'_{4a}(\xi) = -\xi \cdot (2 - 3 \cdot \xi)$

Second derivatives

$\Phi''_{1a}(\xi) = -\dfrac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$

$\Phi''_{2a}(\xi) = -\dfrac{2}{a_1} \cdot (2 - 3 \cdot \xi)$

$\Phi''_{3a}(\xi) = \dfrac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$

$\Phi''_{4a}(\xi) = -\dfrac{2}{a_1} \cdot (1 - 3 \cdot \xi)$

Along dimension *b*

Base functions

$\Phi_{1b}(\eta) = 1 - \eta^2 \cdot (3 - 2 \cdot \eta)$

$\Phi_{2b}(\eta) = \eta \cdot b_1 \cdot (1 - \eta \cdot (2 - \eta))$

$\Phi_{3b}(\eta) = \eta^2 \cdot (3 - 2 \cdot \eta)$

$\Phi_{4b}(\eta) = \eta^2 \cdot b_1 \cdot (-1 + \eta)$

First derivatives

$\Phi'_{1b}(\eta) = -6 \cdot \dfrac{\eta}{b_1} \cdot (1 - \eta)$

$\Phi'_{2b}(\eta) = 1 - \eta \cdot (4 - 3 \cdot \eta)$

$\Phi'_{3b}(\eta) = 6 \cdot \dfrac{\eta}{b_1} \cdot (1 - \eta)$

$\Phi'_{4b}(\eta) = -\eta \cdot (2 - 3 \cdot \eta)$

Second derivatives

$\Phi''_{1b}(\eta) = -\dfrac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$

$\Phi''_{2b}(\eta) = -\dfrac{2}{b_1} \cdot (2 - 3 \cdot \eta)$

$\Phi''_{3b}(\eta) = \dfrac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$

$\Phi''_{4b}(\eta) = -\dfrac{2}{b_1} \cdot (1 - 3 \cdot \eta)$

For vertical displacements *w*

$N_{1,w}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{1b}(\eta)$

$N_{2,w}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{1b}(\eta)$

For rotations θ_x

$N_{1,\theta_x}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{1b}(\eta)$

$N_{2,\theta_x}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{1b}(\eta)$

For rotations θ_y

$N_{1,\theta_y}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{2,\theta_y}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{3,w}(\xi;\eta) = \Phi_{3a}(\xi)\cdot\Phi_{3b}(\eta)$ $N_{3,\theta_x}(\xi;\eta) = \Phi_{4a}(\xi)\cdot\Phi_{3b}(\eta)$ $N_{3,\theta_y}(\xi;\eta) = \Phi_{3a}(\xi)\cdot\Phi_{4b}(\eta)$

$N_{4,w}(\xi;\eta) = \Phi_{1a}(\xi)\cdot\Phi_{3b}(\eta)$ $N_{4,\theta_x}(\xi;\eta) = \Phi_{2a}(\xi)\cdot\Phi_{3b}(\eta)$ $N_{4,\theta_y}(\xi;\eta) = \Phi_{1a}(\xi)\cdot\Phi_{4b}(\eta)$

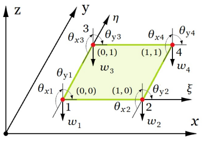
For twist ψ

$N_{1,\psi}(\xi;\eta) = \Phi_{2a}(\xi)\cdot\Phi_{2b}(\eta)$

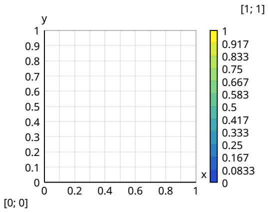
$N_{2,\psi}(\xi;\eta) = \Phi_{4a}(\xi)\cdot\Phi_{2b}(\eta)$

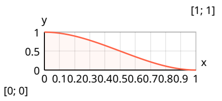
$N_{3,\psi}(\xi;\eta) = \Phi_{4a}(\xi)\cdot\Phi_{4b}(\eta)$

