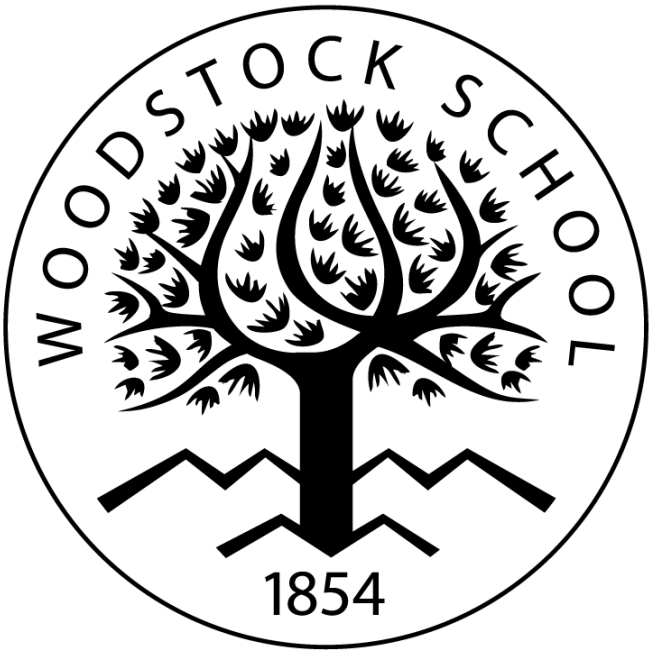
Cryptography Lab

AP Computer Science A



Name: _____

Contents

Background

Ever since one human being has had to send a secret message to another, cryptography has been used. Even today, cryptography, or the art of writing secret messages, is at the core of much that we do throughout our daily lives. Online commerce and other forms of secure digital communication would likely be non-existent without the ability to encrypt (and, as necessary, decrypt) information passed between individuals so that a third-party could not intercept and read that information. In this lab, you will be implementing two different crytographic techniques, including one that is used in modern digital communications.

Historical Encryption Techniques

There have been many cryptographic techniques developed over the years; however, many of the historical methods for encryption can be placed into one of two categories: substitution ciphers or transposition ciphers.

Substitution Ciphers

A substitution cipher is any encryption method which causes one letter in a message to be replaced by another, predetermined one. One such method, called the Caesar Cipher because of Julius Caesar's apparent use of it to encrypt important military messages, prescribes that each letter should be "shifted right" in value by a set number.

Caesar Cipher (Shift = 11)

"MEETAFTERSCHOOL" $\Longrightarrow$ "XPPELQEPCDNSZZW"

Note: "M" becomes "X" because "X" is 11 characters after "M" in the alphabet. Each letter is then shifted according to this same rule, wrapping around back to "A" as necessary.

Decrypting such a message is trivially done if the shift number is known: simply shift the letters to the left in the alphabet, again wrapping around back to "Z" as necessary.

Other substitution ciphers indicate that certain letters should be replaced by set other letters. The Kama Sutra Cipher, for instance, indicates that two keywords should be used to represent letter replacements.

Kama Sutra Cipher (Keywords: *OPULENT*, *DIARYS*)

"MEETAFTERSCHOOL" $\Longrightarrow$ "MRRSUFSRETCHDDA"

Note: Letters found in one word are replaced with their corresponding letters in the other word. Letters which do not appear in either word remain fixed.

Because each letter is replaced with its corresponding letter between the words, decrypting these messages requires the reader to substitute using the same keywords as was used during encryption.

Transposition Ciphers

A transposition cipher does not change any of the letters in the message. Instead, messages are "scrambled" so that the letters appear in a different location than in the original message. One such technique, called the Greek Scytale, involves winding a strip of parchment or leather around a rod of a specific diameter. After writing the message, the strip is then unwound to reveal a seemingly meaningless series of characters.

Scytale (3 letters around, 5 across)

M	E	E	T	A
F	T	E	R	S
C	H	O	O	L

When unwound, this message would reed: "MFCETHEEOTROASL". Note that there are no new letters in the encrypted message; however, the message has been sufficiently scrambled so as to not allow it to be easily read.

Decrypting this message requires the reader to know what size rod was used in its encryption. The message can then be wound the strip around a similarly sized rod and the original message read back.

