



$$N_{3,w}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{3,\theta_x}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{3,\theta_y}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{4b}(\eta)$$

$$N_{4,w}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{4,\theta_x}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{4,\theta_y}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{4b}(\eta)$$

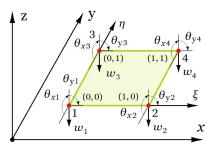
For twist  $\psi$

$$N_{1,\Psi}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{2b}(\eta)$$

$$N_{2,\psi}(\zeta; \eta) = \Phi_{4a}(\zeta) \cdot \Phi_{2b}(\eta)$$

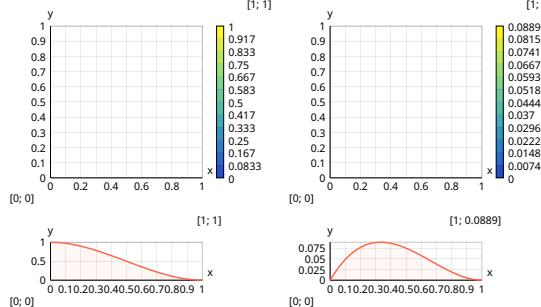
$$N_{3,\psi}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{4b}(\eta)$$

$$N_{4,\psi}(\varsigma; \eta) = \varPhi_{2a}(\varsigma) \cdot \varPhi_{4b}(\eta)$$



### $N_{1,w}$ shape function plot

### $N_{1,\theta_x}$ shape function plot



## **Constitutive matrix** (stress - strain relationship)

$$D_{1,1} = \frac{E \cdot t^3}{12 \cdot (1 - \nu^2)} = \frac{35000 \cdot 0.2^3}{12 \cdot (1 - 0.2^2)} = 24.305556 \text{ kNm}$$

$$\mathbf{D} = \mathbf{D}_{1,1} \cdot \mathbf{h} \mathbf{p} \left( \begin{bmatrix} 1; v; 0 & | & v; 1; 0 & | & 0; 0; \frac{1-v}{2} \end{bmatrix} \right) = 24.305556 \cdot \mathbf{h} \mathbf{p} \left( \begin{bmatrix} 1; 0.2; 0 & | & 0.2; 1; 0 & | & 0; 0; \frac{1-0.2}{2} \end{bmatrix} \right) =$$

$$= \begin{bmatrix} 24.305556 & 4.861111 & 0 \\ 4.861111 & 24.305556 & 0 \\ 0 & 0 & 9.722222 \end{bmatrix} \text{kNm}$$

Element stiffness matrix calculation ... ▾

Element stiffness matrix coefficients (above the main diagonal only)

$$K_e = D_{1,1} \cdot K_e = 24.305556 \cdot K_e =$$

### **Element load vector**

$$\tilde{\mathbf{e}} = \frac{\mathbf{q} \cdot \mathbf{A}_1}{24} \cdot \left[ 6; \mathbf{a}_1; \mathbf{b}_1; \frac{\mathbf{A}_1}{6}; 6; -\mathbf{a}_1; \mathbf{b}_1; \frac{-\mathbf{A}_1}{6}; 6; -\mathbf{a}_1; -\mathbf{b}_1; \frac{\mathbf{A}_1}{6}; 6; \mathbf{a}_1; -\mathbf{b}_1; \frac{-\mathbf{A}_1}{6} \right] = \frac{10 \cdot 0.36}{24} \cdot \left[ 6; 0.6; 0.6; \frac{0.36}{6}; 6; -0.6 \right]$$

$$0.6; \frac{-0.36}{6}; 6; -0.6; -0.6; \frac{0.36}{6}; 6; 0.6; -0.6; \frac{-0.36}{6} \Big] = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 0.9 \ -0.09 \ 0.09 \ -0.009 \\ 0.9 \ -0.09 \ -0.09 \ 0.009 \ 0.9 \ 0.09 \ -0.09 \ -0.009] \text{ kN}$$