$N_{4,\psi}(\xi;\eta) = \Phi_{2a}(\xi)\cdot\Phi_{4b}(\eta)$

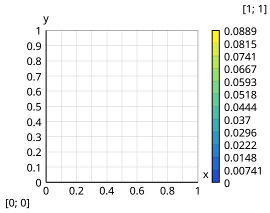


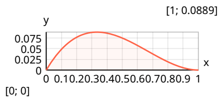
$N_{1,w}$ shape function plot





N_{1,θ_x} shape function plot





Constitutive matrix (stress - strain relationship)

$D_{1,1} = \dfrac{E\cdot t^3}{12\cdot(1-\nu^2)} = \dfrac{35000\cdot 0.2^3}{12\cdot(1-0.2^2)} = 24.305556$ kNm

$\boldsymbol{D} = D_{1,1}\cdot\mathbf{hp}\left(\left[1;\ \nu;\ 0\ |\ \nu;\ 1;\ 0\ |\ 0;\ 0;\ \dfrac{1-\nu}{2}\right]\right) = 24.305556\cdot\mathbf{hp}\left(\left[1;\ 0.2;\ 0\ |\ 0.2;\ 1;\ 0\ |\ 0;\ 0;\ \dfrac{1-0.2}{2}\right]\right) =$

$$\begin{bmatrix} 24.305556 & 4.861111 & 0 \\ 4.861111 & 24.305556 & 0 \\ 0 & 0 & 9.722222 \end{bmatrix} \text{kNm}$$

Element stiffness matrix calculation ... ▼

Element stiffness matrix coefficients (above the main diagonal only)

$\boldsymbol{K}_e = D_{1,1}\cdot\boldsymbol{K}_e = 24.305556\cdot\boldsymbol{K}_e =$

$$\begin{bmatrix}
796.296296 & 135.185185 & 135.185185 & 16.736111 & -391.203704 & 84.953704 & -13.657407 & 4.097222 & -13.888889 & 36.574074 & 36.574074 & -8.541667 & -391.203704 & -13.657407 & 84.953704 & 4.097222 \\
0 & 46.666667 & 21.597222 & 4.666667 & -84.953704 & 14.027778 & -4.097222 & 0.7083333 & -36.574074 & 10.277778 & 8.541667 & -1.625 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 \\
0 & 0 & 46.666667 & 4.666667 & -13.657407 & 4.097222 & 1.944444 & -0.5833333 & -36.574074 & 8.541667 & 10.277778 & -1.625 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 \\
0 & 0 & 0 & 0.9777778 & -4.097222 & 0.7083333 & 0.5833333 & -0.1611111 & -8.541667 & 1.625 & 1.625 & -0.2305556 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 \\
0 & 0 & 0 & 0 & 796.296296 & -135.185185 & 135.185185 & -16.736111 & -391.203704 & 13.657407 & 84.953704 & -4.097222 & -13.888889 & -36.574074 & 36.574074 & 8.541667 \\
0 & 0 & 0 & 0 & 0 & 46.666667 & -21.597222 & 4.666667 & 13.657407 & 1.944444 & -4.097222 & -0.5833333 & 36.574074 & 10.277778 & -8.541667 & -1.625 \\
0 & 0 & 0 & 0 & 0 & 0 & 46.666667 & -4.666667 & -84.953704 & 4.097222 & 14.027778 & -0.7083333 & -36.574074 & -8.541667 & 10.277778 & 1.625 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9777778 & 4.097222 & 0.5833333 & -0.7083333 & -0.1611111 & 8.541667 & 1.625 & -1.625 & -0.2305556 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 796.296296 & -135.185185 & -135.185185 & 16.736111 & -391.203704 & -84.953704 & 13.657407 & 4.097222 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 46.666667 & 21.597222 & -4.666667 & 84.953704 & 14.027778 & -4.097222 & -0.7083333 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 46.666667 & -4.666667 & 13.657407 & 4.097222 & 1.944444 & 0.5833333 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9777778 & -4.097222 & -0.7083333 & -0.5833333 & -0.1611111 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 796.296296 & 135.185185 & -135.185185 & -16.736111 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 46.666667 & -21.597222 & -4.666667 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 46.666667 & 4.666667 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9777778
\end{bmatrix}$$

