

Finite Element Analysis of Flat Slab

Using numerical formulation of Bogner-Fox-Schmit (BFS) plate element

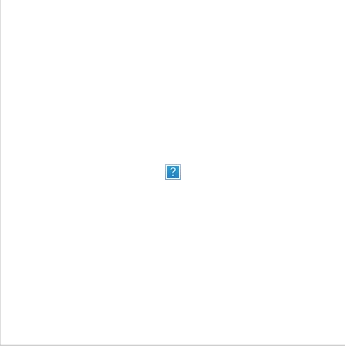
Input data

Span lengths

$r = \mathbf{hp}([3.6; 4.2; 4.2; 3.6]) = [3.6 \ 4.2 \ 4.2 \ 3.6] \text{ m}$

$r = \mathbf{hp}([3; 3.6; 3]) = [3 \ 3.6 \ 3] \text{ m}$

Number of axes - $n_{sa} = \text{len}(r) + 1 = 5$, $n_{sb} = \text{len}(r) + 1 = 4$



Axis coordinates - $\alpha_s = [0 \ 3.6 \ 7.8 \ 12 \ 15.6] \text{ m}$, $\alpha_b = [0 \ 3 \ 6.6 \ 9.6] \text{ m}$

Slab dimensions - $l_a = \alpha_{s,5} = 15.6 \text{ m}$, $l_b = \alpha_{b,4} = 9.6 \text{ m}$

Thickness - $t = 0.2 \text{ m}$

Load - $q = 10 \text{ kN/m}^2$

Modulus of elasticity - $E = 35000 \text{ MPa}$

Poisson's ratio - $\nu = 0.2$

Finite element mesh

We will use Bogner-Fox-Schmit rectangular finite element with $n_{\text{DOFs}} = 16$

Element dimensions - $a_1 = 0.6 \text{ m}$, $b_1 = 0.6 \text{ m}$

Number of elements and joints along a and b -

$n_a = \text{ceiling}\left(\frac{a}{a_1}\right) = \text{ceiling}\left(\frac{a}{0.6}\right) = [6 \ 7 \ 7 \ 6]$, $n_{ea} = \text{sum}(n_a) = 26$, $n_{ja} = n_{ea} + 1 = 26 + 1 = 27$

$n_b = \text{ceiling}\left(\frac{b}{b_1}\right) = \text{ceiling}\left(\frac{b}{0.6}\right) = [5 \ 6 \ 5]$, $n_{eb} = \text{sum}(n_b) = 16$, $n_{jb} = n_{eb} + 1 = 16 + 1 = 17$

Total number of elements - $n_e = n_{ea} \cdot n_{eb} = 26 \cdot 16 = 416$

Total number of joints - $n_j = n_{ja} \cdot n_{jb} = 27 \cdot 17 = 459$

Supported joints count - $n_s = n_{sa} \cdot n_{sb} = 5 \cdot 4 = 20$

Joint coordinates

$j = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.6 \ 0.6 \ \dots \ 15.6] \text{ m}$

$j = [0 \ 0.6 \ 1.2 \ 1.8 \ 2.4 \ 3 \ 3.6 \ 4.2 \ 4.8 \ 5.4 \ 6 \ 6.6 \ 7.2 \ 7.8 \ 8.4 \ 9 \ 9.6 \ 0 \ 0.6 \ 1.2 \ \dots \ 9.6] \text{ m}$

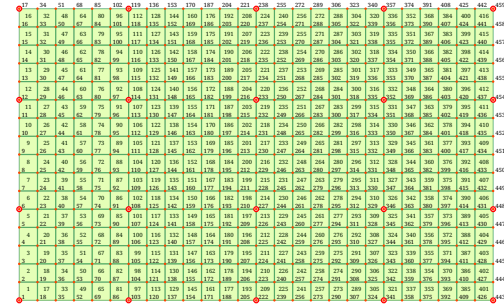
Numbers of joints at elements' corners

$\text{transp}(e_j) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 18 & 19 & 20 & 21 & \dots & 441 \\ 18 & 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 35 & 36 & 37 & 38 & \dots & 458 \\ 19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 & 33 & 34 & 36 & 37 & 38 & 39 & \dots & 459 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 19 & 20 & 21 & 22 & \dots & 442 \end{bmatrix}$

Supported joints

$j = [1 \ 6 \ 12 \ 17 \ 103 \ 108 \ 114 \ 119 \ 222 \ 227 \ 233 \ 238 \ 341 \ 346 \ 352 \ 357 \ 443 \ 448 \ 454 \ 459]$

