



$$N_{3,w}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{3,\theta_x}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{3,\theta_y}(\xi; \eta) = \Phi_{3a}(\xi) \cdot \Phi_{4b}(\eta)$$

$$N_{4,w}(\xi, \eta) = \Phi_{1a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{4,\theta_x}(\xi, \eta) = \Phi_{2a}(\xi) \cdot \Phi_{3b}(\eta) \quad N_{4,\theta_y}(\xi, \eta) = \Phi_{1a}(\xi) \cdot \Phi_{4b}(\eta)$$

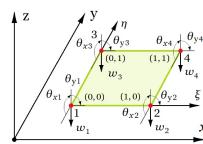
For twist  $\psi$

$$N_{1,\psi}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{2b}(\eta)$$

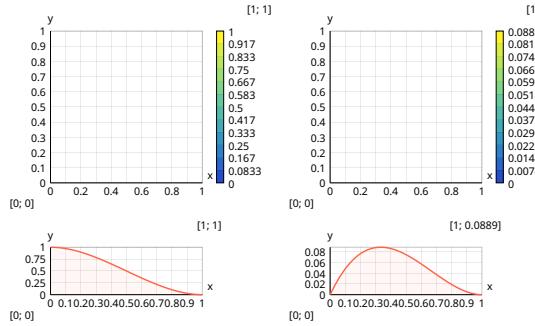
$$N_{2,\Psi}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{2b}(\eta)$$

$$N_{3,\psi}(\xi; \eta) = \Phi_{4a}(\xi) \cdot \Phi_{4b}(\eta)$$

$$N_{4,\psi}(\xi; \eta) = \Phi_{2a}(\xi) \cdot \Phi_{4b}(\eta)$$



### $N_{1,w}$ shape function plot



## Shape functions vector

$N(\vec{E}; \vec{\zeta}, \eta) = \text{take}(\vec{i}, N_{1,w}(\vec{\zeta}, \eta); N_{1,\theta}(\vec{\zeta}, \eta); N_{1,\vartheta}(\vec{\zeta}, \eta); N_{1,\psi}(\vec{\zeta}, \eta); N_{2,w}(\vec{\zeta}, \eta); N_{2,\theta}(\vec{\zeta}, \eta); N_{2,\vartheta}(\vec{\zeta}, \eta); N_{2,\psi}(\vec{\zeta}, \eta); N_{3,w}(\vec{\zeta}, \eta); N_{3,\theta}(\vec{\zeta}, \eta); N_{3,\vartheta}(\vec{\zeta}, \eta); N_{3,\psi}(\vec{\zeta}, \eta); N_{4,w}(\vec{\zeta}, \eta); N_{4,\theta}(\vec{\zeta}, \eta); N_{4,\vartheta}(\vec{\zeta}, \eta); N_{4,\psi}(\vec{\zeta}, \eta))$

## **Constitutive matrix** (stress - strain relationship)

$$D = \frac{E \cdot r^3}{12 \cdot (1 - 0.2^2)} \cdot \mathbf{hp} \left( \begin{bmatrix} 1; v; 0 & | & v; 1; 0 & | & 0; 0; \frac{1-v}{2} \end{bmatrix} \right) = \frac{35000 \cdot 0.2^3}{12 \cdot (1 - 0.2^2)} \cdot \mathbf{hp} \left( \begin{bmatrix} 1; 0.2; 0 & | & 0.2; 1; 0 & | & 0; 0; \frac{1-0.2}{2} \end{bmatrix} \right) = \begin{bmatrix} 24.30556 & 4.861111 & 0 \\ 4.861111 & 24.30556 & 0 \\ 0 & 0 & 9.722222 \end{bmatrix} \text{ kNm}$$