Element load vector

$F_e = \dfrac{q\cdot A_1}{24}\cdot\left[6;\ a_1;\ b_1;\ \dfrac{A_1}{6};\ 6;\ -a_1;\ b_1;\ \dfrac{-A_1}{6};\ 6;\ -a_1;\ -b_1;\ \dfrac{A_1}{6};\ 6;\ a_1;\ -b_1;\ \dfrac{-A_1}{6}\right] = \dfrac{10\cdot 0.36}{24}\cdot\left[6;\ 0.6;\ 0.6;\ \dfrac{0.36}{6};\ 6;\ -0.6;\ 0.6;\ \dfrac{-0.36}{6};\ 6;\ -0.6;\ -0.6;\ \dfrac{0.36}{6};\ 6;\ 0.6;\ -0.6;\ \dfrac{-0.36}{6}\right] = [0.9\ \ 0.09\ \ 0.09\ \ 0.009\ \ 0.9\ \ -0.09\ \ 0.09\ \ -0.009\ \ 0.9\ \ -0.09\ \ -0.09\ \ 0.009\ \ 0.9\ \ 0.09\ \ -0.09\ \ -0.009]$ kN

Solution

Global stiffness matrix

$\boldsymbol{K} =$

$$\begin{bmatrix}
10^{20} & 135.185185 & 135.185185 & 16.736111 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
135.185185 & 46.666667 & 21.597222 & 4.666667 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
135.185185 & 21.597222 & 46.666667 & 4.666667 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
16.736111 & 4.666667 & 4.666667 & 0.9777778 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
-391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
-13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & -391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & -13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0.9777778
\end{bmatrix}$$

Global load vector

$F = [0.9\ \ 0.09\ \ 0.09\ \ 0.009\ \ 1.8\ \ 0.18\ \ 0\ \ 0\ \ 1.8\ \ 0.18\ \ 0\ \ 0\ \ 1.8\ \ 0.18\ \ 0\ \ 0\ \ 1.8\ \ 0.18\ \ 0\ \ 0\ \ \ldots\ \ 0.009]$ kN

Solution of the system of equations

$Z = \mathbf{slsolve}(\boldsymbol{K};F) = [0\ \ 0.5523352\ \ 0.382696\ \ -0.4159901\ \ 0.2028057\ \ 0.3732616\ \ 0.2648531\ \ -0.1936926\ \ 0.2989084\ \ 0.3091046\ \ 0.04831304\ \ -0.02500182\ \ 0.2612117\ \ 0.3426307\ \ -0.1651507\ \ 0.1275183\ \ 0.1211517\ \ 0.4681261\ \ -0.2671228\ \ 0.2293407\ \ \ldots\ \ -0.4159902]$ mm

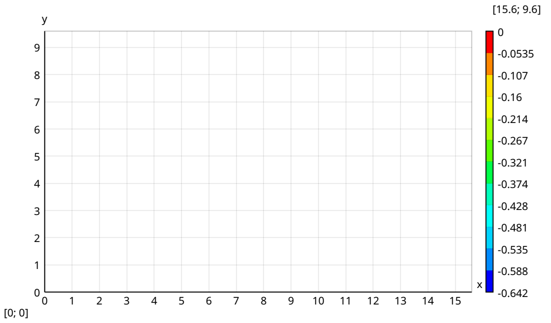
Results

Joint displacements

$\mathbf{transp}(W_z) =$

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	⋯	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	⋯	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	⋯	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	⋯	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	⋯	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	⋯	0
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	⋯	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	⋯	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	⋯	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	⋯	0.299
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	⋯	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	⋯	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	⋯	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	⋯	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	⋯	0.299
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	⋯	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	⋯	0

mm



Bending moments

$Z_j(j) = \mathbf{slice}(Z; k_1 \cdot (j-1)+1; k_1 \cdot j)$

$Z_e(e) = \mathbf{hp}([Z_j(e_{j.e,1}); Z_j(e_{j.e,2}); Z_j(e_{j.e,3}); Z_j(e_{j.e,4})])$