Joints for element e - $j_e(e) = \text{row}(e_j; e)$



Finite element formulation

Shape functions

Along dimension a

Base functions	First derivatives	Second derivatives
$\phi_{1a}(\xi) = 1 - \xi^2 \cdot (3 - 2 \cdot \xi)$	$\phi'_{1a}(\xi) = -6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$	$\phi''_{1a}(\xi) = -\frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$
$\phi_{2a}(\xi) = \xi \cdot a_1 \cdot (1 - \xi) \cdot (2 - \xi)$	$\phi'_{2a}(\xi) = 1 - \xi \cdot (4 - 3 \cdot \xi)$	$\phi''_{2a}(\xi) = -\frac{2}{a_1} \cdot (2 - 3 \cdot \xi)$
$\phi_{3a}(\xi) = \xi^2 \cdot (3 - 2 \cdot \xi)$	$\phi'_{3a}(\xi) = 6 \cdot \frac{\xi}{a_1} \cdot (1 - \xi)$	$\phi''_{3a}(\xi) = \frac{6}{a_1^2} \cdot (1 - 2 \cdot \xi)$
$\phi_{4a}(\xi) = \xi^2 \cdot a_1 \cdot (-1 + \xi)$	$\phi'_{4a}(\xi) = -\xi \cdot (2 - 3 \cdot \xi)$	$\phi''_{4a}(\xi) = -\frac{2}{a_1} \cdot (1 - 3 \cdot \xi)$

Along dimension b

Base functions	First derivatives	Second derivatives
$\phi_{1b}(\eta) = 1 - \eta^2 \cdot (3 - 2 \cdot \eta)$	$\phi'_{1b}(\eta) = -6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$	$\phi''_{1b}(\eta) = -\frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$
$\phi_{2b}(\eta) = \eta \cdot b_1 \cdot (1 - \eta) \cdot (2 - \eta)$	$\phi'_{2b}(\eta) = 1 - \eta \cdot (4 - 3 \cdot \eta)$	$\phi''_{2b}(\eta) = -\frac{2}{b_1} \cdot (2 - 3 \cdot \eta)$
$\phi_{3b}(\eta) = \eta^2 \cdot (3 - 2 \cdot \eta)$	$\phi'_{3b}(\eta) = 6 \cdot \frac{\eta}{b_1} \cdot (1 - \eta)$	$\phi''_{3b}(\eta) = \frac{6}{b_1^2} \cdot (1 - 2 \cdot \eta)$
$\phi_{4b}(\eta) = \eta^2 \cdot b_1 \cdot (-1 + \eta)$	$\phi'_{4b}(\eta) = -\eta \cdot (2 - 3 \cdot \eta)$	$\phi''_{4b}(\eta) = -\frac{2}{b_1} \cdot (1 - 3 \cdot \eta)$

For vertical displacements w
For rotations θ_x
For rotations θ_y

$N_{1,w}(\xi;\eta) = \phi_{1a}(\xi) \cdot \phi_{1b}(\eta)$
 $N_{1,\theta_x}(\xi;\eta) = \phi_{2a}(\xi) \cdot \phi_{1b}(\eta)$
 $N_{1,\theta_y}(\xi;\eta) = \phi_{1a}(\xi) \cdot \phi_{2b}(\eta)$

$N_{2,w}(\xi;\eta) = \phi_{3a}(\xi) \cdot \phi_{1b}(\eta)$
 $N_{2,\theta_x}(\xi;\eta) = \phi_{4a}(\xi) \cdot \phi_{1b}(\eta)$
 $N_{2,\theta_y}(\xi;\eta) = \phi_{3a}(\xi) \cdot \phi_{2b}(\eta)$

$N_{3,w}(\xi;\eta) = \phi_{3a}(\xi) \cdot \phi_{3b}(\eta)$
 $N_{3,\theta_x}(\xi;\eta) = \phi_{4a}(\xi) \cdot \phi_{3b}(\eta)$
 $N_{3,\theta_y}(\xi;\eta) = \phi_{3a}(\xi) \cdot \phi_{4b}(\eta)$

$N_{4,w}(\xi;\eta) = \phi_{1a}(\xi) \cdot \phi_{3b}(\eta)$
 $N_{4,\theta_x}(\xi;\eta) = \phi_{2a}(\xi) \cdot \phi_{3b}(\eta)$
 $N_{4,\theta_y}(\xi;\eta) = \phi_{1a}(\xi) \cdot \phi_{4b}(\eta)$