## Strain-displacement matrix

$$B_1(j; \zeta; \eta) = \text{take}(j; \Phi^1_{1a}(\zeta), \Phi^1_{1b}(\eta); \Phi^2_{2a}(\zeta), \Phi^2_{1b}(\eta); \Phi^3_{1a}(\zeta), \Phi^3_{2b}(\eta); \Phi^4_{2a}(\zeta), \Phi^4_{2b}(\eta); \Phi^5_{3a}(\zeta), \Phi^5_{1b}(\eta); \Phi^6_{4a}(\zeta), \Phi^6_{1b}(\eta); \Phi^7_{3a}(\zeta), \Phi^7_{2b}(\eta); \Phi^8_{4a}(\zeta), \Phi^8_{2b}(\eta); \Phi^9_{3a}(\zeta), \Phi^9_{3b}(\eta); \Phi^{10}_{4a}(\zeta), \Phi^{10}_{3b}(\eta); \Phi^{11}_{3a}(\zeta), \Phi^{11}_{4b}(\eta); \Phi^{12}_{4a}(\zeta), \Phi^{12}_{4b}(\eta); \Phi^{13}_{1a}(\zeta), \Phi^{13}_{3b}(\eta); \Phi^{14}_{2a}(\zeta), \Phi^{14}_{3b}(\eta); \Phi^{15}_{1a}(\zeta), \Phi^{15}_{4b}(\eta); \Phi^{16}_{2a}(\zeta), \Phi^{16}_{4b}(\eta))$$

$$B_2(\cdot; \xi, \eta) = \text{take}(f; \Phi_1(\xi) \cdot \Phi^1(1\eta), \Phi_2(\xi) \cdot \Phi^0(1\eta), \Phi_1(\xi) \cdot \Phi^0(2\eta), \Phi_2(\xi) \cdot \Phi^0(2\eta), \Phi_3(\xi) \cdot \Phi^1(1\eta), \Phi_4(\xi) \cdot \Phi^1(1\eta); \Phi_3(\xi) \cdot \Phi^2(2\eta); \Phi_4(\xi) \cdot \Phi^2(2\eta); \Phi_3(\xi) \cdot \Phi^3(2\eta); \Phi_4(\xi) \cdot \Phi^3(2\eta); \Phi_3(\xi) \cdot \Phi^4(1\eta); \Phi_4(\xi) \cdot \Phi^4(1\eta); \Phi_1(\xi) \cdot \Phi^0(3\eta), \Phi_2(\xi) \cdot \Phi^0(3\eta), \Phi_1(\xi) \cdot \Phi^0(4\eta), \Phi_2(\xi) \cdot \Phi^0(4\eta))$$

$$\Phi'_{4b}(\eta); \Phi'_{4a}(\xi) \cdot \Phi'_{4b}(\eta); \Phi'_{1a}(\xi) \cdot \Phi'_{3b}(\eta); \Phi'_{2a}(\xi)$$

$$B(j; \xi; \eta) = \mathbf{hp}([B_1(j; \xi; \eta); B_2(j; \xi; \eta); B_3(j; \xi; \eta)])$$

$$K_{e,ii} = a_1 \cdot b_1 \cdot \int_{\xi}^1 B_1(\xi; \eta)^T \cdot D \cdot B_1(\xi; \eta) d\xi d\eta$$

0 0

## Element stiffness matrix

(above the main diagonal only)

$$BTDB_e(i; j; \xi; \eta) = \text{transp}(B(i; \xi; \eta)) \cdot D \cdot B(j; \xi; \eta)$$

**K<sub>e</sub>** =   

$$K_e(i,j) = a_1 \cdot b_1 \cdot \int\limits_0^1 \int\limits_0^1 BTDB_E(i,j; \xi, \eta) d\xi d\eta$$

\$Repeat{ \$Repeat{K\_{e,i,j} = K\_e(i,j) \text{ for } j=i...n} \text{ for } i=1...n} = 0.9777823

### Element load vector

$$F_{e,i} = \mathbf{a}_1 \cdot \mathbf{b}_1 \cdot \int_0^1 \int_0^1 \mathbf{N}_i(\xi; \eta)^T \cdot \mathbf{q} \, d\xi \, d\eta$$

$$\mathbf{z}_1 = [0.9, 0.09, 0.09, 0.009, 0.9, -0.09, 0.09, -0.009, 0.9, -0.09, -0.09, 0.009, 0.9, 0.09, -0.09, -0.009] \text{ KN}$$