Evaluating Cryptographic Methods

Although the historical crytographic techniques are interesting, the advent of modern digital computers means they have become extremely vulnerable to attack. Over the last several decades, it has become clear that any new cryptographic techniques must be developed with the ever-increasing computational power of modern devices in mind. *Cryptanalysis* works to find any potential weaknesses in encryption methods. "Breaking" a cryptographic technique usually involves the study and analysis of the specific algorithms used in encrypting a message in the hopes that some method can produce, with high regularity, a decrypted message through the examination and analysis of the encrypted one.

The methods by which cryptanalysis evaluates an ecryption algorithm for weaknesses, as well as the many other methods for attacking enciphered text are beyond the scope of this lab; however, it is an ever-growing field of study and one in which some students might find some interest in researching.

It is also important to note that many of the cryptographic techniques employed today rely on the sheer difficulty of "brute-force" decryption or the difficulty in solving complex mathematical problems. Despite this, they are, theoretically, breakable; however, the time it would take to break many of these methods using modern, or even speculated, computers far outpaces the viability of the information they are protecting. In other words, if it takes several thousand, million, or billion years to read encrypted information, the likelihood that it would be useful to do so becomes increasingly slim.

Applications

Question #1: Use a Caesar-Cipher with shift number 7 to encrypt the following message:

"THERE IS NO SCHOOL TODAY"

Question #2: Encrypt the same message in Question #1 using a Greek Scytale that allows for 4 characters to be written around the rod and 5 characters across.
Note: Do not include the spaces between words.

Question #3: Why are the substitution and transposition ciphers decscribed in the background particularly susceptible to attack using modern computers?

Question #4: The famed German Enigma machine used by the German forces in World War II was a modified form of substitution cipher. One big difference between it and the more primitive techniques is that each time a letter was substituted, the substitution sequence would change. In other words, the first letter 'A' might be replaced by a 'T'; however, a future 'A' could be replaced by a 'C', etc. Explain why this makes the encrypted message far more secure than a simple substitution cipher.

Activity #1

Introduction

A "one-time pad" is an historical method for encrypting messages. Using this method, physical pads or notebooks containing encryption keys or methods are distributed to any party that would need to read or write an encrypted message. These encryption keys would then be discarded after every use or after a designated period of time. As recently as World War II, these types of encryption systems were in wide-spread use by military and commercial organizations around the world. Even the famed German Enigma machine used settings that were changed daily in accordance to their own version of a one-time pad.

Although not considered secure enough for highly sensitive information, the following algorithm based on one-time encryption keys can be used to encrypt digital messages. A random key containing a certain number of bits (128 or 256 are common) is first generated and passed to anyone who would need to encrypt/decrypt a message using this method. A digital message is then encrypted by XORing each bit in the message with the corresponding bit in the key. A message that is longer than the generated key can be encrypted by first breaking it into chunks no more than the size of the key, encrypting each chunk, then concatenating all encrypted chunks together. Below is an example using an 8-bit key, a 16-bit key, and a 40-bit message (encoded in Extended ASCII).

Example: *message:* "HELLO", *key:* `10011101`

$$\textbf{HELLO} \Longrightarrow \left\{ \begin{array}{lcll} 01001000 & \oplus & 10011101 & = 11010101 \\ 01000101 & \oplus & 10011101 & = 11011000 \\ 01001100 & \oplus & 10011101 & = 11010001 \\ 01001100 & \oplus & 10011101 & = 11010001 \\ 01001111 & \oplus & 10011101 & = 11010010 \end{array} \right\} \Longrightarrow \textbf{ÕØÑÑÒ}$$

Example: *message:* "HELLO", *key:* `0110111000110101`

$$\textbf{HELLO} \Longrightarrow \left\{ \begin{array}{lcll} 0100100001000101 & \oplus & 0110111000110101 & = 0010011001110000 \\ 0100110001001100 & \oplus & 0110111000110101 & = 0010001001111001 \\ 01001111 & \oplus & 0110111000110101 & = 00100001 \end{array} \right\} \Longrightarrow \textbf{\&p"y!}$$

Note: Because our key is a binary *string*, the chunk of the "HELLO" message that does not contain a full 16-bits is XORed against the *left-most* bits in the string.