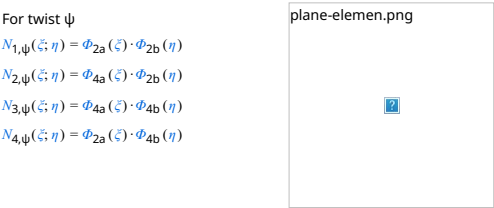
For twist ψ

$N_{1,\psi}(\xi;\eta) = \phi_{2a}(\xi) \cdot \phi_{2b}(\eta)$

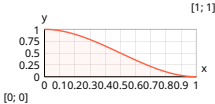
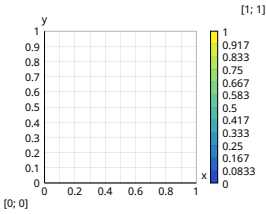
$N_{2,\psi}(\xi;\eta) = \phi_{4a}(\xi) \cdot \phi_{2b}(\eta)$

$N_{3,\psi}(\xi;\eta) = \phi_{4a}(\xi) \cdot \phi_{4b}(\eta)$

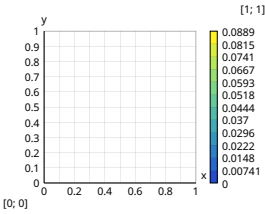
$N_{4,\psi}(\xi;\eta) = \phi_{2a}(\xi) \cdot \phi_{4b}(\eta)$



$N_{1,w}$ shape function plot



N_{1,θ_x} shape function plot



Shape functions vector

$$N(i;\xi;\eta) = \text{take}(i; N_{1,w}(\xi;\eta); N_{1,\theta_x}(\xi;\eta); N_{1,\theta_y}(\xi;\eta); N_{1,\psi}(\xi;\eta); N_{2,w}(\xi;\eta); N_{2,\theta_x}(\xi;\eta); N_{2,\theta_y}(\xi;\eta); N_{2,\psi}(\xi;\eta); N_{3,w}(\xi;\eta); N_{3,\theta_x}(\xi;\eta); N_{3,\theta_y}(\xi;\eta); N_{3,\psi}(\xi;\eta); N_{4,w}(\xi;\eta); N_{4,\theta_x}(\xi;\eta); N_{4,\theta_y}(\xi;\eta); N_{4,\psi}(\xi;\eta))$$

Constitutive matrix (stress - strain relationship)

$$D = \frac{E \cdot t^3}{12 \cdot (1 - \nu^2)} \cdot \text{hp}\left(\left[\begin{array}{c|c} 1; \nu; 0 & \nu; 1; 0 \\ 0; 0; \frac{1-\nu}{2} \end{array}\right]\right) = \frac{35000 \cdot 0.2^3}{12 \cdot (1 - 0.2^2)} \cdot \text{hp}\left(\left[\begin{array}{c|c} 1; 0.2; 0 & 0.2; 1; 0 \\ 0; 0; 0 \end{array}\right]\right)$$

$$\frac{1 - 0.2^2}{2} \left] \right) = \begin{bmatrix} 24.305556 & 4.861111 & 0 \\ 4.861111 & 24.305556 & 0 \\ 0 & 0 & 9.722222 \end{bmatrix} \text{ kNm}$$

Strain-displacement matrix

$$B_1(j;\xi;\eta) = \text{take}(j; \phi'_{1a}(\xi) \cdot \phi_{1b}(\eta); \phi''_{2a}(\xi) \cdot \phi_{1b}(\eta); \phi''_{1a}(\xi) \cdot \phi_{2b}(\eta); \phi''_{2a}(\xi) \cdot \phi_{2b}(\eta); \phi''_{3a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{1b}(\eta); \phi'_{3a}(\xi) \cdot \phi_{2b}(\eta); \phi''_{4a}(\xi) \cdot \phi_{2b}(\eta); \phi''_{3a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{3b}(\eta); \phi''_{3a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{4a}(\xi) \cdot \phi_{4b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{3b}(\eta); \phi''_{2a}(\xi) \cdot \phi_{3b}(\eta); \phi'_{1a}(\xi) \cdot \phi_{4b}(\eta); \phi''_{2a}(\xi) \cdot \phi_{4b}(\eta))$$

