

Finite Element Analysis of Flat Slab

Using analytical formulation of Bogner-Fox-Schmit (BFS) plate element

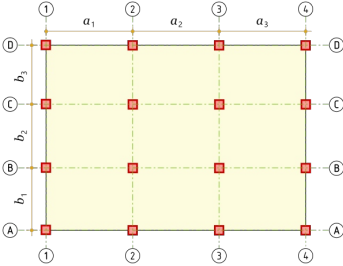
Input data

Span lengths

$\mathbf{r} = \mathbf{hp}([3.6; 4.2; 4.2; 3.6]) = [3.6 \ 4.2 \ 4.2 \ 3.6] \text{ m}$

$\mathbf{r} = \mathbf{hp}([3; 3.6; 3]) = [3 \ 3.6 \ 3] \text{ m}$

Number of axes - $n_{sa} = \text{len}(\mathbf{r}) + 1 = 5$, $n_{sb} = \text{len}(\mathbf{r}) + 1 = 4$



Axis coordinates - $\mathbf{x}_s = [0 \ 3.6 \ 7.8 \ 12 \ 15.6] \text{ m}$, $\mathbf{y}_s = [0 \ 3 \ 6.6 \ 9.6] \text{ m}$

Slab dimensions - $\mathbf{l}_a \Rightarrow \mathbf{x}_{s,5} = 15.6 \text{ m}$, $\mathbf{l}_b \Rightarrow \mathbf{y}_{s,4} = 9.6 \text{ m}$

Thickness - $t = 0.2 \text{ m}$

Load - $q = 10 \text{ kN/m}^2$

Modulus of elasticity - $E = 35000 \text{ MPa}$

Poisson's ratio - $\nu = 0.2$

Finite element mesh

We will use BFS rectangular finite element with $n_{\text{DOFs}} = 16$

Element dimensions - $a_1 = 0.6 \text{ m}$, $b_1 = 0.6 \text{ m}$

Number of elements and joints along a and b -

$n_a = \text{ceiling}(\frac{a}{a_1}) = \text{ceiling}(\frac{a}{0.6}) = [6 \ 7 \ 7 \ 6]$, $n_{ea} = \text{sum}(\mathbf{n}_a) = 26$, $n_{ja} = n_{ea} + 1 = 26 + 1 = 27$

$n_b = \text{ceiling}(\frac{b}{b_1}) = \text{ceiling}(\frac{b}{0.6}) = [5 \ 6 \ 5]$, $n_{eb} = \text{sum}(\mathbf{n}_b) = 16$, $n_{jb} = n_{eb} + 1 = 16 + 1 = 17$

Total number of elements - $n_e = n_{ea} \cdot n_{eb} = 26 \cdot 16 = 416$

Total number of joints - $n_j = n_{ja} \cdot n_{jb} = 27 \cdot 17 = 459$

Supported joints count - $n_s = n_{sa} \cdot n_{sb} = 5 \cdot 4 = 20$

Joint coordinates

$\mathbf{j} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.6 \ 0.6 \ 0.6 \ \dots \ 15.6] \text{ m}$

$\mathbf{j} = [0 \ 0.6 \ 1.2 \ 1.8 \ 2.4 \ 3 \ 3.6 \ 4.2 \ 4.8 \ 5.4 \ 6 \ 6.6 \ 7.2 \ 7.8 \ 8.4 \ 9 \ 9.6 \ 0 \ 0.6 \ 1.2 \ \dots \ 9.6] \text{ m}$

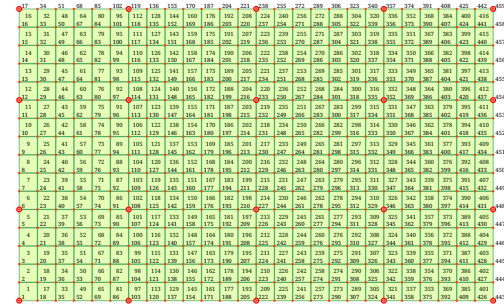
Numbers of joints at elements' corners

$\text{transp}(\mathbf{e_j}) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 18 & 19 & 20 & 21 & \dots & 441 \\ 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 35 & 36 & 37 & 38 & \dots & 458 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 36 & 37 & 38 & 39 & \dots & 459 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 19 & 20 & 21 & 22 & \dots & 442 \end{bmatrix}$

