

Finite Element Analysis of Flat Slab

Using numerical formulation of Bogner-Fox-Schmit (BFS) plate element

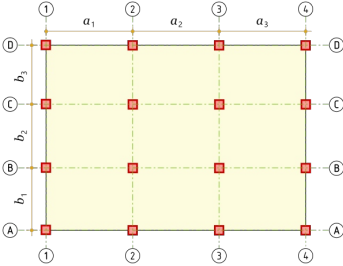
Input data

Span lengths

$r = \mathbf{hp}([3.6; 4.2; 4.2; 3.6]) = [3.6 \ 4.2 \ 4.2 \ 3.6] \text{ m}$

$r = \mathbf{hp}([3; 3.6; 3]) = [3 \ 3.6 \ 3] \text{ m}$

Number of axes -  $n_{sa} = \text{len}(r) + 1 = 5$ ,  $n_{sb} = \text{len}(r) + 1 = 4$



Axis coordinates -  $\alpha_s = [0 \ 3.6 \ 7.8 \ 12 \ 15.6] \text{ m}$ ,  $\eta_s = [0 \ 3 \ 6.6 \ 9.6] \text{ m}$

Slab dimensions -  $l_a \Rightarrow x_{s,5} = 15.6 \text{ m}$ ,  $l_b \Rightarrow y_{s,4} = 9.6 \text{ m}$

Thickness -  $t = 0.2 \text{ m}$

Load -  $q = 10 \text{ kN/m}^2$

Modulus of elasticity -  $E = 35000 \text{ MPa}$

Poisson's ratio -  $\nu = 0.2$

Finite element mesh

We will use Bogner-Fox-Schmit rectangular finite element with  $n_{\text{DOF}_5} = 16$

Element dimensions -  $a_1 = 0.6 \text{ m}$ ,  $b_1 = 0.6 \text{ m}$

Number of elements and joints along  $a$  and  $b$  -

$n_a = \text{ceiling}\left(\frac{a}{a_1}\right) = \text{ceiling}\left(\frac{a}{0.6}\right) = [6 \ 7 \ 7 \ 6]$ ,  $n_{ea} = \text{sum}(n_a) = 26$ ,  $n_{ja} = n_{ea} + 1 = 26 + 1 = 27$

$n_b = \text{ceiling}\left(\frac{b}{b_1}\right) = \text{ceiling}\left(\frac{b}{0.6}\right) = [5 \ 6 \ 5]$ ,  $n_{eb} = \text{sum}(n_b) = 16$ ,  $n_{jb} = n_{eb} + 1 = 16 + 1 = 17$

Total number of elements -  $n_e = n_{ea} \cdot n_{eb} = 26 \cdot 16 = 416$

Total number of joints -  $n_j = n_{ja} \cdot n_{jb} = 27 \cdot 17 = 459$

Supported joints count -  $n_s = n_{sa} \cdot n_{sb} = 5 \cdot 4 = 20$

Joint coordinates

$j = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.6 \ 0.6 \ 0.6 \dots 15.6] \text{ m}$

$j = [0 \ 0.6 \ 1.2 \ 1.8 \ 2.4 \ 3 \ 3.6 \ 4.2 \ 4.8 \ 5.4 \ 6 \ 6.6 \ 7.2 \ 7.8 \ 8.4 \ 9 \ 9.6 \ 0 \ 0.6 \ 1.2 \dots 9.6] \text{ m}$

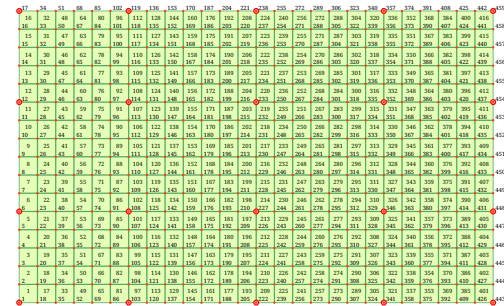
Numbers of joints at elements' corners

$\text{transp}(e_j) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 18 & 19 & 20 & 21 & \dots & 441 \\ 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 35 & 36 & 37 & 38 & \dots & 458 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 36 & 37 & 38 & 39 & \dots & 459 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 19 & 20 & 21 & 22 & \dots & 442 \end{bmatrix}$

Supported joints

$j = [1 \ 6 \ 12 \ 17 \ 103 \ 108 \ 114 \ 119 \ 222 \ 227 \ 233 \ 238 \ 341 \ 346 \ 352 \ 357 \ 443 \ 448 \ 454 \ 459]$