Table 1. Mean values of the parameters

### Solution

## Global stiffness matrix

### Global load vector

$$\mathbf{z} = [0.9, 0.09, 0.09, 0.009, 1.8, 0.18, 0, 0, 1.8, 0.18, 0, 0, 1.8, 0.18, 0, 0, 1.8, 0.18, 0, 0, -0.009]^T \text{ kN}$$

### Solution of the system of equations

$\text{?} = \text{solve}(K, E) = [0, 0.5523352, 0.382696, -0.4159901, 0.2028057, 0.3732616, 0.2648531, 0]$

$\text{sisolve}(\mathbf{R}, \mathbf{y}) = [0.3091046 \ 0.04831304 \ -0.02500182 \ 0.2612117 \ 0.3426]$

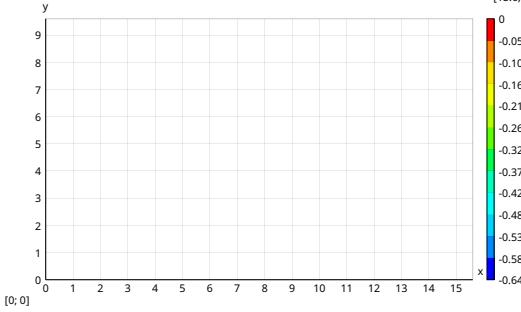
0.4681261

## Results

### Joint displacement

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	0
0.139	0.398	0.556	0.564	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	0.299
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.387	0.489	0.564	0.562	0.485	0.387	0.299	
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	0

mm



#### Bending moments

$$Z_j(j) = \text{slice}(\mathcal{E}; k_1 \cdot (j-1) + 1; k_1 \cdot j)$$

$$Z_e(e) = \text{hp}([Z_j(e_{j,e}, 1); Z_j(e_{j,e}, 2); Z_j(e_{j,e}, 3); Z_j(e_{j,e}, 4)])$$

Average bending moments at joints, kNm/m

$$M_j =$$

1.500275	0.3097747	0.2197379	0.1563434	0.1570478	0.9983225	0.1564233	0.1519636	0.1942465	0.151963	0.1564238	0.9983224	0.1570487	0.1563432	0.2197373	0.3097501	1.500258	8.50221	6.47933	5.777678	1.500075
1.561323	7.805992	9.338126	7.6683	3.333101	-28.361989	3.146397	7.323883	8.895047	7.323882	3.146398	-28.361989	3.333102	7.6683	9.338126	7.805978	1.56134	0.3220403	5.382484	7.126847	1.561519
8.088697	3.766245	4.0861465	-2.479522	-4.459403	0.1550078	4.780402	2.842763	-2.410353e-10^8	-2.842763	4.780402	-0.1550078	4.459403	2.479522	-0.4861463	-3.766235	-8.088697	4.109025	2.572475	0.3669336	8.088698