Average bending moments at joints, kNm/m

M_j =

1.500275	0.3097747	0.2197379	0.1563434	0.1570478	0.9983225	0.1564233	0.1519636	0.1942465	0.151963	0.1564238	0.9983224	0.1570487	0.1563432	0.2197373	0.3097501	1.500258	8.50221	6.47933	5.777678	⋯	1.500075
1.561323	7.805992	9.338126	7.6683	3.333101	-28.361989	3.146397	7.323883	8.895047	7.323882	3.146398	-28.361989	3.333102	7.6683	9.338126	7.805978	1.56134	0.3220403	5.382484	7.126847	⋯	1.561519
8.088697	3.766245	0.4861465	-2.479522	-4.459403	0.1550078	4.780402	2.842763	-2.410353×10^{-8}	-2.842763	-4.780402	-0.1550078	4.459403	2.479522	-0.4861463	-3.766235	-8.088697	4.109025	2.572475	0.3669336	⋯	8.088698

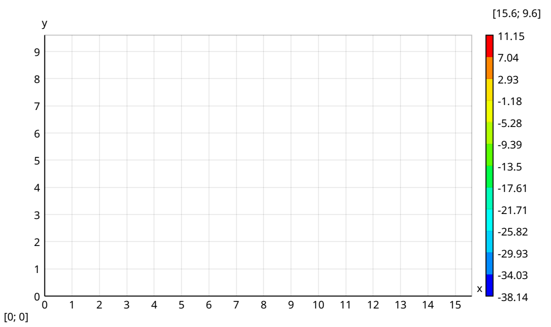
Bending moments for the plate

Bending moments - M_x

transp(*Mx*) =

1.500275	8.50221	11.065783	10.476631	6.796938	0.8844119	-31.234001	0.3020629	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860142	5.919519	8.876859	8.779239	5.643938	0.3020627	⋯	1.500259
0.3097747	6.47933	9.431828	8.876831	4.922389	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360952	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	7.249978	3.808562	-3.260011	⋯	0.3097374
0.2197379	5.777678	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	⋯	0.2197445
0.1563434	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.832661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.832661	⋯	0.1563406
0.1570478	7.26966	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	⋯	0.1570494
0.9983225	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	⋯	0.9983223
0.1564233	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	⋯	0.1564268
0.1519636	5.873203	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.626467	6.469771	2.594271	-4.009025	⋯	0.1519571
0.1942465	5.514146	8.380234	7.692958	3.468366	-2.491073	-5.742609	-3.07336	2.310482	5.979232	6.142664	2.786926	-2.376796	-5.118358	-2.376796	2.786926	6.142664	5.979232	2.310482	-3.07336	⋯	0.194254
0.151963	5.873204	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.626467	6.469771	2.594271	-4.009025	⋯	0.1519564
0.1564238	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	⋯	0.1564266
0.9983224	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	⋯	0.9983221
0.1570487	7.269659	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	⋯	0.1570582
0.1563432	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.832661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.832661	⋯	0.156342
0.2197373	5.777677	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	⋯	0.2197303
0.3097501	6.479335	9.43183	8.876831	4.922388	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360953	4.127543	-2.764018	-9.732553	-2.764019	4.127543	7.360952	7.249978	3.808562	-3.260012	⋯	0.3094814
1.500258	8.502224	11.065783	10.47663	6.796939	0.8844112	-31.234001	0.302063	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860143	5.919519	8.876859	8.779239	5.643938	0.3020627	⋯	1.500075