Joints for element  $e$  -  $j_e(e) = \text{row}(e_j; e)$



Finite element formulation

Shape functions

Along dimension  $a$

Base functions First derivatives Second derivatives

$\Phi_{1a}(\xi) = 1 - \xi^2 \cdot (3 - 2 \cdot \xi)$   $\Phi'_{1a}(\xi) = -6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$   $\Phi''_{1a}(\xi) = -\frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$

$\Phi_{2a}(\xi) = \xi \cdot a_1 \cdot (1 - \xi \cdot (2 - \xi))$   $\Phi'_{2a}(\xi) = 1 - \xi \cdot (4 - 3 \cdot \xi)$   $\Phi''_{2a}(\xi) = -\frac{2}{a_1} \cdot (2 - 3 \cdot \xi)$

$\Phi_{3a}(\xi) = \xi^2 \cdot (3 - 2 \cdot \xi)$   $\Phi'_{3a}(\xi) = 6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$   $\Phi''_{3a}(\xi) = \frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$

$\Phi_{4a}(\xi) = \xi^2 \cdot a_1 \cdot (-1 + \xi)$   $\Phi'_{4a}(\xi) = -\xi \cdot (2 - 3 \cdot \xi)$   $\Phi''_{4a}(\xi) = -\frac{2}{a_1} \cdot (1 - 3 \cdot \xi)$

Along dimension  $b$

Base functions First derivatives Second derivatives

$\Phi_{1b}(\eta) = 1 - \eta^2 \cdot (3 - 2 \cdot \eta)$   $\Phi'_{1b}(\eta) = -6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$   $\Phi''_{1b}(\eta) = -\frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$

$\Phi_{2b}(\eta) = \eta \cdot b_1 \cdot (1 - \eta \cdot (2 - \eta))$   $\Phi'_{2b}(\eta) = 1 - \eta \cdot (4 - 3 \cdot \eta)$   $\Phi''_{2b}(\eta) = -\frac{2}{b_1} \cdot (2 - 3 \cdot \eta)$

$\Phi_{3b}(\eta) = \eta^2 \cdot (3 - 2 \cdot \eta)$   $\Phi'_{3b}(\eta) = 6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$   $\Phi''_{3b}(\eta) = \frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$

$\Phi_{4b}(\eta) = \eta^2 \cdot b_1 \cdot (-1 + \eta)$   $\Phi'_{4b}(\eta) = -\eta \cdot (2 - 3 \cdot \eta)$   $\Phi''_{4b}(\eta) = -\frac{2}{b_1} \cdot (1 - 3 \cdot \eta)$

For vertical displacements  $w$  For rotations  $\theta_x$  For rotations  $\theta_y$

$N_{1,w}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{1,\theta_x}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{1,\theta_y}(\xi; \eta) = \Phi_{1a}(\xi) \cdot \Phi_{2b}(\eta)$

$N_{2,w}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{2,\theta_x}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{1b}(\eta)$   $N_{2,\theta_y}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{2b}(\eta)$

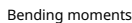


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Global load vector
r = [0.9 0.09 0.09 0.009 1.8 0.18 0 0 1.8 0.18 0 0 1.8 0.18 0 0 1.8 0.18 0 0 ... 0.009] kN
Solution of the system of equations
f = solve(K*f) = [0 0.5523613 0.3827392 -0.4161287 0.2028064 0.3732832 0.2648463 -0.1937261 0.2989047
0.3091182 0.04830843 -0.02501454 0.261207 0.3426386 -0.1651493 0.1275073 0.1211489
0.4681287 -0.2671179 0.2293302 ... -0.4161286] mm