Decrypting a message is as simple as XORing it with the same key that was used for encryption, as seen below:

Example: *message:* "ÕØÑÑÒ", *key:* `10011101`

$$\textbf{ÕØÑÑÒ} \Longrightarrow \left\{ \begin{array}{lcll} 11010101 & \oplus & 10011101 & = 01001000 \\ 11011000 & \oplus & 10011101 & = 01000101 \\ 11010001 & \oplus & 10011101 & = 01001100 \\ 11010001 & \oplus & 10011101 & = 01001100 \\ 11010010 & \oplus & 10011101 & = 01001111 \end{array} \right\} \Longrightarrow \textbf{HELLO}$$

Example: *message:* "&p"y!", *key:* `0110111000110101`

$$\textbf{\&p"y!} \Longrightarrow \left\{ \begin{array}{lcll} 0010011001110000 & \oplus & 0110111000110101 & = 0100100001000101 \\ 0010001001111001 & \oplus & 0110111000110101 & = 0100110001001100 \\ 00100001 & \oplus & 0110111000110101 & = 01001111 \end{array} \right\} \Longrightarrow \textbf{HELLO}$$

Note: Because our key is a binary *string*, the chunk of the "&p"y!" message that does not contain a full 16-bits is XORed against the *left-most* bits in the string.

Exercises

Create the `OneTimePad` class that implements each of the following:

- The `generateRandomKey()` helper method which will generate a random binary string of the desired length.
- A class constructor that will automatically generate and store a random binary string of the desired length.
- The `binaryToString()`, `splitMessage()`, `stringToBinary()`, and `XORStrings()` helper methods useful for the encryption and decryption process.
- The `encryptMessage()` method required by the Encrypter interface.
  Note: This method should return `null` if the message cannot be encrypted due to a previous method having not yet been decrypted.
- The `decryptMessage()` method required by the Decrypter interface.
  Note: This method should generate a new encryption key once a message has been successfully decrypted. It should then allow new messages to be encrypted with this new key.

Questions

Question #5: For an encryption method such as this, why is it a good idea to use a key whose bit-length does *not* match the bit-length of your encoding scheme? (i.e., why is using an 8-bit key not desirable for Extended ASCII, an 8-bit encoding scheme?) You might want to use the examples from the introduction as inspiration for this answer.

Question #6: Why is it important that the encryption key being used with this method be kept *private*?

Activity #2

Introduction

In 1977, Ron Rivest, Adi Shamir, and Leonard Adleman, researchers at MIT, publicly described what is known as an assymmetric public-private key cryptosystem. Known as the RSA cryptosystem, this algorithm allows for a published, widely-distributed encryption key. The decryption key is held private and differs from that used to encrypt a message. It relies on two very important key facts: that testing whether or not a number is prime is "easy" and that prime factorization of a number is "hard".

Generating an RSA Key

The algorithm for generating the key for this method involves the choice of two distinct prime numbers. The steps are as follows:

1. Choose two distinct prime numbers, p and q.
2. Calculate $n = pq$.
3. Choose e relatively prime to $(p-1)(q-1)$.
4. Find d so that $ed \equiv 1 \bmod (p-1)(q-1)$.

The *public key* is then (e, n) and the *private key* is (d, n). Note that primality testing, choosing e, and how to calculate d are not part of this lab. A helper method has been provided for you for the calculation of d.

Encrypting a Message using RSA

Once the encryption key is known, encrypting a message is a fairly straight-forward mathematical procedure:

1. Calculate an integer representation, x, for the message.
   Note: For a traditional RSA cryptosystem, x *must be* smaller than n. More specifically, $\lfloor \lg x \rfloor < \lfloor \lg n \rfloor$. If using an encoding scheme, like Extended ASCII, this number can be calculated by concatenating the characters as a binary string, then converting that binary string to its decimal equivalent.
2. Calculate y such that $x^e \equiv y \bmod n$.

The value, y, is the encrypted message.

Decrypting a Message using RSA

The decryption algorithm is essentially the same as that for encrypting the message; however, the private key, d is now used as the exponent.

1. Calculate an integer representation, y, for the encrypted message.
2. Calculate x such that $y^d \equiv x \bmod n$.

x, or the reencoding of x using your encoding system is the decrypted message.

Example

Although practical RSA cryptosystems use very large prime numbers (with, say, 1024 bits), we can examine how it works to encrypt a message with a simplified set of numbers.

Using $p = 17837$, $q = 102881$ allows us to derive $n = 1835088397$, choose $e = 29$, and find $d = 1075670709$. Because $\lfloor \lg 1835088397 \rfloor \approx \lfloor 30.77 \rfloor = 30$, this RSA cryptosystem can encrypt any message containing 29 bits or less.