Supported joints

$\mathbf{j} = [1 \ 6 \ 12 \ 17 \ 103 \ 108 \ 114 \ 119 \ 222 \ 227 \ 233 \ 238 \ 341 \ 346 \ 352 \ 357 \ 443 \ 448 \ 454 \ 459]$

Joints for element e - $\mathbf{j_e}(e) = \text{row}(\mathbf{e_j}; e)$



Finite element formulation

Shape functions

Along dimension a

Base functions	First derivatives	Second derivatives
$\Phi_{1a}(\xi) = 1 - \xi^2 \cdot (3 - 2 \cdot \xi)$	$\Phi'_{1a}(\xi) = -6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$	$\Phi''_{1a}(\xi) = -\frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$
$\Phi_{2a}(\xi) = \xi \cdot a_1 \cdot (1 - \xi \cdot (2 - \xi))$	$\Phi'_{2a}(\xi) = 1 - \xi \cdot (4 - 3 \cdot \xi)$	$\Phi''_{2a}(\xi) = -\frac{2}{a_1} \cdot (2 - 3 \cdot \xi)$
$\Phi_{3a}(\xi) = \xi^2 \cdot (3 - 2 \cdot \xi)$	$\Phi'_{3a}(\xi) = 6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$	$\Phi''_{3a}(\xi) = \frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$
$\Phi_{4a}(\xi) = \xi^2 \cdot a_1 \cdot (-1 + \xi)$	$\Phi'_{4a}(\xi) = -\xi \cdot (2 - 3 \cdot \xi)$	$\Phi''_{4a}(\xi) = -\frac{2}{a_1} \cdot (1 - 3 \cdot \xi)$

Along dimension b

Base functions	First derivatives	Second derivatives
$\Phi_{1b}(\eta) = 1 - \eta^2 \cdot (3 - 2 \cdot \eta)$	$\Phi'_{1b}(\eta) = -6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$	$\Phi''_{1b}(\eta) = -\frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$
$\Phi_{2b}(\eta) = \eta \cdot b_1 \cdot (1 - \eta \cdot (2 - \eta))$	$\Phi'_{2b}(\eta) = 1 - \eta \cdot (4 - 3 \cdot \eta)$	$\Phi''_{2b}(\eta) = -\frac{2}{b_1} \cdot (2 - 3 \cdot \eta)$
$\Phi_{3b}(\eta) = \eta^2 \cdot (3 - 2 \cdot \eta)$	$\Phi'_{3b}(\eta) = 6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$	$\Phi''_{3b}(\eta) = \frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$
$\Phi_{4b}(\eta) = \eta^2 \cdot b_1 \cdot (-1 + \eta)$	$\Phi'_{4b}(\eta) = -\eta \cdot (2 - 3 \cdot \eta)$	$\Phi''_{4b}(\eta) = -\frac{2}{b_1} \cdot (1 - 3 \cdot \eta)$

For vertical displacements w For rotations θ_x For rotations θ_y

$N_{1,w}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{1,\theta_x}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{1,\theta_y}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{2,w}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{2,\theta_x}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{1b}(\eta)$ $N_{2,\theta_y}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{2b}(\eta)$

$$N_{3,w}(\xi;\eta)=\varphi_{3a}(\xi)\cdot\varphi_{3b}(\eta) \quad N_{3,\theta}(\xi;\eta)=\varphi_{4a}(\xi)\cdot\varphi_{3b}(\eta) \quad N_{3,\theta_1}(\xi;\eta)=\varphi_{3a}(\xi)\cdot\varphi_{4b}(\eta)$$

$$N_{4,w}(\xi;\eta)=\varphi_{1a}(\xi)\cdot\varphi_{3b}(\eta) \quad N_{4,\theta}(\xi;\eta)=\varphi_{2a}(\xi)\cdot\varphi_{3b}(\eta) \quad N_{4,\theta_1}(\xi;\eta)=\varphi_{1a}(\xi)\cdot\varphi_{4b}(\eta)$$