kNm/m

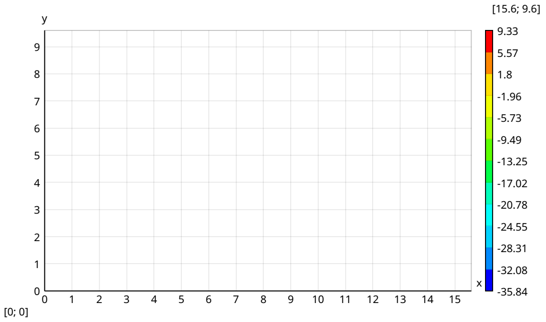


Bending moments M_y

transp(*My*) =

1.561323	0.3220403	0.2354335	0.2167291	0.1693836	0.1786538	1.012971	0.1781616	0.1678576	0.2114171	0.2108095	0.1672762	0.1764527	0.9864448	0.1764533	0.1672753	0.2108105	0.2114162	0.1678582	0.1781613	⋯	1.561345
7.805992	5.382484	4.309253	4.135769	4.742647	6.34008	8.230114	6.28276	4.612133	3.892868	3.868094	4.530706	6.1222	7.985279	6.1222	4.530706	3.868093	3.892869	4.612132	6.28276	⋯	7.80597
9.338126	7.126847	5.800492	5.553422	6.415097	8.11573	9.046504	8.014613	6.188934	5.145102	5.11703	6.090686	7.82112	8.77703	7.82112	6.090686	5.11703	5.145101	6.188934	8.014613	⋯	9.338132
7.6683	5.417781	4.262721	3.920623	4.233829	5.419878	6.344599	5.301132	3.979565	3.47814	3.481043	3.968664	5.222849	6.18144	5.222849	3.968664	3.481043	3.47814	3.979565	5.301132	⋯	7.668298
3.333101	-0.3279945	0.5397481	0.4057356	-1.174273	-3.412893	-0.3301173	-3.53113	-1.399477	0.09976086	0.1610125	-1.210479	-3.234594	-0.2183684	-3.234594	-1.210479	0.1610125	0.09976085	-1.399477	-3.53113	⋯	3.333102
-28.361989	-7.038398	-1.936124	-1.64658	-4.99389	-13.644591	-36.317421	-13.771031	-5.215778	-1.858062	-1.763919	-4.886758	-13.033067	-34.570998	-13.033067	-4.886758	-1.763919	-1.858062	-5.215778	-13.771031	⋯	-28.361989
3.146397	-0.5066875	0.3485373	0.1799829	-1.45618	-3.762354	-0.7232665	-3.922882	-1.765717	-0.2519029	-0.1967723	-1.596231	-3.661551	-0.6660923	-3.661551	-1.596231	-0.1967723	-0.2519029	-1.765717	-3.922882	⋯	3.1464
7.323883	5.094567	3.909	3.489181	3.675526	4.685937	5.498817	4.484059	3.254296	2.794477	2.788653	3.214598	4.35007	5.243754	4.35007	3.214598	2.788653	2.794477	3.254296	4.484059	⋯	7.323878
8.895047	6.740137	5.350891	4.961298	5.608992	6.913134	7.572611	6.693252	5.141881	4.180623	4.149738	5.027895	6.456963	7.213083	6.456963	5.027895	4.149738	4.180623	5.141881	6.693252	⋯	8.895053
7.323882	5.094567	3.908999	3.489181	3.675526	4.685937	5.498817	4.484059	3.254296	2.794477	2.788653	3.214598	4.35007	5.243754	4.35007	3.214598	2.788653	2.794477	3.254296	4.484059	⋯	7.323877
3.146398	-0.5066877	0.3485373	0.1799829	-1.45618	-3.762354	-0.7232665	-3.922882	-1.765717	-0.2519029	-0.1967723	-1.596231	-3.661551	-0.6660923	-3.661551	-1.596231	-0.1967723	-0.2519029	-1.765717	-3.922882	⋯	3.1464
-28.361989	-7.038398	-1.936124	-1.64658	-4.99389	-13.644591	-36.317421	-13.771031	-5.215778	-1.858062	-1.763919	-4.886758	-13.033067	-34.570998	-13.033067	-4.886758	-1.763919	-1.858062	-5.215778	-13.771031	⋯	-28.36199
3.333102	-0.3279946	0.5397482	0.4057355	-1.174273	-3.412893	-0.3301173	-3.53113	-1.399477	0.09976086	0.1610125	-1.210479	-3.234594	-0.2183684	-3.234594	-1.210479	0.1610125	0.09976085	-1.399477	-3.53113	⋯	3.333109
7.6683	5.417781	4.262721	3.920623	4.233829	5.419878	6.344599	5.301132	3.979565	3.47814	3.481043	3.968664	5.222849	6.18144	5.222849	3.968664	3.481043	3.47814	3.979565	5.301132	⋯	7.6683
9.338126	7.126845	5.800491	5.553422	6.415097	8.11573	9.046504	8.014613	6.188934	5.145102	5.11703	6.090686	7.82112	8.77703	7.82112	6.090686	5.11703	5.145101	6.188934	8.014613	⋯	9.338126
7.805978	5.382479	4.309254	4.13577	4.742647	6.340081	8.230114	6.28276	4.612133	3.892868	3.868094	4.530706	6.1222	7.985279	6.1222	4.530706	3.868093	3.892869	4.612132	6.28276	⋯	7.805828
1.56134	0.3220664	0.2354339	0.2167277	0.1693845	0.1786529	1.012971	0.1781618	0.1678575	0.2114172	0.2108094	0.1672763	0.1764527	0.9864448	0.1764534	0.167275	0.2108108	0.211416	0.1678581	0.1781614	⋯	1.561519