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## Joint displacements

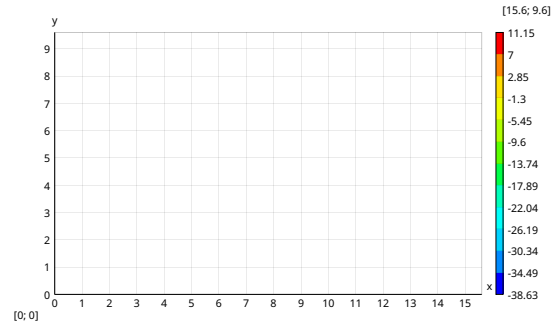

$$Z_i(j) = \text{slice}(Z; k_1 \cdot (j-1) + 1; k_1 \cdot j)$$

Average bending moments at joints, kNm/m

### Bending moments for the plate

$$\text{transp}(Mx) =$$

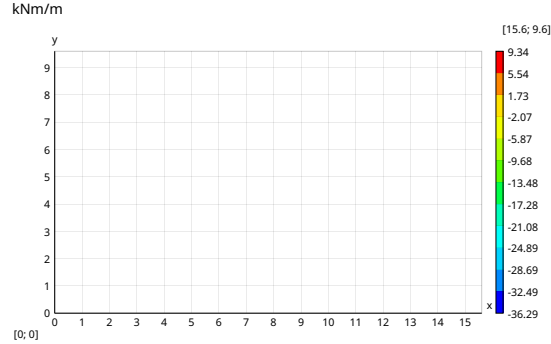
kNm/m



Bending moments  $M_y$

transp ( $M_y$ ) =

1.566914	0.3205972	0.2333587	0.2147852	0.1682816	0.1798035	1.015789	0.1794301	0.1669894	0.2098191	0.2091906	0.1663467	0.1776119	0.9891482	0.1776123	0.1663459	0.2091915	0.2098181	0.1669903	0.1794296	...	1.566846
7.805101	5.382201	4.308412	4.134952	4.742331	6.340395	8.230714	6.28316	4.611981	3.89228	3.867499	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805153
9.33778	7.126518	5.800074	5.553032	6.414898	8.11577	9.046666	8.014704	6.188836	5.14485	5.116777	6.090589	7.821224	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337789
7.668097	5.41756	4.262493	3.920413	4.233692	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
3.333	-0.3280779	0.5396346	0.405616	-1.17438	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964996	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964995	-1.399574	-3.531204	...	3.332998
-28.361735	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.14639	-0.5066696	0.3485146	0.1799353	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146398
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
8.895019	6.740122	5.350863	4.96126	5.608937	6.913063	7.572529	6.693158	5.141784	4.180525	4.149637	5.027791	6.456862	7.212973	6.456862	5.027791	4.149637	4.180525	5.141784	6.693158	...	8.895035
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
3.14639	-0.5066695	0.3485145	0.1799354	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146397
-28.361734	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.332997	-0.3280781	0.5396345	0.405616	-1.174379	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964994	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964996	-1.399574	-3.531204	...	3.333003
7.668093	5.41756	4.262493	3.920414	4.233691	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
9.337791	7.126529	5.800079	5.553029	6.414899	8.115769	9.046667	8.014704	6.188836	5.14485	5.116777	6.090589	7.821225	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337788
7.80518	5.382247	4.308397	4.134958	4.742326	6.340398	8.230713	6.283161	4.61198	3.892281	3.867498	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805067
1.566813	0.3204128	0.2333748	0.2147746	0.1682944	0.1797983	1.015789	0.1794292	0.1669914	0.2098169	0.2091927	0.166345	0.1776128	0.9891482	0.177612	0.1663463	0.209191	0.2098187	0.1669898	0.1794301	...	1.56695