For twist ψ

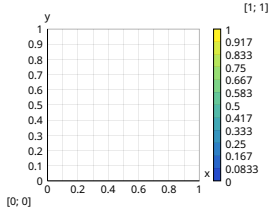
$$N_{1,\psi}(\xi;\eta)=\varphi_{2a}(\xi)\cdot\varphi_{2b}(\eta)$$

$$N_{2,\psi}(\xi;\eta)=\varphi_{4a}(\xi)\cdot\varphi_{2b}(\eta)$$

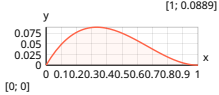
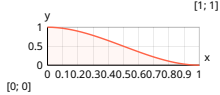
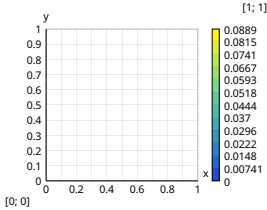
$$N_{3,\psi}(\xi;\eta)=\varphi_{4a}(\xi)\cdot\varphi_{4b}(\eta)$$

$$N_{4,\psi}(\xi;\eta)=\varphi_{2a}(\xi)\cdot\varphi_{4b}(\eta)$$

$N_{1,w}$ shape function plot



$N_{1,\theta}$ shape function plot



Constitutive matrix (stress - strain relationship)

$$D_{1,1}=\frac{E\cdot t^3}{12\cdot (1-\nu^2)}=\frac{35000\cdot 0.2^3}{12\cdot (1-0.2^2)}=24.305556\text{ kNm}$$

$$D=D_{1,1}\cdot \mathbf{hp}\left(\left[\begin{array}{c|c|c|c|c|c} 1; \nu; 0 & \nu; 1; 0 & 0; 0; \frac{1-\nu}{2} \end{array}\right]\right)=24.305556\cdot \mathbf{hp}\left(\left[\begin{array}{c|c|c|c|c|c} 1; 0.2; 0 & 0.2; 1; 0 & 0; 0; \frac{1-0.2}{2} \end{array}\right]\right)=$$

$$\begin{bmatrix} 24.305556 & 4.861111 & 0 \\ 4.861111 & 24.305556 & 0 \\ 0 & 0 & 9.722222 \end{bmatrix} \text{ kNm}$$

Element stiffness matrix calculation ... ▼

Element stiffness matrix coefficients (above the main diagonal only)

$$K_e=D_{1,1}\cdot K_e=24.305556\cdot K_e=$$

796.296296	135.185185	135.185185	16.736111	-391.203704	84.953704	-13.657407	4.097222	-13.888889	36.574074	36.574074	-8.541667	-391.203704	-13.657407	84.953704	4.097222
0	46.666667	21.597222	4.666667	-84.953704	14.027778	-4.097222	0.7083333	-36.574074	10.277778	8.541667	-1.625	-13.657407	1.944444	4.097222	-0.5833333
0	0	46.666667	4.666667	-13.657407	4.097222	1.944444	-0.5833333	-36.574074	8.541667	10.277778	-1.625	-84.953704	-4.097222	14.027778	0.7083333
0	0	0	0.9777778	-4.097222	0.7083333	0.5833333	-0.1611111	-8.541667	1.625	1.625	-0.2305556	-4.097222	0.5833333	0.7083333	-0.1611111
0	0	0	0	796.296296	-135.185185	135.185185	-16.736111	-391.203704	13.657407	84.953704	-4.097222	-13.888889	-36.574074	36.574074	8.541667
0	0	0	0	0	46.666667	-21.597222	4.666667	13.657407	1.944444	-4.097222	-0.5833333	36.574074	10.277778	-8.541667	-1.625
0	0	0	0	0	0	46.666667	-4.666667	-84.953704	4.097222	14.027778	-0.7083333	-36.574074	-8.541667	10.277778	1.625
0	0	0	0	0	0	0	0.9777778	4.097222	0.5833333	-0.7083333	-0.1611111	8.541667	1.625	-1.625	-0.2305556
0	0	0	0	0	0	0	0	796.296296	-135.185185	-135.185185	16.736111	-391.203704	-84.953704	13.657407	4.097222
0	0	0	0	0	0	0	0	0	46.666667	21.597222	-4.666667	84.953704	14.027778	-4.097222	-0.7083333
0	0	0	0	0	0	0	0	0	46.666667	-4.666667	13.657407	4.097222	1.944444	0.5833333	0.7083333
0	0	0	0	0	0	0	0	0	0	0	0.9777778	-4.097222	-0.7083333	-0.5833333	-0.1611111
0	0	0	0	0	0	0	0	0	0	0	0	796.296296	135.185185	-135.185185	-16.736111
0	0	0	0	0	0	0	0	0	0	0	0	46.666667	-21.597222	-4.666667	0.7083333
0	0	0	0	0	0	0	0	0	0	0	0	0	46.666667	4.666667	0.7083333
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9777778