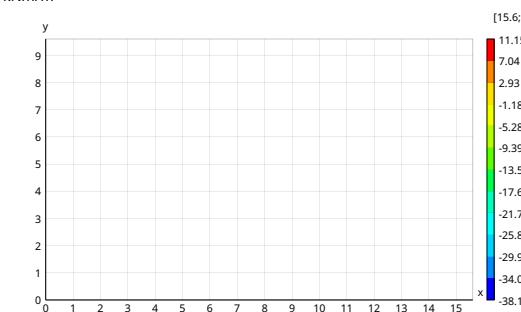
#### Bending moments for the plate

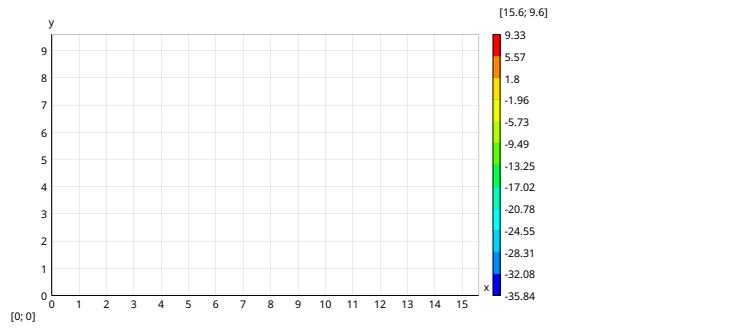
Bending moments -  $M_x$ 

$$\text{transp}(Mx) =$$

1.500275	11.065783	10.476631	6.796938	0.8844119	-31.234001	0.3020629	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860142	5.919519	8.876859	8.779239	5.643938	0.3020627	1.500259	
0.3097747	6.47933	9.431828	8.876831	4.922389	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360952	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	3.808562	-3.260011	0.3097374	
0.2197379	5.777678	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	5.616596	2.93207	-2.819213	0.2197445
0.1563434	5.993171	6.125974	8.492762	8.492762	-3.257726	-7.988419	-8.323661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.318295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.82661	0.1563406
0.1570478	7.269667	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	8.788235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.888235	7.713212	3.766824	-5.865488	0.1570494
0.9983225	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	4.521792	-2.745552	0.9983223	
0.1564233	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	0.1564268
0.1519636	5.873203	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.624647	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.624647	6.469771	2.594271	-4.009025	0.1519571
0.1942465	5.514146	8.380234	7.692958	3.468366	-2.491073	-5.742609	-3.07336	2.914028	5.979232	6.142664	2.786926	-2.376796	-5.118358	-2.376796	2.786926	6.142664	5.979232	2.310482	-3.07336	0.194254
0.151963	5.873204	8.92797	8.229786	3.773483	-3.41909	-8.032715	-4.009025	2.594271	6.469771	6.624647	3.062933	-3.269913	-7.306549	-3.269913	3.062933	6.624647	6.469771	2.594271	-4.009025	0.1519564
0.1564238	7.221066	10.282036	9.436498	4.848272	-5.357982	-16.182847	-5.963002	3.62427	7.560119	7.698867	4.053292	-5.138773	-15.091479	-5.138773	4.053292	7.698867	7.560119	3.62427	-5.963002	0.1564266
0.9983224	9.038567	11.080453	10.122808	5.763875	-2.1353	-38.650028	-2.745552	4.521792	8.190807	8.315047	4.904107	-2.088386	-36.536389	-2.088386	4.904107	8.315047	4.521792	-2.745552	0.9983221	
0.1570478	7.269659	10.37227	9.556617	4.972566	-5.268395	-16.118845	-5.865488	3.766824	7.713212	8.788235	4.157483	-5.096247	-15.085958	-5.096247	4.157483	7.888235	7.713212	3.766824	-5.865488	0.1570582
0.1563432	5.993171	6.125974	8.492762	8.492762	-3.257726	-7.988419	-8.323661	2.913644	6.797415	6.926775	3.304594	-3.205142	-7.318295	-3.205142	3.304594	6.926775	6.797415	2.913644	-3.82661	0.156342
0.2197373	5.777678	8.716196	8.134919	4.039416	-2.257802	-6.060553	-2.819213	2.93207	6.516597	6.640392	3.291629	-2.294127	-5.615407	-2.294127	3.291629	6.640392	5.616596	2.93207	-2.819213	0.2197303
0.3097501	6.479335	9.43183	8.876831	4.922388	-2.695046	-10.242271	-3.260012	3.808562	7.249978	7.360952	4.127543	-2.764018	-9.732553	-2.764018	4.127543	7.360952	3.808562	-3.260012	0.3094814	
1.500258	8.502224	11.065783	10.476631	6.796939	0.8844112	-31.234001	0.3020626	5.643937	8.779239	8.876858	5.91952	0.6860137	-30.131649	0.6860143	5.919519	8.876859	8.779239	5.643938	0.3020627	1.500075