$$B_2(j;\xi;\eta) = \text{take}(j; \phi_{1a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{1a}(\xi) \cdot \phi''_{2b}(\eta); \phi_{2a}(\xi) \cdot \phi''_{2b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{1b}(\eta); \phi_{3a}(\xi) \cdot \phi''_{2b}(\eta); \phi_{4a}(\xi) \cdot \phi''_{2b}(\eta); \phi_{3a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{4a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{3a}(\xi) \cdot \phi''_{4b}(\eta); \phi_{4a}(\xi) \cdot \phi''_{4b}(\eta); \phi_{1a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{2a}(\xi) \cdot \phi'_{3b}(\eta); \phi_{1a}(\xi) \cdot \phi''_{4b}(\eta); \phi_{2a}(\xi) \cdot \phi''_{4b}(\eta))$$

$$B_3(j;\xi;\eta) = 2 \cdot \text{take}(j; \phi'_{1a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{1b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{2b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{3a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{4a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{3b}(\eta); \phi'_{1a}(\xi) \cdot \phi'_{4b}(\eta); \phi'_{2a}(\xi) \cdot \phi'_{4b}(\eta))$$

$$B(j;\xi;\eta) = \text{hp}([B_1(j;\xi;\eta); B_2(j;\xi;\eta); B_3(j;\xi;\eta)])$$

The coefficients of the stiffness matrix will be calculated by using the equation

$$K_{e,ij} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 B_i(\xi;\eta)^T \cdot D \cdot B_j(\xi;\eta) \, d\xi \, d\eta$$

Element stiffness matrix

(above the main diagonal only)

$$BTDB_e(i;j;\xi;\eta) = \text{transp}(B(i;\xi;\eta)) \cdot D \cdot B(j;\xi;\eta)$$

$$K_e(i;j) = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 BTDB_e(i;j;\xi;\eta) \, d\eta \, d\xi$$

$$\text{\textcolor{red}{\$Repeat}\{ \$Repeat\{ $K_{e,i,j} = $K_e(i;j) \text{ for } j = i...n\} \text{ for } i = 1...n\} = 0.9777823}$$

$K_e =$

796.296296	135.185185	135.185185	16.736111	-391.203704	84.953704	-13.657407	4.097222	-13.888889	36.574074	36.574074	-8.541667	-391.203704	-13.657407	84.953704	4.097222
0	46.666667	21.597222	4.666667	-84.953704	14.027778	-4.094824	0.7083333	-36.574074	10.277778	8.541667	-1.624997	-13.657407	1.944444	4.097222	-0.5833333
0	0	46.666667	4.666667	-13.657407	4.094824	1.944444	-0.5833333	-36.574074	8.541667	10.277778	-1.625	-84.953704	-4.097222	14.027778	0.7083333
0	0	0	0.9777823	-4.097222	0.7083333	0.5833333	-0.1611111	-8.541667	1.624997	1.625	-0.2305541	-4.097222	0.5833333	0.7083333	-0.1611111
0	0	0	0	796.296296	-135.185185	135.185185	-16.736111	-391.203704	13.657407	84.953704	-4.097222	-13.888889	-36.574074	36.574074	8.541667
0	0	0	0	0	46.666667	-21.597222	4.666667	13.657407	1.944444	-4.097222	-0.5833333	36.574074	10.277778	-8.541667	-1.624997
0	0	0	0	0	0	46.666667	-4.666667	-84.953704	4.097222	14.027778	-0.7083333	-36.574074	-8.541667	10.277778	1.625
0	0	0	0	0	0	0	0.9777823	4.097222	0.5833333	-0.7083333	-0.1611111	8.541667	1.624997	-1.625	-0.2305541
0	0	0	0	0	0	0	0	796.296296	-135.185185	-135.185185	16.736111	-391.203704	-84.953704	13.657407	4.097222
0	0	0	0	0	0	0	0	0	46.666667	21.597222	-4.666667	84.953704	14.027778	-4.094824	-0.7083333
0	0	0	0	0	0	0	0	0	0	0	46.666667	-4.666667	13.657407	4.094824	1.944444
0	0	0	0	0	0	0	0	0	0	0.9777823	-4.097222	-0.7083333	-0.5833333	-0.1611111	0.5833333
0	0	0	0	0	0	0	0	0	0	0	0	796.296296	135.185185	-135.185185	-16.736111
0	0	0	0	0	0	0	0	0	0	0	0	0	46.666667	-21.597222	-4.666667
0	0	0	0	0	0	0	0	0	0	0	0	0	0	46.666667	4.666667
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9777823

Element load vector

$$F_{e,i} = a_1 \cdot b_1 \cdot \int_0^1 \int_0^1 N_i(\xi;\eta)^T \cdot q \, d\xi \, d\eta$$

$$r_e = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 0.9 \ -0.09 \ 0.09 \ -0.009 \ 0.9 \ -0.09 \ -0.09 \ 0.009 \ 0.9 \ 0.09 \ -0.09 \ -0.009] \text{ kN}$$