Element load vector

$$r_e=\frac{q\cdot A_1}{24}\cdot\left[\begin{array}{c} 6; a_1; b_1; \frac{A_1}{6}; 6; -a_1; b_1; \frac{-A_1}{6}; 6; -a_1; -b_1; \frac{A_1}{6}; 6; a_1; -b_1; \frac{-A_1}{6} \end{array}\right]=\frac{10\cdot 0.36}{24}\cdot\left[\begin{array}{c} 6; 0.6; 0.6; \frac{0.36}{6}; 6; -0.6; \end{array}\right];$$

$$0.6; \frac{-0.36}{6}; 6; -0.6; -0.6; \frac{0.36}{6}; 6; 0.6; -0.6; \frac{-0.36}{6}]=\left[\begin{array}{c} 0.9 \ 0.09 \ 0.09 \ 0.009 \ 0.9 \ -0.09 \ 0.09 \ -0.009 \end{array}\right] \text{ kN}$$

$$0.9 \ -0.09 \ -0.09 \ 0.009 \ 0.9 \ 0.09 \ -0.09 \ -0.009] \text{ kN}$$

Solution

Global stiffness matrix

$$K= \begin{bmatrix} 10^{20} & 135.185185 & 135.185185 & 16.736111 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 135.185185 & 46.666667 & 21.597222 & 4.666667 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 135.185185 & 21.597222 & 46.666667 & 4.666667 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 16.736111 & 4.666667 & 4.666667 & 0.9777778 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & -391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & -13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & -391.203704 & -13.657407 & 84.953704 & 4.097222 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & -13.657407 & 1.944444 & 4.097222 & -0.5833333 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & -4.097222 & 0.5833333 & 0.7083333 & -0.1611111 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -391.203704 & -13.657407 & -84.953704 & -4.097222 & 1592.592593 & 270.37037 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -13.657407 & 1.944444 & -4.097222 & 0.5833333 & 270.37037 & 93.333333 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 84.953704 & 4.097222 & 14.027778 & 0.7083333 & 0 & 0 & 93.333333 & 9.333333 & -84.953704 & -4.097222 & 14.027778 & 0.7083333 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.097222 & -0.5833333 & 0.7083333 & -0.1611111 & 0 & 0 & 9.333333 & 1.955556 & \cdots & 0 \\ \vdots & \vdots \\ 0 & \cdots & 0.9777778 \end{bmatrix}$$

Global load vector

$$r=\left[\begin{array}{c} 0.9 \ 0.09 \ 0.09 \ 0.009 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.009 \end{array}\right] \text{ kN}$$