### Solution

### Global stiffness matrix

K =

## Global load vector

$$\vec{r} = [0.9 \ 0.09 \ 0.09 \ 0.009 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ 1.8 \ 0.18 \ 0 \ 0 \ \dots \ 0.009] \text{ kN}$$

## Solution of the system of equations

$\mathbb{Z} = \text{slslove}(K\mathcal{F}) = [0 \ 0.5523613 \ 0.3827392 \ -0.4161287 \ 0.2028064 \ 0.3732832 \ 0.2648463 \ -0.1937261 \ 0.2989047]$

0.3091182 0.04830843 -0.02501454 0.261207 0.3426386 -0.1651493 0.1275073 0.1211489

0.4681287 -0.2671179 0.2293302 ... -0.4161286] mm

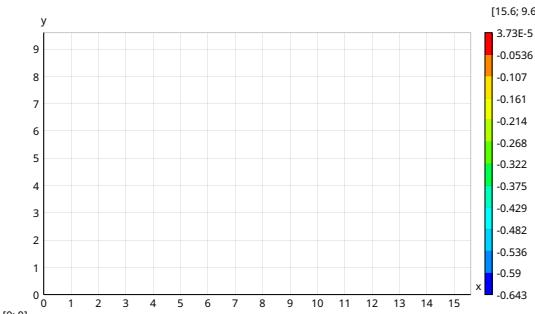
## Results

## Joint displacements

**transp(***W<sub>z</sub>***) =**

0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	0
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	0.203
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	0.299
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	0.261
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	0.121
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.36	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	0
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	0.139
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	0.299
0.363	0.526	0.632	0.635	0.544	0.42	0.35	0.376	0.464	0.536	0.538	0.467	0.373	0.329	0.373	0.467	0.538	0.536	0.464	0.376	0.363
0.299	0.487	0.608	0.612	0.511	0.369	0.287	0.325	0.433	0.516	0.519	0.437	0.324	0.27	0.324	0.437	0.519	0.516	0.433	0.325	0.299
0.139	0.398	0.556	0.566	0.44	0.247	0.129	0.206	0.367	0.478	0.481	0.374	0.211	0.121	0.211	0.374	0.481	0.478	0.367	0.206	0.139
0	0.34	0.527	0.542	0.404	0.173	0	0.135	0.36	0.463	0.467	0.345	0.145	0	0.145	0.345	0.467	0.463	0.336	0.135	0
0.121	0.386	0.55	0.565	0.443	0.253	0.138	0.217	0.381	0.493	0.497	0.389	0.225	0.134	0.225	0.389	0.497	0.493	0.381	0.217	0.121
0.261	0.461	0.593	0.608	0.513	0.375	0.299	0.342	0.455	0.542	0.544	0.46	0.345	0.289	0.345	0.46	0.544	0.542	0.455	0.342	0.261
0.299	0.482	0.605	0.62	0.537	0.417	0.35	0.387	0.485	0.562	0.564	0.489	0.387	0.338	0.387	0.489	0.564	0.562	0.485	0.387	0.299
0.203	0.419	0.562	0.581	0.485	0.337	0.25	0.31	0.438	0.531	0.533	0.443	0.311	0.242	0.311	0.443	0.533	0.531	0.438	0.31	0.203
0	0.303	0.488	0.512	0.383	0.165	0	0.139	0.34	0.469	0.472	0.347	0.146	0	0.146	0.347	0.472	0.469	0.34	0.139	0

mm



### Bending moments

$$Z(i) = \text{slice}(Z; k, (i-1)+1; k, i)$$

$$Z_1(e) = \text{hp}([Z_2(e_{-1}); Z_2(e_{-2}); Z_2(e_{-3}); Z_2(e_{-4})])$$

•  $\text{E}_\text{e}(\text{c}) = \text{NP}([\text{E}](\text{c}).\text{e}, 1), [\text{E}](\text{c}).\text{e}, 2), [\text{E}](\text{c}).\text{e}, 3)$