Encrypting "BYE" (Extended ASCII) using Public Key: $(1835088397, 29)$

$$\textbf{BYE} \implies 010000100101100101000101 \implies 4348229$$

$$4348229^{29} \equiv 372880434 \bmod 1835088397$$

$$372880434 \implies 10110001110011011010000110010$$

Note: Unlike the one-time pad encryption method described and implemented in Activity #1, the RSA cryptosystem often produces an encrypted message with a higher bit-count than the original message.

Decrypting "10110001110011011010000110010" using Private Key: $(1835088397, 1075570709)$

$$10110001110011011010000110010 \Longrightarrow 37880434$$
$$37880434^{1075670709} \equiv 4348229$$
$$4348229 \Longrightarrow 010000100101100101000101$$
$$010000100101100101000101 \Longrightarrow \textbf{BYE} \text{ (Extended ASCII)}$$

Hopefully, this example will make clear that, even for simple versions of the RSA encryption method, we will be dealing with very large numbers. In particular, the calculations of 4348229^{29} and $37880434^{1075670709}$ prove problematic for our traditional exponentiation techniques. Luckily, a particularly clever algorithm for handling large exponents in applications such as these has been developed. A helper method has been provided for you which implements the *Modular Exponentiation* algorithm.

Exercises

1. Create the `RSADecrypter` class that implements each of the following:
   - A class constructor that accepts `p`, `q`, and `e` as parameters and initializes `n` and `d`.
   - The `binaryToString()` helper method that will convert a binary string encoded in Extended ASCII to a string of characters.
   - The `getPublicKey()` method which will return the public key (n, e) as an array.
   - The `decryptMessage()` method required by the Decrypter interface.
2. Create the `RSAEncrypter` class that implements each of the following:
   - A class constructor that accepts `n` and `e`, the public key of an RSA cryptosystem.
   - The `stringToBinary()` helper method that will convert a string of characters into an Extended ASCII encoded binary string.
   - The `encryptMessage()` method required by the Encrypter interface.

Questions

Question #7: Why are *public key* encryption techniques important in modern communications?

Question #8: What major limitation does the fact that the decimal representation of a message must be less than n impose on sending messages using RSA? How would you overcome this limitation?

Final Analysis

Question #9: *Private key* encryption techniques, such as the Advanced Encryption System (AES), provide significant improvements on processing speed and can overcome the message-length problems RSA faces. Nonetheless, one major weakness, that the private encryption key must be exchanged over potentially unsecure communication lines, persists. Explain how *public key* encryption can be used to overcome this weakness.

Question #10: Explain why writing `Encrypter` and `Decrypter` as interfaces to be implemented makes more sense than writing them as classes to be extended.

Question #11: What part of the implementation of the `OneTimePad`, `RSADecrypter`, or `RSAEncrypter` classes did you find most challenging? How did you overcome this challenge?

Question #12: What new programming techniques or knowledge did you learn as a result of this lab?