Solution of the system of equations

$$r=\mathbf{solve}(K;r)=\left[\begin{array}{c} 0 \ 0.5523352 \ 0.382696 \ -0.4159901 \ 0.2028057 \ 0.3732616 \ 0.2648531 \ -0.1936926 \ 0.2989084 \end{array}\right]$$

$$0.3091046 \ 0.04831304 \ -0.02500182 \ 0.2612117 \ 0.3426307 \ -0.1651507 \ 0.1275183 \ 0.1211517$$

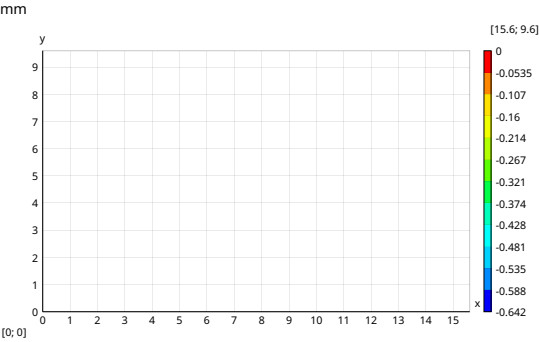
$$0.4681261 \ -0.2671228 \ 0.2293407 \ \cdots \ -0.4159902] \text{ mm}$$

Results

Joint displacements

$$\mathbf{transp}(W_2)=$$

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	...	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	...	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	...	0.299
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0



[0;0]

Bending moments

$$Z_j(j) = \text{slice}(\mathcal{E}; k_1 \cdot (j-1) + 1; k_1 \cdot j)$$

$$Z_e(e) = \text{hp}([Z_j(e_{j,e},1); Z_j(e_{j,e},2); Z_j(e_{j,e},3); Z_j(e_{j,e},4)])$$

Average bending moments at joints, kNm/m

$$M_j =$$

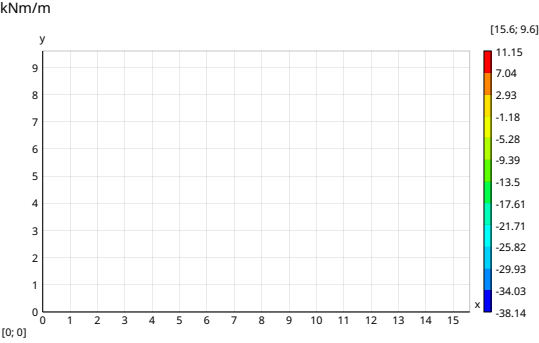
1.500275	0.3097747	0.2197379	0.1563434	0.1570478	0.9983225	0.1564233	0.1519636	0.1942465	0.151963	0.1564238	0.9983224	0.1570487	0.1563432	0.2197373	0.3097501	1.500258	8.50221	6.47933	5.777678	...	1.500075
1.561323	7.805992	9.338126	7.6683	3.333101	-28.361989	3.146397	7.323883	8.895047	7.323882	3.146398	-28.361989	3.333102	7.6683	9.338126	7.805978	1.56134	0.3220403	5.382484	7.126847	...	1.561519
8.088697	3.766245	0.4861465	-2.479522	-4.459403	0.1550078	4.780402	2.842763	-2.410353×10 ⁻⁸	-2.842763	-4.780402	-0.1550078	4.459403	2.479522	-0.4861463	-3.766235	-8.088697	4.109025	2.572475	0.3669336	...	8.088698