## Average

$$M_1 = \begin{pmatrix} 1.498465 & 0.3097372 & 0.2197322 & 0.1563433 & 0.1570519 & 0.9983302 & 0.1564193 & 0.1519741 & 0.19422 & 0.1519742 & 0.1564197 & 0.9983303 & 0.1570475 & 0.1563397 & 0.2197486 & 0.3098808 & 1.49858 & 8.502998 & 6.479199 & 5.777684 & \cdots & 1.498434 \\ 1.566914 & 7.805101 & 9.33778 & 7.668097 & 3.333 & -28.361735 & 3.14639 & 7.323888 & 8.895019 & 7.323888 & 3.14639 & -28.361734 & 3.332997 & 7.668093 & 9.337791 & 7.80518 & 1.566813 & 0.3205972 & 5.382201 & 7.12651 & \cdots & 1.56695 \\ 8.091391 & 3.766896 & 0.4863938 & -2.479308 & -4.459199 & 0.1550864 & 4.780359 & 2.842723 & 1.18852 \times 10^{-8} & -2.842723 & -4.780359 & -0.1550866 & 4.459199 & 2.479312 & -0.4863957 & -3.766967 & -8.09139 & 4.110857 & 2.572942 & 0.367139 & \cdots & 8.09139 \end{pmatrix}$$