kNm/m

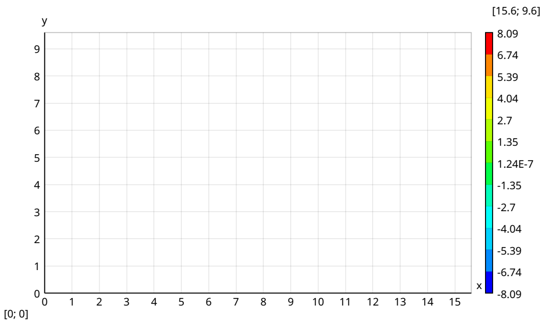


Bending moments M_{xy}

transp(Mxy) =

8.088697	4.109025	1.416336	-0.9453079	-3.232331	-4.781353	0.02176366	4.838587	3.334748	1.141499	-1.025238	-3.194104	-4.673215	9.790543×10^{-9}	4.673215	3.194104	1.025238	-1.141499	-3
3.766245	2.572475	0.9829256	-0.515595	-2.056503	-3.228613	0.05825511	3.354524	2.212955	0.7309473	-0.6696087	-2.124156	-3.225638	-2.241521×10^{-8}	3.225638	2.124156	0.6696087	-0.7309473	-2
0.4861465	0.3669336	0.21034	0.08576266	-0.1194038	-0.3136824	0.0734795	0.4611397	0.2665259	0.05675809	-0.07980332	-0.2737226	-0.431664	4.761034×10^{-9}	0.431664	0.2737226	0.07980332	-0.05675809	-0
-2.479522	-1.796141	-0.601364	0.6167145	1.746922	2.09269	0.07626484	-1.948268	-1.634146	-0.5815058	0.4919668	1.536331	1.879395	-1.027996×10^{-9}	-1.879395	-1.536331	-0.4919668	0.5815058	1
-4.459403	-3.240091	-0.8978376	0.6826157	2.370906	4.525	0.07575709	-4.382355	-2.264704	-0.6817866	0.6007781	2.152138	4.210913	2.395547×10^{-10}	-4.210913	-2.152138	-0.6007781	0.6817866	2
0.1550078	0.1611012	0.1491877	0.1245521	0.09635607	0.07794845	0.07385991	0.06986752	0.05123571	0.0210446	-0.008412859	-0.02659053	-0.02344541	-5.804372×10^{-11}	0.02344541	0.02659053	0.008412859	-0.0210446	-0.
4.780402	3.565976	1.18935	-0.4505739	-2.206794	-4.399329	0.06346754	4.535463	2.379375	0.7252099	-0.6264028	-2.225368	-4.279295	1.444385×10^{-11}	4.279295	2.225368	0.6264028	-0.7252099	-2
2.842763	2.134393	0.8682899	-0.4377384	-1.679619	-2.084094	0.03777839	2.168358	1.796138	0.6296579	-0.5417111	-1.677943	-2.037693	-3.548115×10^{-12}	2.037693	1.677943	0.5417111	-0.6296579	-