Bending moments for the plate

Bending moments - M_x

$$\text{transp}(M_x) =$$

1.500275	8.50221	11.065783	10.476631	6.796938	0.8844119	-31.234001	0.3020629	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860142	5.919519	8.876859	8.779239	5.643938	0.3020627	...	1.500259
0.3097747	6.47933	9.431828	8.876831	4.922389	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360952	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	7.249978	3.808562	-3.260011	...	0.3097374
0.2197379	5.777678	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	...	0.2197445
0.1563434	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.832661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.832661	...	0.1563406
0.1570478	7.26966	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	...	0.1570494
0.9983225	0.938567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	...	0.9983223
0.1564233	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	...	0.1564268
0.1519636	5.873203	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.626467	6.469771	2.594271	-4.009025	...	0.1519571
0.1942465	5.514146	8.380234	7.692958	3.468366	-2.491073	-5.742609	-3.07336	2.310482	5.979232	6.142664	2.786926	-2.376796	-5.118358	-2.376796	2.786926	6.142664	5.979232	2.310482	-3.07336	...	0.194254
0.151963	5.873204	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.626467	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.626467	6.469771	2.594271	-4.009025	...	0.1519564
0.1564238	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	...	0.1564266
0.9983224	0.938567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	8.190807	4.521792	-2.745552	...	0.9983221
0.1570487	7.269659	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	7.838235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.838235	7.713212	3.766824	-5.865488	...	0.1570582
0.1563432	5.993171	9.125974	8.492762	4.057756	-3.257726	-7.988419	-3.832661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.381295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.832661	...	0.156342
0.2197373	5.777677	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	6.516596	2.93207	-2.819213	...	0.2197303
0.3097501	6.479335	9.43183	8.876831	4.922388	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360953	4.127543	-2.764018	-9.732553	-2.764019	4.127543	7.360952	7.249978	3.808562	-3.260012	...	0.3094814
1.500258	8.502224	11.065783	10.47663	6.796939	0.8844112	-31.234001	0.302063	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860143	5.919519	8.876859	8.779239	5.643938	0.3020627	...	1.500075



[0;0]

Bending moments M_y

$$\text{transp}(M_y) =$$

1.561323	0.3220403	0.2354335	0.2167291	0.1693836	0.1786538	1.012971	0.1781616	0.1678576	0.2114171	0.2108095	0.1672762	0.1764527	0.9864448	0.1764533	0.1672753	0.2108105	0.2114162	0.1678582	0.1781613	...	1.561345
7.805992	5.382484	4.309253	4.135769	4.742647	6.34008	8.230114	6.28276	4.612133	3.892868	3.868094	4.530706	6.1222	7.985279	6.1222	4.530706	3.868093	3.892869	4.612132	6.28276	...	7.80597