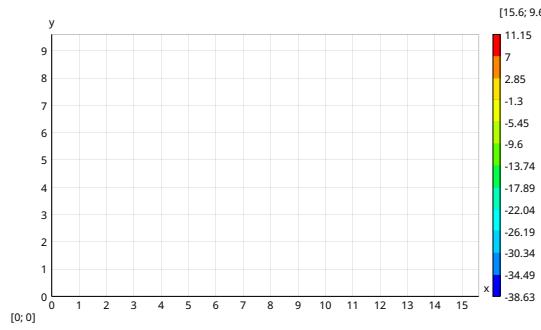
### Bending moments for the plate

### Bending moments - $M_y$

**transp**(*Mx*) =

**transp(Mx) =**

$\mu$  kNm/m

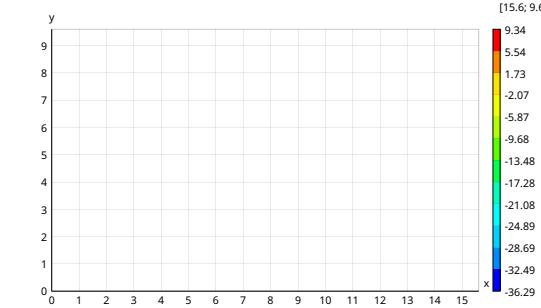


Bending moments  $M_y$

transp( $M_y$ ) =

1.566914	0.3205972	0.2333587	0.2147852	0.1682816	0.1798035	1.015789	0.1794301	0.1669894	0.2098191	0.2091906	0.1663467	0.1776119	0.9891482	0.1776123	0.1663459	0.2091915	0.2098181	0.1669903	0.1794296	...	1.566846
7.805101	5.382201	4.308412	4.134952	4.742331	6.340395	8.230714	6.28316	4.611981	3.89228	3.867499	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.611981	6.28316	...	7.805153
9.337778	7.126518	5.800074	5.553032	6.414898	8.11577	9.046666	8.014704	6.188836	5.14485	5.116777	6.090589	7.821224	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337789
7.668097	5.41756	4.262493	3.920413	4.233692	5.419822	6.344591	5.301204	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
3.333	-0.3280779	0.5396346	0.405616	-1.17438	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964996	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964995	-1.399574	-3.531204	...	3.332998
-28.361735	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.03174	-34.571125	-13.03174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.14639	-0.5066696	0.3485146	0.1799353	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146398
7.323888	5.094544	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
8.895091	6.740122	5.350863	4.96126	5.608937	6.913063	7.572529	6.693158	5.141784	4.180525	4.149637	5.027791	6.456862	7.212973	6.456862	5.027791	4.149637	4.180525	5.141784	6.693158	...	8.895035
7.323888	5.094544	3.908978	3.489139	3.675468	4.685865	5.49873	4.483969	3.254199	2.794376	2.788549	3.214495	4.349964	5.243651	4.349964	3.214495	2.788549	2.794376	3.254199	4.483969	...	7.323874
3.14639	-0.5066695	0.3485145	0.1799354	-1.456247	-3.762435	-0.7233495	-3.922981	-1.765819	-0.2520091	-0.1968807	-1.596341	-3.661657	-0.6661947	-3.661657	-1.596341	-0.1968807	-0.2520091	-1.765819	-3.922981	...	3.146397
-28.361735	-7.038366	-1.936162	-1.646652	-4.993974	-13.644678	-36.317523	-13.771126	-5.215881	-1.858171	-1.764027	-4.886867	-13.03174	-34.571125	-13.03174	-4.886867	-1.764027	-1.858171	-5.215881	-13.771126	...	-28.361736
3.332997	-0.3280781	0.5396345	0.405616	-1.174379	-3.412974	-0.3301829	-3.531204	-1.399574	0.09964994	0.1609008	-1.210571	-3.234664	-0.2184219	-3.234664	-1.210571	0.1609008	0.09964996	-1.399574	-3.531204	...	3.333003
7.668093	5.41756	4.262493	3.920414	4.233691	5.419822	6.344591	5.301104	3.979478	3.477998	3.480904	3.968582	5.222837	6.181459	5.222837	3.968582	3.480904	3.477998	3.979478	5.301104	...	7.668093
9.337791	7.126529	5.800079	5.553029	6.414899	8.115769	9.046667	8.014704	6.188836	5.14485	5.116777	6.090589	7.821225	8.777225	7.821225	6.090589	5.116777	5.14485	6.188836	8.014704	...	9.337788
7.80518	5.382247	4.308397	4.134958	4.742326	6.340398	8.230713	6.283161	4.61198	3.892281	3.867498	4.530534	6.12258	7.985896	6.12258	4.530534	3.867499	3.89228	4.61198	6.28316	...	7.805067
1.566813	0.3204128	0.2333748	0.2147746	0.1682944	0.1797983	1.015789	0.1794292	0.1669914	0.2098169	0.2091927	0.1663435	0.1776128	0.9891482	0.177612	0.1663436	0.209191	0.2098187	0.1669898	0.1794301	...	1.56695