Template Class & Test Cases

```
/**
 * Cryptography Lab (Template Classes and Test Cases)
 * These are the template classes and test cases for the Cryptography Lab.
 * Written for the Woodstock School in Mussoorie, Uttarakhand, India.
 *
 * @author Jeffrey Santos
 * @version 1.0
 */

public class Cryptography {
  public static void main(String[] args) {
    //  Tests for OneTimePad
    //    Note: These tests assume the use of each of the required helper
    //          methods and do not test them directly.
    //    Output: Output should vary based on the random nature of the
    //            generated keys.
    OneTimePad otp = new OneTimePad();
    String encryptedMessage = otp.encryptMessage("Hello, World!");
    //  Output: Should output 'gibberish' of equal length to 'Hello, World!'
    System.out.println(encryptedMessage);
    //  Output: Should output null
    System.out.println(otp.encryptMessage("Goodbye, World!"));
    //  Output: Should output 'Hello, World!'
    System.out.println(otp.decryptMessage(encryptedMessage));
    //  Output: Should output 'gibberish' of equal length to 'Hello, World!';
    //          however, the output should be distinct from the first call to
    //          the encryptMessage() method.
    System.out.println(otp.encryptMessage("Hello, World!"));

    //  Tests for RSADecrypter and RSAEncrypter
    //    Note: These tests assume the use of each of the required helper
    //          methods and do not test them directly.
    //    Output: Unlike the OneTimePad, keys are not generated randomly, so
    //            outputs should match exactly.
    RSADecrypter rsad = new RSADecrypter(1013839, 67866833, 47);
    //  Output: "68806042101887, 47"
    long[] publicKey = rsad.getPublicKey();
    System.out.println(publicKey[0] + ", " + publicKey[1]);

    //  Output: "10101000001110101100111110101101101110111110100"
    RSAEncrypter rsae = new RSAEncrypter(publicKey);
    encryptedMessage = rsae.encryptMessage("Tests");
    System.out.println(encryptedMessage);

    //  Output: "Tests"
    System.out.println(RSADecrypter.decryptMessage(encryptedMessage));
  }
}

/**
 * The Encrypter interface provides access to the encryptMessage method.
 */
interface Encrypter {
  /**
   * Encrypts a given message and returns the cipher text.
   *
   * @param message The message to be encrypted.
   * @return  The ciphertext after applying the encryption method.
   */
  public String encryptMessage(String message);
}

/**
```

```
 * The Decrypter interface provides access to the decryptMessage method.
 */
interface Decrypter {
  /**
   * Decrypts a given message and returns the original text.
   *
   * @param message The ciphertext to be decrypted.
   * @return  The deciphered message text.
   */
  public String decryptMessage(String message);
}

/**
 * The OneTimePad class implements the one-time pad digital encryption technique
 * decribed in Activity #1. The class implements both the Encrypter and
 * Decrypter interfaces.
 */
class OneTimePad implements Encrypter, Decrypter {
  private String encryptionKey;   // the current encryption key
  private boolean isReady;        // indicates readiness for encryption

  /**
   * OneTimePad class constructor. The class constructor should generate a
   * random key to be used the first time a message is encrypted with this
   * class.
   */
  public OneTimePad() {
    //  To be implemented in Activity #1
  }

  /**
   * The encryptMessage method required by the Encrypter interface.
   *
   * @param message The message to be encrypted.
   * @return  The ciphertext after applying the one-time pad encryption
   *          technique described in Activity #1 or null if the encryption
   *          process cannot take place.
   */
  public String encryptMessage(String message) {
    //  To be implemented in Activity #1
    return null;
  }

  /**
   * The decryptMessage method required by the Decrypter interface.
   *
   * @param message The ciphertext to be decrypted.
   * @return  The deciphered message text.
   */
  public String decryptMessage(String message) {
    //  To be implemented in Activity #1
    return null;
  }

  /**
   * Helper method to generate a random binary string to act as a key.
   *
   * @param bitLength The number of bits the binary string should contain.
   * @return  A random binary string of bitLength bits.
   */
  private String generateRandomKey(int bitLength) {
    //  To be implemented in Activity #1
    return null;
  }

  /**
```

```
    * Helper method to create an Extended ASCII encoded string from a given
    * binary string.
    *
    * @param msgBitString The binary string to encode.
    *  Precondition: msgBitString is a binary string.
    * @return An string representing the binary string encoded in Extended
    *         ASCII.
    */
   private String binaryToString(String msgBitString) {
     //  To be implemented in Activity #1
     return null;
   }

   /**
    * Helper method to split a binary string into strings of a given length.
    *
    * @param msgBitString The binary string representing the message.
    * @param bitLength The number of bits to divide the string into.
    * @return  An array of binary strings, each <= bitLength in length.
    */
   private String[] splitMessage(String msgBitString, int bitLength) {
     //  To be implemented in Activity #1
     return null;
   }

   /**
    * Helper method to create a binary string out of a given string of
    * characters.
    *
    * @param msgString The message to convert to a binary string.
    *  Precondition: msgString is encoded in Extended ASCII
    * @return  A binary string representing the message string.
    */