9.338126	7.126847	5.800492	5.553422	6.415097	8.11573	9.046504	8.014613	6.188934	5.145102	5.11703	6.090686	7.82112	8.77703	7.82112	6.090686	5.11703	5.145101	6.188934	8.014613	...	9.338132
7.6683	5.417781	4.262721	3.920623	4.233829	5.419878	6.344599	5.301132	3.979565	3.47814	3.481043	3.968664	5.222849	6.18144	5.222849	3.968664	3.481043	3.47814	3.979565	5.301132	...	7.668298
3.333101	-0.3279945	0.5397481	0.4057356	-1.174273	-3.412893	-0.3301173	-3.53113	-1.399477	0.09976086	0.1610125	-1.210479	-3.234594	-0.2183684	-3.234594	-1.210479	0.1610125	0.09976085	-1.399477	-3.53113	...	3.333102
-28.361989	-7.038398	-1.936124	-1.64658	-4.99389	-13.644591	-36.317421	-13.771031	-5.215778	-1.858062	-1.763919	-4.886758	-13.033067	-34.570998	-13.033067	-4.886758	-1.763919	-1.858062	-5.215778	-13.771031	...	-28.361989
3.146397	-0.5066875	0.3485373	0.1799829	-1.45618	-3.762354	-0.7232665	-3.922882	-1.765717	-0.2519029	-0.1967723	-1.596231	-3.661551	-0.6660923	-3.661551	-1.596231	-0.1967723	-0.2519029	-1.765717	-3.922882	...	3.1464
7.323883	5.094567	3.909	3.489181	3.675526	4.685937	5.498817	4.484059	3.254296	2.794477	2.788653	3.214598	4.35007	5.243754	4.35007	3.214598	2.788653	2.794477	3.254296	4.484059	...	7.323878
8.895047	6.740137	5.350891	4.961298	5.608992	6.913134	7.572611	6.693252	5.141881	4.180623	4.149738	5.027895	6.456963	7.213083	6.456963	5.027895	4.149738	4.180623	5.141881	6.693252	...	8.895053
7.323882	5.094567	3.908999	3.489181	3.675526	4.685937	5.498817	4.484059	3.254296	2.794477	2.788653	3.214598	4.35007	5.243754	4.35007	3.214598	2.788653	2.794477	3.254296	4.484059	...	7.323877
3.146398	-0.5066877	0.3485373	0.1799829	-1.45618	-3.762354	-0.7232665	-3.922882	-1.765717	-0.2519029	-0.1967723	-1.596231	-3.661551	-0.6660923	-3.661551	-1.596231	-0.1967723	-0.2519029	-1.765717	-3.922882	...	3.1464
-28.361989	-7.038398	-1.936124	-1.64658	-4.99389	-13.644591	-36.317421	-13.771031	-5.215778	-1.858062	-1.763919	-4.886758	-13.033067	-34.570998	-13.033067	-4.886758	-1.763919	-1.858062	-5.215778	-13.771031	...	-28.36199
3.333102	-0.3279946	0.5397482	0.4057355	-1.174273	-3.412893	-0.3301173	-3.53113	-1.399477	0.09976086	0.1610125	-1.210479	-3.234594	-0.2183684	-3.234594	-1.210479	0.1610125	0.09976085	-1.399477	-3.53113	...	3.333109
7.6683	5.417781	4.262721	3.920623	4.233829	5.419878	6.344599	5.301132	3.979565	3.47814	3.481043	3.968664	5.222849	6.18144	5.222849	3.968664	3.481043	3.47814	3.979565	5.301132	...	7.6683
9.338126	7.126845	5.800491	5.553422	6.415097	8.11573	9.046504	8.014613	6.188934	5.145102	5.11703	6.090686	7.82112	8.77703	7.82112	6.090686	5.11703	5.145101	6.188934	8.014613	...	9.338126
7.805978	5.382479	4.309254	4.13577	4.742647	6.340081	8.230114	6.28276	4.612133	3.892868	3.868094	4.530706	6.1222	7.985279	6.1222	4.530706	3.868093	3.892869	4.612132	6.28276	...	7.805828
1.56134	0.3220664	0.2354339	0.2167277	0.1693845	0.1786829	1.012971	0.1781618	0.1678575	0.2114172	0.2108094	0.1672763	0.1764527	0.9864448	0.1764534	0.1672725	0.2108108	0.211416	0.1678581	0.1781614	...	1.561519