Bending moments  $M_{xy}$

transp ( $M_{xy}$ ) =																			
8.091391	4.110857	1.417137	-0.9458016	-3.233923	-4.783472	0.02175397	4.840696	3.336369	1.142106	-1.025808	-3.195694	-4.675291	8.203632×10 <sup>-9</sup>	4.675291	3.195694	1.025808	-1.142106	-4.675291	-8.203632×10 <sup>-9</sup>
3.766896	2.572942	0.9831798	-0.5157529	-2.056961	-3.229076	0.05823395	3.354953	2.213405	0.7311327	-0.6697977	-2.124616	-3.226088	-1.924384×10 <sup>-8</sup>	3.226088	2.124615	0.6697977	-0.731132	-3.226088	-1.924384×10 <sup>-8</sup>
0.4863938	0.367139	0.2104715	0.08573235	-0.1195606	-0.3138301	0.07348128	0.4612966	0.2667083	0.05683469	-0.07988453	-0.2739108	-0.4318273	4.135681×10 <sup>-9</sup>	0.4318273	0.2739109	0.07988453	-0.0568346	-4.135681×10 <sup>-9</sup>	
-2.479308	-1.795977	-0.6012575	0.6167458	1.746893	2.092654	0.07628287	-1.948193	-1.634066	-0.5814737	0.4919345	1.536254	1.879329	-9.399297×10 <sup>-10</sup>	-1.879329	-1.536254	-0.4919345	0.5814737	1.879329	9.399297×10 <sup>-10</sup>
-4.459199	-3.239942	-0.8977427	0.6826669	2.370922	4.524997	0.07578146	-4.382303	-2.264662	-0.6817699	0.6007663	2.152105	4.210878	2.347034×10 <sup>-10</sup>	-4.210878	-2.152105	-0.6007663	0.6817699	4.210878	2.347034×10 <sup>-10</sup>
0.1550864	0.1611718	0.1492474	0.1245987	0.09639145	0.07797703	0.07388198	0.06988651	0.05125056	0.02105121	-0.008413693	-0.02659669	-0.02344966	-6.046931×10 <sup>-11</sup>	0.02344966	0.02659669	0.008413693	-0.0210512	-0.02344966	-6.046931×10 <sup>-11</sup>
4.780359	3.565968	1.189369	-0.4505428	-2.206755	-4.399285	0.06348492	4.535454	2.37937	0.7252095	-0.6263983	-2.225358	-4.279277	1.578955×10 <sup>-11</sup>	4.279277	2.225358	-0.6263983	-0.725209	-4.279277	-1.578955×10 <sup>-11</sup>
2.842723	2.13438	0.8682924	-0.4377229	-1.679594	-2.084069	0.03778704	2.16835	1.796129	0.6296557	-0.5417072	-1.677932	-2.037682	-4.744667×10 <sup>-12</sup>	2.037682	1.677932	0.5417072	-0.629655	-2.037682	-4.744667×10 <sup>-12</sup>
1.18852×10 <sup>-8</sup>	-6.576042×10 <sup>-9</sup>	-7.016847×10 <sup>-10</sup>	1.244056×10 <sup>-10</sup>	2.191792×10 <sup>-10</sup>	-7.501514×10 <sup>-11</sup>	1.8954×10 <sup>-11</sup>	-1.177769×10 <sup>-13</sup>	3.047327×10 <sup>-12</sup>	1.092988×10 <sup>-12</sup>	-4.777445×10 <sup>-12</sup>	5.435255×10 <sup>-12</sup>	-3.554444×10 <sup>-12</sup>	3.228026×10 <sup>-12</sup>	-2.990397×10 <sup>-12</sup>	1.375859×10 <sup>-12</sup>	-1.298123×10 <sup>-12</sup>	7.488428×10 <sup>-13</sup>	-3.228026×10 <sup>-12</sup>	3.554444×10 <sup>-12</sup>
-2.842723	-2.13438	-0.8682924	0.4377229	1.679594	2.084069	-0.03778704	-2.16835	-1.796129	-0.6296557	0.5417072	1.677932	2.037682	-7.622464×10 <sup>-12</sup>	-2.037682	-1.677932	-0.5417072	0.6296555	-2.037682	-7.622464×10 <sup>-12</sup>
-4.780359	-3.565968	-1.189369	0.4505428	2.206755	4.399285	-0.06348492	-4.535454	-2.37937	-0.7252095	0.6263983	2.225358	4.279277	2.756043×10 <sup>-11</sup>	-4.279277	-2.225358	-0.6263983	0.725209	4.279277	2.756043×10 <sup>-11</sup>
-0.1550866	-0.1611716	-0.1492474	-0.1245987	-0.09639145	-0.07797703	-0.07388197	-0.06988651	-0.05125056	-0.02105121	0.008413693	0.02659669	0.02344966	-1.059221×10 <sup>-10</sup>	-0.02344966	-0.02659669	-0.008413693	0.0210512	-1.059221×10 <sup>-10</sup>	
4.459199	3.239941	0.8977427	-0.6826669	-2.370922	-4.524997	-0.07578147	4.382303	2.264662	0.6817699	-0.6007663	-2.152105	-4.210878	4.094768×10 <sup>-10</sup>	4.210878	2.152105	0.6007663	-0.681769	-4.094768×10 <sup>-10</sup>	
2.479312	1.795976	0.6012571	-0.6167458	-1.746893	-2.092654	-0.07628286	1.948193	1.634066	0.5814737	-0.4919345	-1.536254	-1.879329	-1.611647×10 <sup>-9</sup>	1.879329	1.536254	0.4919345	-0.581473	-1.611647×10 <sup>-9</sup>	
-0.4863957	-0.3671418	-0.2104716	-0.08573185	0.1195605	0.3138302	-0.07348132	-0.4612966	-0.2667083	-0.05683469	0.07988452	0.2739109	0.4318273	6.850159×10 <sup>-9</sup>	-0.4318273	-0.2739109	-0.07988453	0.0568346	6.850159×10 <sup>-9</sup>	
-3.766967	-2.57294	-0.9831755	0.5157536	2.056962	3.229076	-0.05823381	-3.354953	-2.213405	-0.7311327	0.6697977	2.124615	3.226088	-3.125512×10 <sup>-8</sup>	-3.226088	-2.124615	-0.6697977	0.731132	3.226088	-3.125512×10 <sup>-8</sup>
-8.09139	-4.110792	-1.417137	0.9457983	3.233922	4.783472	-0.02175395	-4.840696	-3.336369	-1.142106	1.025808	3.195694	4.675291	1.254052×10 <sup>-8</sup>	-4.675291	-3.195694	-1.025808	1.142106	4.675291	1.254052×10 <sup>-8</sup>