-2.410353×10^{-8}	3.062399×10^{-8}	-4.397001×10^{-9}	3.066824×10^{-10}	-8.12632×10^{-11}	3.636829×10^{-11}	-1.325189×10^{-11}	4.764349×10^{-12}	-7.14584×10^{-13}	1.790365×10^{-13}	-3.8029×10^{-13}	2.910935×10^{-13}	0	1.375317×10^{-13}	-9.949168×10^{-13}	2.393×10^{-12}	-3.712996×10^{-12}	2.380218×10^{-12}	3.02
-2.842763	-2.134393	-0.8682899	0.4377384	1.679619	2.084094	-0.03777839	-2.168358	-1.796138	-0.6296579	0.5417111	1.677943	2.037693	4.127238×10^{-12}	-2.037693	-1.677943	-0.5417111	0.6296579	1
-4.780402	-3.565976	-1.18935	0.4505739	2.206794	4.399329	-0.06346754	-4.535463	-2.379375	-0.7252099	0.6264028	2.225368	4.279295	-1.911611×10^{-11}	-4.279295	-2.225368	-0.6264028	0.7252099	2
-0.1550078	-0.1611013	-0.1491877	-0.1245521	-0.09635607	-0.07794845	-0.07385991	-0.06986752	-0.05123571	-0.0210446	0.008412859	0.02659053	0.02344541	7.954439×10^{-11}	-0.02344541	-0.02659053	-0.008412859	0.0210446	0.0
4.459403	3.240091	0.8978376	-0.6826157	-2.370906	-4.525	-0.07575709	4.382355	2.264704	0.6817866	-0.6007781	-2.152138	-4.210913	-3.335117×10^{-10}	4.210913	2.152138	0.6007781	-0.6817866	-2
2.479522	1.796141	0.601364	-0.6167145	-1.746922	-2.09269	-0.07626484	1.948268	1.634146	0.5815058	-0.4919668	-1.536331	-1.879395	1.414276×10^{-9}	1.879395	1.536331	0.4919668	-0.5815058	-
-0.4861463	-0.3669331	-0.21034	-0.08576274	0.1194038	0.3136824	-0.07347951	-0.4611397	-0.2665259	-0.05675809	0.07980332	0.2737226	0.431664	-6.33653×10^{-9}	-0.431664	-0.2737226	-0.07980332	0.05675809	0.
-3.766235	-2.572475	-0.9829263	0.5155949	2.056503	3.228613	-0.05825506	-3.354524	-2.212955	-0.7309473	0.6696087	2.124156	3.225638	2.942834×10^{-8}	-3.225638	-2.124156	-0.6696087	0.7309473	2
-8.088697	-4.109035	-1.416336	0.9453084	3.232331	4.781353	-0.0217637	-4.838587	-3.334748	-1.141499	1.025238	3.194104	4.673215	-1.247931×10^{-8}	-4.673215	-3.194104	-1.025238	1.141499	3

kNm/m



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