Solution

Global stiffness matrix

$K =$

Global load vector

$$\vec{r} = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \dots 0.009] \text{ kN}$$

Solution of the system of equations

$$\vec{r} = \text{solve}(\mathbf{K}, \vec{r}) = [0 \ 0.5523613 \ 0.3827392 \ -0.4161287 \ 0.2028064 \ 0.3732832 \ 0.2648463 \ -0.1937261 \ 0.2989047 \ 0.3091182 \ 0.04830843 \ -0.02501454 \ 0.261207 \ 0.3426386 \ -0.1651493 \ 0.1275073 \ 0.1211489 \ 0.4681287 \ -0.2671179 \ 0.2293302 \dots -0.4161286] \text{ mm}$$

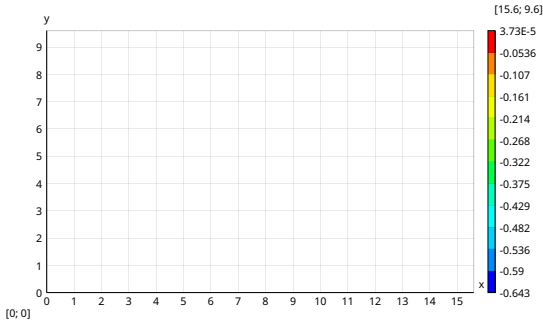
Results

Joint displacements

$$\text{transp}(W_Z) =$$

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.435	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.435	0.325	...	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	...	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.435	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.435	0.325	...	0.299
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	...	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.336	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	...	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	...	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	...	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	...	0.299
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	...	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	...	0

mm



Bending moments

$$Z_j(j) = \text{slice}(\mathcal{Z}; k_1 \cdot (j-1) + 1; k_1 \cdot j)$$

$$Z_e(e) = \mathbf{hp}([Z_j(e_{j,e,1}); Z_j(e_{j,e,2}); Z_j(e_{j,e,3}); Z_j(e_{j,e,4})])$$

Average bending moments at joints, kNm/m

$M_i =$

1.498465	0.3097372	0.2197322	0.1563433	0.1507519	0.9983302	0.1564193	0.1519741	0.19422	0.1519742	0.1564197	0.9983303	0.1570475	0.1563397	0.2197486	0.3098808	1.49858	8.502998	6.479199	5.777684	...	1.498434
1.566914	7.805101	9.33778	7.668097	3.333	-28.61735	3.14639	7.323888	8.89519	7.323888	3.14639	-28.61734	3.33297	7.668093	9.337791	7.80518	1.566813	8.5029972	5.382201	7.126518	...	1.56695
8.091391	3.766896	0.4863938	-2.479308	-4.459199	0.1550864	4.780359	28.84723	1.18852-10 ⁻⁸	-28.84723	-4.780359	-0.1550866	4.459199	2.479312	-0.4863957	-3.766697	-8.09139	4.110857	2.572942	0.367139	...	8.09133