Bending moments M_{xy}

transp (M_{xy}) =

8.088697	4.109025	1.416336	-0.9453079	-3.232331	-4.781353	0.02176366	4.838587	3.334748	1.141499	-1.025238	-3.194104	-4.673215	9.790543×10 ⁻⁹	4.673215	3.194104	1.025238	-1.141499	-∞
3.766245	2.572475	0.9829256	-0.515595	-2.056503	-3.228613	0.05825511	3.354524	2.212955	0.7309473	-0.6696087	-2.124156	-3.225638	-2.241521×10 ⁻⁸	3.225638	2.124156	0.6696087	-0.7309473	-∞
0.4861465	0.3669336	0.21034	0.08576266	-0.1194038	-0.3136824	0.0734795	0.4611397	0.2665259	0.05675809	-0.07980332	-0.2737226	-0.431664	4.761034×10 ⁻⁹	0.431664	0.2737226	0.07980332	-0.05675809	-0
-2.479522	-1.796141	-0.601364	0.6167145	1.746922	2.09269	0.07626484	-1.948268	-1.634146	-0.5815058	0.4919668	1.536331	1.879395	-1.027996×10 ⁻⁹	-1.879395	-1.536331	-0.4919668	0.5815058	1
-4.459403	-3.240091	-0.8978376	0.6826157	2.370906	4.525	0.07575709	-4.382355	-2.264704	-0.6817866	0.6007781	2.152138	4.210913	2.395547×10 ⁻¹⁰	-4.210913	-2.152138	-0.6007781	0.6817866	2
0.1550078	0.1611012	0.1491877	0.1245521	0.09635607	0.07794845	0.07385991	0.06986752	0.05123571	0.0210446	-0.008412859	-0.02659053	-0.02344541	-5.804372×10 ⁻¹¹	0.02344541	0.02659053	0.008412859	-0.0210446	-0.
4.780402	3.565976	1.18935	-0.4505739	-2.206794	-4.399329	0.06346754	4.535463	2.379375	0.7252099	-0.6264028	-2.225368	-4.279295	1.444385×10 ⁻¹¹	4.279295	2.225368	0.6264028	-0.7252099	-∞
2.842763	2.134393	0.8682899	-0.4377384	-1.679619	-2.084094	0.03777839	2.168358	1.796138	0.6296579	-0.5417111	-1.677943	-2.037693	-3.548115×10 ⁻¹²	2.037693	1.677943	0.5417111	-0.6296579	-1
-2.410353×10 ⁻⁸	3.062399×10 ⁻⁸	-4.397001×10 ⁻⁹	3.066824×10 ⁻¹⁰	-8.12632×10 ⁻¹¹	3.636829×10 ⁻¹¹	-1.325189×10 ⁻¹¹	4.764349×10 ⁻¹²	-7.14584×10 ⁻¹³	1.790365×10 ⁻¹³	-3.8029×10 ⁻¹³	2.910935×10 ⁻¹³	0	1.375317×10 ⁻¹³	-9.949168×10 ⁻¹³	2.393×10 ⁻¹²	-3.712996×10 ⁻¹²	2.380218×10 ⁻¹²	3.02
-2.842763	-2.134393	-0.8682899	0.4377384	1.679619	2.084094	-0.03777839	-2.168358	-1.796138	-0.6296579	0.5417111	1.677943	2.037693	4.127238×10 ⁻¹²	-2.037693	-1.677943	-0.5417111	0.6296579	1
-4.780402	-3.565976	-1.18935	0.4505739	2.206794	4.399329	-0.06346754	-4.535463	-2.379375	-0.7252099	0.6264028	2.225368	4.279295	-1.911611×10 ⁻¹¹	-4.279295	-2.225368	-0.6264028	0.7252099	2
-0.1550078	-0.1611013	-0.1491877	-0.1245521	-0.09635607	-0.07794845	-0.07385991	-0.06986752	-0.05123571	-0.0210446	0.008412859	0.02659053	0.02344541	7.954439×10 ⁻¹¹	-0.02344541	-0.02659053	-0.008412859	0.0210446	0.
4.459403	3.240091	0.8978376	-0.6826157	-2.370906	-4.525	-0.07575709	4.382355	2.264704	0.6817866	-0.6007781	-2.152138	-4.210913	-3.335117×10 ⁻¹⁰	4.210913	2.152138	0.6007781	-0.6817866	-∞
2.479522	1.796141	0.601364	-0.6167145	-1.746922	-2.09269	-0.07626484	1.948268	1.634146	0.5815058	-0.4919668	-1.536331	-1.879395	1.414276×10 ⁻⁹	1.879395	1.536331	0.4919668	-0.5815058	-1
-0.4861463	-0.3669331	-0.21034	-0.08576274	0.1194038	0.3136824	-0.07347951	-0.4611397	-0.2665259	-0.05675809	0.07980332	0.2737226	0.431664	-6.33653×10 ⁻⁹	-0.431664	-0.2737226	-0.07980332	0.05675809	0.
-3.766235	-2.572475	-0.9829263	0.5155949	2.056503	3.228613	-0.05825506	-3.354524	-2.212955	-0.7309473	0.6696087	2.124156	3.225638	2.942834×10 ⁻⁸	-3.225638	-2.124156	-0.6696087	0.7309473	2
-8.088697	-4.109035	-1.416336	0.9453084	3.232331	4.781353	-0.0217637	-4.838587	-3.334748	-1.141499	1.025238	3.194104	4.673215	-1.247931×10 ⁻⁸	-4.673215	-3.194104	-1.025238	1.141499	3

kNm/m