kNm/m



Bending moments  $M_{xy}$

transp( $M_{xy}$ ) =

8.091391	4.110857	1.417137	-0.9458016	-3.233923	-4.783472	0.02175397	4.840696	3.336369	1.142106	-1.025808	-3.195694	-4.675291	8.203632×10 <sup>-9</sup>	4.675291	3.195694	1.025808	-1.142106
3.766896	2.572942	0.9831798	-0.5157529	-2.056961	-3.229076	0.05823395	3.354953	2.213405	0.7311327	-0.6697977	-2.124616	-3.226088	-1.924384×10 <sup>-8</sup>	3.226088	2.124615	0.6697977	-0.731132
0.4863938	0.367139	0.2104715	0.08573235	-0.1195606	-0.3138301	0.07348128	0.4612966	0.2667083	0.05683469	-0.07988453	-0.2739108	-0.4318273	4.135681×10 <sup>-9</sup>	0.4318273	0.2739109	0.07988453	-0.0568346
-2.479308	-1.795977	-0.6012575	0.6167458	1.746893	2.092654	0.07628287	-1.948193	-1.634066	-0.5814737	0.4919345	1.536254	1.879329	-9.39927×10 <sup>-10</sup>	-1.879329	-1.536254	-0.4919345	0.581473
-4.459199	-3.239942	-0.8977427	0.6826669	2.370722	4.524997	0.07578146	-4.382303	-2.264662	-0.6817699	0.6007663	2.152105	4.210878	2.347034×10 <sup>-10</sup>	-4.210878	-2.152105	-0.6007663	0.681769
0.1550864	0.1611718	0.1492474	0.1245987	0.09639145	0.07797703	0.07388198	0.06988651	0.05125056	0.02105121	0.008413693	-0.02659669	-0.02344966	-6.046931×10 <sup>-11</sup>	0.02344966	0.02659669	0.008413693	-0.0210512
4.780359	3.565968	1.189369	-0.4505428	-2.206755	-4.399285	0.06348492	4.535454	2.37937	0.7252095	-0.6263983	-2.225358	-4.279277	1.578955×10 <sup>-11</sup>	4.279277	2.225358	0.6263983	-0.725209
2.842723	2.13438	0.8682924	-0.4377229	-1.679594	-2.084069	0.03778074	2.16835	1.796129	0.6296557	-0.5417072	-1.677932	-2.037682	-4.744667×10 <sup>-12</sup>	2.037682	1.677932	0.5417072	-0.629655
1.18852×10 <sup>-8</sup>	-6.576042×10 <sup>-9</sup>	-7.016847×10 <sup>-10</sup>	1.244056×10 <sup>-10</sup>	2.191792×10 <sup>-10</sup>	-7.501514×10 <sup>-11</sup>	1.8954×10 <sup>-11</sup>	-1.177769×10 <sup>-11</sup>	3.047327×10 <sup>-12</sup>	1.092988×10 <sup>-12</sup>	-4.777445×10 <sup>-12</sup>	5.435255×10 <sup>-12</sup>	-3.554444×10 <sup>-12</sup>	3.228026×10 <sup>-12</sup>	-2.990397×10 <sup>-12</sup>	1.375859×10 <sup>-12</sup>	-1.298123×10 <sup>-12</sup>	7.488428×10 <sup>-12</sup>
-2.842723	-2.13438	-0.8682924	0.4377229	1.679594	2.084069	-0.03778074	-2.16835	-1.796129	-0.6296557	0.5417072	1.677932	2.037682	-7.622464×10 <sup>-12</sup>	-2.037682	-1.677932	-0.5417072	0.629655
-4.780359	-3.565968	-1.189369	0.4505428	2.206755	4.399285	-0.06348492	-4.535454	-2.37937	-0.7252095	0.6263983	2.225358	4.279277	2.756043×10 <sup>-11</sup>	-4.279277	-2.225358	-0.6263983	0.725209
-0.1550866	-0.1611716	-0.1492474	-0.1245987	-0.09639145	-0.07797703	-0.07388197	-0.06988651	-0.05125056	-0.02105121	0.008413693	0.02659669	0.02344966	-1.059221×10 <sup>-10</sup>	-0.02344966	0.02659669	-0.008413693	0.0210512
4.459199	3.239941	0.8977427	-0.6826669	-2.370722	-4.524997	-0.07578147	4.382303	2.264662	0.6817699	-0.6007663	-2.152105	-4.210878	4.094768×10 <sup>-10</sup>	4.210878	2.152105	0.6007663	-0.681769
2.479312	1.795976	0.6012571	-0.6167458	-1.746893	-2.092654	0.07628286	1.948193	1.634066	0.5814737	-0.4919345	-1.536254	-1.879329	-1.616467×10 <sup>-9</sup>	1.879329	1.536254	0.4919345	-0.581473
-0.4863957	-0.3671418	-0.2104716	-0.085732185	0.1195605	0.3138302	-0.07348132	-0.4612966	-0.2667083	-0.05683469	0.07988452	0.2739109	0.4318273	6.850159×10 <sup>-9</sup>	-0.4318273	-0.2739109	-0.07988453	0.0568346
-3.766967	-2.57294	-0.9831755	0.5157536	2.056962	3.229076	-0.05823381	-3.354953	-2.213405	-0.7311327	0.6697977	2.124615	3.226088	-3.125512×10 <sup>-8</sup>	-3.226088	-2.124615	-0.6697977	0.731132
-0.809139	-4.110792	-1.417137	0.9457983	3.233922	4.783472	-0.02175395	-4.840696	-3.336369	-1.142106	1.025808	3.195694	4.675291	1.254052×10 <sup>-8</sup>	-4.675291	-3.195694	-1.025808	1.142106

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