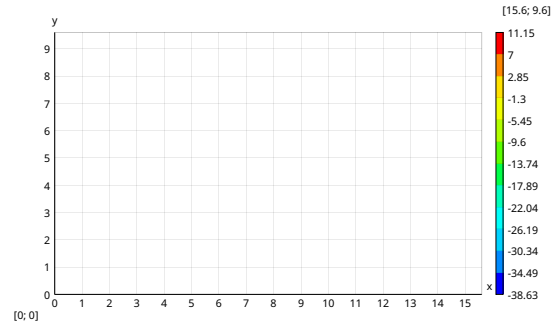
Bending moments for the plate

Bending moments - M_x

$$\text{transp}(\mathbf{M}\mathbf{x}) =$$

1.498465	8.502998	11.06652	10.47741	6.797447	0.888327	-1.235271	0.301501	5.644442	8.779998	8.877612	5.920001	0.685447	-30.133041	0.685445	9.920001	8.877618	8.779995	5.644443	0.301504	...	1.498453
0.309732	6.479199	9.44232	8.87725	4.922637	-2.659317	-1.042892	-3.26094	3.808797	7.250442	7.361393	1.277522	-7.264349	-9.733242	-2.764349	4.127752	7.361393	7.250426	3.808797	-3.260293	...	0.3098324
0.219732	5.777684	8.716293	8.135055	4.039483	-2.257879	-6.060711	-1.2819291	2.932144	6.516758	6.640543	3.291674	-2.294253	-5.616642	-2.294253	3.291674	6.640546	6.516758	2.932144	-1.2819291	...	0.2197447
0.156343	5.931716	9.125993	8.496278	4.07577	-3.257751	-7.98846	-3.832677	2.913674	6.979469	6.926822	3.304597	-3.205198	-7.381383	-3.205198	3.304597	6.926822	6.979469	2.913674	-3.832677	...	0.1563395
0.157019	7.269644	10.372249	9.566604	4.972555	-2.68409	-6.118864	-8.565491	3.766832	7.713228	7.838243	1.457472	-5.096284	-15.086013	-5.096284	1.457472	7.838243	7.713228	3.766832	-8.565491	...	0.1570491
0.9983302	9.038525	11.080418	10.122777	5.76385	-2.135317	-38.650059	-2.745557	4.52179	8.190804	8.315041	4.904088	-2.088413	-36.536458	-2.088413	4.904088	8.315041	8.190804	4.52179	-2.745557	...	0.9983301
0.1564193	7.221047	10.282001	9.436465	4.848424	-3.589009	-6.18287	-5.96302	3.624262	7.560111	6.698855	4.05327	-5.180058	-15.091525	-5.180058	4.05327	6.698855	7.560111	3.624262	-5.96302	...	0.1564295
0.1519741	8.573186	8.92795	8.229755	3.775343	-3.419119	-8.032747	-4.009042	2.594259	6.469761	6.628453	3.062913	-3.269947	-7.306587	-3.269947	3.062913	6.628453	6.469761	2.594259	-4.009042	...	0.1519596
0.19422	5.514447	8.830213	7.692391	3.468337	-2.491105	-5.742638	-3.073384	2.310468	5.979222	6.14265	2.786902	-3.736825	-5.118402	-3.736825	2.786902	6.14265	5.979222	2.310468	-3.073384	...	0.1942406
0.1519742	8.573186	8.92795	8.229755	3.775343	-3.419119	-8.032747	-4.009042	2.594259	6.469761	6.628453	3.062913	-3.269947	-7.306587	-3.269947	3.062913	6.628453	6.469761	2.594259	-4.009042	...	0.1519563
0.1564197	7.221047	10.282001	9.436465	4.848424	-3.589009	-6.18287	-5.96302	3.624262	7.560111	6.698855	4.05327	-5.180058	-15.091525	-5.180058	4.05327	6.698855	7.560111	3.624262	-5.96302	...	0.1564287
0.9983303	9.038524	11.080418	10.122777	5.76385	-2.135317	-38.650059	-2.745557	4.52179	8.190804	8.315041	4.904088	-2.088413	-36.536458	-2.088413	4.904088	8.315041	8.190804	4.52179	-2.745557	...	0.9983303
0.1570475	7.269644	10.372249	9.566604	4.972554	-2.68409	-6.118864	-8.565491	3.766832	7.713228	7.838243	1.457472	-5.096284	-15.086013	-5.096284	1.457472	7.838243	7.713228	3.766832	-8.565491	...	0.1570551
0.1563397	5.993177	9.125994	8.492789	4.07577	-3.257751	-7.98846	-3.832677	2.913674	6.979469	6.926822	3.304597	-3.205198	-7.381383	-3.205198	3.304597	6.926822	6.979469	2.913674	-3.832677	...	0.1563387
0.2197486	5.777685	8.716292	8.135052	4.039483	-2.257878	-6.060711	-1.2819291	2.932144	6.516758	6.640543	3.291674	-2.294253	-5.616642	-2.294253	3.291674	6.640543	6.516758	2.932144	-1.2819291	...	0.2197395
0.3098808	6.479177	9.432178	8.877282	4.922634	-2.659318	-1.042892	-3.260929	3.808797	7.250446	7.361393	1.277522	-7.264349	-9.733242	-2.764349	4.127752	7.361393	7.250426	3.808797	-3.260293	...	0.309677
1.49858	8.502893	11.066537	10.477401	6.797448	0.888363	-1.235227	0.301501	5.644443	8.779994	8.877613	5.920009	0.6854455	-30.133041	0.6854449	9.920011	8.87761	8.779995	5.644442	0.301507	...	1.498434