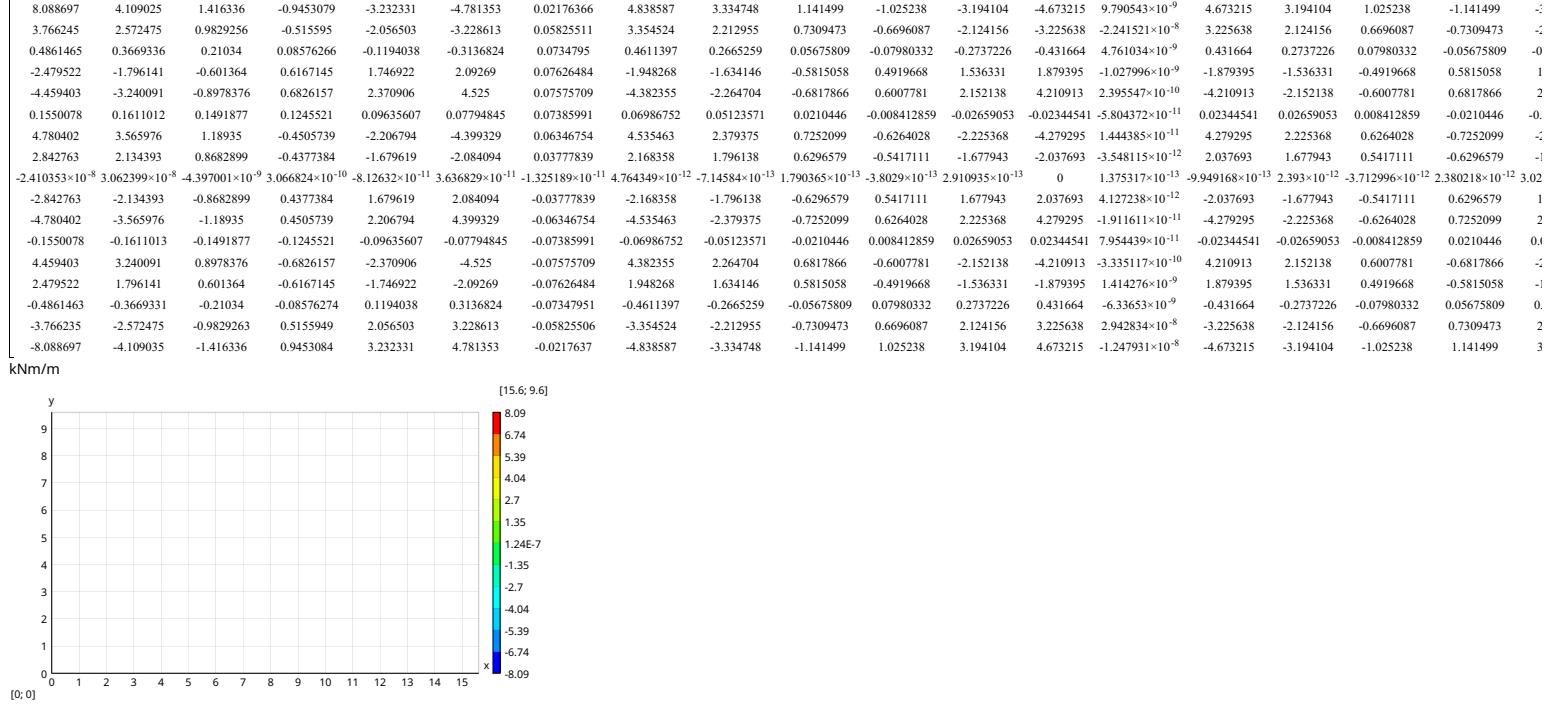
kNm/m





Bending moments  $M_{xy}$

transp( $M_{xy}$ ) =



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