   private String stringToBinary(String msgString) {
     //  To be implemented in Activity #1
     return null;
   }

   /**
    * Helper method to XOR bits in an array of binary strings
    * with the corresponding bits in the encryptionKey.
    *
    * @param msgBitStrings An array of binary strings to XOR.
    *  Precondition: Each string is a binary string of length <= the length of
    *                the encryptionKey binary string.
    * @return  An array of binary strings resulting from XORing the given
    *          strings with the encryptionKey string.
    */
   private String[] XORStrings(String[] msgBitStrings) {
     //  To be implemented in Activity #1
     return null;
   }
}

/**
 * The RSADecrypter class implements the private side of the RSA cryptosystem.
 * It implements the Decrypter interface.
 */
class RSADecrypter implements Decrypter {
   private final long n, e, d; // relevant information for the public-private
                               // keys of the RSA cryptosystem

   /**
    * RSADecrypter class constructor. Accepts the two prime numbers and desired
    * exponent for the public key. n and d should be calculated based on these
    * parameters.
```

```
     *
     * @param p The first prime number to use for the RSA cryptosystem.
     *  Precondition: p is prime
     * @param q The second prime number to use for the RSA cryptosystem.
     *  Precondition: q is prime
     * @param e The exponent for the public-key of the RSA cryptosystem.
     *  Precondition: e is relatively prime to (p - 1)(q - 1)
     */
    public RSADecrypter(long p, long q, long e) {
      //  To be implemented in Activity #2
      n = 0; this.e = 0; d = 0;
    }

    /**
     * Accessor method for the public-key pair: (n, e).
     *
     * @return  An array containing the public-key pair: (n, e).
     */
    public long[] getPublicKey() {
      //  To be implemented in Activity #2
    }

    /**
     * The decryptMessage method required by the Decrypter interface.
     *
     * @param message The ciphertext to be decrypted.
     * @return  The deciphered message text.
     */
    public String decryptMessage(String message) {
      //  To be implemented in Activity #2
      return null;
    }

    /**
     * Helper method to create an Extended ASCII encoded string from a given
     * binary string.
     *
     * @param msgBitString The binary string to encode.
     *  Precondition: msgBitString is a binary string.
     * @return An string representing the binary string encoded in Extended
     *         ASCII.
     */
    private String binaryToString(String msgBitString) {
      //  To be implemented in Activity #2
      return null;
    }
}

/**
 * The RSAEncrypter class implements the public side of the RSA cryptosystem.
 * It implements the Encrypter interface.
 */
class RSAEncrypter implements Encrypter {
    private final long n, e;  // public-key information for the RSA cryptosystem

    /**
     * RSAEncrypter class constructor. Accepts the components of the public-key
     * for the cryptosystem.
     */
    public RSAEncrypter(long n, long e) {
      //  To be implemented in Activity #2
      this.n = 0; this.e = 0;
    }

    /**
     * The encryptMessage method required by the Encrypter interface.
```

```
   *
   * @param message The message to be encrypted.
   * @return  The ciphertext after applying the RSA encryption
   *          technique described in Activity #2.
   */
  public String encryptMessage(String message) {
    //  To be implemented in Activity #2
    return null;
  }

  /**
   * Helper method to create a binary string out of a given string of
   * characters.
   *
   * @param msgString The message to convert to a binary string.
   *  Precondition: msgString is encoded in Extended ASCII
   * @return  A binary string representing the message string.
   */
  private String stringToBinary(String msgString) {
    //  To be implemented in Activity #2
    return null;
  }
}

/**
 * RSAHelper class. Declared final with a private constructor so that it cannot
 * be subclassed or instantiated. This is a common technique for creating
 * classes whose sole purpose is the access to static methods. These two methods
 * in particular will aid in the development of the RSAEncrypter and
 * RSADecrypter classes.
 */
final class RSAHelper {
  /**
   * RSAHelper class constructor. Declared private so that objects cannot
   * be instantiated.
   */
  private RSAHelper() { }

  /**
   * Calculates the value, d, such that ed == 1 mod m using Euclid's Extended
   * Algorithm.
   *
   * @param e The value for which an inverse is requested.
   *  Precondition: e < m
   * @param m The modulus for finding the inverse.
   * @return  The value, d, such that ed == 1 mod m.
   */
  public static long calcModInverse(long e, long m) {
    long modulus = m, x = 0, y = 1;
    long buffer, quotient;

    if (m == 1)
      return 0;

    while (e > 1) {
      quotient = e / m;
      buffer = m;
      m = e % m;
      e = buffer;
      buffer =  x;
      x = y - quotient * x;
      y = buffer;
    }
    if (y < 0)
      y += modulus;
    return y;
```

```
  }

  /**
   * Performs modular exponentiation.
   *
   * @param base The base for modular exponentiation.
   * @param exponent The exponent for modular exponentiation.
   * @param m The modulus for modular exponentiation.
   * @return The result of base^exponent mod m.
   */
  public static long modExp(long base, long exponent, long m) {
    if (exponent == 0)
      return 1;
    long temp = modExp(base, exponent / 2, m);
    temp = (temp * temp) % m;
    if (exponent % 2 == 0)
      return temp;
    return base * temp;
  }
}
```