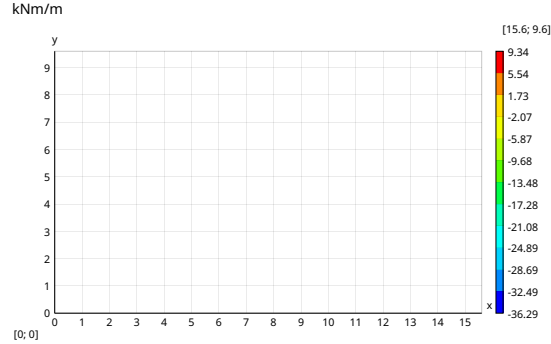
kNm/m



Bending moments M_y

transp (M_y) =

1.566914	0.3205972	0.2333587	0.2147852	0.1682816	0.1798035	1.015789	0.1794301	0.1669894	0.2098191	0.2091906	0.1663467	0.1776119	0.9891482	0.1776123	0.1663459	0.2091915	0.2098181	0.1669903	0.1794296	...	1.566846
7.805101	5.382201	4.308412	4.134952	4.742331	6.340395	8.230714	6.28316	4.611981	3.89228	3.867499	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805153
9.33778	7.126518	5.800074	5.553032	6.414898	8.11577	9.046666	8.014704	6.188836	5.14485	5.116777	6.090589	7.821224	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337789
7.668097	5.41756	4.262493	3.920413	4.233692	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
3.333	-0.3280779	0.5396346	0.405616	-1.17438	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964996	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964995	-1.399574	-3.531204	...	3.332998
-28.361735	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.14639	-0.5066696	0.3485146	0.1799353	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146398
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
8.895019	6.740122	5.350863	4.96126	5.608937	6.913063	7.572529	6.693158	5.141784	4.180525	4.149637	5.027791	6.456862	7.212973	6.456862	5.027791	4.149637	4.180525	5.141784	6.693158	...	8.895035
7.323888	5.094548	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
3.14639	-0.5066695	0.3485145	0.1799354	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146397
-28.361734	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.033174	-34.571125	-13.033174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.332997	-0.3280781	0.5396345	0.405616	-1.174379	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964994	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964996	-1.399574	-3.531204	...	3.333003
7.668093	5.41756	4.262493	3.920414	4.233691	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
9.337791	7.126529	5.800079	5.553029	6.414899	8.115769	9.046667	8.014704	6.188836	5.14485	5.116777	6.090589	7.821225	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337788
7.80518	5.382247	4.308397	4.134958	4.742326	6.340398	8.230713	6.283161	4.61198	3.892281	3.867498	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805067
1.566813	0.3204128	0.2333748	0.2147746	0.1682944	0.1797983	1.015789	0.1794292	0.1669914	0.2098169	0.2091927	0.166345	0.1776128	0.9891482	0.177612	0.1663463	0.209191	0.2098187	0.1669898	0.1794301	...	1.56695



Bending moments M_{xy}

transp (M_{xy}) =																			
8.091391	4.110857	1.417137	-0.9458016	-3.233923	-4.783472	0.02175397	4.840696	3.336369	1.142106	-1.025808	-3.195694	-4.675291	8.203632×10 ⁻⁹	4.675291	3.195694	1.025808	-1.142106	-4.675291	-8.203632×10 ⁻⁹
3.766896	2.572942	0.9831798	-0.5157529	-2.056961	-3.229076	0.05823395	3.354953	2.213405	0.7311327	-0.6697977	-2.124616	-3.226088	-1.924384×10 ⁻⁸	3.226088	2.124615	0.6697977	-0.731132	-3.226088	-1.924384×10 ⁻⁸
0.4863938	0.367139	0.2104715	0.08573235	-0.1195606	-0.3138301	0.07348128	0.4612966	0.2667083	0.05683469	-0.07988453	-0.2739108	-0.4318273	4.135681×10 ⁻⁹	0.4318273	0.2739109	0.07988453	-0.0568346	-4.135681×10 ⁻⁹	
-2.479308	-1.795977	-0.6012575	0.6167458	1.746893	2.092654	0.07628287	-1.948193	-1.634066	-0.5814737	0.4919345	1.536254	1.879329	-9.399297×10 ⁻¹⁰	-1.879329	-1.536254	-0.4919345	0.5814737	1.879329	9.399297×10 ⁻¹⁰
-4.459199	-3.239942	-0.8977427	0.6826669	2.370922	4.524997	0.07578146	-4.382303	-2.264662	-0.6817699	0.6007663	2.152105	4.210878	2.347034×10 ⁻¹⁰	-4.210878	-2.152105	-0.6007663	0.6817699	4.210878	2.347034×10 ⁻¹⁰
0.1550864	0.1611718	0.1492474	0.1245987	0.09639145	0.07797703	0.07388198	0.06988651	0.05125056	0.02105121	-0.008413693	-0.02659669	-0.02344966	-6.046931×10 ⁻¹¹	0.02344966	0.02659669	0.008413693	-0.0210512	-0.02344966	-6.046931×10 ⁻¹¹
4.780359	3.565968	1.189369	-0.4505428	-2.206755	-4.399285	0.06348492	4.535454	2.37937	0.7252095	-0.6263983	-2.225358	-4.279277	1.578955×10 ⁻¹¹	4.279277	2.225358	-0.6263983	-0.725209	-4.279277	-1.578955×10 ⁻¹¹
2.842723	2.13438	0.8682924	-0.4377229	-1.679594	-2.084069	0.03778704	2.16835	1.796129	0.6296557	-0.5417072	-1.677932	-2.037682	-4.744667×10 ⁻¹²	2.037682	1.677932	0.5417072	-0.629655	-2.037682	-4.744667×10 ⁻¹²
1.18852×10 ⁻⁸	-6.576042×10 ⁻⁹	-7.016847×10 ⁻¹⁰	1.244056×10 ⁻¹⁰	2.191792×10 ⁻¹⁰	-7.501514×10 ⁻¹¹	1.8954×10 ⁻¹¹	-1.177769×10 ⁻¹³	3.047327×10 ⁻¹²	1.092988×10 ⁻¹²	-4.777445×10 ⁻¹²	5.435255×10 ⁻¹²	-3.554444×10 ⁻¹²	3.228026×10 ⁻¹²	-2.990397×10 ⁻¹²	1.375859×10 ⁻¹²	-1.298123×10 ⁻¹²	7.488428×10 ⁻¹³	-3.228026×10 ⁻¹²	3.554444×10 ⁻¹²
-2.842723	-2.13438	-0.8682924	0.4377229	1.679594	2.084069	-0.03778704	-2.16835	-1.796129	-0.6296557	0.5417072	1.677932	2.037682	-7.622464×10 ⁻¹²	-2.037682	-1.677932	-0.5417072	0.6296555	-2.037682	-7.622464×10 ⁻¹²
-4.780359	-3.565968	-1.189369	0.4505428	2.206755	4.399285	-0.06348492	-4.535454	-2.37937	-0.7252095	0.6263983	2.225358	4.279277	2.756043×10 ⁻¹¹	-4.279277	-2.225358	-0.6263983	0.725209	4.279277	2.756043×10 ⁻¹¹
-0.1550866	-0.1611716	-0.1492474	-0.1245987	-0.09639145	-0.07797703	-0.07388197	-0.06988651	-0.05125056	-0.02105121	0.008413693	0.02659669	0.02344966	-1.059221×10 ⁻¹⁰	-0.02344966	-0.02659669	-0.008413693	0.0210512	-1.059221×10 ⁻¹⁰	
4.459199	3.239941	0.8977427	-0.6826669	-2.370922	-4.524997	-0.07578147	4.382303	2.264662	0.6817699	-0.6007663	-2.152105	-4.210878	4.094768×10 ⁻¹⁰	4.210878	2.152105	0.6007663	-0.681769	-4.094768×10 ⁻¹⁰	
2.479312	1.795976	0.6012571	-0.6167458	-1.746893	-2.092654	-0.07628286	1.948193	1.634066	0.5814737	-0.4919345	-1.536254	-1.879329	-1.611647×10 ⁻⁹	1.879329	1.536254	0.4919345	-0.581473	-1.879329	-1.611647×10 ⁻⁹
-0.4863957	-0.3671418	-0.2104716	-0.08573185	0.1195605	0.3138302	-0.07348132	-0.4612966	-0.2667083	-0.05683469	0.07988452	0.2739109	0.4318273	6.850159×10 ⁻⁹	-0.4318273	-0.2739109	-0.07988453	0.0568346	0.4318273	6.850159×10 ⁻⁹
-3.766967	-2.57294	-0.9831755	0.5157536	2.056962	3.229076	-0.05823381	-3.354953	-2.213405	-0.7311327	0.6697977	2.124615	3.226088	-3.125512×10 ⁻⁸	-3.226088	-2.124615	-0.6697977	0.7311327	3.226088	-3.125512×10 ⁻⁸
-8.09139	-4.110792	-1.417137	0.9457983	3.233922	4.783472	-0.02175395	-4.840696	-3.336369	-1.142106	1.025808	3.195694	4.675291	1.254052×10 ⁻⁸	-4.675291	-3.195694	-1.025808	1.142106	4.675291	1.254052×10 ⁻⁸

